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On Power Minimization for Non-orthogonal Multiple Access (NOMA)

Lei Lei, Di Yuan, and Peter Värbrand

Abstract—We formulate a power optimization problem for non-orthogonal multiple access (NOMA) systems mathematically, and prove its NP-hardness. For tackling the problem, we first identify a convex problem by relaxation. Based on this convexity, we then propose an efficient “relax-then-adjust” algorithm and provide results of performance evaluation.

Index Terms—Non-orthogonal multiple access, optimization.

I. INTRODUCTION

A power-domain non-orthogonal multiple access (NOMA) scheme has been proposed to LTE Release 13, under a study called “downlink multiuser superposition transmission (MUST)” [1]. Unlike orthogonal multiple access (OMA), NOMA allows multiple users to simultaneously access the same time-frequency resource, by applying superposition coding and successive interference cancellation (SIC) [2]. It has been shown that NOMA is able to improve system throughput and fairness [1]–[7]. In [2], the authors characterized the capacity regions for downlink and uplink NOMA. In [3], it has been shown that NOMA yields better ergodic sum rate than OMA, though the outage performance depends highly on user demand and power allocation. The authors of [4] investigated power allocation for maximizing total mutual information in a two-user NOMA system. The authors of [1] and [5] demonstrate the potential of NOMA for improving throughput and fairness, respectively. In [6], the authors study several utility maximization problems for NOMA. In [7], a monotonic optimization approach is proposed for resource allocation in NOMA. The approach has the potential of approaching global optimum, though the numbers of constraints and variables increase exponentially in the candidate number of users to be multiplexed on the same subchannel. The authors of [8] investigated the important aspect of user pairing in NOMA, and showed that, for optimizing the outage probability, the users sharing a channel shall have large difference in power gains.

In addition to spectral efficiency, another key requirement is power consumption. We remark that maximizing utility with the presence of power bound (such as in [6]) and minimizing power subject to demand requirements are referred to as rate adaptive and margin adaptive resource allocation, respectively. Both are of significance and studied in their own right [9], and the solution of one does not generalize to the other due to the

difference in problem structure. For example, the algorithm idea in [6] does not fit into the problem of power minimization. For OMA, we refer to [10]–[12] and the references therein for papers that perform either rate adaptive or margin adaptive optimization. For NOMA, margin adaptive optimization schemes remain a significant topic of study. Indeed, recently there is a growing interest in power optimization for NOMA. In [13], the authors take a game-theoretic approach for studying the Nash equilibrium of power control for uplink NOMA. In [14], the authors provide insights of NOMA power control using the framework of standard interference functions.

II. SYSTEM MODEL

We consider a downlink NOMA cellular system consisting of K user equipments (UEs) and one base station (BS). The total bandwidth B is divided into N subchannels, each with bandwidth $\frac{B}{N}$. The sets of UEs and subchannels are denoted as $\mathcal{K}$ and $\mathcal{N}$, respectively. We use $\mathcal{K}_n$ to denote the set of multiplexed UEs on subchannel n . The composition of set $\mathcal{K}_n$ is subject to the optimization process. Hence, for OMA, $|\mathcal{K}_n| \leq 1$ applies in channel allocation, whereas for NOMA $|\mathcal{K}_n|$ may be greater than one at optimum. Power $p_{kn} > 0$ if and only if $k \in \mathcal{K}_n$.

The BS, by applying superposition coding, superposes and transmits the signals of the UEs in $\mathcal{K}_n$ on subchannel n . Each UE has capabilities of multi-user detection (MUD) and SIC. Based on the principle of NOMA [1], we sort all K UEs on each subchannel in descending order of power gains. We use $(k)_n$ to represent the position of UE k in the sorted sequence on subchannel n . We also introduce a mapping $(i)'_n$ to indicate the index of the UE in the i th position of the sorted sequence on subchannel n , where $(0)'_n = 0$. The interference I_{kn} after SIC for UE k on subchannel n is shown below,

$$I_{kn} = \sum_{h \in \mathcal{K}_n \setminus \{k\} : (h)_n < (k)_n} p_{hn} g_{kn}, \quad (1)$$

where g_{kn} is the power gain from BS to UE k on subchannel n . UE k on subchannel n decodes and subtracts the received signals of the UEs in $\{\mathcal{K}_n \setminus \{k\} : (h)_n > (k)_n\}$, before decoding the signal of interest. UE k treats the signals of the UEs in $\{\mathcal{K}_n \setminus \{k\} : (h)_n < (k)_n\}$ as noise [15]. For power allocation, following the NOMA system model in [1], [2], [4], it is assumed that more power is allocated to UEs with poor channel condition. As a result, for any user, the interfering signals that are intended for users with poorer channel condition and hence subject to interference cancellation, are received with higher strength. This is coherent with a common assumption of SIC, namely, decoding takes place in the order of signal strength. Also, the scheme of decoding a

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strong signal first (which is easier than decoding a weak one) addresses imperfect SIC implementation in practice. However, we remark that the analysis and algorithmic solution in the coming sections are easily adapted to power allocation without the assumption.

III. PROBLEM FORMULATION AND COMPLEXITY

NOMA power minimization (NPM) consists of determining UE grouping for each subchannel and power allocation. For UE grouping, both the number and the composition of UEs to be multiplexed for each individual subchannel are to be optimized in the mathematical formulation, and hence our model is different from that in [8].

We use continuous power variables $p_{kn} \geq 0$. In NPM, the objective (2a) is to minimize the sum power. Constraints (2b) formulate the rate demand, where $\frac{B}{N}$ is normalized to be 1.0, $\log$ is the natural logarithm, and η is the noise power. Inequalities (2c) limit, for each subchannel, the cardinality of the subset of UEs having positive power allocation, or equivalently, the number of multiplexed users in NOMA, to be at most L , where L is a parameter with $L \leq K$, to consider the practical limitations due to the receiver's design complexity and the signal processing time for SIC [15]. Thus L can be used as a parameter to address the delay in NOMA. Constraints (2d) are imposed to allocate more power to the UE with lower power gain. Specifically, for any two UEs k and k' on subchannel n with $g_{kn} \geq g_{k'n}$, the constraint requires $p_{kn} \leq p_{k'n}$ if $p_{k'n} > 0$. If the user with lower gain is not using the subchannel (i.e., $p_{k'n} = 0$), (2d) becomes void and hence does not impose restriction on the power relation.

p_{kn} = power allocated to UE k on subchannel n .

$$\text{NPM: } \min_{p_{kn} \geq 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{kn} \quad (2a)$$

$$\text{s.t. } \sum_{n \in \mathcal{N}} \log\left(1 + \frac{p_{kn}g_{kn}}{\sum_{h \in \mathcal{K} \setminus \{k\}: (h)_n < (k)_n} p_{hn}g_{kn} + \eta}\right) \geq D_k, \quad k \in \mathcal{K} \quad (2b)$$

$$|\{k \in \mathcal{K} : p_{kn} > 0\}| \leq L, \quad n \in \mathcal{N} \quad (2c)$$

$$p_{kn}p_{k'n} \leq p_{k'n}p_{k'n}, \quad k, k' \in \mathcal{K}, n \in \mathcal{N} : g_{kn} \geq g_{k'n} \quad (2d)$$

We remark that constraints (2d) affect the optimal power allocation. To illustrate this aspect, consider two UEs and one single subchannel with parameters $g_1 = 0.7, g_2 = 0.3, d_1 = 3, d_2 = 0.5$, and $\eta = 0.1$. We omit the channel index as there is no ambiguity. Without (2d), $p_2 < p_1$ at optimum, with the specific values of $p_1 = 1$ and $p_2 = 0.552$. The observation leads to the presence of (2d).

Problem complexity for NPM is formalized below.

Theorem 1. *NPM is NP-hard.*

Proof: We establish a polynomial-time transformation from an OMA minimum-power channel allocation (MPCA) problem to NPM. The NP-hardness of the former has been proved in [12]. For $L = 1$, we conclude that NPM is NP-hard since it is equivalent to MPCA.

We provide the proof for $L = 2$ by constructing a special case NPM_S of NPM; the proof is easily extended to any $L >$

2. Consider $N < K < 2N$, and set $\epsilon = \frac{1}{e}^{KN}$. The noise is $\eta = \epsilon^{KN}$. The rate requirement $D_k = \epsilon$ is uniform $\forall k \in \mathcal{K}$. The UE set $\mathcal{K} = \{1, \dots, K\}$ consists of two sets $\mathcal{K}' = \{1, \dots, N\}$ and $\mathcal{K}'' = \{N+1, \dots, K\}$. In set $\mathcal{K}''$, the values of power gain satisfy $g_{kn} \leq \epsilon^{KN}, \forall k \in \mathcal{K}'', \forall n \in \mathcal{N}$. In set $\mathcal{K}'$, we define UEs $k = 1, \dots, N$ with power gain $g_{11} = g_{22}, \dots, = g_{NN} = 1$, whereas every UE $k \in \mathcal{K}'$ on subchannels $\forall n \in \mathcal{N} \setminus \{k\}$ with $g_{kn} \leq \epsilon$. As a consequence, one can observe that the optimal allocation for the UEs in $\mathcal{K}'$ is to assign UEs $k = 1, \dots, N$ to subchannels $n = 1, \dots, N$, respectively, and with uniform power $p^* = (e^\epsilon - 1)\epsilon^{KN}$ to meet rate requirements $D_k = \epsilon$. The remaining problem is to allocate N subchannels among the UEs in $\mathcal{K}''$, where $N > |\mathcal{K}''|$. Each subchannel can multiplex one extra UE at most due to $L = 2$, then NPM_S is equivalent to MPCA. Note that constraints (2d) are in fact redundant for the defined instance of NPM_S, as every UE in $\mathcal{K}''$ has extremely inferior power gains $g_{kn} \leq \epsilon^{KN}$ over all subchannels. Moreover, the UEs in $\mathcal{K}''$ have received co-channel interference from the UEs in $\mathcal{K}'$. Thus the UEs in $\mathcal{K}''$ require more power than p^* on each subchannel. Therefore, a special case NPM_S is equivalent to MPCA in [12], and NPM is NP-hard and intractable. ■

From the proof, we obtain the following corollary.

Corollary 2. *NPM is NP-hard even if D_k is uniform.*

IV. ALGORITHMIC SOLUTION

The intractability of NPM justifies the development of suboptimal solutions. The proposed RTA for solving NPM includes two components, relaxation and adjustment. We consider a relaxed version NPM_R without (2c) and (2d), and prove its convexity in Theorem 3.

$$\begin{aligned} \text{NPM}_R: \quad & \min_{p_{kn} \geq 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{kn} \\ \text{s.t.} \quad & \sum_{n \in \mathcal{N}} \log\left(1 + \frac{p_{kn}g_{kn}}{\sum_{h \in \mathcal{K} \setminus \{k\}: (h)_n < (k)_n} p_{hn}g_{kn} + \eta}\right) \geq D_k, \quad k \in \mathcal{K} \end{aligned}$$

Theorem 3. *NPM_R is convex.*

Proof: We identify and prove the convexity by reformulation. Define rate variable $R_{kn} = \log\left(1 + \frac{p_{kn}g_{kn}}{\sum_{h \in \mathcal{K} \setminus \{k\}: (h)_n < (k)_n} p_{hn}g_{kn} + \eta}\right)$. The p -variables can be then expressed in the rate variables by successive variable substitution. Consider as an example one subchannel (and hence we can omit the channel index) with UEs $1, 2, \dots$, with $g_1 > g_2 > \dots$, and hence $R_1 = \log(1 + \frac{p_1g_1}{\eta})$, $R_2 = \log(1 + \frac{p_2g_2}{p_1g_2 + \eta})$, and so on. For UEs 1 and 2, we have $p_1 = \frac{(\exp(R_1)-1)\eta}{g_1}$ and $p_2 = \frac{(\exp(R_2)-1)(p_1g_2 + \eta)}{g_2}$. In the expression of p_2 , we substitute p_1 with $\frac{(\exp(R_1)-1)\eta}{g_1}$, and obtain $p_2 = \frac{(\exp(R_2)-1)(\frac{(\exp(R_1)-1)\eta}{g_1}g_2 + \eta)}{g_2}$. For UE 3, the definition of p_3 contains p_1 and p_2 , which now can be substituted using the above expressions that contain the rate variables only, and the procedure generalizes to any number of UEs. Following the above procedure, the resulting reformulation of NPM_R in the rate variables is given in (4).

$$\min_{R_{kn} \geq 0} \sum_{i=1}^K \sum_{n=1}^N \left(\frac{\eta}{g(i)_{n,n}'} - \frac{\eta}{g(i-1)_{n,n}'} \right) \exp \left(\sum_{h=i}^K R_{(h)_{n,n}'} \right) - \frac{\eta}{g(K)_{n,n}'} \quad (4a)$$

$$\text{s.t.} \quad \sum_{n=1}^N R_{kn} \geq D_k, \quad k \in \mathcal{K} \quad (4b)$$

In (4a), we can omit a constant $-\sum_{n=1}^N \frac{\eta}{g(K)_{n,n}'}$ which does not affect optimality. This term is however accounted for in obtaining the numerical results in the next section. One can observe that $\frac{\eta}{g(i)_{n,n}'} - \frac{\eta}{g(i-1)_{n,n}'} \geq 0$ due to the descending order of power gains. Therefore the objective is a *sum-exp* function which is convex. The constraints are linear. Hence NPM_R is convex. ■

Formulation (4) can be efficiently solved by standard convex optimization tools or by applying Karush-Kuhn-Tucker (KKT) conditions [16]. We note that variable substitution is used also in [6]. However the purpose is merely for tractability analysis of a special case, whereas here the result above constitutes the core of algorithm design. Our algorithm RTA, presented in Algorithm 1, uses the optimum of NPM_R to derive a feasible though possibly suboptimal solution for NPM. In the algorithm, the optimal power and rate values of NPM_R are formed into $K \times N$ matrices $\mathbf{p}^*$ and $\mathbf{r}^*$, respectively. If $\mathbf{p}^*$ does not satisfy (2c) or (2d), RTA adjusts $\mathbf{p}^*$ to derive a feasible solution in two phases. In Phase 1 (Lines 5 to 11), RTA adjusts UE-subchannel allocation using greedy selection, such that the UE-subchannel pair with the smallest fraction of allocated demand is discarded first, until at most L UEs are allocated on each subchannel. By the end of Phase 1, $\mathbf{p}^*$ is updated by re-indexing the UEs, such that all the positive elements of each column are sorted in descending order of gain, and all zero elements are placed in arbitrary order after the positive ones. In Phase 2 (Lines 12 to 19), power adjustment is performed for $\mathbf{p}^*$, first by power increase to ensure that the power solution satisfies constraints (2d) and every UE's demand is delivered, followed by scaling down the power for each UE as much as possible while maintaining solution feasibility. We use $u_{i,n}$ to indicate the UE index of $p_{i,n} \in \mathbf{p}^*$ in Line 18. We remark that, by adapting the rule of channel-user selection, the algorithm is easily extended to the case where each UE has a UE-specific limit on the number of decoding; this is equivalent to introducing a UE index to parameter L . Such an extension is useful for considering the delay metric.

We remark that NPM_R can be solved by methods with polynomial-time complexity [16]. Next, we observe that Phase 1 is clearly polynomial, and the complexity of Phase 2 is bounded by performing $K C_{max}$ times one-dimensional bisection search, which is typically polynomial.

V. PERFORMANCE EVALUATION

Table I summarizes the key simulation parameters. The UEs are randomly and uniformly distributed in a cell. We generate one thousand instances and consider the average performance. In Fig. 1, we provide performance comparison between NOMA and OMA, as well as algorithm evaluation with respect to the global optimum of NOMA using the monotonic optimization (MO) approach [7]. As this approach

Algorithm 1 RTA for NPM

Input: $K, N, L, D_k, \beta_k = P_{max}, \forall k \in \mathcal{K}$

Output: $\mathbf{P}^*$

```

1: Solve (4) and obtain optimum  $\mathbf{p}^*, \mathbf{r}^*$ , and  $\mathcal{K}_n, \forall n \in \mathcal{N}$ 
2: if  $\mathbf{p}^*$  satisfies (2c) and (2d) then
3:    $\mathbf{P}^* \leftarrow \|\mathbf{p}^*\|_1$ , break
4: else
5:   for  $n = 1 : N$  do
6:     while  $|\mathcal{K}_n| > L$  do
7:        $k' \leftarrow \{k | \min(\frac{r_{kn}}{D_k}), \forall r_{kn} > 0\}, r_{kn} \in \mathbf{r}^*$ 
8:        $\mathbf{p}^* \leftarrow \{\mathbf{p}^* | p_{k'n} = 0\}, \mathbf{r}^* \leftarrow \{\mathbf{r}^* | r_{k'n} = 0\}$ 
9:        $\mathcal{K}_n = \mathcal{K}_n \setminus \{k'\}$ 
10:    for  $n = 1 : N$  do
11:      For the positive elements of column  $n$  of  $\mathbf{p}^*$ , sort the
        corresponding UEs in descending order of gain.
12:    for  $n = 1 : N$  do
13:      for  $i = 2 : L$  do
14:         $p_{i,n} = \max(p_{i,n}, p_{i-1,n})$ , for all  $p_{i,n}, p_{i-1,n} > 0$ 
15:       $p_{i,n} = p_{i,n} + P_{max}$ , for all  $p_{i,n} > 0$  in  $\mathbf{p}^*$ 
16:    repeat
17:      for  $k = 1 : K$  do
18:        Bisection search for  $\beta_k$ , such that (2d) and UE  $k$ 's
          demand are satisfied, update  $p_{i,n} = p_{i,n} - \beta_k, \forall n \in \mathcal{N}$ ,
          where  $i$  is subject to  $u_{i,n} = k$ 
19:    until  $C_{max}$  iterations, or both (2b) and (2d) hold
20:  $\mathbf{P}^* \leftarrow \|\mathbf{p}^*\|_1$ 

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is developed for small L , we set $L = 2$ as in [7]. To accurately evaluate the potential improvement of NOMA over OMA, we consider scenarios with $K = N$ for which OMA power minimization is polynomial-time solvable as in this case it reduces to bi-partite matching [12]. Here, $K = N = 10$. Fig. 1 shows the sum power with uniform D_k which is successively increased. In addition to Fig. 1(a) that shows the total transmit power, in Fig. 1(b) the power of the decoding operations is accounted for to obtain a more complete performance picture, using the model and parameters for decoding power in [17].

Table I
SIMULATION PARAMETERS.

Parameter	Value
Cell radius	300 m
Carrier frequency	2 GHz
Bandwidth per subchannel	180 KHz
Subchannels (N)	5, 10
UEs (K)	10
Path loss	COST-231-HATA
Shadowing	Log-normal, 8 dB standard deviation
Fading	Rayleigh flat fading [15]
Noise power	-173 dBm/Hz

From the results of Fig. 1(a), NOMA can achieve significant power-saving over OMA. The average improvement is more than 60%. Moreover, the rate of power increase in demand is clearly lower for NOMA. When the power consumption of signal decoding is included, we observe that at very low demand, the power consumption of NOMA is in fact slightly higher than that of OMA, because NOMA has more decoding operations. However, when demand increases, NOMA remains superior with very significant improvement in total power. We

also observe that the proposed RTA algorithm performs well with respect to global optimality; the average deviation from global optimum is less than 15%. In view of that the proposed algorithm scales well in L , whereas MO has exponential complexity in L , the algorithm provides a complementary approach to [7].

Figure 1. Performance evaluation for $L = 2$, $K = 10$, and $N = 10$.

In Fig. 2, we evaluate the influence of parameter L , with $K = 10$ and $N = 5$. Note that OMA is infeasible for this case. The curve “ $|K_n| = 10$ ” shows the results where all users are allocated positive power to all subchannels in NOMA. From Fig. 2, we notice that the sum power is reduced when more UEs are allowed to (but not necessarily have to) be multiplexed on each subchannel, though the improvement of power saving in NOMA is marginal when L becomes large. This observation is consistent with [3]. On the other hand, multiplexing all UEs to all subchannels with positive power is not optimal (in particular if decoding power is accounted for); this is because for each UE, the gain varies by subchannel. The observation demonstrates the importance of UE-channel allocation in our optimization process.

Figure 2. Power comparison in respect of L , with $K = 10$ and $N = 5$.

Indexing UEs in their gain for each subchannel, we consider the difference of the UE indices in the global optimal solution. As $K = 10$, the maximum possible distance is 9 (if the users with the highest and lowest gains are grouped together). The percentage values for distance intervals $[7, 9]$, $[4, 6]$, and $[1, 3]$ are approximately 30%, 50%, and 20%, respectively. Hence UEs with large or relatively large gain difference are more like to be multiplexed (this is line with [8]), yet optimal grouping is not necessarily to pair UEs with the highest and

lowest gains, illustrating the significance of optimal grouping for power minimization.

In addition to power, throughput is an important system performance objective [6], [7]. We have made a comparison of sum rate, by first solving our problem of NOMA power minimization for the given throughput requirement, followed by imposing the resulting total power as a constraint in maximizing OMA throughput. The average improvement is in the range $[40\%, 100\%]$ (whereas the specific value depends on K and L). This range is in line with the results in [6], [7] that focus on the throughput objective.

VI. CONCLUSIONS

We have addressed complexity, algorithm development, and performance evaluation for power minimization in NOMA. An extension of the work is to incorporate NOMA in scheduling along the time dimension with a mix of service types to more explicitly address other metrics, such as the overall delay.

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