Children's early mathematics learning and development

Number game interventions and number line estimations

Jessica Elofsson

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Abstract

Children's early mathematics learning and development have become a topic of increasing interest over the past decade since early mathematical knowledge and skills have been shown to be a strong predictor of later mathematics performance. Understanding how children develop mathematical knowledge and skills and how they can be supported in their early learning could thus prove to be a vital component in promoting learning of more formal mathematics.

In light of the above, with this thesis I sought to contribute to an increased understanding of children's early mathematics learning and development by examining effects of playing different number games on children's number knowledge and skills, and by investigating children's representations of numbers on number line tasks.

Two number game intervention studies were performed, and effects of three different number game conditions (linear number, circular number and nonlinear number) were investigated by examining 5- and 6-year-old children's pre- and posttest performance on different numerical tasks. The findings indicate that playing number games in general support children's development of number knowledge and skills, where the specific learning outcomes are affected differently depending on the type of number game utilized.

To elucidate children's representations of numbers, their performance on two different number line tasks have been analyzed using a latent class modeling approach. The results reveal that there is a heterogeneity in 5- and 6-year-old children's number line estimations and subgroups of children showing different estimation patterns were distinguished. In addition, it is shown that children's number line estimations can be associated to their number knowledge as well as to task specific aspects.

The findings presented in this thesis contribute to the discussion of the value of selecting game activities in a conscious way to support children's early mathematics learning and development. They also add to the discussion regarding the number line task and how children's number line estimations can be analyzed and interpreted.

Sammanfattning på svenska

Barns tidiga lärande och utveckling av kunskaper i matematik är ett område som har uppmärksammats allt mer i samhällsdebatten och i forskning under det senaste decenniet. En anledning till detta kan vara att barns tidiga kunskaper i matematik har visat sig vara en stark prediktor för senare prestationer i matematik. Att förstå hur barn utvecklar kunskaper i matematik och hur de kan stödjas i sitt tidiga lärande kan vara en viktig och betydelsefull komponent för att främja barns fortsatta lärande av mer formell matematik.

Mot bakgrund av ovanstående så är syftet med denna avhandling att bidra till en ökad förståelse kring barns tidiga lärande och utveckling av kunskaper i matematik genom att undersöka hur barns lärande och utveckling av kunskaper i matematik påverkas av att spela olika numeriska spel, samt genom att undersöka barns representationer av tal på tallinjesuppgifter.

Två spelinterventionsstudier har genomförts där effekter av att spela tre olika spel (linjära numeriska, cirkulära numeriska och icke-linjära numeriska) har undersökts. Analyserna av 5- och 6-åriga barns resultat på olika numeriska uppgifter före och efter interventionen indikerar att numeriska spel stödjer barns utveckling av kunskaper om tal och tals relationer. Vidare så visar resultaten att barns lärande påverkas olika beroende på vilket numeriskt spel som används.

För att undersöka barns representationer av tal så har barns skattningar av tal på två olika tallinjesuppgifter analyserats med hjälp av latent klassregressionsanalys. Resultaten visar att det finns en heterogenitet i 5- och 6-åriga barns representationer av tal på tallinjer och subgrupper av barn som visade olika skattningsmönster urskildes. Dessutom visas att det verkar finnas en relation mellan barns kunskaper om tal och hur de representerar tal på tallinjer samt att barnen också verkar påverkas av uppgiftsspecifika aspekter när de skattar tal på tallinjer.

Resultaten som presenteras i denna avhandling bidrar till diskussion om värdet av att välja spelaktiviteter på ett medvetet sätt för att stödja barns tidiga lärande och utveckling i matematik. Vidare så bidrar de också till diskussionen kring tallinjesuppgifter och hur barns representationer av tal på tallinjer kan analyseras, tolkas och förstås.

List of appended papers

I.

Elofsson, J., Gustafson, S., Samuelsson, J., & Träff, U. (2016).
Playing number board games supports 5-year-old children's early
mathematical development. *The Journal of Mathematical Behavior*,
43, 134-147.

II.

Elofsson, J., Samuelsson, J., Gustafson, S., & Träff, U. (submitted).
Effects of playing number games on 6-year-old children's number
knowledge and skills.

III.

Elofsson, J., Samuelsson, J., Gustafson, S., & Träff, U. (submitted).
Investigating children's number line estimation patterns using a Latent
class regression analysis.

IV.

Elofsson, J. (submitted).
Children's number line estimation patterns on a 0-10 number line task.

Preface

This thesis is the result of my doctoral studies carried out between 2011 and 2017 in the Division of Education, Teaching and Learning at the Department of Behavioural Sciences and Learning, Linköping University, Sweden. This research has been financially supported by the Swedish Research Council (721-2010-4932).

A handwritten signature in black ink, reading "Jessica Elofsson". The script is cursive and fluid, with the first name "Jessica" and the last name "Elofsson" clearly distinguishable.

Linköping, May 2017.

Acknowledgements in Swedish

I maj 2011 började jag min vandring. Under flera år har jag nu befunnit mig på en kunskapsresa under vilken jag lärt mig otroligt mycket om forskning, barns lärande och utveckling i matematik och inte minst om mig själv. Under vandringen har jag mött personer som inspirerat, influerat och utmanat mig. Jag har brottats med olika frågor, kämpat på i uppförsbackar och ibland vandrat i långa, djupa dalar och funderat på meningen med att fortsätta. Men att ge upp vandringen kändes aldrig som ett alternativ, för hela tiden drevs jag på av min nyfikenhet för vad jag skulle komma att möta längre fram under färderna. Våren 2017. Fåglarna kvittrar och de första blommorna har tittat fram. Nu sitter jag här i solen med min dator i knät och skriver på de sista raderna som ska läggas in i den bok som är det fysiska beviset på den resa som jag gjort och som startade för ungefär sex år sedan. Jag ser mållinjen framför mig nu och vill ta tillfället i akt att tacka alla er som på ett eller annat sätt funnits med mig under min utmanande, utvecklande och fantastiska vandring.

Först av allt vill jag tacka alla härliga barn som gjort denna avhandling möjlig. Tack för att jag fick besöka er och tack för att ni ställde upp på att genomföra olika uppgifter och att spela spel. Under tiden jag fick vara hos er lärde jag mig massor!

Jag vill tacka mina handledare, professor Ulf Träff, professor Joakim Samuelsson och biträdande professor Stefan Gustafson för er insats. Jag vill tacka för att ni funnits med under min resa och för att ni låtit mig gå på egna upptäcktsfärder lite då och då under vilka jag fått möjligheten att hitta intressanta saker som vi sedan kunnat diskutera och jobba vidare med. Tack för att ni delat med er av era kunskaper och erfarenheter kring forskning. Det finns mycket jag skulle vilja skriva men som ni så många gånger sagt till mig, man måste begränsa och försöka hålla det kort. Så för att nu följa detta råd, ett STORT TACK till er!

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med dig under doktorandtiden. Tillsammans har vi brottats med frågor och funderingar kring både forskning, undervisning och livet. Tack för att du hejat på mig! Och så ett extra tack till alla underbara personer i matematikdidaktikgruppen. Margareta Engvall, Pether Sundström, Joakim Samuelsson, Mats Bevemyr, Cecilia Sveider, Rickard Östergren och Sofie Arnell. Tack för samtal och diskussioner om allt mellan himmel och jord. Ni inspirerar mig och har gjort det extra roligt att gå till jobbet! Jag har också haft förmånen att befinna mig på vandring tillsammans med ett gäng fantastiska doktorandkollegor (ingen nämnd, ingen glömd). Även om vi allesammans intresserat oss för olika saker i våra doktorandprojekt och befunnit oss i olika faser så har vi delat medgångar och motgångar med varandra och hjälpts åt på vägen. Ni har satt extra guldkant på min resa!

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egen bok – jag sa ju att jag skulle göra det redan för länge, länge sedan. Den blev kanske i en annan variant än vad jag tänkte då, men det spelar här och nu ingen som helst roll. Min underbara, fina Viktor! We did it – two down! Du är min allra bästa vän och jag är så glad och tacksam över att få dela livet med dig. Tack för att du stöttat mig. Jag älskar dig! Linnea, du är den underbaraste, smartaste och roligaste människan på jorden! Du får mig att skratta och har verkligen fått mig att förstå vad kärlek är. Jag älskar dig! Och du, kom ihåg att ingenting är omöjligt! Och så mina fyrbenta vänner. Conrad, du lärde mig mycket under tiden vi fick. Bulan, min älskade, busiga Bulis! Tack för alla fantastiska tänkarpromenader, uppmuntrande pussar och roliga träningspass under det gångna året. Framöver ska de förhoppningsvis bli ännu fler nu när jag får mer tid över om kvällarna.

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PLAYING NUMBER BOARD GAMES SUPPORTS 5-YEAR-OLD CHILDREN'S
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CHAPTER 1

Introduction

Mathematics has a long history stretching back thousands of years. Human curiosity and desire to conquer and develop new knowledge to understand the world has influenced the development of mathematics, as you and I know it. In today's society, we are surrounded by numbers and we use number skills and mathematics daily, many times without awareness. During the last decade there has been an increased recognition of the importance of mathematical knowledge and skills for everyday life in our modern society, and this has also been incorporated in preschool and school curriculums (Lembrer & Meaney, 2014). When children grow up in Sweden today they are expected to learn and develop mathematical knowledge and skills and to be able to use these in different contexts and situations (Skolverket, 2011). During elementary school, children should develop their understanding for and ability to use mathematics in everyday life and for further studies (Skolverket, 2011). In relation to this, Krajewski and Schneider (2009) highlight the importance of paying attention to and support young children's precursory and informal mathematical knowledge and skills and their desire to learn mathematics already in preschool, both from a preparatory as well as from a lifelong learning perspective.

Since the beginning of the 21st century, young children's early mathematics learning and development has been increasingly highlighted (Tallberg Broman, 2010). Research has revealed that young children often show (spontaneous) interest in mathematics before entering school and that children's free play contains a considerable amount of mathematical activities such as classification, magnitude comparison, enumeration, and discovery of patterns and spatial relations (Björklund, 2007; Seo & Ginsburg, 2004). On the other hand, research also indicates that not all young children pay attention to numbers spontaneously (Hannula & Lehtinen, 2005). Hannula and Lehtinen's (2005) research show that there seems to be a relationship between children's spontaneous focus and attention to numerosity and children's later mathematical skills. Based on this,

Hannula and Lehtinen (2005) argue that it is important to capture young children's natural interest and curiosity in mathematics in order to allow them to develop (in)formal mathematical knowledge and skills important for further mathematics learning and development.

Since the revision of the Swedish preschool curriculum in 2010 (Skolverket, 2010), there has been an increased attention towards young children's early mathematics learning and development with focus on development of number sense and spatial skills. During early childhood, most children make (mathematical) experiences and develop number knowledge and skills that will provide the foundation for learning of formal mathematics (LeFevre, Skwarchuk, Smith-Chant, Fast, Kamawar, & Bisanz, 2009; von Aster & Shalev, 2007). It has been shown that children's early number knowledge and skills are important since children entering school with a mathematical foundation benefit in further learning, not only in mathematics, but also in science, literacy and technology (Duncan et al., 2007). This indicates that it is important to find effective activities that can stimulate and help children to develop number knowledge and skills in a conscious way with the intention to promote later and more complex learning in mathematics. However, it is not obvious and easy to identify activities and experiences that contribute to establish this development. What mechanisms underlie children's learning and development of number knowledge and skills and early mathematical understanding? How can we support young children's early mathematics learning in a conscious and systematic way?

Humans seem to have an inherent ability to represent and manipulate numbers, a preverbal sense of number, and ability that is shared by humans from different cultural backgrounds as well as with other species (Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004). This preverbal sense is often referred to as core systems of number and are described as an important foundation for children's mathematics learning and development (Dehaene, 2011; Feigenson et al., 2004; von Aster & Shalev, 2007). As young children make (mathematical) experiences, develop their language, and gets familiar with the symbolic number system with verbal counting words and written Arabic numerals, it is believed that the symbolic number system gradually becomes mapped onto the inherent ability to represent number (Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010; von

Aster & Shalev, 2007) and that children develop a (symbolic) mental number line with numerals spatially presented in relation to their magnitude (von Aster & Shalev, 2007). The development of a mental number line is described as a vital step for the development of mathematical knowledge and skills, and not least for arithmetic thinking (von Aster & Shalev, 2007). The precision of children's representation of numerical magnitude is described to be characterized by a developmental shift, where children's representations develop from being logarithmically compressed to become linear with increasing age and experience (e.g., Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Booth, 2004; Siegler & Opfer, 2003). Further, the linearity of children's representations of number on number line tasks has been found to correlate with children's overall mathematics achievement (e.g., Booth & Siegler, 2006; Siegler & Booth, 2004). These findings suggest that children's representations of numerical magnitude are important and influence children's mathematics learning and development. Siegler and Booth (2004) hypothesized that in order to support children's mathematics learning and development, children could benefit by participating in (game) activities and interventions focusing on supporting number knowledge and skills and (more) linear representations of numerical magnitude.

Research has shown that playing board games that contain numbers, especially games where the numbers are being presented in a visual-linear structure, stimulates and improves children's number knowledge and skills and their development of a (more) linear representation of numerical magnitude (Ramani & Siegler, 2008; Siegler & Ramani, 2009; Whyte & Bull, 2008). Children's number line estimations have also been found to be closely related to their counting skills and arithmetic ability (Booth & Siegler, 2006; Siegler & Ramani, 2009). Further, longitudinal studies have revealed that children's mathematical knowledge and skills at the age of four and a half years are related to children's achievement in mathematics at the end of elementary school (Duncan et al., 2007). Taken together, these empirical findings emphasize the importance of supporting children's early mathematics learning and development in a conscious way to prepare them for learning of formal mathematics important for everyday life as well as for further studies. Considering the importance of the mental number line (i.e., representation of numerical magnitude)

(Siegler & Booth, 2004; Siegler & Ramani, 2009; von Aster & Shalev, 2007), teachers should work with activities where children are given opportunities to develop their number knowledge and skills and linear representations of numerical magnitude to support further learning and development in mathematics. Another important area to continue to explore in relation to previous research focusing on children's representations of numerical magnitude that has also been highlighted in previous studies (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Rouder & Geary, 2014), is how children represent numbers on number line tasks and how their number line estimations should be analyzed, understood and interpreted.

Overall aim

The overall aim of this thesis is to contribute to the understanding of young children's early mathematics learning and development by examining effects of playing different number games on children's number knowledge and skills, and by investigating children's representations of numbers on number line tasks. Two main questions will be addressed: 1) How can young children be supported in their learning and development of number knowledge and skills by playing number games? and 2) How can children's representations of numbers on number line tasks be described and interpreted? This thesis consists of four studies. In Study I and II, I address the first main question by investigating how number games supports 5- and 6-year-old children's learning and development of number knowledge and skills. In Study III and IV, I address the second main question by examining 5- and 6-year-old children's number line estimations on two different number line tasks.

Outline of this thesis

The present thesis consists of five chapters. The first chapter gives an introduction to this thesis and the overall aim is presented. In chapter two, research and theories regarding children's early mathematics learning and development are presented. The third chapter focuses on how children's early mathematics learning and development can be supported through mathematics programs and educational

interventions. In the fourth chapter, the empirical studies included in this thesis are presented. The fifth and final chapter contains a general discussion of the main research findings presented in this thesis. Limitations will be discussed and implications and suggestions for future research are presented.

CHAPTER 2

Early mathematics learning and development

Mathematics can be described as a human construction with number systems and rules helping us to understand and solve different kinds of problems. Counting, calculation and other formal mathematical activities are culturally constructed and learned (Dehaene, 2011). However, the underlying mechanisms supporting our mathematical thinking and understanding seem to be innate and shared by humans from different cultures as well as with other species (Dehaene, 2011; Feigenson et al., 2004). Dehaene (2011) describes that we have an inherent preverbal sense of number. Children enter the world equipped not only with general learning abilities, but also with specialized learning mechanisms, or mental structures. These mechanisms, together with theories of how children acquire understanding of the symbolic number system, are described in the following chapter.

Number and quantitative thinking

One important mathematical capacity to develop during early childhood is the understanding of numerical concepts (Feigenson et al., 2004; von Aster & Shalev, 2007). This understanding provides the foundation for children to be able to learn formal mathematics and acquire mathematical knowledge and skills important for life in today's society. As mentioned earlier, the mechanisms that are believed to support our ability to learn and use mathematics seem to be inherent (e.g., Dehaene, 2011; von Aster & Shalev, 2007). The human brain seems to be endowed with two cognitive systems for non-verbal representation of number, the *Approximate number system* and the *Object tracking system* (Dehaene, 2011; Feigenson et al., 2004; Hyde & Spelke, 2011). These systems are often referred to as *core systems of number* (Feigenson et al., 2004) and “account for our basic numerical intuitions, and serve as the foundation for the more sophisticated numerical concepts that are uniquely human” (Feigenson et al., 2004, p. 307). As I will describe in the following sections these two core

knowledge systems are believed to serve different impressions of number and seem to be important for children's acquisition of the symbolic number system and further mathematics learning (Dehaene, 2011; Feigenson et al., 2004; von Aster & Shalev, 2007).

The Approximate Number System

Multiple studies provide evidence for a core knowledge system in our brains that allows us to nonverbally represent number without actually counting, often referred to as the *Approximate number system* (ANS) (Butterworth, 2010; Dehaene, 2011; Feigenson et al., 2004; Hyde & Spelke, 2011; Piazza, 2010). This cognitive system lets us represent and detect differences in magnitude between sets of objects. For example, the ANS inform us that one queue in the grocery store is shorter than the other or that our friend has more popcorns left in her bowl than we have in ours. In these situations, we do not actually count the number of people lined up to pay their groceries or the number of popcorns left, we just approximate the number.

The ANS is described as a basic and innate preverbal non-symbolic ability to apprehend and manipulate quantities in an approximate mode (Feigenson et al., 2004; Halberda, Mazocco, & Feigenson, 2008; Piazza, 2010). It forms approximate analog magnitude representations of numbers (Dehaene, 2011). Empirical support for an inherent ability to represent and discriminate between sets of objects (quantities) comes from habituation studies on infants. It has been shown that 6-month-old infants can discriminate between sets of objects with a ratio of 1:2, meaning that they can discriminate between sets containing for example, 8 vs. 16 objects (Xu & Spelke, 2000) or 16 vs. 32 objects (Xu, Spelke, & Goddard, 2005). Xu and Spelke's (2000) habituation experiment also revealed that infants were unable to discriminate between sets with a ratio of 2:3 (i.e., 8 vs. 12 objects) at the age of 6 months. Also, Xu, Spelke and Goddard (2005) found that infants failed to discriminate sets containing 8 vs. 12 objects and 16 vs. 24 objects. These results indicate that there are limitations in infants' representations of number, and the ANS is therefore described as imprecise, ratio dependent and noisy (Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005; see also Feigenson et al., 2004). Since the ANS represents number in an approximate and compressed fashion, the relation between the stimulus and the internal representation could be

described as being logarithmically compressed. That is, larger numbers are being presented closer together than smaller numbers. Researchers have shown that the ANS acuity can be described by Weber's law and is calculated by a Weber fraction $((A-B)/B)$ where A and B represent the quantities being compared (Halberda et al., 2008) indexing how much two sets needs to differ so that an individual can notice the difference between them (Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2011). "Because of the inexactness of ANS representations, two quantities cannot be distinguished when the distance between them is too small" (Halberda et al., 2008, p. 1457). The acuity of the ANS changes from infancy to adulthood and empirical findings suggest a refinement of the internal number representation as a function of age and maturation (Halberda & Feigenson, 2008; Piazza, 2010; see also Feigenson et al., 2004). Studies have shown a decrease in the ratio needed for children and adults in order to be able to discriminate between sets of objects (Halberda et al., 2008; Xu & Spelke, 2000). Young children at the age of three years can discriminate between sets of objects with a ratio of 3:4, while five-year-olds can discriminate sets with a ratio of 5:6 (Halberda & Feigenson, 2008). The acuity of the ANS saturate in adulthood, somewhere around the age of 20, and adults are able to discriminate between sets of objects with a ratio of 7:8 (Halberda & Feigenson, 2008; Piazza, 2010).

The *mental number line* can be used as a metaphor for describing the function of the ANS (Dehaene, 2011). The mental number line can be described as a mental picture with numbers spatially ordered from left to right with increasing magnitude. The ANS represents numbers with decreasing accuracy with increasing magnitude. The larger the number is, the more noisy and imprecise is our mental representation (Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010). The ANS has been hypothesized to constitute the foundation for humans to acquire the symbolic number system (Dehaene, 2011; Piazza, 2010; von Aster & Shalev, 2007) used for enumeration, calculation and formal mathematics. Empirical evidence for this interpretation comes from effects observed in experimental settings, where parallels between representations of non-symbolic and symbolic numerical magnitudes have been found. The same *distance effect* that have been shown with non-symbolic magnitudes have also been shown with symbolic information. Moyer and Landauer (1967) describe the distance effect

that can be observed when individuals are asked to determine which of two written Arabic numerals that is the largest. When presented with two numerals it takes longer time to decide which numeral that is the largest when the numerical distance between the numerals being compared are small (e.g., 4 and 5) compared to large (e.g., 4 and 9). Another finding that gives support for the notion that the ANS constitutes the foundation for acquiring the symbolic number system (Dehaene, 2011; Piazza, 2010; von Aster & Shalev, 2007) is the *problem size effect* (Moyer & Landauer, 1967); when asked to select the largest of two numerals, the choice is made faster when the numerals being presented are small (e.g., 3 and 4) compared to large (e.g., 8 and 9). Together, these two effects demonstrate that there is a link between the ANS and the symbolic number system. It is believed that the ability to represent number constitutes the foundation for acquiring the symbolic number system (e.g., Dehaene, 2011; Geary, 2013; von Aster & Shalev, 2007). As children make early (mathematical) experiences and evolve knowledge of our culturally constructed and language-based symbolic number system with both verbal counting words and written Arabic numerals, it is believed that this knowledge gradually becomes mapped onto the ANS (e.g., Dehaene, 2011; Geary, 2013; von Aster & Shalev, 2007). The link between the ANS, the symbolic number system and children's mathematics learning and development is described in theoretical models of number representations (e.g., Geary, 2013; von Aster & Shalev, 2007) as presented later in this chapter.

The Object Tracking System

The Approximate number system described above is part of our preverbal cognitive system for representing and manipulating large numbers in an approximate manner. Evidence suggest that humans also have a second core system for precise representation of number that helps us to quickly identify and keep track of small number of individual objects (i.e., 1-3(4) objects) (Piazza, 2010; Van de Walle, Carey, & Prevor, 2000; Wynn, 1992). This system is called the *Object tracking system* (OTS) or *Parallel individuation system* (Carey, 2009; Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010). In this thesis, I will use the term Object tracking system. The OTS is limited to sets of less than three objects (for infants) or four objects (for older children

and adults) (Piazza, 2010; Spelke, 2000). This core system help us to differentiate between sets of objects containing different number of objects by tracking multiple individual objects at the same time and encoding the exact numerical identity of the objects. For example, imaging a situation where three dogs run behind a house and after a few seconds two of the dogs come back. This infer that the third dog must still be behind the house. Your mind register that two of three dogs came back. In this situation, there is a mismatch with your mental representation (dog, dog, dog) and the actual state of the world (dog, dog). This can be described as a one-to-one correspondence process between the mental representation and the actual objects (Carey, 2004). The OTS gives an exact mental model of what happens when one object is added or subtracted to a small set (Carey, 2004; Piazza, 2010).

The presence of an Object tracking system can explain some data from habituation experiments where infants were able to distinguish between sets containing 1, 2 or 3 objects (i.e., 1 vs. 2, 2 vs. 3, 1 vs. 3) but not between sets of 1 and 4 objects (Feigenson & Carey, 2005; Feigenson, Carey, & Hauser, 2002), despite the fact that the ratio between these two sets suggests that the ANS should be able to distinguish these (Xu, 2003). Infants are able to discriminate between small sets of objects (1-3) but fail to discriminate between small sets when one of the sets contains more than three objects (Feigenson & Carey, 2005; Feigenson et al., 2002). Further evidence for the existence of an OTS comes from studies using enumeration tasks. That is, when asked to determine the number of objects in a set, individuals can tell the number of objects with high speed and accuracy for sets containing up to 3 (4) items while it takes longer for sets consisting of more than four items (Piazza, 2010), suggesting that we have a system specified for representation of small numbers. The ability to quickly determine the number of objects in a set is called *subitizing* (Feigenson et al., 2004; Piazza, 2010). Subitizing could be described as an ability that allows immediate and accurate recognition of small numbers without actually counting them (Feigenson et al., 2004; Piazza, 2010).

To summarize, humans seem to have two core systems (i.e. ANS and OTS) that is involved in representing number and these two systems seem to be specified to serve different impressions of number. The ANS and the OTS are described as inherent preverbal number

abilities that are present from birth and these systems are suggested to play a central role in the acquisition of the symbolic number system and learning in mathematics (Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010; von Aster & Shalev, 2007). The core systems of number are described as the foundation for learning of mathematics and Dehaene (2011) describes that the symbolic number system becomes mapped onto the core systems of number which could be described as the child's "first step on the way to higher mathematics" (Dehaene, 2011, p. 260).

Acquiring the meaning of the symbolic number system

As described above, humans seem to be endowed with an inherent preverbal sense of number (Dehaene, 2011), before learning and acquiring the concepts of the culturally constructed symbolic number system with verbal counting words and written Arabic numerals. The ability to apprehend and manipulate non-symbolic quantities are described as the foundation for children's further development of number knowledge and learning in mathematics (Feigenson et al., 2004; Jordan, Glutting, & Ramineni, 2010; Krajewski & Schneider, 2009). As young children gain experience with the symbolic number system, they gradually develop their number knowledge, a process that is ongoing over many years (Opfer & Siegler, 2012; von Aster & Shalev, 2007). Before entering school most children are familiar with numbers and have basic understanding of the symbolic number system (Butterworth, 1999; Gelman & Gallistel, 1978). But what is the relationship between the inherent core systems as described in previous sections and the symbolic number system? What role does the core systems of number have for children's learning and development of number knowledge and their learning in mathematics? In the following, I will describe how the core systems of number are presumed to serve as a foundation for children's development of number knowledge and skills, and thereby their learning of formal mathematics.

There is an ongoing debate whether the mechanisms for representing numbers are composed by one or two systems (e.g., Geary, 2013). Irrespectively, there is a common understanding that there must be some kind of mapping between the symbolic number system (with verbal counting words and written Arabic numerals) and

the inherent non-symbolic ability to represent number (Dehaene, 2011; Feigenson et al., 2004; Gallistel & Gelman, 1992; von Aster & Shalev, 2007; Xu & Spelke, 2000). Dehaene (1992) describes that we have three main codes for representations of number; a verbal code, a visual Arabic code, and analogue magnitude. Children initially learn to associate quantities with their verbal referent, for example * * - “two” (verbal code) and later they learn to associate written Arabic numerals to these (visual Arabic code), for example * * - “two”- 2 and that the understanding of these codes develop hierarchically (von Aster & Shalev, 2007; Dehaene, 1992). The visual code, the verbal code and the inherent number representation system (i.e., the analogue magnitude) constitute the so-called triple-code model (Dehaene, 1992), a model that has gained a lot of empirical support (see Dehaene, Piazza, Pinel, & Cohen, 2003). Most researchers agree upon that children learn to associate verbal counting words with corresponding quantities before developing understanding for written Arabic numerals. Number knowledge can be described as the ability to process number words and written Arabic numerals when they have been mapped onto the inherent number representation system (Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006). This ability could also be referred to as number sense (Jordan et al., 2006).

As mentioned earlier in this chapter, different theoretical models have been proposed to describe the integration of the inherent core systems of number and the symbolic number system. In one of these models, von Aster and Shalev (2007) have structured a hierarchical four-step developmental pathway model of numerical cognition (see Figure 1 for an illustration). According to this model, the concept of number is given its meaning through 1) non-symbolic quantity representation, 2) verbal quantity representation, 3) written Arabic-symbolic quantity representation, and 4) integration of the former steps to form a (symbolic) mental number line. In the first step in von Aster and Shalev’s (2007) model, the inherited core systems of number and related functions (e.g., approximation and subitizing) are described as providing the basic meaning of number during infancy. This step is described as a precondition for the second step in the model where objects becomes associated with verbal counting words, and for the third step where children learn to associate objects and counting words to written Arabic numerals during preschool and early school years. In

the fourth and final step, the former steps in the model have been integrated and a (symbolic) mental number line is developed. To be able to construct, automatize and extend numerical representations, children must link their understanding of quantities (core systems of number) with the symbolic number system (with both verbal counting words and written Arabic numerals) (Dehaene, 1992, 2011; Geary, 2013; Jordan et al., 2006; von Aster and Shalev, 2007). In relation to the development of numerical cognition, working memory is also described as a domain-general ability that is also important for children’s mathematics learning and development (e.g., von Aster & Shalev, 2007).

Figure 1. Hierarchically organized four-step developmental model describing the development of cognitive representation of number. The area below the dotted grey arrow illustrates demand of working memory. Inspired by von Aster and Shalev (2007).

Another developmental model describing the integration of the inherent core systems of number and the symbolic number system is presented by Geary (2013). Geary (2013) has presented a three-step developmental model describing children’s early mathematics learning in which the first step constitutes the core systems of number. Further, as children learn verbal number words and written Arabic numerals

these acquire meaning as they become mapped onto the inherent number representation system (Geary, 2013). The final step in the model consists of developing understanding for relations among numerals (Geary, 2013). In both von Aster and Shalev's (2007) and Geary's (2013) models the innate ability to manipulate and represent numbers is described as the foundation for learning and understanding the symbolic number system. Von Aster and Shalev (2007) suggest that the mental number line is a vital step in the development of number representation and that a well-functioning mental number line is believed to support children's development of arithmetic abilities (Siegler & Ramani, 2009; von Aster & Shalev, 2007). As outlined by von Aster and Shalev (2007), the development of a mental number line seems to be a result of an experience based developmental process that requires more than just well-functioning core systems. The mental number line is described as developing successively with increasing experience and knowledge of the symbolic number system (von Aster & Shalev, 2007).

The (mental) number line

A physical number line is most often illustrated as a horizontal straight line with numbers arranged from left to right with larger numbers to the right (Heeffer, 2011). Heeffer (2011) describes a number line as “a *representation* of numbers on a straight line where points represent integers or real numbers and the distance between the points match the arithmetical difference between the corresponding numbers” (Heeffer, 2011, p. 2). The same description can be used to illustrate the idea of the mental number line. As described previously, a mental number line can be used as a metaphor in thinking about encoding and representing number. The mental number line provides a non-verbal mechanism for a true number concept (Dehaene, 1992). Long before infants and young children learn verbal counting words and written Arabic numerals they can convert, for example, a set of five apples or a set of five tones to the same position on the mental number line. That is, they seem to recognize “fiveness” to which the verbal counting word “five” and the written Arabic numeral “5” later can be associated (Opfer & Siegler, 2012). As children are being exposed to and make experiences of the symbolic number system they gain knowledge and evolve new ways of thinking about numbers. Children's acquisition of the

symbolic number system profoundly expands their mathematical abilities and their (symbolic) mental number line representations (Dehaene, 2011; Opfer & Siegler, 2012; von Aster & Shalev, 2007). As children gain experience with the symbolic number system with verbal counting words and written Arabic numerals they associate these with quantities and the symbolic number system becomes mapped onto the mental number line (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003) as illustrated in von Aster and Shalev's (2007) developmental model presented above. With increasing age and experience, children seem to develop their mental representation of number from being logarithmically compressed to become (more) linear (e.g., Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003) as will be further described later in this chapter.

Number line estimation

Estimation can be described as a translation between representations (Siegler & Booth, 2004; Siegler & Opfer, 2003). One example is numerical-to-numerical translation, for example when estimating the sum of two multi-digit numerals in an addition problem. Another type of estimation involves numerical-to-non-numerical translation, for example when locating (estimating) the position of a number on a number line. This last estimation task, the number line task is commonly used in research to investigate children's number knowledge and to describe children's representations of numerical magnitude (e.g., Siegler & Booth, 2004; Siegler & Opfer, 2003). The number line task consists of a straight horizontal line with fixed and labeled endpoints (e.g., 0 and 10 or 0 and 100) on which individuals are asked to estimate the position of given numbers (number-to-position task) or to tell what numbers that corresponds to marked positions on the number line (position-to-number task). The number line task is described as revealing about individuals' representations of number (numerical magnitude) since it transparently reflects the ratio characteristics of the symbolic number system (Siegler & Opfer, 2003; Siegler & Ramani, 2009). For example, just as 8 is twice as great as 4, the estimated position of 8 should be twice as far from 0 as the estimated position of 4 on the number line. That is, the estimated magnitude of numbers should increase linearly with the actual

magnitude of the numbers being presented generating a linear estimation pattern (see Figure 2a for an illustration of a linear estimation pattern on a 0-10 number line task).

A number of cross-sectional studies using different number line tasks have shown a developmental shift in children’s representations of numerical magnitude from initially being logarithmically compressed (Figure 2b) to become linear (Figure 2a) with children’s increasing age and experience of number (Booth & Siegler, 2006; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Siegler, Thompson, & Opfer, 2009). Siegler and Opfer (2003) have studied children’s (age from 7-12 years) and adults’ estimation patterns using both number-to-position tasks and position-to-number tasks within different numerical ranges (i.e., 0-100 and 0-1000). Differences in estimation patterns were found between younger children and older children/adults. Younger children’s representations of number on number lines between 0-100 were best described as being logarithmically compressed (Siegler & Opfer, 2003). In contrast, the estimates of older children and adults on number lines within the same numerical range (i.e., 0-100) were described as being linear. That is, the given estimates of older children/adults corresponded to the actual magnitudes of the numbers being presented on the number line tasks.

Figure 2. Illustration of (a) a linear representation of numerical magnitude and (b) a logarithmic representation of numerical magnitude on a 0-10 number line task.

Research has shown that a developmental shift from a logarithmically compressed representation of numerical magnitude to a linear representation of numerical magnitude seems to emerge within

different numerical ranges as a function of increasing age and experience with numbers (see Table 1). Between the age of 3-5 years, most children develop a linear representation of small whole numbers within the numerical range between 0 and 10, between kindergarten and class 2, most children develop a linear representation of larger numbers in the range between 0 and 100 and between second grade and fourth grade, most children develop a linear representation of numerals between 0 and 1000 (Berteletti et al., 2010; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). The developmental shifts in representations within different numerical ranges seem to be related to the period where children are being exposed to numbers within that specific numerical range. The shift from a logarithmic to a linear representation of number with increasing age has also been shown using other kinds of number estimation tasks. When asked to generate N dots on a computer screen where 1 and 1000 dots were shown as references and when asked to draw lines of approximate X units when lines of 1 and 1000 units were shown as referents have revealed the same age-related developmental shift from a logarithmic to a linear representation as found in research using number line task (Booth & Siegler, 2006; see also Booth & Siegler, 2008). Further, both non-symbolic and symbolic representations of number have been found to be related to children's mathematics achievement. Halberda et al. (2008) showed that the results on a non-symbolic magnitude comparison task was related to children's mathematics achievement test scores. Holloway and Ansari (2009) found that 6- to 8-year-old children's individual differences in distance effect on a symbolic number comparison task were related to children's mathematics achievement. Further, the linearity of children's number line estimates on number line tasks has been found to be related to children's overall mathematics achievement test scores (Booth & Siegler, 2006, 2008; Laski & Siegler, 2007). Thus, a linear representation of numerical magnitude seems to be related to children's mathematical knowledge and skills.

Table 1. *Children’s development of a linear representation of numerical magnitude of whole numbers within different numerical ranges (0-10, 0-100, and 0-1000) between the age of 3 to 12 years.*

Age	Numerical range
~3 to 5 years	Small whole numbers
~5 to 7 years	Larger whole numbers
~7 to 12 years	Even larger whole numbers

Children’s representations of number on number line tasks has been investigated extensively, and when analyzing children’s number line estimates, different mathematical models have been fitted and proposed to describe children’s representations of number (e.g., Ashcraft & Moore, 2012; Barth & Paladino, 2011; Booth & Siegler, 2006; Bouwmeester & Verkoeijen, 2012; Opfer & Siegler, 2007; Rouder & Geary, 2014; Siegler & Booth, 2004; Siegler & Opfer, 2003). One of the most frequently used methods when analyzing children’s number line estimates is to use group median estimates and predefined functions (i.e., linear, logarithmic, and exponential) to describe children’s estimation patterns (e.g., Bouwmeester & Verkoeijen, 2012; Siegler & Opfer, 2003). The group median estimate is used as the dependent variable and the actual number to be represented as the independent variable. Predefined functions are then fitted in order to describe children’s estimation patterns and the model with the highest R^2 -value (variance explained) is selected to describe children’s representations of numerical magnitude (Bouwmeester &

Verkoeijen, 2012; Siegler & Opfer, 2003). As described above, several studies using both non-symbolic and symbolic magnitude tasks have shown that children's representations of number develop from being logarithmically compressed to become (more) linear with increasing age and experience. Despite the mounting evidence supporting this developmental shift model of numerical magnitude representations presented by Siegler and colleagues (e.g., Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003), shortcomings of the model have been identified and discussed (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). Even though the logarithmic-to-linear developmental shift in children's representations of number have been replicated in several studies using number line tasks, researchers have questioned if this model gives the best explanation and description of children's number line estimates (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). Alternative model-fitting approaches and analyzes have been proposed and tested in order to describe children's number line estimation patterns and representations of numerical magnitude (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Ebersbach et al., 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009). The use of alternative model-fitting approaches has generated interesting research findings and has contributed to a discussion about the number line task and that different aspects may affect children's number line estimations and that children's estimation patterns on number line tasks may not be a reflection of their internal representation of number (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). For example, task specific properties such as marked and fixed endpoints (reference points) (Barth & Paladino, 2012; Rouder & Geary, 2014; White & Szűcs, 2012) as well as children's number knowledge (Ebersbach et al., 2008; LeFevre, Sowinski, Cankaya, Kamawar, & Skwarchuk, 2013) and estimation strategies (Ashcraft & Moore, 2012; Rouder & Geary, 2014) may affect their ability to estimate number on number line tasks. The possible influence of individual children's strategies when estimating numbers on number line tasks was mentioned by Siegler and Booth (2004), but Barth and Paladino (2011) and Peeters, Degrande, Ebersbach, Verschaffel and Luwel (2015) discussed children's use of different estimation strategies and argue for the need of alternative model fitting approaches when analyzing

children's number line estimates in order to capture the variation and characteristics in children's estimations of number.

Ebersbach and colleagues (2008) found that children's estimation patterns were better described by a segmented linear model, consisting of two linear segments, as compared to logarithmic and linear functions. Their research showed that the breakpoint between the two linear segments in the model were found to be correlated with children's familiarity with numbers (Ebersbach et al., 2008). Moeller and colleagues (2009) also propose a segmented linear model to describe children's representations of numerical magnitude on number line task. However, based on their findings, they suggest a fixed breakpoint between singular and tens (Moeller et al., 2009).

Barth and Paladino (2011) propose that children's estimates of number on number line tasks might be affected by anchors (i.e., reference points) and that children need to make proportional judgments in order to give accurate (linear) estimations on number line tasks. They hypothesized that children's estimation patterns follow a cyclical power model: children overestimate the distance between small numbers while underestimating the distance between large numbers, but this underestimation decreases close to referents. In order to be able to capture nuances in children's estimations, Barth and Paladino (2011) applied power functions to better capture nuances in children's number line estimates as compared to fitting only logarithmic and linear functions. In line with their hypothesis, Barth and Paladino (2011) found that "proportion-judgment models provide a unified account of estimation patterns that have previously been explained in terms of a developmental shift from logarithmic to linear representations of number" (Barth & Paladino, 2011, p. 125). Furthermore, White & Szűcs (2012) found that children employ various strategies when estimating numbers on number lines. The 6-year-old children included in their study mainly used the lower reference point (i.e., 0) when making estimations of number on a 0-20 number line task while somewhat older children used more advanced estimation strategies and were able to use more referents making their number line estimations.

In relation to the discussion of how to analyze and interpret children's number line estimation patterns, Bouwmeester and Verkoeijen (2012) have addressed five methodological issues

regarding the logarithmic-to-linear model fitting approach commonly used when describing children's representation of numerical magnitude and they introduce a latent variable modeling approach (see e.g., Bouwmeester, Sijtsma, & Vermunt, 2004; Bouwmeester & Verkoeijen, 2012; Vermunt & Magidson, 2005; Wedel & DeSarbo, 1994) as an alternative method to use when analyzing and describing children's number line estimation patterns. Two of the five methodological issues discussed by Bouwmeester and Verkoeijen (2012) highlight the fact that important information can be missed when averaging across data (Ding, 2006; Siegler, 1987; Simon, 1975). They problematize the use of group median estimates as a summary score to describe children's estimation patterns. The use of group median estimates as a summary score might result in a description of children's number line estimates that is not representative for any individual in the sample and the variability in children's estimates is seen as error rather than as interesting and useful information (Bouwmeester & Verkoeijen, 2012). They also discuss the use of predefined age groups to study the potential development shift from logarithmic to linear representations of numerical magnitude and state that "although a developmental shift may be observed in children's mental representation of numbers, it is a rather strong assumption that this shift is clearly visible between two predefined age groups" (Bouwmeester & Verkoeijen, 2012, p. 250). The use of group median estimates and predefined age or grade groups conceal individual differences that could be due to intelligence or experience with numbers for example (Bouwmeester & Verkoeijen, 2012). The three other methodological issues presented by Bouwmeester and Verkoeijen (2012) address the use of predefined functions and the procedure used to decide what function that best describes children's representations of number. In previous studies, predefined functions (i.e., linear, logarithmic, and exponential) have been fitted in order to describe children's representations of number (Siegler & Booth, 2004, 2008; Siegler & Opfer, 2003). However, Bouwmeester and Verkoeijen (2012) highlight that it is possible that other less restrictive functions could provide better descriptions of children's number line estimations. They also problematize that the absolute fit of two regression lines (e.g., logarithmic and linear) are compared and that the difference in fit between these are interpreted absolutely. In relation to this,

Bouwmeester and Verkoeijen (2012) also discuss that two logarithmic or two linear functions themselves can be completely different and that this needs to be considered when deciding what function that best describes children's estimation patterns.

To be able to capture the heterogeneity and characteristics in 6- to 8-year-old children's estimation patterns on a 0-100 number line task (number-to-position), Bouwmeester and Verkoeijen (2012) used a latent class regression analysis (see e.g., Bouwmeester et al., 2004; Bouwmeester & Verkoeijen, 2012; Vermunt & Magidson, 2005; Wedel & DeSarbo, 1994) to identify latent (unobserved) classes (subgroups) of children showing similar number line estimation patterns. They presented a latent class regression model consisting of five subgroups of children showing different estimation patterns on the 0-100 number-to-position estimation task. Children in one of the five subgroups in their model showed a rather accurate linear relationship between the estimated positions and the actual numerals presented and this group was described by a rather accurate linear regression function. Another subgroup was described by a curve with a gentle S-shape where children's estimates close to the reference points (i.e. 0 and 100) was rather accurate. One subgroup was described by a logarithmic-like curve and children in this group tended to overestimate small numbers and underestimate larger numbers. Bouwmeester and Verkoeijen (2012) also identified one group that included children that overestimated small numbers, underestimated large number but that gave rather accurate estimates around the middle of the number line. The fifth subgroup was described by an almost horizontal regression curve. Children within this group provided all estimates around the midpoint of the number line. An interesting result in Bouwmeester and Verkoeijen (2012) study was that no statistically significant age developmental trend in the shape of 6- to 8-year-old children's estimation patterns were found. The results from their study provide that children's estimation patterns on the 0-100 number line task (number-to-position) can be described by various functions, some that do not correspond to or cannot be accounted for by logarithmic or linear functions as described in previous research (e.g., Siegler & Booth, 2004; Siegler & Opfer, 2003). Further, Bouwmeester and Verkoeijen's (2012) research revealed that children within the

same age group showed accurate as well as inaccurate estimation patterns on the 0-100 number line task.

As described above, different methods and models can be used to analyze and describe children's estimations of number on number line tasks. Depending on what model to interpret, children's estimation patterns can be described, explained and interpreted in different ways. Despite the fact that a number of studies have been carried out, further discussion of how to analyze, describe, and interpret children's pre-linear estimation patterns on number line tasks is needed. Nevertheless, as described previously in this thesis, the linearity of children's representations of number seems to be important for children's mathematics learning and achievement. Therefore, the question of how to support the linearity of children's representation of number is an important issue for both researchers and teachers.

CHAPTER 3

Supporting children's mathematics learning and development

As described in previous sections, children's mathematics learning and development start in early years. Children build foundational knowledge and understanding of numbers during early childhood that is important for future learning of formal mathematics (LeFevre et al., 2009; von Aster & Shalev, 2007). Young children have great capacity to learn mathematics and children's own knowledge can be used as a starting point for their learning during preschool (Björklund, 2013; Björklund Boistrup, 2006; Palmér, 2011). In the Swedish curriculum for preschools (Skolverket, 2010), play is described as an important activity for children's learning and development. A conscious use of play can promote children's learning and development and generate new experience, knowledge and skills (Skolverket, 2010). Play and other informal activities provide important opportunities for children to develop interest and curiosity for mathematics and extend their conceptual understanding (Gersten, Jordan, & Flojo, 2005; Ginsburg, Lee, & Boyd, 2008; Siegler & Booth, 2004).

Research has shown that there is a variation in children's early mathematics performance before formal schooling and that children's early mathematical knowledge and skills are a strong predictor of later mathematical achievement (Duncan et al., 2007; Hornung, Schiltz, Brunner, & Martin, 2014; Krajewski & Schneider, 2009). Hence, it is important to support preschool children's learning and development in mathematics in a conscious way to prepare them for learning of more formal mathematics. Early mathematics support, by means intervention, is one way to support and improve children's mathematics learning and prepare them for learning of formal mathematics and also for diminishing the possibility of mathematical learning difficulties later on. Clements and Sarama (2011) state that "there is much to gain, and little to lose, by engaging young children in mathematical experiences" (Clements & Sarama, 2011, p. 968) and game-based learning is described as one way to support children's learning in an enjoyable, motivating and interesting way (Clements &

Sarama, 2011; Ramani, Siegler, & Hitti, 2012). In the following section I will present mathematics programs and educational interventions designed and used for supporting young children's early number knowledge and skills and their early mathematics learning and development.

Mathematics programs and educational interventions

During the last decade, research-based mathematics programs and educational interventions have been designed and used in order to support children's mathematics learning and development in playful but purposeful ways. These programs and interventions have focused on different mathematical content and been designed for children within different age groups and with different precursory knowledge. A number of these programs and interventions focus on supporting young children's early learning and development of number knowledge and skills. For instance, two mathematics programs, *Big math for little kids* and *Number worlds*, have been developed in order to support children's early learning in mathematics. *Big math for little kids* aim to stimulate learning of numbers, shapes, patterns, reasoning, measures, procedures and spatial concepts. In order to stimulate this learning, different activities, such as stories and play, is used in this program (Ginsburg, Greenes, & Balfanz, 2003; Greenes, Ginsburg, & Balfanz, 2004). The *Number worlds* (Griffin, 2004) is a mathematics program focusing on concepts and procedures in mathematics while playing games and performing different activities. In order to stimulate mathematics learning, this program is based on five instructional principles; (1) build upon children's current knowledge, (2) follow the natural developmental progression when selecting new knowledge to be taught, (3) teach computational fluency as well as conceptual understanding, (4) provide plenty of opportunity for hands-on exploration, problem-solving and communication, and (5) expose children to the major ways number is represented and talked about in developed societies (Griffin, 2004, p 175-178). Both *Big math for little kids* and *Number worlds* are programs that reflect the intuition that playing games might promote children's mathematics learning and development (Greenes et al., 2004; Griffin, 2004). However, a variety

of activities (e.g., counting, arithmetic, number comparison and board games) are included in both of these programs and therefore it is not possible to interpret and specify the contribution of each of these components to children's learning and increased numerical understanding after participating in these mathematics programs. There are also other educational interventions that are more focused on supporting specific mathematics abilities using computer software and number board games. In the following sections I will present computer assisted interventions and number board game interventions that focus on supporting young children's early number knowledge and skills.

Computer assisted interventions

Wilson and colleagues (2006) presented the *Number race*, a computer game designed for 3- to 8-year-old children with focus on supporting children's learning of basic concepts of number and arithmetic. When playing the game, the child is racing against the computer in order to reach the end of a number line first. In each trial, the child is asked to select the larger of two quantities and then move their token the equivalent number of steps forward on the race track marked with numerals. The game is described as a program that "train every important aspect of early arithmetic: the rapid assessment of quantities, the counting routine, the quick link between symbols and quantities, and the understanding that number and space are closely linked" (Dehaene, 2011, p. 276). The results from a study using the *Number race* have shown moderate effects after the intervention, where children improved their performance on subitizing, numerical comparison and subtraction (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). However, no control group was included in the study making it difficult to tell if it was the intervention that had effects on children's performance or if other aspects affected the results.

In Räsänen and colleagues (2009) computer assisted intervention study, two different games were used: the *Number race* based on the interpretation that numerical skills develop from approximate representations of numerical magnitudes and *Graphogame-math* that focus on exact representation of numerosity and symbols (* * * – "three" – 3). Children in kindergarten received daily training for approximately 15 minutes during a three-weeks period. The results showed that children in both game intervention groups improved on

number comparison when compared to the control group. No significant improvement was found on verbal counting, object counting or arithmetic (Räsänen et al., 2009). Dehaene (2011) highlight that the design of the *Number race* includes many different aspects of number knowledge such as subitizing, counting, and estimation and therefore, it is not easy to interpret which of the aspects built in the game that is essential for children's learning. A more subtle and refined game intervention design has been presented by Siegler and Ramani (2008, 2009; Ramani and Siegler, 2008) using simple linear number board games to support children's early learning and development of number knowledge and skills (Dehaene, 2011).

Number board game interventions

Based on theories and empirical research of the mental number line and children's representations of numerical magnitude, Siegler and colleagues designed theoretically based educational interventions with focus on supporting and enhancing children's early number knowledge and skills with focus on developing linear representation of numerical magnitude by playing number board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; see also Whyte & Bull, 2008). As described in previous chapters the mental number line seems to be an important structure for mathematics learning and therefore it should be a goal in (early) education to provide children with the best possible conditions to develop functional internal representations of numbers (Ramani & Siegler, 2008; see also Siegler & Booth, 2004). Siegler and Booth (2004) hypothesized that children could construct linear representations of numerical magnitudes by playing *linear number board games* with linearly arranged, equal-sized, and numbered squares such as *Chutes and Ladders*¹. Siegler and Booth (2004) describe that the design of such a game provides multiple cues of numbers, both about order of numbers, their magnitudes and how small and large numbers are related to each other.

¹ The game *Chutes and Ladders* consists of a square board made up of 100 squares (arranged ten squares by ten squares). To play the game, players throw a die and advance their token the number of spaces indicated by the dice, and the arrival at the end of the board - the Hundredth Square - is the ultimate goal.

In order to test whether playing number board games could promote children's early number knowledge and their representations of numerical magnitude, Ramani and Siegler (2008) and Siegler and Ramani (2008, 2009) performed intervention studies with 4- and 5-year-olds using linear number board games, linear color board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008), and circular number board games (Siegler & Ramani, 2009). The linear number board game and the linear color board game both consisted of twelve linearly arranged, equally sized and colored squares. The first and the last square was marked with start and finish, respectively. The only difference between the two games was that the linear number board game also had the numerals 1-10 ordered from left to right in the squares on the game board. The circular number board game consisted of a circle divided into twelve equal-sized circle sectors. Both clockwise and counter clockwise number board games was used with the numerals 1-10 arranged between the location of 1 and 11 on an analogue clock (Siegler & Ramani, 2009). A spinner marked with 1 and 2 was used for the linear and circular number board game and a spinner with colors was used for the linear color board game. When playing the linear and circular number board games, children were asked to spin the spinner and move their token forward while naming the numerals in the squares/circle sectors that they passed out loud. For example, a child who had its token on the square/circle sector marked with 3 and spun a 2, would move forward on the number game board and count "four, five". This counting procedure is called the count-on-procedure (e.g., Laski & Siegler, 2014). Children who played the color board game were asked to move the token forward to the next square that had the same color as the spinner showed and name the colors in the squares that they passed (e.g., "blue, yellow").

The results from Siegler and Ramani's (2008) study where 36 children ($M_{age} = 4.6$ years) participated and a linear number board game and a linear color board game was used, showed that children playing the linear number board game showed a more linear representation of numerical magnitude on a 0-10 number line task (number-to-position) after playing the number game for approximately one hour divided on four sessions during a two-weeks period. Children in the linear color board game group showed no improvement on the number line task after the game intervention. In another study where 124 children

($M_{age}=4.9$ years) participated and the same board games were used, Ramani and Siegler (2008) showed that children improved their performance on several tasks (numerical magnitude comparison, number line estimation 0-10, verbal counting 0-10, and numeral identification) after playing the linear number board game for approximately one hour (four 15 minute sessions during a two-weeks period). Children in the linear color board game group did not improve their performance between pre- and posttest on any of the numerical tasks. Based on these findings, Ramani and Siegler (2008) concluded that playing linear number board games increase children's number knowledge and support children's development of more linear representation of numerical magnitude, as hypothesized by Siegler and Booth (2004). In a later intervention study, Siegler and Ramani (2009) included three different intervention groups: a linear number board game group, a circular number board game group, and a nonlinear numerical activity group. In total, 88 children ($M_{age}=4.8$ years) participated in the study and played linear or circular number board games or participated in nonlinear numerical activities during four 15-20-minute-long sessions over two weeks. Post intervention children also received one session with arithmetic training. The results from Siegler and Ramani's (2009) study revealed that children who had played the linear number board game improved their performance most on the two numerical tasks that measured numerical magnitude understanding. They improved their performance on the numerical magnitude comparison task and the 0-10 number line task (number-to-position) as compared to children in both the circular number board game group and the nonlinear numerical activities group included in the study. Further, Siegler and Ramani's (2009) results showed that children who had played the linear number board game produced more correct answers in response to subsequent training on verbal arithmetic problems. Based on these findings, Siegler and Ramani (2009) discuss the implication of children's representation of numerical magnitude in terms of being an important retrieval structure for numerical information that can be used when solving arithmetic problems. Another goal of Siegler and Ramani's (2009) board game intervention study was to test the representational mapping hypothesis. The representational mapping hypothesis implies that the more transparent the mapping between the physical number board game and the desired

mental representation of number is, the greater learning of the desired mental representation will occur (Siegler & Ramani, 2009). Therefore, the linear number board game was hypothesized to have greater effects as compared to the circular number board game and the nonlinear numerical activities and the results from their study supported this hypothesis since children in the linear number board game group improved their performance on the numerical magnitude task and the number line task while children in the other number game intervention groups did not (Siegler & Ramani, 2009).

Whyte and Bull (2008) have also performed an intervention study using number games and they included three different games in their study; linear number board games, linear color board games and nonlinear numerical activities. Fifty-five preschool children ($M_{age}=3.8$ years) participated in this study and each child played one of the three games during four intervention sessions (25 minutes each time) during a two-weeks period. Children in the linear number board game group played a linear number board game with numerals 1-10 during the first session, 1-20 during the second session, 1-30 during the third session, and 1-40 during the last session. After the intervention, both children who had played the linear number board game and children who had participated in the nonlinear numerical activity condition improved on verbal counting between 1-20, numerical identification 1-9, and on symbolic numerical magnitude comparison, but the linear number board game group improved the most. In addition, children who had played the linear number board game also improved their performance on the number line task 0-10 (number-to-position). Before the intervention, 31% percent of the children in the linear number board game group had an estimation pattern that was best fitted by a logarithmic function, but after the intervention all children in the linear number board game group showed estimation patterns that were best fit by linear functions. The results from Whyte and Bull's (2008) study supports previous finding by Siegler and colleagues (Laski & Siegler, 2014; Siegler & Ramani, 2008, 2009; Ramani & Siegler, 2008) that the experiences acquired when playing linear number board games support children's development of more linear representation of number.

In a recent study, Laski and Siegler (2014) tested the hypothesis that children's encoding of numerical-spatial relations on number board games is an important process affecting children's learning while

playing (linear) number board games. They suggest that the cognitive alignment framework can be used “for identifying how physical materials and mental and physical actions with those materials produce learning” (Laski & Siegler, 2014, p. 853). The count-on procedure used when playing the linear number board games (e.g., Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008) may be crucial for the learning outcome and minor changes in counting procedure might affect children’s encoding of numerical-spatial relations and thereby the results from the number game intervention (Laski & Siegler, 2014). In order to test this hypothesis, two groups of children ($N_{tot}=42$, $M_{age}=5.8$ years) played a number board game with numerals 1-100 arranged in a 10 * 10 square matrix and used different counting procedures while playing the number game for four sessions during a two-weeks period (the game was played twice during each session). One of the groups used the count-on procedure as described above while the other group used the count-from-1 procedure (i.e., children started to count from 1 each time they moved their token and counted forward the number of steps that the spinner showed). Laski and Siegler (2014) found that children in the game intervention group that had used the count-on procedure while playing the number board game improved their performance on number line estimation 0-100, numerical identification and count-on skills. Children in this group also promoted greater encoding of spatial positions of numerals post intervention as compared to children in the group using the count-from-1 procedure during the intervention period. Laski and Siegler’s (2014) results indicate that the counting procedure used playing number board games are important for the learning outcome.

In Table 2 on the following page, the number board game intervention studies previously presented are summarized with respect to participants (N , M_{age} , SES), games used, duration, and improvement on numerical tasks post intervention.

Table 2. *Summary of number board game intervention studies including linear number board games with respect to participants (N, M_{age}, SES), games used, duration, and improvement on numerical tasks post intervention.*

Study	Participants	Games and duration	Improvement
Siegler & Ramani (2008)	N=36 children M _{age} = 4.6 years Low-income	<i>Linear number 1-10^a</i> (LN) <i>Linear color</i> (LC) 4 *15 minute sessions over 2 weeks	Number line task 0-10 (LN)
Ramani & Siegler (2008)	N=124 children M _{age} = 4.9 years Low-income	<i>Linear number 1-10^a</i> (LN) <i>Linear color</i> (LC) 4*15-20 minute sessions over 2 weeks	Number line task 0-10 (LN) Numerical magnitude comparison (LN) Verbal counting (LN) Numeral identification (LN)
Siegler & Ramani (2009)	N=88 children M _{age} = 4.8 years Low-income	<i>Linear number 1-10^a</i> <i>Circular number 1-10^a</i> <i>Numerical activity</i> 4*15-20-minute session over two weeks + 1*15-20-minute session with arithmetic training	Number line task 0-10 (LN) Numerical magnitude comparison (LN) Arithmetic (addition) (LN)
Whyte & Bull (2008)	N=55 children M _{age} = 3.8 years Middle class	<i>Linear number 1-40^a</i> (LN) <i>Linear color</i> (LC) <i>Numerical activity</i> (NA) 4*25 minute sessions over 4 weeks	Number line task 0-10 (LN) Numerical magnitude comparison (LN) (NA) Verbal counting (LN) (NA) Numeral identification (LN) (NA)
Laski & Siegler (2014)	N=42 children M _{age} = 5.8 years Low-to lower-middle-income	<i>Linear number 1-100^b</i> (arranged in a 10*10 matrix) 4 sessions over 2 weeks (the game was played twice during each session)	Children in the count-on-procedure group improved on: Number line task 0-100 Count on Numeral identification

^aCount-on procedure used, ^bCount-on procedure and count-from-1 procedure used.

The results from the number board game studies presented above support the assumption that playing number board games with numerals presented in a linear manner improve children's number knowledge and linear representations of numbers. After playing linear number board games during a limited amount of time, children improve their number knowledge and skills, especially verbal counting skills, identification of Arabic numerals and their representations of numerical magnitude (Ramani & Siegler, 2008; Siegler & Ramani, 2009; Whyte & Bull, 2008). Children's visuospatial, kinesthetic, auditive and temporal experiences playing linear number board games seem to build a broad multisensory base supporting the ability to develop a linear representation of numerical magnitude, a linear mental number line (Ramani & Siegler, 2008). The count-on-procedure seems to be important for the learning outcome playing linear number board games (Laski & Siegler, 2014). Griffin (2007) describes the development of the mental number line (i.e., representation of numerical magnitude) as a process that gives structure to natural numbers, referring to the connection between quantity, verbal counting words and written Arabic numerals, together with the order relationships between numerals (see also Geary, 2013; von Aster & Shalev, 2007).

Concluding remarks

In previous chapters I have presented research on children's early mathematics learning and development as well as research on how children's early mathematics learning can be supported through mathematics programs and educational interventions. From the review, we can conclude that children's early number knowledge and skills and children's representations of numerical magnitude seem to be important for further learning in mathematics. Therefore, it is important to investigate and contribute with knowledge and understanding of children's mathematics learning and development and how children can be supported in their mathematics learning in a conscious way already from an early age. In the following chapter the four studies included in this thesis will be presented. The first two studies (Study I and II) focus on how 5- and 6-year old children can be supported in their learning and development of number knowledge and

skills by playing number games. Study III and IV focus on children's number line estimations on two different number line tasks.

CHAPTER 4

Empirical studies

In this chapter, I initially describe the general method used in this thesis in terms of participant recruitment and overall design of the research project. I also highlight ethical issues and considerations in this chapter. After that, a summary of each of the four studies included in this thesis will be presented, giving a brief description of background and aims, methods, and results.

General method

Participants and ethical considerations

In the process of recruiting participants for this research project the ethical guidelines presented by the Swedish research council for research in social science regarding information, consent, confidentiality and use have been followed (Vetenskapsrådet, 2002, 2011). The process of getting access to and recruiting participants for research is an important part of the research process. The recruitment process of young children to research projects is controlled by both individual (e.g., parents/caregivers) and institutional (professionals) gatekeepers (Einarsdóttir, 2007; McFadyen & Rankin, 2016; Powell & Smith, 2009). In order to get access to young children and make it possible for them to participate in research, these gatekeepers need to give their consent (Powell & Smith, 2009). For this study, I initially contacted principals and outlined the broad aim of the research project and presented the criteria for selection of children to the project (no specific criteria were listed for participation, with exception of children's age). When recruiting preschools and preschool classes², I recruited these within different catchment areas in a middle-sized city

² From the age of 6, children in Sweden are entitled to begin preschool class. Preschool class is a voluntary form of schooling the year before the formal school start, but municipalities are obligated to offer preschool class placements, and most 6-year-old children attend (Skolverket, 2016).

with more than 135,000 inhabitants. After initial consent was given by the principals, I met with preschool and preschool class teachers and gave them a short presentation of the project and asked for their help to distribute information letters to parents/caregivers. In this study, we distributed information letters written in Swedish and attached our contact information (phone number and e-mail) so that parents/caregivers could contact us if they had any questions about the project. The teachers also had some knowledge about the project and could help parents/caregivers to get in contact with us if needed. A specific information letter directed to the children was also enclosed with the information letter sent out to the parents/caregivers. The adults were encouraged to read that information letter to their child and ask if he or she wanted to participate in the research project before signing the address form giving informed consent for participation. Children whose parents/caregivers gave written informed consent had the opportunity to participate in the research project.

In this research, a total of one hundred and fourteen 5-year-old preschool children from 18 different preschools and ninety-four 6-year-old preschool class children from 5 different schools participated throughout the research project during the fall of 2011 and 2012. The research period consisted of individual pretest sessions, an intervention period over three weeks and individual posttest sessions. During the research period, a few children (six 5-year-olds and two 6-year-olds) did not complete their participation because they were absent for an extended period of time. At the start of the research period we wanted to ascertain that the participating children had been asked about participation and wanted to take part in the research study. Children (participants) themselves have the right to decide about their participation and need to get information about the project (aim, what the study involves, what will happen and for how long) in an appropriate way to be able to make an informed decision about participation (Bogdan & Biklen, 1998; Einarsdóttir, 2007; Morrow & Richards, 1996; Vetenskapsrådet, 2002, 2011). Therefore, we started the research period with giving all participating children the same information as provided in the information letter directed to the children and asked if they wanted to participate. During the research period we also asked each child if he/she wanted to participate before the start of each session since the question about consent is an ongoing

process in every stage of the research process and the child has the right to withdraw from participating at any time (Alderson, 2000; Einarsdóttir, 2007; Flewitt, 2005; Vetenskapsrådet, 2002, 2011). None of the participating children included in the research project chose to intermit their participation.

In relation to the recruitment process, dilemmas can arise when children for example want to participate but their parents/caregivers have not approved (Danby & Farrell, 2004) or when the child wants to stay with its friends and play instead of for example perform a test with the researcher. The researcher needs to be sensitive and listen to the child in these situations and it is important to know how to handle situations like these (Danby & Farrell, 2004). For example, is it really the fact that the child does not want to participate anymore or is it that you interrupt the child and ask the child to come with you at the “wrong” time? In this research project, we informed children about when we were going to visit them each week and tried not to interrupt them if they were in the middle of something. However, most children came directly to us when we visited them and asked when it was their turn to perform tests or play numerical games. At a few occasions children said that they wanted to take the test later or play games after they had finished the activity that they were doing at the moment and at these occasions we were able to meet these wishes. Einarsdóttir (2007) describe that, as an adult, the researcher may have an advantage in the balance of power that is natural in the relation between children and adults. Hence, it is extra important to be attentive and responsive to what children express and to their consent to participate in research.

The ethical principles regarding confidentiality and use (Vetenskapsrådet, 2002, 2011) was followed in this research project. We have kept the identity of the participating children confidential in the process of analyzing the data and presenting the research results. The data has been handled with caution and only the participating researchers have had access to the collected data material that is kept in locked cabinets and to the data files containing results from pre- and posttest sessions. The participants’ identities have been protected and it is not possible to recognize individual children or groups of children included in this research. Further, the collected data material has only been used for research purpose; for this thesis, articles and conference proceedings. Information regarding confidentiality and use of the

research material collected during the project was given to the parents/caregivers and children in the information letters describing the research project.

Overview of the research project

The intervention study and data collection for this research project was performed during the fall of 2011 with children in preschool class and during the fall of 2012 with children in preschool. The author and three pre-service teachers administered all test and game sessions during the fall of 2011, and the author and six pre-service teachers (hereafter called experimenters) administered all test and game sessions during the fall of 2012. Initially, the experimenters received training in test administration and game intervention procedures in workshops and they were instructed to follow the same procedure and use the written instructions during test and game sessions to increase the equivalence during these.

The results presented in each of the four studies included in this thesis are based on children's pre- and posttest performance (Study I and II) or just on children's pretest performance (Study III and IV) on different numerical tasks from the two data collection periods in 2011 and 2012. An overview of the four studies included in this thesis is presented in Table 3, where the time for data collection, the sample, the methods for data collection and the methods for data analysis used in each study are briefly summarized. More specific descriptions of participants, methods for data collection, and methods for data analysis for each study are provided in the summary section of the four empirical studies presented later in this chapter as well as in each of the original studies.

Table 3. *Overview of the four studies included in this thesis. The time for data collection, the sample, the method for data collection and the methods for data analysis are presented.*

	Study I	Study II	Study III	Study IV
Time	Fall 2012	Fall 2011	Fall 2011 Fall 2012	Fall 2011 Fall 2012
Sample	<i>N</i> = 114 preschool children	<i>N</i> = 94 preschool class children	<i>N</i> = 202 children* (111 preschool children and 91 preschool class children)	<i>N</i> = 208 children (114 preschool children and 94 preschool class children)
Method for data collection	Individual performance on pre- and posttests	Individual performance on pre- and posttests	Individual performance on pretest	Individual performance on pretest
Method for data analysis	MANCOVA, ANCOVA, Regression analysis	Repeated measure ANOVA, Regression analysis	Latent class regression analysis, ANOVA	Latent class regression analysis, ANOVA, Regression analysis

* Six children out of the total sample included in the research project did not perform the 0-100 number line task at the pretest session.

Summary of the studies

In the following sections I will present the background and aims, the methods and results briefly for each of the four studies included in this thesis.

Study I:

Playing number board games supports 5-year-old children's early mathematical development.

Study II:

Effects of playing number games on 6-year-old children's number knowledge and skills.

Study III:

Investigating children's number line estimation patterns using a Latent class regression analysis.

Study IV:

Children's number line estimation patterns on a 0-10 number line task.

Study I

Playing number board games supports 5-year-old children's early mathematical development.

Background and aim

Children start to develop mathematical skills at an early age (Fuson, 1992; Gasteiger, 2012; Ramani & Siegler, 2011). Research has shown that there is a variation in children's mathematical skills before formal schooling and that these skills are a strong predictor of later mathematical achievement (Duncan et al., 2007; Hornung et al., 2014; Krajewski & Schneider, 2009). Therefore, it is important to support children's early mathematics learning and development in a conscious way to prepare them for learning of more formal mathematics.

This study is based on the assumption that humans have an innate sense of number (Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010) that constitute the foundation for later acquiring the symbolic number system with both verbal counting words and written Arabic numerals (von Aster & Shalev, 2007). To be able to construct, automatize and extend number knowledge and skills, children must link their understanding about quantities with the symbolic number system (von Aster & Shalev, 2007) and understand that number and space are closely linked. In this, the mental number line seems to be an important structure for mathematics learning and development (e.g., von Aster & Shalev, 2007) and therefore it should be a goal in (early) education to provide children with the best possible conditions to develop functional internal representations of numerical magnitude (Ramani & Siegler, 2008; Siegler & Booth, 2004). Siegler and Booth (2004) hypothesized that playing linear number board game could support children's development of linear representation of numerical magnitude. Positive effects of playing linear number board games on children's representation of numerical magnitude and development of number knowledge and skills have been presented (Laski & Siegler, 2014; Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008).

The aim of this study was to examine effects of playing different number games with focus on Swedish 5-year-old children's development of early number knowledge and skills. Two hypotheses

and one research question were addressed. The first hypothesis was that children playing linear number board games should develop a more linear representation of numerical magnitude. The second hypothesis was that all numerical activities should have a positive effect on children's counting ability and ability to name written Arabic numerals. The research question addressed in this study was whether playing number board games would result in a positive effect on children's early arithmetic ability or not.

Method

Participants

In this study, one hundred and fourteen 5-year-old preschool children (61 girls, 53 boys, $M_{age} = 5.37$ years, $SD = 0.31$) participated. Children were assigned to four different intervention groups: a linear number board game group ($N=30$), a circular number board game group ($N=24$), a nonlinear numerical activities group ($N=27$) and a passive control group ($N=33$). A few children were absent for an extended period during the intervention, resulting in a small variation in the number of children in the different groups. The preschools where children were recruited varied with respect to location, size, number of groups on the preschools and the number of 5-year-old children going there. Due to this, it was not possible to randomize children within the same preschool group to all four intervention groups included in this study. Instead, each preschool was randomly assigned to one of the four conditions, meaning that all children at the same preschool were included in the same game intervention group.

Materials and procedure

The study consisted of pretest sessions, a three-weeks intervention period consisting of six game sessions and posttest sessions, all taking place at the respective preschools in an environment familiar to the children. After the initial individual pretest was performed, children assigned to one of the three game conditions (i.e., linear number, circular number, nonlinear numerical) met, in pairs, with the experimenter for six 10-min sessions during a period of three weeks (twice a week) and played games together with one peer. Each child performed an individual posttest after the game intervention period approximately four weeks after the initial pretest.

Interventions

The game interventions used in this study were designed as follows.

Linear and circular number board games: Children in the linear and circular number board game groups played the same type of game (i.e. linear or circular; see illustration in Table 4) during each of the six game sessions but within different numerical ranges during these (see Table 4). When playing the linear or circular number board game, children were asked to take turn and throw a dice with numerals 1-3 and move their token forward on the game board the number of steps that the dice showed, and at the same time name the numerals in the squares/circle sectors that they passed out loud (count-on-procedure). The child that first reached the square/circle sector marked “Finish” won the game.

Table 4. *Illustration of the design of the linear number board game and the circular number board game and presentation of the numerical ranges for the game boards used during each of the six game sessions during the intervention period.*

Game	Session					
	1	2	3	4	5	6
Linear number board game	1–10	1–20	1–30	1–30	1–40	1–40

Circular number board game	1–10	1–20	1–30	1–30	1–40	1–40

Nonlinear numerical activities: For children in the nonlinear numerical activities group, each of the six intervention sessions consisted of two

parts: counting activities and playing nonlinear number games. The activities for each session is summarized in Table 5.

Table 5. *Counting activities and nonlinear number game activities used in the nonlinear numerical activities group during each of the six sessions through the intervention period.*

Activities	Session					
	1	2	3	4	5	6
<i>Part 1</i>						
Rote counting	1–10	1–20	1–30	1–30	1–40	1–40
Counting forward	3–10	5–11	8–20	10–22	15–30	19–31
Counting backwards	4–0	6–0	9–3	11–6	16–8	21–14
Naming Arabic numerals	1–10	1–10	1–10			
Counting objects				1–10	1–10	1–10
<i>Part 2</i>						
Number game (nonlinear)	Memory	Memory	Bingo	Bingo	Go fish	Go fish
	1–10	1–10	1–10	1–10	1–10	1–10

Passive control group: Children included in the passive control group participated in the regular preschool activities during the intervention period and only met one-to-one with one experimenter to perform pre- and posttest.

Pre- and posttest measures

Children performed a *0–10 number line task* (number-to-position) and estimated the positions of the numerals between 1 and 9 twice on 18 separate number lines and the numerals were presented in random order. Children also *counted forwards and backwards* (3–8; 7–12; 14–19; 18–24; 26–32; 47–53; 90–94; 96–102; 4–0; 5–0; 8–4; 10–6; 12–8; 15–9; 18–13 and 23–17.) and *named Arabic numerals* between 1–99 (1–9, 10–20, and 9 numerals between 21–99). They also solved *verbal arithmetic* tasks (3 + 1; 4 + 2; 2 + 2; 8 + 1; 7 + 2; 4 + 3; 4 + 4; 6 + 3; 5 + 4; 10 + 3; 10 + 6; 13 + 1; 17 + 2; 11 + 5; 12 + 7; 6 + 4; 8 + 2; 6 + 6; 8 + 5; 7 + 4; 9 + 6; 3–1; 5–2; 6–1; 8–2; 6–3; 5–4; 8–3; 7–4; 10–4; 10–7; 13–1; 16–3; 18–2; 17–4; 11–4; 16–6; 12–5; 15–6; 14–8 and 17–9) that were presented orally in the form “If you have N_1 cookies/buns and I have N_2 cookies/buns, how many cookies/buns do we have

together?” or “If you have N_1 cookies/buns and then you eat N_2 cookies/buns, how many cookies/buns do you have left?”, where N_1 and N_2 represent the two numerals in the arithmetic tasks. Children received no specific feedback during the test sessions except of general praise and encouragement.

Results and discussion

The results from the intervention study revealed that playing number games supported 5-year-old children’s early mathematics learning and development. Our results revealed that depending on intervention, the learning outcome was affected in different ways. Children who played the linear number board game improved their performance on the number line task significantly more as compared to the other groups. This finding is consistent with previous findings, where positive effects have been obtained for children playing linear number board games (Ramani & Siegler, 2008, 2011; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008). The frequency of children whose estimates were best described by a linear function compared to a logarithmic function increased for the linear number board game group post intervention while there were no or very small differences post intervention for children in the other three groups. With respect to the results on the number line task, our results support the representational mapping hypotheses (Siegler & Ramani, 2009) stating that in order to support children’s development of a (more) linear representation of numerical magnitude, number board games that are transparently mapped to the desired mental number line representation should be used.

On the verbal arithmetic task children who played the linear number board game improved their results significantly more compared to children in the passive control group post intervention. The linear number board game group enhanced with 4.94 points between pre- and posttest while the other groups showed changes between -1.45-1.58 points. Earlier research has presented similar results as found in this study, showing that children who had played linear number board games were supported in their ability to solve simple arithmetic problems (Ramani & Siegler, 2011; Siegler & Ramani, 2009).

Children who played the circular number board game improved their performance on the naming Arabic numerals task significantly more as compared to children in the nonlinear numerical activities

group and the passive control group. This result is somewhat surprising since children in both the linear and circular number board game groups were exposed to written numerals within the same numerical range for approximately the same amount of time and therefore there was no particular reason to predict that the circular number board game group should improve more on this task than children in the linear number board game group. Children in the linear number board game group also improved their performance on this task, but not to the same extent as children in the circular number board game group.

Further, children in all three game intervention groups improved their performance on the count forwards and backwards task. However, children in the nonlinear numerical activities group improved significantly more as compared to the other three groups. This result is not so surprising since counting forwards and backwards were activities that children in the nonlinear numerical activities group were exposed to during the intervention sessions.

Study II

Effects of playing number games on 6-year-old children's number knowledge and skills.

Background and aim

Research have shown that several foundational skills such as subitizing, quantity comparison, estimation and representation of numerical magnitude seem to underlie children's development in mathematics and learning of arithmetic (e.g., Berch, 2005; Dehaene, 2011; von Aster & Shalev, 2007). Children's ability to represent numerical magnitude in a linear fashion have been found to be a strong predictor of mathematical skills and arithmetic performance at school start (Booth & Siegler, 2006; Siegler & Booth, 2004). Given this, it has been suggested that children should be given opportunities to participate and engage in numerical activities that supports their number knowledge and mathematical skills (e.g., Gersten et al., 2005; Siegler & Booth, 2004). Since a linear representation of numerical magnitude seem to play an important role in children's mathematics learning and development (von Aster & Shalev, 2007; Ramani & Siegler, 2008; Siegler & Booth, 2004), activities that support construction of a (more) linear representation of number should be used (Gersten et al., 2005; Ramani & Siegler, 2008; Siegler & Booth, 2004; Siegler & Ramani, 2009). Siegler and Booth (2004) hypothesized that linear number board games could be used to support children's development of a more linear representation of numerical magnitude. Siegler and colleagues have designed simple linear number board games and used these in a number of studies and their research show that children can improve their number knowledge and skills and not least, their representations of numerical magnitude playing linear number board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; see also Whyte & Bull, 2008).

The aim of this study was to examine effects of playing number games (i.e., linear number, circular number, nonlinear number) on Swedish 6-year-old children's number knowledge and skills. Based on previous research, two hypotheses were formulated. The first hypothesis was that children in the linear number board game group would develop a more linear representation of numerical magnitude compared with children in the other two game conditions. The second

hypothesis was that children who played number games would develop their counting skills and ability to name Arabic numerals. Further, we also examined effects of playing number games on children's ability to solve verbal arithmetic problems.

Method

Participants

Ninety-four 6-year-old children (50 girls, 44 boys, $M_{age} = 6.36$ years, $SD = .32$) participated in this study. The participating children were recruited from five schools (9 different preschool classes) and children within each preschool class were randomly assigned to one of the three different game intervention groups: a linear number board game group ($N = 32$), a circular number board game group ($N = 30$) and a nonlinear numerical activities group ($N = 32$).

Materials and procedure

The study consisted of a pretest, a game intervention period and a posttest. At pre- and posttest, children met individually with an experimenter and performed different tasks focusing on number knowledge and skills. During the intervention period, children played number games together with their peers in smaller groups (3-4 children) for twenty minutes twice a week during three weeks.

Interventions

In this study children were assigned to three different numerical game intervention groups and they were designed as follows.

Linear and circular number board games: For a presentation of the linear and circular number board games and the game playing procedure used in this study, see description in the summary of Study I under the heading *Linear and circular number board games*.

Nonlinear numerical activities: For a description of the nonlinear numerical activities used in this study, see description in the summary section of Study I under the heading *Nonlinear numerical activities*.

Pre- and posttest measures

Children met individually with an experimenter and performed different tasks pre- and post the number game intervention period. Children performed a *0-10 number line task* (number-to-position) where they were asked to estimate the position of the numerals between 1 and 9 two times on 18 separate number lines (the numerals were presented in random order), a *rote counting task* where children were asked to start to count from 1 and continue to count as far as possible (maximum limit 100), a *naming Arabic numerals task* where children were asked to name the numerals 1-9, 10-20 and nine numerals between 21-99, and a *verbal arithmetic task* where children were asked to give answers to addition problems ($3+1$; $4+2$; $2+2$; $8+1$; $7+2$; $3+5$; $6+4$; $5+6$; $7+8$; $9+12$; $13+12$; $14+11$; $12+13$; $14+13$; $17+15$; $13+19$; $23+18$; $21+17$) presented verbally in the form “If you have N_1 cookies/buns and I have N_2 cookies/buns, how many cookies/buns do we have together?”, where N_1 and N_2 represent the numerals in the addition tasks.

Results and discussion

The results from this research revealed no significant interaction effect, nor any main effect of time with respect to children’s performance on the 0-10 number line task (number-to-position). That no effects were found could be explained by the fact that the 6-year-old children gave rather accurate estimates on the number line task already at pretest. Children’s performance on pretest gave them limited opportunities to improve their performance on the 0-10 number line task post intervention.

The analysis of children’s performance on the rote counting task, the naming Arabic numerals task, and the verbal addition task revealed no significant interaction effects. Although, significant main effects of time were found for all three tasks which indicate that children in all three game intervention groups improved their performance post intervention. There were larger effect sizes for the rote counting task and the naming Arabic numerals task compared to the verbal addition task. One possible explanation for these results could be that the different game activities used in this study more explicitly focused on counting and number naming than arithmetic. Our findings are consistent with previous studies showing that children’s counting

ability and ability to name Arabic numerals can be improved participating in number games activities (Laski & Siegler, 2014; Ramani & Siegler, 2008; Whyte & Bull, 2008).

No passive control group or active non-numerical control group was included in this study, and this limits the conclusions that can be drawn regarding effects of playing number games on 6-year-old children's learning and development of number knowledge and skills. It is not possible for us to reject the fact that the main effects revealed could be a result of ordinary activities at the schools, test-retest effects or special treatment effects.

Study III

Investigating children's number line estimation patterns using a Latent class regression analysis.

Background and aim

The linearity of children's representation of numerical magnitude has in earlier research been described as important for future learning in mathematics (e.g., Laski & Siegler, 2014; Siegler & Booth, 2004; von Aster & Shalev, 2007). Number line tasks have been used to assess children's representations of number and children's number line placements have been shown to correlate with their mathematics achievement test scores (Booth & Siegler, 2006; Schneider, Grabner, & Paetsch, 2009; Siegler & Booth, 2004; Siegler, Thompson, & Schneider, 2011).

Siegler and colleagues have used number line tasks in several studies in order to describe children's representations number (e.g., Booth & Siegler, 2006, 2008; Ramani & Siegler, 2008; Ramani et al., 2012; Siegler & Opfer, 2003; Siegler & Ramani, 2009). Based on findings from their research, they describe that children's representations of numerical magnitudes develop from being logarithmically compressed to become linear with increasing age and experience (e.g., Booth & Siegler, 2006; Siegler & Opfer, 2003). When analyzing children's estimation patterns on number line tasks, Siegler and colleagues have mainly used group median estimates and predefined functions (i.e., linear, logarithmic, and exponential) to describe children's representations. However, this way of analyzing and describing children's number line estimations have been questioned and researchers have presented alternative ways of analyzing and describing children's number line estimates (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Ebersbach et al., 2008; Moeller et al., 2009). In a previous work, Bouwmeester and Verkoeijen (2012) highlight five methodological issues regarding the more conventional way of analyzing children's number line estimations and they suggest that a latent class modeling approach (Bouwmeester et al., 2004; Bouwmeester & Verkoeijen, 2012; Vermunt & Magidson, 2005; Wedel & DeSarbo, 1994) could be used as an alternative way of analyzing children's number line estimations.

A latent class regression analysis can be used to study differences in children's estimation patterns and makes it possible to identify latent classes (subgroups) of children in which children are similar to each other in their patterns of responses on the number line task (Bouwmeester & Verkoeijen, 2012). A latent class regression analysis can be described as a tool for identifying unobserved heterogeneity that makes it possible to distinguish differences and characteristics in children's estimations of number on number line tasks. The responses of all participants to all items are analyzed and groups of children who show similarities in their responses will be identified. A latent class modeling approach is an exploratory analysis and different criteria can be used to decide the number of latent classes that captures the heterogeneity and characteristics in the data material (Bouwmeester et al., 2004; Bouwmeester & Verkoeijen, 2012; Ding, 2006; Vermunt & Magidson, 2005). Log-likelihood statistics (e.g. LL and BIC), class sizes, classification errors and substantive considerations can influence the choice of what model to present (Bouwmeester et al., 2004; Ding, 2006).

The aim of the present study was to investigate children's estimation patterns on a 0-100 number line task (position-to-number). In this study, special attention is directed towards the unobserved heterogeneity and characteristics in children's number line estimation patterns and a latent class regression analysis have been used to analyze children's number line estimations. Further, the counting ability and ability to name Arabic numerals for children in different latent classes were investigated in this study.

Method

One hundred and eleven preschool children (65 girls, $M_{age}= 5.35$ years, $SD=.30$) and 91 preschool class children (49 girls, $M_{age}= 6.36$ years, $SD=.32$), in total 202 children (114 girls, $M_{age}= 5.80$ years, $SD=.59$) participated in this study. Children performed a *0-100 number line task* (position-to-number) where they were asked to name numerals that they thought corresponded to marked positions on 16 number lines. The marked positions presented on the number lines that children were asked to give answers to in this study corresponded to the actual position of 3, 4, 6, 8, 15, 23, 27, 29, 33, 48, 53, 58, 66, 73, 85 and 90 and were presented in random order on separate number lines.

Children also performed a *number word list counting* task and were asked to start to count from 1 and continue to count as far as he or she could (maximum limit 100). Children also performed a *naming Arabic numerals* task and were asked to name numerals between 1-99 (1-9, 10-20 and nine numerals between 21-99).

Results and discussion

When we analyzed children's performance on the 0-100 number line task (position-to-number) using a latent class regression analysis, models with 1-7 classes was investigated. The model consisting of 6 classes had the lowest BIC values and could be suggested to be the best fitting model (Bouwmeester et al., 2004; Ding, 2006). However, after substantive considerations, inspection of class sizes, classification errors and graphs for the estimated models, the model with four classes was selected to describe the heterogeneity and characteristics in children's number line estimations. Scrutinizing the four classes in our model, differences was found with respect to the regression curves describing each latent class (subgroup). The estimation pattern for children in class 1 ($N=129$) in our model could be described as being somewhat linear but within a limited numerical range and with a slope far from 1. Most of the answers given by children in this class were in the numerical range between 0 and 40. This indicate that children in this class only used one reference point (i.e., 0) when making their estimates. Class 2 ($N=50$) was described by a regression curve with a gentle S-shape around the reference line (i.e., $y=x$), indicating that children in this class gave rather accurate estimates on the number line task. The estimates between 10 and 50 given by children in this class tended to be underestimated and children's estimations of larger numbers (>50) showed a large spread. Children's estimates close to the endpoints on the number line (i.e., 0 and 100) was rather accurate which might indicate that children in this class were able to use these as referents when giving their answers. The regression curve describing the estimation patterns for children in class 3 ($N=15$) was rather similar to the curve describing the estimation pattern for children in class 1. However, class 3 differ from class 1 with respect to the numerical range in which the answers were given. Children in class 3 gave their answers in the numerical range between 0 and 25 while children in class 1 provided most of their answers between 0 and 40.

Class 4 ($N=8$) is the smallest class in the latent class regression model. Children in this class provided most of their answers in the numerical range between 0 and 20. Taking a closer look at the individual estimates given by children in this class it could be seen that children's answers were limited to a few numerals on all tasks.

Children within the four latent classes in our model did not only differ with respect to estimation patterns, they also differed with respect to counting ability and ability to name Arabic numerals. The analyzes showed that children in class 2 performed significantly better on the verbal counting task and the naming Arabic numerals task compared to children in the other three classes indicating that children's familiarity with numbers are related to children's performance on the 0-100 number line task (position-to-number).

The results from our study showed that there is a heterogeneity in 5- and 6-year-old children estimation patterns on the 0-100 number line task (position-to-number). Children's estimation patterns were described by different regression curves that not only could be described as being logarithmically compressed or linear. Our findings strengthen the statement that a latent class regression analysis provides a more nuanced picture of children's estimation patterns (see also Bouwmeester & Verkoeijen, 2012) as compared to using group median estimates and predefined functions (i.e., logarithmic, linear, exponential) in order to describe children's estimation patterns (Booth & Siegler, 2006, 2008; Siegler & Opfer, 2003).

Study IV

Children's number line estimation patterns on a 0-10 number line task.

Background and aim

In the last decade, children's number line estimations have attracted intense research interest (e.g., Barth & Paladino, 2011; Booth & Siegler, 2006; Bouwmeester & Verkoeijen, 2012; Ebersbach et al., 2008; Moeller et al., 2009; Siegler & Booth, 2004; Siegler & Opfer, 2003). To analyze and describe number line estimations, group median estimates are commonly employed (e.g., Booth & Siegler, 2006; Siegler & Opfer, 2003) and predefined functions have been used to describe children's representations of number as being logarithmically compressed or linear (e.g., Siegler & Opfer, 2003; Booth & Siegler, 2006; Whyte & Bull, 2008). However, when using group median estimates, variability is assumed to be errors rather than seen as giving important information regarding children's individual estimation patterns, thus yielding oversimplified descriptions of children's estimation patterns (Bouwmeester & Verkoeijen, 2012). The use of predefined functions have also been questioned since it is possible that other (less restrictive) functions could provide better descriptions of children's number line estimations (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). In a previous work (Bouwmeester and Verkoeijen, 2012), it has been suggested that a latent class regression analysis (Bouwmeester et al., 2004; Bouwmeester & Verkoeijen, 2012; Ding, 2006; Vermunt & Magidson, 2005; Wedel & DeSarbo, 1994) can overcome these issues by analyzing individual children's number line estimations and identify unobserved latent classes (subgroups) of children showing similar estimation patterns (Bouwmeester & Verkoeijen, 2012). This analysis approach was utilized in the present study to investigate and describe 5- and 6-year-old children's number line estimation patterns on a 0-10 number line task (number-to-position). The main purpose was to investigate if multiple representations of number on the number line task could be distinguished with respect to individual children's estimations of number. Further, the performance on three different numerical tasks (number word list counting, naming written Arabic numerals, and

verbal addition) were investigated for children in the different latent classes.

Method

Participants were 208 Swedish preschool and preschool class children (110 girls, 98 boys, $M_{age} = 5.82$ years, $SD = 0.58$ years). Each child met individually with an experimenter and performed different tasks: a paper pencil version of a *0–10 number line* task (number-to-position) where the child was asked to estimate the position of the numerals between 1 and 9 (presented in random order) two times on 18 separate number lines, a *number word list counting* task where the child was asked to count from 1 and onwards as far as he or she could (maximum limit 100), a *naming Arabic numerals* task where the child was asked to name the numerals between 1 and 9 presented in random order, and a *verbal arithmetic* task where the child was asked to give answers to addition problems (3+1; 4+2; 2+2; 8+1; 7+2) presented orally in the form “If you have N_1 cookies/buns and I have N_2 cookies/buns, how many cookies/buns do we have together?”, where N_1 and N_2 represents the numerals in the addition tasks. No feedback regarding children’s performance was given during the test session, only general encouragement.

Results and discussion

The latent class regression model with 9 classes had the lowest BIC value and could be suggested to be the best fitting model (Bouwmeester et al., 2004; Ding, 2006). However, when taking a closer look at classification errors, class sizes and graphs for the estimated models, the latent class regression model consisting of seven classes (subgroups) was selected to describe the heterogeneity and characteristics in children’s number line estimates on the 0-10 number line task. The results revealed that children within the same age group varied and showed accurate (i.e., accurate linear) as well as inaccurate estimation patterns on the number line task. Class 1 ($N=61$) showed a rather accurate linear relationship between estimated positions and the actual numbers presented and where described by a rather linear function (i.e., $y=x$), while class 2 ($N=45$) consisted of children that estimated numbers randomly on the number line and where described

by an almost horizontal regression curve indicating no relationship between the numbers presented and children's number line estimations. In two of the subgroups (class 3 and 5) in the latent class regression model, children underestimated all numbers 1–9 (class 3, $N=34$) or numbers between 1–7 (class 5, $N=23$). Class 4 ($N=27$) were described by a regression curve that was almost accurate linear, but when taking a closer look at children's estimations it could be seen that children in this class gave rather accurate estimates close to the marked endpoints (i.e., 0 and 10) while their estimates in-between varied a lot, indicating that children might have used the endpoints as references when making their estimations. Class 6 ($N=11$) in the model were described by a regression curve indicating that there was no relationship between children's estimations and the actual number being presented. Children in this class gave all their estimates within the numeral range between 4 and 6. The smallest class (class 7, $N=7$) included children who provided all their estimations between the actual positions of number 7 and 10 irrespective of what number that were presented. Further, the results also revealed that children showing more accurate linear estimation patterns (i.e., class 1 and 5) also performed better on the three numerical tasks (i.e., number wordlist counting, naming Arabic numerals, and verbal addition).

The result from the latent class regression analysis is to be compared to conventional regression analysis where only the median child is taken into account and where predefined functions are fitted to describe children's number line estimations. When using this approach in this study it was found that children's estimates were best described by a linear function as compared to a logarithmic function. This work presented in this study contributes towards understanding children's early mathematics development and aids to the discussion about the number line task and how to analyze and interpret children's number line estimations. Based on the results from this study it can be concluded that the latent class regression analysis captures nuances in children's estimation patterns and generates interesting information that would have been missed if averaging across data and using the more conventional way of analyzing children's number line estimations.

CHAPTER 5

General discussion

In this final chapter I will discuss the main research findings from the four studies included in this thesis in relation to current theories and previous research on children's early mathematics learning and development. Limitations and implications will be discussed and suggestions for future research will be presented.

Main research findings

The overall aim of this thesis has been to contribute to the understanding of young children's early mathematics learning and development by examining effects of playing different number games on children's number knowledge and skills, and by investigating children's representations of numbers on number line tasks. Two main research questions have been addressed; 1) How can young children be supported in their learning and development of number knowledge and skills by playing number games? and 2) How can children's representations of number on number line tasks be described and interpreted? In the first two studies (Study I and II) the first main research question has been focused and effects of playing number games on 5- and 6-year-old children's number knowledge and skills has been investigated. In Study III and IV, the second main research question has been addressed and children's number line estimations on two different number line tasks has been investigated.

The findings presented in Study I and II contribute to our understanding of what kind of activities that supports children's number knowledge and skills. Our results revealed that playing different number games affect 5-year-old children's learning in different ways, with respect to performance on the 0-10 number line task (number-to-position), the verbal arithmetic task, the naming Arabic numerals task and the counting forwards and backwards task. Playing linear number board games improved 5-year-old children's performance on the number line task significantly more as compared to playing circular number board games, participating in nonlinear numerical activities or participating in the regular preschool activities.

In contrast to the results on the number line task for the 5-year-old children, no significant effect was found with respect to performance on the number line task for the 6-year-old children in any of the game intervention groups post intervention. This could be explained by the fact that the 6-year-old children gave rather accurate linear estimations already at pretest, giving them limited opportunities to improve their performance on this specific task. The 5-year-old children who played the linear number board game also improved their performance on the verbal arithmetic task with 4.94 points between pre- and posttest. This could be compared with the other groups that showed changes between -1.45 and 1.58 points. The post hoc test revealed that the linear number board game group improved their performance significantly more on this task than children in the passive control group.

The 5-year-olds that played the circular number board game improved their performance on the naming Arabic numerals task significantly more than children in the nonlinear numerical activities group and the passive control group. Children in the linear number board game group also improved their performance on this task but not to the same extent as children in the circular number board game group. Further, the 5-year-old children who played number games during the intervention period improved their performance on the count forwards and backwards task. However, children in the nonlinear numerical activities group improved their performance significantly more as compared to the other three groups. The 6-year-olds participating in the three different game intervention groups improved their performance on the rote counting task, the naming Arabic numerals task, and the verbal addition task irrespective of game condition. Our analyzes revealed significant main effects of time on all three tasks.

The main findings presented in Study III and IV contribute to our understanding of children's representations of number on two different number line tasks. Our results revealed that children's estimation patterns vary and that there is a heterogeneity in 5- and 6-year-old children's representations of numbers on number line tasks. Subgroups of children showing different estimation patterns were distinguished in both studies. When analyzing children's estimation patterns on the 0-100 number line task (position-to-number), a model consisting of four latent classes were selected to describe children's number line

estimations. Class 1 in the model could be described as being somewhat linear but with a slope far from 1. Most of the answers provided by children in this class were given in the numerical range between 0 and 40. Class 2 was described by a regression curve with a gentle S-shape located around the reference line (i.e., $y=x$). Children's estimations of number between 10 and 50 tended to be underestimated while estimates close to the endpoints (i.e., 0 and 100) on the number line was rather accurate. The estimation pattern for class 3 was rather similar to class 1, but these two classes differ with respect to the numerical range in which the answers were given by the children (i.e., 0-25 as compared to 0-40). Children in class 4 provided most of their answers in the numerical range between 0 and 20, and children's answers were limited to a few numerals on all number line tasks. Children in the four classes did not only differ with respect to the regression curves describing their estimation patterns, they also differed with respect to counting ability and ability to name Arabic numerals. Class 2 performed significantly better on both tasks as compared to the other three classes. That is, children in class 2 counted further than children in the other classes and were also able to name more written Arabic numerals between 1 and 99.

When analyzing children's estimation patterns on the 0-10 number line task (number-to-position), a model with seven latent classes were selected to describe the heterogeneity and characteristics in children's number line estimation patterns. Class 1 in the model were described by a rather accurate linear regression function (i.e., corresponded close to $y=x$), meaning that children in this class estimated the position of the presented numerals rather accurate on the number lines. Class 2 include children that estimated numbers randomly on the number lines. The regression curves for class 3 and class 5 indicate that children in class 3 underestimated all numbers between 1-9 and that children in class 5 underestimated numbers between 1-7. Children in class 4 gave rather accurate estimates close to the endpoints (i.e., 0 and 10) while their estimates between 3-6 did not correspond closely to the actual numbers being presented. Children in class 6 gave all estimates around the midpoint on the number lines, and children in class 7 gave all their estimates between the actual location of number 7 and 10. Further, our findings indicate that children in classes with more linear estimation patterns (i.e., class 1 and 5) performed better on the three numerical

tasks (i.e., number wordlist counting, naming Arabic numerals, and verbal addition) investigated.

In study IV, we also analyzed children's number line estimates on the 0-10 number line task using group median estimates and predefined functions (i.e., logarithmic and linear) were fitted to describe children's estimation patterns. It was found that children's estimation patterns were best described by a linear function as compared to the logarithmic function. However, in relation to the results from the latent class regression analysis, it became clear that a lot of information regarding children's number line estimations was lost when averaging across data and using predefined functions to describe their estimations. Our findings in Study III and IV could be interpreted as indicating that children are affected by different aspects such as task specific properties (e.g., endpoints) and number knowledge when making estimates on number line tasks, as will be further discussed later in this chapter.

In sum, the research findings presented in this thesis add to previous research on effects of playing number games with children to support their learning and development of number knowledge and skills, and to the literature on and the discussion about children's estimations of number on number line tasks and how these could be analyzed and interpreted. In the following sections, the main findings outlined above will be discussed in relation to current theories and previous research on children's mathematics learning and development. The discussion consists of two parts. In the first part I will focus on the results from the two game intervention studies and after that I will discuss the main research findings from the two studies focusing on investigating the heterogeneity and characteristics in children's estimates of number on number line tasks.

Effects of number game interventions

The effects of the number game interventions used in Study I and II will be discussed with respect to the development of children's number knowledge and skills. I will discuss the development of children's number line estimations and their development of counting skills, ability to name Arabic numerals and ability to solve verbal arithmetic tasks.

Development of children's number line estimations

When analyzing the effects of the three game conditions (i.e., linear number board game, circular number board game, and nonlinear numerical activities) on 5-year-old preschool children's number line performance it was found, in consistence with the representational mapping hypothesis (Siegler & Ramani, 2009), that children in the linear number board game group improved their performance on the 0-10 number line task significantly more as compared to children in the three other groups. Similar results have been found in previous game intervention studies (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008). However, the development of the 5-year-old children's number line estimation performance found in our study was not of the same magnitude as previously reported (see e.g., Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008). In previous research, greater development on the number line task has been reported post intervention for children playing linear number board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008). The differences in results may be explained by the sample selection and children's precursory number knowledge and skills. When we recruited preschool children for participation in this research, all children whose parents/caregivers gave written consent where eligible for participation, and 114 five-year-olds where included in the study. These children were then admitted to the different intervention groups, resulting in training groups covering a range of prior number knowledge and skills among children. This is to be compared with previous studies using linear number board games where mainly children with confined initial number knowledge and skills have been included (e.g., from low-income families in the US, see Ramani & Siegler, 2008, 2009; Siegler & Ramani, 2008).

The results in study II showed an increase in the number of 5-year-old children whose estimates were best described by a linear function instead of a logarithmic function post intervention for children in the linear number board game group. This result is consistent with previous research findings showing that the frequency of children who's estimates where best fit by a linear function (and not a logarithmic) increased for children who had played linear number board games but not for children who had played linear color board

games or participated in nonlinear numerical activities (Whyte & Bull, 2008). This result also give support to the representational mapping hypothesis: the greater transparency between the physical game and the desired internal representation is, the greater learning of the desired representation will occur (Siegler & Ramani, 2009; see also Ramani & Siegler 2008). Thus, the design of the number board games seems to be an important aspect to consider when deciding on what games to use in order to support young children's development of more linear representations of numbers.

In contrast to the results on the number line task presented in Study I (5-year-olds), no significant effects were found on the number line task in Study II (6-year-olds) for any of the intervention groups (i.e., linear number board games, circular number board games, and nonlinear numerical activities). When analyzing the performance of the 6-year-olds on the 0-10 number line task it became evident that the group of children included in this study provided rather accurate linear estimates on the number line task already at pretest, which gave them very limited opportunities to improve their performance on this specific task post intervention. The results indicate that the linear number board game was not strong enough to support refinements of children's representations of number on the number line task. It is possible that the number line training in form of a linear number board game as used in this intervention study mainly is beneficial for children with lower initial number knowledge and skills and limited experiences of number (Ramani & Siegler, 2008).

The results from Study I on the number line task indicate that the design of the number game have impact on children's performance on the number line task. In a recent study, Laski and Siegler (2014) also call attention to the counting procedure used playing number board games. Laski and Siegler (2014) highlight the importance of the count-on procedure when playing number board games, a procedure that differ as compared to the way board games are usually played using the ordinary count-from-1 procedure (Laski & Siegler, 2014; see also Ramani & Siegler, 2008; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008). Laski and Siegler's (2014) research showed that the count-on procedure promoted greater encoding of numerals' spatial positions on the game board among children as compared to the count-from-1 procedure while paying the same number board game. The

count-on procedure was also used in our research, but how this counting procedure affected children's learning and development was not an issue focused in our study. Nevertheless, based on the results from Laski and Siegler's (2014) study it is reasonable to believe that both the design of the games and the count-on procedure used in this research had positive effects on children's learning and development, helping them to focus on the numerals while playing the number board game.

Development of counting skills, ability to name Arabic numerals and ability to solve verbal arithmetic tasks

It has been found that children who use the count-on procedure when playing linear number board games improve their ability to name written Arabic numerals and their count-on-skills (Laski & Siegler, 2014). The results from Study I in this thesis partially strengthen these results. In our investigation of 5-year-old children development of number knowledge and skills, we found that children in the circular number board game group improved their performance on the naming Arabic numerals task significantly more as compared to the nonlinear numerical activities group and the passive control group. Also, children in the linear number board game group improved their performance but not to the same extent as children in the circular number board game group. The fact that children in the circular number board game group showed larger improvement on the naming Arabic numbers task than children in the linear number board game group is surprising since both groups used the same counting procedure (i.e., count-on) while playing the game and were exposed to written Arabic numerals in the same numerical range (1-40) for approximately the same amount of time during the game intervention. In order to understand this result further research is needed.

An important finding in Study I was that the 5-year-old preschool children in the linear number board game group improved their performance on the arithmetic task more between pre- and posttest than children in the other three groups. This result corroborates previous findings showing that 4- and 5-year-olds playing linear number board games improve their ability to give correct answers to simple arithmetic problems (Siegler & Ramani, 2009). One plausible explanation for this result could be that playing linear number board

games supports children's representations of number on the mental number line and help them to maintain equal spacing between numbers and organize numerical information around number's magnitudes as a result of their game playing experience, which would help them in their calculation of answers to arithmetic problems (Ramani & Siegler, 2011; Siegler & Ramani, 2009). Von Aster and Shalev (2007) describe the number line as an important structure for arithmetic thinking. The symbolic number system with verbal counting words and written Arabic numerals as representations for quantities are important precursory knowledge for the construction of a mental number line as a spatial image of numbers important for arithmetic thinking (von Aster & Shalev, 2007). The linear number board game used in this research focus on aspects included in step 2, 3, and 4 in the developmental model presented by von Aster and Shalev (2007) while the other game activities used in the training sessions mainly focus on aspects of step 2 and 3 in the model. It is possible that the design of the linear number board game and the count-on procedure used when playing the game gave children experiences of ordinal and spatial relations among numerals and that children could use these experiences when solving the verbal arithmetic tasks after the game intervention.

When analyzing the effects of the three game conditions on 6-year-old preschool class children's performance on the counting task, the naming Arabic numerals task, and the verbal addition task, no significant interaction effects were found. That is, no game condition was found to support children's learning more than the other. However, significant main effects of time were revealed on all three measures. These results show that independent of game condition, children improved their performance on these three tasks post intervention. As described above, the games used in this research did not explicitly focus on arithmetic, while counting and number naming was part of the training in all three game conditions. The training of counting and number naming during the game sessions could explain why we get larger effects on the counting task and the naming Arabic numerals task than on the arithmetic task. However, an important issue to highlight in relation to the results presented in Study II is that no active non-numerical control group or passive control group was included in the research design. Ideally, in addition to the three game intervention groups, the study design should also have included an

active non-numerical control group and/or a passive control group in order to make it possible for us to fully reject the possible explanation that the main effects of time could be a result of the ordinary activities at the schools, test-retest effects or special treatment effects. However, our findings are in line with previous findings showing that children's counting ability and ability to name Arabic numerals can be improved by playing number games during a limited amount of time (Laski & Siegler, 2014; Ramani & Siegler, 2008; Whyte & Bull, 2008).

Summary

The findings from the two number game intervention studies presented in this thesis add to the body of research suggesting that engaging in number game activities can support children's mathematics learning and development, at least among preschool children. The results demonstrate that playing number games supports young children's development of number knowledge and skills. Depending on what game and activities that are used, the learning outcome seem to be affected in different ways and support children's development of different skills to different degrees. However, further research is needed to increase our knowledge and understanding of how children can be supported in their early mathematics learning and development by playing number games, as further discussed later in this chapter.

Children's number line estimation patterns

Children's representations of number on number line tasks will be discussed with respect to diversity and characteristics in children's number line estimations and with respect to different aspects that may affect children's number line estimations.

Diversity in children's number line estimations

Within the research field on children's early mathematics learning and development, there is an ongoing discussion of how to analyze, understand, and interpret children's estimations of number on number line tasks (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Rouder & Geary, 2014). When investigating children's number line estimation patterns on two different number line tasks (i.e., a 0-100 position-to-number task and a 0-10 number-to-position task) in

Study III and IV, an alternative analysis approach, a latent class regression analysis, was used to describe children's number line estimations. The results revealed that a number of latent classes (subgroups) of children could be distinguished in both studies which were characterized by showing different estimation patterns and that differed with respect to the regression functions describing these patterns. The latent class regression analysis made it possible for us to identify subgroups of children showing distinguishable patterns of performance on the two number line tasks. The hypothesis by Siegler and Booth (2004) that children within an age group show different kinds of representations of number was confirmed. One interesting finding was that none of the latent class regression models presented in Study III or IV included one or more subgroups that were clearly described by exponential or logarithmic functions as found in previous research (e.g., Siegler & Opfer, 2003; Siegler & Booth, 2004). The findings presented in Study III and IV show that children's estimation patterns were described by varying functions that would not have been found if only fitting predefined functions (i.e., logarithmic, linear or exponential) to describe children's number line estimations.

In study IV, we also analyzed children's number line estimation patterns using the more commonly employed analysis with group median estimates as the dependent variable and the numbers to be estimated as the independent variable and then fitted predefined functions (i.e., logarithmic and linear) in order to describe children's estimation patterns. Our analyzes revealed that a linear function best described children's estimation patterns on the 0-10 number line task (number-to-position) as compared to a logarithmic function. However, in relation to the results from the latent class regression analysis, it became evident that a lot of information regarding children's estimations of number on the number task were lost when averaging across data using group median estimates and fitting predefined functions. In study IV, one of the subgroups (class 1) were described by a close to perfect accurate linear regression function indicating that children in this class gave rather accurate estimates on the 0-10 number line task. This class included 61 children out of the total sample of 208 children. This tells that about 3/10 of the total sample gave estimates that corresponded closely to an accurate linear representation (i.e., $y=x$) while the other children showed other characteristics in their

number line estimations. Together, these results highlight the importance of problematizing methodological issues of the general procedure commonly employed when analyzing and describing children's estimations of number on number line tasks, which has been highlighted in previous research (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). Bouwmeester and Verkoeijen (2012) discuss that the use of group median estimates is one of the issues that needs to be problematized. If children's estimates of number on a number line task differ substantively, the use of group median estimates over the entire group may generate scores that is not representative for any of the children and important information would be lost, providing a limited description of the data (Bouwmeester & Verkoeijen, 2012; Siegler, 1987; Simon, 1975). When using group median estimates to describe children's estimations of number on number line tasks, individual deviations are seen as errors (Bouwmeester & Verkoeijen, 2012). However, just because children's estimations vary, it does not mean that their estimates are errors and that the heterogeneity in children's estimation performance should be overlooked. Instead, individual children's estimates could give interesting and valuable information about children's representations of and understanding about numbers (Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; LeFevre et al., 2013). Thus, the latent class modeling approach used in Study III and IV captured heterogeneity and characteristics in 5- and 6-year-old children's estimation patterns and gave us the opportunity to study characteristics in children's estimation patterns closer, and these will be further discussed later in this chapter.

In Study III and IV, we have also examined children's number knowledge using a verbal counting task and a naming Arabic numerals task. In Study IV we also examined the performance on a verbal addition task of children included in the different latent classes. LeFevre and colleagues (2013) found that there is a relationship between children's number line estimation performance and their number knowledge in their research. Also, Ebersbach and colleagues (2008) found that children gave rather accurate linear estimations within the numerical range that they were familiar with. Many studies focusing on describing children's number line estimations have not assessed children's knowledge of numbers more specifically. The

results from our research indicate that children that we interpret as showing more accurate linear estimation patterns also perform better on the counting task and the naming Arabic numerals task. Children's number knowledge may affect their number line estimations and their use of reference points when making estimations (see e.g., Ebersbach et al., 2008; Peeters et al., 2015). Aspects that might affect children's number line estimations will be further discussed in the following section.

Aspects affecting children's number line estimation patterns

Study III and IV in this thesis take another view on the debate that has emerged regarding children's number line estimation patterns and the comparison of different models to describe children's (pre-linear) number line estimations. The most important thing may not be to find the model that most accurately describes children's number line estimations, but to understand how children represent numbers and how they make estimations of number on number line tasks. What aspects and strategies that affect children's number line estimations needs to be further investigated and discussed.

In the discussion about children's number line estimations it is important to realize that children's estimation patterns on number line tasks may not be the same as their internal representations of number (e.g., Bouwmeester & Verkoeijen, 2012; Ebersbach et al., 2008). The number line task is often described as revealing about children's internal representation of number (e.g., Siegler & Opfer, 2003), but this assumption has also been challenged (e.g., Barth & Paladino, 2011; Ebersbach et al., 2008; Moeller & Nuerk, 2011; White & Szűcs, 2012). The number line task is a complex measure (LeFevre et al., 2013) and there may be other explanations for children's estimation patterns apart from that it is a reflection of their internal representations of numbers (Ashcraft & Moore, 2012; Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012). But if children's estimation patterns are not just a reflection of their internal representations of numbers, what do their estimation patterns reflect? Several researchers (e.g., Barth & Paladino, 2011; Ebersbach et al., 2008; Bouwmeester & Verkoeijen, 2012) suggest that the number line task does not purely reflect children's internal representation of number since cues in the number line task such as reference points and

fixed endpoints, might affect children's number line estimations. The design of number line tasks might influence children's estimations and their estimation strategies and therefore, children's estimation patterns may be biased and not equivalent to their internal representations of number. Further, children's familiarity with numbers have also been found to be related to the accuracy of children's number line estimations (Ebersbach et al., 2008). These aspects need to be highlighted and discussed since inaccurate representations of number may only be one explanation of inaccurate estimation patterns on number line tasks (e.g., Bouwmeester & Verkoeijen, 2012; Siegler & Booth, 2004).

In Study III and IV, we did not focus on observing or interviewing children about their estimation strategies on number line tasks, but when taking a closer look at the different latent classes presented in the two latent class regression models, the results could be interpreted as showing that children may be affected by task specific cues as well as their familiarity with numbers when making estimations of number on number line tasks, which has been discussed also in previous research (e.g., Barth & Paladino, 2011; Bouwmeester & Verkoeijen, 2012; Ebersbach et al., 2008). Our findings indicate that children seem to use the marked endpoints (i.e., 0 & 10 and 0 & 100) on the number lines as reference points in different ways when making estimations which affect children's number line estimation patterns in different ways. More specifically, some children seem to use no reference points when making their estimates, resulting in random estimation patterns, while other children are more likely to have used one or more reference points resulting in more (accurate) linear estimation patterns. For example, when taking a closer look at class 4 in Study III and class 2 in Study IV, children in these classes seem to give random estimates of number, indicating that they seem to use no reference points when making their estimates. Another possible explanation for the estimation patterns of children in these classes could be that these children possess a representation of number that only consists of a list of counting words without having knowledge about spatial relations between these which means that these children are not fully aware of an important aspect of our symbolic number system, namely that whole numbers are evenly spaced (Booth & Siegler, 2008; Dehaene, 2011; Siegler & Opfer, 2003). It is also possible that children in this

subgroup did not understand the number line task and therefore gave random estimates of number. A more thorough inspection of class 1 and class 3 in Study III, suggest that it is possible that children in both these classes used a single reference point (i.e., 0) when making their estimates of numbers on the 0-100 position-to-number task, resulting in somewhat linear estimation patterns but with slopes far from 1 (i.e., their estimates did not correspond to the actual number being presented). Nevertheless, these estimation patterns could be interpreted as indicating that children in these classes were, to some extent, aware of ordinality and spatial relations between numbers, but that they were not able to use the second reference point (i.e., 100) when making their estimates. Children in class 2 in Study III and children in class 4 in Study IV seem to be able to use both reference points (i.e., 0 and 100 and 0 and 10) when making their estimates, and children in both these classes gave rather accurate estimates close to the reference points while their estimates in between varied a lot.

Our research findings add to the body of literature arguing that children's number line estimates are affected by reference points to different extent and that children's use of these affect their estimation patterns. Results by Rouder and Geary (2014) indicate that young children initially use one reference point (i.e., 0) when estimation numbers on number line tasks, and as they developed and become older, they start to use both endpoints and later on, they also start to use the midpoint as an additional reference point. Rouder and Geary (2014) also found that the more reference points children used, the more linear and accurate where their estimation patterns on the number line task. Barth and Paladino (2011) found that children used the lower and upper reference point as an orientation resulting in more accurate estimation patterns close to the reference points which we also found in our research. Ashcraft and Moore's (2012) research indicate that children seem to use midpoint-based strategies making estimates on number line tasks. For example, based on number knowledge, children know that 50 is half of 100 and then uses 50 as an additional reference point when making their estimates (Ashcraft & Moore, 2012). Further, White and Szűcs (2012) examined children's use of reference point based strategies on a 0-20 number line task and found that 6-year-olds only used the lower reference as an orientation for their estimates which we also interpret that children in our research did (e.g., class 1

and 3 in Study III). In order to be able to use reference points when making number line estimations, children need to possess some basic number knowledge (Aschcraft & Moore, 2012; Ebersbach, Luwel, & Verschaffel, 2015) and children's experiences with numbers is believed to be a main reason for development of linear representations of numbers (Siegler & Booth, 2004; von Aster & Shalev, 2007). Therefore, it is possible that children's number knowledge may have influenced children's estimation patterns on the number line tasks used in this research. As mentioned earlier, the results presented in Study III could be interpreted as that children's number knowledge affected their ability to give answers on the 0-100 position-to-number estimation task where children had to name the numerals that corresponds to marked positions instead of making hatch marks on number lines that corresponds to presented numerals. If children are only familiar with a narrow set of numbers (verbal counting words and/or written Arabic numerals), then children have limited opportunities to provide correct answers on the 0-100 position-to-number task. It is also possible that children due to limited number knowledge may not be able to use the upper reference point on the number line tasks and it is possible that these children apply other estimation strategies generating alternative estimation patterns.

Summary

The results from Study III and IV add to the discussion about the number line task and how to analyze and interpret children's number line estimations. The results indicate that the number line task might not purely measure children's representations of number since children might use and be affected by different aspects such as reference points and/or their number knowledge making number line estimates. Further studies are needed to more thoroughly investigate children's number line estimations and their strategies used in order to make it possible for us to understand the processes by which children are capable of making (correct) estimates of number. While it was not performed in the present research, the use of a latent class modeling approach could also be supplemented with qualitative method such as interviews and observations to gain richer understanding regarding children's estimations and potential strategies used by children included in different latent classes. Investigation of children's estimation strategies

combined with different modeling techniques could help us to increase our understanding of children's representations of number.

Limitations

While conducting the research presented in this thesis, some remarks were made that needs to be addressed. These remarks concern both practical and methodological issues and will be discussed in the following section.

One remark concern the complexity of performing research with young children in preschools and preschool classes. Regardless of careful planning and preparation prior to the start of the research project, different aspects that the researcher cannot fully control might affect the test sessions and intervention sessions and thereby the outcome and results of the research. During intervention sessions as well as test sessions children's motivation and engagement may have affected their performance. When performing pre- and posttest and when playing games with children during the intervention sessions it is not always easy for the experimenter to determine how focused and engaged each child really is. No continues measurement of motivation or engagement was included in this research but it is possible that such aspects may have affected children's performance and thereby the results presented in this thesis. A measure of motivation and engagement had enabled us to control for influence of such aspects during the test and game sessions. However, the overall experience is that most children were engaged during the game sessions and tried to their best at the pre-and-posttests.

Another remark concern the sample selection. The sample selection may have been one of the most critical factors for the difference in outcome from our intervention studies (Study I and II) as compared to results from earlier game intervention studies using (linear) number board games as discussed earlier in this thesis. In previous studies, children with limited experiences of numbers and number games was mainly selected for participation, while we chose to include all children in public preschools and preschool classes. Several of the participating children in our studies had such good number knowledge and skills prior to the intervention that their opportunities to develop were limited by participating in the game intervention. It is possible

that a more restrictive sample selection that mainly had included children with more limited number knowledge and skills or that had mainly included younger children could have generated other effects on children's learning and development if number knowledge and skills by playing the different number games used in Study I and II.

Implications and suggestions for future research

The results presented in this thesis give rise to insights regarding 5- and 6-year-old children's learning and development of number knowledge and skills by playing number games and insights about children's representations of number on number line tasks that could have significance for both researchers and teachers. The results also yield more questions with regard to the topics studied. These questions concern benefits of playing number games with children of different age and skill levels and the importance of further investigating how children represent number on number line tasks and what aspects that might influence their number line estimates.

Two out of four studies in this thesis (Study I and II) focus on effects of playing number games on 5- and 6-year-old children's number knowledge and skills. The results demonstrate that number games can be used to support children's development of number knowledge and skills. However, depending on age and what game used, the learning outcome seems to be affected in different ways and support children's mathematics learning and development to different degrees. The results from the two studies have significance for educational practice, adding to the discussion of the value of selecting games and activities in a conscious way in order to be able to support young children's early mathematics learning and development. Teachers should offer opportunities for children to learn and develop number knowledge and skills and to learn mathematics in different ways. In this, planned learning situations using number games could be one way to provide carefully prepared activities focusing on supporting children's number knowledge and skills important for further mathematics learning. Simple number games can be used together with young children allowing them to participate in implicit mathematics learning while playing together with their peers. However, in order to further understand how we can support young children's learning and

development playing different number games in preschool and preschool class, further investigation is needed. The findings in this thesis suggest that not all number game activities benefit children equally, generating different gains on different numerical tasks. To add to these findings, more studies are needed to assess merits of different number game activities and different approaches playing number games with children with different initial number knowledge and skills. Another important study is to investigate effects on children's number knowledge and skills if combining different number games/activities that direct children's attention towards specific aspects in one intervention compared with using only one specific game/activity during the intervention. It is possible that combined training could lead to additive effects on children's number knowledge and skills and affect children's engagement and motivation in a positive way and thereby also the outcome of the intervention.

The other two studies (Study III and IV) in this thesis focus on the heterogeneity and characteristics in children's representations of numbers on number line tasks. The results showed that several classes could be distinguished and that children in these classes not only differed with respect to estimation pattern, but also with respect to counting ability, ability to name Arabic numerals and ability to solve verbal addition tasks. From the results of this research it can be concluded that Swedish 5- and 6-year-old children show a various estimation patterns in number line tasks. The results presented have implications for the discussion about how to analyze, understand and interpret children's number line estimations. Our findings reveal that children represent numbers in different ways on number line tasks and that their number line estimations might be affected by different aspects. When children show inaccurate representations of number on number line tasks it is possible that these children could benefit practicing more on numbers, number relations and spatial relations between these to support more linear representation of number important for further learning in mathematics. However, further research is needed to understand how children make estimations of number on number line tasks and to understand what aspects that affects their estimations. In future research, interviews and observations with children regarding their number line estimations would give an additional contribution to the field which could provide

new perspectives on and immerse our understanding about children's number line estimates and representations of number (see also Peeters et al. 2015). Further, eye-tracking (Schneider et al., 2008; van Viersen, Slot, Kroesbergen, van't Noordende, & Leseman, 2013) could also be used to confirm the use of reference points and possible additional referents assumed to influence children's number line estimations.

The latent class modelling approach used in Study III and IV generated a nuanced description of children's number line estimations and gave us information about the diversity in children number line estimation patterns. From a methodological point of view, one possible implication of this way of analyzing children's number line estimates could be that individual children with inaccurate estimation patterns could be identified and selected for interventions, aiming at supporting their number knowledge and skills and early mathematics learning and development, for example by playing linear number board games. Based on the findings presented in Study III and IV, further research could also focus on investigating if there is a causal relationship between children's familiarity with numbers and children's number line estimation skills.

General conclusions

The overall aim of this thesis was to contribute to the understanding of young children's early mathematics learning and development by examining effect of playing different number games on children's number knowledge and skills, and by investigating children's representations of number on number line tasks. The results presented contribute to the body of literature on how children can be supported in their learning and development of number knowledge and skills by playing number games. The results also add to the literature and discussion on children's estimations of numbers on number line tasks.

The results from this thesis allows some general conclusions to be drawn. One conclusion is that the number games used in this research can be used to support children's learning and development of number knowledge and skills, at least among preschoolers. Children's learning can be affected in different ways depending on what game activities used. However, further research is needed to advance and immerse our understanding of how different number games can be used to support

children of different age and with different precursory number knowledge in their learning of mathematics as discussed earlier in this chapter.

Another conclusion that can be drawn is that a more exploratory analysis such as latent class regression analysis can be used to distinguish heterogeneity and characteristics in children's representations of number on number line tasks and capture nuances in children's estimation patterns that otherwise would have been missed using more conventional regression analyzes. A more nuanced description of children estimates could help us to identify children who could benefit from targeted interventions with focus on supporting children's learning and development of number knowledge and skills. Our findings also highlight the importance of further research on children's number line estimations to understand the essence of their estimation patterns.

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