Estimation of inertial parameters for automatic leveling of an underwater vehicle

Feras Faez Elias
Master of Science Thesis in Electrical Engineering

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Abstract

The use of underwater systems has grown significantly, and they can be used both for military and civilian purposes. Many of their parts are replaceable. An underwater vehicle can be equipped with different devices depending on the task it should carry out. This can make the vehicle unbalanced, which means that the demand for balancing systems will increase in line with the increasing use of underwater systems.

The goal of the thesis is to deliver a method for balancing based on parameters estimated both in static and dynamic operation. The parameters define a nonlinear physical model that can describe the underwater vehicle in different environments and conditions.

The main idea in the proposed method for parameter estimation based on static operation data is to solve equilibrium equations when the on-board control system is used to maintain two different orientations. The balancing can then be done by solving an optimisation problem that gives information about where additional weights or float material should be installed.

The static parameter estimation has been evaluated successfully in simulations together with three ways of solving the balancing problem. The dynamic parameter estimation has also been evaluated in simulations. In this case, the estimated parameters seem to have the same sign as the true ones but it seems difficult to obtain accurate estimates of some of the parameters. However, the total dynamic model was good except the prediction of the vertical movements. In particular, the model could explain the rotations of the vehicle well. The reason for the worse performance for the vertical movements might be some difficulties when generating suitable excitation signals.

The work done by Feras Faez Elias in connection to this master thesis made a contribution to a patent application that Saab AB has filed where Feras Faez Elias was one of the inventors.
Acknowledgments

The first I would like to thank is God. God says in Revelation 1:8 “I am the Alpha and the Omega, says the Lord God, who is and who was and who is to come, the Almighty.”

Matt. 6:19 "Do not store up for yourselves treasures on earth, where moth and rust consume and where thieves break in and steal; 20 but store up for yourselves treasures in heaven, where neither moth nor rust consumes and where thieves do not break in and steal. 21 For where your treasure is, there your heart will be also.

Matt. 5:38 "You have heard that it was said, ‘An eye for an eye and a tooth for a tooth.’ 39 But I say to you, Do not resist an evildoer. But if anyone strikes you on the right cheek, turn the other also; 40 and if anyone wants to sue you and take your coat, give your cloak as well; 41 and if anyone forces you to go one mile, go also the second mile. 42 Give to everyone who begs from you, and do not refuse anyone who wants to borrow from you.

Matt. 5:33 "Again, you have heard that it was said to those of ancient times, ‘You shall not swear falsely, but carry out the vows you have made to the Lord.’ 34 But I say to you, Do not swear at all, either by heaven, for it is the throne of God, 35 or by the earth, for it is his footstool, or by Jerusalem, for it is the city of the great King. 36 And do not swear by your head, for you cannot make one hair white or black. 37 Let your word be ‘Yes, Yes’ or ‘No, No’; anything more than this comes from the evil one.

Matt. 5:21 "You have heard that it was said to those of ancient times, ‘You shall not murder’; and ‘whoever murders shall be liable to judgment.’ 22 But I say to you that if you are angry with a brother or sister, you will be liable to judgment; and if you insult a brother or sister, you will be liable to the council; and if you say, ‘You fool,’ you will be liable to the hell of fire. 23 So when you are offering your gift at the altar, if you remember that your brother or sister has something against you, 24 leave your gift there before the altar and go; first be reconciled to your brother or sister, and then come and offer your gift. 25 Come to terms quickly with your accuser while you are on the way to court with him, or your accuser may hand you over to the judge, and the judge to the guard, and you will be thrown into prison. 26 Truly I tell you, you will never get out until you have paid the last penny.

Rom. 1:28 "And since they did not see fit to acknowledge God, God gave them up to a debased mind and to things that should not be done. 29 They were filled with every kind of wickedness, evil, covetousness, malice. Full of envy, murder, strife, deceit, craftiness, they are gossips, 30 slanderers, God-haters, insolent, haughty, boastful, inventors of evil, rebellious toward parents, 31 foolish, faithless, heartless, ruthless. 32 They know God's decree, that those who practice such things deserve to die emdash;yet they not only do them but even applaud others who
practice them."

1Cor. 6:9 "Do you not know that wrongdoers will not inherit the kingdom of God? Do not be deceived! Fornicators, idolaters, adulterers, male prostitutes, sodomites*, 10 thieves, the greedy, drunkards, revilers, robbers, none of these will inherit the kingdom of God."

The second I would thank is my family for the support and help I got during the life time and studying time. I also would like to thank Saab Dynamics for the opportunity to do my thesis in the company and the thesis has been a success for me. I also thank the colleagues at Saab Dynamics.

I also thank the examiner Martin Enqvist for support and the advising during the thesis and Martin Lindfors who has been my supervisor at the university.

Linköping, June 2017
Feras Faez Elias
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# Notation

## Abbreviations

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<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>ROV</td>
<td>Remotely operated vehicle</td>
</tr>
<tr>
<td>AUV</td>
<td>Autonomous underwater vehicle</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial measurement unit</td>
</tr>
<tr>
<td>NED</td>
<td>North east down coordinate system</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical user interface</td>
</tr>
<tr>
<td>AftSB</td>
<td>After starboard</td>
</tr>
<tr>
<td>AftP</td>
<td>After port</td>
</tr>
<tr>
<td>FrontSB</td>
<td>Front starboard</td>
</tr>
<tr>
<td>FrontP</td>
<td>Front port</td>
</tr>
<tr>
<td>CG</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>CB</td>
<td>Center of buoyancy</td>
</tr>
<tr>
<td>CM</td>
<td>Center of mass</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman filter</td>
</tr>
<tr>
<td>MPC</td>
<td>Model predictive control</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo-random binary sequence</td>
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Saab Seaeye Limited is a wholly owned subsidiary of Saab Underwater Systems AB. The company was formed in 1986 by Ian and Janet Blamire to specialise in the manufacturing of electrically powered Remotely Operated Vehicles, ROVs, for the offshore oil and gas industry.

The following chapter will introduce the idea of the master thesis. The purpose of the thesis is explained and what kind of problems that have investigated and solved. The first problem is the balancing of an underwater vehicle and was studied by estimating static parameters and later the balancing was implemented by using a mathematical optimisation problem. The optimisation problem was implemented in three different ways depending of what is preferred by the operator. To verify the solution of the re-balancing problem, the optimisation problem was studied in two different ways that result in the same solution.

Furthermore, dynamic parameter estimation was studied and it can also be used for the proposal on balancing. However, in the following thesis only the static parameter estimation was used for the proposal on balancing. The reason is that for the application it is better to separate the two problems to get an accurate solution for the static parameter estimation that is most important for the proposal on balancing.

The purpose of this project is to implement an automatic method that can be used to find out where additional mass should be placed to get the system balanced before operating in water. The process of analysing the collected data will help making a decision about the system. If the system is not balanced from the beginning then we can excite the system and collect data that can be used to estimate some parameters and later apply the re-balancing algorithm.
1.1 Background

Some of Seaeye’s unmanned underwater vehicles can operate with 360 degree movements in all directions. To get good performance during operation, the underwater craft should be balanced and weighted. For example, when the system changes environment between freshwater and saltwater, if the system should be in the same state as before, more mass has to be added because saltwater has higher density. The problem in this case is a static problem that can be solved by making the system density equal to the density of water. When some additional equipment or weights are installed to the system, the system changes in behaviour because the additional parts act as a disturbance. This will affect the operation and be more costly in terms of energy consumption because we will need to use the controller more often just for stabilising the craft, affecting the motion in water.

The interesting parameters for the static parameter estimation are the mass \(m\), the external mass \(m_{\text{Ext}}\), the center of mass that in this case is the same as the center of gravity \(CG\) and the center of buoyancy \(CB\). The parameters that can be interesting when estimating the dynamic model are the Coriolis effect matrix with added mass, the matrix for hydrodynamic damping and the inertia matrix. The dynamic parameters are interesting because they are important to get a physical model that can be used to get a better controller. If we add some additional external parts to the whole system that has an effect on the estimation for making it balanced, then the included parts have to be taken into consideration.

1.2 An underwater vehicle

In this thesis an AUV (Autonomous underwater vehicle) called Sabertooth was used in a simulation environment and the idea was to use another vehicle called Sea Wasp for real data tests. However, no real were carried out during the project. Figure 1.1 shows a Sabertooth double hull vehicle that goes both autonomously and manually with or without a tether. Most of the missions today are autonomous. The vehicle has big batteries that can be used when operating autonomously, without a tether.
1.3 Problem formulation

It takes time when the pilot does not have a method for balancing the craft. Very often new equipment is installed in the craft, which requires stabilisation of the craft. This thesis should deliver a useful and time-efficient method that can be used for re-balancing.

The problem that is studied here can be divided in two parts. The first part is the static parameters that can be used to get a proposal on balancing. The static part contains the method for estimation of the center of gravity CG and the external mass $m_{\text{Ext}}$ that can be inserted into special holes in the system, or the estimation of the buoyancy center CB and the nominal mass $m$. The two combinations occur together and because of identifiability they have to be estimated separately. The combinations are relevant because when we estimate the center of gravity, the estimate can be used for balancing and the external mass $m_{\text{Ext}}$ tells us where the extra weights are inserted that makes the system unbalanced. The external mass $m_{\text{Ext}}$ will be used for the proposal on balancing because the most simple solution is to place a weight in the opposite direction or to place buoyant material that counteract the external mass $m_{\text{Ext}}$. $m_{\text{Ext}}$ is used to make the system unbalanced and is known. The external mass $m_{\text{Ext}}$ is used to ensure that we can estimate the effect of the unbalanced system and with mass and the direction of the unbalancing by the vector from origin to the center of gravity CG.

The second part of the thesis is to estimate the parameters of a physical dynamic model and use that information to get a complete physical description of the vehicle that can be used for the proposal on balancing and for the dynamic modelling. The two parts can be done in the same experiment but here they will be done in two experiments.

Considering only the static part will reduce the parameters that are needed to be estimated. The static part of the motion model will help to get information about the center of mass and in some cases even the whole mass can be estimated when a known mass is installed in a known position, see the appendix in Linder (Thesis. No.1681, 2014).
1.4 Purpose and goal

The goal of the thesis is to deliver a method for balancing after estimating the parameters that are needed both in static and dynamic operation. The parameters of the nonlinear physical model are needed to describe the craft in different environments and conditions. By estimating the parameters, a complete model will be delivered that can be used for the proposal on balancing or for control design in future work.

1.5 Related research

The most related research for this project is Linder (Thesis. No.1681, 2014), in particular pages 115-116 where he describes how to estimate the mass and center of mass when adding external mass. The issue of identifiability is also described in Linder (Thesis. No.1681, 2014).


The dynamics model for the underwater system is derived in Fossen (2011) and is written in Gustafsson (2010). Glad and Ljung (2006) have a description for state space modelling. In Linder et al. (2015) the article concerns online estimation of the mass and the center of mass for ships, and a similar method can maybe be used to estimate the parameters for the ROV. The book Meriam and Kraige (2011) was used to find out the center of gravity CG who is the same as the center of buoyancy CB for the underwater system. Generally, Meriam and Kraige (2011) was used when using classical mechanics. Gustafsson et al. (2010) was used for understanding the methods for validation of the used model. Other information about the possibilities to use extended and unscented Kalman filters for estimation of parameters is given in Sabet et al. (2014).

By reading the thesis Axelsson that has been applied in the same underwater system Sabertooth DH2, the reader could understand the behaviour of the underwater system and how it could be used.

1.6 Limitations

The methods have only been evaluated in simulations. The simulation environment is better than using real data to ensure that the method is correct. By work-
ing in the simulation environment, the user can verify if the method can deliver results and if the parameters that have been estimated are near the original parameters that were used in the simulation. Furthermore, a real underwater vehicle was not available for testing. For the balancing issue it is easier to know how much force each thruster can give when using the simulation environment because the forces and moments can be calculated or modelled. This is harder for the real system.

For the proposal on balancing, the holes can only take a limited mass and buoyant material. The solution of negative estimated mass $m_{\text{Ext}}$ can be a problem because the system has a limitation of the four places for inserting mass $m_{\text{Ext}}$ or buoyant material and there are also limitations on how much material of mass and buoyant material that can be inserted into the system. For information about the fixed weights in Sabertooth DH2, see Figure 4.2.

1.7 Outline

The master thesis consists of the following chapters. Chapter 2 contains modelling fundamentals for the studied applications and identifiability analysis. It also includes how to validate the methods, and the description of the coordinate transformation in the local coordinate system. Chapter 3 contains the method and experiment design for both static and dynamic modelling. Furthermore, it contains a description of how data was collected. Chapter 4 contains the experiment design with flowcharts of the methods. This chapter contains a description of experiments for both the static and dynamic modelling. Chapter 5 contains results for both the static and dynamic estimation. The last chapter presents the summary, conclusion and future work.
The following chapter will give a description of the modelling fundamentals. The dynamic model is written as a state space model (Glad and Ljung, 2006). The parameters that will be estimated are included in the dynamic model and by simplifying the model the static parameters that will be used for the proposal on balancing can be estimated. The identifiability analysis will be done and some basic transformations between the coordinate systems will be introduced.

### 2.1 Modelling

In the following section the dynamic model that comes from the second law of Newton’s equation of motion and moments will be introduced. The equation

\[ M \dot{\nu} + C(\nu) \nu + D(\nu) \nu + g(\eta) = \tau_{\text{motor}} + \tau_{\text{error}} \]  \hspace{1cm} (2.1)

has been taken from Fossen (2011) p.168 or from Ahmad (2015) p.76 and is defined from the second law of Newton’s motion equation-s. The equation (2.1) will be used for both static and dynamic modelling and will be rewritten as a state space model for dynamic parameter estimation. The equation is a non-linear physical equation and contains physical parameters that will be estimated.

The position and orientation are defined by \( \eta = [n, e, d, \theta, \phi, \psi]^T \) and the velocities and angular velocities are given by \( \nu = [v, \omega]^T = [u, v, w, p, q, r]^T \) where \( \nu = [u, v, w]^T \) is the translation velocity vector and \( \omega = [p, q, r]^T \) is the rotational velocity vector.
2.1.1 Static modelling

In the static case there is no velocities or angular velocities and the model equation (2.1) can be simplified by setting $v$ and $\dot{v}$ to zero. All the derivatives were set to zero in static parameter estimation because there is no motion or it is assumed negligible. The description of each term in (2.1) follows later. The resulting expression for static modelling after simplifying equation (2.1) is

$$g(\eta) = \tau_{motor}$$

if no disturbances $\tau_{error}$ caused by the environment exist.

The total gravitational force according to Meriam and Kraige (2011) p. 113 is given by $W = mg + m_{Ext}g$, where the gravitational force caused by external mass $m_{Ext}$ and the nominal mass $m$ are included. The buoyancy force on the ROV that is proportional to the volume according to Meriam and Kraige (2011) p. 312 is given by $B = \rho g V$. The gravitational force vector

$$g(\eta) = \begin{bmatrix} (W - B)\sin \theta \\ -(W - B)\cos \theta \sin \phi \\ -(W - B)\cos \theta \cos \phi \\ -(y_g W - y_b B)\cos \theta \cos \phi + (z_g W - z_b B)\cos \theta \sin \phi \\ (z_g W - z_b B)\sin \theta + (x_g W - x_b B)\cos \theta \cos \phi \\ -(x_g W - x_b B)\cos \theta \sin \phi - (y_g W - y_b B)\sin \theta \end{bmatrix}$$

(2.3)

is a function of the direction and orientation of the ROV. For more details see Fossen (2011) p.60. The control input vector of forces and moments is $\tau_{motor}$ and $\tau_{error}$ is a vector of environmental disturbance-s or added noise. See Gustafsson (2010) p. 357 for more details.

The driving forces and moments are in the vector

$$\tau_{motor} = \left[ \tau_1 = \sum_i F_{x,i}, \tau_2 = \sum_i F_{y,i}, \tau_3 = \sum_i F_{z,i}, \tau_4 = \sum_i M_{x,i}, \tau_5 = \sum_i M_{y,i}, \tau_6 = \sum_i M_{z,i} \right]^T$$

(2.4)

which can be calculated as a combination of some of the six thrusters. For more details see Chin and Lum (2011), Ch. 2 and Skoglund et al. (2012) p. 950.

The first three terms in (2.4) include the forces in the $x$-, $y$- and $z$-axis and the rest three terms are moments for each coordinate in $x$-, $y$- and $z$-axis. The forces and moments are calculated in the simulation environment. In a real data test, (2.4) is approximated as a linear combination of the thruster signals. The index $i$ in (2.4) represents the thrusters in the system.

2.2 Dynamic modelling

All the equations in the static model are included in the dynamic modelling and the main physical model (2.1) is a complete model for parameter estimation. Here follows a description of the parameters that are included in Equation
that have not been introduced in the static modelling. $C(v)$ is the Coriolis effect and $M$ is the inertia matrix and they contain of two terms, the first is with consideration of a rigid body $C_{RB}$, $M_{RB}$ and the second term contains added mass $C_A$, $M_A$. The term $D(v)$ is hydrodynamic damping matrix.

For simplicity the ROV is assumed symmetrical around the $xy$, $xz$ and $yz$ planes and it is assumed that

$$M = M_{RB} + M_A \quad (2.5)$$

can be written as sum of

$$M_{RB} = \text{diag}[m, m, m, I_x, I_y, I_z] \quad (2.6)$$

and

$$M_A = \text{diag}[X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}]$$

Hence, the final expression for $M$ is

$$M = \text{diag}[m + X_{\dot{u}}, Y_{\dot{v}}, m + Z_{\dot{w}} + K_p, I_y + M_A] \quad (2.8)$$

For more details about the last expression see Fossen (2011), p. (122-173).

The rigid body Coriolis term

$$C_{RB}(v) = \begin{bmatrix} mS(\omega) & 0_{3x3} \\ 0_{3x3} & -S(I\omega) \end{bmatrix} = \begin{bmatrix} m\omega \times v \\ -(I\omega) \times \omega \end{bmatrix} \quad (2.9)$$

can be simplified by the use of the skew formula from linear algebra. The skew formula is defined by $aS(A)B = aA \times B$. For more details see Fossen (2011) p. (122-173).

The added mass Coriolis term can be written as

$$C_A(v) = \begin{bmatrix} 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{fr} & M_{\dot{q}q} \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{fr} & 0 & -K_p\dot{r} \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}q} & K_p\dot{r} & 0 \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (2.10)$$
and the damping term as
\[ D(\mathbf{v}) = -\text{diag}[X_u, Y_v, Z_w, K_p, M_q, N_r] \mathbf{v} \]
\[ -\text{diag}[X_u|u|, Y_v|v|, Z_w|w|] \]
\[ (K_p + K_p|p|) \mathbf{p}, (M_q + M_q|q|) \mathbf{q}, (N_r + N_r|r|r) \mathbf{r} \]
\[ (2.11) \]

To get a state space model
\[ \dot{\mathbf{v}} = f_0(\mathbf{v}, \eta, \tau_{motor}) \]  
\[ (2.12) \]

the expressions (2.1), (2.3) - (2.11) can be written as
\[ \dot{u} = \frac{X_u + X_u|u|}{m + X_u} u - \frac{mZ_w}{m + X_u} w - \frac{mY_v}{m + Y_v} v - \frac{(W-B)\sin(\theta)}{m + X_u} + \frac{\tau_1}{m + X_u} \]
\[ (2.13a) \]
\[ \dot{v} = \frac{Y_v + Y_v|v|}{m + Y_v} v + \frac{mZ_w}{m + Y_v} w - \frac{mX_u}{m + Y_v} u - \frac{(W-B)\cos(\theta)\sin(\phi)}{m + Y_v} + \frac{\tau_2}{m + Y_v} \]
\[ (2.13b) \]
\[ \dot{w} = \frac{Z_w + Z_w|w|}{m + Z_w} w + \frac{mX_u}{m + Z_w} u - \frac{mY_v}{m + Z_w} v - \frac{(W-B)\cos(\theta)\cos(\phi)}{m + Z_w} + \frac{\tau_3}{m + Z_w} \]
\[ (2.13c) \]
\[ \dot{p} = \frac{K_p + K_p|p|}{I_x + K_p} \left( q - \frac{N_r - N_r - I_z}{I_x + K_p} p - \frac{Z_w}{I_x + K_p} \right) w \]
\[ (2.13d) \]
\[ \dot{q} = \frac{M_q + M_q|q|}{I_y + M_q} \left( \frac{N_r - N_r - I_z}{I_y + M_q} q - \frac{Z_w}{I_y + M_q} \right) u - \frac{X_u - X_u|u|}{I_y + M_q} \frac{w}{I_x + K_p} + \frac{\tau_4}{I_x + K_p} \]
\[ (2.13e) \]
\[ \dot{r} = \frac{N_r + N_r|r|}{I_z + N_r} r - \frac{K_p - M_q + I_z}{I_z + N_r} \left( \frac{X_u - X_u|u|}{I_z + N_r} q - \frac{Z_w}{I_z + N_r} \right) u - \frac{w}{I_x + K_p} \frac{u}{I_y + M_q} + \frac{\tau_5}{I_z + N_r} \]
\[ (2.13f) \]

where the right-handed side is a function of \( \mathbf{v}, \eta \) and \( \tau_{motor} \). The last expression contains acceleration and angular acceleration and how they can be expressed relative to other variables.

If any disturbances exist the term \( \tau_{error} \) has to be added in Equation (2.12).

The matrix
\[ J(\phi, \theta, \psi) = \begin{bmatrix} R(\phi, \theta, \psi) & 0 \\ 0 & T(\phi, \theta, \psi) \end{bmatrix} \]  
\[ (2.14) \]

contains the transformation matrices \( R \) that later will be expressed in Euler angles in (2.18) and \( T \) that will be expressed in Euler angles in (2.19). \( J \) can be used for transformation to the NED-coordinate system from the body-fixed coordinate system that is used in the algorithm. \( J \) is defined in both quaternions and Euler angular but here just Euler angles are used.

The relation between velocities in the body-fixed and the NED reference frame can now be written as
\[ \dot{\eta} = J(\eta) \mathbf{v} \]  
\[ (2.15) \]
The state space equation

\[
\dot{x} = f(x, \tau_{\text{motor}}) = \begin{bmatrix}
    J(\phi, \theta, \psi)v \\
    f_b(v, \eta, \tau_{\text{motor}}) \\
    T(\phi, \theta, \psi)w \\
    f_b(v, \eta, \tau_{\text{motor}})
\end{bmatrix},
\]

(2.16)

is the final nonlinear equation for dynamic modelling that will be used in nonlinear grey-box approach for estimating the physical parameters. See Fossen (2011) p.168-169 for more details.

A black-box model would not give a physical description to the system. In this thesis the physical parameters are important because studying these parameters will be useful for controlling the system or developing the proposal on balancing if they are accurately estimated.
2.3 Validation of the model

In the following section the theory of how to validate the estimated static or dynamic model will be described. For static modelling the theory of fundamental mechanics will be used and in dynamic modelling the idea is to compare the model that was estimated from the estimation data with the validation data.

2.3.1 Static validation

To validate the static parameter estimation the rigid body formula

\[
\vec{r}_{g,i}(m_{\text{Ext}}) = \sum_i m_i \vec{r}_{gi} \sum_i m_i \tag{2.17}
\]

for calculating center of gravity of multiple bodies was used. Here the index \(i = [\text{AftSB, AftP, FrontSB, FrontP}]\) represents the holes where the weights can be inserted in the ROV called Sabertooth. See Figures 3.1 and 4.3.

After inserting the external mass \(m_{\text{Ext}}\), the system would be unbalanced because the center of gravity \(CG_0\) has been shifted to the new center of gravity \(CG_1\). The center of gravity \(CG_1\) can be calculated by using (2.17) and can then be compared with the estimated center of gravity \(CG\) that contains of the parameters \(x_g\), \(y_g\) and \(z_g\).

The center of mass \(CM\) which is in the following case same as the center of gravity \(CG\) has been shifted when inserting weights in the ROV. The weights were placed in four places in the ROV because the system has defined places where weights can be inserted. When calculating in theory how much effect the weights will have the formula for calculating center of mass \(CM\) for multiple bodies was used. The reason is to find out the offset that was caused by the weights.

The vector \(\vec{r}_{g,i} = [x_g, y_g, z_g]^T\) can be written as an offset \(x_g\), \(y_g\) and \(z_g\) in each axis relative to the center of gravity \(CG_0\), caused by the external inserted mass \(m_{\text{Ext}}\).

2.3.2 Dynamic validation

There are many methods to validate an grey-box model and many of the methods are well described in Holst et al. (1992). The Matlab command \(\text{compare}\) was used to get a comparison between the estimated output \(\hat{y}(t|\theta)\) of the grey-box model with the real output \(y(t|\theta)\) to see how good they match in percent. The generated plots in Chapter 5 also show the normalised root mean square (NRMSE) measure of the goodness of the fit. Furthermore, to validate the physical parameters a comparison between the estimated and real parameters can be done if the data is collected from the simulation environment, because in the simulation environment the physical parameters are known. There are other methods to validate a
model and they are described in Gustafsson et al. (2010) p. 251.

2.4 Coordinate systems

The simulation environment uses a right-handed coordinate system with the $z$-axis pointing up for some of the data and the solved equations will be described in a right-handed local coordinate system but where the $z$-axis will be pointing down. The reason for different coordinate systems is that the equations for parameter estimation were defined in a right-handed coordinate system where the $z$-axis was pointing down so all of the data have to follow a right-handed coordinate system with $z$-axis pointing down to ensure a correct estimation of the parameters.

To get a correct translation between the two local coordinate systems a transformation vector was used. This transformation vector was used for transformation from the coordinate system that has $z$-axis pointing up to the coordinate system that has $z$-axis pointing down. The transformations vector was used for transformation of forces and moments in each axis by using the vector product of the transformation vector with forces and moment vector.

The angles are defined as a NED-coordinate system so there is no need to change the sign because the algorithm is defined in NED-coordinate system that is the global coordinate system described in next section.

The translation velocities has a right handed local coordinate system with $z$-axis pointing up. This will cause a change in the sign for $y$- and $z$-axis for adapting to the local coordinate system.

The angular velocities has a right handed coordinate system with $z$-axis pointing up. Here also can be used the same method that was used for the velocities. After changing the sign for the $y$- and $z$-axis the angular velocities are now adapted to the local coordinate system.

2.4.1 Transformation matrices

In this section two coordinate systems are described. One of the coordinate systems is the one where the motion equations of Newton can be defined and it is called a global coordinate. This system is defined on the surface of the water and is defined as a right handed coordinate system with $X$-axis in the north direction, $Y$-axis pointing in east direction and $Z$-axis pointing down. The other coordinate system is the one that is defined locally in the ROV and is centred in the ROV.
and following the ROV. The local coordinate system has the origin in the center of the ROV with the $x$-axis pointing in the front direction and the $y$-axis pointing to the right and the $z$-axis pointing down. For more details about the coordinate systems and the transformation matrices see Chen et al. (2007) p.428

Tow rotation matrices

$$R(\phi, \theta, \psi) = \begin{bmatrix}
  c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\
  c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\
  -s\theta & s\phi c\theta & c\phi c\theta
\end{bmatrix} \quad (2.18)$$

and

$$T(\phi, \theta, \psi) = \begin{bmatrix}
  1 & s\phi t\theta & c\phi t\theta \\
  0 & c\phi & -s\phi \\
  0 & s\phi/c\theta & c\phi/c\theta
\end{bmatrix} \quad (2.19)$$

are used for the transformation of velocities respective angular velocities from the local coordinate system to the global coordinate system.

The rotation matrices can be expressed by Euler angles or as quaternions, but here only the Euler angles are used.
The following chapter includes the method-s of the static parameter estimation with the proposal on balancing and the dynamic parameter estimation. The experiment design will be introduced in Chapter 4 for both static and dynamic estimation with flowchart-s that describe all steps from collecting data, solving the estimation problem to applying the proposal on balancing and dynamic parameter estimation.
3.1 System identification and parameter estimation

The following section will describe the static and dynamic parameter estimation and introduce identification theory for investigation of identifiability issues in both static and dynamic modelling.

3.1.1 Static parameter estimation

The parameters that are needed to be estimated in the static model (2.2) and (2.3) are $x_g, y_g, z_g$ and $m_{\text{Ext}}$. The $x_g, y_g, z_g$ are the center of gravity (CG).

Actually, all parameters in (2.2) and (2.3) would be estimated at the same time but the results will not be correct because of identifiability issues. This will result in a choice between estimating the whole mass $m$ or external mass $m_{\text{Ext}}$ and estimating the "center of gravity" or the "center of buoyancy". For the application of the proposal on balancing it is more preferred to estimate $m_{\text{Ext}}$ because by estimating it the proposal of balancing can be applied. The description of each parameter can be seen in Table 3.1. By solving the Equation (2.2) the parameters $x_g, y_g, z_g$ and $m_{\text{Ext}}$ can be estimated assuming that the buoyancy center is unchanged. Each of the parameters has a physical significance and can be verified by measurements or by specific experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>The mass of the ROV</td>
</tr>
<tr>
<td>$m_{\text{Ext}}$</td>
<td>The external mass in the ROV</td>
</tr>
<tr>
<td>$g$</td>
<td>The gravitational acceleration</td>
</tr>
<tr>
<td>$V$</td>
<td>The volume of the ROV</td>
</tr>
<tr>
<td>$\rho$</td>
<td>The density of the water</td>
</tr>
<tr>
<td>$x_g$</td>
<td>The center of the gravitational force in x-direction</td>
</tr>
<tr>
<td>$y_g$</td>
<td>The center of the gravitational force in y-direction</td>
</tr>
<tr>
<td>$z_g$</td>
<td>The center of the gravitational force in z-direction</td>
</tr>
<tr>
<td>$x_b$</td>
<td>The center of the buoyancy force in x-direction</td>
</tr>
<tr>
<td>$y_b$</td>
<td>The center of the buoyancy force in y-direction</td>
</tr>
<tr>
<td>$z_b$</td>
<td>The center of the buoyancy force in z-direction</td>
</tr>
</tbody>
</table>

*Table 3.1: The parameters that are used for the static model.*
3.1 System identification and parameter estimation

3.1.2 Identification

The static model (2.3) can be extended by inserting $W = mg + m_{\text{Ext}}g$ and $B = \rho g V$ in (2.3) and by using Equation (2.2) the parameters can be estimated. This will result in the following equation

$$
\begin{bmatrix}
(m + m_{\text{Ext}})g \sin \theta - B \sin \theta \\
-(m + m_{\text{Ext}})g \cos \theta \sin \phi + B \cos \theta \sin \phi \\
-(m + m_{\text{Ext}})g \cos \theta \cos \phi + B \cos \theta \cos \phi \\
-z_g (m + m_{\text{Ext}})g \sin \theta - z_b B \sin \theta + x_g (m + m_{\text{Ext}})g \cos \theta \cos \phi - x_b B \cos \theta \cos \phi \\
-x_g (m + m_{\text{Ext}})g \cos \theta \sin \phi + x_b B \cos \theta \sin \phi - y_g (m + m_{\text{Ext}})g \sin \theta + y_b B \sin \theta
\end{bmatrix}
= \begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3 \\
\tau_4 \\
\tau_5 \\
\tau_6
\end{bmatrix}
\tag{3.1}
$$

Equation (3.1) cannot be solved directly for the four unknown parameters. Instead, the number of equations has to be extended. The extension was done by adding six equations with another roll angle $\phi$ and pitch angle $\theta$ that are different from the first setup. By collecting data from two experiments with different vehicle configuration and later solving the equation system a solution can be found.

Equation (3.1) can be turned into a linear equation system if the selected estimated parameters are

$$
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
= \begin{bmatrix}
x_g \\
y_g \\
z_g \\
m_{\text{Ext}}
\end{bmatrix}
\tag{3.2}
$$

otherwise the equation can be nonlinear depending on which of the parameters that are selected as unknowns. If the estimated parameters are chosen in a way that the resulting equations include a multiplication of two estimated parameters (e.g. $x_1 x_3$) that can not be separated the expression will be nonlinear. The linear expression can be expressed as a matrix equation formula $Ax = b$ that can be solved by a linear solver or by the least squares method depending on the collected data.

By choosing the following parameters in (3.2) as unknown parameters the matrix equation can be solved by a linear solver but in the following thesis a nonlinear solver was used by combining two experiment where roll $\phi$ and pitch $\theta$ where inserted to zero and another arbitrary chosen roll $\phi$ and pitch $\theta$. The twelve equations gave a solution for the unknowns.

One identification issue was that the mass $m$ and the external mass $m_{\text{Ext}}$ could not be estimated at the same time. To avoid the problem, one of the variables has to be fixed. The expression $W = (m + m_{\text{Ext}})g$ in Equation (3.1) shows how the external mass $m_{\text{Ext}}$ and the total mass $m$ are related to each other and because of the sum one of them has to be fixed.

There are other variables that are related to each other. The center of gravity
has three components $x_g, y_g$ and $z_g$ and the buoyancy $x_b, y_b$ and $z_b$. Estimating $x_g$ at the same time as $x_b$, $y_g$ at the same time as $y_b$ or $z_g$ at the same time as $z_b$ will result in problems with identifiability like the problem that was described above when estimating two masses at the same time. So the best is to assume that the center of buoyancy is known and one of the masses $m$ or $m_{Ext}$.

### 3.1.3 Dynamic parameter estimation

For each value of the unknown parameter vector $\theta$ the model gives a prediction of $y(t)$ that can be written as $\hat{y}(t|\theta)$. Hence, the prediction error can be written as $\epsilon(t, \theta) = y(t) - \hat{y}(t|\theta)$.

After collecting the input and output signals over a period $t = 1, ..., N$, the model can be evaluated for a particular choice of parameters $\theta$ by using $V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} (y(t|\theta) - \hat{y}(t|\theta))^2$. A common way to estimate the parameters is the expression $\hat{\theta}_N = \arg \min_{\theta} V_N(\theta)$ because this expression will find the $\theta$ that minimises the error, see Glad and Ljung (2004) p. 287 and p. 309.

The parameters that need to be estimated in the dynamic model (2.13a) - (2.13f) are explained in Table 3.2. Each of the parameters has a physical significance and can be verified by measurements or by specific experiments. For more details about each parameter, see Chapter 2 in Sabet et al. (2014).
Table 3.2: The parameters that are used for the dynamic model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u, Y_v, Z_w$</td>
<td>Linear translation damping coefficients</td>
</tr>
<tr>
<td>$X_{uu}, Y_{vv}, Z_{ww}$</td>
<td>Quadratic translation damping coefficients</td>
</tr>
<tr>
<td>$X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}$</td>
<td>Added mass coefficients</td>
</tr>
<tr>
<td>$K_p, M_q, N_r$</td>
<td>Linear rotational damping coefficients</td>
</tr>
<tr>
<td>$K_{pp}, M_{qq}, N_{rr}$</td>
<td>Quadratic rotational damping coefficients</td>
</tr>
<tr>
<td>$K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}$</td>
<td>Added inertia coefficients</td>
</tr>
<tr>
<td>$I_x, I_y, I_z$</td>
<td>Moment of inertia coefficients</td>
</tr>
<tr>
<td>$W = mg + m_{\text{Ext}}g$</td>
<td>The total gravitational force</td>
</tr>
<tr>
<td>$B = \rho g V$</td>
<td>The buoyancy force</td>
</tr>
<tr>
<td>$x_g$</td>
<td>The center of the gravitational force in $x$-direction</td>
</tr>
<tr>
<td>$y_g$</td>
<td>The center of the gravitational force in $y$-direction</td>
</tr>
<tr>
<td>$z_g$</td>
<td>The center of the gravitational force in $z$-direction</td>
</tr>
<tr>
<td>$x_b$</td>
<td>The center of the buoyancy force in $x$-direction</td>
</tr>
<tr>
<td>$y_b$</td>
<td>The center of the buoyancy force in $y$-direction</td>
</tr>
<tr>
<td>$z_b$</td>
<td>The center of the buoyancy force in $z$-direction</td>
</tr>
</tbody>
</table>
3.2 Balancing methods

Here, the methods for the proposal on balancing will be introduced in Chapter 2 in section 2.3.1. First, the distances to the places where the weights are will be used to calculate the center of gravity CG for the ROV called Sabertooth. The calculations are based on where the weights can be inserted in the Sabertooth system. There are four places for placing external mass \( m_{\text{Ext}} \) and the calculation of the center of gravity are presented in Equations (3.3a), (3.3b), (3.3c) and (3.3d) for each place depending on where the weights will be inserted.

For after starboard "AftSB":

\[
\begin{align*}
    x_{g,AftSB}(m_{\text{Ext}}) &= \frac{-1.031 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    y_{g,AftSB}(m_{\text{Ext}}) &= \frac{0.6 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    z_{g,AftSB}(m_{\text{Ext}}) &= \frac{0 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} = 0
\end{align*}
\] (3.3a)

For after port "AftP":

\[
\begin{align*}
    x_{g,AftP}(m_{\text{Ext}}) &= \frac{-1.031 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    y_{g,AftP}(m_{\text{Ext}}) &= \frac{-0.6 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    z_{g,AftP}(m_{\text{Ext}}) &= \frac{0 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} = 0
\end{align*}
\] (3.3b)

For front starboard "FrontSB":

\[
\begin{align*}
    x_{g,FrontSB}(m_{\text{Ext}}) &= \frac{0.78 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    y_{g,FrontSB}(m_{\text{Ext}}) &= \frac{0.6 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    z_{g,FrontSB}(m_{\text{Ext}}) &= \frac{0 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} = 0
\end{align*}
\] (3.3c)

For front port "FrontP":

\[
\begin{align*}
    x_{g,FrontP}(m_{\text{Ext}}) &= \frac{0.78 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    y_{g,FrontP}(m_{\text{Ext}}) &= \frac{-0.6 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} \\
    z_{g,FrontP}(m_{\text{Ext}}) &= \frac{0 \cdot m_{\text{Ext}}}{m + m_{\text{Ext}}} = 0
\end{align*}
\] (3.3d)

The equations above includes the \( m_{\text{Ext}} \) that has to be estimated to get a solution for the proposal on balancing. The purpose of estimating the external mass \( m_{\text{Ext}} \) in the following thesis was to validate the estimated center of gravity as above and to re-balance the system. The external mass \( m_{\text{Ext}} \) was used for the proposal on balancing in the balancing methods.
3.2 Balancing methods

3.2.1 Method for proposal on balancing without bounds on gravity and buoyancy force is not included

After estimating the static parameters the proposal on balancing can be applied. The static parameters were estimated, which gave of the new center of gravity \(CG_1\) obtained with the external mass \(m_{\text{Ext}}\).

Theory of how to place the weights is based on the moment equations in the \(x\)-, \(y\)- and \(z\)-axis if needed because if the weights are considered to be placed symmetrically around the \(z\)-axis then it is not needed to take this degree of freedom into account for the solution of the equation system. Figure 3.1 shows the illustration of the weights with different masses \(M_1, M_2, M_3\) and \(M_4\) with the new center of gravity point \(CG_1\) caused by inserting an external mass \(m_{\text{Ext}}\) in \(AftSB\). In real system the external mass \(m_{\text{Ext}}\) will not be inserted to the system because the system would be unbalanced. \(m_{\text{Ext}}\) will be estimated because estimating the external mass with the center of gravity \(CG_1\) gives a step to solve the optimisation expression below that makes the system balanced.

The general balancing problem can be expressed as

\[
\begin{align*}
\min_{M_i} \quad & m_{\text{tot}} \\
= & m + m_{\text{Ext}} + M_1 + M_2 + M_3 + M_4 \\
\sum \text{Moment}_x &= 0 \\
\sum \text{Moment}_y &= 0 \\
\sum \text{Moment}_z &= 0 \\
M_i &\geq 0
\end{align*}
\]

(3.4)

which can be extended by inserting the parameters to

\[
\begin{align*}
\min_{M_i} \quad & m_{\text{tot}} \\
= & m + m_{\text{Ext}} + M_1 + M_2 + M_3 + M_4 \\
M_2(Y_1 - y_g) - M_3(Y_1 + y_g) + M_1(Y_1 - y_g) - M_4(Y_1 + y_g) - mv_g &= 0 \quad (3.5) \\
M_2(X_1 + x_g) + M_3(X_1 + x_g) - M_1(X_2 - x_g) - M_4(X_2 - x_g) + mx_g &= 0 \\
M_i &\geq 0
\end{align*}
\]

for the ROV Sabertooth. The specialization is based on calculation of moment only around the \(x\)- and \(y\)-axis for \(CG_1\). The mass \(m\) is the nominal mass and the external mass \(m_{\text{Ext}}\) is the mass that caused the shifting of the center of gravity from \(CG_0\) who was the position of the nominal mass \(m\) to the new center of gravity \(CG_1\) who is the position of \(m + m_{\text{Ext}}\). All the calculations are done in the ROV.
local coordinate system. For details see Figure 3.1.

### 3.2.2 A simple method for proposal on balancing without taking the nominal mass $m$ into consideration

By applying basic algebra, another simple method that is more efficient and easier to implement in a real system can be defined. When implementing the proposal on balancing it is necessary to use some programmable processor and the amount of code can be limited depending on the type of processor.

After inserting a weight in $AftSB$ the center of gravity has been shifted. Figure 3.2 shows the vector representation after estimation of the new center of gravity $CG_1$. The vector representation can be used to define a line called $L$ that goes from the origin where the center of gravity $CG_0$ point was to the new center of gravity $CG_1$.

The line $L$ will be used to find out how much weight has to be inserted in any point on that line to get the same reaction in moments. By using this line the proposal on balancing can be applied by using the opposite vector and finding the intersection between the line $L$ with the most nearby line between two weights in the opposite direction. The line $L$ here will be used to find out where to insert weights on the opposite direction of the external mass $m_{Ext}$ on that line for rebalancing.

The last step is to rebalance the vehicle by using the moment equation but just including the weight-s that are involved in the intersection between the line $L$ and the line between two places where mass can be inserted. The moment of equation will find a solution that replaces the amount of the moment on that point to the points where the weights can be inserted.

Let us consider an example. The vector $\vec{r}_g = [-0.0086, 0.0050]^T$ that gives the line $L$ in Figure 3.2 can be written as: $L = [0.0086, -0.0050]^T t + [0, 0]^T$. The unknown intersection point between $L$ and the line that connects the two weights will be calculated. By using the x-component, the $t$ in $L$ can be calculated and can then be inserted in $L$ to find out the $y$-value for the intersection point. This results in $0.7800 = 0.0086 t$ which gives $t = \frac{0.0086}{0.7800}$. Inserting $t$ in the y-component gives $y = -0.0050 \frac{0.0086}{0.7800} = -0.45$. The intersection points between the two lines is $[0.78, -0.45]$.

On each point on the line $L$ a mass can be inserted that gives the same action as the external mass $m_{Ext}$ that was inserted in point $[x, y] = [-1.031, 0.600]$. In this case a product formula $m_{Ext} \vec{r}_1 = M_{unknown} \vec{r}_2$ can be used, where $\vec{r}_1 = [-1.031, 0.600]$, $\vec{r}_2 = [0.7800, -0.4500]$ and $m_{Ext} = 10$ gives that $M_{unknown} = -13.2100$. 
3.2 Balancing methods

The last step was to use the moment equation about the point $CG_0$ which gives a solution of how much mass has to be inserted. The equation where written in matrix equation formula with

$$A = \begin{bmatrix} Y_1 & -Y_1 \\ -X_1 & -X_1 \end{bmatrix}$$

and

$$C = \begin{bmatrix} (-M_{unknown}0.4500) \\ (-M_{unknown}0.7800) \end{bmatrix}$$

and the unknowns in

$$B = \begin{bmatrix} M_2 \\ M_3 \end{bmatrix}$$

. The solution of the equation system $AB = C$ gives $B$ with values $M_2 = 1.65$ and $M_3 = 11.56$ which is reasonable.
Figure 3.1: The figure shows the estimated offset which created the vector $\vec{r}_g$ that goes from $CG_0$ to $CG_1$. It also shows where the weights can be placed in the ROV and the external masses $M_1$, $M_2$, $M_3$ and $M_4$. The original mass is $m$. The distances that are in the figure are in relation to $CG_0$ and they are $X_1$, $X_2$ and $Y_1$.

Figure 3.2: The figure shows the estimated offset which created the vector $\vec{r}_g$ that goes from $CG_0$ to $CG_1$. It also shows where the weights can be placed in the ROV and the external masses $M_1$, $M_2$, $M_3$ and $M_4$. The original mass is $m$. The distances that are in the figure are in relation to $CG_0$ and they are $X_1$, $X_2$ and $Y_1$. A line $L$ is introduced to find the point of intersection between $L$ and the line between two weights.
3.2 Balancing methods

3.2.3 A general method for the proposal on balancing

Here, a linear optimisation problem is extended from the original one that was described in Section 3.2.1. The new optimisation problem contains all the limitations that come from the fact that the amount of weights and buoyant material are limited. Hence, the linear optimisation problem includes the gravity force and buoyancy force with limitations in the added mass and buoyant material.

The following problem has been solved by using the linear programming algorithm that is included in Matlab.

\[
\begin{align*}
\min_{M_i, B_i} & \quad m_{\text{tot}} \\
= & \quad m + m_{\text{Ext}} + M_1 + M_2 + M_3 + M_4 + B_1 + B_2 + B_3 + B_4 \\
\text{subject to} & \\
\sum & \quad \text{Moment}_x = 0 \\
\sum & \quad \text{Moment}_y = 0 \\
\sum & \quad \text{Moment}_z = 0 \\
M_i & \geq 0, \quad i = 1, \ldots, 4. \\
B_i & \geq 0, \quad i = 1, \ldots, 4. \\
F_b - F_g & \geq 0 \\
F_b & = \alpha (B_1 + B_2 + B_3 + B_4) \\
F_g & = (M_1 + M_2 + M_3 + M_4) \\
0 & \leq M_i \leq 10, \quad i = 1, \ldots, 4. \\
0 & \leq B_i \leq 10, \quad i = 1, \ldots, 4.
\end{align*}
\]

(3.6)

When applying the LP-solver to the problem a solution will be found that contains how much mass and buoyant material has to be inserted that minimise the expression \(m_{\text{tot}}\) and thus makes the proposal on balancing as optimal as possible. The total buoyant force \(F_b\) here is introduced as a factor \(\alpha\) multiplied with the sum of \(B_i\).

The general formula can be expressed in (3.6) which can be specialised to (3.7) for the ROV Sabertooth. The extension is based on calculation of moment around
the x- and y-axis for $CG_1$.

\[
\min_{M_i, B_i} m_{\text{tot}} = m + m_{\text{Ext}} + M_1 + M_2 + M_3 + M_4 - B_1 - B_2 - B_3 - B_4
\]

subject to

\[
\begin{align*}
M_2(Y_1 - y_g) - M_3(Y_1 + y_g) + M_1(Y_1 - y_g) - M_4(Y_1 + y_g) \\
+ B_2(Y_1 - y_g) - B_3(Y_1 + y_g) + B_1(Y_1 - y_g) - B_4(Y_1 + y_g) - m y_g = 0 \\
M_2(X_1 + x_g) + M_3(X_1 + x_g) - M_1(X_2 - x_g) - M_4(X_2 - x_g) \\
+ B_2(X_1 + x_g) + B_3(X_1 + x_g) - B_1(X_2 - x_g) - B_4(X_2 - x_g) + m x_g = 0
\end{align*}
\]

\[
M_i \geq 0, \ i = 1, \ldots, 4.
\]

\[
B_i \geq 0, \ i = 1, \ldots, 4.
\]

\[
F_b - F_g \geq 0
\]

\[
F_b = \alpha (B_1 + B_2 + B_3 + B_4)
\]

\[
F_g = (M_1 + M_2 + M_3 + M_4)
\]

\[
0 \leq M_i \leq 10, \ i = 1, \ldots, 4.
\]

\[
0 \leq B_i \leq 10, \ i = 1, \ldots, 4.
\]

(3.7)

The calculations are done in the ROV local coordinate system. Figure 3.3 shows the different external masses $M_i$ for $i = 1, 2, 3, 4$ and the buoyant materials $B_i$ for $i = 1, 2, 3, 4$. The expressions in Equation (3.7) are theoretically calculated as moment around the $CG_1$ point.

**Figure 3.3:** The figure shows the estimated offset which created the vector $\vec{r}_g$ that goes from $CG_0$ to $CG_1$. It also shows where the weights can be placed in ROV and the external masses $M_1, M_2, M_3$ and $M_4$ with the buoyant materials $B_1, B_2, B_3$ and $B_4$. The original mass is $m$. The distances that are in the figure are in relation to $CG_0$ and they are $X_1$, $X_2$ and $Y_1$. 
In the following chapter the design of the tests will be introduced. After introducing the way of collecting data from the simulation environment, this chapter will also introduce the tests that have been made for estimation of parameters for both static and dynamic models. The chapter includes the tests and the way of choosing the test and why the test has been made. This chapter also includes the way of collecting data and re-sample it.

4.1 Simulation experiments

The experiment design for the static and dynamic part begins as follows. By using the simulation environment, data has been collected. Later on the collected data was handled in Matlab. The algorithm was written in Matlab and was solved by a different solver depending on if the problem is static or dynamic. The prediction error \( e(t, \theta) = y(t) - \hat{y}(t|\theta) \) that includes the real output \( y(t) \) and the estimated \( \hat{y}(t, \theta) \) has to be as small as possible and the unknown parameters are in the estimated parameter vector \( \theta \).

Depending on which parameters in the static problem will be estimated the last expression in Equation (3.1.2) would be different. As described in Section 3.1.2 the last expression can be linear or nonlinear, which was the reason for choosing a nonlinear solver \texttt{fsolve} in the estimation of the static parameters.
4.1.1 Static experiment in simulator

The system was balanced from the beginning in the simulation environment in the case of static estimation. Extra functionality has been implemented to make the system unbalanced. When starting the system and collecting data for analysis, the system should be unbalanced and the control system will be used to balancing the vehicle. After balance with the controller, the data will be collected and analysed. The collected data has to be informative to get a solution, which is guaranteed by collecting at least two datasets with different roll $\phi$ and pitch $\theta$ angles.

The extra weights on the system will be inserted in known positions and the extra weights have a mass that can also be estimated or weighed before. The center of gravity CG was estimated with the external mass $m_{\text{Ext}}$ and the center of buoyancy CB was in the same position as before because no bouyant material was inserted into the system. If both masses are estimated at the same time then the identifiability issue will occur which was described better in Section 3.1.2. For more details about the experiment see Equation (2.2) and Figure 4.1.

Figure 4.1 shows a flow chart of what happens when inserting a external mass $m_{\text{Ext}}$. If the ROV would be unbalanced after inserting the external mass $m_{\text{Ext}}$, data has to be logged from the navigation. The collected data will be inserted in equation (2.2) to obtain a solution with a solver. The solver gives the estimated center of gravity $CG_1$ and the external mass $m_{\text{Ext}}$. After estimating the parameters the proposal of balancing can be applied by solving the optimisation problem in Equation (3.4).
Figure 4.1: The flow chart describes the static experiment design. The input is the external mass $m_{\text{Ext}}$ and the output would be the proposal on balancing. It also shows how data goes in the chain. The estimated parameters are the external mass $m_{\text{Ext}}$ and the center of gravity. The method can be used both for simulation and real experiments.
Figure 4.2: The figure shows where the external mass $m_{\text{Ext}}$ can be inserted in the system. It also shows how the local coordinate system is defined. The figure contains two figures that is a sketch of the ROV from two sides. The first figure illustrates the ROV from the top and the second shows the ROV from the port side.

Figure 4.2 shows the four positions where the weights can be placed in this specific ROV, which is Sabertooth DH2; front starboard (FrontSB), after starboard (AftSB), front port (FrontP) and after port (AftP). The positions where the weights are located in the ROV are according to a local defined coordinate system in the center of the ROV. In this case when both the roll $\phi$ and pitch $\theta$ angles are zero the local coordinate system is the same as the global coordinate system.

The center of gravity $CG_0$ will be shifted to a new place $CG_1$ with the components $x_g$, $y_g$ and $z_g$ after a weight has been inserted. How to calculate the new center of gravity $CG_1$ is explained mathematically in Equation (2.17) according to the original coordinate system in $CG_0$. All the static experiments has been made by inserting weights in the after starboard (AftSB) position. Figure 4.3 shows the vector $\vec{r}_{g,i}$ that was created after inserting the weight. The figure also shows that a new gravity vector $\vec{F}_{g_1}$ in the new $CG_1$ center of gravity point that contains both the mass $m$ and the external mass $m_{\text{Ext}}$ will replace the old $CG_0$ center of gravity and the buoyancy force $\vec{F}_{b_0}$ will be the same and in the same position $CB_0$ if no float materials were added to the system.
Figure 4.3: The figure shows what the external mass $m_{\text{Ext}}$ has for effect on the system. It also includes the new gravity force $\vec{F}_{g_1}$ that occurs after inserting the mass. The old gravity force vector $\vec{F}_{g_0}$ will not be included in the calculations after inserting the external mass $m_{\text{Ext}}$. The buoyancy force $\vec{F}_{b_0}$ is not affected and stays in the same position $CB_0$ as before.
4.1.2 Dynamic experiment in simulator

Figure 4.4 describes the experiment design for estimating parameters in a physical model that can be used both for a real system and in the simulation environment for dynamic modelling. The choice of the model was based out of the description in Ljung and Söderström (1983) where they describe which type of model is good for the application.

The dynamic model can also be used to estimate the center of gravity $CG$ or to estimate the center of buoyancy $CB$. Static parameter estimation is sufficient for the proposal on balancing and there is no need to estimate the static parameters in the dynamic estimation if they are already known. Figure 4.4 shows that if the ROV is not balanced after the first estimation, an iterative approach can be used. In the case of balanced ROV this is not needed.

Figure 4.4 shows the flow chart of the parameter estimation approach for dynamic motion modelling. When doing dynamic parameter estimation the first step is to balance the ROV for static parameter estimation and then to begin with the dynamic motion. Figure 4.4 contains also a sketch of static parameter estimation included in dynamic estimation but here the ROV has been balanced and there is no need to estimate static parameters. The data was collected and inserted in Equation (2.1). By using grey-box modelling the parameters in Equation (2.1) can be estimated. The idea is to use data to estimate parameters that minimise the prediction error this solving an optimisation problem. When the parameters have been estimated the model will be compared with validation data to verify whether the estimated parameters in the physical model are good for the application. The parameters can be verified because they have a physical description and if they are physically correct with correct sign then the estimation is considered acceptable.

The six degrees of freedom can be controlled with inputs. The value of the inputs is in a range between $-1 \leq u \leq 1$. 
4.1 Simulation experiments

Figure 4.4: The flow chart contains an approach to estimate the parameters and to choose the model in Matlab for data collected from a real system or from a simulation environment.
Figure 4.5: A flow chart of how data is being collected from the ROV computer and sent to the algorithm. There are two algorithms where the first gives a solution for the proposal on balancing and the second estimates the dynamic model parameter.

4.2 Collecting data

In the simulation environment there are many ways that use sensors for logging data. In each of the messages there is some interesting data that can be used in equations or just for analysing. For more information about the system see Axelsson. Velocities and angular velocities are collected from the IMU. There are forces and moments for each motor but also pitch and roll angles. To control the ROV in specific condition a widget that was used in GUI was developed, see Figures A.7, A.8 and A.9, where the signals were used for estimation of the dynamic parameters.

4.2.1 Re-sampling data

When re-sampling zero-order hold was used which is Matlab own re-sampling function. For more information about the re-sampling see Institutionen för systemteknik (2010) p. 18. The re-sampling was applied in forces and moments because they had a lower sampling rate than the Euler angles roll $\phi$, pitch $\theta$ and yaw $\psi$. 
4.3 Static test design

The purpose of the static test design is to ensure that the static parameters can be estimated since they are needed for the proposal on balancing. Knowledge of the external mass $m_{\text{Ext}}$ will be used to verify that the answer from the proposal of balancing is reasonable because when $m_{\text{Ext}} = 10$ kg by using one of the theoretical solutions that have been shown in Section 3.2.1 or in Section 3.2.2, then 10 kg mass has to be inserted in the opposite direction if the ROV has symmetrical holes where the weights can be inserted. Another solution is presented in Section 3.2.3 and later in Chapter 5 there are results for a proposal on balancing that uses the algorithm in Section 3.2.1.

Figures A.1 and A.2 show the same simulated experiment but the first one is without noise and the second one is with noise. In both the figures the external mass $m_{\text{Ext}}$ is 1 kg. In both of the figures the experiments begin with zero in both roll $\phi$ and pitch $\theta$, after a while a step that goes from 0 to 20 degrees is sent in both roll $\phi$ and pitch $\theta$. In the beginning of the tests it takes time until the controller can stabilise the signal to the reference signal, which is the reason of the initial overshoot. The steps result in overshoot that will be damped by the controller after a short time. In each experiment the collected data that would be inserted in Equation (2.2) are roll angle $\phi$, pitch angle $\theta$, forces and moments from the motors calculated directly in simulation environment. Figures A.3, A.4, A.5 and A.6 show the same type of experiment but here $m_{\text{Ext}}$ is chosen as $m_{\text{Ext}} = 5, 10, 15, 20$ kg.

Equation (2.17) can be used for calculation of the center of mass CM or center of gravity CG. The purpose is to ensure that the estimation gives a correct answer. Different tests with different $m_{\text{Ext}}$ give different offset of the estimated center of gravity CG. Some tests have been done for studying the affect of noise on the experiments and the affect of changed external mass has been studied also. The results of experiments with different external masses $m_{\text{Ext}}$ is shown in Figures A.1, A.3 and A.5 that were done without added noise. Figures A.2, A.4 and A.6 shows the same experiments but with added noise. The reason for studying these situations was to ensure that Equation (2.2) gives a right estimated solution by using different external mass $m_{\text{Ext}}$ and collecting data with noise and without noise.

4.3.1 Experiment design for estimation of parameter and proposal on balancing

After estimating the center of mass in the static experiment, the proposal of balancing can now be applied by the following steps. First, by using data that has been collected from one of the data tests in Figures A.1, A.3 or A.5 for a special external mass $m_{\text{Ext}}$, e.g $m_{\text{Ext}} = 10$ kg in Equation (3.1), the solution of the param-
The last algorithm gives a general solution by using linear programming in Matlab. The algorithm includes both the weights and the buoyant material and has to satisfy the condition that the buoyancy force $F_b \geq F_g$.

### 4.4 Dynamic test design

The dynamic experiment design begins with flow chart in Figure 4.4 that describes the whole process in dynamic parameter estimation. The estimation of parameters begins with sending signals to excite the interesting states. Equation (2.12) shows how each signal is related to the speed in the $x$-, $y$- and $z$-axis or the angular velocities around the $x$-, $y$- and $z$-axis.

The choice of signal was based on what other have done like Ahmad (2015) p.77 where he describes the signals that can be used for excitation of a dynamic system. In this experiment the PRBS-based signal described on Ahmad (2015) has been used and the signal contains of multi-step input and doublet.

Figure A.7 shows the signal that was used for excitation of the system in uvr, which means sending signal in the $x$-, $y$-axis and rotation in the yaw angle. It is clear that the other states will be zero. The signal that was sent has to be a signal that contains enough energy in different frequencies to get good results in parameter estimation. See Glad and Ljung (2004) in Chapter 14 where the details of how the signal has to be chosen are described. Figure A.8 shows how the signal would be for the excitation of the system in uwq, which means sending signal in the $x$- and $z$-axis and rotation in the pitch angle. The states that would not be excited have to be zero to ensure that they will not disturb the excited dynamics. The last step was the excitation in all of the states by sending signals in uvwpqr and the signal for the experiment can be seen in Figure A.9.

The choice of the states was based on identifiability issues. The first step was to excite the states $\dot{u}$, $\dot{v}$ and $\dot{r}$ in Equations (2.13a) - (2.13f). The excitation was made by sending a signal as it was described above in $u$, $v$ and $r$. This step will result in estimating the parameters $X_u$, $X_u|u|$, $X_{\dot{u}}$, $Y_v$, $Y_v|v|$, $N_r$, $N_r|r|$, $I_z + N_r$. 
In the next experiment the states $\dot{u}$, $\dot{w}$ and $\dot{q}$ in (2.13a) - (2.13f) have been excited. This will result in estimation of the parameters $Z_w$, $Z_w|w|$, $Z_{\dot{w}}$, $M_{\dot{q}}$, $M_{\dot{q}|q}$ and $I_y + M_{\dot{q}}$.

In the last experiment all the states in (2.13a) - (2.13f) have been excited and that resulted in estimating the parameters $K_p$, $K_{p|p|}$, $I_x + K_{\dot{p}}$, $M_{\dot{q}} - N_r - I_y + I_z$, $N_r - K_{\dot{p}} - I_z + I_x$ and $K_{\dot{p}} - M_{\dot{q}} - I_x + I_y$. 
In the following chapter the results will be presented. The results are split in two categories. First the results of the static experiment will be described and later the dynamic experiment.

5.1 Static model

The theory for static parameter estimation was introduced in Section 2.3.1 where Equation (2.17) shows how to validate the results of static parameter estimation. Here follows a description of static modelling.

In the hole of after starboard (AftSB) depending on how much weights would be inserted via the external mass $m_{\text{Ext}}$, the offset of the shifted center of gravity CG will be more shifted from $CG_0$ if more mass is added and different $m_{\text{Ext}}$ cause different offset. All of the experiments were made in after starboard (AftSB) in this thesis. Equation (3.3a) shows the offset that was caused after inserting a weight in after starboard (AftSB) in the ROV that been used. There are other places to insert weights and for that the Equations (3.3b) - (3.3d) will be used to calculate the offset when weights are inserted in (AftP), (FrontSB) and (FrontP).

Figure A.10 shows the offset calculated theoretically $x_{g,AftSB}$ as a function of the external mass $m_{\text{Ext}}$ inserted in the ROV. Figure A.11 shows the same results but for the estimated parameter $\hat{x}_{g,AftSB}$. The experiment design is based on data that have been collected and are represented in the figures in Appendix A. Figures A.11, A.13 and A.15 show simulation experiment results that were made as described in Chapter 4.3 and Figures A.10, A.12 and A.14 show the theoretical results after inserting the external mass $m_{\text{Ext}}$. Comparing the theoretical results with experiments it is clear that the results are accurate.
Figure A.14 and A.15 show theoretical results and estimation results based on simulations. It has been different results between real $z_g$ and the estimated one, the reason for that is unknown. The parameter $z_g$ could not be estimated from the equation system (3.1) but it does not matter. When using the ROV called Sabertooth all the weights can be inserted from the middle, which means that $z_g$ is not needed for the proposal on balancing.

The experiment of collecting data with two different roll $\phi$ and pitch $\theta$ was done to be able to estimate $z_g$. The estimation has not been right and there were no time to investigate the reason.

### 5.1.1 Estimation error

Many experiments have been made in the static part and lots of data has been collected which can be used for analysing the results, or for defining the error between the real and estimated value as a function of roll $\phi$ and pitch $\theta$ angles. The idea was that the different choices of angles would result in some conclusion about which angle is best optimised to collect data from and to estimate the parameters.

In the following, Figures A.16, A.17 and A.18 include the result of the error of the estimated parameters. The residual $\epsilon_t(\theta, \phi)$ that is defined as the differential between the real and estimated value see Gustafsson et al. (2001) p.228. $\epsilon_t(\theta, \phi)$ has been plotted as a function of roll $\phi$ and pitch $\theta$ angles for the total mass $m$, the external mass $m_{Ext}$, the offset in $x$-axis defined as $x_{g,AftSB}$ and the offset in $y$-axis defined as $y_{g,AftSB}$.

### 5.1.2 Proposal on balancing

The proposal on balancing was introduced in Chapter 3 where, three methods were introduced and one of them is a general method that includes the buoyancy force $\vec{F}_b$, the gravity force $\vec{F}_g$ with constraints and the most important that $\vec{F}_b \geq \vec{F}_g$, otherwise the underwater system will sink by power failure. All the three methods are represented as mathematical optimisation problems. See Chapter 3 for more details. The general algorithm was introduced in (3.6) and after inserting the distances this will result in (3.7) which was the special case for the ROV called Sabertooth.

The proposal on balancing is done by estimating the vector $\vec{r}_g$. If the estimation of the gravity vector gives $x_g = -0.0086$ (m) and $y_g = 0.005$ (m) then the way to eliminate this shift is to insert mass in some positions and specially in the opposite direction of the gravity vector $\vec{r}_g$. If the vector $\vec{r}_g$ is in $AftSB$ then the weights have to be inserted in $FrontP$, $FrontSB$ and $AftP$ to eliminate the effect of the
offset and to get roll and pitch angles to zero because when they are zero the ROV becomes balanced. For the example here it was enough to just insert weights in FrontSB and FrontP because the external mass $m_{\text{Ext}}$ was inserted in AftSB. The best algorithm to use out of simplicity in calculation for the proposal on balancing is the algorithm that was presented in Section 3.2.2 because it shows where the weights should be inserted by using simple linear algebra without using linear programming when searching for the solution. The most general algorithm to use for the proposal on balancing is presented in Section 3.2.3.

The optimisation problem (3.5) is defined by considering the moment around the $x$- and $y$-axis with the total mass of the ROV. The moments are defined around $CG_1$, see Figure 3.1 where the distances to each force from the center of gravity $CG_1$ are defined and by using the forces the moments can be calculated which results in Equation (3.5). The equation is defined in the local coordinate system of the ROV.

An special approach can be used to solve the balancing problem without solving the optimisation problem (3.5) when only the FrontSB and FrontP are used for inserting weights. This special case occurs when the unbalancing is due to weights in AftSB and AftP and the results is that the balancing problem can be solved by solving equation system written in matrix form as

$$AB = C \iff B = A^{-1}C \quad (5.1)$$

where

$$A = \begin{bmatrix} (Y_1 - y_g) & -(Y_1 + y_g) \\ X_1 + x_g & X_1 + x_g \end{bmatrix}$$

$$C = \begin{bmatrix} my_g \\ -mx_g \end{bmatrix}$$

and

$$B = \begin{bmatrix} M_2 \\ M_3 \end{bmatrix}$$

With the offset $x_g = -0.0086$ (m) and $y_g = 0.005$ (m) the solution of how much weight we can place is $M_2 = 1.613359 \ kg$ and $M_3 = 11.640611 \ kg$.

There is no need to extend Equation (3.5) with moments around the $z$-axis because the places where the weight can be inserted are fixed around the $z$-axis. If they are not fixed one more equation has to be added.

### 5.2 Dynamic results

Using the dynamic model in (2.1) with the experiment design in Section 4.1.2 the parameters can be estimated by using the method that is described in Section 3.1.3.
The choice of the model was based on Sjöberg et al. (1995) where an introduction of different types of models and how to solve the estimation issues were described. There was a description in Sjöberg et al. (1995) in the introduction section with the argument and reasons of why a specific model is better than the others depending on the application. For this project the physical modelling was important for the proposal of balancing and for the dynamic model estimation. The used dynamic model for parameter estimation written in state space model was presented in Equation (2.13a)-(2.13f).

The signals for excitation of the system were described in Chapter 4 in Section 4.4. The choice of the excitation states for the identification was based of the identifiability issues that were described in Linder (Thesis. No.1681, 2014) in Chapter 4 and 5. Here, the method to solve the problem of the identifiability was described earlier in Section 4.4.

5.2.1 Model verification using simulation environment

The first step was to estimate the uvr directions in Equation (2.13a)-(2.13f). This means that a signal was sent without using the controller so the ROV will move in the \( x \) and \( y \) directions and rotate in yaw, see Figure A.7. This will result in the parameters that are presented in Table 5.1 that shows the real value compared to the estimated parameter value.

By comparing the real values with the estimated values is seems that the sign for all the parameters is correctly estimated and that most values are accurate. Some values are bigger than the estimated values which means that these parameters are difficult to estimate exactly.

By using the weight function \( V_N(\theta) \) to estimate the parameters \( \theta \), the errors \( \epsilon(t, \theta) \) between the simulated output and the data are as small as possible. For more details see Chapters 2 and 3.

The nonlinear physical model has been implemented in Matlab and by using the Matlab command \texttt{pem} the parameters have been estimated. For more details, read about \texttt{nlgreyest} in Matlab.

Figure 5.1 shows the result when comparing the estimated model parameters in Table 5.1 with validation data from the experiment in the uvr directions. The results are good because the fit in \( x \) is about 95%, in \( y \) about 86% and in yaw \( \psi \) it is about 77%.
Table 5.1: Table describing the parameters that were estimated when exciting the uvr directions in the state space model (2.13a)-(2.13f).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u$</td>
<td>-100</td>
<td>-117.81</td>
</tr>
<tr>
<td>$Y_v$</td>
<td>-300</td>
<td>-289.06</td>
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<tr>
<td>$X_{uu}$</td>
<td>-300</td>
<td>-280.73</td>
</tr>
<tr>
<td>$Y_{vv}$</td>
<td>-1000</td>
<td>-979.28</td>
</tr>
<tr>
<td>$X_{\dot{u}}$</td>
<td>118</td>
<td>416.43</td>
</tr>
<tr>
<td>$Y_{\dot{v}}$</td>
<td>118</td>
<td>307.46</td>
</tr>
<tr>
<td>$N_r$</td>
<td>-5000</td>
<td>-4356.30</td>
</tr>
<tr>
<td>$N_{rr}$</td>
<td>-5000</td>
<td>-2417.30</td>
</tr>
<tr>
<td>$I_z + N_{f}$</td>
<td>1104.81</td>
<td>1534.30</td>
</tr>
</tbody>
</table>

Figure 5.1: Comparison of the estimated model outputs with the simulated data. The excitation is in the uvr directions. The other outputs have not been excited and that is the reason of why they do not have an amplitude of the estimated nonlinear model. The other outputs have some disturbances.
Table 5.2 shows the estimated parameters $\theta$ for the second experiment in the uwq directions. By comparing the real values with the estimated values it seems that the sign for all the parameters is correctly estimated.

Figure 5.2 shows the result when comparing the estimated model parameters in Table 5.2 with validation data from the experiment in the uwq direction. The results are good because the fit in $x$ is about 56%, in $z$ about 68% and in pitch $\theta$ it is about 83%.

Table 5.2: Table describing the parameters that were estimated when exciting the uwq directions. This means that a signal was sent without using the controller so the ROV will move in the $x$ and $z$ directions and rotate in pitch $\theta$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_w$</td>
<td>-600</td>
<td>-0.91486</td>
</tr>
<tr>
<td>$Z_{ww}$</td>
<td>-2000</td>
<td>-5320</td>
</tr>
<tr>
<td>$M_q$</td>
<td>-10000</td>
<td>-245.50</td>
</tr>
<tr>
<td>$M_{qq}$</td>
<td>-10000</td>
<td>-1.0504e+05</td>
</tr>
<tr>
<td>$Z_\dot{w}$</td>
<td>118</td>
<td>828.7</td>
</tr>
<tr>
<td>$I_y + M_\dot{q}$</td>
<td>930.58</td>
<td>3314.8</td>
</tr>
</tbody>
</table>
Figure 5.2: Comparison of the estimated model outputs with the simulated data. The three signals that are excited are uwq. The other outputs have not been excited and that is the reason of why they do not have an amplitude of the estimated nonlinear model. The other outputs have some disturbances.
The last step was to estimate the uvwpqr directions, which means that a signal was sent without using the controller so the ROV will move in the $x$, $y$ and $z$ directions with rotation in roll $\phi$, pitch $\theta$ and yaw $\psi$.

Table 5.3 shows the estimated parameters $\theta$ for the last experiment in the uvwpqr directions. By comparing the real values with the estimated values it seems that the sign for all the parameters is correctly estimated and that some estimates are accurate. Some true values differ a lot from the estimated values which means that these parameters are difficult to estimate exactly because of the construction of the ROV that makes it difficult when sending a signal that excite in $z$. When sending a signal in $z$ the pitch $\theta$ will also be excited because the motor in the $z$-axis is not placed in the center of the ROV.

Figure 5.3 shows the result when comparing the estimated model parameters in Table 5.3 with validation data from the experiment in the uvwpqr directions. The results are good except for the $z$ direction because of the placement of the $z$-motor. The fit in $x$ is about 52%, in $y$ about 41%, in $z$ about -25%, in roll $\phi$ about 82%, in pitch $\theta$ about 82% and in yaw $\psi$ it is about 78%.

**Table 5.3**: Table describing the parameters that were estimated when exciting the uvwpqr states.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>-1000</td>
<td>-1065.30</td>
</tr>
<tr>
<td>$K_{pp}$</td>
<td>-1000</td>
<td>-1054.40</td>
</tr>
<tr>
<td>$I_x + K_p$</td>
<td>205.51</td>
<td>376.23</td>
</tr>
<tr>
<td>$M_{\dot{q}} - N_{\dot{r}} - I_{\dot{y}} + I_z$</td>
<td>Not calculated</td>
<td>0.00020199</td>
</tr>
<tr>
<td>$N_{\dot{r}} - K_p - I_z + I_x$</td>
<td>Not calculated</td>
<td>-0.00070024</td>
</tr>
<tr>
<td>$K_p - M_{\dot{q}} - I_x + I_y$</td>
<td>Not calculated</td>
<td>5040.30</td>
</tr>
</tbody>
</table>
Figure 5.3: Comparison of the estimated model outputs with the simulated data. The six outputs in the uvwpqr directions have been excited. The z-state has worse fit because the ROV has a motor for the excitation in z-direction but it is not centred in the CG point. This will result in excitation in pitch instead.
In the following chapter the summary of this thesis will be presented. Both conclusions and future work for the static parameter estimation that implies in the proposal on balancing and the dynamic parameter estimation will be discussed to improve the process in the future.

The static experiment of collecting data was well organised, see Figures A.1-A.6. All processes need the same sample rate both for static and dynamic parameter estimation to solve an equation system or for analysing data. The collected data for static parameter estimation was inserted in Equation (2.2) that is the same as Equation (3.1) after inserting the parameters. The equation system (3.1) was solved by using an equation system solver. A solution of the estimated center of gravity $CG_1$ with the external mass $m_{Ext}$ was generated. Figure 3.1 and 4.2 show where the weights can be placed relative to the original center of gravity $CG_0$.

Validation of the estimated static parameters was done using Equation (2.17) which is a general equation and for the ROV Sabertooth the Equations (3.3a)-(3.3d) were used.

The solution for proposal on balancing after inserting a external mass $m_{Ext}$ is reasonable and has been validated in the simulation environment. The approach makes the ROV balanced with roll angle $\phi = 0$ degrees and pitch angle $\theta = 0$ degrees by inserting the masses in the holes for $FrontSB$ and $FrontP$ because the external mass $m_{Ext}$ was inserted in $AftSB$. Fore more details see Section 5.1.2 in Chapter 5 and Section 3.2.2 in Chapter 3.

The dynamic model has been estimated and validated with new data and seems to be good step to make a better controller for the ROV in the future. The exper-
imement began with collecting data and re-sampling data. The data was inserted in state space Equations (2.13a)-(2.13f). Using the weight function $V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} (y(t|\theta) - \hat{y}(t|\theta))^2$, the parameters were estimated as $\hat{\theta}_N = \arg \min_{\theta} V_N(\theta)$. In this way, the algorithm solver tries to estimate the model output $\hat{y}(t|\theta)$ by choosing the unknown parameters $\hat{\theta}_N$ that minimise the expression $V_N(\theta)$.

A solution was found for the estimated parameters. The process of estimating parameters was done in three steps. There are other ways but this approach was chosen because of the identifiability issues, see Section 4.4 in Chapter 4. The excitation was done by sending signals separately in three experiments as in Figures A.7-A.9.

6.1 Conclusions

For static parameter estimation it is easy to estimate the parameters and get a view of how the center of the gravity of the ROV is being shifted. The shifted point can be used to calculate where the weights should be placed to get an balanced ROV. The whole mass $m$ can be estimated but the most important parameter is the external mass $m_{\text{Ext}}$.

The estimation results concerning static parameters were good which implies that the proposal on balancing can be solved as mentioned above as an optimisation problem by using linear programming in Matlab with satisfactory results. The optimisation problem (3.4) can be used to solve the proposal on balancing problem with some modifications depending on how many places that would be used for inserting weights.

Another very simple solution has been investigated for the proposal on balancing and is based on linear algebra. The solution does not need to know any information about the nominal mass $m$, but it needs information about the external mass $m_{\text{Ext}}$ and the method is very easy to implement in a real system. The solution is a direct solution of an equation system and is based on moment equations as all of the methods for proposal on balancing. The method is presented in Chapter 3 in Section 3.2.2.

A very general solution for proposal on balancing that has many constraints and that can be solved using linear programming was presented in Chapter 3 in Section 3.2.3. The method is implemented to find a general solution that includes both the gravitation force $F_g$ and the buoyancy force $F_b$ that has to satisfy the expression $F_b \geq F_g$ and the limitations that the system has on limited weights and buoyant material. The calculated solution gives a value for each of the following parameters $M_i$ for $i = 1, 2, 3, 4$ and $B_i$ for $i = 1, 2, 3, 4$.

The issues that occur when installing or re-installing weights in the ROV have
been solved in the simulation environment for the ROV called Sabertooth. The extra inserted mass $m_{\text{Ext}}$ in this case will give a solution of where the weights have to be inserted to get a perfectly balanced ROV or a solution that is well optimised to be balanced. The external mass $m_{\text{Ext}}$ in this project represents the weights of the installed or re-installed equipment.

The results when estimating $z_g$ are less accurate since $z_g$ could not be estimated from the equation system (3.1) and it does not matter when using the ROV called Sabertooth because $z_g$ is not needed for the proposal on balancing.

The dynamic parameter estimation has been carried out using simulation data and the estimated parameters seem to have the same sign as the real parameters that were used in the simulation environment. The parameters are important when estimating a physical model and for this case it seems that they are good enough for the application. One exception is that model quality in the $z$ direction and the reason is because of the construction of the ROV called Sabertooth where the $z$-motor is not placed at the center of gravity. When exciting the $z$-axis the pitch angle $\theta$ will be affected instead of affecting the $z$-axis, see Figure 5.3.

Some of the parameters in the dynamic model are difficult to estimate. The reason is because they include derivatives. Another reason, which was mentioned before, is the construction which makes it difficult to excite in a specific direction. It is best to estimate the parameters without using the controller. If the controller is used when estimating the parameters it might affect the values of the parameters.

6.2 Future work

Real test data from the ROV are preferred to ensure that the algorithms of estimating the parameters and proposal on balancing work correctly.

Collection of the data for the moment is done by using the ROV system and the data is then used offline. In the future it world be better to estimate the parameters online and for this there are many other results like Hong et al. (2013). The estimation of the parameters in the dynamic case can be used to get a controller for the ROV. A bad model is less problematic than good controller because, the affect of modelling can be taken in the controller. For this use the Institutionen för systemteknik (2010) compendium can be used because it contains some of the new control design methods such as MPC.

There are other methods for estimating the parameters of a nonlinear physical model. For example, the EKF can be used to estimate parameters Sabet et al. (2014), or online estimation of the parameters can be used as mentioned above
Hong et al. (2013). Other methods are instead of using a physical model can each degrees of freedom which means $x$, $y$, $z$, roll, pitch and yaw being estimated separately or by combining some of them together depending on the purpose of the modelling. Good results are seen in Savaresi et al. (2004) for the purpose of estimating each degrees of freedom separately. A black box model instead of a non-linear physical model is another option Sjöberg et al. (1995).
Appendix

In this appendix, the signals for the experiments on static parameter estimation are represented in Figures A.1, A.2, A.3, A.4, A.5 and A.6. The signals for dynamic parameter estimation are shown in Figures A.7, A.8 and A.9. Figures A.10, A.11, A.12, A.13, A.14 and A.15 illustrate the estimated parameters of the center of gravity as function of the external mass $m_{\text{Ext}}$. The estimation error $\epsilon$ between real parameter and estimated parameter for the static parameters is shown in Figures A.16, A.17 and A.18.
Figure A.1: Roll and pitch angles as functions of sample number and for $m_{\text{Ext}} = 1$ kg in AftSB.

Figure A.2: Roll and pitch angles with noise as functions of sample number and for $m_{\text{Ext}} = 1$ kg in AftSB.
Figure A.3: Roll and pitch angles as a functions of sample number and for $m_{\text{Ext}} = 5$ kg above and $m_{\text{Ext}} = 10$ kg below in AFSB.

Figure A.4: Roll and pitch angles with noise as functions of sample number and for $m_{\text{Ext}} = 5$ kg above and $m_{\text{Ext}} = 10$ kg below in AFSB.
Figure A.5: Roll and pitch angles as functions of sample number and for $m_{\text{Ext}} = 15$ kg above and $m_{\text{Ext}} = 20$ kg below in AFSB.

Figure A.6: Roll and pitch angles with noise as functions of sample number and for $m_{\text{Ext}} = 15$ kg above and $m_{\text{Ext}} = 20$ kg below in AFSB.
Figure A.7: The signal that was used for excitation in the uvr direction is a translation in the x- and y-axes and a rotation around the z-axis.
Figure A.8: The signal that was used for excitation in the uwq direction is a translation in the x- and z-axes and a rotation around the y-axis.
Figure A.9: The signal that was used for excitation in the uvwpqr direction is a translation in the x-, y- and z-axes. And a rotation around the x-, y- and z-axis.
The center of mass shifted in x-axis $X_g$ (m) as a function of external mass $m_{Ext}$ calculated theoretically for AftSB.

$$X_g = -1.031 \times m_{Ext}/(m + m_{Ext})$$

<table>
<thead>
<tr>
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<th>$Y$</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>5</td>
<td>-0.004336</td>
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<tr>
<td>10</td>
<td>-0.008635</td>
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<tr>
<td>15</td>
<td>-0.0129</td>
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<tr>
<td>20</td>
<td>-0.01713</td>
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Figure A.10: The center of mass displacement in the x-axis as a function of external mass calculated theoretically for AftSB.

The center of mass displacement in the x-axis $X_g$ (m) as a function of external mass $m_{Ext}$.

$$X_g = -0.0008556 \times m_{Ext} - 4.469e-05$$

<table>
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<tr>
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<tbody>
<tr>
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Figure A.11: The figure shows simulation data results. The center of mass displacement in the x-axis is plotted as a function of external mass $m_{Ext}$. The experiments were done in a simulation environment for AftSB without noise and with some noise.
Figure A.12: The center of mass displacement in the y-axis as a function of external mass calculated theoretically for AftSB.

Figure A.13: The figure shows simulation data results. The center of mass displacement in the y-axis is plotted as a function of external mass $m_{\text{Ext}}$. The experiments were done in a simulation environment for AftSB without noise and with some noise.
The center of mass shifted in z-axis $Z_g$ (m)

The center of mass displacement in the z-axis as a function of external mass

$$Z_g = \frac{0^* m_{Ext}}{(m + m_{Ext})}$$

**Figure A.14:** The center of mass displacement in the z-axis as a function of external mass calculated theoretically for AftSB.

The center of mass shifted in z-axis $Z_g$ (m)

The center of mass displacement in the z-axis as a function of external mass

$$Z_g = -1.633e^{-05}m_{Ext} + 0.01999$$

$$Z_{g\text{ brusig}} = -4.884e^{-05}m_{Ext} + 0.01983$$

**Figure A.15:** The figure shows simulation data results. The center of mass displacement in the z-axis is plotted as a function of external mass $m_{\text{Ext}}$. The experiments were done in a simulation environment for AftSB without noise and with some noise.
Figure A.16: Prediction error $\epsilon_t(\theta, \phi) = m_{\text{Ext}} - \hat{m}_{\text{Ext}}(\theta, \phi)$ for the external mass $m_{\text{Ext}}$ as function of pitch $\theta$ and roll $\phi$ angles for collected data without noise. Collection of data was made with different values for roll $\phi$ and pitch $\theta$ as can be seen in the figure. No matter of the chosen value for roll $\phi$ and pitch $\theta$ angles, the reason is that the estimated prediction error seems to be the same. The unit of the y-axis is in $[\text{kg}]$. 

The estimated error in the external mass as a function of pitch $\theta$ and roll $\phi$.
The estimated error in $X_g$ as function of pitch $\theta$ and roll $\phi$

$$\epsilon_t(\theta) = X_g - \hat{X}_g(\theta, \phi)$$

$\times 10^{-6}$

Figure A.17: Prediction error $\epsilon(\theta, \phi) = x_g - \hat{x}_g(\theta, \phi)$ for the estimated offset $x_g$ as function of pitch $\theta$ and roll $\phi$ angles for collected data without noise. Collection of data was made with different values for roll $\phi$ and pitch $\theta$ as seen in the figure. No matter of the chosen value for roll $\phi$ and pitch $\theta$ angles, the reason is that the estimated prediction error seems to be the same. The unit of the y-axis is in [m].
Figure A.18: Prediction error $\epsilon_t(\theta, \phi) = y_g - \hat{y}_g(\theta, \phi)$ for the estimated offset $y_g$ as function of pitch $\theta$ and roll $\phi$ angles for collected data without noise. Collection of data was made with different value for roll $\phi$ and pitch $\theta$ as seen in the figure. No matter of the chosen value for roll $\phi$ and pitch $\theta$ angles, the reason is that the estimated prediction error seems to be the same. The unit of the y-axis is in [m].
Bibliography


