LINKÖPING UNIVERSITY

Master’s programme in Economics, majoring in Financial Economics at Linköping University

Performance of alternative option pricing models during spikes in the FTSE 100 volatility index

– Empirical evidence from FTSE100 index options

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Abstract

Derivatives have a large and significant role on the financial markets today and the popularity of options has increased. This has also increased the demand of finding a suitable option pricing model, since the ground-breaking model developed by Black & Scholes (1973) have poor pricing performance. Practitioners and academics have over the years developed different models with the assumption of non-constant volatility, without reaching any conclusions regarding which model is more suitable to use. This thesis examines four different models, the first model is the Practitioners Black & Scholes model proposed by Christoffersen & Jacobs (2004b). The second model is the Heston’s (1993) continuous time stochastic volatility model, a modification of the model is also included, which is called the Strike Vector Computation suggested by Kilin (2011). The last model is the Heston & Nandi (2000) Generalized Autoregressive Conditional Heteroscedasticity type discrete model. From a practical point of view the models are evaluated, with the goal of finding the model with the best pricing performance and the most practical usage. The model’s robustness is also tested to see how the models perform in out-of-sample during a high respectively low implied volatility market. All the models are effected in the robustness test, the out-sample ability is negatively affected by a high implied volatility market. The results show that both of the stochastic volatility models have superior performances in the in-sample and out-sample analysis. The Generalized Autoregressive Conditional Heteroscedasticity type discrete model shows surprisingly poor results both in the in-sample and out-sample analysis. The results indicate that option data should be used instead of historical return data to estimate the model’s parameters. This thesis also provides an insight on why overnight-index-swap (OIS) rates should be used instead of LIBOR rates as a proxy for the risk-free rate.

Keywords: option pricing, stochastic volatility, implied volatility, GARCH, risk-neutral, characteristic functions, Gauss-Laguerre quadrature, Nelder-Mead search algorithm.
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1. Introduction

In 1973, Black & Scholes (1973) published a ground-breaking report alongside Robert Merton where they provided the financial world with a new type of model to price options. Until their report was published, it did not exist any universal model to set prices on options contracts. One of the key assumption in the model which is a strength but also a weakness, is the assumption of constant volatility, which does not exist on the financial market because volatility is time depending. This problem was first discovered in 1987 after the financial turmoil on the 19th October, the so called “Black Monday”. After the turmoil, investors did not price options contracts with a constant volatility factor across all strike levels and time-to-maturity levels as they did before. This was the beginning of the volatility smile that was discovered in stock index options after the crisis, which means that investors have a different belief of what the volatility should be at different strike levels and time-to-maturity levels, this was the first insight of the term implied volatility.

Derivatives have a large and significant role on financial markets, and to put in perspective are the total amount of all the outstanding Over-the-Counter (OTC) contracts approximately US$ 553 trillion dollar according to Bank for international settlements (BIS)\(^1\), and according to Bloomberg, the market capitalization of all the world stocks approximately US$ 65.6 trillion dollar. Options and derivatives has become more important for investors after the 2008 financial crisis, since options can be written on many underlying asset classes, such as stocks, stock indexes, commodities, currencies and interest rates, therefore can options offer an effective way for investors to manage the risk exposure in their portfolios. Options have become a key component in many investors portfolios, therefore it is crucial that options are priced as accurate as possible.

The assumption of constant volatility is what makes the theoretical price of the Black & Scholes model to differ significantly from the market prices and that affects investors who want to measure the market risk in their portfolios or price exotic options. If they cannot price their contracts accurately, the risk exposure will be difficult to estimate. As the market is pricing

\(^1\) A visualisation of the size over the derivative market is provided by the money project: http://money.visualcapitalist.com/all-of-the-worlds-money-and-markets-in-one-visualization/
implied volatility, models have emerged that include the volatility as a stochastic process and has more unknown parameters than the Black & Scholes model, which has only one, volatility.

This thesis aims to conduct research on the models that emerged over the years and four models are chosen to be included in the thesis. The first model is the Practitioners Black & Scholes model (PBS) proposed by Christoffersen & Jacobs (2004b), and will act as a benchmark in this thesis. The second model is the Heston’s (1993) continuous time stochastic volatility model (SV), a modification of the SV-model will also be included, which is called a Strike Vector Computation (SV-SVC) suggested by Kilin (2011). The forth one is the Heston & Nandi (2000) Generalized Autoregressive Conditional Heteroscedasticity type discrete model (HN-GARCH). The complexity of the models has increased significantly and the financial community has not reached a consensus regarding which option pricing model should be used today, and the question remains, which of the models can price option contracts most accurately?

1.1 Purpose

The main purpose of this thesis is to contribute to the existing literature on the subject. Christoffersen, Jacobs & Heston (2009), Su, Chen & Huang (2010), Christoffersen, Jacobs & Ornthanalai (2012), Kanniainen, Lin. & Yang (2014), Siu-Hang & Cheuk-Yin (2015), Singh & Dixit (2016) and Bhat & Arekar (2016) have all compared models in the same categories, but reports that compare stochastic volatility and GARCH models are rare. Lehar, Scheicher & Schittenkopf (2002) compared alternative groups of option pricing models against each other, but only the GARCH model suggested by Duan (1995) and the SV-model, in the original from. The models are not in closed form and simulated prices are used in their GARCH model. Moyaert & Petitjean (2011) did a similar comparison as Lehar, Scheicher & Schittenkopf (2002) and used simulated prices of the HN-GARCH model and compared the SV-model, in its original form, the empirical analysis was performed on Eurostoxx 50 index options. In recent years has it not emerged any published reports that compare the models in closed form. The main contribution of this thesis is to fill that literature gap, to study how stochastic volatility and GARCH models differ from each other, how they perform during times of high respectively low volatility on the financial markets and look at how they can be implemented efficiently. Three research questions will be in focus in this thesis and these will be answered to better evaluate the models:
1. How does high respectively low market volatility influence the model’s performance?
2. Which model included in this thesis has the best performance?
3. Which model included in this thesis is suitable for practitioners to use on the financial market?

1.2 Data & Method

Index options that are written on the FTSE100 index are included, and it is chosen since it is Europe’s largest stock index on a country level and 2016 have been a year were the index has had times of spikes in the FTSE100 volatility index (VIX), and one of the main aspects of this thesis is to see how the models perform during distress times in the FTSE100 index. Three main data components have been collected to price option contracts, the first component is the performance of the FTSE 100 index, obtained from DATASTREAM, the second one is the historical option data, obtained from iVolatility (2017) and the third one is the risk-free rate, obtained from the Bank of England (2017). The dataset consists of all the call options written on the FTSE100 index from the 1st of January 2016 to 31st of December 2016 and the in-sample parameters will be re-calibrated every Wednesday, as Wednesdays has the least probability to be a holiday, this procedure will follow the work of Christoffersen, Jacobs & Heston (2009). The out-of-sample is created by identifying three spikes events in the VIX during 2016. To create a series of days were the option contracts could be priced are two days before and two days after the spike day used to create a sample week. In a similar way are three days in the VIX index chosen with the lowest values. Three high volatility weeks and three low volatility weeks are created where the out-of-sample ability is tested. The importance of choosing the correct loss function when evaluating these models is crucial, this is needed since the model’s parameters will be estimated in closed form and therefore will a loss function be chosen as an objective function. The Nelder-Mead algorithm is used to find the optimal set of parameters that minimize the error in objective function, as this is an effective method to find near the global optimum se parameters in excel.

1.3 Limitations

This thesis will have a practical perspective and will focus on the practitioners on the financial markets. The pricing aspect of options will be the focus and not the hedging perspective, hedging parameters can be extracted from the models since all models will be evaluated in closed form, but this aspect will be excluded in this thesis. This thesis focus on the pricing
aspect of plain vanilla options and not on exotic options, but to engage into the more sophisticated fields of exotic option pricing one must first find the parameters estimated from plain vanilla options, because exotic option is not publicly traded, instead traded directly between large financial institutions – so called over-the-counter or OTC deals. But the field of pricing exotic option is left for further studies. This thesis will not include any transition cost and that is important to have in mind in the analysis of the results. The results in this thesis are estimated with excel and therefore the results are bound to the limitations of excel, but precautions have been made to ensure that the search algorithm find results near the global optimal values in the parameters estimation.

1.4 Outline
This thesis start with explaining some of the expression included in stochastic calculus to give the reader a better understanding of the mathematical framework that underlies all the models in this thesis, the theoretical part will continue and describe the models in depth. This part will be a technical part of the thesis, and the aim of this thesis is to provide the reader with enough information in the other parts to understand the purpose, and therefore can these parts be left out without any significant loss of information; however, it is important to understand the main differences between the models in the theoretical part. The literature review will be presented after that and provide the reader with an overview of both recent and historical research on the field of stochastic volatility models. Chapter five includes the data section, with both option data and FTSE 100 return series, the risk-free rate and includes a description on why overnight-index-swap (OIS) rates should be used instead of LIBOR rates as proxies for the risk-free rate. Chapter six is the method part that includes a detailed presentation of the method behind the calibration of the models, how the parameters are estimated and how the models are evaluated. Chapter seven, includes the empirical results and the last part presents the conclusion of the results.
2. Theory of the models

Chapter two will give the reader an in-depth analysis of the theory, the assumptions and the mathematical framework behind the models included in this thesis. As the SV, SV-SVC and HN-GARCH models all have a characteristic function with complex numbers, these parts will be a technical part of the theory segment. The aim of this thesis is to provide the reader with sufficient information to oversee those parts without any significantly loss of information and still understand the purpose of the thesis. Have in mind that all the models will be presented in their closed form and the assumption of the risk-neutral dynamics of the stock price that underlies all models. If the reader need a brief introduction to the basics behind options or if the more experience reader need to be reminded of the basics, are you referred to Appendix C.

2.1 Theoretical framework

The underlying mathematical framework that all the models in this thesis are based upon will be presented in the 2.1 section. The framework is used to describe the movements of the underlying assets. This is based upon stochastic calculus, and the geometric Brownian motion, risk natural valuation and a description on how this is connected to the price dynamics of stocks is presented. It also includes information of what affect the option price, an insight on what implied volatility is and how the implied volatility can be extracted.

2.1.1 Geometric Brownian motion

In all the models, it is assumed that the stock price follows a geometric Brownian motion, which is defined as a continuous-time stochastic process, where the logarithm of the underlying asset \( (S_t) \) follows a generalized Wiener process.

\[
    dS_t = \mu S_t dt + \sigma S_t dW_t
\]  

(2.1)

The equation describes the price movements of the underlying asset, \( \mu \) is the percentage drift term, \( W_t \) is the Wiener process and \( \sigma \) is the volatility of the underlying asset. In the introduction, it was mentioned that the Black Scholes model assumes that the \( \sigma \) term is constant and this is where the main problem is. The PBS model uses a deterministic volatility function instead when deriving the local volatility and replacing the assumption of constant term \( \sigma \) in the equation with a deterministic volatility term. HN-GARCH and SV model assumes that \( \sigma \) term in the equation is a stochastic variable which is determined by estimated parameters (Hull 2011).
2.1.2 Risk-neutral valuation

One important framework of pricing derivatives such as options, is the risk neutral valuation. It states that, when valuation of options contract is made, the assumption of risk-neutral investors can be implemented, this means that investors do not increase their expected return as a compensation for an increase in risk. This is referred to a risk-neutral world, and in the world of options the assumption give an accurate picture of option prices. One can wonder why that is, but that is because in the world of options a person’s risk preference is not important, investors are clearly not risk neutral in the real world, instead investors risk preferences are reflected in the price of the underlying asset. As investors become more risk averse, the price of the underlying asset declines, but the formulas relating to option prices of the underlying asset remain the same (Hull 2011).

Cox, Ross & Rubinstein (1979) introduced the Girsanov’s theorem which shows that moving from a world with a specific risk preference to another with different risk preference, the expected growth rates in the variable changes, but the volatility will stay the same. Because of this result, it is shown that option prices also hold in a risk averse world.

2.1.3 Price Dynamics

In the previous two sections the Geometric Brownian motion and the risk-neutral world have been introduced. These two frameworks are the link to the stock price dynamic and the risk-neutral valuation of option contracts. In a risk-neutral-world cannot probabilities be used to determine if the stock price can move up or down, since investors have no risk preference. Instead stock prices are described according to the geometric Brownian motion formula (2.1).

In a risk-neutral world, the stock prices will move according to the risk-neutral probabilities, meaning that the drift term ($\mu$) is the risk-free rate. Since the risk premium is eliminated in a risk neutral world, compared to the real world where an unknown risk premium will be added to the stock price. To clarify, the dynamics of the stock price under risk-neutral probabilities will move randomly with a drift term that is equal to the risk-free rate. Risk-neutral valuation of option contracts, a stochastic process is required to determine the underlying asset price and not the real-world probability measure. The parameters in a risk-neutral model can either be

---

2 Think of a binomial tree model where the probabilities are derived to determine up or down movements for the stock price.
estimated by looking at historical stock price data or option data. The problem with the first approach is that parameters such as the market price of volatility risk can be difficult to detect and the estimation of the parameters in the model becomes difficult. The later approach, which is more common, is to derive the parameters from observed option price data, since parameters such as the market price of volatility can be detected in the option data (Hull 2011).

2.1.4 Implied Volatility & Volatility Surface

There are different factors that affect the option price, the spot price ($S_t$), the strike price of an option ($K$), time to maturity ($T$), the risk-free interest rate ($r$), possible dividend the underlying asset pays out ($q$) and the volatility of the underlying asset ($\sigma$). All these factors affect the option price and there is a consensus regarding the first five parameters. But the volatility ($\sigma$) factor is a parameter that has created a great debate in the financial world, and the estimation of the volatility ($\sigma$) factor, which is a science in itself.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Call Option</th>
<th>Put Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase $S$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Increase $K$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Increase $T$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Increase $r$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Increase $Q$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Increase $\sigma$</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

The volatility of the underlying asset cannot be observed on the market as mentioned above, the assumption of constant volatility in B&S model makes the empirical results poor. Traders use what can be observed on the market, which is all the parameters in B&S model except the volatility term $\sigma$. Instead they use the B&S model to estimate implied volatility, this is done by leaving the $\sigma$ term unknown and then use the equation to estimate implied volatility. On the option market, the dilemma of constant volatility reveals itself quite clearly in figure 1. The surface shows how implied volatility decreases as the strike level moves further away from the spot price. If the assumption about constant volatility in the B&S model would hold, the volatility would be constant over all strike prices, therefore should the volatility surface be flat.

---

3 Implied volatility is observed volatility and act as a proxy for future volatility for investors.
Instead a classic pattern reveals itself, that is called a volatility smirk. In figure 1, it is seen that call options that are OTM has higher implied volatility and is lower for options that are ITM. Figure 1 is clear indicator on why models that uses stochastic volatility or local volatility is more suitable to use in option pricing, the models estimated implied volatility will try to create a similar volatility surface as figure 1, in Appendix F figure 2 are the model’s volatility surface. Because If the models have the same shape as the market implied volatility surface, will the theoretical price of the option contract match the market price of the option contract.

2.2 Black & Scholes model
In the 1970’s, a breakthrough was made in the research area of pricing European option contracts. Black & Schols (1973) included Robert Merton work in their article and he provided a crucial insight in how option can be used eliminated the systematic risk through hedging. The formula they derived for call price was,

\[ C_{BS}(K) = S_0N(d_1) - Ke^{-rT}N(d_2) \]  

(2.2)

where \( d_1 \) and \( d_2 \) can be expressed in the following way,

\[ d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \]  

(2.3)
\[ d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \] or

\[ d_2 = d_1 - \sigma\sqrt{T} \]

\( S_0 \) is the spot price for the underlying asset, \( K \) is strike level, \( T \) is time to maturity for the option contract, \( r \) is the risk-free rate and \( \sigma \) is the volatility, which in the B&S model is assumed to be constant in the Geometric Brownian motion. The \( N(d_{1-2}) \) expression in [2.2] is the cumulative probability distribution function for a standardized normal distribution. The complete derivation of the B&S model is widely known and is not included in this thesis and acts only as an introduction to the other formulas, due to their similarities. The reader is referred to the original article by Black & Schols (1973) if the derivation of B&S model is requested.

### 2.3 The Practitioners Black & Scholes model

The PBS model was introduced as the Ad-hoc model in Dumas, Fleming & Whaley (1998), they used a deterministic volatility function to capture the implied volatility. The parameters were derived from an ordinary least square method (OLS) and their result showed that the Ad-hoc model outperformed the B&S model. Later Christoffersen & Jacobs (2004b) put the Ad-hoc model to test against the SV model and the Ad-hoc B&S model performed poorly. They suggested that the parameters in the Ad-hoc model should be estimated through a loss function in the same way as the parameters in the SV-model. This improved the results significantly and the model became widely used by practitioners on the markets, that is why it is called the practitioners B&S model. By implementing a deterministic function of strike price and time to maturity, the implied volatility is determined in PBS model in the following way:

\[ \sigma = \alpha_0 + \alpha_1 K + \alpha_2 K^2 + \alpha_3 T + \alpha_4 T^2 + \alpha_5 KT \]

The price of the plain vanilla option is derived by inserting the implied volatility calculated from (2.6) into the B&S formula (2.2).
2.4 Heston stochastic volatility model

As many stochastic volatility models exist, one is more recognized than others, Heston (1993) developed a closed form solution to the valuation of option contracts with a stochastic volatility factor, which means that the assumption of constant-volatility was not included. The SV-model is made from a bivariate system of stochastic differential equations.

\[ dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_1 \]  
\[ dv_t = \kappa(\theta - v_t) + \sigma \sqrt{v_t} dW_2 \]

Notice that the first diffusion process has the same expression as in (2.1), but instead of the constant volatility factor \( \sigma \), the \( \sqrt{v_t} \) term is included, which is the stochastic volatility term. These diffusion processes are described in a historical measure, also called the physical measure, denoted as \( \mathbb{P} \). As the pricing procedure is done in a risk neutral world, the process of \( v_t \) and \( S_t \) need to be expressed under the risk-neutral measure, \( \mathbb{Q} \).

\[ dS_t = \mu S_t dt + \sqrt{v_t} S_t d\bar{W}_1 \]  
\[ dv_t = \kappa^*(\theta^* - v_t) + \sigma \sqrt{v_t} d\bar{W}_2 \]  
\[ \text{Cov}(dW_t^{Q(1)}, dW_t^{Q(2)}) = \rho dt \]

The following parameters are included in the model:
- \( \mu \) = the drift term of the underlying asset, and is equal to risk free rate under \( \mathbb{Q} \).
- \( \kappa > 0 \) the mean reversion speed of the variance
- \( \theta > 0 \) the mean reversion level of the variance (long term variance)
- \( \sigma > 0 \) the volatility of the variance in underlying asset
- \( v_0 > 0 \) the initial level of the variance (at time zero)
- \( \rho \in [-1,1] \) describes the correlation between the two Brownian motions \( W_1 \) and \( W_2 \).
The risk neutral parameters of the variance process in the second diffusion process are \( \kappa^* = k + \lambda \) and \( \theta^* = \frac{k \theta}{(k + \lambda)} \). The \( \lambda \) term is the volatility risk parameter, and as the parameter estimation is done in a risk neutral world, the \( \lambda \) term will be equal to 0 and the \( \lambda \) term do not need to be estimated. As the \( \lambda \) is 0, the parameters of the physical and risk-neutral world will be the same, as \( \kappa^* = k \) and \( \theta^* = \theta \). Next, the dentation of the call price is presented, only call options are included in this thesis, the put option notion will be excluded, but can be derived with the put-call parity principle\(^4\).

\[
C_{SV}(K) = S_t P_1 - Ke^{-r\tau} P_2
\] (2.12)

This denotation of the call price is like the B&S model (2.2), the difference is that \( d_1 \) and \( d_2 \) is changed to \( P_1 \) and \( P_2 \). The parameters of \( P_j \) describe the probability that the call option expires ITM. The \( P_j \) term, which \( j = 1,2 \) can be obtained by the following formula,

\[
P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re \left[ \frac{e^{-i\theta \ln(K)} f_j}{i\theta} \right] d\theta
\] (2.13)

where

\[
f_j = \exp(C_j + D_j v_t + i \theta x)
\] (2.14)

The \( C_j \) and \( D_j \) are coefficients and has the following expression.

\[
C_j = r\theta i\tau + \frac{k \theta}{\sigma^2} \left\{ (b_j - \rho \sigma \theta i + d_j) \tau - 2 \ln \left[ \frac{1 - g_j e^{d_j \tau}}{1 - g_j} \right] \right\}
\] (2.15)

\[
D_j = \frac{b_j - \rho \sigma \theta i + d_j}{\sigma^2} \left[ \frac{1 - e^{d_j \tau}}{1 - g_j e^{d_j \tau}} \right]
\] (2.16)

and

\[
d_j = \sqrt{(\rho \sigma \theta i - b_j)^2 - \sigma^2 (2u_j i \theta - \theta^2)}
\] (2.17)

\(^4\) \( C + PV(x) = P + S \) where \( C = \) Call price, \( PV(x) = \) present value of the strike price, \( P = \) Put price and \( S = \) spot price or market value of underlying asset.
\[ g_j = \frac{b_j - \rho \sigma \phi i + d_j}{b_j - \rho \sigma \phi i + d_j} \]  

(2.18)

\[ u_1 = \frac{1}{2}, \ u_2 = -\frac{1}{2}, \ b_1 = \kappa + \lambda - \rho \sigma, \ b_2 = \kappa + \lambda \] and note that \( i = \sqrt{-1} \) is an imaginary unit.

Both the SV and the SV-SVC models have two small modifications compared to the original model. The first one is called “the little Heston trap” where Albrecher et al. (2007) developed a change to formation of \( C_j \) and \( D_j \), they suggest that (2.16) is multiplied by \( e^{-d_j \tau} \) in the numerator and denominator, this leads to:

\[ D_j = \frac{b_j - \rho \sigma \phi i + d_j}{\sigma^2} \left[ \frac{1 - e^{-d_j \tau}}{1 - c_j e^{-d_j \tau}} \right] \]  

(2.19)

\[ c_j = g_j \frac{1}{g_j} = \frac{b_j - \rho \sigma \phi i - d_j}{b_j - \rho \sigma \phi i + d_j} \]  

(2.20)

This is a simple and an effective modification of the original model. By implementing this we ensure that the parameters estimation creates stable values as the integrand in (2.13) becomes more accurate, and we can easier find a near global minimum when estimating the parameters in Excel. The second modification is the use of the Gauss-Laguerre quadrature which is used to approximate the value of the integral in (2.13). As the limit reaches to infinity in the integral we need an approximation for the limit, the method is used to reduce the computational time as it avoids domains that are needlessly wide, by assigning abscessas and weights to the integrand are no end points needed to be estimated. Both modifications are used in the SV-model and the SV-SVC models (Rouah & Heston 2015).

Next is a brief explanation of the modification suggested by Kilin (2011). The VBA code for of the objective function is presented in Appendix G code 2, and the reader is referred to Kilin (2011) for the mathematical evidence behind the modification. This is both mathematical and programming techniques, but essentially Kilin (2011) found that the integrand in (2.13) – the characteristic function:

\[ \int_0^\infty \text{Re} \left[ \frac{e^{-i \phi \ln(\tau)} f_j}{i \phi} \right] \]  

(2.21)
Is not dependent of the strike level \((K)\) instead it is dependent of the maturity date \((\tau)\), as in (2.13) this implies that the integrals \(f_1(\emptyset)\) and \(f_2(\emptyset)\) can be estimated for every maturity date and the integrals do not need to be re-estimated for every strike level, meaning that the \(f_j(\emptyset)\) term is estimated, stored and can be reused for every strike level on that specific maturity date. Because \(f_j(\emptyset)\) is the term that needs the most computation time, it will reduce the estimation time of the parameters significantly, according to average estimation presented in Table 2 in Appendix E are the estimation time of the parameters in the in-sample approximately three times faster compared to the SV model.

### 2.5 Heston & Nandi GARCH model

To get a clear view of how the Heston & Nandi GARCH model derive the call price of a call option, we must first understand the underlying concept of both the autoregressive conditional heteroscedasticity (ARCH) model and the GARCH model, since the GARCH model has its origin from the ARCH model. The idea behind the ARCH model is that in financial time series data, the conditional variance is not constant over time, the volatility is clustered which means that times with high volatility will likely be followed by high volatility. This is useful because the ARCH process can therefore be used to predict variance. The ARCH \((q)\) process has the following formula:

\[
Y_t = \beta x_t + \varepsilon_t \sim D(0, h_t) \tag{2.22}
\]

\[
h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{2.23}
\]

The mean equation is (2.22) and the variance function is (2.23). The \(x_t\) in (2.22) is the explanatory variable in an AR or ARMA process and to find the variance of \(Y_t\) we first have to ensure that it is a stationary variable\(^5\), because the residual need to be white noise in the mean equation\(^6\). The variance equation (2.23) shows an autoregressive process in the variance of \(\varepsilon_t\)

\(^5\) A stationary variable has a constant mean regardless of the time-period.

\(^6\) If the residuals are not white noise will the variance series in the GARCH process be explosive, meaning that GARCH process cannot be used to estimate future variance
i.e. the ARCH process captures the influence of $\epsilon_{t-1}$ error term to estimate the variance $h_t$. In the GARCH $(q,p)$ model it is similar to the ARCH$(q)$ model but is instead described as:

$$Y_t = \beta x_t + \epsilon_t \sim D(0,h_t)$$ \hspace{1cm} (2.24)

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta h_{t-i}$$ \hspace{1cm} (2.25)

In the variance equation (2.24), we can see that we have lagged value of the variance ($h_{t-1}$), this is the GARCH term that captures the influence of the previous variance. To ensure that the variance series becomes stationary, the estimated coefficients of GARCH model must fulfil the following:

$$\alpha + \beta < 1$$ \hspace{1cm} (2.26)

If this is not fulfilled then the GARCH process becomes an explosive process which means that the variance increases steadily and therefore become non-stationary, the variance will not be mean reverting. The closer $\alpha + \beta$ it is to zero, the faster will the mean reverting process of the variance be.

Next will be an introduction of the Heston & Nandi GARCH (1,1) closed form model, where the single lag structure is motivated by the characteristics of the returns series and the fact that, multiple lags does not yield better results and the single lag is sufficiently powerful (Heston & Nandi 2000) & (Christoffersen, Jacobs & Ornthanalai 2012). The model assumes that the logarithmic return of the spot price follows the GARCH (1,1) process:

$$R_t = r + \lambda \sigma_t^2 + \sigma_t z_t$$ \hspace{1cm} (2.27)

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha (z_{t-1} - \gamma \sigma_{t-1})^2$$ \hspace{1cm} (2.28)
The following parameters are included in the model:

\[ R = \ln \left( \frac{S_{t+1}}{S_t} \right) \]
\[ r = \text{risk-free interest rate} \]
\[ \sigma_t^2 = \text{conditional variance} \]
\[ z_t = \text{error term distributed as a standard normal variable, } z_t \sim N(0,1) \]
\[ \lambda = \text{the volatility risk parameter} \]
\[ \omega, \beta, \alpha, \gamma = \text{model parameters, will be estimated by the loss function approach.} \]

The equation is similar to the original GARCH (1, 1), wherein the variance equation (2.25) the term \( \varepsilon_{t-1}^2 \) is multiplied with \( \alpha \) and compared to (2.28), this term is instead replaced with the term \( (z_t - \gamma \sigma_t)^2 \). The process of this model will be mean reverting if \( \beta + \alpha \gamma^2 < 1 \). Because of the rearrangement in the original model that drives \( \sigma_{t+1}^2 \), they show that \( \alpha \) determines kurtosis and that \( \gamma \) determines the skewness of the probability distribution of the underlying asset returns. In this model, as with the Heston SV model, a \( \lambda \) term exist, which is the volatility risk parameter, therefore must the equation be determined in its risk-neutral version:

\[ R = r + \lambda \sigma_t^2 + \sigma_t z_t^* \] \hspace{1cm} (2.29)

\[ \sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha (z_{t-1}^* - \gamma^* \sigma_{t-1})^2 \] \hspace{1cm} (2.30)

By replacing \( \lambda \) with \( -1/2 \) and \( \gamma \) with \( \gamma^* = \gamma + \lambda + \frac{1}{2} \) are the risk-neutral process in the same form as the real process. The call price can be derived from the same equation (2.12) as for the SV-model:

\[ C_{HN}(K) = S_t P_1 - Ke^{-rT}P_2 \] \hspace{1cm} (2.31)

where \( P_1 \) and \( P_2 \) are risk-neutral probabilities where \( P_1 \) represents the delta of the call option and \( P_2 \) represents the probability that the price of the underlying asset at maturity is greater than the
strike price. To derive the price, $P_1$ and $P_2$ must be estimated and this is done in a similar way as in the SV-model (Rouah & Vainberg 2007):

$$P_1 = \frac{1}{2} + \frac{e^{-rT}}{\pi S_t} \int_0^\infty \text{Re} \left[ \frac{K^{-i\emptyset}f^*(i\emptyset + 1)}{i\emptyset} \right] d\emptyset$$

(2.32)

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left[ \frac{K^{-i\emptyset}f^*(i\emptyset)}{i\emptyset} \right] d\emptyset$$

(2.33)

$\text{Re}[]$ in the formula denotes the real part of a complex number. Note that $i = \sqrt{-1}$ is an imaginary unit. $f^*(i\emptyset)$ represents the conditional characteristic function of the log asset price using the risk neutral probabilities, the HN-GARCH model changes in a risk neutral world, which means that the generating function gets the following expression $f^*(i\emptyset) = E_t[S^{\emptyset}_{t+T}]$ and Heston & Nandi (2000) shows that the moment generating function takes the following log-linear form.

$$f(\emptyset) = S_t^\emptyset \exp(A_t + B_t \sigma^2_{t+1})$$

(2.34)

where

$$A_t = A_{t+1} + \emptyset r + B_{t+1} \omega - \frac{1}{2} \log(1 - 2\alpha B_{t+1})$$

(2.35)

$$B_t = \emptyset (\lambda + \gamma) - \frac{1}{2} \gamma^2 + \beta B_{t+1} + \frac{\gamma(\lambda+\gamma)^2}{1-2\alpha B_{t+1}}$$

(2.36)

$A_t$ and $B_t$ are defined recursively, it is done by working backward from the maturity date of the option and using the terminal conditions.

$$A_t = B_t = 0$$

(2.37)

The risk-neutral distribution must be extracted and this is done from the risk neutral generating function $f^*(i\emptyset)$, where the $\lambda$ term is replaced with $-1/2$ and $\gamma$ term with $\gamma^* = \gamma + \lambda + \frac{1}{2}$ in $A_t$ (2.35) and $B_t$ (2.36).

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$^7 P_2 = P_r[S_{t+T} > K]$
3. Literature review

This segment will include the research findings on different GARCH models, stochastic volatility models and give the reader a clear overview of historical and recent research in the subject of option pricing models.

3.1 Stochastic volatility

Hull & White (1987) extended the work of Black & Scholes (1973) and introduced time varying volatility into the B&S model. This was the first introduction to stochastic volatility in option valuation, both the asset price and the assets volatility follow their own diffusion process\(^8\). Later Heston (1993) presented a new type of model which had a closed form solution and was built upon the work from Hull & White, this was called Heston’s stochastic volatility model and was widely acknowledged when it was presented by both the academic and financial community. Over the years, some have presented other models that introduce more factors and jumps to the original model. Bakshi, Cao, and Chen (1997) introduced a modification to the SV model by adding jumps to the diffusion driving process of the asset price, but this did not add any significant improvement over the original model. Christoffersen, Heston & Jacobs (2009) presented a two-factor model, and showed that this has an advantage both on short-term and long-term options. The pricing errors are smaller than the single-factor, but the parameter estimation is more complicated and has therefore not been established as a model that outperform the original model.

The SV-model has shown promising empirical results over the years, but the model has its flaws and is much more complicated than the B&S model. Therefore, have the latest research regarding option valuation with stochastic volatility looked at how the estimation of the parameters in the original model can be done more accurately and faster. Kilin (2011) introduced a way to speed up the optimization process required for a loss function. Kilin (2011) suggested that the characteristic function in the SV-model should depend on time to maturity and not on strike price as in the original model, therefore it can be calculated for each maturity instead of every strike level, this is called a Strike-Vector computation.

\(^8\) In probability theory, is this a solution to a stochastic differential equation. The Markow process which the Markow chain proceeds from, and the geometric Brownian motion are both examples of a diffusion process.
Storn & Price (1997) presented a new type of algorithm that could in a simple and efficient way find the global minimum solution to optimizations problems called the differential evolution algorithm. Vollrath & Wendland (2009) then applied the differential evolution algorithm to option pricing and included the algorithm in the SV-model to find the global minimum in the parameter space. This increased the accuracy of the parameters which minimize the pricing errors, but at a cost of high computational time. He & Zhu (2016) presented an affine solution to the optimization problem by using partial differential equation of the option price. With necessary conditions imposed on their parameters they found promising empirical results, it showed similar results in the pricing errors as in the original model, but mostly, their version is much faster to implement compared to the original model.

Over the years, different parameters estimations have been developed and Christoffersen & Jacobs (2004b) have highlighted the importance of choosing the right loss function when faced with the optimization problem that the closed form requires. Others have looked at how some of the five parameters in the SV-model can be hold constant or estimated by a different approach than parameters estimation by loss function. This means that not all parameters have to be estimated in the optimization procedure, which will ease the componential burden. Janek et al. (2010) estimated the $V_0$ parameter by looking at ATM implied volatility in FX market and set the $\kappa$ parameter to fulfil the Feller’s condition $2\kappa \theta > \sigma^2$, this ensures that $V_0$ is positive and that leaves, $\sigma$ and $\rho$ parameters to be calibrated. Guillaume and Schoutens (2012) choose to hold $V_0$ and $\theta$ fixed and estimated their values by looking at the CBOE VIX index$^9$ and the $\sigma$, $\rho$ and $\kappa$ parameters are calibrated by the loss function approach. Their results were close to the original model, they argued that their method gave more stable values for the parameters and reduced the estimation time of parameters in comparison to the original model.

### 4.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH)

Duan (1995) was one of the first to use the GARCH model for option valuation, he presented how options can be priced in a local risk-neutral valuation relationship, but this was not in closed form and therefore was a Monte-Carlo approach used to simulate the underlying asset price. This was both time consuming and slow, later Duan et al. (1999) developed a Markov

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$^9$ Chicago Board Options exchange look at S&P 500 index option and use them to derive the volatility index(VIX)
Chain\textsuperscript{10} approach with a single lag in the GARCH process to speed up the process. One of the important highlights of Duan (1995) was that he presented how option can be priced in a local risk neutral valuation relationship which layed the foundation that Heston & Nandi (2000) could work from when they presented their GRACH model. The model that Heston & Nandi introduced did not only have the closed form solution but they also presented mathematical evidence for a HN-GARCH \((p, q)\) model, a model that allows for multiple lags. The closed form solution improved accuracy drastically and the HN-GARCH \((1, 1)\) model becom widely used in the financial community.

Under development in the research area of option valuation is whether or not GARCH models with Non-Gaussian\textsuperscript{11} innovations can improve the accuracy. Different approaches have been suggested by Christoffersen, Heston, & Jacobs (2006), Christoffersen et al. (2010a) and Liu, Siu-Hang & Cheuk-Yin (2015) to capture skewness in the data but the evidence is not compelling if a Non-Gaussian approach improve the Gaussian approach. In the literature review by Christoffersen, Jacobs & Ornthanalai (2012) where they look at the research performed on GARCH models, they formulate:

\begin{quote}
While the literature has expanded significantly, it seems to us that much remains unknown. Even within the class of models discussed in this survey, our knowledge is limited, and many outstanding questions have not been addressed. For instance, do non-normal innovations or multiple volatility components offer the best chances of improving empirical fit? How many volatility components are needed? Even more importantly, what is the nature of the trade-off between the convenience offered by the affine structure and the resulting deterioration in fit? Can this be addressed by additional volatility components or does the resulting increase in parameters come at too high a price? \\
\end{quote}

This is good description of the current situation, many questions remain unanswered in both areas, such as which modification of the Heston model performs the best? Is the two-factor model better? Which distribution should be used when implementing GARCH models? Which model can price option better, the HN-GARCH or the Heston model? Should GARCH models be in an affine or non-affine solution?

\textsuperscript{10} A randomized process that give future estimations values based on today’s value, the process is “memoryless”.

\textsuperscript{11} Non-normal distribution of stock returns, can include distributions that has heavier tails or skewed for example.
Table 2
Literature overview

Presents a summary of previous studies that have investigated the subject presented in this thesis, the findings are based on what is relevant for this thesis.

<table>
<thead>
<tr>
<th>PAPER</th>
<th>DATA/TIME PERIOD</th>
<th>PARAMETER TIME SPAN</th>
<th>FINDINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dumas, Fleming &amp; Whaley (1998)</td>
<td>S&amp;P 500 call and put options</td>
<td>1 week</td>
<td>Ad-hoc has better out-of-sample performance than the deterministic volatility function, the Ad-hoc model gives more reliable hedge ratios and is more reliable. The DVF model has larger prediction errors and has problem with overfitting, they conclude the Ad-hoc is better.</td>
</tr>
<tr>
<td>Lehar, Scheicher &amp; Schittenkopf (2002)</td>
<td>FTSE 100 Call &amp; Put options</td>
<td>10 days</td>
<td>Evaluates the classical BS, GARCH model by Duan (1995) and Heston SV model the paper has risk management perspective. The GARCH model by Duan has the smallest pricing errors and no difference between the models’ value at risk forecast.</td>
</tr>
<tr>
<td>Chrissoffersen &amp; Jacobs (2004a)</td>
<td>S&amp;P 500 Call options</td>
<td>250 days</td>
<td>Evaluates different types of GARCH models, they found that their GARCH density model gives superior out of sample results (weekly update), but could not capture the skewness in option prices.</td>
</tr>
<tr>
<td>Christoffersen &amp; Jacobs (2004b)</td>
<td>S&amp;P 500 Call Options</td>
<td>1 day</td>
<td>Describes how important it is with the loss function when comparing different models specially in the out-sample analysis, also show that the PBS model gives good results.</td>
</tr>
<tr>
<td>Christoffersen, Jacobs &amp; Heston (2009)</td>
<td>S&amp;P 500 Call Options</td>
<td>1 week</td>
<td>Look at multi factor stochastic models with jumps and show how the MFSV model with jumps outperform the regular model and that the jump model better catches the skewness in the option prices.</td>
</tr>
<tr>
<td>Su, Chen &amp; Huang (2010)</td>
<td>FTSE 100 Call options</td>
<td>1 week</td>
<td>Compare the Ad-hoc BS model against the Heston &amp; Nandi GARCH model, they use T-Bills as risk free, no dividend rate and they use the RMSE to measure error of the models, conclusion is that HN-GARCH performs better.</td>
</tr>
<tr>
<td>Christoffersen, Jacobs &amp; Ornthanalai (2012)</td>
<td>Literature review and proof of the mathematics behind the models.</td>
<td></td>
<td>The authors discuss different GARCH models and suggest how both European index options and American index options can be valued using Monte Carlo simulations and they also look at the different stochastic volatility models. They suggest how the empirical research should look like and what is the main problem with the existing research.</td>
</tr>
<tr>
<td>Liu, Siu-Hang &amp; Cheuk-Yin (2015)</td>
<td>S&amp;P 500 Call &amp; Put options</td>
<td>1 day</td>
<td>Use canonical valuation to the sample paths in the model and can therefore use the EGARCH model with a modified t-student distribution. The model shows good pricing results and capture the skewness in the option prices.</td>
</tr>
<tr>
<td>He &amp; Zhu (2016)</td>
<td>S&amp;P 500 Call Options</td>
<td>1 week</td>
<td>Suggest a new method to estimate the parameters in the Heston stochastic volatility model.</td>
</tr>
</tbody>
</table>
4. Data

This part aim is to clearly specify for the reader what kind of data has been used to test the different models and it also includes a longer segment regarding what kind of risk-free rate should be used, as this thesis uses overnight index swap (OIS) rates, which is a different approach compared to the recent literature in option pricing.

4.1 FTSE 100 index & option data

The FTSE 100 index is market capitalized weighted index that consist of blue chip companies listed in the United Kingdom and traded on the London Stock exchange. The FTSE 100 volatility index (VIX) is comprised of OTM put and call options with the FTSE 100 as underlying asset, the prices reflect the market’s expectation of future volatility. (FTSE Russel 2016). 2016 has been an interesting year with events that have set of spikes in the FTSE 100 VIX, in January there were signals of a declining economic growth in China, in February the crude oil hit $26,05/barrel which was the lowest point since May 2003 and in June, Great Britain voted to leave the European Union which was a surprise since many polls suggested that the No-side would win. The FTSE 100 VIX level have been on high levels during these events, but has also been on record lows during 2016, the five-year average is around seventeen and the VIX level have been close to ten in 2016, see figure 2, which is extreme low levels. In a low implied volatility market, the market is strong, since there is a negative correlation between the VIX index and the returns series. This can be seen in figure 2 & 3, as the implied volatility goes down, the index level rises. All these events will be used in the out-of-sample to evaluate the robustness of the models, and to see if the pricing performance is affected when the VIX is low or high.
Figure 2
The FTSE 100 VIX index
The time-period is January-2016 to December-2016.
■ Represents days with high implied volatility
○ Represents days with low implied volatility

Figure 3
FTSE 100 index and return series
The grey line is the FTSE100 return plot and the black line is the FTSE100 Index level. The time-period is from Jan-2015 to Dec-2016.

The options dataset consists of 381 501 call options written on FTSE 100 index from the 1st of January 2016 to the 31st of December 2016. The information regarding strike level, time to maturity, exercise date, current index level and bid & ask quotes is all gathered from the options data that is provided from iVolatility (2017). The mid-price was calculated and used as the market price, this was done by averages. To produce a good sample of options contract that
could be used in the models, was a cleaning procedure performed on the option data, this procedure follows the suggestion made by Bakshi, Cao & Chen (1997) and Dumas, Fleming & Whaley (1998). The steps include removing options contract with no trading volume, options with shorter than 6 days to maturity and options with moneyness above 110% and below 90%. These are three out of the ten steps included and an in-depth description of all steps are in Appendix D. The cleaning procedure reduced the number of call option contracts to 14,279 possible contracts to value in 2016. In table 3 and 4 are the options contracts that were used in the in-sample and out-of-sample analysis, since the two sample periods did not cover the whole year, was 3,436 option contracts priced of the possible 14,279.

### Table 3
**Option data Out-Of-Sample**

All call options written on the FTSE 100 index, six weeks where three are weeks with high implied volatility and three with low volatility, these periods range from 2016-01-18 to 2016-12-20. O=Out-of-money, A=At-the-money and I=In-the-money. T represents days to maturity of the option contract.

<table>
<thead>
<tr>
<th>T&lt;40</th>
<th>0.90&lt;S/K&lt;0.97(O)</th>
<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imp.Vol</td>
<td>18.91</td>
<td>17.96</td>
<td>23.10</td>
<td>18.71</td>
</tr>
<tr>
<td>Option Price</td>
<td>25.32</td>
<td>101.88</td>
<td>339.05</td>
<td>89.90</td>
</tr>
<tr>
<td>Observations</td>
<td>299</td>
<td>423</td>
<td>57</td>
<td>779</td>
</tr>
<tr>
<td>40&lt;T&lt;70</td>
<td>0.90&lt;S/K&lt;0.97(O)</td>
<td>0.97≤S/K≤1.03(A)</td>
<td>1.03&lt;S/K&lt;1.10(I)</td>
<td>ALL</td>
</tr>
<tr>
<td>Imp.Vol</td>
<td>15.12</td>
<td>14.08</td>
<td>17.39</td>
<td>14.60</td>
</tr>
<tr>
<td>Option Price</td>
<td>38.58</td>
<td>129.18</td>
<td>318.31</td>
<td>87.86</td>
</tr>
<tr>
<td>Observations</td>
<td>223</td>
<td>243</td>
<td>5</td>
<td>471</td>
</tr>
<tr>
<td>T&gt;70</td>
<td>0.90&lt;S/K&lt;0.97(O)</td>
<td>0.97≤S/K≤1.03(A)</td>
<td>1.03&lt;S/K&lt;1.10(I)</td>
<td>ALL</td>
</tr>
<tr>
<td>Option Price</td>
<td>69.02</td>
<td>207.45</td>
<td>422.20</td>
<td>152.26</td>
</tr>
<tr>
<td>Observations</td>
<td>286</td>
<td>315</td>
<td>23</td>
<td>624</td>
</tr>
</tbody>
</table>

### Table 4
**Option data In-Sample**

All call options written on the FTSE 100 index from 2016-01-04 to 2016-07-04, every Wednesday. O=Out-of-money, A=At-the-money and I=In-the-money. T represents days to maturity of the option contract.

<table>
<thead>
<tr>
<th>T&lt;40</th>
<th>0.90&lt;S/K&lt;0.97(O)</th>
<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imp.Vol</td>
<td>16.11</td>
<td>16.55</td>
<td>20.79</td>
<td>16.56</td>
</tr>
<tr>
<td>Option Price</td>
<td>20.10</td>
<td>102.44</td>
<td>346.63</td>
<td>79.20</td>
</tr>
<tr>
<td>Observations</td>
<td>262</td>
<td>343</td>
<td>27</td>
<td>632</td>
</tr>
<tr>
<td>40&lt;T&lt;70</td>
<td>0.90&lt;S/K&lt;0.97(O)</td>
<td>0.97≤S/K≤1.03(A)</td>
<td>1.03&lt;S/K&lt;1.10(I)</td>
<td>ALL</td>
</tr>
<tr>
<td>Imp.Vol</td>
<td>14.46</td>
<td>15.06</td>
<td>16.16</td>
<td>14.88</td>
</tr>
<tr>
<td>Option Price</td>
<td>31.57</td>
<td>119.53</td>
<td>298.69</td>
<td>91.94</td>
</tr>
<tr>
<td>Observations</td>
<td>178</td>
<td>192</td>
<td>27</td>
<td>397</td>
</tr>
<tr>
<td>T&gt;70</td>
<td>0.90&lt;S/K&lt;0.97(O)</td>
<td>0.97≤S/K≤1.03(A)</td>
<td>1.03&lt;S/K&lt;1.10(I)</td>
<td>ALL</td>
</tr>
<tr>
<td>Imp.Vol</td>
<td>13.75</td>
<td>15.08</td>
<td>16.92</td>
<td>14.67</td>
</tr>
<tr>
<td>Option Price</td>
<td>54.63</td>
<td>192.39</td>
<td>441.71</td>
<td>153.08</td>
</tr>
<tr>
<td>Observations</td>
<td>199</td>
<td>309</td>
<td>25</td>
<td>533</td>
</tr>
</tbody>
</table>
4.2 Risk Free rate

Traditionally in the academic world, a proxy for the risk-free rate is usually T-bill rates and by practitioners it was common to use the LIBOR swap rates. But recently it has been a shift, instead of using LIBOR swap rates, are practitioners and major clearing houses now using OIS rates and it has become the market standard for calculating collateralized deals (Hull & White 2013). But still in the academic world it is not used as a proxy for the risk-free rate. This thesis has the perspective of looking at the models from the practitioner’s perspective and therefore has the OIS rate been chosen. But the reader can wonder why this shift has occurred? Because since the financial crisis in 2008, a spread has emerged between the LIBOR rate and the OIS rate and until the crisis it was a minimal spread. This spread happened because in the mid-2007, before the Lehman crash, banks started to have concerns regarding the creditworthiness and suspicions of bad loans in other banks assets, and the LIBOR rate increased sharply due to this. Remember that the LIBOR rate describes the average rate that the banks charge each other for short term loans with no collateral, only the bank creditworthiness is used, and if the creditworthiness starts to be questioned, then it is only logical that the rate start to rise, and this is exactly what happened in 2007, nowadays is the spread between LIBOR and OIS used to measure credit risk (Hull & White 2013).

The OIS swap rate is where a fixed rate of interest is exchanged for a floating rate, the floating rate is gathered from an overnight rate index which is calculated by looking at the geometric mean of a daily overnight rate. The overnight rate is set by the bank of England, is closely related to the repo rate, and it is used to control the monetary system in the Great Britain, the fixed rate is set by the two parties that initiate the swap contract. The spot rate of the OIS rate is collected from the bank of England website, their estimated spot yield curve is used as a proxy and they provide monthly rates up to five years’ maturities on the OIS rates. Since the risk-free rate is linear interpolated to match the days to maturity for all option contracts, is the OIS rate better to use, because it is easier to match the different days to maturity for the different option contracts and that gives a more accurate approximation of the risk-free rate.
5. Method

This will give the reader a detailed presentation of the method behind the calibration of the models, how the parameters are estimated and how the models are evaluated. To better understand the calibration procedure for the specific models, has the models their own segment, because the calibration method has been different for all models. The VBA programming code for the Nelder-Mead search algorithm is included in Appendix G, code 1, and is used to clarify how the parameters were estimated.

5.1 Calibration of the models

To find the true unbiased parameters for all models is an impossible task because some of the variables are unobservable and must be approximated. This can be done in two ways, either by looking at historical values or at the cross-sectional options data. The advantage of looking at options data is that it contains information about the future, as implied volatility contains information about the future beliefs among investors. Heston & Nandi (2000) argue that the use of only option data can cause a problem with overfitting parameters\(^{12}\) and the information about the underlying asset can be missed. For example, historical patterns in the index return series will be missed if only option data is used. In this thesis are some of the starting values in the Nelder-Mead search algorithm estimated from historical returns on the FTSE100 index, except for the PBS-model that instead uses an OLS regression to obtain the starting values for the estimation. As mentioned will Appendix G include the VBA programming for the Nelder-Mead search algorithm and the reader is referred to Nelder & Mead (1965) or Dréo, Nunes, & Siarry (2009) for the full mathematical evidence behind the algorithm. The use of the Nelder-Mead algorithm is common practice in the literature to solve the optimization problem in the models. Gilli & Schumann (2010) and Singh & Dixit (2016) use it to estimate the parameters in the SV model. Su, Chen & Huang (2010) and Kanniainen, Lin. & Yang (2014) use it to estimate the parameters in a maximum likelihood estimation for different GARCH models.

5.2 Parameter Estimation

To ensure that the parameter estimation is near the global minimum, some precautions must be made in the Nelder-Mead algorithm. The parameters that could be observed was extracted from

\(^{12}\) Overfitting problem occurs when a model is overstated complex, for example having too many parameters relative to the number of observations. A model that has an overfitting problem has a poor predictive ability, as it overreacts to small changes in the sample data.
the FTSE100 index historical return series, ranging from one to five years back, and used as starting values. For the HN-GARCH, SV-SVC and SV models was the historical starting values used to estimate the parameters for the first period both in the in-sample and the out-of-sample analysis. To ensure that stable parameters are found in period $t$, are the parameters from period $t - 1$ used as starting values. Meaning that the historical starting values are only used once in the in-sample analysis and in the out-of-sample analysis, after that are the previous periods optimal set of parameters used as starting values. The estimated starting values that was estimated historical and based on previous research is listed in Table 1, Appendix E. The estimation of the parameters is performed multiple times to ensure that the search algorithm find an error in the objective function that is as small as possible.

5.2.1 PBS
As the PBS-model only has $K$ and $T$ as variables it is not possible to obtain any useful information from the FTSE100 index return series, as described above is the PBS model starting values instead estimated by an OLS regression. This is done by looking at the option data and collect $K$ and $T$ as variables from period $t - 1$, period $t - 1$ parameters are used as starting values in period $t$. This is a simple equation that derive the local volatility, therefore are no restrictions made on the parameters in the objective function.

5.2.2 SV & SV-SVC
Both the SV and SV-SVC models has five parameters that needs to be estimated to price the option contract, a reminder from the theory section (2.4)

- $\kappa > 0$ The mean reversion speed of the variance
- $\theta > 0$ The mean reversion level of the variance (long term variance)
- $\sigma > 0$ The volatility of the variance in underlying asset
- $v_0 > 0$ (Volatility) The initial level of the variance (at time zero)
- $\rho \in [-1,1]$ Describes the correlation between the two Brownian motions $W_1$ and $W_2$.

$\theta$ is the long run variance that was collected from the five years returns series of the FTSE 100 index. $\sigma$ is the volatility of the variance and it was collected by analysing the constant volatility for the 30-day volatility between 2010-2015, and this was transformed to the variance. $v_0$ is the initial variance term and $\kappa$ is the parameter that drives $v_0$ to the long run variance ($\theta$), but it is
debatable if \( v_0 \) can be observed in the market or only approximated, in thesis is \( v_0 \) set to the constant variance from the day before the parameter estimation. It is difficult to find information in the returns series on the values that should be used for \( \rho \) and \( \kappa \), this thesis is setting the approximation of \( \rho \) and \( \kappa \) close to previous findings of Eraker (2004), Christoffersen, Heston, & Jacobs (2009) and He & Zhu (2016).

Restrictions have been imposed on the parameters to get parameters in the search algorithm that are reasonable and in range of the parameters extracted from historical returns. The restriction cannot be to tight, as this can limit the search algorithm ability to find parameters near the global minimum. The following restrictions are based on Eraker (2004), Christoffersen, Heston, & Jacobs (2009), He & Zhu (2016) and own measurement from the historical returns series. \( \kappa \geq 15, \theta \geq 1, \sigma \geq 5, v_0 \geq 2 \) and the feller condition was also imposed on the parameters:

\[
2\kappa \theta > \sigma^2
\]  

(5.1)

### 5.2.3 HN-GARCH

There are five model parameters that exist in the HN-GARCH model, the parameters can be described in the following way:

- \( \omega = \) Intercept
- \( \alpha = \) Represent the kurtosis
- \( \beta = \) Effect of previous variance
- \( \gamma = \) Skewness/Correlation factor
- \( \lambda = \) The volatility risk parameter

The starting values are determined by analysing the GARCH (1,1) process for the 2015 returns series of the FTSE 100 index, the returns series descriptive statics are included in Appendix E table 3. The \( \beta \) and \( \omega \) is gathered from the historical return series. The \( \gamma \) parameter, which is not the same as the physical skewness parameter, cannot be estimated from historical returns and the parameter is crucial, as it produces the HN-GARCH model’s ability to capture the volatility smirk. The starting value the \( \gamma \) parameter is therefore based on the previous findings from Heston & Nandi (2000), Christoffersen, Heston & Ornthanalai (2012) and Kannianen, Lin. & Yang (2014). Alpha is determined to fit the mean reverting process (6.2). Restrictions in the
HN-GARCH are used to get stable values. The following restrictions are made, $0 > \beta > 1$ and $0 > \alpha > 1$, because else will variance process become explosive, meaning that the variance process will not revert to its mean. $\lambda$ is not estimated as this represents the risk parameter and Heston & Nandi (2000) set this to $-1/2$ while Kanniainen, Lin. & Yang (2014) includes the $\lambda$ term in the parameter estimation. In this thesis, the $\lambda$ parameter is set to $-1/2$, compared to Kanniainen, Lin. & Yang (2014) approach, as it produces better results and the parameters values was more consistent. This cannot be concluded, since the conclusion was reached after some test runs and not after an in-depth analysis. The first mean reverting process must also be fulfilled for the parameters estimated\(^{13}\) and used to price the option contracts:

$$\beta + \alpha \gamma^2 < 1$$ (5.2)

### 5.3 Objective function

To find the optimal set of parameters it is essential that an objective function is chosen in the Nelder-Mead search algorithm which can minimize the error in the function. Next will be a presentation of the three most commonly used loss functions in the literature:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} w_i (C_i - \hat{C}_i)^2$$ (5.3)

$$MSE \% = \frac{1}{N} \sum_{i=1}^{N} w_i \left(\frac{C_i - \hat{C}_i}{C_i}\right)^2$$ (5.4)

$$IV\ MSE = \frac{1}{N} \sum_{i=1}^{N} w_i (\sigma_i - \hat{\sigma}_i)^2$$ (5.5)

Where

$C_i =$ Market call price

$\hat{C}_i =$ Theoretical call price

$\sigma_i =$ Market implied volatility

$\hat{\sigma}_i =$ Model implied volatility

---

\(^{13}\) Since the variance series will be explosive and the results will be poor if $\beta + \alpha \gamma^2 > 1$
Functions (5.3) and (5.4) are used as loss functions in the objective function and the same functions are also used to evaluate the performance of the models. One disadvantage of (5.3) is options that are deep out-of-the money (OTM) and have a short maturity date, those options are cheap and add little value to the sum in (5.3), the search algorithm will therefore fit options that are in-the-money options and with a long maturity date, expensive options will over contribute to the optimization. The solution for this is to use the (5.4) loss function, but the opposite will occur, cheap options will over contribute in the optimization. Solutions can be to assign weights or as used in this thesis a selective selection of options included in the optimization. Option that are in-the-money (ITM), out-of-the money (OTM) and at-the money (ATM) have been selected and maturities that are $t < 40$, $40 < t < 70$ and $t > 70$ where included. This procedure was followed for all models and to the extent that the option data could be divided in to these groups. This procedure allowed for control of overly in-expensive and expensive options that would offset the sum in the optimization problem. Instead of using the bid-ask weights, suggested by Mikhailov & Nögel (2003), was the conclusion reached to use selective selection after test runs, where the selective selection yielded better results. The (5.5) loss function do not have any known patterns of favouring any options, as the other two. The main disadvantages of the loss function for implied volatility (5.5) is that the implied volatility must be extracted for every option contract and this increase the computation time with approximately 40% (own estimate). Instead is (5.5) used to evaluate the models but not included in the objective function. As the HN-GARCH model can take 1 800 seconds to estimate the parameters and therefore has it been a time saving decision to exclude (5.5) as an objective function in the Nelder-Mead search algorithm.

Christoffersen & Jacobs (2004b) argues that no loss function is better than the other, but emphasise that it is important to choose the same loss function in the objective function as in the evaluation of the model. In the case of comparison of models should the same loss function be used for all the models. Since this thesis does not follow this urging from Christoffersen & Jacobs (2004b), as (5.5) is used to evaluate the models, and not included as an objective function, this is true for all models, they are all evaluated in the same way and therefore is the (5.5) be a good evaluation method for the models.
5.4 Sample Procedure

The main purpose for the out-of-sample is to test the robustness of the models, therefore are three spikes events identified in the VIX during 2016 (China January, Oil February and Brexit June), to create a series of days were the option contracts could be priced, the spike day are chosen, two days before and two days after the spike day is used to create a sample week. Three days in the VIX index are chosen with the lowest values and a sample week created in a similar way. Six weeks were formed and tested, three with financial turmoil and three calm weeks. The models were tested in two separate ways. The model’s parameters are estimated at period \( t - 1 \) to price option at period \( t \).\(^{14}\) Second approach is that the models’ parameters were estimated at the beginning of the week and then used to price all the options in the sample week. This means that 1-day parameters and 5-days parameters are created to price the option contracts. The in-sample estimation is also included in this thesis, this follow the same methodology as in He & Zhu (2016), Christoffersen, Heston, & Jacobs (2009) and Heston & Nandi (2000). Every Wednesday, the option contracts are priced as this has the lowest probability historically to be a holiday and be subject to end-of-the week bias, option contracts issued between 2016-01-04 to 2016-07-06 are used as sample in this thesis.

\(^{14}\) Yesterday’s option data used to estimate parameters that were used to price today’s options contract.
6. Results & Discussion

In this section, the empirical results will be presented. It begins with the average parameter values estimated for both in-sample and out-of-sample analysis. Second part, presents the pricing performance for both samples. Third part, is the out-of-sample analysis that is divided in a high and a low volatility sample, as explained above this is done to measure the robustness of the models. All three parts are closely related to the research questions. The last part connects to the purpose of the thesis, to give the reader a better understanding of the models. This part aims to discuss the difference between the HN-GARCH and SV-model, because the HN-GARCH model uses historical return data and the SV-model only uses option data when pricing option contacts.

6.1 Parameter estimation

The average parameters estimation values and the standard deviation from all four models are gathered from 27 observations of each loss function in the in-sample analysis and 30 observation of each loss functions in the out-of-sample analysis, a total of 114 observations. The results are presented in table 5 & 6. In total, there were 3436 option contracts priced in both in the in-sample and out-of-sample. The average parameter estimation for the 5-day out-of-sample are presented in Appendix B.

The results for the SV-model and SV-SVC model show negative values of the \( \rho \) term, meaning it is a negative correlation between returns and volatility. This is expected since volatility rise when financial markets go down, and the results show a range between -0.75 to -0.99 in both the in-sample and out-of-sample. This indicates that both stochastic models can generate shapes of their calculated implied volatility that are close to the volatility smirk observed in the market. The results for the mean reversion (\( \kappa \)) term in the out-of-sample is significantly higher than the results Christoffersen, Jacobs & Heston (2009) presented, the speed of the mean reverting process of the variance in the long run is faster. This means that volatility shocks do not have a significant impact on investors, but this difference emerge because of the weeks with low volatility raised the \( \kappa \) term significantly. In the in-sample the value of the \( \kappa \) term is lower, which is reasonable since the first six months in 2016 had more volatile periods, which indicates that volatility shocks had an impact on investors.
Table 5
Average Parameter Estimates In-Sample

Average parameters estimation calculated from 2016-01-04 to 2016-07-04 every Wednesday, a total of 27 observation for both objective functions. The SV = Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents the different loss functions used in the optimization problem. The λ term in HN-GARCH model is not estimated and set to -1/2, more details in section 5.2.3.

<table>
<thead>
<tr>
<th></th>
<th>κ</th>
<th>θ</th>
<th>σ</th>
<th>V₀</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV-SVC%</td>
<td>3.87</td>
<td>0.08</td>
<td>0.40</td>
<td>0.03</td>
<td>-0.78</td>
</tr>
<tr>
<td></td>
<td>(4.99)</td>
<td>(0.10)</td>
<td>(0.32)</td>
<td>(0.03)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>SV-SVC£</td>
<td>4.83</td>
<td>0.07</td>
<td>0.43</td>
<td>0.03</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td>(5.60)</td>
<td>(0.11)</td>
<td>(0.43)</td>
<td>(0.03)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>SV%</td>
<td>7.03</td>
<td>0.04</td>
<td>0.31</td>
<td>0.03</td>
<td>-0.73</td>
</tr>
<tr>
<td></td>
<td>(5.43)</td>
<td>(0.07)</td>
<td>(0.19)</td>
<td>(0.02)</td>
<td>(0.51)</td>
</tr>
<tr>
<td>SV£</td>
<td>4.05</td>
<td>0.03</td>
<td>0.36</td>
<td>0.03</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>(3.27)</td>
<td>(0.05)</td>
<td>(0.36)</td>
<td>(0.02)</td>
<td>(0.23)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>ω</th>
<th>β + αγ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>HN-GARCH%</td>
<td>1.05E-06</td>
<td>0.81</td>
<td>297.08</td>
<td>4.65E-06</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>(7.59E-05)</td>
<td>(0.04)</td>
<td>(31.01)</td>
<td>(1.67E-07)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>HN-GARCH£</td>
<td>1.07E-06</td>
<td>0.82</td>
<td>286.26</td>
<td>4.42E-06</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>(5.46E-08)</td>
<td>(0.04)</td>
<td>(17.56)</td>
<td>(3.32E-07)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>α₀</th>
<th>α₁</th>
<th>α₂</th>
<th>α₃</th>
<th>α₄</th>
<th>α₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>PBS%</td>
<td>3.78</td>
<td>-1.08E-03</td>
<td>8.00E-08</td>
<td>-1.34</td>
<td>0.15</td>
<td>1.89E-04</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(4.33E-05)</td>
<td>(4.33E-09)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(7.17E-06)</td>
</tr>
<tr>
<td>PBS£</td>
<td>3.77</td>
<td>-1.06E-03</td>
<td>7.72E-08</td>
<td>-1.30</td>
<td>0.16</td>
<td>1.90E-04</td>
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<td>(0.18)</td>
<td>(3.50E-05)</td>
<td>(2.09E-09)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(3.12E-06)</td>
</tr>
</tbody>
</table>

The value of the θ term, long term variance, ranges from 0.01 to 0.08 in table 5 & 6, there is a difference between the SV the SV-SVC-model in the in-sample that is hard to explain, the SV-SVC has a bigger interval of the θ term. The range of the parameters is not visibly in the out-sample and the values of θ are in line with previous research, He & Zhu (2016), Christoffersen, Jacobs & Heston (2009) and Schoutens, Simons, & Tistaert (2004) have all similar values. Indicating that the long-term variance does not change. The σ term, the volatility of the variance, is not as high as expected, since 2016 has been a year with volatility spikes. Christoffersen, Jacobs & Heston (2009) have similar results during years with low volatility and He & Zhu (2016) have higher results during distress times, 2007 and 2012. The explanation for this can be that despite that 2016 have been a year with high spikes in the VIX, the market has been strong and on average have 2016 been a year with low volatility. The average VIX level for 2016 (17.4) has been below the five-year average (17.7). This is a reasonable explanation to why the V₀ term, initial level of variance, has been low, ranges between 0.03 to 0.05. Since there have been more days with low implied volatility than days with high implied volatility.
volatility in the in-sample period, the values of the $V_0$ term increases in the out-of-sample period, as more days with high volatility are included.

Table 6

Average Parameter Estimates Out-Of-Sample, 1-Day.

Average parameters estimation calculated from six different weeks, three low volatility weeks and three high volatility weeks, ranging from 2016-01-18 to 2016-12-20, see figure 2, a total of 30 observation for both objective functions. The model’s parameters are estimated at period $t - 1$ and used to price option at period $t$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents the different loss functions used in the optimization problem. The $\lambda$ term in HN-GARCH model is not estimated and is set to $-1/2$, see more details in section 6.2.3.

<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$V_0$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV-SVC%</td>
<td>9.82</td>
<td>0.01</td>
<td>0.59</td>
<td>0.05</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>(4.64)</td>
<td>(0.01)</td>
<td>(0.80)</td>
<td>(0.05)</td>
<td>(0.19)</td>
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<tr>
<td>SV-SVÇE</td>
<td>7.97</td>
<td>0.02</td>
<td>0.48</td>
<td>0.04</td>
<td>-0.87</td>
</tr>
<tr>
<td></td>
<td>(5.61)</td>
<td>(0.02)</td>
<td>(0.82)</td>
<td>(0.04)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>SV%</td>
<td>9.87</td>
<td>0.02</td>
<td>0.41</td>
<td>0.04</td>
<td>-0.87</td>
</tr>
<tr>
<td></td>
<td>(5.79)</td>
<td>(0.01)</td>
<td>(0.61)</td>
<td>(0.04)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>SV£</td>
<td>7.75</td>
<td>0.04</td>
<td>0.55</td>
<td>0.05</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>(5.91)</td>
<td>(0.07)</td>
<td>(0.58)</td>
<td>(0.04)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>HN-GARCH%</td>
<td>1.13E-06</td>
<td>0.69</td>
<td>354.60</td>
<td>5.05E-06</td>
<td>0.78</td>
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<tr>
<td></td>
<td>(1.34E-05)</td>
<td>(0.15)</td>
<td>(88.85)</td>
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<td>HN-GARCH£</td>
<td>1.08E-06</td>
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<td>(8.92E-08)</td>
<td>(0.11)</td>
<td>(70.71)</td>
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<tr>
<td>PBS%</td>
<td>3.53</td>
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<td>(0.78)</td>
<td>(2.32E-04)</td>
<td>(1.75E-08)</td>
<td>(0.72)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>PBS£</td>
<td>3.63</td>
<td>-1.00E-03</td>
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<tr>
<td></td>
<td>(0.87)</td>
<td>(2.49E-04)</td>
<td>(1.77E-08)</td>
<td>(0.72)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

In the results of the HN-GARCH model are the mean reverting process presented, it is important the condition $\beta + \alpha \gamma^2 < 1$ is fulfilled. The mean reversion process is weaker in the in-sample, and in the out-of-sample it is stronger. The results indicate that the volatility is more persistent during high levels in the VIX since the mean reversion process is weaker in the in-sample, and stronger in the out-of-sample, because it includes days where the VIX has extreme low levels. The value of $\alpha$ and $\beta$ are in line with previous findings of Su, Chen & Huang (2010a) and Heston & Nandi (2000), the value of $\alpha$ is consistent in both in-sample and out-of-sample, $\beta$ has the same characteristic as the mean reversion process, as it is lower in the out-of-sample. The $\gamma$ term is hard to detect, since risk-neutral skewness parameter cannot be estimated from the returns series, to estimate the risk-neutral skewness must the risk neutral distribution first be estimated from the option data. This is a science in itself and a complicated procedure. The
reader is referred to Stephen, Pradeep & Yuanyuan (2009) for more details on how the estimation procedure of the $\gamma$ term can be performed. This thesis will instead set the $\gamma$ term close to Heston & Nandi (2000) and Moyaert & Petitjean (2011) average estimated values of the $\gamma$ term. The starting value was set to 400 for the first estimation and previous parameters value from the optimization was then used as starting values throughout, this is described in detail in the method part. In the optimization procedure was the average value of $\gamma$ term approximately 280 in-sample and 350 in the out-of-sample. The estimation of the skewness parameters is not performed and this does not seem to affect the results since it is only used for the first estimation, and the interval of the of $\gamma$ term is 200 to 600 in previous reports. The positive values mean that when the index level falls will the variance rise, the positive values suggest that the variance rises more during time of distress. During times of high volatility have the increase in the $\gamma$ term been higher as seen in standard deviation in the out-of-sample.

The PBS model’s parameters are harder to analyse since the starting values are estimated with an OLS regression and is not connected to historical indicators. The results show that the standard deviation of the parameter values is reduced compared to Christoffersen & Jacobs (2004b) results which had larger fluctuations in their parameter values. This can be explained by the method followed in this thesis, where the starting values are dependent on previous findings and not estimated with OLS, only in the beginning, therefore have the parameters values less fluctuations. The PBS-model can construct the volatility smirk, the results in Appendix F shows that, the average parameter values that are in line with previous findings also indicates this, since $\alpha_1$ is negative and $\alpha_2$ positive, the coefficients $\alpha_3-\alpha_5$ have a downward slope, because $\alpha_3$ is negative, and the convex structure appears since $\alpha_4-\alpha_5$ is positive.

The results of the parameters for all models are in-line with expectations of the FTSE 100 returns series and from previous findings. The empirical results and the visualization in Appendix F shows that the model’s abilities to capture the volatility term structure and generate a volatility smirk seems to be successful especially the SV model and SV-SVC model, but the PBS and HN-GARCH model have problems creating a similar volatility surface as in figure 2.
6.2 Pricing performance

The loss functions described in the method section are all in squared form or in absolute value form, and in the evaluation of the models are the same loss functions used in the same form except for the mean error sum of squares (MSE £), this instead referred to as the RMSE£, meaning the root-squared form is used instead to analyse the results.

6.2.1 Out-Of-Sample performance

The purpose of an out-of-sample analysis must first be discussed, it can be used to find the optimal set of parameters that can price option contracts precise regardless of time-period and market conditions. This is difficult and no optimal set of parameters exists for the models included in this thesis. The perspective of this thesis is practical and does not aim to search for the optimal set of parameters in the models, instead is the purpose to test if the models can successfully be used to price tomorrows option price based on the information we have today.

The out-of-sample analysis is not a true out-of-sample analysis since that is time consuming and the data material would be limited, instead is a pseudo-out-of-sample analysis is performed. This means that a date is chosen where the information is gathered from and used to price the option contracts on the following day. The main interest of this analysis is the performance of the 1-day parameter in the out-of-sample period, since this is the closet we get to an out-sample evaluation under similar markets conditions to when the models was estimated, the 5-day parameter in the out-of-sample has the purpose of testing the robustness of the models.

Table 7 shows that the SV model yields has the best pricing performance closely followed by the SV-SVC model, the SV model outperform all other models in both objective functions and in all loss functions. In table 2 Appendix A, are the results divided in to moneyness category and different maturity-periods. Table 2 in Appendix A shows that the results are mixed as the SV-SVC model performs best in some moneyness categories and in different maturity periods. The SV model does have better pricing performance with short maturity options, measured in MSE%, because cheaper options is overrepresented in $T < 40$, therefore are the results measured in MSE% in favour for the SV-model in the %-objective function, but the results are similar in the £-objective function. According to the results are the pricing performance similar but with an advantage for the SV model, according to table 2, are there no specific patterns where one model outperforms the other. In table 8 are the results similar, the difference is broader in the £-objective compared to the %-objective function. Kilin (2011) presents a small
sample as the empirical evidence and only compare the implied volatility. Based on his results he implicates that the modification is suitable for practical use, because the results are close to the original model and the modification significantly reduce the estimation time. According to the empirical results in this thesis are the results indeed very close to the SV-model measured in IV MSE, the difference is small but significant for the other loss functions, but remember that original model in this thesis is also modified\textsuperscript{15} with the goal to improve the empirical results. He & Zhu (2016) includes both a modified model and the Heston model, the models yield better results compared to the SV-model and SV-SVC model on S&P 500 index options, the time-period was 2011, the difference measured in MSE % is around three percentage points in the out-of-sample.

### Table 7
**Average Out-Of-Sample Error, 1-Day.**

Average loss function calculated from six different weeks, three low volatility weeks and three high volatility weeks, ranging from 2016-01-18 to 2016-12-20, see figure 2, a total of 30 observation for both objective functions. The model’s prices are estimated with period $t-1$ options data and used to price options at period $t$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (5.4) and (5.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: %</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.04</td>
<td>0.04</td>
<td>0.13</td>
<td>0.27</td>
</tr>
<tr>
<td>(0.10)</td>
<td>(0.15)</td>
<td>(0.27)</td>
<td>(0.56)</td>
<td></td>
</tr>
<tr>
<td>MSE%</td>
<td>6.00</td>
<td>7.5</td>
<td>14.15</td>
<td>16.37</td>
</tr>
<tr>
<td>(16.37)</td>
<td>(18.31)</td>
<td>(38.16)</td>
<td>(58.56)</td>
<td></td>
</tr>
<tr>
<td>RMSE£</td>
<td>11.11</td>
<td>11.31</td>
<td>16.30</td>
<td>20.14</td>
</tr>
<tr>
<td>(12.40)</td>
<td>(14.49)</td>
<td>(16.34)</td>
<td>(34.50)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.04</td>
<td>0.06</td>
<td>0.13</td>
<td>0.34</td>
</tr>
<tr>
<td>(0.10)</td>
<td>(0.22)</td>
<td>(0.26)</td>
<td>(0.84)</td>
<td></td>
</tr>
<tr>
<td>MSE%</td>
<td>6.69</td>
<td>7.40</td>
<td>14.94</td>
<td>13.23</td>
</tr>
<tr>
<td>(24.10)</td>
<td>(21.73)</td>
<td>(41.16)</td>
<td>(39.66)</td>
<td></td>
</tr>
<tr>
<td>RMSE£</td>
<td>10.48</td>
<td>11.25</td>
<td>16.17</td>
<td>20.37</td>
</tr>
<tr>
<td>(11.87)</td>
<td>(15.23)</td>
<td>(16.13)</td>
<td>(34.45)</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{15} The little Heston trap and the Gauss-Laguerre quadrature procedure, see section 2.5.
### Table 8
Average Out-Of-Sample Error, 5-Days.

Average loss function calculated from six different weeks, three low volatility weeks and three high volatility weeks, ranging from 2016-01-18 to 2016-12-20, see figure 2, a total of 30 observation for both objective functions. The model’s prices are estimated with period \( t - 1 \) options data and used to price options at period \( t \) to \( t + 4 \). The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: %</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV RMSE</td>
<td>0.06</td>
<td>0.07</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.19)</td>
<td>(0.31)</td>
<td>(0.71)</td>
</tr>
<tr>
<td>RMSE%</td>
<td>7.74</td>
<td>10.93</td>
<td>15.31</td>
<td>12.63</td>
</tr>
<tr>
<td></td>
<td>(21.36)</td>
<td>(29.43)</td>
<td>(49.90)</td>
<td>(40.22)</td>
</tr>
<tr>
<td>RMSE£</td>
<td>13.36</td>
<td>14.88</td>
<td>16.92</td>
<td>16.92</td>
</tr>
<tr>
<td></td>
<td>(17.64)</td>
<td>(17.07)</td>
<td>(16.53)</td>
<td>(32.46)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV RMSE</td>
<td>0.06</td>
<td>0.07</td>
<td>0.15</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.19)</td>
<td>(0.31)</td>
<td>(0.59)</td>
</tr>
<tr>
<td>MSE%</td>
<td>10.52</td>
<td>10.07</td>
<td>16.59</td>
<td>11.30</td>
</tr>
<tr>
<td></td>
<td>(34.77)</td>
<td>(29.35)</td>
<td>(54.89)</td>
<td>(42.18)</td>
</tr>
<tr>
<td>MSE£</td>
<td>12.19</td>
<td>12.84</td>
<td>16.41</td>
<td>15.72</td>
</tr>
<tr>
<td></td>
<td>(12.74)</td>
<td>(13.87)</td>
<td>(16.38)</td>
<td>(31.57)</td>
</tr>
</tbody>
</table>

The HN-GARCH shows weaker results than expected in table 7 & 8, it outperforms the PBS-model, both in the IV MSE and the MSE£, but have significantly weaker results compared to the SV-model and SV-SVC-model. This is concluded for both objective functions, but the HN-GARCH model have strong results in the category of options with \( T > 70 \) days to maturity, which can be seen in table 2 Appendix A and visually in Appendix F. The 5-day out-of-sample shows similar results, according to table 3 in Appendix A, are the HN-GARCH model consequent with outperforming all models in all loss functions and both objective functions for options with \( T > 70 \) days to maturity. While the results are poor for the HN-GARCH model, are the results mixed compared to previous findings, according to Su, Chen & Huang (2010) it underperforms, in MSE%, and compared to Moyaert & Petitjean (2011) is it outperforming their version of the HN-GARCH model measured in MSE%. The PBS-model underperforms, as expected, according to table 2 & 3 in Appendix A. If studied, it is shows that the model has problems with long dated options, \( (T > 70) \). The results could be improved if the long-dated options were to be excluded, on the other hand this is not fair. The IV MSE, especially shows that the PBS model has troubles with capturing market implied volatility on long dated options, the graphs in Appendix F also indicates this. The HN-GARCH model outperform the PBS
model when the objective function and the loss function is the same. Interesting is that the opposite occurs in table 8, but the HN-GARCH model outperforms the PBS-model in IV MSE, but the PBS model have better results measured in pricing performance, the opposite occurs in table 7. The HN-GARCH model shows poor results in table 2 & 3 in Appendix A, IV MSE with short maturity \(T < 40\) options, this can be an indicator that the model has a problem with constructing the volatility smile of short maturity options, and short maturity options contains the most information about future beliefs, as the trading volumes are higher. The HN-GARCH model do not exclusively use option data, it also uses historical returns in the pricing method, crucial information is therefore lost and that is a probable explanation to why the 5-day sample has poor pricing performance. Both HN-GARCH model and the PBS model implied volatility is not close to market implied volatility in option with \(T<40\) days of maturity, this can also be seen in the graphs in Appendix F.

### 6.2.2 In-Sample Performance

In-sample results shows similar results as the out-of-sample analysis, the performance of the SV model and SV-SVC model are similar. The % objective function shows no significant difference between the models in the IV RMSE. While the difference in the pricing performance depends on the SV-SVC-model poor results with short maturity options \((T < 40)\), and long-dated options \((T > 70)\) that are OTM, according to table 1 in Appendix A. While option that are ATM and ITM, the results are similar between the models. In the £-objective function is the difference not as clear and the SV-SVC model yield better results in both the IV MSE and the MSE £, this is because the problems with short maturity options \((T < 40)\) does not occur in the £-objective function. Table 9 shows that the HN-GARCH model outperform the PBS model in the in-sample compared to the out-sample analysis, both objective function and in all loss functions. While the HN-GARCH model is, as in the out-of-sample analysis, significantly outperformed by both the SV model and SV-SVC model. Furthermore, table 1 in Appendix A shows that the HN-GARCH model underperform for options that are ITM, compared to the PBS-model, the HN-GARCH model has, measured in average, better results.
Table 9
Average In-Sample Error

Average loss function calculated from 2016-01-04 to 2016-07-04 every Wednesday, a total of 27 observation for both objective functions. The model’s prices are estimated at period $t$ and the parameters are used to price options at period $t$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss function used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION; %</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.04</td>
<td>0.04</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>MSE%</td>
<td>3.99</td>
<td>6.52</td>
<td>10.04</td>
<td>12.68</td>
</tr>
<tr>
<td>RMSE£</td>
<td>9.09</td>
<td>11.50</td>
<td>13.96</td>
<td>22.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.05</td>
<td>0.04</td>
<td>0.11</td>
<td>0.18</td>
</tr>
<tr>
<td>MSE%</td>
<td>5.47</td>
<td>5.91</td>
<td>10.37</td>
<td>12.83</td>
</tr>
</tbody>
</table>

The two objective functions performances differ, the pricing performance is better for the loss function that uses the same objective function, meaning that the %-objective function yield better results for the models in the loss function MSE % and the £-objective function yield better results in the RMSE £ loss function, which is in line with the findings from Christoffersen & Jacobs (2004b). Both loss functions disadvantages appear clearly in the results.

6.3 Robustness Performance

In the beginning of this thesis was it describe that 2016 has been a year with high spikes in the VIX, but also extreme lows, this created an interesting scenario to analyse, since implied market volatility has a large effect on option prices. The hypothesis is that high implied market volatility creates prices that are not based on rational beliefs, that cause a significant difference between theoretical prices and market prices compared to when the market is not in distress. According to IV MSE are the models that uses stochastic volatility negatively affected by a high implied volatility market, and the PBS model are negatively affected by a low implied volatility market. The results seem to support the hypothesis, the results show that the IVMSE error is larger in a high implied volatility market compared to a low implied volatility market.
The data shows higher trading volumes and open interest\textsuperscript{16} in the lower implied volatility market\textsuperscript{17}, this is explained by figure 2 & 3. In a low implied volatility market is the market strong and that creates larger trading volumes and higher open interest\textsuperscript{18}. To clarify, in a market with low implied volatility are traders more active, because of the increase in trading volume and open interest, the opposite occurs in a high implied volatility market. Therefore, are prices more accurate in a low implied volatility market because of the increase in volume.

The difference in pricing performance indicates another conclusion, the same loss function and objective function are used in the analysis as these two contradicts each other. The RMSE£ does support the findings of the IV MSE, the average pricing error is higher in the high volatility sample in table 10 & 11. The MSE % show the opposite, expect for the SV-SVC model pricing performance which becomes worst when implied market volatility is low.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: %</th>
<th>SV</th>
<th>SY-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.07</td>
<td>0.05</td>
<td>0.21</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.08)</td>
<td>(0.32)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>MSE%</td>
<td>5.28</td>
<td>7.15</td>
<td>14.29</td>
<td>10.34</td>
</tr>
<tr>
<td></td>
<td>(9.28)</td>
<td>(17.33)</td>
<td>(23.69)</td>
<td>(21.61)</td>
</tr>
<tr>
<td>RMSE£</td>
<td>14.64</td>
<td>13.63</td>
<td>22.27</td>
<td>20.06</td>
</tr>
<tr>
<td></td>
<td>(14.29)</td>
<td>(15.75)</td>
<td>(18.42)</td>
<td>(36.80)</td>
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</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SY-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.07</td>
<td>0.10</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.25)</td>
<td>(0.32)</td>
<td>(0.36)</td>
</tr>
<tr>
<td>MSE%</td>
<td>6.31</td>
<td>8.56</td>
<td>14.10</td>
<td>8.75</td>
</tr>
<tr>
<td></td>
<td>(25.55)</td>
<td>(22.37)</td>
<td>(23.51)</td>
<td>(21.22)</td>
</tr>
<tr>
<td>RMSE£</td>
<td>12.39</td>
<td>14.75</td>
<td>22.14</td>
<td>22.10</td>
</tr>
<tr>
<td></td>
<td>(13.26)</td>
<td>(17.70)</td>
<td>(18.41)</td>
<td>(39.83)</td>
</tr>
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</table>

Table 10
Average Out-Of-Sample Error High Volatility, 1-Day.
Average loss function calculated from three different weeks with high implied volatility, ranging from 2016-09-06 to 2016-12-20, see figure 2, total of 15 observation for both objective functions. The model’s prices are estimated with period $t - 1$ options data and used to price options at period $t$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss function used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

\textsuperscript{16} Open interest is a measure for the flow of money in to the option market, higher number indicates more money in to the market.

\textsuperscript{17} High implied volatility sample: Average trading volume = 290 and average open interest = 3154

Low implied volatility sample: Average trading volume = 315 and average open interest = 3383
Table 11  
Average Out-Of-Sample Error Low Volatility, 1-Day

Average loss function calculated from three different weeks with low implied volatility, ranging from 2016-01-18 to 2016-06-20, see figure 2, total of 15 observation for both objective functions. The model’s prices are estimated with period \( t - 1 \) options data and used to price options at period \( t \). The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: %</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.21</td>
</tr>
<tr>
<td>MSE%</td>
<td>(0.03)</td>
<td>(0.20)</td>
<td>(0.13)</td>
<td>(0.67)</td>
</tr>
<tr>
<td>RMSE£</td>
<td>(21.94)</td>
<td>(19.41)</td>
<td>(50.16)</td>
<td>(82.70)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>0.56</td>
</tr>
<tr>
<td>MSE%</td>
<td>(0.04)</td>
<td>(0.17)</td>
<td>(0.12)</td>
<td>(1.14)</td>
</tr>
<tr>
<td>RMSE£</td>
<td>(22.27)</td>
<td>(20.88)</td>
<td>(55.15)</td>
<td>(53.38)</td>
</tr>
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</table>

The result indicates that high implied volatility affects the models’ ability to both price and capture the implied volatility, negatively. Table 12 & 13 has similar results as the 1-day sample. The results of MSE% and RMSE £ depends on the properties of the loss functions. The RMSE £ will increase in the high implied volatility sample because prices are higher and the MSE % will instead have better results in the high implied volatility sample. This indicates that IV MSE should be used to evaluate the model’s performance in both samples. The HN-GARCH model is the model that is being affected the most by a high-implied volatility market. The difference in results are remarkably high, especially when it is measured in IV MSE and RMSE£.

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19 Average option price and average implied volatility in the low sample is, 98.93 and 15.71%. Average option price and average implied volatility in the high sample is, 119.62 and 20.22%.
Table 12
Average Out-Of-Sample Error High volatility, 5-Day
Average loss function calculated from three different weeks with high implied volatility, ranging from 2016-01-18 to 2016-06-20, see figure 2, total of 15 observation for both objective functions. The model’s prices are estimated with period \( t - 1 \) options data and used to price options at period \( t \) to \( t + 4 \). The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: %</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.10</td>
<td>0.10</td>
<td>0.25</td>
<td>0.17</td>
</tr>
<tr>
<td>(0.23)</td>
<td>(0.17)</td>
<td>(0.37)</td>
<td>(0.41)</td>
<td></td>
</tr>
<tr>
<td>MSE%</td>
<td>7.10</td>
<td>12.61</td>
<td>14.39</td>
<td>7.34</td>
</tr>
<tr>
<td>(17.50)</td>
<td>(32.62)</td>
<td>(20.26)</td>
<td>(23.33)</td>
<td></td>
</tr>
<tr>
<td>RMSE£</td>
<td>16.46</td>
<td>17.92</td>
<td>23.36</td>
<td>16.52</td>
</tr>
<tr>
<td>(20.98)</td>
<td>(19.03)</td>
<td>(18.30)</td>
<td>(36.98)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
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<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.09</td>
<td>0.10</td>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>(0.22)</td>
<td>(0.17)</td>
<td>(0.37)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>MSE%</td>
<td>10.79</td>
<td>10.97</td>
<td>13.91</td>
<td>6.17</td>
</tr>
<tr>
<td>(38.50)</td>
<td>(33.17)</td>
<td>(19.76)</td>
<td>(20.07)</td>
<td></td>
</tr>
<tr>
<td>RMSE£</td>
<td>14.00</td>
<td>15.82</td>
<td>23.20</td>
<td>15.97</td>
</tr>
<tr>
<td>(12.75)</td>
<td>(13.88)</td>
<td>(16.31)</td>
<td>(31.65)</td>
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</tbody>
</table>

Table 13
Average Out-Of-Sample Error Low Volatility, 5-Day
Average loss function calculated from three different weeks with low implied volatility, ranging from 2016-01-18 to 2016-06-20, see figure 2, total of 15 observation for both objective functions. The model’s prices are estimated with period \( t - 1 \) options data and used to price options at period \( t \) to \( t + 4 \). The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds.

<table>
<thead>
<tr>
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<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.35</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.20)</td>
<td>(0.13)</td>
<td>(0.94)</td>
<td></td>
</tr>
<tr>
<td>MSE%</td>
<td>8.35</td>
<td>8.71</td>
<td>17.30</td>
<td>18.54</td>
</tr>
<tr>
<td>(25.16)</td>
<td>(25.02)</td>
<td>(70.34)</td>
<td>(53.05)</td>
<td></td>
</tr>
<tr>
<td>RMSE£</td>
<td>9.70</td>
<td>11.29</td>
<td>7.82</td>
<td>17.39</td>
</tr>
<tr>
<td>(11.57)</td>
<td>(13.59)</td>
<td>(8.33)</td>
<td>(26.15)</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>OBJECTIVE FUNCTION: £</th>
<th>SV</th>
<th>SV-SVC</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.39</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.20)</td>
<td>(0.12)</td>
<td>(0.80)</td>
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<tr>
<td>MSE%</td>
<td>10.21</td>
<td>9.01</td>
<td>19.76</td>
<td>17.35</td>
</tr>
<tr>
<td>(29.78)</td>
<td>(24.04)</td>
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</tr>
<tr>
<td>RMSE£</td>
<td>10.05</td>
<td>9.32</td>
<td>8.38</td>
<td>15.44</td>
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<tr>
<td>(9.90)</td>
<td>(9.77)</td>
<td>(8.16)</td>
<td>(23.21)</td>
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</table>
6.4 Comparison of the GARCH and the Stochastic volatility model

The research of GARCH models has lately revolved around maximum likelihood estimation of and the use of Monte-Carlo simulations. The results are mixed, the GARCH model included in this thesis is in closed form, and the HN-GARCH have outperformed and underperformed compared to previous studies. Even though the SV model, SV-SVC model and the HN-GARCH models are in closed form and uses stochastic volatility to better capture the implied volatility term structure, does it exist difference between the models. The difference is that the SV-models uses option data in the estimation of the parameters and in the pricing formula. The HN-GARCH model uses constant volatility from historical returns, the historical returns series and option data in the estimation of the parameters. To price the option contract must the variance term \( (h_{t+1}) \) be calculated, this is done with the model’s estimated parameters and the error term estimated at period \( t \), the error term includes the returns and the GARCH term at period \( t - 1 \).

Therefore, would it be interesting to continue the work of Kanniainen, Lin. & Yang (2014) and include the VIX returns series instead of the FTSE100 return series, to see if it can improve the results. According to Heston & Nandi (2000), if an investor strictly uses option data can significant information be lost, as historical returns provides crucial information about the underlying asset, and to only use option data can cause overfitting problems to the model. This statement is not supported by the results in this thesis as the SV-models outperform the HN-GARCH model and previous GARCH models estimated with maximum likelihood estimation. According to the results, provide option data more useful information, because the SV-models that only uses option data price option contracts more accurately compared to the GARCH models that uses returns series. The results of Kanniainen, Lin. & Yang (2014) also support this, as the VIX is constructed from option data. But to draw a conclusion about this would additional research be needed on the HN-GARCH model, that would include the VIX return series, use both maximum likelihood estimation and the loss function approach to fully investigate if only option data should be used and which type of models perform has the best performance.

The difference between the VIX data and historical return series data displayed itself during the Brexit vote, on Thursday 23 of June, Britain voted to leave the European Union. On Friday, 24th of June, began the fall of stock markets around the world and the stock market continued to fall on Monday, 27th of June, from Thursday to Monday had the FTSE100 index fallen almost six percent. Naturally, would one think that the spike of the VIX index would occur on these
days, but the spike occurred on Thursday, one week earlier, 16th of June. The VIX index then slowly decreased on Friday and on Monday did it rose but not close to the spike one week earlier. This indicates that the option market had already reacted and was not as surprised as the stock market was when the no-side won, because the VIX did not jump back to previous level, and the VIX index then showed signs that the market was going to bounce back, as it also did20. After the vote, the VIX gave signals that a strong market would begin for the FTSE 100 index, because of the significant decrease in the VIX, this can be seen in figure 2 & 3. The VIX have interesting characteristics, as it contains valuable information about future beliefs among investors that could not otherwise be obtained. As mentioned above, should more research be performed in order to reach a conclusion regarding if the VIX return series (option data) is superior to use in option pricing, but the results in thesis does support that conclusion.

20 The VIX spike occurred 16th of June and one week after the vote, 30th of June, had the VIX fallen approximately 40%.
7. Conclusion

This thesis has compared two stochastic volatility models (SV & SV-SVC), one GARCH model (HN-GARCH) and one Ad-hoc local volatility model (PBS). The results indicate that the PBS has the worst pricing performance of all the models, both the in-sample and the out-of-sample analysis. The problems seem to be severe with long dated option, the ability of the PBS-model to accurately price long dated options are significantly worse compared to the other models. The results are in line with other reports, although other reports have found better results for the PBS-model, Chen & Huang (2010) and Moyaert & Petitjean (2011) yield better results in their PBS-models, while the PBS results in this thesis outperforms the Christoffersen, Jacobs & Heston (2009) model. The significant difference seems to be in the fluctuations of the parameter values, since this thesis uses a method where the starting values are dependent on the previous periods values, and is not re-estimated with an OLS-regression, this have a significant impact on the fluctuations of the parameters.

The PBS-model is closely followed by the HN-GARCH regarding pricing performance, the results are not in line with the initial expectations of the HN-GARCH model. Since the SV-models outperform the HN-GARCH model in both in-sample and out-of-sample, the pricing performance is not even close. It exists limited research on the closed form of the HN-GARCH model, but the results are in-line with previous findings, since it underperforms against Su, Chen & Huang (2010 and outperforms Moyaert & Petitjean (2011) model. The main problem seems to be short-dated option where the HN-GARCH have significantly worse results compared to the SV-model and the SV-SVC-model, while at the same time shows promising results for long-dated option. However, the long-dated options have a small sample size and that is why the HN-GARCH model shows such disappointing results in the average pricing performance.

Both stochastic volatility models have significantly better results both in the in-sample and out-sample compared to the PBS model and HN-GARCH model. Overall the performance and the results are good across all maturity categories and moneyness categories. To conclude that both models outperform the other models is not difficult, the problem arises when you want to conclude which of the stochastic volatility models has the best performance, which model is the least affected by high and low implied volatility market and which one is more practical to use. Starting with the pricing performance, it is very close, overall the results are in favour of
the SV model, both in the in-sample and out-of-sample analysis, IV MSE is the decisive factor and it shows a small advantage for the SV model. Next is the factor of high and low implied volatility market, all models out-of-sample ability seems to be affected by high implied volatility market, except for the PBS-model that performs better during high implied volatility market. The 5-day sample shows similar results as the 1-day sample, the difference is that the pricing performance does not support the hypothesis as strong, since the RMSE£ shows that all models performs worse in a high implied volatility market. This is because of properties of the loss function as the error will increase as option prices increases. But according to IV MSE, are the results negatively affected by a high implied volatility market, which is line with the initial beliefs. Moving on to which model is more practical to use according to table 2 in Appendix E, the estimation of the SV-SVC model parameters is approximately three times faster in the in-sample, this is important if it is a large dataset of option. The modification of Kilin (2011) introduced a smart modification of the original SV-model and the results are very similar, Kilin (2011) had a small sample size when he tested his model and concluded that the results was similar measured in IV MSE. This thesis has also included modifications of the original model to enhance the performance, the modifications are the little Heston trap and the Gauss-Laguerre quadrature procedure, both helps to improve the results of the original model compared to other studies. The results of the SV-SVC model are similar to the SV model, it shows stable values across different maturity categories and moneyness categories, good performance in the out-sample analysis and it significantly outperform the SV-model in the estimation time of the parameters. This make the SV-SVC model an attractive model for practitioners to use to determine prices on stock index options.

The difference between the two loss functions are mixed but the literature does not give any recommendation regarding which function that should be used, as neither seem to have a significant advantage over the other. The disadvantages of the loss functions mentioned above, display itself in the results and that is why the IV MSE is mainly used to evaluate the models.

7.1 Further research
To continue the work of this thesis, the next step would be to include both affine and non-affine GARCH models that use FTSE 100 VIX index series instead of returns series data and compare them against affine stochastic volatility models. Include Maximum Likelihood estimations, Kaeck and Alexander (2012) and Liu, Siu-Hang & Cheuk-Yin (2014) are implementing
GARCH models with non-affine solutions\textsuperscript{21} and gives better results than affine solutions. But Christoffersen, P, Jacobs, K. & Mimouni, K (2010b) argues that model with non-affine solutions gives better results. The N-GARCH, T-GARCH and E-GARCH models can be implemented with the non-affine solutions. Further research in optimizing the HN-GARCH model in its closed form is something that is missing in the literature and it would be interesting to see if the same progress could be made for GARCH models as the optimizing of the Heston stochastic volatility model, where He & Zhu (2016) and Kilin (2011) and others have made progress on that subject.

Stochastic volatility models yield the best results in the affine solution and the SV-model shows promising results, especially with the modifications included in this thesis. Previous reports have proposed adding volatility jumps and additional parameters to improve the results of the SV model, the disadvantage of adding jumps and more parameters is the significant increase in complexity and estimation time. It seems that practitioners are searching for a model that is accurate, but also simple and fast. Because if an investor is working with intraday data it is not possible to have a model with an estimation time of hours, instead it is important that model can price contracts in a matter of seconds. Therefore, it would be interesting to see a further study on the He & Zhu (2016) developed model, to see if the original model can be optimized even further and to test if He & Zhu (2016) developed model can yield good results on different type of data.

\textsuperscript{21} Non-affine solution implicates that stock prices must be simulated via Monte Carlo simulation.
8. References


Appendix A

Table 1

Average in-sample error. Divided in moneyness and to maturity categories.

Average loss function calculated from 2016-01-04 to 2016-07-04 every Wednesday, total of 27 observation for both objective functions. The model’s prices are estimated at period t, as the parameters used to price options at period t. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss functions used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds. O=Out-of-money, A=At-the-money and I=In-the-money. T represents days to maturity.

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>Model</th>
<th>0.90&lt;S/K&lt;0.97(O)</th>
<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>T&lt;40</td>
<td>T&lt;70</td>
<td>T&gt;70</td>
</tr>
<tr>
<td>IV MSE%</td>
<td>SV-SVC</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
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<td></td>
<td>SV</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
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<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
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<td>0.05</td>
<td>0.08</td>
</tr>
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<td>8.79</td>
<td>14.74</td>
</tr>
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<td></td>
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<td>7.01</td>
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<td></td>
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<td>4.27</td>
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</tr>
<tr>
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<td>2.60</td>
<td>3.84</td>
<td>10.07</td>
</tr>
<tr>
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<td>7.95</td>
<td>8.42</td>
<td>18.80</td>
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</table>

<table>
<thead>
<tr>
<th>Objective Function: £</th>
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<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T&lt;40</td>
<td>T&lt;70</td>
<td>T&gt;70</td>
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<tr>
<td>IV MSE</td>
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<td></td>
<td>PBS</td>
<td>7.32</td>
<td>7.91</td>
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The % and £ observation for both objective functions. The model’s prices are estimated with period £–1 options data and used to price options at period t. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss function used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds. O=Out-of-money, A=At-the-money and I=In-the-money. T represents days to maturity.

<table>
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</tr>
<tr>
<td></td>
<td>15.26</td>
<td>18.65</td>
<td>25.18</td>
</tr>
<tr>
<td></td>
<td>7.50</td>
<td>9.74</td>
<td>17.14</td>
</tr>
<tr>
<td></td>
<td>24.60</td>
<td>27.76</td>
<td>16.88</td>
</tr>
<tr>
<td></td>
<td>10.31</td>
<td>3.14</td>
<td>27.33</td>
</tr>
</tbody>
</table>
Table 3
Average out-of-sample error, 5-Day. % or £ per moneyness category, £ or % loss function.
Average loss function calculated from six different weeks, three low volatility weeks and three with high volatility, ranging from 2016-01-18 to 2016-12-20, see figure 2, total of 30 observation for both objective functions. The model’s prices are estimated with period $t−1$ options data and used to price options at period $t$ to $t+4$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of the Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents formula (6.4) and (6.3) the different loss function used in the search algorithm. IV MSE and MSE % are presented in percent and MSE £ is presented in British pounds. O=Out-of-money, A=At-the-money and I=In-the-money. $T$ represents days to maturity.

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>Model</th>
<th>0.90&lt;S/K&lt;0.97(O)</th>
<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.90&lt;S/K&lt;0.97(O)</td>
<td>0.97≤S/K≤1.03(A)</td>
<td>1.03&lt;S/K&lt;1.10(I)</td>
</tr>
<tr>
<td>IV MSE</td>
<td>SV-SVC</td>
<td>0.07</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>0.24</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>0.15</td>
<td>0.06</td>
<td>0.73</td>
</tr>
<tr>
<td>MSE%</td>
<td>SV-SVC</td>
<td>55.76</td>
<td>15.02</td>
<td>11.59</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>21.82</td>
<td>9.61</td>
<td>9.29</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>48.10</td>
<td>16.07</td>
<td>3.94</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>21.05</td>
<td>12.83</td>
<td>26.72</td>
</tr>
<tr>
<td>RMSE£</td>
<td>SV-SVC</td>
<td>7.81</td>
<td>9.38</td>
<td>16.76</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>6.46</td>
<td>6.95</td>
<td>15.32</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>13.22</td>
<td>11.71</td>
<td>9.12</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>7.45</td>
<td>6.46</td>
<td>18.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>Model</th>
<th>0.90&lt;S/K&lt;0.97(O)</th>
<th>0.97≤S/K≤1.03(A)</th>
<th>1.03&lt;S/K&lt;1.10(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV MSE</td>
<td>SV-SVC</td>
<td>0.07</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>0.07</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>0.24</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>0.15</td>
<td>0.12</td>
<td>0.72</td>
</tr>
<tr>
<td>MSE%</td>
<td>SV-SVC</td>
<td>32.52</td>
<td>12.95</td>
<td>11.59</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>37.26</td>
<td>12.76</td>
<td>7.82</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>48.06</td>
<td>16.52</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>18.43</td>
<td>8.99</td>
<td>29.09</td>
</tr>
<tr>
<td>RMSE£</td>
<td>SV-SVC</td>
<td>6.80</td>
<td>8.40</td>
<td>13.98</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>7.26</td>
<td>7.16</td>
<td>10.32</td>
</tr>
<tr>
<td></td>
<td>HN-GARCH</td>
<td>13.03</td>
<td>11.60</td>
<td>9.14</td>
</tr>
<tr>
<td></td>
<td>PBS</td>
<td>6.81</td>
<td>4.97</td>
<td>18.06</td>
</tr>
</tbody>
</table>
Appendix B

Table 1
Average Parameter Estimates Out-Of-Sample, 5-Day.
Average parameters estimation calculated from six different weeks, three low volatility weeks and three high volatility weeks, ranging from 2016-01-18 to 2016-12-20, a total of 30 observation for every loss function. The model’s parameters are estimated at period $t$ and used to price option at period $t + 4$. The SV= Heston Stochastic volatility model, SV-SVC = Strike Vector Computation modification of Heston Stochastic volatility model, HN-GARCH = Heston & Nandi GARCH model and PBS = Practitioners Black & Scholes model. The % and £ represents the different loss functions used in the optimization problem. The $\lambda$ term in HN-GARCH model is not estimated and is set to -1/2, more details in section 6.2.3.

<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$V_0$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV-SVC%</td>
<td>8.01</td>
<td>0.01</td>
<td>0.14</td>
<td>0.04</td>
<td>-0.99</td>
</tr>
<tr>
<td></td>
<td>(3.83)</td>
<td>(0.01)</td>
<td>(0.09)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>SV-SVCF</td>
<td>10.43</td>
<td>0.02</td>
<td>0.41</td>
<td>0.05</td>
<td>-0.97</td>
</tr>
<tr>
<td></td>
<td>(4.89)</td>
<td>(0.01)</td>
<td>(0.40)</td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>SV%</td>
<td>9.47</td>
<td>0.01</td>
<td>0.15</td>
<td>0.04</td>
<td>-0.93</td>
</tr>
<tr>
<td></td>
<td>(5.52)</td>
<td>(0.01)</td>
<td>(0.10)</td>
<td>(0.02)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>SV£</td>
<td>7.47</td>
<td>0.02</td>
<td>0.26</td>
<td>0.04</td>
<td>-0.88</td>
</tr>
<tr>
<td></td>
<td>(6.32)</td>
<td>(0.01)</td>
<td>(0.23)</td>
<td>(0.03)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>HN-GARCH%</td>
<td>1.14E-06</td>
<td>0.61</td>
<td>386.50</td>
<td>5.08E-06</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(7.46E-05)</td>
<td>(0.20)</td>
<td>(49.22)</td>
<td>(6.97E-07)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>HN-GARCH£</td>
<td>1.07E-06</td>
<td>0.71</td>
<td>378.13</td>
<td>4.51E-06</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(6.81E-08)</td>
<td>(0.11)</td>
<td>(23.62)</td>
<td>(4.35E-07)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\gamma$</td>
<td>$\omega$</td>
<td>$\beta + \alpha \gamma^2$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>-9.46E-04</td>
<td>6.72E-08</td>
<td>-0.79</td>
<td>0.12</td>
<td>1.11E-04</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>(0.79)</td>
<td>(2.33E-04)</td>
<td>(1.72E-08)</td>
<td>(0.05)</td>
<td>(1.04E-04)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>3.52</td>
<td>-9.60E-03</td>
<td>6.75E-08</td>
<td>-0.80</td>
<td>0.12</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>(0.89)</td>
<td>(2.52E-04)</td>
<td>(1.79E-08)</td>
<td>(0.05)</td>
<td>(1.04E-04)</td>
</tr>
</tbody>
</table>
A call option, which is the right to buy the underlying asset for a specified price, is called the strike price and the contract has an expiration date which is called the maturity date. A put option gives the holder of the option contract the right to sell the underlying asset. An option gives you the right to exercise the contract and not the obligation to do so. If you write an option contract, which means that you are the seller of the contract and you receive a risk premium, as it represents a compensation for the writer for bearing the risk. Writing a call option on a stock, means that you will receive a risk premium and if the strike price of the option is higher than the market price you will keep that premium and will make a profit. Because there is no point to exercise the option for the buyer, since they cannot go to the market and buy the underlying option for a better price. But if the strike price is below the market price, then the option contract is in favour for the buyer and not for you as a writer of the option contract, therefore your loss will be the premium minus the strike price. (Hull 2011)

The reader should be aware of a terminology when it comes to describing the current position of the option contract. If the underlying asset are in-the-money (ITM) the underlying asset price is above the strike price, at-the-money (ATM) the underlying asset is equal to the strike price, and out-of-the money (OTM) the underlying asset price is below the strike price.
Appendix D

Table 1
Cleaning Procedure.
The table describes a summarize of the cleaning procedure that was performed to produce the data set. Every contract was eliminated in the order describe in the table, from 1 to 10.

<table>
<thead>
<tr>
<th>CLEANING STEPS</th>
<th>STEP-WISE REMOVALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Remove options with no traded volume or open interest</td>
</tr>
<tr>
<td>2</td>
<td>Remove options with shorter than 6 days to maturity</td>
</tr>
<tr>
<td>3</td>
<td>Remove options with the bid price greater than the ask price</td>
</tr>
<tr>
<td>4</td>
<td>Remove options where bid or ask price greater than the index level ( (S_t) )</td>
</tr>
<tr>
<td>5</td>
<td>Remove options with negative time value</td>
</tr>
<tr>
<td>6</td>
<td>Remove options with bid or ask price less than 50 pence</td>
</tr>
<tr>
<td>7</td>
<td>Remove options with moneyness ( (S_0/K) ) lower than 90% and higher than 110%</td>
</tr>
<tr>
<td>8</td>
<td>Remove options where the ask price is more than 50 % higher than bid price</td>
</tr>
<tr>
<td>9</td>
<td>Remove options with higher implied vol. than 100%</td>
</tr>
<tr>
<td>10</td>
<td>Remove options that heavily violate the requirement that call prices are monotonically decreasing in strike</td>
</tr>
</tbody>
</table>

Total number of removed options | 367 222 |
Total number of options in the data | 381 501 |
Remaining options | 14 279 |

To produce a good sample of options contract that could be valuated, a cleaning procedure suggested by Bakshi, Cao & Chen (1997) and Dumas, Fleming & Whaley (1998) was followed. It was executed on the on raw data collected from iVolatility (2017). Options contracts with no trading volume can have invalid market prices and removing options with shorter than 6 days to maturity can have liquidity-related biases (Bakshi, Cao & Chen 1997). Steps 3 to 6 ensure that we get an arbitrage-free condition and exclude option with less than 50 pence, as the models can set negative prices, these precautions are accordingly to Bakshi, Cao & Chen (1997). Step 7 is because options that are very deep (OTM) or deep (ITM), have little contribution to the volatility term structure, this is based on the recommendation from Fleming & Whaley (1998). Step 8-10 is performed to ensure that no options are included that are illiquid or produce irrational market prices, as the market price is determined by taking average between bid and ask, mid-price. These steps have been checked from 1 to 10 and after has the 14 279 options been used in the selection of in the in-sample and out-of-sample analysis.
Appendix E

Table 1
Average Parameter Starting values.
The values of the parameters used as starting values in the Nelder-Mead search algorithm for the parameter estimation. They are based on the historical returns series of the FTSE 100 index and on previous research.

<table>
<thead>
<tr>
<th>Models</th>
<th>$\kappa$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$V_0$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV-SVC</td>
<td>5</td>
<td>0.01</td>
<td>0.05</td>
<td>0.03</td>
<td>-0.75</td>
</tr>
<tr>
<td>SV</td>
<td>5</td>
<td>0.01</td>
<td>0.05</td>
<td>0.03</td>
<td>-0.75</td>
</tr>
<tr>
<td>HN-GARCH</td>
<td>1.18E-06</td>
<td>0.81</td>
<td>400</td>
<td>4.49E-06</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Table 2
Average estimation time of the parameters.
Average time for the parameter estimation in the Nelder-Mead search algorithm, the results are presented in seconds and based on the average estimation time of the parameters included in the in-sample data. The computer used was an Macbook Air, Intel Core i5-5250U, 1.6 GHz x 2, 4GB RAM.

<table>
<thead>
<tr>
<th>Models</th>
<th>SV-SVC</th>
<th>SV</th>
<th>HN-GARCH</th>
<th>PBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation Time</td>
<td>8.7</td>
<td>24.8</td>
<td>699.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3
Descriptive statistics
Descriptive statistics of the FTSE 100 index returns series from Jan-2015 to Dec-2016

| Values         | 0.0001                  | 0.0006                  | 0.0352                  | -0.0477                  | 0.0107                  | -0.1293                  | 4.5623                  | 52.8749                  |
Appendix F

Figure 1
Implied volatility graph
The graphs include both market implied volatility and the models theoretical implied volatility. The date is 2016-06-17 and is the 1-day out-of-sample analysis, this day had the highest implied volatility during 2016.

28-days

63-days

91-days
Figure 2

Implied Volatility Surface

The volatility surface constructed in MATLAB by interpolating the option data collected 2016-06-29, the in-sample data is used. X-axel= Moneyness, Y-axel= Time to maturity, Z-axel=Implied volatility. The models are listed in the following order: SV, SV-SVC, HN-GARCH and PBS model.
Appendix G

VBA-Code 1
The Nelder-Mead Search Algorithm

The Nelder-Mead Search Algorithm was used to find near global optimum parameters for all models included in this thesis. The specific code in the snippet below is for the PBS-model.

```vba
Function NelderMead(fname As String, startParams As Variant, Tolerance As Double, MaxIters As Integer, PC, S, K, r, T, realPrice)
    ' Nelder Mead Algorithm
    ' From "Option Pricing Models and Volatility Using Excel-VBA"
    ' INPUTS
    ' fname = name of the objective function
    ' startParams = a vector of starting values
    ' Tolerance = Tolerance on objective function value (stops when value < tolerance)
    ' Shock = shock to create starting vertices (start = start + start*RandomShock%)
    ' MaxIter = Maximum number of iterations
    ' OUTPUTS
    ' Vector of parameter estimates
    ' Value of objective function
    ' Actual number of iterations used
    ' Required settings
    Dim rho As Double, xi As Double, gam As Double, sigma As Double
    rho = 1
    xi = 2
    gam = 0.5
    sigma = 0.5
    ' Number of parameters
    n = Application.Count(startParams)
    Dim x1() As Double, xn() As Double, x1n1() As Double, xbar() As Double, xr() As Double, xe() As Double, xo() As Double
    Dim passParams() As Double
    ReDim resmat(n + 1, n + 1) As Double
    ReDim x1(n) As Double, xn(n) As Double, x1n1(n) As Double, xbar(n) As Double, xr(n) As Double, xe(n) As Double, xo(n) As Double
    ReDim passParams(n)
    For i = 1 To n
        resmat(1, i + 1) = startParams(i)
    Next i
    resmat(1, 1) = Run(fname, startParams, PC, S, K, r, T, realPrice)
    ' Randomize and initialize the starting values
    For j = 1 To n
        For i = 1 To n
            Randomize
            random = 2 * Shock * Rnd - Shock
            resmat(j + 1, i + 1) = (1 + random) * startParams(i)
            passParams(i) = resmat(j + 1, i + 1)
        Next i
        resmat(j + 1, 1) = Run(fname, passParams, PC, S, K, r, T, realPrice)
    Next j
    ' Start the timer
    Dim StartTime As Double, EndTime As Double
    StartTime = Timer
    n_stop_iter_ps = 25
    For iter = 1 To MaxIters
        If (iter = n_stop_iter_ps) Then
            Debug.Print iter
        End If
        Sleep (100)
    Next iter
    ' Sort the functional values
End Function
```
resmat = BubSortRows(resmat)
If (Abs(resmat(1, 1) - resmat(n + 1, 1)) < Tolerance) Then
  Exit For
End If
'The best functional value and point
f1 = resmat(1, 1)
For i = 1 To n
  x1(i) = resmat(1, i + 1)
Next i
'The second-to-worst functional value and point
f2 = resmat(n, 1)
For i = 1 To n
  x2(i) = resmat(n, i + 1)
Next i
'The worst functional value and point
f3 = resmat(n + 1, 1)
For i = 1 To n
  x3(i) = resmat(n + 1, i + 1)
Next i
'The center of gravity
xbar(i) = 0
For j = 1 To n
  xbar(i) = xbar(i) + resmat(j, i + 1)
Next j
xbar(i) = xbar(i) / n
Next i
'Reflection point
For i = 1 To n
  xr(i) = xbar(i) + rho * (xbar(i) - x3(i))
Next i
fr = Run(fname, xr, PC, S, K, r, T, realPrice)
shrink = 0
If ((fr >= f1) And (fr < f2)) Then
  newpoint = xr
  newf = fr
ElseIf (fr < f1) Then
  'Expansion point
  xe(i) = xbar(i) + xi * (xr(i) - xbar(i))
  Next i
  fe = Run(fname, xe, PC, S, K, r, T, realPrice)
  If (fe < fr) Then
    newpoint = xe
    newf = fe
  Else
    newpoint = xr
    newf = fr
  End If
ElseIf (fr >= f2) Then
  'Outside contraction
  xoc(i) = xbar(i) + gam * (xr(i) - xbar(i))
  Next i
  foc = Run(fname, xoc, PC, S, K, r, T, realPrice)
  If (foc <= fr) Then
    newpoint = xoc
    newf = foc
  Else
    shrink = 1
  End If
Else
  'Inside contraction
  xic(i) = xbar(i) - gam * (xbar(i) - x3(i))
  Next i
  fic = Run(fname, xic, PC, S, K, r, T, realPrice)
  If (fic < f3) Then
    newpoint = xic
  Else
    shrink = 1
  End If
Else
  'Inside contraction
  xic(i) = xbar(i) - gam * (xbar(i) - x3(i))
  Next i
  fic = Run(fname, xic, PC, S, K, r, T, realPrice)
  If (fic < f3) Then
    newpoint = xic
  Else
    shrink = 1
  End If
Else
  'Outside contraction
  xoc(i) = xbar(i) + gam * (xr(i) - xbar(i))
  Next i
  foc = Run(fname, xoc, PC, S, K, r, T, realPrice)
  If (foc <= fr) Then
    newpoint = xoc
    newf = foc
  Else
    shrink = 1
  End If
Else
  'Expansion point
  xe(i) = xbar(i) + xi * (xr(i) - xbar(i))
  Next i
  fe = Run(fname, xe, PC, S, K, r, T, realPrice)
  If (fe < fr) Then
    newpoint = xe
    newf = fe
  Else
    newpoint = xr
    newf = fr
  End If
ElseIf (fr >= f1) Then
  Exit For
End If
newf = fic
Else
shrink = 1
End If
End If
End If
If (shrink = 1) Then
For scnt = 2 To n + 1
For i = 1 To n
'Shrinkage step
resmat(scnt, i + 1) = x1(i) + sigma * (resmat(scnt, i + 1) - x1(1))
passParams(i) = resmat(scnt, i + 1)
Next i
resmat(scnt, 1) = Run(fname, passParams, PC, S, K, r, T, realPrice)
Next scnt
Else
For i = 1 To n
resmat(n + 1, i + 1) = newpoint(i)
Next i
resmat(n + 1, 1) = newf
End If
Next iter
'End the timer and measure the elapsed time
Dim ElapsedTime As Double
EndTime = Timer
ElapsedTime = EndTime - StartTime
'Sort the results
resmat = BubSortRows(resmat)
'Output the parameter estimates
ReDim output(n + 9) As Double
For i = 1 To n
output(i) = resmat(1, i + 1)
Next i
'Output the value of the objective function
output(n + 1) = resmat(1, 1)
'Output the actual number of iterations uzed
output(n + 2) = iter
'Output the run time
output(n + 3) = ElapsedTime
NelderMead = Application.Transpose(output)
End Function
Function BubSortRows(passVec)
Dim tmpVec() As Double, temp() As Double
uVec = passVec
rownum = UBound(uVec, 1)
colnum = UBound(uVec, 2)
ReDim tmpVec(rownum, colnum) As Double
ReDim temp(colnum) As Double
For i = rownum - 1 To 1 Step -1
For j = 1 To i
If (uVec(j, 1) > uVec(j + 1, 1)) Then
For K = 1 To colnum
temp(K) = uVec(j + 1, K)
uVec(j + 1, K) = uVec(j, K)
uVec(j, K) = temp(K)
Next K
Next j
Next i
BubSortRows = uVec
End Function
VBA-Code

Kilin’s Objective function

The code was included in the SV-SVC model as a modification to the original model, Kilin suggested that the characteristic function was not dependent of the strike level \( (K) \) instead is it dependent of the maturity date \( (\tau) \), this implies that the integrals \( f_1(0) \) and \( f_2(0) \) can be estimated for every maturity date and the integrals do not need to be re-estimated for every strike level. Meaning that the \( f_j(0) \) term is estimated, stored and can be reused for every strike level on that specific maturity date.

```vba
Function HestonObjFunSVC(params As Variant, S As Double, rf As Double, q As Double, MktPrice As Variant, K As Variant, T As Variant, PutCall As String, MktIV As Variant, Abscissas As Variant, Weights As Variant, Trap As Integer, ObjFun As Integer, a As Double, b As Double, Tol As Double, MaxIter As Integer) As Variant
    ' params = vector of parameters to estimate
    ' S = Spot price
    ' rf = risk free rate
    ' q = dividend yield
    ' MktPrice = vector of market prices of puts or calls
    ' K = vector of strikes.
    ' T = vector of maturities.
    ' MktIV = vector of market implied volatilities
    ' x = abscissas for Gauss Laguerre integration
    ' w = weights for Gauss Laguerre integration
    ' trap = 1 L Little Trap formulation of c.f / 0 = Heston formulation
    ' a = Bisection algorithm, small initial estimate
    ' b = Bisection algorithm, large initial estimate
    ' Tol = Bisection algorithm, tolerance
    ' MaxIter = Bisection algorithm, maximum iterations

    Dim NK As Integer, NT As Integer, NX As Integer
    NK = WorksheetFunction.Count(K)
    NT = WorksheetFunction.Count(T)
    NX = WorksheetFunction.Count(Abscissas)
    Dim Errors() As Double, Strike As Double, Mat As Double, ModelPrice As Double, marketprice As Double
    Dim phi As Double, wt As Double
    ReDim Errors(NK, NT) As Double
    Dim f2() As cNum, f1() As cNum, CharFun As Variant
    ReDim f2(NX) As cNum, f1(NX) As cNum
    Dim int1() As Double, int2() As Double
    ReDim int1(NX) As Double, int2(NX) As Double
    pi = WorksheetFunction.pi()
    Dim i As cNum, Denominator As cNum, expiphi As cNum, iphi As cNum
    i = Complex(0, 1)

    Dim tt As Integer, j As Integer, kk As Integer
    For tt = 1 To NT
        Mat = T(tt)
        For j = 1 To NX
            ' Store the characteristic function at each time step
            phi = Abscissas(j)
            wt = Weights(j)
            CharFun = HestonCF(Complex(phi, 0), params, Mat, S, rf, q, Trap)
            f2(j) = Complex(CharFun(1, 1), CharFun(2, 1))
            CharFun = HestonCF(cSub(Complex(phi, 0), i), params, Mat, S, rf, q, Trap)
            Denominator = Complex(S * Exp((rf - q) * Mat), 0)
            f1(j) = cDiv(Complex(CharFun(1, 1), CharFun(2, 1)), Denominator)
        Next
    Next tt
    For j = 1 To NK
        Strike = K(kk)
        For j = 1 To NX
            phi = Abscissas(j)
            wt = Weights(j)
            ' int1(j) = w(j) * real(exp(-i*phi*log(K(k))))11(j)/i/phi;
            iphi = cProd(Complex(phi, 0), i)
            expiphi = cExp(cProd(Complex(-phi * Log(K(kk)), 0), i))
\[ I_1 = \text{cDiv}(\text{cProd}(\text{expiphi}, f_1(i)), \text{iphi}) \]
\[ I_2 = \text{cDiv}(\text{cProd}(\text{expiphi}, f_2(i)), \text{iphi}) \]
\[ \text{int1}(j) = \text{wt} \times \text{cReal}(I_1) \]
\[ \text{int2}(j) = \text{wt} \times \text{cReal}(I_2) \]
\[ \text{Next } j \]

* The Call Price

\[ P_1 = 0.5 + \frac{1}{\pi} \times \text{Ssum}(\text{int1}, \text{NX}) \]
\[ P_2 = 0.5 + \frac{1}{\pi} \times \text{Ssum}(\text{int2}, \text{NX}) \]
\[ \text{CallPrice} = S \times \text{Exp}(-q \times \text{Mat}) \times P_1 - \text{Strike} \times \text{Exp}(-rf \times \text{Mat}) \times P_2 \]

If PutCall = "C" Then
\[ \text{ModelPrice} = \text{CallPrice} \]
Else
\[ \text{ModelPrice} = \text{CallPrice} - S \times \text{Exp}(-q \times \text{Mat}) + \text{Exp}(rf \times \text{Mat}) \times K(kk) \]
End If