Identification and Prediction in Dynamic Networks with Unobservable Nodes

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Abstract

The interest for system identification in dynamic networks has increased recently with a wide variety of applications. In many cases, it is intractable or undesirable to observe all nodes in a network and thus, to estimate the complete dynamics. If the complete dynamics is not desired, it might even be challenging to estimate a subset of the network if key nodes are unobservable due to correlation between the nodes. In this contribution, we will discuss an approach to treat this problem. The approach relies on additional measurements that are dependent on the unobservable nodes and thus indirectly contain information about them. These measurements are used to form an alternative indirect model that is only dependent on observed nodes. The purpose of estimating this indirect model can be either to recover information about modules in the original network or to make accurate predictions of variables in the network. Examples are provided for both recovery of the original modules and prediction of nodes.

Keywords: Dynamic networks, closed-loop identification, identifiability, system identification

Identification and Prediction in Dynamic Networks with Unobservable Nodes

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The interest for system identification in dynamic networks has increased recently with a wide variety of applications. In many cases, it is intractable or undesirable to observe all nodes in a network and thus, to estimate the complete dynamics. If the complete dynamics is not desired, it might even be challenging to estimate a subset of the network if key nodes are unobservable due to correlation between the nodes. In this contribution, we will discuss an approach to treat this problem. The approach relies on additional measurements that are dependent on the unobservable nodes and thus indirectly contain information about them. These measurements are used to form an alternative indirect model that is only dependent on observed nodes. The purpose of estimating this indirect model can be either to recover information about modules in the original network or to make accurate predictions of variables in the network. Examples are provided for both recovery of the original modules and prediction of nodes.

Contents

1 Introduction

Large complex systems, such as electrical power networks or telecommunication networks, can be found around us in our daily life. In a world of ever increasing demands on efficiency and reliability, model based estimation and control can be introduced to better understand and control the states of these systems. Making complete models or centrally controlling these complex systems are difficult, or even intractable, tasks, for instance, due to the size of the network or difficulties to measure all relevant signals. To decrease complexity or computational cost, these systems are typically broken down into subsystems that are individually modeled and controlled.

Modeling of dynamic networks, i.e. modeling of a set of internal variables (nodes) that are interconnected through dynamic subsystems (modules), has recently gained in popularity. A common approach to modeling is data-based inference using the system identification methodology, see for instance, Chiuso and Pillonetto (2012), Van den Hof et al. (2013), Everitt et al. (2014), Gunes et al. (2014), Dankers (2014) and Weerts et al. (2015). The data-based modeling field can be divided in two groups depending on the knowledge of the topology, i.e. the interconnection structure.

In the first group, topology detection, the interconnection structure is estimated, commonly assuming that all nodes are observable, and many methods are based on causality or sparsity conditions, see for instance, Yuan et al. (2011).

In the second group, the topology is assumed to be known and the focus is typically on estimating a part of the network or specific subsystems. In this setting, consistency, identifiability and variance properties have been studied (Van den Hof et al., 2013; Dankers, 2014; Gevers and Bazanella, 2015; Weerts et al., 2015). Commonly, nodes relevant to the desired part of the network are assumed to be observable. However, in some situations, certain nodes might be intractable or undesirable to observe, for example, due to, cost or inconvenience.

The observability requirement was relaxed in Dankers et al. (2016). It was shown that not all nodes have to be observable in order to get consistent estimates of a part of the network and that the conditions guaranteeing consistency are based on the interconnection structure. However, these results indirectly showed that some nodes are crucial to get the desired consistency properties.

In this report, we will discuss the case when some crucial nodes are unobservable. The proposed approach uses additional measurements that depend on the unobservable nodes and thus contain indirect information about them. These extra measurements can be used as a remedy in certain situations by “flipping the arrows” to create an indirect model. This indirect model only depends on observable nodes and can under certain conditions be used to estimate modules in the original network. In addition, we will discuss the benefits of using the indirect model for predicting internal variables.

The structure of this report is as follows. In Section 2, the problem is formulated and the notation is established. Indirect node observations are introduced and the indirect model is derived in Section 3. In Section 4, estimation of a specific module is discussed in terms of identifiability and properties of the indirect model. In Section 5, prediction of internal variables using the indirect model is presented and finally, the report is concluded in Section 6.

2 Problem Formulation

There are many ways of modeling a dynamic network and here, the framework described in Van den Hof et al. (2013) and Dankers (2014) will be used. In this framework, the network is assumed to consist of L internal signals, or nodes, $w_k,\ k \in \mathcal{N}$ where $\mathcal{N} = \{1, \dots, L\}$. These nodes are assumed to be dynamically dependent on each other and in addition, there are external user-controllable signals r_k, $k \in \mathcal{N}$, and unmeasured disturbances v_k, $k \in \mathcal{N}$, that could enter at each node. Note that some v_k or r_k could be zero for all times. The j^{th} node can thus be described by

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G_{jk}(\mathsf{q}) w_k(t) + r_j(t) + v_j(t) \tag{1}$$

where q is the shift operator, $G_{jk}(\mathsf{q}), k \in \mathcal{N}_j$, are transfer functions and the set $\mathcal{N}_j$ is the indices $k \in \mathcal{N} \setminus \{j\}$ (i.e. no self-loops) for which $G_{jk}(\mathsf{q}) \neq 0$. To simplify notation, we will call r_j and $w_k,\ k \in \mathcal{N}_j$, predictor inputs to w_j.

It is convenient to talk about the behavior of all nodes. The descriptions (1) of each node can be combined in vector notation and the entire network can be described by

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & \dots & G_{1L} \\ G_{21} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & G_{L-1L} \\ G_{L1} & \dots & G_{LL-1} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_L \end{bmatrix} \tag{2}$$

where the non-zero entries of the j^{th} row are defined by $\mathcal{N}_j$ and the dependencies of q, $s_k = r_k + v_k$ and t have been dropped for notational convenience. Equation (2) can also be written on the compact form

$$w = Gw + s \tag{3}$$

The network is assumed to satisfy the following conditions.

Assumption 1 (Assumption 1 of Dankers et al. (2016))

(a) The network is well-posed in the sense that all principal minors of $\lim_{z \to \infty}(I - G(z))$ *are non-zero.*

(b) $(I - G)^{-1}$ *is stable.*

(c) All $r_m, m \in \mathcal{N}$ *are uncorrelated with all* $v_k, k \in \mathcal{N}$. □

In addition, all $v_k, k \in \mathcal{N}$ are assumed to be independent. To simplify notation we will use the path and loop concepts.

Definition 1 (Path and loop) *There exist a* path *between the nodes* w_i *and* w_j *if there exist a sequence of integers* $n_1, \dots, n_k$, *such that* $G_{jn_1} G_{n_1 n_2} \dots G_{n_k i} \neq 0$. *There exist a* direct path *between* w_i *and* w_j *if* $G_{ji} \neq 0$. *A path from* w_j *to* w_j *is called a* loop *and a direct path from* w_j *to* w_j *is called* self-loop. □

In this contribution we are interested in a part of the network around one node, here denoted with the index j. We might either be interested in finding an estimate of a specific part of the network, for instance, the module $G_{ji}(\mathsf{q})$, or be interested in predicting w_j with good accuracy.

2.1 Unobservable Nodes

It is common that only a subset of the nodes are observable, for instance, due to the size of the network, cost or inconvenience. The impact of not observing certain nodes is different depending on the properties of the nodes and the usage of the measurements. For instance, if the measurements are used to build a model of a certain module, some nodes might be neglected without affecting the consistency properties (Dankers et al., 2016). However, neglecting signals will typically decrease the signal-to-noise ratio which will affect the variance properties of the estimator. In this report, we will focus on the case when crucial nodes $w_k, k \in \mathcal{N}_j$ are unobservable with the desired sensor setup.

Nodes will be grouped and reordered into sets denoted by large calligraphic letters to simplify notation. The variable $w_{\mathcal{X}}$ is the vector containing all $w_k,\ k \in \mathcal{X}_j$ and similar for $s_{\mathcal{X}}$, $r_{\mathcal{X}}$ and $v_{\mathcal{X}}$. The ordering is not important as long as all vectors and matrices are ordered consistently. Given a sensor setup, the sets of indices of all (directly) observable and unobservable nodes are denoted $\mathcal{S}_j$ and $\mathcal{U}_j = \mathcal{N} \setminus \mathcal{S}_j$, respectively. The set of observable nodes that are predictor inputs to w_j is defined as $\mathcal{K}_j = \mathcal{N}_j \cap \mathcal{S}_j$. The set $\mathcal{A}_j$ is the indices of all additional nodes that are observable, i.e. the set $\mathcal{S}_j \setminus (\mathcal{K}_j \cup \{j\})$. With this notation, (2) can be written

$$\begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \\ w_{\mathcal{U}} \end{bmatrix} = \begin{bmatrix} 0 & G_{j\mathcal{K}} & 0 & G_{j\mathcal{U}} \\ G_{\mathcal{K}j} & G_{\mathcal{K}\mathcal{K}} & G_{\mathcal{K}\mathcal{A}} & G_{\mathcal{K}\mathcal{U}} \\ G_{\mathcal{A}j} & G_{\mathcal{A}\mathcal{K}} & G_{\mathcal{A}\mathcal{A}} & G_{\mathcal{A}\mathcal{U}} \\ G_{\mathcal{U}j} & G_{\mathcal{U}\mathcal{K}} & G_{\mathcal{U}\mathcal{A}} & G_{\mathcal{U}\mathcal{U}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \\ w_{\mathcal{U}} \end{bmatrix} + \begin{bmatrix} s_j \\ s_{\mathcal{K}} \\ s_{\mathcal{A}} \\ s_{\mathcal{U}} \end{bmatrix} \tag{4}$$

where $G_{\mathcal{K}\mathcal{K}}$, $G_{\mathcal{A}\mathcal{A}}$ and $G_{\mathcal{U}\mathcal{U}}$ are zero on the diagonals.

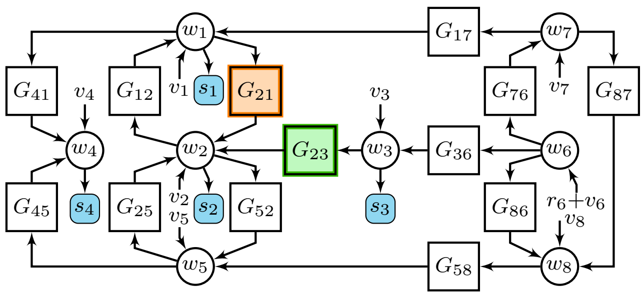

Figure 1: Example of a dynamic network. The circles, the boxes and the (blue) rounded boxes correspond to the nodes, the dynamics and measurements, respectively.

Example 1 *Some of the aspects in this report will be illustrated by the example seen in Figure 1 and described by*

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \hline w_4 \\ \hline w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix} = \left[\begin{array}{ccc|c|cccc} 0 & G_{12} & 0 & 0 & 0 & 0 & G_{17} & 0 \\ G_{21} & 0 & G_{23} & 0 & G_{25} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{36} & 0 & 0 \\ \hline G_{41} & 0 & 0 & 0 & G_{45} & 0 & 0 & 0 \\ \hline 0 & G_{52} & 0 & 0 & 0 & 0 & 0 & G_{58} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{76} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{86} & G_{87} & 0 \end{array}\right] \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \hline w_4 \\ \hline w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \hline v_4 \\ \hline v_5 \\ r_6+v_6 \\ v_7 \\ v_8 \end{bmatrix} \tag{5}$$

Here the node of focus is w_2*, i.e.* $j = 2$ *and* $\mathcal{N}_2 = \{1, 3, 5\}$*. There is only a limited set of sensors in the network and the set of observable nodes is given by* $\mathcal{S}_2 = \{1, 2, 3, 4\}$ *which means that* $\mathcal{U}_2 = \{5, 6, 7, 8\}$ *is unobservable. Hence,* $\mathcal{K}_2 = \{1, 3\}$ *and* $\mathcal{A}_2 = \{4\}$*.* □

2.2 The Immersed Network

The changes to the dynamics among the remaining nodes when certain variables are unobservable can be evaluated by looking at the immersed network. This is the equivalent network, from all external signals s to the remaining nodes $w_{\mathcal{S}}$, when the nodes $w_{\mathcal{U}}$ are eliminated (Dankers, 2014; Dankers et al., 2016). The immersed network of (4) can be formed by solving row four of (4) for $w_{\mathcal{U}}$ and inserting into the other rows which after normalization gives

$$\begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \end{bmatrix} = \begin{bmatrix} 0 & \breve{G}_{j\mathcal{K}} & \breve{G}_{j\mathcal{A}} \\ \breve{G}_{\mathcal{K}j} & \breve{G}_{\mathcal{K}\mathcal{K}} & \breve{G}_{\mathcal{K}\mathcal{A}} \\ \breve{G}_{\mathcal{A}j} & \breve{G}_{\mathcal{A}\mathcal{K}} & \breve{G}_{\mathcal{A}\mathcal{A}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \end{bmatrix} + \breve{F}s \tag{6}$$

where $\breve{G}_{\mathcal{K}\mathcal{K}}$ and $\breve{G}_{\mathcal{A}\mathcal{A}}$ are zero on the diagonal. Note that the external signals $r_n,\ n \in \mathcal{P}_j$, where $\mathcal{P}_j$ is the set of indices such that $\breve{F}_{jn} \neq 0$, and the nodes $w_k,\ k \in \mathcal{A}_j$, such that $\breve{G}_{jk} \neq 0$, are needed in addition to the nodes $w_k,\ k \in \mathcal{K}_j$ to describe w_j after $w_{\mathcal{U}}$ has been eliminated. The transfer function matrices $\breve{F}$ will typically depend on the dynamics of the eliminated variables. Furthermore, the dynamics $\breve{G}_{j\mathcal{K}}$ and $\breve{G}_{j\mathcal{A}}$ are not necessarily equal to $G_{j\mathcal{K}}$ and $G_{j\mathcal{A}} = 0$ if crucial nodes are unobserved and thus eliminated. Conditions for guaranteeing equality of $\breve{G}_{ji}$ and G_{ji} were given in Dankers et al. (2016) and are restated below.

Proposition 1 (Proposition 4 of Dankers et al. (2016))
The transfer function $\breve{G}_{ji}$ *in the immersed network is equal to* G_{ji} *if* $\mathcal{K}_j$ *satisfies the following conditions:*

(a) $i \in \mathcal{K}_j$*,* $j \notin \mathcal{K}_j$

(b) every path w_i *to* w_j*, excluding the path* G_{ji}*, goes through a node* $w_k, k \in \mathcal{K}_j$

(c) every loop w_j *to* w_j *goes through a node* $w_k, k \in \mathcal{K}_j$*.* □

Note that Proposition 1 is fulfilled if all predictor inputs to w_j are observable, i.e. $\mathcal{K}_j = \mathcal{N}_j$.

Example 2 *The immersed network of Example 1 is*

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & 0 & 0 \\ \frac{G_{21}}{1-G_{25}G_{52}} & 0 & \frac{G_{23}}{1-G_{25}G_{52}} & 0 \\ 0 & 0 & 0 & 0 \\ G_{41} & G_{45}G_{52} & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} + \begin{bmatrix} v_1+G_{17}(G_{76}(r_6+v_6)+v_7) \\ \frac{v_2+G_{25}(v_5+G_{58}((G_{86}+G_{87}G_{76})(r_6+v_6)+G_{87}v_7+v_8))}{1-G_{25}G_{52}} \\ v_3+G_{36}(r_6+v_6) \\ v_4+G_{45}(v_5+G_{58}((G_{86}+G_{87}G_{76})(r_6+v_6)+G_{87}v_7+v_8)) \end{bmatrix} \tag{7}$$

which means that $\mathcal{P}_2 = \{6\}$. The transfer functions $\breve{G}_{21} \neq G_{21}$ and $\breve{G}_{23} \neq G_{23}$ due to the feedback between w_5 and w_2 which violates condition (c) of Proposition 1. In addition, a significant part of the eliminated dynamics related to the unobserved nodes is contained in $\breve{F}$. □

3 Indirect Node Observations

If the additional nodes $w_{\mathcal{A}}$ contain information about all the unobservable nodes, an alternative solution is to use these measurements to form an indirect model (Linder and Enqvist, 2016). If we assume that $G_{\mathcal{AU}}$ has full column-rank and that there exists a filter $f_{\mathcal{UA}}$ such that

$$f_{\mathcal{UA}} G_{\mathcal{AU}} = I, \tag{8}$$

then the indirect model can be derived by first solving row three of (4) for $G_{\mathcal{AU}} w_{\mathcal{U}}$ and filter with $f_{\mathcal{UA}}$, i.e.

$$w_{\mathcal{U}} = f_{\mathcal{UA}}(w_{\mathcal{A}} - G_{\mathcal{A}j} w_j - G_{\mathcal{AK}} w_{\mathcal{K}} - G_{\mathcal{AA}} w_{\mathcal{A}} - s_{\mathcal{A}}) \tag{9}$$

Inserting (9) into (4) gives the indirect model

$$\begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \end{bmatrix} = \begin{bmatrix} 0 & \mathring{G}_{j\mathcal{K}} & \mathring{G}_{j\mathcal{A}} \\ \mathring{G}_{\mathcal{K}j} & \mathring{G}_{\mathcal{KK}} & \mathring{G}_{\mathcal{KA}} \\ \mathring{G}_{\mathcal{A}j} & \mathring{G}_{\mathcal{AK}} & \mathring{G}_{\mathcal{AA}} \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{A}} \end{bmatrix} + \begin{bmatrix} \mathring{F}_{jj} & 0 & \mathring{F}_{j\mathcal{A}} & 0 \\ 0 & \times & \times & 0 \\ 0 & 0 & \times & \times \end{bmatrix} \begin{bmatrix} s_j \\ s_{\mathcal{K}} \\ s_{\mathcal{A}} \\ s_{\mathcal{U}} \end{bmatrix} \tag{10}$$

after normalization, where $\times$ represents a possibly non-zero entry and $\mathring{G}_{\mathcal{KK}}$ and $\mathring{G}_{\mathcal{AA}}$ have zeros on the diagonal. Note that the indirect model only is dependent on the observable nodes $w_{\mathcal{S}}$ and the external variables $r_{\mathcal{S}}$ and $v_{\mathcal{S}}$. Furthermore, the observable nodes $w_{\mathcal{A}}$ contain information about all excitation that enters into the network and has paths to $w_{\mathcal{A}}$, also from the external unmeasured disturbances $v_{\mathcal{U}}$. By using this extra excitation, the variance can potentially be reduced in comparison to (6).

The rank assumption on $G_{\mathcal{AU}}$ can be restrictive in a dynamic network setting since all unobservable nodes $w_{\mathcal{U}}$ have to be indirectly observed, even nodes far away from the j^{th} node, i.e. the node of interest. To relax the rank assumption, the unobservable nodes are first categorized and the set $\mathcal{A}_j$ is split into the sets $\mathcal{I}_j$ and $\mathcal{O}_j$.

Definition 2 *The unobservable nodes are divided into the set of unobservable predictor inputs, i.e. $\bar{\mathcal{U}}_j = \mathcal{U}_j \cap \mathcal{N}_j$, and the remaining unobservable nodes, i.e. $\tilde{\mathcal{U}}_j = \mathcal{U}_j \setminus \bar{\mathcal{U}}_j$.* □

Definition 3 *The set $\mathcal{I}_j$ is the indices $i \in \mathcal{A}_j$ such that $G_{ik} \neq 0$ and $G_{in} = 0$ for some $k \in \bar{\mathcal{U}}_j$ and all $n \in \tilde{\mathcal{U}}_j$. The measurements of $w_{\mathcal{I}}$ will be called* indirect node observations *since they contain indirect information about the unobservable predictor inputs $w_{\bar{\mathcal{U}}}$. The set $\mathcal{O}_j = \mathcal{A}_j \setminus \mathcal{I}_j$ is the indices of the remaining observable nodes.* □

Note that $w_{\mathcal{I}}$ is the additional nodes that are indirectly dependent of the unobserved predictor inputs $w_{\bar{\mathcal{U}}}$ but not on the remaining unobservable nodes $w_{\tilde{\mathcal{U}}}$.

Now, if we assume that all the unobservable predictor inputs $w_{\bar{\mathcal{U}}}$ are indirectly observable, i.e. that $G_{\mathcal{I}\bar{\mathcal{U}}}$ have full column-rank, and that there exists a filter $f_{\bar{\mathcal{U}}\mathcal{I}}$ such that

$$f_{\bar{\mathcal{U}}\mathcal{I}} G_{\mathcal{I}\bar{\mathcal{U}}} = I, \tag{11}$$

then $w_{\bar{\mathcal{U}}}$ can be eliminated using $w_{\mathcal{I}}$ in a similar way as the full elimination. The unobservable nodes $w_{\tilde{\mathcal{U}}}$ can be neglected since they are not predictor inputs to w_j. The resulting indirect model after normalization is

$$\begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{O}} \\ w_{\mathcal{I}} \end{bmatrix} = \begin{bmatrix} 0 & \tilde{G}_{j\mathcal{K}} & \tilde{G}_{j\mathcal{O}} & \tilde{G}_{j\mathcal{I}} \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \begin{bmatrix} w_j \\ w_{\mathcal{K}} \\ w_{\mathcal{O}} \\ w_{\mathcal{I}} \end{bmatrix} + \begin{bmatrix} \tilde{F}_{jj} & 0 & 0 & \tilde{F}_{j\mathcal{I}} & 0 & 0 \\ 0 & \times & 0 & \times & 0 & \times \\ 0 & 0 & \times & \times & 0 & \times \\ 0 & 0 & 0 & \times & \times & \times \end{bmatrix} \begin{bmatrix} s_j \\ s_{\mathcal{K}} \\ s_{\mathcal{O}} \\ s_{\mathcal{I}} \\ s_{\bar{\mathcal{U}}} \\ s_{\tilde{\mathcal{U}}} \end{bmatrix} \tag{12}$$

Note that the same result is obtained if elimination is reversed, i.e. firstly forming the immersed network by eliminating $w_{\tilde{\mathcal{U}}}$ and then eliminating $w_{\bar{\mathcal{U}}}$ using the indirect observations. Furthermore, since the indirect node observations were assumed to contain information about all unknown predictor inputs $w_{\bar{\mathcal{U}}}$, the j^{th} node will only depend on locally observable nodes and external signals.

Example 3 *For the dynamic network of Example 1, $\mathcal{I}_2 = \{4\}$ and the indirect observation is given by*

$$w_4 = G_{41} w_1 + G_{45} w_5 + v_4 \tag{13}$$

A full elimination of $\mathcal{U}_2 = \{5, 6, 7, 8\}$ is thus not possible, but a partial elimination of w_5 using $f_{\bar{\mathcal{U}}\mathcal{I}} = G_{45}^{-1}$ gives

$$\begin{aligned} \left[\begin{array}{c} w_1 \\ w_2 \\ w_3 \\ \hdashline w_4 \\ w_6 \\ w_7 \\ w_8 \end{array}\right] &= \left[\begin{array}{ccc:c} 0 & G_{12} & 0 & 0 \\ G_{21} - G_{25} G_{45}^{-1} G_{41} & 0 & G_{23} & G_{25} G_{45}^{-1} \\ 0 & 0 & 0 & 0 \\ \hdashline G_{41} & G_{45} G_{52} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} w_1 \\ w_2 \\ w_3 \\ \hdashline w_4 \end{array}\right] \\ &+ \left[\begin{array}{ccc} 0 & G_{17} & 0 \\ 0 & 0 & 0 \\ G_{36} & 0 & 0 \\ \hdashline 0 & 0 & G_{45} G_{58} \\ 0 & 0 & 0 \\ G_{76} & 0 & 0 \\ G_{86} & G_{87} & 0 \end{array}\right] \begin{bmatrix} w_6 \\ w_7 \\ w_8 \end{bmatrix} + \left[\begin{array}{c} v_1 \\ v_2 - G_{25} G_{45}^{-1} v_4 \\ v_3 \\ \hdashline v_4 + G_{45} v_5 \\ r_6 + v_6 \\ v_7 \\ v_8 \end{array}\right] \end{aligned} \tag{14}$$

The indirect model, i.e. the immersed network of this partially eliminated model, is given by

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \left[\begin{array}{ccc:c} 0 & G_{12} & 0 & 0 \\ G_{21} - G_{25}G_{45}^{-1}G_{41} & 0 & G_{23} & G_{25}G_{45}^{-1} \\ 0 & 0 & 0 & 0 \\ G_{41} & G_{45}G_{52} & 0 & 0 \end{array}\right] \left[\begin{array}{c} w_1 \\ w_2 \\ w_3 \\ \hdashline w_4 \end{array}\right] + \begin{bmatrix} v_1 + G_{17}(G_{76}(r_6+v_6)+v_7) \\ v_2 - G_{25}G_{45}^{-1}v_4 \\ v_3 + G_{36}(r_6+v_6) \\ v_4 + G_{45}(v_5 + G_{58}((G_{86}+G_{87}G_{76})(r_6+v_6)+G_{87}v_7+v_8)) \end{bmatrix} \tag{15}$$

Note that the second row is unaffected by the immersion and that the other three rows are equivalent to (7). *Furthermore,* w_2 *only depends on* $w_{\mathcal{S}}$, v_2 *and* v_4. *The variables* w_1, w_3 *and* w_4 *capture all excitation from* r_6 *and more importantly the disturbances* v_5, v_6, v_7 *and* v_8. □

4 Recovering Specific Modules

In this section, the possibilities of estimating a specific part of the network dynamics will be discussed. Two subjects are addressed, identifiability and properties of the predictor model to ensure that the estimator is consistent.

Unique estimation of a model for a chosen model structure is both connected to identifiability of the selected model structure and the informativity of the data set used for identification (Bellman and Åström, 1970; Bazanella et al., 2010). For a discussion of identifiability and informativity in dynamic networks, see Gevers and Bazanella (2015). The focus in this report is on structural properties of the indirect model since elimination of unobservable nodes typically will change the structure. In addition to the identifiability issues, properties of the resulting model that are important for estimation will also be presented.

4.1 Structural changes due to unobservable nodes

To understand the structural changes due to elimination of unobservable nodes, it is convenient to look at the details of the indirect model (12). The first row of (4) is given by

$$w_j = G_{j\mathcal{K}} w_{\mathcal{K}} + G_{j\bar{\mathcal{U}}} w_{\bar{\mathcal{U}}} + r_j + v_j$$

and the indirect node observations of Definition 3 are

$$w_{\mathcal{I}} = G_{\mathcal{I}j} w_j + G_{\mathcal{I}\mathcal{K}} w_{\mathcal{K}} + G_{\mathcal{I}\mathcal{O}} w_{\mathcal{O}} + G_{\mathcal{I}\mathcal{I}} w_{\mathcal{I}} + G_{\mathcal{I}\bar{\mathcal{U}}} w_{\bar{\mathcal{U}}} + r_{\mathcal{I}} + v_{\mathcal{I}}$$

If all the unobservable predictor inputs $w_{\bar{\mathcal{U}}}$ are indirectly observable, then the indirect model is given by

$$\begin{aligned} w_j &= \tilde{G}_{j\mathcal{K}} w_{\mathcal{K}} + \tilde{G}_{j\mathcal{O}} w_{\mathcal{O}} + \tilde{G}_{j\mathcal{I}} w_{\mathcal{I}} + \tilde{r}_j + \tilde{v}_j \\ &= (I + G_{j\bar{\mathcal{U}}} f_{\bar{\mathcal{U}}\mathcal{I}} G_{\mathcal{I}j})^{-1} \left[(G_{j\mathcal{K}} - G_{j\bar{\mathcal{U}}} f_{\bar{\mathcal{U}}\mathcal{I}} G_{\mathcal{I}\mathcal{K}}) w_{\mathcal{K}} \right. \\ &\quad \left. - G_{j\bar{\mathcal{U}}} f_{\bar{\mathcal{U}}\mathcal{I}} G_{\mathcal{I}\mathcal{O}} w_{\mathcal{O}} + G_{j\bar{\mathcal{U}}} f_{\bar{\mathcal{U}}\mathcal{I}} (I - G_{\mathcal{I}\mathcal{I}}) w_{\mathcal{I}} \right] + \tilde{r}_j + \tilde{v}_j \end{aligned} \tag{16}$$

The indirect predictor model of w_j is unaffected by the elimination of $w_{\bar{\mathcal{U}}}$ since $w_{\bar{\mathcal{U}}}$ is not present in (16). However, the elimination of $w_{\bar{\mathcal{U}}}$ by using the indirect node observations will alter the dynamics according to (16). Sufficient but not necessary conditions for guaranteeing module equality $\tilde{G}_{ji} = G_{ji}$ for an observable predictor input $i \in \mathcal{K}_j$ are formalized below.

Proposition 2 *Consider the dynamic network* (2)*. The transfer function* $\tilde{G}_{ji}, i \in \mathcal{K}_j$ *of the indirect model* (12) *will be equal to* G_{ji} *of* (2) *if the following conditions are satisfied:*

(a) There exist a filter $f_{\bar{\mathcal{U}}\mathcal{I}}$ *such that* $f_{\bar{\mathcal{U}}\mathcal{I}} G_{\mathcal{I}\bar{\mathcal{U}}} = I$

(b) $G_{\mathcal{I}j} = 0$ *(otherwise a self-loop will be introduced)*

(c) $G_{\mathcal{I}i} = 0$ □

Proof 1 *Condition (a) implies that all predictor inputs* w_k*,* $k \in \mathcal{N}_j$ *are directly or indirectly observed (*$w_{\bar{\mathcal{U}}}$ *are not predictor inputs by Definition 2). Hence, no change of* G_{ji} *will occur when the remaining unobservable nodes* $w_{\bar{\mathcal{U}}}$ *are eliminated. Conditions (b) and (c) ensures that* G_{ji} *is unaltered when the indirect observations are used to eliminate* $w_{\bar{\mathcal{U}}}$*. Direct insertion of conditions (b) and (c) into* (16) *gives*

$$w_j = \tilde{G}_{j\mathcal{D}\setminus i} w_{\mathcal{D}\setminus i} + G_{ji} w_i + \tilde{G}_{j\mathcal{O}} w_{\mathcal{O}} + \tilde{G}_{j\mathcal{I}} w_{\mathcal{I}} + \tilde{r}_j + \tilde{v}_j \tag{17}$$

which shows that $\tilde{G}_{ji} = G_{ji}$*.* □

Example 4 *In Example 1,* $\mathcal{K}_2 = \{1, 3\}$*,* $\bar{\mathcal{U}}_2 = \{5\}$ *and* $\mathcal{I}_2 = \{4\}$*. The indirect observation is given by*

$$w_4 = G_{41} w_1 + G_{45} w_5 \tag{18}$$

which means that $G_{42} = 0$*,* $G_{41} \neq 0$ *and* $G_{43} = 0$*. Proposition 2 gives* $\tilde{G}_{23} = G_{23}$ *which was shown in Example 3.* □

Proposition 2 gives conditions on the interconnection structure of the network, i.e. that links between certain nodes cannot exist, but it does not require knowledge about any of the modules. When the input w_i to the module of interest is unobservable, i.e. $i \in \bar{\mathcal{U}}_j$, more information about the modules in the network is typically needed. Instead of listing a number of special cases, it is perhaps more natural to talk about identifiability of the indirect model. Assume that (4) is parameterized with ϑ, then row one is given by

$$w_j = G_{j\mathcal{K}}(\vartheta) w_{\mathcal{K}} + G_{j\bar{\mathcal{U}}}(\vartheta) w_{\bar{\mathcal{U}}} + r_j + v_j$$

and the parameterized indirect node observations are

$$w_{\mathcal{I}} = G_{\mathcal{I}j}(\vartheta) w_j + G_{\mathcal{I}\mathcal{K}}(\vartheta) w_{\mathcal{K}} + G_{\mathcal{I}\mathcal{O}}(\vartheta) w_{\mathcal{O}} + G_{\mathcal{I}\mathcal{I}}(\vartheta) w_{\mathcal{I}} + G_{\mathcal{I}\bar{\mathcal{U}}}(\vartheta) w_{\bar{\mathcal{U}}} + r_{\mathcal{I}} + v_{\mathcal{I}}$$

Then the module $G_{ji}(\vartheta)$ can be recovered if the resulting indirect model

$$w_j = \tilde{G}_{j\mathcal{K}}(\vartheta) w_{\mathcal{K}} + \tilde{G}_{j\mathcal{O}}(\vartheta) w_{\mathcal{O}} + \tilde{G}_{j\mathcal{I}}(\vartheta) w_{\mathcal{I}} + \tilde{r}_j(\vartheta) + \tilde{v}_j(\vartheta) \tag{19}$$

is identifiable with respect to ϑ.

4.2 Confounding variables – Correlation of noise

As in the previous section, assume that (4) is parameterized with ϑ. Then the immersed network predictor is given by

$$w_j = \breve{G}_{j\mathcal{K}}(\vartheta)w_{\mathcal{K}} + \breve{G}_{j\mathcal{A}}(\vartheta)w_{\mathcal{A}} + \breve{r}_j + \breve{v}_j$$

and if it is identifiable with respect to ϑ, the module $G_{ji}(\vartheta)$ can be recovered even if Proposition 1 is not fulfilled. However, neglecting predictor inputs might lead to correlation between $\breve{v}_j$ and $\breve{v}_k,\ k \in (\mathcal{K}_j \cup \mathcal{A}_j)$, that can give a biased estimator if it is not carefully considered. The disturbance of the l^{th} node is given by

$$\breve{v}_l = \breve{F}_{ll}v_l + \breve{F}_{l\mathcal{U}}v_{\mathcal{U}} \tag{20}$$

and the correlation is due to a *confounding variable* $v_n, n \in \mathcal{U}_j$ having paths to both the j^{th} and $k^{\text{th}}, k \in (\mathcal{K}_j \cup \mathcal{A}_j)$, nodes, see Dankers et al. (2016) for details.

As mentioned in Section 3, the indirect node observations contain information about all excitation that enters the network and has paths to $w_{\mathcal{I}}$. If all unobservable predictor inputs $w_{\bar{\mathcal{U}}}$ are indirectly observed, this implies that all excitation that enters into $w_{\bar{\mathcal{U}}}$ will be described by $w_{\mathcal{I}}$ and hence, cannot act as confounding variables.

Example 5 *Due to the elimination of* $\mathcal{U}_5 = \{5, 6, 7, 8\}$ *in Example 2,* r_6 *and* v_6 *enter directly into all remaining nodes while* v_7 *enters into* w_1 *and* w_2*. This could give a bias since the external disturbances are not available and thus act as confounding variables. One solution is to use* r_6 *as an instrumental variable. However, this will not utilize the excitation that the external disturbances provide which could increase the variance of the estimator. In contrast,* $\tilde{v}_2$ *in the indirect model of Example 3 is not correlated with neither* $\tilde{v}_1$ *nor* $\tilde{v}_3$*.* □

4.3 Properties of the indirect model

The indirect model (12) will get specific properties due to the usage of the indirect node observations. These properties are important to consider in the choice of parameter estimation method and certain methods might be better suited than others (Linder and Enqvist, 2016).

Firstly, there are artificial paths used to form the indirect model and the model is thus not representing a part of the actual physical system. These artificial paths might introduce direct terms in $\tilde{G}_{j\mathcal{K}}$, $\tilde{G}_{j\mathcal{O}}$ or $\tilde{G}_{j\mathcal{I}}$ even if the physical system has delays in all modules. The reason is that the propagation of a certain signal to several nodes in the network might take equally long time. Consider, for example, if G_{25} and G_{45} of Example 3 have the same order. Direct terms are potentially a problem and some system identification methods, such as the direct prediction error method, might fail if this is not considered (Dankers, 2014).

Secondly, the indirect node observations $w_{\mathcal{I}}$ are not actual predictor inputs to the j^{th} node. Even if the external disturbances $v_k, k \in \mathcal{N}$, are all uncorrelated with each other, the disturbance in the indirect model $\tilde{v}_j = \tilde{F}_{jj}v_j + \tilde{F}_{j\mathcal{I}}v_{\mathcal{I}}$ will be correlated with $w_{\mathcal{I}}$ which means that it will be an errors-in-variables problem.

Finally, as noted in Section 3, the indirect model will depend only on locally observable nodes. If instead the signals are neglected, then depending on the interconnection structure, $\mathcal{P}_j \cap \mathcal{U}_j$ might be non-empty which potentially means that a larger part of the network dynamics has to be modeled. For instance, consider the model in Example 2 compared to the indirect model of Example 3.

5 Prediction of Internal Variables

There are many possible uses for a model of a dynamical system, for example, control, design, diagnosis, prediction or simulation (Ljung, 1999). In some of these application areas, an exact representation of a part of the network, for example, G_{ji} as discussed in the previous section, is needed. For other use cases, it might be more important to accurately predict a set of nodes, for instance, the j^{th} node, in the network, and it might be sufficient to work with a black-box model. Assume that we have obtained an accurate model of $G_{j\mathcal{K}}$ and $G_{j\bar{\mathcal{U}}}$. A straightforward output error predictor is given by

$$\hat{w}_j = G_{j\mathcal{K}} w_{\mathcal{K}} + r_j \tag{21}$$

and the output error residual becomes

$$\varepsilon_j = w_j - \hat{w}_j = G_{j\bar{\mathcal{U}}} w_{\bar{\mathcal{U}}} + v_j \tag{22}$$

Note that since $w_{\bar{\mathcal{U}}}$ is unknown, it is not obvious how to use the knowledge about $G_{j\bar{\mathcal{U}}}$. Rather than simply neglecting the unknown inputs, a better approach is to work with the immersed model. Assuming that the model is known, the output error predictor for the j^{th} node of the immersed model is given by

$$\hat{\breve{w}}_j = \breve{G}_{j\mathcal{K}} w_{\mathcal{K}} + \breve{G}_{j\mathcal{A}} w_{\mathcal{A}} + \breve{r}_j \tag{23}$$

and the output error residual is

$$\breve{\varepsilon}_j = w_j - \hat{\breve{w}}_j = \breve{F}_{jj} v_j + \breve{F}_{j\mathcal{U}} v_{\mathcal{U}} \tag{24}$$

The predictor based on the immersed model gives more accurate predictions since part of the unknown excitation is described by the "up stream nodes" $w_{\mathcal{A}}$ and part is described by the external user controllable signals $r_{\mathcal{U}}$. However, some of the external disturbances will be unobserved unless all predictor inputs $\mathcal{N}_j$ are observable.

Example 6 *Consider the network of Example 1. If w_7 is measured instead of w_4, then the immersed network is*

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_7 \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & 0 & G_{17} \\ \frac{G_{21}}{1-G_{25}G_{52}} & 0 & \frac{G_{23}}{1-G_{25}G_{52}} & \frac{G_{25}G_{58}G_{87}}{1-G_{25}G_{52}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_7 \end{bmatrix} + \begin{bmatrix} v_1 \\ \frac{v_2 + G_{25}v_5 + G_{25}G_{58}(v_8 + G_{86}(r_6+v_6))}{1-G_{25}G_{52}} \\ v_3 + G_{36}(r_6+v_6) \\ v_7 + G_{76}(r_6+v_6) \end{bmatrix} \tag{25}$$

The observable node w_7 thus supply additional information and a larger part of the excitation that enters in the network is described compared with (7) *where w_7 is unobservable.* □

Similarly, a black-box model of the indirect model might be useful for predictive purposes. Consider the output error predictor for the j^{th} node of the indirect model given by

$$\hat{\tilde{w}}_j = \tilde{G}_{j\mathcal{K}} w_{\mathcal{K}} + \tilde{G}_{j\mathcal{O}} w_{\mathcal{O}} + \tilde{G}_{j\mathcal{I}} w_{\mathcal{I}} + \tilde{r}_j \tag{26}$$

The output error residual becomes

$$\tilde{\varepsilon}_j = w_j - \hat{\tilde{w}}_j = \tilde{F}_{jj} v_j + \tilde{F}_{j\mathcal{I}} v_{\mathcal{I}} \tag{27}$$

Note that the residual $\tilde{\varepsilon}_j$ is uncorrelated with the external disturbances $v_{\mathcal{U}}$ since the indirect node observations contain information about them as noted and discussed in Sections 3 and 4.3. However, $\tilde{\varepsilon}_j$ is correlated with $v_{\mathcal{I}}$ due to $w_{\mathcal{I}}$ being used as predictor input.

Example 7 *To summarize the discussion of this section, let us look the performance of the discussed predictors for Example 1. The modules are all of first order described by*

$$G_{nk} = \frac{\beta_{nk} q^{-1}}{1 - \alpha_{nk} q^{-1}} \tag{28}$$

with the parameters given in Table 1. The external disturbances $v_k,\ k \in \mathcal{N}$ *and the external user-controllable signal* r_6 *were white zero-mean Gaussian noise with unit variance while all other external user-controllable signals* $r_n,\ n \in \mathcal{N} \setminus \{6\}$ *were zero for all times.* $1\,000$ *samples were created by simulating* $(I - G)^{-1}$ *with* $r + v$ *as input and the resulting signals can be seen in Figure 2.*

Four predictors were tested, one based on (21) *(*$\hat{w}_2 = G_{21} w_1 + G_{23} w_3$*) where the node* w_5 *is simply neglected, two based on* (23) *with the immersed networks corresponding to* (7) *(*$\hat{\breve{w}}_2$*) and* (25) *(*$\hat{\breve{w}}_2^7$*), and one based on* (26) *with the indirect model corresponding to* (15) *(*$\hat{\tilde{w}}_2$*). The output of the predictors were simulated with the signals in Figure 2 as inputs and no filtering were performed.*

The results can be seen in Figure 3 where the numbers in the legend describes the fit. As expected, the predictor $\hat{w}_2$ *gives the worst result. The predictor* $\hat{\breve{w}}_2$ *partially compensates for the unobservable nodes but the excitation from* $v_n, n \in \{2, 5, 6, 7, 8\}$ *is not fully captured. The predictor* $\hat{\breve{w}}_2^7$ *does considerably better by including* w_7 *as an input. It partially compensates for the unobservable nodes but the excitation from* $v_n, n \in \{2, 5, 6, 8\}$ *is still not fully captured which can be seen around the peaks. The indirect predictor* $\hat{\tilde{w}}_2$ *follows the true signal*

Table 1: The values of the transfer function parameters in Example 7.

Module	β_{nk}	α_{nk}	**Module**	β_{nk}	α_{nk}
G_{12}	0.07	0.11	G_{45}	0.17	0.75
G_{17}	0.20	0.92	G_{52}	0.11	0.41
G_{21}	0.19	0.91	G_{58}	0.17	0.93
G_{23}	0.18	0.53	G_{76}	0.18	0.89
G_{25}	0.20	0.89	G_{86}	0.21	0.61
G_{36}	0.17	0.77	G_{87}	0.19	0.92
G_{41}	0.16	0.93			

well. The variation around the true signal is due to v_4 entering the predictor and the unknown v_2. Note that the performance of the indirect model predictor is dependent on a sufficiently large signal-to-noise ratio. Finally, both $\hat{\tilde{w}}_2$ and $\hat{\tilde{w}}_2^7$ require knowledge about more modules than $\hat{\tilde{w}}_2$ as mentioned in Section 4.3. □

6 Conclusions

In this contribution we have discussed the benefits of using indirect node observations in dynamic network estimation and prediction. The possible benefits are that only a local part of the network has to be modeled and that the variance of the estimator can be decreased since the indirect input measurements contain partial information about the unknown disturbances. The predictive properties of the indirect model are good but since the indirect node observations are used as input, disturbances entering in $w_{\mathcal{I}}$ are propagated to the predicted output. Here we have presented the case when all unobservable predictor inputs $w_{\bar{\mathcal{U}}}$ can be eliminated by the indirect node observations. Partial elimination of the unobservable predictor inputs $w_{\bar{\mathcal{U}}}$ is also interesting. This would be similar to predictor input selection discussed in Dankers et al. (2016) but it is left as future work.

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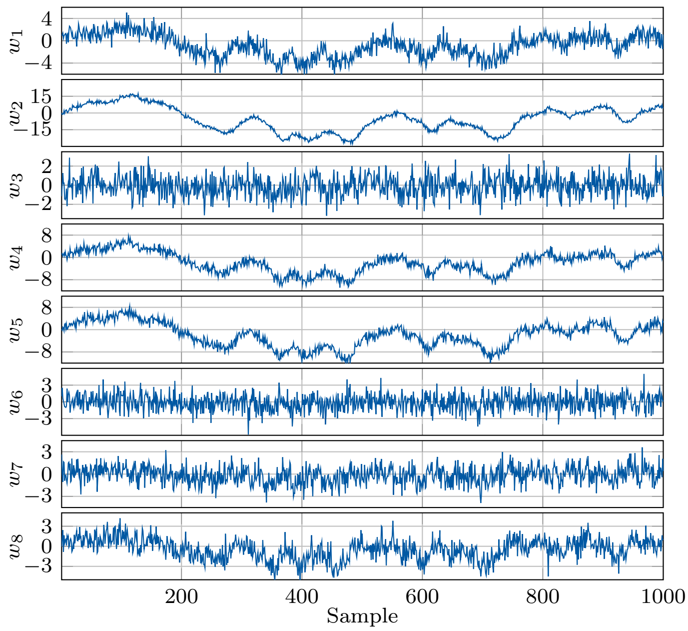

Figure 2: A simulation of the network in Example 7.

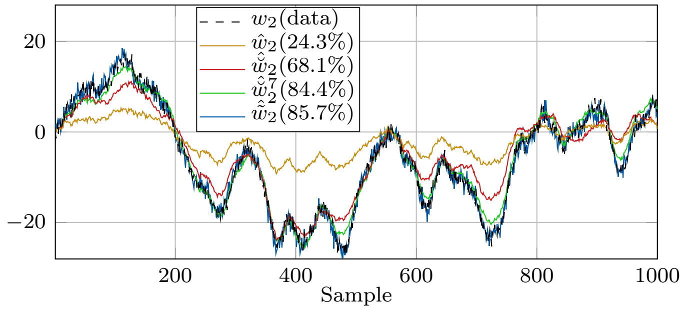

Figure 3: A comparison of the predictors in Example 7.

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Identification and Prediction in Dynamic Networks with Unobservable Nodes

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Sammanfattning
Abstract

The interest for system identification in dynamic networks has increased recently with a wide variety of applications. In many cases, it is intractable or undesirable to observe all nodes in a network and thus, to estimate the complete dynamics. If the complete dynamics is not desired, it might even be challenging to estimate a subset of the network if key nodes are unobservable due to correlation between the nodes. In this contribution, we will discuss an approach to treat this problem. The approach relies on additional measurements that are dependent on the unobservable nodes and thus indirectly contain information about them. These measurements are used to form an alternative indirect model that is only dependent on observed nodes. The purpose of estimating this indirect model can be either to recover information about modules in the original network or to make accurate predictions of variables in the network. Examples are provided for both recovery of the original modules and prediction of nodes.

Nyckelord
Keywords

Dynamic networks, closed-loop identification, identifiability, system identification