

GRAB-ECO for Minimal Fuel Consumption Estimation of Parallel Hybrid Electric Vehicles

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Abstract — As a promising solution to the reduction of fuel consumption and CO₂ emissions in road transport sector, hybrid electric powertrains are confronted with complex control techniques for the evaluation of the minimal fuel consumption, particularly the excessively long computation time of the design-parameter optimization in the powertrain's early design stage. In this work, a novel and simple GRaphical-Analysis-Based method of fuel Energy Consumption Optimization (GRAB-ECO) is developed to estimate the minimal fuel consumption for parallel hybrid electric powertrains in light- and heavy-duty application. Based on the power ratio between powertrain's power demand and the most efficient engine power, GRAB-ECO maximizes the average operating efficiency of the internal combustion engine by shifting operating points to the most efficient conditions, or by eliminating the engine operation from poorly efficient operating points to pure electric vehicle operation. A turning point is found to meet the requirement of the final state of energy of the battery, which is charge-sustaining mode in this study. The GRAB-ECO was tested with both light- and heavy-duty parallel hybrid electric vehicles, and validated in terms of the minimal fuel consumption and the computation time. Results show that GRAB-ECO accurately approximates the minimal fuel consumption with less than 6% of errors for both light- and heavy-duty parallel hybrid electric powertrains. Meanwhile, GRAB-ECO reduces computation time by orders of magnitude compared with PMP-based (Pontryagin's Minimum Principle) approaches.

INTRODUCTION

Hybrid Electric Vehicle (HEV) has been regarded as a promising solution to reducing fuel consumption and CO₂ emissions, and other air pollutant emissions in road transport sector. The fuel consumption is reduced through energy recuperation during the regenerative braking operation and improvement of the operating efficiency of powertrain systems.

In parallel HEVs (see [Figure 1](#)), the main powertrain components consist of internal combustion ENGINE (ENG), TRANsmiSSion (TRA), BATtery (BAT), and Electric Motor/Generator (EMG). In addition to these components, Energy Management Strategy (EMS) is mandatory to coordinate the energy flows between ENG and EMG.

Thanks to the additional electrified drivetrain, fuel consumption can be minimized with the optimized EMS, which results from optimal control techniques, such as Dynamic Programming (DP), [1,2], Pontryagin's Minimum Principle (PMP) [3,4], and combined optimization techniques [5,6].

Optimal control techniques are also implemented to minimize the fuel consumption by optimizing the design of a hybrid powertrain, which refers to the co-optimization of control and design for hybrid powertrains. The co-optimization problem solved through bi-level optimization approach is characterized by high computational load, thereby taking much computation time. In [5], powertrain topology, transmission, and dimension of EMG are

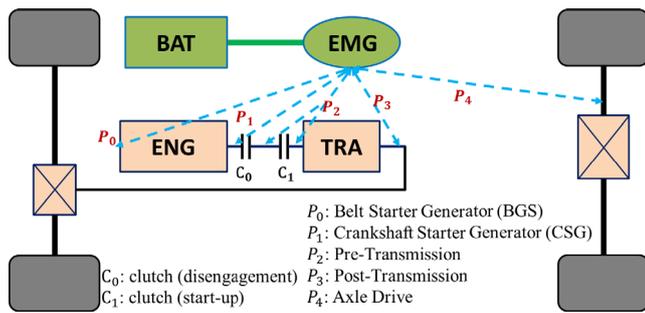


Figure 1

Scheme of parallel hybrid electric vehicle with several possible sub-configurations.

optimized; whereas the fuel consumption is minimized by DP in each evaluation. Despite lack of the computation time specified in the problem of co-optimization, the computation time of the optimized combined optimization technique still needs 9 seconds for each evaluation. As reported in [7], DP even takes more than 2 hours to evaluate without considering the design optimization.

To reduce the computational time, researchers investigate different approaches to achieve the co-optimization of control and design for hybrid powertrains. One is to apply convex optimization to solve the problem of co-optimization [8]. However, discrete variables have to be defined heuristically. Another one is to apply analytic technique in order to fast evaluate the minimal fuel consumption. In [9], the analytic technique is applied to DP, hence leading to orders of magnitude reduction of computation time. The analytic technique is also applied to PMP in [10]. The method, namely Selective Hamiltonian Minimization (SHM), reduces computation time by orders of magnitude for parallel and series HEVs.

The main contribution of this paper is to further reduce the computation time of the evaluation of the minimal fuel consumption for parallel HEVs in the context of co-optimization of design and control for hybrid electric powertrains. The computation time is significantly diminished with a novel approximation method that is designated as GRaphical-Analysis-Based method of fuel Energy Consumption Optimization (GRAB-ECO). Based on maximization of the average operating efficiency of the internal combustion engine, the minimal fuel consumption is approximated within a few several milliseconds. A counterpart of GRAB-ECO is introduced solely for series HEVs in [11].

Performance of GRAB-ECO is benchmarked against two variants based on PMP, which are the standard PMP with array operation (simplified as PMP hereafter) and the semi-analytical SHM, respectively. GRAB-ECO is applied to approximate the minimal fuel consumption of a light-duty parallel HEV. The fuel consumption and evaluation time are

correspondingly compared with those of PMP and SHM. With a success in the light-duty application, GRAB-ECO is further investigated with a heavy-duty parallel hybrid-electric truck.

In addition to the comparison among GRAB-ECO, PMP, and SHM, parametric models of main powertrain components are investigated to verify the improvement of computation time and the accuracy of minimal fuel consumption. Both simple analytic models and mapped data are comparatively implemented to PMP, and then to GRAB-ECO. Impacts on minimal fuel consumption and computation time are comparatively analyzed.

After this section, hybrid electric vehicles are analytically modeled in Section 1. PMP and SHM are briefly introduced in Section 2. Details of GRAB-ECO are introduced in Section 3. After that, results are presented and discussed in Section 4. Conclusions are summarized in the last section.

1 MODELING OF HYBRID ELECTRIC VEHICLES

HEVs are analytically modeled for two aspects, which refer to powertrain components and vehicle load.

1.1 Modeling of Hybrid Powertrain Components

In the context of the co-optimization of design and control for hybrid-electric powertrains, powertrain components are analytically modeled to describe energy losses and to predict such losses of powertrain components with different dimensions, but the same family of technology.

Because this study focuses on the development of a fast running method for minimal fuel consumption evaluation, closed-form parametric models are introduced for internal combustion engine, battery, electric motor/generator, and transmission. As a comparison to the mapped data, these analytic models will be applied to evaluate the minimal fuel consumption as well.

1.1.1 Internal Combustion ENGINE (ENG)

Parametric models of internal combustion engines are developed in analytical expressions from fuel consumption maps that are obtained from *IFPEN* and *Scania* for the light-duty vehicles and the heavy-duty vehicles, respectively.

As for the light-duty engines, the analytic model is expressed as

$$P_{ef}(\omega_e) = \begin{cases} \frac{P_{e0}(\omega_e) + P_e}{k_e(\omega_e)}, & P_e \leq P_{ec}(\omega_e), \\ \frac{P_{e0}(\omega_e) + P_{ec}(\omega_e)}{k_e(\omega_e)} + \frac{P_e - P_{ec}(\omega_e)}{k_f}, & P_e \geq P_{ec}(\omega_e), \end{cases} \quad (1)$$

where P_{ef} is the chemical power of burned fuel, P_{e0} is the friction power in function of engine speed ω_e , P_e is the engine brake power, P_{ec} is the engine corner power of the best efficiency that is in function of engine speed ω_e , k_e is the indicated efficiency in function of engine speed ω_e when engine power is no greater than engine corner power, and k_f is the constant indicated efficiency when engine power is greater than engine corner power [10,11].

Concerning the heavy-duty engines, (for example, the large turbocharged diesel engines), the engine parametric model is a quadratic function expressed as

$$P_{ef}(\omega_e) = k_{e0}(\omega_e) + k_{e1}(\omega_e)P_e + k_{e2}(\omega_e)P_e^2, \quad (2)$$

where coefficients $k_i = k_i(\omega_e)$ ($i = e0, e1, e2$) are functions of engine speed ω_e [12].

The corner power that has the best efficiency at each engine speed is analytically expressed as

$$P_{ec}(\omega_e) = \sqrt{\frac{k_{e0}(\omega_e)}{k_{e2}(\omega_e)}}. \quad (3)$$

1.1.2 BATTERY (BAT)

Internal resistances of battery cells are identified from the on-line available data-sheets of lithium-ion battery cells [13] through the method proposed in [14]. With the discretization of charging and discharging current of a battery cell, battery electrochemical power and terminal power are evaluated based on the zeroth-order equivalent circuit battery model.

After that, parameterization is performed between the electrochemical power and the terminal power. As a result, the parametric model of a lithium-ion battery is expressed as

$$P_{be} = k_{b0} + k_{b1}P_{bt} + k_{b2}P_{bt}^2, \quad (4)$$

where P_{be} is the electrochemical power, P_{bt} is the terminal power that is positive in discharging phase and negative in charging phase, $k_i = k_i(N_{bc})$ ($i = b0, b1, b2$) are the coefficients as a function of battery cell number N_{bc} [10,12]. The analytic model of battery is developed at room temperature.

1.1.3 Electric Motor/Generator (EMG)

Similar to engines, efficiency maps are used to build up the closed-form parametric models for electric motor/generators.

Parametric model of an EMG in terms of a permanent magnet synchronous machine is expressed as

$$P_{me} = k_{m0} + k_{m1}\omega_m + k_{m2}\omega_m^2 + k_{m3}P_{mm} + \frac{k_{m4}}{\omega_m^2}P_{mm}^2, \quad (5)$$

where P_{me} is the electric power, P_{mm} is the mechanical power at motor shaft, and coefficients k_i ($i = m0, \dots, m4$) are identified based on mapped data of EMGs [10,12].

Note that, power losses of power electronics and EMG have been lumped into the parametric model of EMG in Equation (5).

1.1.4 TRANSMISSION (TRA)

The transmission in this work is composed of a multi-speed gearbox between the engine and the drive shaft (denoted with subscript t), and a single-speed gear train between motor shaft and the torque coupler (denoted with subscript m).

Usually, transmission is characterized by gear ratio and gear efficiency. The gear ratio of a multi-speed transmission γ_t is in function of gear number. The one of a simple transmission between EMG and torque coupler γ_m is a constant value. Combining the vehicle speed, wheel radius, final drive, and gear ratio of transmissions, the engine speed and motor speed are correspondingly evaluated by

$$\omega_e = \frac{\gamma_t \gamma_d}{R_w} v, \quad (6)$$

$$\omega_m = \frac{\gamma_t \gamma_d \gamma_m}{R_w} v, \quad (7)$$

where R_w is the radius of wheels, and γ_d is the ratio of final drive.

Concerning the efficiency of transmission, constant values are assumed and implemented separately for a light- and heavy-duty parallel HEV.

1.2 Vehicle Load Models

Besides the aforementioned powertrain-component power losses, powertrain's power demand is calculated from the vehicle longitudinal dynamics by

$$P_{tot}(t) = (F_a(t) + F_r(t) + F_g(t) + ma(t))v(t), \quad (8)$$

where a is the acceleration, m is the vehicle mass without considering the inertia of rotating parts in a powertrain, F_a is the aerodynamic resistive force, F_r is the rolling resistive force, F_g is the force caused by gravity of a vehicle on a non-flat road, as depicted in Figure 2.

1.3 Power Flows and Balance

In a parallel hybrid electric powertrain of P_2 configuration (see Fig. 1), power supplies include engine power and mechanical power of EMG in the traction phase, whereas an additional mechanical brake power is required if EMG cannot fully recuperate the braking power in braking phase. Therefore, a general power balance form is imposed by

$$P_e(t) + P_{mm}(t) = P_d(t), \quad (9)$$

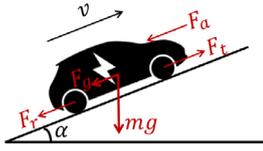


Figure 2

A schematic representation of forces acting on a vehicle in motion.

$$P_d(t) = (P_{tot}(t) - P_{brk}(t))\eta_t^{-\text{sign}(P_{tot}(t))}, \quad (10)$$

where P_d is the power demand with consideration of transmissions efficiency, P_{brk} is the dissipated power of a mechanical break in braking phase, and η_t is the efficiency of the transmission of stepped gearbox.

When an EMG cannot fully recuperate the braking power during braking, an additional brake power is provided by a mechanical brake system. Therefore, the brake power P_{brk} is expressed as

$$P_{brk}(t) = \begin{cases} P_{tot}(t) - \frac{P_{mmL}(t)}{\eta_t}, & P_{mmL}(t) > P_d(t)\eta_t, \\ 0, & P_{mmL}(t) \leq P_d(t)\eta_t, \end{cases} \quad (11)$$

where P_{mmL} is the lower boundary of the admissible mechanical power of an EMG.

1.4 Driving Cycles

Vehicle load and power demand depend on not only powertrains, but the prescribed driving cycle as well. A driving cycle contains trajectories of vehicle speed and road slope, sometimes of gear number.

The applied driving cycles for both light- and heavy-duty vehicles are briefly summarized in Table 1, including trip distance and duration, average speed and maximal speed. New European Driving Cycle (NEDC), Federal Test Procedure for urban driving condition (FTP-72), and High Way Fuel Economy Test cycle (HYWFET) are standardized driving cycles that are dedicated to fuel consumption evaluation for the light-duty vehicles.

Concerning the heavy-duty vehicle application, one virtual drive cycle and two standardized driving cycles are utilized for the minimal fuel consumption evaluation. The reference driving cycles are the Heavy-Duty Södertälje–Norköping Drive Cycle (HDSNDC), the Heavy-Duty Urban Dynamometer Driving Schedule (HDUDDS), and Heavy Heavy-Duty Diesel Truck (HHDDT) cycle. In particular, the HDSNDC is a virtual driving profile derived from the benchmark problem in IFAC ACC Conference 2016 [15], as illustrated in Figure 3.

2 OPTIMAL CONTROL OF PARALLEL HYBRID ELECTRIC VEHICLES

The minimal fuel consumption of a parallel hybrid electric powertrain is achieved through an optimal control technique. As an optimal control technique, PMP and SHM are briefly introduced for the sake of benchmark.

2.1 Optimal Control Problem Formulation

In a parallel HEV, the optimal control problem is formulated as to find the control variable $u(t) = P_{mm}(t)$, such that

$$\min_{u \in U} J = \int_{t_0}^{t_f} P_{ef}(u(t), t) dt, \quad (12)$$

with $x := \Delta E_{be}$, subject to

$$\begin{aligned} \dot{x}(t) &= P_{be}(t) \\ P_{btL}t &\leq P_{bt}t \leq P_{et} \\ P_{mmL}t &\leq P_{mm}t \leq P_{mmU}t, \\ \omega_{mL} &\leq \omega_m t \leq \omega_{mU} \\ P_{eL}t &\leq P_e t \leq P_{eU}t \\ \omega_{eL} &\leq \omega_e t \leq \omega_{eU} \end{aligned} \quad (13)$$

where subscripts L and U indicate the lower and upper boundaries, respectively; t_0 and t_f are the beginning and ending time of the investigated driving cycle.

In this work, the constrains on state variable, *i.e.* state of energy of battery, are not taken into account. A charge-sustaining mode of the battery, *i.e.* $E_{be}(t_f) = E_{be}(t_0)$, is investigated.

2.2 Solutions Based on Pontryagin's Minimum Principle

Based on Pontryagin's Minimum Principle, the minimization of burned fuel energy is converted to the local minimization of Hamiltonian, which are

$$H(u, t, x, s) = P_{ef}(t) + sP_{be}(t), \quad (14)$$

$$u^*(t) = \underset{u \in U}{\operatorname{argmin}} H(u, t, x^*, s^*), \quad (15)$$

where u^* is the optimal solution of the control variable, s is the adjoint state that is assumed to be constant since Hamiltonian function is almost independent from state variable.

A proper s satisfying the final state requirement is found via shooting method. Array operation is applied based on the mapped data of powertrain components for the purpose of fast computation [4]. This approach is indicated by PMP.

TABLE 1
Main features of investigated driving cycles for light- and heavy-duty parallel hybrid electric powertrains.

	D_{trip}	T_{trip}	V_{avg}	V_{max}
	[km]	[s]	[km/h]	[km/h]
NEDC	10.93	1180	33.4	120
FTP-72	12.07	1370	31.5	91.2
HYWFET	16.45	765	77.7	96.4
HDSNDC	119	5347	80.0	84.6
HDUDDS	8.94	1059	30.3	93.3
HHDDT	41.92	3003	50.2	95.4

Combining Equations (4) and (5), analytic model an Electric Drive Unit (EDU), containing a BAT and an EMG, is expressed as

$$P_{be}(\omega_m) = k_{u0}(\omega_m) + k_{u1}(\omega_m)P_{mm} + k_{u2}(\omega_m)P_{mm}^2, \quad (16)$$

where coefficients k_{ui} ($i=0, 1, 2$) are in function of motor speed ω_m .

Combining Equation (1) for light-duty vehicles (or (2) for heavy-duty vehicles), Equations (6), (7), and (16), the closed-form Hamiltonian can be found in [10,12]. Then, Selective Hamiltonian Minimization (SHM) is developed based on analytic solutions to the closed-form Hamiltonian.

Without considering constraints, the optimal control variable is obtained by solving $\frac{\partial H}{\partial u} = 0$, thereby leading to $P_{mm,0} = f(s, t)$.

Considering constraints of operating limits, the possible solutions of the control variable $u(t)$ are enumerated as

$$\left\{ \begin{array}{l} P_{mm1}P_d t - P_e L t \\ P_{mm2}P_d t - P_e U t \\ P_{mm3}P_{mm} L t \\ P_{mm4}P_{mm} L t \end{array} \right\}, \quad (17)$$

Note that, the operating limits of battery are assumed to have a larger operating range than the matched electric motor/generator.

Apart from the unconstrained optima and constraints in Equation (17), two discontinuous cases, which are $P_{mm,5} = 0$ and $P_{mm,6} = P_d(t) - P_{ec}(t)$, are taken into account because power losses of electric powertrain components are assumed to be zero when vehicle is at a standstill.

In summary, the full control space U of SHM is reduced, which is $u(t, s) \in \{P_{mm,i}(t, s) \mid i=0, \dots, 6\}$. With a properly determined s , the minimal fuel consumption is evaluated. This approach is named as SHM.

Despite further reduction of computation time via SHM [10,12], a much faster method to approximate the minimal fuel consumption is developed, which is the GRAB-ECO.

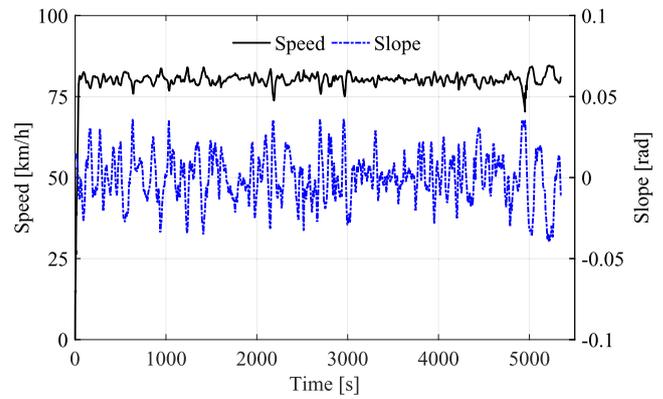


Figure 3

A typical driving cycle generated by a cruise controller to maintain the long haulage truck to meet the desired speed along the highway from Södertälje to Norköping in Sweden.

3 MINIMAL FUEL CONSUMPTION APPROXIMATION BY GRAB-ECO

As the main contribution of this study, GRAB-ECO is developed to approximate the minimal fuel consumption for parallel HEVs regardless of light- or heavy-duty application. Besides, GRAB-ECO can approximate the minimal fuel consumption of parallel HEVs with an extremely fast computation.

3.1 Principle of GRAB-ECO

GRAB-ECO approximates the minimal fuel consumption based on the maximization of average operating efficiency of the internal combustion engine. The maximization of average engine operating efficiency depends on a sorted power ratio R_e and a turning point τ^* , which are detailed in the following section.

Based on the power ratio R_e , GRAB-ECO maximizes the average operating efficiency of the internal combustion engine by either shifting operating points of an engine to their most efficient condition, or eliminating the operating points by turning off the engine.

Moreover, battery is maintained in a desired condition, such as the charge sustaining mode in this study, which is achieved by finding the turning point τ^* such that the varied electrochemical energy of battery is equal to zero over the investigated driving cycle.

3.2 Essential Steps of GRAB-ECO

Essentials of GRAB-ECO are explained with an example of a light-duty parallel HEV as follows.

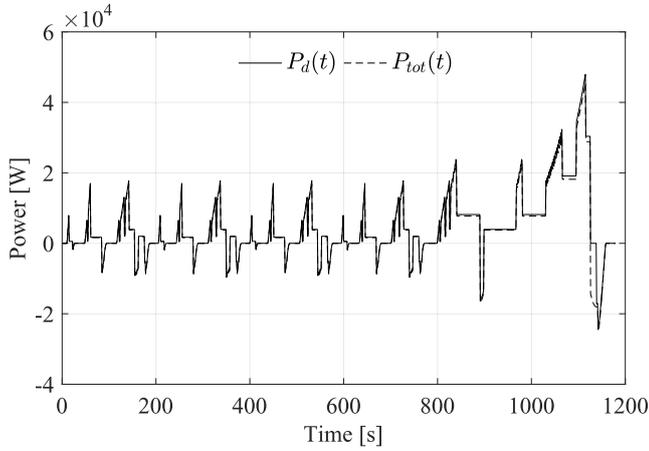


Figure 4

Power demand of a light-duty parallel HEV over NEDC with operating constraints.

Step 1: Effective Power Analysis

Depending on vehicle parameters and given discretized mission profile (time $t \in [t_0, t_f]$), the vehicle power demand with consideration of drivetrain efficiency, P_d , is constrained and saturated by the physical limits of powertrain components, such as ENG, BAT, and EMG.

The power demand of an example of a light-duty parallel hybrid electric powertrain is depicted in Figure 4. Over NEDC, the power demand indicated by black curve is constrained to zero due to the physical constraints of the electric motor/generator, which is the maximal allowable motor speed in this case. The other dashed curve represents the vehicle demand at wheels.

The power ratio, R_e , between the power P_d and the engine corner power P_{ec} , is expressed by

$$R_e(t) = \begin{cases} \frac{P_d(t)}{P_{ec}(t)}, & \omega_e(t) > \omega_{el}, \\ 0, & \omega_e(t) \leq \omega_{el}, \end{cases} \quad (18)$$

where ω_{el} is the launch speed of an engine.

Taking the constraints of powertrain components into account, the essential power factor, R_e , is forced to be 1 when the pure electric vehicle mode alone cannot satisfy the power demand. Then, the power demand is satisfied by the hybrid operating mode in this case. Similarly, the essential power factor is forced to be 0 when any hybrid mode would violate the lower boundary limits of the EMG.

Step 2: Operating Modes Identification

The minimal fuel consumption is approximated with four optimal operating modes in GRAB-ECO. These optimal operating modes are those that the average operating efficiency of the internal combustion engine is maximized over a driving cycle. The average operating efficiency is

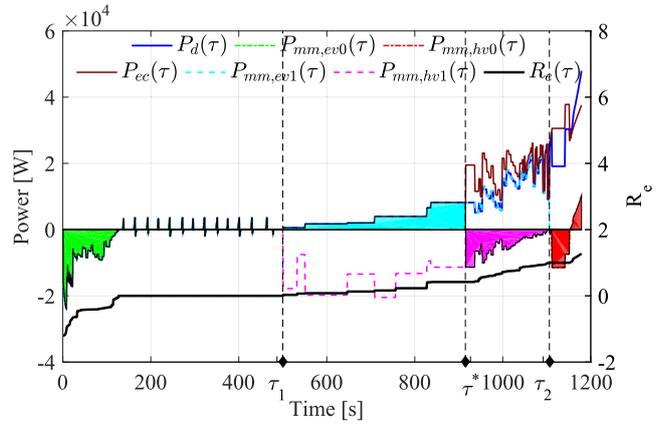


Figure 5

Powertrain working modes identification.

maximized by an altering or suspending the operation of an engine. The altering operation shifts an engine operating point to the best efficiency ($P_e = P_{ec}$), whereas the suspending operation suppresses the engine operation ($P_e = 0$).

The switch of engine operation between altering and suspending operation is controlled by the power ratio $R_e(\tau)$ in Equation (18), where time τ is the permutation of time t , such that

$$R_e(\tau) \geq R_e(\tau - \Delta\tau), \quad \forall \tau \in \mathbb{R}_+, \quad (19)$$

where $\Delta\tau$ is the time step.

Then, the control variable $u(\tau) := P_{mm}(\tau)$ of each operating mode is determined by

$$u(\tau) := \begin{cases} P_{mm,ev0}(\tau) = P_d(\tau), & \tau \leq \tau_1, \\ P_{mm,hv0}(\tau) = P_d(\tau) - P_{ec}(\tau), & \tau \geq \tau_2, \\ P_{mm,ev1}(\tau) = P_d(\tau), & \tau_1 < \tau \leq \tau^*, \\ P_{mm,hv2}(\tau) = P_d(\tau) - P_{ec}(\tau), & \tau^* < \tau < \tau_2, \end{cases} \quad (20)$$

where τ^* is the turning point defined in the next step, τ_1 is the time instant such that $R_e(\tau_1) \leq 0 \cap R_e(\tau_1 + \Delta\tau) > 0$, and τ_2 is time instant such that $R_e(\tau_2) < 1 \cap R_e(\tau_2 + \Delta\tau) \geq 1$.

As shown in Figure 5, the power ratio $R_e(\tau)$ is illustrated in the right vertical axis, whereas the left vertical axis is about the sorted power relating to power demand and mechanical power of the electric motor generator. The power $P_{mm}(\tau)$ shaded with green is in pure electric vehicle mode (ev0), in which the main operation of regenerative braking; the power $P_{mm}(\tau)$ shaded with cyan is in pure electric traction mode (ev1); the power $P_{mm}(\tau)$ shaded with magenta is in the hybrid mode (hv1), where battery is charged; and the power $P_{mm}(\tau)$ shaded with red is the hybrid mode (hv0), where the vehicle is boosted. As can be seen, the engine load is shifted from the power demand to the most efficient operating conditions in mode hv0 and hv1.

Step 3: Turning Point Determination

At each time instant, τ , a parallel hybrid electric powertrain can work only in one operating mode. Thus, a turning point, τ^* , must be determined to maintain the battery in charge-sustaining condition.

Using Equations (4) and (5), the electrochemical power of the battery in a function of the mechanical power of EMG is calculated by $P_{be}(\tau) = \varphi(P_{mm}(\tau))$.

According to the control variable in Equation (20), the resultant electrochemical energy of the battery, ΔE_{be} , is calculated by

$$E_{be,ev0} = \sum_{\tau'=t_0}^{\tau_1} \varphi(P_d(\tau')) \Delta\tau, \quad (21)$$

$$E_{be,hv0} = \sum_{\tau'=\tau_2}^{t_f} \varphi(P_d(\tau') - P_{ec}(\tau')) \Delta\tau, \quad (22)$$

$$E_{be,ev1}(\tau) = \sum_{\tau'=\tau_1}^{\tau} \varphi(P_d(\tau')) \Delta\tau, \quad (23)$$

$$E_{be,hv1}(\tau) = \sum_{\tau'=\tau}^{\tau_2} \varphi(P_d(\tau') - P_{ec}(\tau')) \Delta\tau. \quad (24)$$

To maintain the battery in the charge-sustaining mode, a turning point, τ^* , is determined by

$$\Delta E_{be}(\tau^*) = E_{be,ev0} + E_{be,hv0} + \dots + E_{be,ev1}(\tau^*) + E_{be,hv1}(\tau^*) = 0. \quad (25)$$

As depicted in Figure 6, the turning point, τ^* , is indicated by a marker of dot, where $\Delta E_{be}(\tau^*) = 0$. Other terms, *i.e.* $E_{be,hv0}$, $E_{be,ev0}$, $E_{be,hv1}$, and $E_{be,ev1}$, refer to the resultant electrochemical energies in Equations (21)–(24).

Step 4: Optimal Fuel Consumption Evaluation

As a consequence, the control variable is simplified as

$$u^*(\tau) = \begin{cases} P_d(\tau), & \tau \leq \tau^*, \\ P_d(\tau) - P_{ec}(\tau), & \tau > \tau^*. \end{cases} \quad (26)$$

The engine power is accordingly determined by

$$P_e(\tau) = \begin{cases} 0, & \tau \leq \tau^*, \\ P_{ec}(\tau), & \tau > \tau^*. \end{cases} \quad (27)$$

The minimal fuel energy consumption of a parallel hybrid electric powertrain is calculated by

$$E_{ef} = \sum_{\tau=t_0}^{t_f} \psi(P_e(\tau)) \Delta\tau. \quad (28)$$

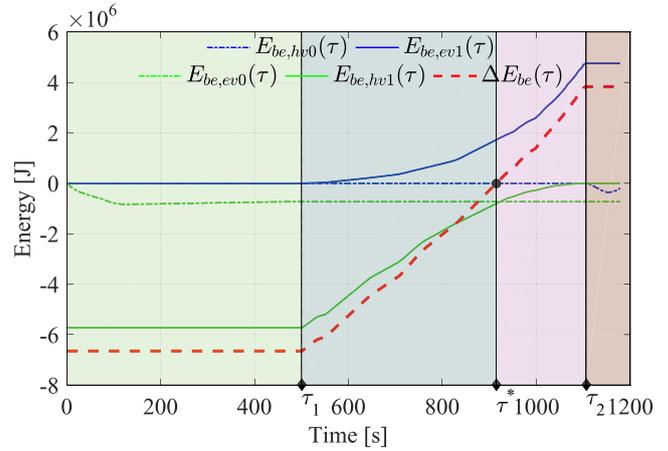


Figure 6
Energy analysis and turning point determination.

4 RESULTS AND DISCUSSIONS

The developed GRAB-ECO is tested and validated with a light-duty and a heavy-duty parallel hybrid electric vehicle over several driving cycles.

To benchmark the performance of GRAB-ECO, a standard PMP-based approach with array operation (PMP) and SHM are utilized to compare the fuel consumption and the computation time.

4.1 Simulation Setup

Two types of parallel hybrid electric vehicles are tested in this work: (1) a light-duty parallel hybrid electric vehicle, whose electric motor is engaged in the second gear of its gearbox; (2) a virtual heavy-duty truck from [15] that is hybridized with real powertrain components. The main characteristics of the light- and heavy-duty parallel hybrid electric powertrains are summarized in Table 2.

Apart from GRAB-ECO, both PMP and SHM, are tested in the platform of MATLAB 2015b on a DELL laptop of i7-4810MQ CPU @ 2.80 GHz and 16 Gb RAM. The computation time is the execution time of each evaluation obtained with MATLAB function.

When the minimal fuel consumption is under evaluation, it can be based on either mapped data or parametric models of powertrain components. Note that, the semi-analytical approach, SHM, depends on full parametric models.

Additionally, corresponding driving cycles for light- and heavy-duty vehicle application are summarized in Section 1.4.

4.2 Results

Results of GRAB-ECO are presented in terms of the minimal fuel consumption and the average computation time, which are compared with the ones of PMP and SHM.

TABLE 2

Specifications of light- and heavy-duty parallel hybrid electric powertrains.

	Light-duty vehicle	Heavy-duty vehicle
Powertrain type	P_2	P_2
$C_d A_f$ [m ²]	0.88	5
C_r [kg/t]	5.3	7.5
m [kg]	1814	32000
R_w [m]	0.32	0.5
TRA Type	MT	AMT
γ_m	3.3	1
γ_d	2.7	3
γ_t	5.8/3.0/2.2/	13/10/9/7/5/4.6/
	1.6/1.3	3.7/3/2.4/1.9/1.5/
		1.2/1/0.8
ENG type	SI	CI
V_{ed} [L]	1.4	13
T_{emax} [Nm]	130	2300
P_{emax} [kW]	60	330
MOT Type	PMSM	PMSM
T_{mmax} [Nm]	28	1050
P_{mmax} [kW]	37	150
BAT Type	Li-ion	Li-ion
Energy [kWh]	7	20

In particular, results based on mapped data and parametric models through PMP are correspondingly denoted by PMP^m and PMP^p. As for through GRAB-ECO, they are denoted by GRAB-ECO^m and GRAB-ECO^p, respectively.

4.3 Light-Duty Parallel HEV

For the light-duty parallel HEV, the minimal fuel consumption is depicted in Figure 7. The minimal fuel consumption is evaluated separately over NEDC, FTP-72, and HYWFET.

Concerning the minimal fuel consumption, mapped-data-based PMP^m and GRAB-ECO^m, and the parametric-model-based PMP^p, SHM, and GRAB-ECO^p presented similar minimal fuel consumption without significant discrepancy. The errors were about 1.0% between PMP and GRAB-ECO both based on the same type of powertrain data.

In detail, GRAB-ECO^m had the least error of the minimal fuel consumption compared with PMP^m over each investigated driving cycle, which was -1.0%, 0.4%, and 0.9% over NEDC, FTP-72, and HYWFET, respectively. However,

GRAB-ECO^p presented higher error than GRAB-ECO^m over the corresponding driving cycle, which were 1.8%, 3.7%, and 4.0% over NEDC, FTP-72, and HYWFET, respectively.

As shown in Figure 8, the average computation time is illustrated over NEDC, FTP-72, and HYWFET. In general, the evaluation based on mapped data took slightly more computation than the one based on analytic models. GRAB-ECO^p and GRAB-ECO^m took the least and the penultimate computation time, respectively. The average computation time of GRAB-ECO^p was 307, 368, and 234 times less than the one of PMP^m over NEDC, FTP-72, and HYWFET, respectively. As for GRAB-ECO^m, the average computation time was 46, 53, and 31 times less than those of PMP^m.

Suppose the co-optimization of control and design takes 20000 function evaluations, the total computation time will be 1.4 h, 41 min, and 0.27 min corresponding to PMP^p, SHM, and GRAB-ECO^p over NEDC. Therefore, co-optimization with GRAB-ECO^p can significantly reduce the total computation time.

In addition, the trajectory of the state of charge of the battery is exemplified over FTP-72, as depicted in Figure 9. The charge-sustaining mode was well reserved by PMP, SHM, and GRAB-ECO.

4.4 Heavy-Duty Parallel HEV

The newly developed GRAB-ECO is also implemented to a virtual heavy-duty hybrid-electric truck, and compared with PMP and SHM as well.

Results of the minimal fuel consumption are depicted in Figure 10 over various driving cycles. Minimal fuel consumption of PMP^p was almost the same as the one of PMP^m; whereas GRAB-ECO^m has similar minimal fuel consumption to that of GRAB-ECO^p over the same driving cycle. This indicated that analytic models were capable of evaluating the minimal fuel consumption with high accuracy. However, compared with PMP, GRAB-ECO always had larger errors than SHM. Specifically, the errors of GRAB-ECO^m were 4.3%, 5.8%, and 4.6% over HDSNDC, HDUDDS, and HHDDT, respectively; whereas, the ones of GRAB-ECO^p 4.4%, 5.9%, 5.4%, accordingly.

Figure 11 summarizes the average computation time of each method. The average computation time was significantly reduced by GRAB-ECO compared with PMP and SHM over each driving cycle. Particularly, the average computation time of GRAB-ECO^p was about 540 times less than that of PMP^m over HDSNDC.

Suppose the co-optimization of control and design takes 20000 function evaluations, the total computation time will be 6.6 h and 0.8 min over HDSNDC. Note that, a real-world driving cycle for heavy-duty vehicles is usually much longer than HDSNDC, which means much more computation time is required.

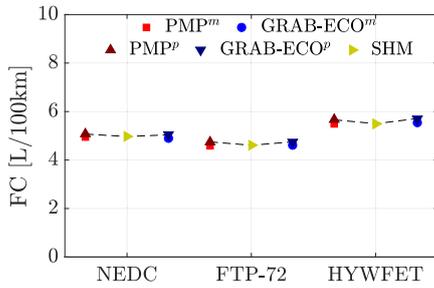


Figure 7
Minimal fuel consumption of the light-duty HEV.

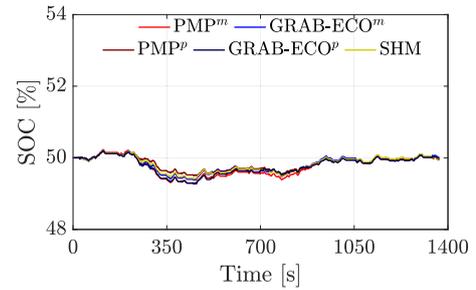


Figure 9
Trajectory of battery state of charge over FTP-72.

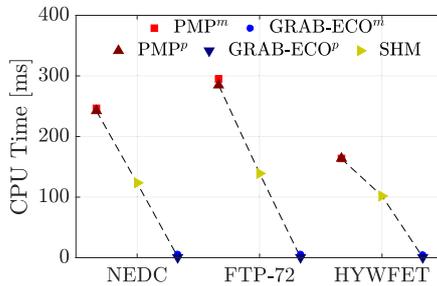


Figure 8
Average computation time of the evaluation of minimal fuel consumption for the light-duty HEV.

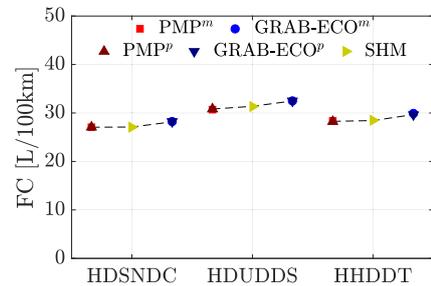


Figure 10
Minimal fuel consumption of the heavy-duty HEV.

The state of charge of the battery is exemplified over HDSNDC, as shown in Figure 12. The charge-sustaining mode is strictly maintained with GRAB-ECO. However, the slight difference of final state of charge of PMP and SHM has been taken into account during the evaluation of minimal fuel consumption.

5 DISCUSSIONS

In general, GRAB-ECO, maximizing the average engine operating efficiency, approximates the minimal fuel consumption with high accuracy and significantly reduced computation time for light-duty parallel HEVs. Despite significantly diminished computation time, the maximal error of the minimal fuel consumption is 5.9% in heavy-duty application, which should be further improved.

5.1 Accuracy of GRAB-ECO

In preceded part, results prove that both mapped-data- and parametric-model-based GRAB-ECO are capable of approximating the minimal fuel consumption for light-duty parallel hybrid electric powertrain within 4.0% error compared with the mapped-data-based benchmark PMP^m.

However, GRAB-ECO in the heavy-duty application has slightly higher error (5.9%) than that in the light-duty application. The reason is due to the optimal operating

modes of GRAB-ECO cannot fully represent the operating conditions of PMP and SHM. This is probably caused by the low power/weight ratio R_e of the heavy-duty truck and the flat high efficiency zone in the engine efficiency map.

The minimal fuel consumption of the light-duty HEV obtained through GRAB-ECO^m is slightly less than the one through PMP^m. This is most probably caused by the numeric errors of the discretized time τ in GRAB-ECO^m and the discretized adjoint state variable s in PMP^m.

To summarize, GRAB-ECO can easily and accurately approximate the minimum fuel consumption for light-duty HEVs by maximizing the average operating efficiency of internal combustion engine. As for heavy-duty HEVs, it can be used as a surrogate model of the approximation of minimal fuel consumption to optimize the powertrain design with significantly diminished computation time.

5.2 Computation Efficiency of GRAB-ECO

Compared with the baseline benchmark PMP, the average computation time of the minimal fuel consumption has been reduced by SHM due to the size reduction of control space U .

Compared with SHM, GRAB-ECO further reduced the average computation time by eliminating the evaluation of adjoint state s , and simply maximizing the average operating efficiency of the engine with respect to the power ratio R_e .

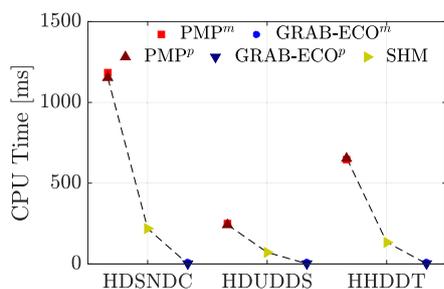


Figure 11

Average computation time of the evaluation of minimal fuel consumption for the heavy-duty HEV.

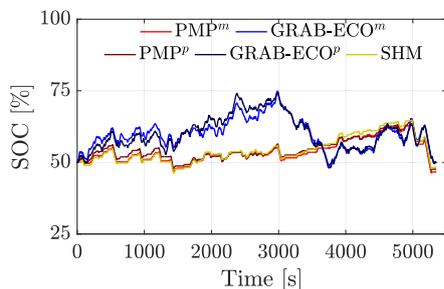


Figure 12

Trajectory of battery state of charge over HDSNDC.

With the notably reduced computation time, GRAB-ECO is promising to be implemented in powertrain design optimization with massive case evaluations.

5.3 Average Operating Efficiency of Engine

In contrast to the global optimization by PMP, GRAB-ECO solely maximizes the average operating efficiency of the internal combustion engine. The average operating efficiency of internal combustion engine is summarized in Table 3.

The average operating efficiency of the engine through PMP was always less than that through GRAB-ECO, indicating that the overall efficiency of a powertrain systems is substantially different from the average operating efficiency of a single component that has the worst efficiency.

CONCLUSION

GRAB-ECO, short for GRaphical-Analysis-Based method of fuel Energy Consumption Optimization, is developed, validated and benchmarked in this paper. Taking operating constraints into account, GRAB-ECO maximizes the average operating efficiency of internal combustion engine through four optimal operating modes according to the power ratio R_e that is between power demand and engine corner power.

TABLE 3

Average operating efficiency of internal combustion engine over tested driving cycles.

Drive cycle	PMP ^m	GRAB-ECO ^m
	[%]	[%]
NEDC	33.24	33.69
FTP-72	33.00	33.78
HYWFET	32.42	33.81
HDSNDC	44.25	44.90
HDUDDS	44.52	45.24
HHDDT	44.21	45.32

Results prove that GRAB-ECO is capable of approximating the minimal fuel consumption accurately for light-duty HEVs but with slightly higher error for heavy-duty HEVs. Besides, the average computation time of GRAB-ECO is reduced by orders of magnitude compared with PMP and SHM. In the context of co-optimization of design and control for parallel HEVs, the total computation time can be reduced from several hours to a few minutes with GRAB-ECO. However, constraints on battery state of charge is not considered in this work.

Concerning future work, GRAB-ECO will be investigated with case studies and compared other control optimizations, such as PMP and SHM. Apart from the implementation of state constraint, GRAB-ECO will be further improved in terms of accuracy particularly for heavy-duty HEVs.

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