Pathways to arithmetic fact retrieval and percentage calculation in adolescents

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Abstract

Background: Developing sufficient mathematical skills is a prerequisite in order to function adequately in society today. Given this, an important task is to increase our understanding regarding the cognitive mechanisms underlying young people's acquisition of early number skills and formal mathematical knowledge.

Aims: The purpose was to examine if the Pathways to mathematics model provides a valid account of the cognitive mechanisms underlying symbolic number processing and mathematics in adolescents. The Pathways model states that the three pathways should provide independent support to symbolic number skill. Each pathway's unique contribution to formal mathematics varies depending on the complexity and demand of the tasks.

Sample: The study used a sample of 114 adolescents (71 girls). Their mean age was 14.60 years (SD = 1.00).

Methods: The adolescents were assessed on tests tapping the three pathways and general cognitive abilities (e.g., working memory). A structural equation path analysis was computed.

Results: Symbolic number comparison was predicted by the linguistic pathway, the quantitative pathway, and processing speed. The linguistic pathway, quantitative pathways and symbolic number comparison predicted arithmetic fact retrieval. The linguistic pathway, working memory, visual analogies and symbolic number comparison predicted percentage calculation.

Conclusions: There are both similarities and differences in the cognitive mechanisms underlying arithmetic fact retrieval and percentage calculation in adolescents. Adolescents' symbolic number processing, arithmetic fact retrieval and percentage calculation continues to rely on the linguistic pathways whereas the reliance upon the spatial pathway has ceased. The reliance upon the quantitative pathway varies depending on the task.

Background

The Pathways to mathematics model developed by LeFevre et al. (2010) states that young children's acquisition of early numeracy skills and formal mathematical knowledge is founded on three pathways, the quantitative pathway, the linguistic pathway, and the spatial pathway.

The quantitative pathway refers to a pre-verbal core number system assumed to form the basis for the language-based symbolic-number system, and support early formal mathematic skills via the symbolic-number system (Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Gelman & Butterworth, 2005; Lyons & Beilock, 2011; Piazza, 2010; von Aster & Shalev, 2007).

According to the *approximate number system* (ANS) model, the spatial pathway plays an important role in human number processing, as numerosities are represented on a spatially oriented mental number line (analog magnitude representations) in an ascending left to right order (Dehaene, 1992; Dehaene, Bossini, & Giraux, 1993; Feigenson et al., 2004; Piazza, 2010). Moreover, mathematical problems that are complex, novel, or ill-defined or require transformations are assumed to be represented in spatial formats in order to be efficiently and accurately solved (Kaufmann, 1990; Mix & Cheng, 2012).

The linguistic pathway is assumed to subserve the acquisition of the language-based symbolic-number system (LeFevre et al., 2010), that is, learning the counting words and Arabic numerals (Dehaene, 1992; Geary, 2013; von Aster & Shalev, 2007). According to LeFevre et al. (2010) learning the symbolic-number system is similar to mastering the symbolic writing system. *The Triple-code model* states that when these symbols connect to the core number representation system (analogue magnitude representation) a verbal-phonological number code and a visual Arabic number code become established (Dehaene, 1992). The verbal-phonological number code is used for counting and solving single-digit arithmetic problems by means of counting strategies, and establishing and retrieving arithmetic facts (Dehaene, 1992; Geary, 1993; Logie & Baddeley, 1987; Mix & Cheng, 2012; von Aster & Shalev, 2007). The visual Arabic number code is assumed to support written multi-digit calculation (Dehaene, 1992; Träff & Passolunghi, 2015; Vukovic, Lesaux, & Siegel, 2010). Theoretically, the linguistic pathway should contribute to most aspects of mathematics that involve the symbolic-number system (LeFevre et al., 2010).

The purpose of the study was to examine if the Pathways model is valid for mathematics in adolescents. More specifically, the two hypotheses provided by the model were tested in relation to two aspects of mathematics, arithmetic fact retrieval and percentage calculation, constituting important skills that are required to function adequately in society today. The first hypothesis states that the three pathways provide independent support to basic symbolic-number processing skills. The second hypothesis states that each pathway's unique and relative contribution to formal mathematics varies depending on the complexity and demand of the tasks.

Existing research directly addressing the two hypotheses of the Pathways model have produced inconsistent empirical findings. LeFevre et al's (2010) findings were consistent with both hypotheses, while Cirino (2011) and Sowinski et al. (2015) obtained support for the first hypothesis, and the second hypothesis, respectively. In contrast, Cirino, Tolar, Fuchs, & Huston-Warren (2016) did not find support for either of the two hypotheses in their sample of sixth graders.

A large number of studies, not explicitly testing the Pathways model per se, indicate that the quantitative pathway support children's acquisition of the symbolic number system (Bartelet, Vaessen, Blomert, & Ansari, 2014; Gilmore, McCarthy, & Spelke, 2010; Göbel, Watson, Lervåg & Hulme, 2014; Hornung, Schiltz, Brunner & Martin, 2014; Sasanguie, Göbel, Moll, Smets & Reynvoet, 2013; van Marle, Chu, Li, & Geary, 2014). Results reported by Koponen, Salmi, Eklund, and Aro (2013) and Krajewski and Schneider (2009) show that the linguistic pathway in the form of phonological awareness and rapid automatic naming (RAN) contributes to early number knowledge skills (e.g., counting; digit knowledge). Evidence of a link between the spatial pathway and early symbolic-number skills (counting, number identification, symbolic-number magnitude; number line estimation) have been obtained by Verdine, Irwin, Golinkoff and Hirsh-Pasek (2014) and Gunderson, Ramirez, Beilock, and Levine (2012). The combined results of these studies provide support to the first hypothesis of the Pathways model.

Consistent with the Pathways model and other models (e.g., Triple code model; Dehaene, 1992), a number of studies show that phonological awareness contributes to simple arithmetic problem-solving (e.g., 3 + 4) via counting strategies or via direct and automatic retrieval of arithmetic facts (De Smedt, Taylor, Archibald, & Ansari, 2010; Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Passolunghi, & Lanfranchi, 2012; Simmons, Singleton, & Horne, 2008). Recently, Koponen, Salmi, Torppa, Eklund, Aro, Aro, Poikkeus, Lerkkanen, and Nurmi (2016) observed that RAN in kindergarten was predictive of arithmetic fluency one year later (see also Koponen, Aunola, Ahonen, & Nurmi, 2007).

Research performed on 3-6-year-olds demonstrates that the quantitative pathway (i.e., non-symbolic number processing) contribute directly to one-digit arithmetic performance (Libertus, Feigenson, & Halberda, 2011; Libertus, Feigenson, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011; Passolunghi, Cargnelutti, & Pastore, 2014; Vanbinst, Ghesquière & De Smedt, 2015). The contribution from the quantitative pathway to one-digit arithmetic skills in older children and adults is, however, only indirect via the symbolic number system (Bartelet et al., 2014; Göbel et al., 2014; Hornung et al., 2014; Lyons & Beilock, 2011; Sasanguie et al., 2013; van Marle et al., 2014). Other studies have not found any direct or indirect contribution from the quantitative pathway (Cirino et al., 2016;

Sasanguie et al., 2013; Szücs, Devine, Soltesz, Nobes & Gabriel's, 2014). This empirical picture has been challenged by recent intervention studies showing that training non-symbolic number discrimination enhances the arithmetical skills of adults and children (Kuhn & Holling; 2014; Park & Brannon, 2013; Wang, Odic, Halberda, & Feigenson, 2016). Thus, it appears that the quantitative pathway may continue to directly support arithmetic achievement, even after the symbolic number system has been established.

Available research shows that spatial processing skills sub-serve children's mathematical proficiency. Zhang, Koponen, Räsänen, Aunola, Lerkkanen, and Nurmi (2014) found that kindergarten spatial processing ability was predictive of early arithmetic skills and its' growth up to grade three. In contrast, Skagerlund and Träff (2016) found recently that spatial processing did not provide an unique contribution to eight to ten-year-olds arithmetic fact retrieval performance. Similar lack of association was obtained by Cirino et al. (2016) in their study on twelve-year-olds. Contrary to the negative findings concerning the spatial processing–arithmetic fact retrieval link in older children, studies show that complex, novel, or ill-defined mathematical problems are support by spatial processing (Hegarty & Kozhevnikov, 1999; Lachance & Mazzocco, 2006; Mazzocco & Myers, 2003; Skagerlund & Träff, 2016; van Garderen, 2006; Szücs et al., 2014).

Although no study has yet explicitly examined the mechanism underlying percentage calculation; a few studies have addressed this issue in the related domain of fractions. These studies show that whole number arithmetic fluency or computation, and whole number line estimation independently contribute to fraction concepts and fraction calculation in middle school children (e.g., Namkung, & Fuchs, 2016; Seethaler, Fuchs, Star, & Bryant, 2011; Vukovic, Fuchs, Geary, Jordan, Gersten, & Siegler, 2014). General cognitive abilities beyond math-related skills also have a role in fraction learning. Language skills contribute to both understanding of fraction concepts and fraction calculation (e.g., Namkung & Fuchs, 2016;

Seethaler et al., 2011; Vukovic et al., 2014). Verbal and visual-spatial working memory and attentive behaviour have consistently been found to support fractions skills (e.g., Hansen, Jordan, Fernandez, Siegler, Fuchs, Gersten, & Micklos, 2015; Jordan, Hansen, Fuchs, Siegler, Gersten, & Micklos, 2013; Vukovic et al., 2014). In addition, a few studies suggest that processing speed and nonverbal reasoning have a role in children's fractions performance (Namkung & Fuchs, 2016; Jordan et al., 2013; Seethaler et al., 2011).

All in all, prior studies provide support to the second hypothesis of the Pathways model as the relative contribution from the pathways varies depending on the tasks at hand (i.e., arithmetic fact retrieval or percentage calculation) and the age of the participants.

The present study examined if the Pathways model is valid for mathematical processing in adolescents. This is an important research task as the model was primarily developed to account for young children's mathematical development and the majority of studies examining the cognitive mechanism of mathematics have been performed on 5- to 10-year-old children. It is quite possible that the underlying mechanism changes as a function of skill development. According to Ackerman's general theory of skill acquisition (1988), simple arithmetic (e.g., 3+4) should draw heavily on general cognitive abilities during the early phase of skill development (i.e., younger children), but not during later phases (i.e., older children) when a high level of automatization has been reached (Ashcraft, 1995; Geary & Hoard, 2005). More advanced mathematical skills (e.g., percentage calculation) acquired and learned in later phases of mathematical development may rely on somewhat different sets of cognitive abilities than earlier acquired skills (Cirino et al., 2016; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Träff, 2013). A test-battery consisting of ten tasks (phonological awareness, colour-naming, non-symbolic number comparison, symbolic number comparison, mental rotation, nonverbal reasoning ability; working memory; general processing speed; arithmetic fact retrieval, percentage calculation) was used to assess the three pathways, symbolic number processing, general cognitive abilities and mathematics. Structural equation path analyses were computed to test the direct and indirect contributions from the pathways.

The following four hypotheses were tested:

1) All three pathways should provide independent support to basic symbolic number processing skills (LeFevre et al., 2010).

2) The linguistic pathway is expected to provide direct contribution to arithmetic fact retrieval (De Smedt et al., 2010; Fuchs et al., 2005; Passolunghi, & Lanfranchi, 2012; Simmons et al., 2008), whereas it is less clear whether the quantitative pathway will contribute directly or indirectly, via symbolic number processing, to arithmetic fact retrieval (Göbel et al., 2014; Hornung et al., 2014; Passolunghi et al., 2014). The contribution of the spatial pathway to arithmetic fact retrieval in the present age group is unclear.

3) A novel and important question is to what extent the three pathways will account for individual differences in percentage calculation. In view of prior studies, the linguistic pathway and arithmetic fact retrieval are expected to provide direct contributions (cf. Vukovic et al., 2014), whereas the contributions from the other two pathways are unclear due to lack of prior studies.

4) Working memory and nonverbal reasoning ability are expected to support percentage calculation as percentage calculation may involve transformation operation which should draw on these two abilities (cf. Jordan et al., 2013; Namkung & Fuchs, 2016).

Method

Participants

An unselected sample of 114 adolescents (71 girls) in southern Sweden participated in this study. Participants were recruited by means of a letter of consent that they brought home to their parents from school. All participants with parental consent were included. Their mean age was 14.60 years (SD = 1.00, min = 13.10, max = 17.00). All participants were fluent speakers of Swedish, had normal or corrected-to-normal visual acuity, and no hearing loss. Participants diagnosed with neuropsychological disturbances (e.g., ADHD) were excluded from the study.

Measures

Arithmetic fact retrieval. The participant had to solve simple arithmetic problems (e.g. 5 + 4) by direct retrieval from long-term memory. The test material consisted of twelve additions (e.g., 5 + 4; 3 + 8), and twelve subtractions (e.g., 9 – 4; 6 – 2), and twelve multiplications problems (e.g., 4 x 5; 9 x 8). The operation types were administered in three separate blocks. A single problem was presented horizontally on the computer screen, each problem was preceded by the word “READY”. The experimenter pressed the mouse button and a problem was displayed on the computer screen until the participant gave an oral response. Then, by pressing the mouse button again the problem disappeared. To ensure that the task tapped fact retrieval, the participant was instructed to provide an answer immediately by recalling the answer and was encouraged to guess if not able to recall the answer immediately. The number of correctly solved problems with response times within three seconds was used as the dependent measure (cf. Russell & Ginsburg, 1984).

Percentage calculation. This task consisted of 22 problems. The participant's task was to solve as many problems as possible in five minutes, in any order. The problems were presented horizontally. The adolescents responded in writing. Only paper and pencil were allowed. Some tasks were simple arithmetic calculations, such as "How many percent are 25 of 50?", “How much is 1% of 3500?”; How much is 7% of 4300?” Other tasks required calculations in several stages to be solved, such as " A school has 625 students and 52% are boys. Thirteen percent of the girls are in a riding club. How many girls are in the riding club?”. The maximum score was 22.

Phonological awareness. The test consisted of 25 problems divided into 5 different subtasks: deletion of individual phonemes (“If we have the word dog and remove the d, what will remain?), deletion of phoneme sequence (If we have the word crocodile and remove the dile, what will remain?), saying words backwards (“Say the word live backwards (evil)”, saying nonwords backwards, and spoonerism (eye–ball; bye–all). If the adolescent responded correctly within 10 seconds he/she received 2 points, responses within 20 seconds generated 1 point. Thus, the maximum test score was 50.

Colour-naming. The material consisted of a sheet of paper, containing 50 red, green, blue, black and yellow squares, presented in 5 rows with 10 in each column. The task was to name the colour of the squares as quickly as possible without making any errors. A stopwatch was used to measure the total time it took to name the 50 squares. The response time was used as a measure of rapid and automatic access to phonological representations connected to familiar visual stimuli (Denckla & Rudel, 1976).

Non-symbolic number comparison. The computer program Panamath, version 1.21 (see Halberda, Mazzocco & Feigenson, 2008) was utilized for this task. Two arrays of blue and yellow dots, ranging from 5 to 21, was displayed on the screen. The adolescent was then asked to determine which array contained more dots, quickly but as accurately as possible. The adolescent indicated his/her response by pressing either F- or L-key, corresponding blue/yellow respectively. The arrays only remained visible for 984 ms, to discourage enumeration. However, the adolescent was had an unlimited amount of time to indicate his/her response. To enable the next trial, the adolescent pressed the space bar which produced a fixation cross on the center of the screen prior to each trial. Initially, two practice trials were performed prior to the experimental trials. A total of 104 trials were displayed, utilizing four ratios (1.24; 1.37; 1.60; 2.60), each presented 26 times. Half of the trials contained more blue dots and vice versa. To control for confounding variables and attempt to ensure attention to numerosity, surface area and dot size varied on half of the trials. Each participant's accuracy (percent correct) and response time measures were utilized as dependent measures.

Symbolic number comparison. This task was used to assess symbolic number processing. Two digits were simultaneously displayed on the screen. The task was to decide which of the digits was numerically larger and respond by pressing the key corresponding to the appropriate side of the screen. Before each trial, a fixation cross was displayed for 1000 ms, after which two digits were presented and remained until the participant pressed a key. The task was presented in two blocks: one for single-digit numbers (e.g., 1-2; 4-5; 1-6; 4-9) and one for double-digit numbers (e.g., 21-22; 86-87; 31-36; 94-99). Each digit pair was presented twice (e.g., 4-5; 5-4), resulting in a total of 64 trials. For each adolescent, a mean total response time (correct responses) was calculated based on the two blocks, and used as the dependent measure.

Mental rotation. This task was used as a measure of spatial processing (Rüsseler, Scholz, Jordan, & Quasier-Pohl, 2005). The test material consisted of alphabetic letters, one letter per trial, a total of 16 trials. A target letter was located on the left side, accompanied by four comparison letters located on the right side, adjacent to the target. The comparison letters always consisted of two "correct" and two "incorrect" letters. The task was to identify the two matching letters, which prompted a mental rotation, and by marking them with a pen. Inverted instances of the target (i.e., visually mirrored) were used as incorrect comparison stimuli. All letters were rotated only on the picture-plane and in one of six rotation angles (45°; 90°; 135°; 225°; 270°; 315°). Both correct comparison letters had to be marked to yield a point for each trial, yielding a maximum score of 16. Number of correctly solved trials divided by the time to complete the 16 trials (maximum 120 sec) was used as dependent measure.

Complex word repetition. In this task, the adolescent was presented with sequences of words. The task was to decide whether each presented word was an animal (“YES”) or not (“NO”). The adolescent had to answer before the next word was presented. At the end of the sequence the adolescent had to recall in correct serial order the words in the sequence. The first span size employed was two, the next was three, four, five, six and seven. Testing continued until the participant failed both trials of the same span length. Forty-three percent of the words were animals. The experimenter read the sequences of words from a sheet of paper. The longest sequence of words, plus 0.5 point extra if the participant managed to recall both trials was used as an index of verbal working memory.

Visual analogies. This measure of non-verbal reasoning consisted of 34 problems and four exercise problems (McCarthy & Kirk, 1961). In this study, only problems 12-34 were used. Each problem consisted of four pictures, numbered 1-4, and an un-numbered picture in the middle. The participant's task was to say which of the four images belonged to the unnumbered image in the center. The first problems involved connecting images of objects and people. For example, a picture of a dog and a picture of a dog bone, but the pictures became increasingly more abstract and difficult throughout the test. After three consecutive errors the testing was stopped. The maximum score was 35.

Visual digit-matching. This task tapped rapid identification of digits. The participant received a sheet of paper with 15 rows of digits (1-9). Each row consisted of seven digits, with two of the digits being identical. The task was to mark the two identical digits in each row as quickly and accurately as possible. A practice trial of three rows was performed before the actual task started. Performance was measured by the time needed to complete all 15 rows.

General procedure

All ten tasks were administered in one session lasting for approximately 60 minutes. The test order was the same for all adolescents and all testing was performed at the adolescent's school. All instructions regarding the tasks were presented orally. Four experimenters performed all data collection. The software program SuperLab Pro 4.5 was used to administer the symbolic number comparison and the arithmetic fact retrieval tasks.

Results

Descriptive statistics (means, standard deviations, correlations) for all measures used in the study are presented in Table 1 and 2.

Insert Table 1 and Table 2 here

Analysis

The present hypotheses were tested by computing a path analysis using the Mplus 7 program with maximum likelihood estimator (Muthén & Muthén, 2012). According to theory and prior studies symbolic number comparison and arithmetic fact retrieval were used as mediators in the model. The indirect contributions (the links via the mediators) were tested according to the recommendation in Kelloway (2014) with both bootstrapping and bias-corrected confidence interval. To identify the model, the intercept of the arithmetic fact retrieval task was fixed to zero.

The path analysis

The model (see Figure 1) showed a good model fit, $\chi^2(1, N= 114) = 0.809$, $p = .368$, RMSEA= 0.00, 90% CI 0.00-0.24, CFI = 1.00, SRMR = 0.01. The model predicted 46% ($R^2 = .46$) of the variance in symbolic number comparison, 43% of the variance in arithmetic fact retrieval ($R^2 = .43$), and 53% of the variance in percentage calculation ($R^2 = .53$).

Insert Figure 1

Phonological awareness, colour-naming, non-symbolic number comparison (speed), and visual digit-matching predicted symbolic number comparison. Phonological awareness, non-symbolic number comparison (accuracy), and symbolic number comparison contributed directly to arithmetic fact retrieval.

Age, phonological awareness, working memory, visual analogies, and arithmetic fact retrieval contributed directly to percentage calculation. Symbolic number comparison accounted for 1% variance indirectly via arithmetic fact retrieval.

Discussion

The present study examined if the Pathways to mathematics model provides a valid account of the cognitive mechanisms underlying symbolic number processing, arithmetic fact retrieval and percentage calculation in adolescents.

It was found that phonological awareness, colour-naming (linguistic pathway), non-symbolic number comparison (speed; quantitative pathway) and visual digit-matching emerged as significant predictors of symbolic number comparison. Arithmetic fact retrieval was directly predicted by phonological awareness, non-symbolic number comparison (accuracy), visual digit-matching and symbolic-number comparison. Percentage calculation was directly predicted by arithmetic fact retrieval, phonological awareness, working memory, visual analogies, age and indirectly by symbolic number comparison via arithmetic fact retreival. These results will be discussed in relation to the hypotheses of the study.

Pathways to symbolic number processing

The predictor pattern of symbolic number comparison is not fully consistent with the first hypothesis or with previous research performed on 5-to-10 year olds as only two pathways contributed uniquely to symbolic number comparison (Cirino, 2011; LeFevre et al., 2010). Symbolic number comparison in adolescents appears to rely upon linguistic-phonological processes (phonological awareness, colour-naming) and quantitative processes (non-symbolic number comparison), but not spatial processes (cf. Bartelet et al., 2014; Gilmore et al., 2010; Göbel et al., 2014; Hornung et al., 2014; Koponen et al., 2013; Krajewski & Schneider, 2009; Sasanguie et al., 2013; van Marle et al., 2014).

A theoretically plausible interpretation of the pattern of predictors is that the cognitive mechanism underlying symbolic number comparison in adolescents comprises of several different processes, consistent with the Triple code model (Dehaene, 1992). The initial process, reflecting the contribution from the visual digit-matching task, concerns rapid identification of the two digits which should involve activation of visual Arabic number codes (e.g., 1 vs. 2). The support from the linguistic pathway (i.e., phonological awareness and colour-naming) suggests activation of verbal-phonological number codes (e.g., “one” vs. “two”) during symbolic number comparison (cf. LeFevre et al., 2010). More specifically, rapid and automatic access (i.e., colour-naming) to high quality phonological representations (i.e., phonological awareness) should, theoretically, enable efficient activation of verbal-phonological number codes (cf. Koponen et al., 2013; Krajewski & Schneider, 2009). The contribution from the speed measure of the non-symbolic number comparison task suggests that rapid access to the ANS (i.e., analog magnitude representation) is a crucial process during symbolic number comparison (cf. Bartelet et al., 2014; Dehaene, 1992; Hornung et al., 2014; LeFevre et al., 2010; van Marle et al., 2014).

The lack of contribution from the spatial pathway (mental rotation) to symbolic number comparison contradicts the Pathway model and is at odds with research performed on younger children (Bull, Espy, & Wiebe, 2008; Cirino, 2011; De Smedt, Janssen, Bouwens, Verschaffel, Boets, & Ghesquière, 2009; Gunderson et al., 2012; Verdine et al., 2014). However, this finding is consistent with Cirino et al.'s (2016; see also Bailey, Siegler, & Geary, 2014) study performed on sixth graders, and indicates that spatial processes are not critically involved in symbolic number comparison in older children and adolescents, with well-developed symbolic number processing skills.

Pathways to arithmetic and percentage calculation

As expected, the linguistic pathway (phonological awareness) provided direct contribution to arithmetic fact retrieval (De Smedt et al., 2010; Fuchs et al., 2005; LeFevre et al., 2010; Passolunghi, & Lanfranchi, 2012; Simmons et al., 2008) as well as percentage calculation (LeFevre et al., 2010; Namkung & Fuchs, 2016; Seethaler et al., 2011; Vukovic et al., 2014). Similar to research on young children but contrary to older children and adults, the quantitative pathway contributed directly to arithmetic fact retrieval (Bartelet et al., 2014; Dehaene, 1992; Hornung et al., 2014; LeFevre et al., 2010; Libertus et al., 2011; Mazzocco et al., 2011; Passolunghi et al., 2014; Vanbinst et al., 2015; van Marle et al., 2014), but not to percentage calculation. Contrary to prior research, neither arithmetic fact retrieval nor percentage calculation were supported by the spatial pathway (cf. Lachance & Mazzocco, 2006; Mazzocco & Myers, 2003; Skagerlund & Träff, 2016; Szücs et al., 2014; Zhang et al., 2014; see also Cirino et al., 2016 for similar results). Similar to some prior studies, symbolic number comparison supported arithmetic fact retrieval directly (Bartelet et al., 2014; Cirino et al., 2016; Martin, Cirino, Sharp, Barnes, 2014; Träff, 2013) and percentage calculation indirectly. Percentage calculation was, as expected, supported by arithmetic fact retrieval, verbal working memory, and non-verbal reasoning (i.e., visual analogies; Jordan et al., 2013; Seethaler et al., 2011; Vukovic et al., 2014).

In accordance with the second hypothesis of the Pathway model (LeFevre et al., 2010), the two sets of predictors demonstrate that different cognitive mechanisms underlie arithmetic fact retrieval and percentage calculation in adolescents. Another important finding was that the linguistic pathway (i.e., phonological awareness) contributed to both mathematical tasks. This finding supports LeFevre et al.'s (2010) position that the linguistic pathway should support all mathematical skills that involves symbolic number knowledge, possibly by providing high quality verbal-phonological number codes that can be manipulated during task performance. This conclusion is further supported as the linguistic pathway (i.e., phonological awareness) contributed to symbolic number comparison which in turn predicted both mathematical tasks, directly or indirectly. Speed of access to high quality verbal-phonological number codes appears, on the other hand, not be a critical process during arithmetic fact retrieval or percentage calculation in adolescents as the colour naming task did not provide unique contribution to either of the tasks (cf. Koponen et al., 2007; Koponen et al., 2016 for opposite results).

The pattern of predictors regarding arithmetic fact retrieval is theoretically important as it suggests that solving visually presented single-digit number combinations (3+4) through direct retrieval involves both the linguistic and quantitative pathways but not the spatial pathway. Regarding the validity of the Triple code model, arithmetic fact retrieval in adolescents appears to engage all three number codes; the verbal-phonological number code (phonological awareness), the visual Arabic number code (symbolic number comparison; visual-digit matching) and the analog magnitude representation (non-symbolic number comparison). Thus, contrary to the Triple code model this arithmetic skill does not exclusively rely on the verbal-phonological number code (Dehaene, 1992). The involvement of all three number codes leads us to suggest that arithmetic fact retrieval in adolescents entails several processes. The initial process involves rapid identification of the two digits and the activation of visual Arabic number codes. A second process involves accessing the two digits' (in the arithmetic combination) analogue magnitude representations (for similar reasoning, see Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Träff, 2013). These two processes would be reflected by the contribution from the visual-digit matching and symbolic number comparison tasks. A third process, involving both retrieval and response, activates the analog magnitude representations tapped by the non-symbolic number comparison task and the verbal phonological number code inferred by the phonological awareness task. Thus, this arithmetic task is sub-served by both the analog magnitude representations (i.e., the quantitative pathway; Träff, 2013) and the verbal phonological number code (i.e., the linguistic pathway). If these two number representations are of good quality, arithmetic facts can be correctly established and stored, as well as quickly retrieved. As the present arithmetic fact retrieval task required oral responses, it is likely that this also engages the verbal-phonological number code and/or the linguistic pathway.

Working memory did not predict arithmetic fact retrieval, which is reasonable assuming that the adolescents performed the arithmetic task with a high level of automatization (Ackerman, 1988; Ashcraft, 1995; Geary & Hoard, 2005).

Percentage calculation was directly supported by the linguistic pathway, arithmetic fact retrieval, and indirectly by symbolic number comparison via arithmetic fact retrieval. Thus, percentage calculation in adolescents seems to rely directly on the verbal-phonological number code (phonological awareness) and the visual Arabic number code (symbolic number comparison; arithmetic fact retrieval). Furthermore, performing percentage calculation appears to involve partially similar process as arithmetic fact retrieval. The direct and indirect contribution from the arithmetic fact retrieval and symbolic number comparison tasks suggests a digit identification process, activating visual Arabic number codes. A second process involves accessing the analog magnitude representation (ANS) of the digits in the percentage calculation problem. The calculation process might be subserved by the linguistic pathway, working memory, nonverbal reasoning ability (visual analogies), in combination with arithmetic fact retrieval. Efficient percentage calculation might thus require the activation of verbal-phonological number codes and access to the analog magnitude representation (ANS) via the visual Arabic number codes (i.e., arithmetic fact retrieval; symbolic number comparison). Given the complexity of calculating percentages, the importance of reasoning ability and verbal working memory is theoretically plausible.

In contrast to Vukovic et als (2014) study on 10-year-olds but consistent with Cirino et al. (2016), the spatial pathway did not contribute to percentage calculation. The present finding in conjunction with prior studies suggest that calculating percentages and fractions do not rely on mental rotation abilities (Cirino et al., 2016), but on spatial working memory/spatial attention processes (Vukovic et al., 2014).

Conclusion

The present findings should be interpreted with some care because even though the sample size was reasonably good, it would be ideal to have a larger sample for an analysis of this nature. Thus, an important task for future research would be replicate this study with a larger sample. Nevertheless, this study presents new and theoretically important findings concerning the cognitive mechanisms underlying symbolic number processing and mathematics in adolescents. In view of the Pathways model, the overall results indicate that adolescents' symbolic number processing and mathematics continues to rely on the linguistic pathway whereas the reliance upon the spatial pathway has ceased. The reliance upon the quantitative pathway is uneven as it subserves symbolic number processing and the basic skill arithmetic fact retrieval, but not the more advanced skill percentage calculation. Furthermore, symbolic number comparison underpinned both mathematical tasks, and verbal working memory and nonverbal reasoning ability supports percentage calculation. Thus, the study demonstrates that there are both similarities and differences in the cognitive mechanisms underlying arithmetic fact retrieval and percentage calculation in adolescents. The finding that the linguistic pathway supports both mathematical tasks are in line with LeFevre et al.'s (2010) position, that this pathway should support all mathematical skills that involves symbolic number knowledge.

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Table 1

Descriptive statistics for the measures used in the study

Measures	Mean (SD)	Min–Max	Skewness (SE)	Kurtosis (SE)
AGE (in years)	14.60 (1.00)	13.10–17.00	0.04 (0.23)	-0.83 (0.45)
Phonological awareness (PA)	33.54 (10.22)	0.00–50.00	-0.71 (0.23)	0.65 (0.45)
Colour-naming (CN)	37.70 (8.16)	23.00–74.00	1.24 (0.23)	3.37 (0.45)
Non-symbolic number comparison accuracy (NSNC)	86.55 (7.90)	62.50–100.00	-0.27 (0.23)	0.18 (0.45)
Non-symbolic number comparison speed in ms (NSNCs)	701.18 (213.71	397–1559	1.73 (0.23)	3.61 (0.45)
Mental rotation (MR; accuracy/time)	0.10 (0.04)	0.01–0.19	-0.156 (0.23)	-0.44 (0.45)
Symbolic number comparison (SNC) ms	723.43 (138.10)	432.50–1321.03	1.09 (0.23)	2.30 (0.45)
Complex word repetition (CWR)	4.13 (0.81)	2.00–7.00	0.17 (0.23)	0.96 (0.45)
Visual analogies (VA)	28.15 (4.18)	16.00–35.00	-0.82 (0.23)	0.37 (0.45)
Visual digit-matching (VDM)	43.75 (12.07)	23.19–90.00	1.23 (0.23)	2.40 (0.45)
Arithmetic fact retrieval (AFR)	24.38 (8.53)	1.00–36.00	-0.95 (0.23)	0.45 (0.45)
Percentage calculation (PC)	4.42 (3.92)	0.00–18.00	0.90 (0.23)	0.44 (0.45)

Table 2

Correlations among measures and variance for each measure

	1	2	3	4	5	6	7	8	9	10	11	12
1. AGE	1.00											
2. PA	-.04	104.52										
3. CN	-.04	-.32*	66.58									
4. NSNC	-.05	-.20*	-.05	7.90								
5. NSNCs	-.14	-.08	.28*	.24*	45671.43							
6. MR	.10	.49*	-.28*	.18	-.20*	0.01						
7. SNC	-.16	-.39*	.48*	-.06	.35*	-.42*	19072.92					
8. CWR	-.08	.14	-.02	.19*	.07	.11	-.12	0.66				
9. VA	.09	.39*	-.21*	.11	-.13	.43*	-.21*	.17	17.51			
10. VDM	-.11	-.23*	.38*	-.19*	.02	-.38*	.47*	-.16	-.28*	145.69		
11. AFR	.14	.39*	-.36*	.32*	-.12	.30*	-.53*	.08	.26*	-.44*	72.79	
12. PC	.32*	.46*	-.33*	.20*	-.17	.39*	-.45*	.23*	.40*	-.35*	.56*	15.34

Variance of the measures are presented in the diagonal. * $p < .05$

PA: Phonological awareness, CN: Colour naming, NSNC: Non-symbolic number comparison percent accuracy, NSNCs: Non-symbolic number comparison speed, MR: Mental rotation, SNC: Symbolic number comparison, CWR: Complex word repetition, VA: Visual analogies, VDM: Visual digit-matching, AFR: Arithmetic fact retrieval, PC: Percentage calculation

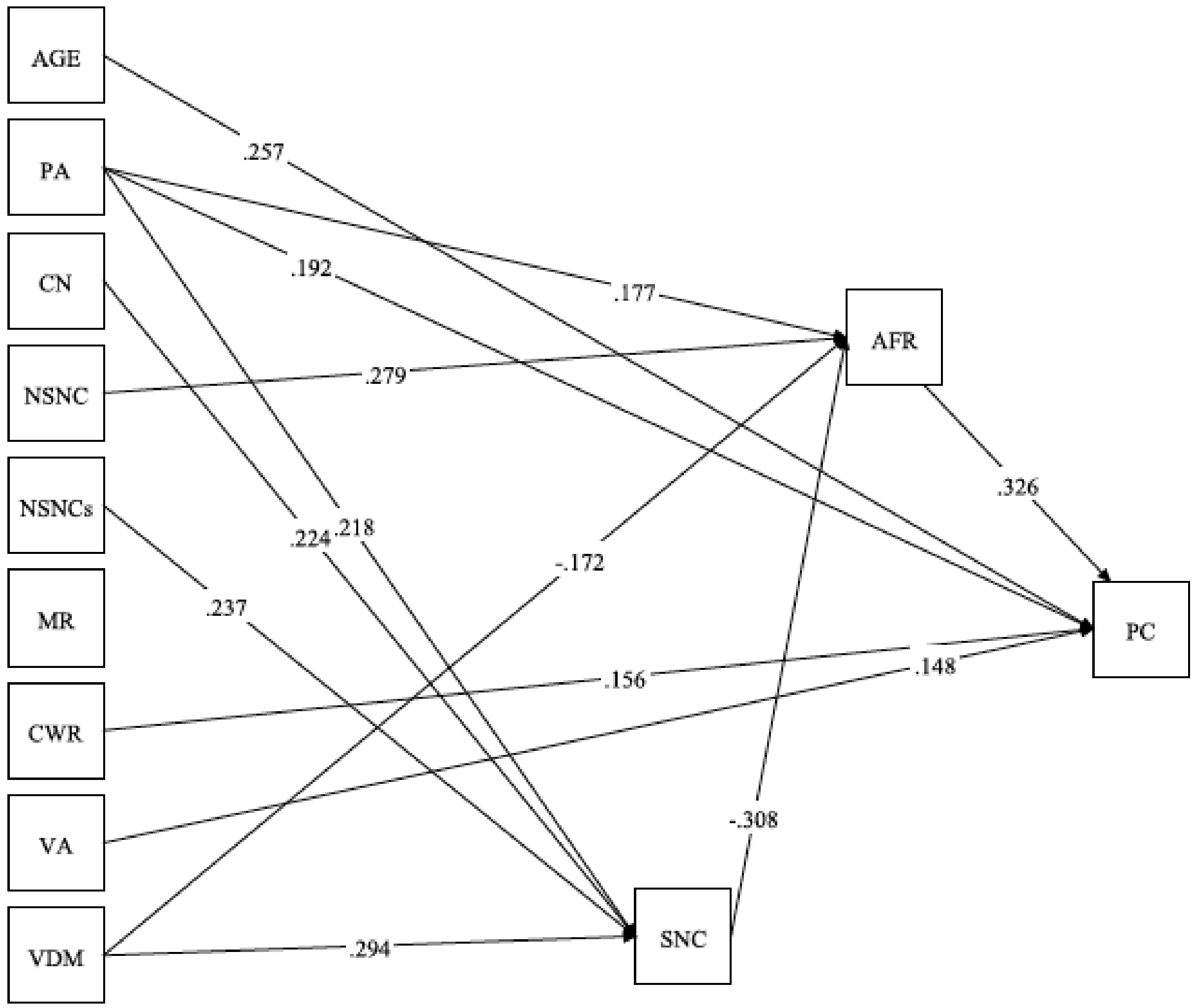

Figure 1. Path model, displaying the different paths to SNC, AFR, and PC. Only paths (standardized estimates) with p <.05 are displayed. Error of SNC, AFR and PC is not shown, of visual purposes. PA: Phonological awareness, CN: Colour naming, NSNC: Non-symbolic number comparison percent accuracy, NSNCs: Non-symbolic number comparison speed, MR: Mental rotation, SNC: Symbolic number comparison, CWR: Complex word repetition, VA: Visual analogies, VDM: Visual digit-matching, AFR: Arithmetic fact retrieval, PC: Percentage calculation