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Martin Falk, A. Seizinger, F. Sadlo, M. Üffinger and D. Weiskopf

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TRAJECTORY-AUGMENTED VISUALIZATION OF LAGRANGIAN COHERENT STRUCTURES IN UNSTEADY FLOW

Martin Falk, Alexander Seizinger, Filip Sadlo, Markus Üffinger, Daniel Weiskopf
VISUS – Visualization Research Center, Universität Stuttgart

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ABSTRACT: The finite-time Lyapunov exponent (FTLE) field can be used for many purposes, from the analysis of the predictability in dynamical systems to the topological analysis of time-dependent vector fields. In the topological context, the topic of this work, FTLE ridges represent Lagrangian coherent structures (LCS), a counterpart to separatrices in vector field topology. Since the explicit vector field behavior cannot be deduced from these representations, they may be augmented by line integral convolution patterns, a computational flow visualization counterpart to the surface oil flow method. This is, however, strictly meaningful only in stationary vector fields. Here, we propose an augmentation that visualizes the LCS-inducing flow behavior by means of complete trajectories but avoids occlusion and visual clutter. For this we exploit the FTLE for both the selection of significant trajectories as well as their individual representation. This results in 3D line representations for 2D vector fields by treating 2D time-dependent vector fields in 3D space-time. We present two variants of the approach, one easing the choice of the finite advection time for FTLE analysis and one for investigating the flow once the time is chosen.

1 Introduction

Different and even contradicting definitions were proposed in the last decades for the term coherent structure. For example, Robinson identified them as regions of the domain exhibiting correlation between flow variables [11]. In recent years an almost complementary definition is becoming increasingly popular: the Lagrangian coherent structures (LCS), which separate regions of qualitatively different flow behavior. Thus, the LCS feature a lower dimensionality than the flow domain they are embedded in, typically they are of codimension one. As shown by Haller [7], LCS can be identified as ridges (local maximizing curves or surfaces) in the finite-time Lyapunov exponent (FTLE), also called direct Lyapunov exponent (DLE). We proposed [12] to extract the ridges as height ridges according to Eberly [4]. The FTLE is nowadays typically derived from the gradient of the so-called flow map at a given time, which maps the seeding positions of trajectories to their end points after finite advection time. LCS have been receiving increasing attention in analysis and visualization of unsteady flow. In the context of our work, they serve as a time-dependent variant of the concept of vector field topology [10]. Vector field topology deals with structures that are invariant under the action of a steady or instantaneous vector field or dynamical system [1]. These structures can be defined in terms of distinguished streamlines. In this sense, critical points, the isolated zeros of a vector field, can be interpreted as degenerate streamlines, and the sets of streamlines converging to these points in forward...
or reverse time are the separatrices both in 2D [8] and 3D [9] vector fields. Similar to LCS, separatrices represent invariant manifolds in vector fields, separating regions of different behavior. However, separatrices play this role only in steady flow or at isolated time steps in unsteady flow because they are computed from streamlines. In contrast, LCS allow this interpretation also in the time-dependent case since these are based on pathlines. Therefore, we proposed a new variant of vector field topology for time-dependent 2D vector fields by replacing the role of streamlines by streaklines [13]. This leads to space-time streak manifolds representing invariant manifolds in unsteady flow [6]. In this sense, space-time streak manifolds are the time-dependent counterpart to separatrices, and hyperbolic trajectories [6] the counterpart to critical points.

Despite their effectiveness in representing the high-level structure of vector fields, separatrices convey in many respects only insufficient information about the flow: speed as well as the orientation of velocity is neither represented along the separatrices nor in their vicinity, which is even more important. It is therefore a widely used approach for steady 2D flow to augment the topology-based visualization by line integral convolution (LIC) [3], a computational counterpart to the surface oil flow method. Although this approach proved successful in many applications, it does not lend itself to the augmentation of LCS visualizations because LIC is based on streamlines whereas LCS is computed from pathlines. Instead, these pathlines may be directly rendered as geometric curves.

To analyze the trajectories that give rise to a specific LCS region, selected trajectories are typically visualized in an interactive manner, since visualization of large sets of pathlines tends to produce visual clutter due to potential intersection and the fact that trajectories may pass the same regions at different times. We therefore propose to visualize only those trajectories that are instrumental for LCS generation.

For steady flow, several approaches for streamline placement are available, irrespective of the LCS context, such as the control of the overall line density [16] or the consideration of the topology of the flow [17]. However, only few approaches are known so far for the placement of pathlines. One example is the work by Salzbrunn et al. [14] on pathline predicates and the concept of anchor lines by Bürger et al. [2], where pathlines are seeded according to the FTLE value, an approach similar to ours. A related method was suggested by Garth et al. [5], who interpret the FTLE as a probability distribution function for particle seeding. But whereas Bürger et al. concentrate on the visualization of the flow behavior along pathlines by seeding particles along these, and Garth et al. only address the seeding of particles in general, our approach concentrates on the visualization of LCS and its augmentation based on pathlines.

In this work, we introduce a twofold approach to visualizing pathlines in the context of LCS generation: the selection of significant trajectories and their individual visualization. We apply these techniques to 2D unsteady flow, treating time as an additional dimension. The resulting visualizations strongly benefit from the aforementioned visualization technique, greatly reducing clutter and occlusion. We demonstrate the value of our approach for augmented LCS visualization for a 2D analytic example and computational fluid dynamics (CFD) results.

2 Our Approach

We first review the basics of LCS (see Sec. 2.1) and then extend the concept of the FTLE to compute several intermediate flow maps (Sec. 2.2). Afterward, we discuss our trajectory-augmented visualization technique for two-dimensional vector fields (Sec. 2.3).

2.1 FTLE

The FTLE is a measure of growth of perturbations, or in other words, it relates to the maximum separation that two infinitesimally close points can undergo under the action of a vector field when seeded
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at arbitrary orientation at the FTLE sample. The FTLE is computed from the flow map \( \phi_{t_0}^{t_0+T}(x) \), which maps a sample point \( x \) at time \( t_0 \) to its advected position after time span \( T \). We sample the flow map on a regular grid. The direction of maximum separation is given as the major eigenvector of the (right) finite-time Cauchy-Green deformation tensor

\[
\Delta_{t_0}^{T} = \left( \nabla \phi_{t_0}^{t_0+T}(x) \right)^\top \cdot \nabla \phi_{t_0}^{t_0+T}(x).
\]  

With \( \lambda_{\text{max}} \) being the largest eigenvalue, the FTLE is defined [7] as the logarithm of the spectral norm of the flow map gradient, normalized by advection time \( T \):

\[
\sigma_{t_0}^{T}(x) = \frac{1}{|T|} \ln \sqrt{\lambda_{\text{max}}(\Delta_{t_0}^{T}(x))}.
\]

2.2 Maximum Separation Factor

When using the FTLE alone, no information is provided about the temporal development of trajectories. To visualize the behavior of trajectories with respect to FTLE generation, we want to measure the distance between the trajectories. Although the FTLE is computed only from the final positions at advection time \( t = t_0 + T \), i.e. the end points of the trajectories, one can utilize it to visualize that property along the complete trajectories for an overview (see below). For this, we trace the trajectories in smaller intervals of \( \Delta t \) and measure their mutual distance after each time step. However, since the FTLE is computed from the maximum possible separation factor of adjacently seeded trajectories, we do not evaluate the true Euclidean distance between the intermediate end points but take the Eulerian-Lagrangian property of the FTLE into account. This can be achieved by computing the FTLE from the end points of each intermediate flow map \( \phi_{t_0}^{t_0+i\Delta t} \), \( i \in \{1 \ldots n\} \). The resulting FTLE values are then mapped onto the trajectories at their respective positions. To avoid interpretation difficulties, we omit the logarithm and the normalization by advection time \( T \) in (2), resulting in the spectral norm of \( \nabla \phi_{t_0}^{t_0+i\Delta t} \), which represents the maximum separation factor (MSF). For example, the MSF can be visualized by color coding or varying tube radius of the trajectories.

2.3 Trajectory-Augmented Visualization of LCS in 2D Vector Fields

Basically, there are two main approaches to conveying flow behavior together with LCS visualizations: by geometry such as arrow-glyphs or trajectories, or by texture advection techniques such as LIC. Both methods, geometric approaches and texture-based techniques, typically suffer from occlusion and clutter in 3D renderings. In this work, we employ a geometric approach with pathlines where occlusion is reduced by discarding some of the trajectories that have been used for FTLE computations. However, we aim at selecting only interesting trajectories to reduce visual clutter and lower occlusion. In this work, we follow the approach of selecting pathlines starting in the vicinity of LCS by user-defined thresholding depending on the FTLE value. For example, trajectories leading to a small FTLE value are omitted, whereas trajectories near LCS remain. The resulting visualization highlights the flow behavior that leads to the FTLE ridges. Another possible method is that in regions with low FTLE, only a few trajectories are selected by discarding e.g. every second trajectory.

The remaining trajectories are visualized with tubes in space-time. They originate from their corresponding seeding positions in the FTLE field. For the investigation of the flow with respect to a given FTLE, we use a constant tube radius per trajectory. The radius is chosen according to the corresponding FTLE value. High FTLE values are represented by thick tubes whereas low values lead to fine ones. The FTLE field is shown as color-coded plane located at \( t_0 \) in space-time.

In the following, we describe how a slight variation of this approach can serve as an overview representation for choosing appropriate advection times \( T \), a crucial parameter for FTLE analysis. For
Fig. 1 Trajectory-augmented space-time visualization of LCS. The FTLE field of the flow is shown in the $x-y$ plane at $t = t_0$ (color coding: light colors for low FTLE values and dark colors for high FTLE values). Time is directed upward. Only trajectories near FTLE ridges are visualized. The maximum separation factor (MSF) is mapped to color (green: low; red: high) and tube radius of the trajectories. this, the visualization of the trajectories is augmented by information from the respective intermediate flow maps $\phi_{t_0 + i \Delta T}$, i.e. the corresponding MSF is mapped onto the tubes in terms of radius and color. This supports depth perception of the overall structure in important space-time regions exhibiting high FTLE and hence pronounced occlusion. In that way, trajectories at small FTLE values, i.e. coherent regions, are suppressed and the remaining trajectories representing high FTLE values emphasize the separating flow behavior corresponding to the LCS (see Fig. 1).

The choice of the finite advection time is supported by interactively moving a plane along the time axis. The FTLE field shown at $t_0$ is updated according to the selected time using the FTLE from the corresponding intermediate flow map. Trajectories above the selected time are shown with reduced opacity. Our representation, i.e. the MSF on trajectories, is simply a space-time stack of flow maps over a given time interval combined with a space-time stack of FTLE (or more precisely MSF).

2.4 Implementation Details

The core of our system, the FTLE computations and the pathline seeding and filtering algorithms are implemented as separate AVS (Advanced Visualization System) modules that are connected using an AVS visualization network. ParaView is used for interactive exploration of the resulting data. The generation of the flow maps and the pathlines is the most time-consuming part. However, the involved particle integration is an embarrassingly parallel problem, making it a perfect match for modern many-core architectures. In our implementation, the computation of the flow maps is done by a CUDA kernel running on NVIDIA graphics processing units (GPUs). A fourth-order Runke-Kutta solver is used for particle integration. Additionally, the instationary vector field data is streamed to the GPU to enable the efficient handling of large data sets and long integration times $T$. For our numerical experiment, we used a PC with an Intel Xeon X5550 CPU, 12 GB RAM, and a NVIDIA Tesla C1060 GPU with 4 GB graphics memory. FTLE computations took only a few minutes for reasonable resolutions and hence our approach is also feasible with equipment of lower performance.

3 Results

We demonstrate the effectiveness of our approach for LCS augmentation for two examples: an analytic double-gyre and a buoyant CFD flow. Both examples can be interactively explored in ParaView on recent hardware. A video illustrating our visualization technique can be found at http://www.vis.uni-stuttgart.de/texflowvis.
3.1 Double-Gyre

Our first example is a time-dependent analytic field introduced by Shadden et al. [15]. The vector field of the double-gyre flow is two-dimensional, divergence-free, features a saddle point, and exhibits FTLE ridges due to non-linearity. It is a common example for the investigation of flow topology in time-dependent vector fields [15, 13]. Its instantaneous vector field topology substantially differs from LCS or the related concept of manifolds of hyperbolic trajectories [6, 13]. In Fig. 2, the overview visualization of the double-gyre flow is shown in space-time with augmentation by trajectories. Time is directed upward. The cut plane indicates the currently selected advection time and the corresponding FTLE field is shown in the x–y plane. Moving the cut plane allows one to quickly skim through all intermediate flow maps. The formation of the ridges is indicated in the FTLE field and the trajectories show the respective spatial development. The local MSF is mapped to both tube radius and color. Low MSF values are indicated by thin green tubes, whereas high values by thick red ones. Thereby, the evolution of the FTLE over time is shown along each trajectory. Coherent regions exhibiting low FTLE values are less dominant as the tubes in these regions are thin.

For Fig. 3, the advection time, i.e. the cut plane position, was chosen so that FTLE ridges are clearly visible and trajectories originating in the center ridge separate toward the boundaries. To reduce occlusion only a subset of the trajectories is shown. Trajectories near FTLE ridges, i.e. with high FTLE, are selected to highlight the separating flow behavior. We apply a constant tube radius, depending on the final FTLE, to facilitate tracking of individual pathlines. One can clearly see the separating flow toward left and right boundaries leading to the FTLE ridge in the center, clarifying the origin of this LCS.

3.2 Buoyant Flow

We now exemplify our approach for the CFD simulation of a buoyant unsteady 2D flow. All boundaries impose a no-slip condition. The bottom wall is heated to 75°C whereas the upper is cooled to 5°C. The side walls are supplied with adiabatic boundary conditions. Together with gravity acting downward, this gives rise to convective flow due to buoyant effects. To avoid the onset of a simple circular flow, two barriers are introduced, a short one on the bottom wall and a longer on one of the side walls.

The overview visualization is depicted in Fig. 4 (left). In Fig. 4 (right), the separating flow is revealed by trajectories at high FTLE values. The underlying flow behavior can be seen in Fig. 5. A sparse representation is given in the left image. The right image shows how a dense set of trajectories can be used to emphasize the separating flow. In the examined time interval, the influence of buoyancy has not reached the top left corner, leading to negligible advection and hence almost straight trajectories.

4 Conclusions and Future Work

We have presented a technique for augmenting visualizations of the FTLE to show LCS-inducing flow behavior. The FTLE field of the underlying flow is combined with visualizations of trajectories in space-time. In the overview visualization, the maximum separation factor is mapped to geometric properties of the trajectories, thereby easing the choice of advection time for the FTLE analysis. After choosing the advection time, further investigations are facilitated by pronounced visualization of trajectories in the vicinity of FTLE ridges. This allows one to identify the origin of the respective LCS. The suitability of our approach has been shown for time-dependent examples of an analytic flow and a CFD simulation. Interesting directions of future research could include the investigation of more advanced seeding strategies for the trajectories and the extension to 3D flow. Since the visualization in space-time is no option in case of 3D flow, we consider a direct visualization in 3D space by projecting from 4D
Fig. 2 Double-gyre. Selection of the finite advection time supported by space-time visualization (overview technique). Some of the pathlines used for the FTLE computation are plotted in space-time. Time evolution points upward in the vertical direction. The advection time can be adjusted by moving a cut plane along the time axis. The FTLE field corresponding to the selected advection time is shown in the $x - y$ plane. MSF is mapped to line color and thickness.

Fig. 3 Double-gyre. For a given advection time $T$, only trajectories featuring high FTLE values (mapped to color) are selected to highlight the flow behavior leading to FTLE ridges.
**Fig. 4** Buoyant flow. Left: visualization-augmented selection of the advection time. Right: only trajectories featuring high FTLE values are selected to highlight the flow behavior leading to FTLE ridges. Time evolution points upward in the vertical direction.

**Fig. 5** Buoyant flow. Left: sparse representation of trajectories indicating the flow behavior without occluding the underlying FTLE field. Right: dense representation emphasizing the separation of the flow. The seeding density is controlled by the FTLE.

space-time. Additional visual cues will be necessary to illustrate temporal properties of the flow. The partial visualization of trajectories for reducing visual clutter will play an even more important role than in 2D flow.

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**References**


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