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PITFALLS IN CROSS-SECTION STUDIES WITH INTEGRATED REGRESSORS: A SURVEY AND NEW DEVELOPMENTS

Stelios Bekiros, Bo Sjö, and Richard J. Sweeney

ABSTRACT

In cross-section studies, if the dependent variable is $I(0)$ but the regressor is $I(1)$, the true slope must be zero in the resulting "unbalanced regression." A spuriously significant relationship may be found in large cross sections, however, if the integrated regressor is related to a stationary variable that enters the DGP but is omitted from the regression. The solution is to search for the related stationary variable, in some cases the first difference of the integrated regressor, in other cases a categorical variable that can take on limited number of values which depend on the integrated variable. We present an extensive survey, new developments and applications in finance.

Keywords: unbalanced regressions; unit roots; categorical variables; stock appreciation; survey

JEL Classification: C31; C32; G1

1. INTRODUCTION AND LITERATURE REVIEW

Cross-section studies frequently use "unbalanced" regressions, where a stationary dependant variable is regressed on an integrated explanatory variable, for example, a rate of return is regressed on size. There are severe problems in unbalanced regressions, however: In an unbalanced regression, the true slope on the integrated regressor is necessarily zero. Thus, in the many studies that estimate unbalanced regressions and report statistically significant slopes, the significance is necessarily spurious. This paper shows that, aside from sampling variability, the spurious significance arises under two conditions. First, the data contain a true relationship between the stationary dependent variable and an omitted stationary explanatory variable that is related to the integrated regressor - the integrated regressor is a proxy variable for the omitted stationary independent variable. Second, the omitted stationary variable and the related integrated variable show small but true correlation. For example, the stationary explanatory variable in the true relationship may be the first difference of the integrated regressor, for example, the first difference of earnings rather than the level of earnings. In finite samples the change in earnings must show a small but true, positive correlation with the level of earnings; the true finite-sample correlation depends inversely on the number of changes in earnings that compose the level of earnings, and goes to zero as this number grows large. Or the stationary explanatory variable in the true relationship may be a categorical variable which can take on a limited number of values that are related to the integrated regressor, for example, "large" versus "small" firms rather than firm size, or "young" versus "older" firms rather than firm age. Size does not affect returns but does affect the category into which a firm falls. Unsurprisingly, the larger is the cross section, the more likely is the spurious relationship to show significance. Further, as is to be expected, all these points fully apply to panels with large cross sections N but few time-series observations T , where asymptotic results rely on N growing large; this situation applies to the great majority of corporate finance studies. Finally, this paper provides step-by-step procedures to deal

with unbalanced regressions: It discusses how to avoid problems, to diagnose problems and to remedy problems.

The problem discussed here arises frequently in finance. For example, many prestigious Journals in finance and econometrics had 86 articles and notes e.g., *Journal of Finance* (2005); 62 are cross-section studies or panel studies, and 42 of these are subject to the problem discussed here (please see Appendix II). All use stationary dependent variables, and 37 use regressors that are non-stationary, such as price, size in market value or accounting terms, or cash flows, invested capital or earnings, as Table 1 shows. 15 use trend-like explanatory variables, such as time or age. 18 use ratios that may well be non-stationary, given that data cannot reject the null that the rate of return on equity and also the rate of return on invested capital are integrated (Siddique and Sweeney, 2006);¹ all but four of these 18 articles are also included in one of the other categories.

Regressing a stationary variable on a integrated variable is a misspecification, which can lead the researcher astray in two ways. (a) The researcher may find no relationship—the true slope is necessarily zero in the unbalance regression. The researcher may abandon this avenue, thus missing any true relationship between the stationary dependent variable and a stationary explanatory variable which is related to the integrated regressor. (b) If the integrated regressor is related to an omitted stationary variable that enters the true relationship, the slope may be spuriously significant, as discussed in detail and illustrated by examples below; this is more likely to arise the larger is the cross section. The researcher may stop his search here, thus missing the true relationship between the stationary dependent variable and the omitted stationary variable.

This paper's points are theoretical and must hold under specified, general conditions; these points are presented as theorems, but examples below with actual data and with simulated data illustrate the theoretical points. One empirical example investigates OLS regressions of share-price

¹ Common microstructure variables, not included in the sample may be $I(1)$. For volumes of Dow-30 firms the data cannot reject the unit-root null; if volume is $I(1)$, the number of trades or shares per trade must be $I(1)$.

appreciation on the change in earnings versus the levels of earnings; the level of earnings is supposed to capture earnings-persistence effects on price appreciation, as Ohlson and Shroff (1992) suggest and Easton and Harris (1991) suggest and investigate. Another example is a simulation where the true relationship's conditional expectation is between rates of return and the size category into which a firm falls: The mean is larger for small firms in the lower half of the size distribution than for large firms in the upper half of the size distribution, but within each half returns are uncorrelated with size. Because the rate of return is stationary but log size is integrated, the true slope is zero, but a OLS regression is likely to find a significant but spurious relationship.

The researcher may argue that the time series available are too short for meaningful unit-root tests; some panel approaches, however, do not require long time series (Breitling and Meyer, 1994; Levin and Lin, 1992; Im, Pesaran and Shin, 2003). (Later approaches allow for cross-sectional dependence, for example, Bai and Ng, 2004; Pesaran, 2007; Demetrescu and Hanck, 2012; Hanck and Czuda, 2015) Further, the literature provides substantial time-series evidence on some of the variables used in cross section studies, for example, firm size, assets, etc. In addition, in doubtful cases the researcher might replace the suspect variable with a related stationary variable, for example, the first difference or a related categorical variable.

In the following, Sections 2 and 4 show that if the dependent variable is $I(0)$ and an explanatory variable is $I(1)$, then the true slope on the $I(1)$ regressor is necessarily zero. The estimated slope may be significant, however. Suppose an $I(0)$ explanatory variable enters the true model importantly, but is omitted in estimation. Instead, the $I(1)$ regressor is included and is related to the omitted $I(0)$ regressor; the $I(1)$ regressor acts as a proxy variable for the omitted $I(0)$ explanatory variable. Unsurprisingly, the relationship is more likely to be significant the larger is the cross section. Section 2 discusses the case where the dependent variable, the omitted $I(0)$ variable and the $I(1)$ regressor are continuous (i.e., can take on any real value); in section 3, data on stock prices and earnings illustrate

how spurious significance can arise in this case in cross-section regressions. Alternatively, section 4 shows that the omitted variable may be a categorical variable (i.e., can take on a limited number of values) where the category a firm is in is related to the I(1) regressor; section 5 provides an example from simulation, where a firm's rate of return depends on the half of the size distribution the firm is in, but the researcher regresses returns on lagged size. Section 6 offers a summary and some conclusions.

2. "UNBALANCED REGRESSIONS": AN I(0) VARIABLE IS REGRESSED ON AN I(1) VARIABLE

In cross-section and panel studies, researchers often do not consider the persistence of the explanatory variables. Indeed, Wooldridge (2002, section 7.8.3, p. 175) offers the view: "[W]e do not need to restrict the dynamic behavior of our data in any way because we are doing fixed-T, large-N asymptotics."² This view is incorrect in the unbalanced regression case.

The Model: Let the dependent and independent variables be $y_{jt} \sim I(0) \sim \Delta x_{jt}$, the conditional expectation $E(y_{jt} | \Delta x_{jt})$ be

$$y_{jt} = \alpha + \beta \Delta x_{jt}, \quad j=1, \dots, N, \quad t=T', \dots, T \quad (1)$$

$\beta > 0$, $\alpha > / < 0$, and the error be $u_{jt} \sim \text{iid}(0, \sigma_u^2)$, $E(\Delta x_{jt} u_{jt}) = 0$.³ Thus, the Data Generating Process (DGP) is

$$y_{jt} = \alpha + \beta \Delta x_{jt} + u_{jt}, \quad j=1, \dots, N, \quad t=T', \dots, T \quad (2)$$

This DGP is internally consistent in the sense that all variables are I(0).

Intuition for the Slope of an I(1) Regressor: (2) holds for all j,t. Focus on unit j (a household or

² Hayashi (2000, pp. 198, 261), however, assumes the unique, non-constant variables are ergodic and stationary. For panel estimation, Hsiao (2003, pp. 107-108) presents a demonstration that "The GMM estimator is consistent and asymptotically normally distributed as $N \rightarrow \infty$ if all the roots ... fall outside the unit circle, but breaks down if some roots are equal to unity."

³ With cross correlations of the errors, OLS understates the slope's standard error and overstates its t-value; *mutatis mutandis* this paper's key points remain valid. Note that if cross correlations are due to fixed-time effects affecting all firms, the cross-section regression automatically removes such effects, and in a panel the researcher can remove fixed-time effects (Hsiao 2003, pp. 28-33; Im, Pesaran and Shin, 2003; Levin and Lin, 1992; Levin *et al.*, 2002); the panel is unlikely to be large enough to allow removal of time-constant firm effects, but the researcher can likely remove industry effects.

firm, etc.) in the panel. Denote the sample variance and covariance operators by var and cov ; then $\text{var}(y_{jt}) \sim I(0) \sim \text{var}(u_{jt})$ because $\text{var}(u_{jt}) \rightarrow^p \sigma_u^2$ and $\text{var}(y_{jt}) \rightarrow^p \beta \sigma_{\Delta x}^2 + \sigma_u^2$. $\text{var}(x_{jt}) \sim I(1)$, however: as t grows large, $\text{var}(x_{jt})$ grows without limit because $x_{jt} \sim I(1)$.⁴ Thus, in a relationship of y_t to x_t the true slope must be zero (Banerjee *et al.*, 1993).

THEOREMS: For the DGP in (2), consider two OLS cross-section or panel regressions,

$$y_{jt} = a + b \Delta x_{jt} + z_{jt}, \quad j=1, \dots, N, \quad t=T', \dots, T$$

$$y_{jt} = a + g x_{jt} + z_{jt}, \quad j=1, \dots, N, \quad t=T', \dots, T$$

where z_{jt} is a residual,

$$\hat{b} = \beta + \text{cov}(\Delta x_{jt}, u_{jt}) / \text{var}(\Delta x_{jt})$$

$$t_b = [\beta \text{cov}(\Delta x_{jt}, \Delta x_{jt}) + \text{cov}(x_{jt}, u_{jt})] / N^{-1/2} \text{var}(\hat{z}_{jt})^{1/2} \text{var}(\Delta x_{jt})^{1/2}$$

$$\hat{g} = \text{cov}(x_{jt}, u_{jt}) / \text{var}(x_{jt}) = [\text{cov}(x_{jt}, \beta \Delta x_{jt}) / \text{var}(x_{jt})] + [\text{cov}(x_{jt}, u_{jt}) / \text{var}(x_{jt})]$$

$$= [\text{cov}(\Delta x_{jt}, \beta \Delta x_{jt}) / \text{var}(x_{jt})] + [\text{cov}(x_{jt-1}, \Delta x_{jt}) / \text{var}(x_{jt})] + [\text{cov}(x_{jt}, u_{jt}) / \text{var}(x_{jt})]$$

$$t_g = [\beta \text{cov}(\Delta x_{jt}, \Delta x_{jt}) + \beta \text{cov}(x_{jt-1}, \Delta x_{jt}) + \text{cov}(x_{jt}, u_{jt})] / N^{-1/2} \text{var}(\hat{z}_{jt})^{1/2} \text{var}(x_{jt})^{1/2}$$

It is useful to state results in the theorem-proof format; *the proofs are presented in Appendix I.*

THEOREM 1 (included for completeness): *plim $\hat{b} = \beta > 0$; as $N \rightarrow \infty$, then $\hat{b} \rightarrow \beta > 0$. Moreover, $\text{plim } t_g = \beta N^{1/2} > 0$; as $N \rightarrow \infty$, then $t_g \rightarrow^d \beta N^{1/2} + N(0, 1)$.*

Next, write $x_{jT} = \sum_{h=T'}^T \Delta x_{jh}$, where the sample's initial time period T' , $x_{j,T'-1} = 0$ for convenience, and the process generating x_{jT} has run G_T periods, $G_T = T - T'$, $T' \leq T$. Thereafter, the subscript T in G_T is dropped where this causes no confusion.

THEOREM 2: *plim $\hat{g} = \beta / G > 0$; as $N \rightarrow \infty$, then $\hat{g} \rightarrow^d (\beta / G) + (N G)^{-1/2} (\sigma_u^2 / \sigma_{\Delta x}) N_{u, \Delta x}(0, 1) + (N G_T)^{-1/2} (1 - 1/G)^{-1/2} \sigma_{\Delta x} N_{\Delta x, x}(0, 1)$. Furthermore, $\text{plim } t_g = \beta (\sigma_{\Delta x} / \sigma_u) (N / G)^{1/2} / [(1 - 1/G) (\sigma_{\Delta x}^2 / \sigma_u^2) + 1]^{1/2} > 0$; as $N \rightarrow \infty$, then $t_g \rightarrow^d \beta (\sigma_{\Delta x} / \sigma_u) (N / G)^{1/2} / [(1 - 1/G) (\sigma_{\Delta x}^2 / \sigma_u^2) + 1]^{1/2} + N(0, 1)$.*

⁴ The assumptions about the error u_{jt} and Δx_{jt} (below) imply y_t shows no persistence. In cases where the $y_{jt} \sim I(0) \sim \Delta x_{jt}$ show persistence, a footnote below illustrates that nevertheless $\text{plim } \hat{g} = 0$.

$$1) \{(1 + \sigma_{\Delta x}^2 / \sigma_u^2) / [(1 - 1/G_T) (\sigma_{\Delta x}^2 / \sigma_u^2) + 1]\}^{1/2}$$

(The notation $N_{u,\Delta x}(0, 1)$ signifies the standard normal associated with the interaction of u_{jt} with Δx_{jt})

Use an I(0) or I(1) Regressor?: If the I(1) variable's estimated slope is significant, the researcher should not stop there. In general the true relationship is between the dependent variable and an omitted I(0) independent variable that shows sample correlation with the included I(1) regressor. Use of the correct I(0) regressor gives a slope that is asymptotically larger [$\beta > (\beta / G)$ for $G \geq 2$], and a t-value with a larger plim.

I(1)Regressors and Empirical Puzzles: Using an I(1) regressor can cause needless puzzles in empirical results. As noted, if the regressor is I(1), the estimated slope is expected to be relatively small (β / G rather than β). Frequently, the researcher reports a slope that seems puzzlingly small; this is to be expected because $\text{plim } \hat{g} = \beta / G$ rather than β (Theorem 2). Related, in reviewing studies that use moderate cross sections, say $N = 50$ or $N = 100$, researchers may find that $\hat{g}$ tends to be positive (from $\beta > 0$) across samples but with insignificant t-values, and may find that significance at conventional levels is associated with surprisingly large samples; both phenomena are expected, however, because t_g depends on $(N / G)^{1/2}$ rather than $N^{1/2}$.

Simulation results in Table 2 illustrate these points. In the DGP, $\beta = 0.177701$, an estimate that Table 3 reports and section 4 discusses, for an experiment where both variables are stationary; the variances of y_{jt} , Δx_{jt} and u_{jt} are taken from the same experiment. Results are reported for cross-section unbalanced regressions where N ranges from 50 to 1000, and G is either 10 or 20. For a moderate cross section $N = 100$ with the larger $G = 20$, the mean $\hat{g}$ is positive but small—only 0.008600—and the mean t-value is only 0.59697. Keeping $G = 20$, the cross section must increase to $N = 1000$, for the mean t-value to rise to 1.95214, borderline significant at the 5% level.⁵ Thus, the researcher who works diligently to make N very large is likely to find significant results—though the

true slope is zero. A key empirical issue, taken up in section 4, is the likely value of G .

Unbalanced Regressions: Time-Series vs. Cross-Section Problems In time-series analysis, asymptotically the distributions of $\hat{g}$ and t_g are generally non-normal; in cross-section analysis, $\hat{g}$ and t_g are normal but $E \hat{g}$, $E t_g$ do not go to zero (for $\beta \neq 0$). Consider again the DGP

$$y_{jt} = \alpha + \beta \Delta x_{jt} + u_{jt}, \quad j=1, \dots, N, \quad t=T', \dots, T$$

For time-series purposes, let $j=1$. Then, in the regression

$$y_{jt} = a + g x_{jt} + z_{jt}, \quad j=1, \quad t=T', \dots, T$$

$\hat{g}$ is super-consistent and as T grows large, $E \hat{g} \rightarrow 0$; the distribution of $\hat{g}$ is normal, however, only if the innovation in x_{jt} (i.e., Δx_{jt} in this case) has zero long-run correlation with the error u_{jt} . Related, the distribution of t_g is asymptotically standard normal only if Δx_{jt} has zero long-run correlation with the error u_{jt} . For cross-section purposes, let $t=T$. Then, in the regression

$$y_{jt} = a + g x_{jt} + z_{jt}, \quad j=1, \dots, N, \quad t=T$$

from Theorem 2, as N grows large $\hat{g} \rightarrow (\beta / G > 0, \text{plim } t_g \rightarrow \infty$, and both $\hat{g}$ and t_g are normal.

3. AN EMPIRICAL EXAMPLE: STOCK PRICE APPRECIATION AND EARNINGS

Earnings play a key role in virtually all valuation models. In the Residual Income (RI) model, earnings and book are the drivers of a firm's equity value. In free cash flow (FCF) models of overall firm value, FCFs are a key driver; ceteris paribus, earnings have a one-to-one effect on FCFs. Authors sometimes consider two channels through which earnings may affect share prices (Easton and Harris, 1991; Ohlson and Shroff, 1992) and present results from testing both (Easton and Harris, 1991). In one channel, a firm's stock price appreciation depends positively on its contemporaneous growth rate in earnings. In a second channel price appreciation depends positively on earnings

⁵ The mean $\hat{g}$ remains quite small at 0.008907, as expected from $\text{plim } \hat{g} = \beta / G$, independent of N .

persistence. The first channel suggests the cross-section estimating equation

$$\Delta \ln P_{jt} = a + b \Delta \ln X_{jt} + z_{jt}, \quad X_{j,t}, X_{j,t-1} > 0, \quad j=1,N, \quad t=1996 \quad (3)$$

and the second channel suggests

$$\Delta \ln P_{jt} = a + g \ln X_{jt} + z_{jt}, \quad X_{j,t} > 0, \quad j=1,N, \quad t=1996 \quad (4)$$

where z_{jt} is a residual, P_{jt} is firm j 's share price at time t and $X_{jt} > 0$ its earnings per share, $\Delta \ln P_{jt} \equiv \ln P_{jt} - \ln P_{j,t-1}$, $\Delta \ln X_{jt} \equiv \ln X_{jt} - \ln X_{j,t-1}$; thus, this paper considers only firms with positive earnings, a common restriction.⁶ The errors are assumed homoskedastic with no cross correlation.⁷ Because $\Delta \ln P_{jt} \sim I(0) \sim \Delta \ln X_{jt}$, and $\ln X_{jt} \sim I(1)$, (3) is a balanced regression and (4) is unbalanced.

As an example, Table 3 (panels A and B) shows results for 1996; this year was chosen randomly from among the data set: The earnings, price and book data used here are from thirty-one-years worth of data drawn from Compustat, for firms with yearly observations in 1980-2000. In (3), with $N = 991$, the slope (t-value, p-value) is 0.177701 (13.70773, 0.0000); in (4), with the same firms, the slope (t-value, p-value are) 0.020529 (1.972933, 0.0488), a weaker relationship but significant at the 5% level. Compared to (3), the slope in (4) is only 11.55% as large, the t-value only 14.39% as large and the R^2 only 2.455% as large.⁸ Results for the 1995 cross section are similar, though the slope on $\ln X_{j,1995}$ has a larger t-value, 3.259458. For a panel that includes both years, or $t = 1995, 1996$, results in Table 3.C illustrate that this paper's points regarding cross sections apply directly to panels with small T .^{9, 10}

⁶ Many papers focus on positive earnings; negative earnings, if considered, are virtually always treated separately.

⁷ In estimation, only firms with $X_{jt}, X_{j,t-1} > 0$ are considered; further, book values $B_{j,t-1} > 0$ and $B_{j,t-2} > 0$ are also required, jointly implying that the return on equity is positive, $ROE_{jt} = X_{jt} / B_{j,t-1} > 0$, for t and $t-1$. Siddique and Sweeney (2006) find empirically that firms with negative book behave differently from firms with positive book.

⁸ Sometimes these channels are formulated as the *changes* in price ΔP_{jt} and earnings ΔX_{jt} and the un-logged level X_{jt} . The variables are then typically normalized by $P_{j,t-1}$ to give $\Delta P_{jt} / P_{j,t-1}$, $\Delta X_{jt} / P_{j,t-1}$, $X_{jt} / P_{j,t-1}$; Easton and Harris (1991) use this approach. This paper's specifications avoid problems that arise from dividing variables by the same (lagged) integrated variable, P_{jt} . This paper focus on $\Delta \ln P_{jt}$, but Easton and Harris include dividends.

⁹ Year effects are captured by allowing differing intercepts for the two years.

¹⁰ If both $\Delta \ln X_{j,t}$ and $\ln X_{j,t}$ are included in an OLS regression, the slope on $\ln X_{j,t}$ is unstable and often insignificant. For 1996 data, the slope and t-value for $\Delta \ln X_{j,1996}$ are virtually unchanged, but $\ln X_{j,1996}$ has the slope (t-value) of -0.019061 (-1.911287), a sign reversal. For 1995 data, the slope and t-value for $\Delta \ln X_{j,1995}$ are virtually unchanged, but $\ln X_{j,1995}$ has a sign reversal and a t-value of only -0.141844.

Estimates of G: The R^2 in a regression of $\ln X_{j,1996}$ on $\Delta \ln X_{j,1996}$ is 0.087517, or the cross-section variability of $\Delta \ln X_{j,1996}$ ($\equiv \ln X_{j,1996} - \ln X_{j,1995}$) explains about 9% of the variability in $\ln X_{j,1996}$. For $\ln X_{j,1995}$ regressed on $\Delta \ln X_{j,1995}$, the R^2 is 0.091374. These R^2 s suggest $G \approx 11$ years ($1 / 0.087517 = 11.4264$ and $1 / 0.091374 = 10.9440$). Thus, in Table 2 the results for $G = 10$ appear more appropriate.

4. THE INTEGRATED REGRESSOR AS A PROXY VARIABLE FOR AN OMITTED CATEGORICAL REGRESSOR: NEW DEVELOPMENTS

Consider the case where the true relationship is between an $I(0)$ variable and a categorical variable that can take only a limited number of values. For example, in an event study discussed below, returns may have a larger mean for the small firms than for large firms, where small (large) firms are those in the lower (upper) half of the size distribution, but within each half returns are uncorrelated with size. Many researchers, however, regress the rate of return on (log) size. If the researcher uses the $I(1)$ variable size as a regressor, its true coefficient is necessarily zero. If returns depend on the categorical variable in size, then in a regression where the categorical variable is omitted and size is a proxy for it, the expectation of the estimated slope of size is negative, and as the number of firms in the cross section grows large the plim of the slope's t-value diverges to negative infinity. In such a case, the solution is to use the correct categorical variable, "small" versus "large." Similar problems arise in studies where the categorical variable is say "newer" versus "older" firms but the researcher uses a time trend or trend-like explanatory variable such as firm "age." In an analysis consistent with the above, firms are analytically and empirically divided into a limited number of categories based on log size (or on age or other trend-type variables). For example, the researcher believe that firms fall into five size categories ranging from low to high, along with say five categories of the book-to-market ratio. In this view, movements of the firm's log size from one category to another affect its

conditional rate of return, but movements within the category do not. Without loss of generality, suppose there are two categories, and let the conditional relationship be

$$y_{jt} = \alpha + \alpha_{sm} D_{jt} \quad (5)$$

where y_{jt} can be taken as the firm's rate of return, x_{jt-1} is log size (or another stochastic or deterministic trend-type variable), the firms are divided into halves based on x_{jt-1} relative to the median value x_{mt-1} , with $D_{jt} = 1$ if $x_{jt-1} \leq x_{mt-1}$ and $D_{jt} = 0$ if $x_{jt-1} > x_{mt-1}$, and for specificity $\alpha_{sm} > \alpha > 0$.¹¹ For trend-like variables such as firm age, a conditional relationship similar to (5) is consistent with the category approach.¹² In either case, let the error be $u_{jt} \sim \text{iid}(0, \sigma_u^2)$, $E(D_{jt} u_{jt}) = 0$.

Instead of focusing on the true categorical relationship in (5), suppose the researcher believes the firm's predetermined log size [or age] continuously and linearly affects its conditional rate of return. The appropriate regression specification, associated with the true DGP (5), is

$$y_{jt} = a + a_{sm} D_{jt} + z_{jt} \quad (6)$$

Nevertheless, in the OLS regression,

$$y_{jt} = a + g x_{jt-1} + z_{jt} \quad j=1, \dots, N, \quad t=T', \dots, T \quad (7)$$

generally $\text{plim } \hat{g} \neq 0$ and in a large cross section is likely to be significant. Let $x_{jt-1} = [\sum_{t=T'+1}^{T+h} \Delta x_{j,T+h}] \sim G^{1/2} \sigma_{\Delta x} \text{iid } N(0, 1)$. Intuitively, y_{jt} is unrelated to x_{jt-1} *within* each category, $x_{jt} < x_{mt}$ and $x_{jt} \geq x_{mt}$. But x_{jt} is correlated with the category into which a firm falls, and thus in regression (8) $\hat{g}$ is likely to show a spurious continuous relationship between y_{jt} and x_{jt-1} .

THEOREM 3 (included for completeness): *For the conditional expectation in (5) and the error u_{jt} , in the OLS regression $R_{jt} = a + a_{sm} D_{jt} + z_{jt}$, $\text{plim } a = \alpha$, $\text{plim } a_{sm} = \alpha_{sm}$ and the $\hat{a}$, $\hat{a}_{sm}$ are asymptotically distributed normal around their true values.*

In the OLS regression, the slope estimate is

$$\hat{g} = [\alpha_{sm} \text{cov}(D_{jt}, x_{jt}) / \text{var}(x_{jt})] + [\text{cov}(u_{jt}, x_{jt}) / \text{var}(x_{jt})].$$

Let $\hat{m}_{sm}$, $\hat{m}_{la}$ denote are the means of the halves of the sample with small and large values of x_{jt} relative to the median x_{mt} , and let $m_{sm} = -m_{la} < 0$ denote the means' asymptotic values.

THEOREM 4: *plim $\hat{g} = \alpha_{sm} m_{sm} / G \sigma_{\Delta x}^2 = - (0.675 G^{1/2} \sigma_{\Delta x}) \alpha_{sm} / G \sigma_{\Delta x}^2 = - 0.675 \alpha_{sm} / G^{1/2} \sigma_{\Delta x} < 0$. As $N \rightarrow \infty$, $\hat{g} \rightarrow^d \alpha_{sm} m_{sm} / G \sigma_{\Delta x}^2 + (N G)^{-1/2} (\sigma_u / \sigma_{\Delta x}) N_{\Delta, x}(0, 1) = [- 0.675 \alpha_{sm} / G^{1/2} \sigma_{\Delta x}] + (N G)^{-1/2} (\sigma_u / \sigma_{\Delta x}) N_{\Delta, x}(0, 1)$. From $\alpha_{sm} > 0$, $\hat{g}$ is asymptotically normal around its mean, $-0.675 \alpha_{sm} / G^{1/2} \sigma_{\Delta x} < 0$. Moreover, $\text{plim } t_g = -0.675 N^{1/2} (\alpha_{sm} / \sigma_u) < 0$; as $N^{1/2} \rightarrow \infty$, then $\text{plim } t_g$ diverges to negative infinity. As $N \rightarrow \infty$, then $[V(\hat{g})]^{1/2} \rightarrow (N G)^{-1/2} (\sigma_u / \sigma_{\Delta x})$ and $t_g \rightarrow -0.675 N^{1/2} (\alpha_{sm} / \sigma_u) + N_{\Delta, x}(0, 1)$.*

Interpretation: As in the continuous-variable case, $\text{plim } \hat{g}$ depends inversely on G ; $\text{plim } t_g$ does not depend on G in this categorical-variable case, but t_g does in the continuous case.

The data must contain a true relationship between the $I(0)$ dependent variable and an unincluded $I(0)$ variable for the unbalanced regression to show a spurious relationship between the $I(0)$ dependent variable and the included $I(1)$ variable that shows sample correlation with the unincluded variable. If small and large firms have the same mean, α , then $\alpha_{sm} = 0$ and $\text{plim } \hat{g} = [\alpha_{sm} m_{sm} / G \sigma_{\Delta x}^2] = 0$. If D_{jt} , x_{jt} are uncorrelated, as N grows large $\text{cov}(D_{jt}, x_{jt}) \rightarrow^p 0$ and thus $\text{plim } \hat{g} = 0$.

A number of empirical puzzles, regarding instability of results and non-linear relationships, are likely to arise if the researcher uses the misspecification in (7) rather than the appropriate specification in (6); section 5 presents illustrative simulation results.

Remedies for the Misspecification: The remedy for the misspecification in (7) is to use the size-based categorical specification in (6). Using Δx_{jt-1} is not the remedy. Let the researcher consider the

¹¹ In the DGP $y_{jt} = \alpha + \alpha_{sm} D_{jt} + u_{jt}$, the categorical variable $\alpha + \alpha_{sm} D_{jt}$ is stationary but may well not be ergodic. The mean across firms is $[\alpha + (1/2) \alpha_{sm}]$. If the size-change process Δx_{jt} and the error u_{jt} are highly likely to leave the firm in the smaller half over time, the firm's mean time-series return is likely $[\alpha + \alpha_{sm}] > [\alpha + (1/2) \alpha_{sm}]$.

regression

$$y_{jt} = a + g \Delta x_{jt} + z_{jt} \quad (8)^{13}$$

Both (7) and (8) are misspecifications, but generally (7) gives stronger results than (8), tempting the researcher to use the unbalanced regression (7).¹⁴

Use of a regressor in which x_{jt} enters in ratio form is generally inappropriate. First, in this case, the true conditional expectation in (5) includes a *categorical, discrete* variable in x_{jt} , not a *continuous* variable in x_{jt} . Second, the ratio may be $I(1)$. For example, in log terms the ratio of size to its median is $x_{jt} - x_{mt}$; unless x_{jt} and x_{mt} are cointegrated, then $(x_{jt} - x_{mt}) \sim I(1)$. This is related to the issue of $ROE \sim I(1)$. Let earnings $X_t \sim I(1)$ and book $B_t \sim I(1)$ be related through $X_t = \gamma B_{t-1} + B_{t-1} \varepsilon_t$, where $\gamma > 0$ is a parameter and ε_t an error with effects proportional to B_{t-1} . Then, $ROE_t = X_t / B_{t-1} = \gamma + \varepsilon_t$, and $ROE_t \sim [I(0), I(1)]$ as $\varepsilon_t \sim [I(0), I(1)]$. Failure to reject the null that $ROE \sim I(1)$ is equivalent to failure to reject the null that X_t and B_t are not cointegrated.¹⁵

Does Theory Specify Continuous Measures? The researcher may acknowledge the problem of regressing $y_{jt} \sim I(0)$ on $x_{jt} \sim I(1)$ but still insist that theory specifies size as the explanatory variable. Often, such arguments confuse continuous with categorical measures of x_{jt} . As an example, the null “Size does not affect the rate of return” and the alternative “Size negatively affects the rate of return”

¹² 15 of the 42 papers discussed in section 1 include either time-trends or trend-like variables such as age of firm.

¹³ Intuitively, y_{jt} depends in part on $D_{jt} \alpha_{sm}$; because $x_{jt-1} = \sum \Delta x_{jt-h}$ and D_{jt} depends on x_{jt-1} , likely Δx_{jt-1} shows a weak correlation with D_{jt} and thus may weakly enter (10).

¹⁴ In section 4 where the true regressor is $\Delta \ln X_{jt} \sim I(0)$, the $\Delta \ln X_{jt}$ -specification produces stronger results than the $\ln X_{jt}$ -specification. In this section the $\ln X_{jt}$ -specification produces stronger results than the $\Delta \ln X_{jt}$ -specification: the true explanatory variable here is categorical, neither Δx_{jt} nor x_{jt} .

¹⁵ Researchers often assume that financial and accounting ratios are $I(0)$. For example, some deflate an $I(1)$ variable by another $I(1)$ variable to “remove trends.” When researchers apply standard tests of the unit-root null to ratios, sometimes the data can reject the null but often they cannot. Generally, it appears wiser to treat a ratio as $I(1)$ if the data in the sample being examined cannot reject the unit-root null. Often, however, data in panels have few time-series observations, and of course a cross section contains only a single observation for each element of the cross section. Thus, unit-root tests on such panels may have little power. But, for such cases, consider the DGPs for the ratios, and compare results for a regression with Δx_{jt} with results for a regression with x_{jt} . If the DGP generates $x_{jt} \sim I(1)$, and the true relationship is between y_{jt} and Δx_{jt} , then the text’s discussion implies the researcher will find stronger results using the specification with Δx_{jt} rather than x_{jt} . If the DGP generates $x_{jt} \sim I(0)$, and the true relationship is between y_{jt} and x_{jt} , the specification with x_{jt} will yield stronger results than Δx_{jt} . Thus, in the case of few time-series observations on x_{jt} , trying specifications with both Δx_{jt} and x_{jt} should allow the researcher to infer the true relationship as well as the order of integration of x_{jt} .

do not imply the continuous test equation (8), but are consistent with the categorical-variable specification (7). If the relationship to size is continuous (and monotonic), doubling all firms' sizes reduces all expected y_{jt} ; if categorical, doubling all firms' size may leave all expected rates of return unchanged, but with $\alpha_{sm} > \alpha$ before and after the doubling.

5. SIMULATION STUDY WITH A CATEGORICAL VARIABLE

To explore the results in section 4, consider a simulation study of the effect of size on returns. Figure 1 illustrates a simulation for 1,000 firms. The errors are $u_{jt} \sim \sigma_u$ iid $N(0, 1)$, size is $S_{jt-1} = [\Sigma^{T'+t-1}_{T+h=T'+1} \Delta S_{j,T'+h}] \sim G^{1/2} \sigma_{\Delta S}$ iid $N(0, 1)$, $G^{1/2} = 10$ and $\sigma_{\Delta S} = \sigma_u$. The DGP for the rate of return R_{jt} is

$$R_{jt} = D_{jt} \alpha_{sm} + (1 - D_{jt}) \alpha_{la} + u_{jt} = D_{jt} 0.20 \times \sigma_u - (1 - D_{jt}) 0.20 \times \sigma_u + u_{jt} \quad (9)$$

where $\alpha_{sm} = 0.20 \sigma_u$, $\alpha_{la} = -0.20 \sigma_u$, and the dummy variable is $D_{jt} = \{1, 0\}$ as $\{S_{jt-1} < S_{mt-1}, S_{jt-1} \geq S_{mt-1}\}$ where the median size is S_{mt-1} . Table 4 shows that for the misspecification

$$R_{jt} = a + g S_{jt-1} + z_{jt} \quad (10)$$

the slope (t-value) [probability] are -0.016406 (-4.812320) [0.0000], and $R^2 = 0.022679$.

I(1) Regressors and Empirical Puzzles: Researchers often report puzzling instability and non-linearities over sub-sets of a cross section or over differing data sets. Note that if the true model is the qualitative model (5), then the regression in (10) is likely to show instabilities and non-linearities over subsets. Using the simulation data shown in Figure 1, Table 4 presents regression results that illustrate some puzzles. In a regression over the whole sample, $\hat{g} < 0$ in the linear specification in (10), as reported above; the linear specification seems to "work." If the researcher examines only firms in the same half of the size distribution, the detected relationship tends to vanish, making $\hat{g}$ appear to be unstable and insignificant. For the smallest 500 firms the results are 0.000971

(0.121879) [0.9030], and for the largest 500 are -0.003759 (-0.488689) [0.6253]. This instability arises from using the misspecified estimating equation (10).

Spurious Threshold and Satiation Effects: The researcher might fit a piece-wise linear relationship over three sub-samples, one with the 250 smallest firms, one with the 250 largest firms, and one with a mixture of small and large firms. (Alternatively, the researcher may fit R_{jt} as a non-linear function of S_{jt-1} , with continuous derivatives, e.g., a cubic.) For example, in Table 4, for the middle 500 firms in a linear regression the results are -0.041741 (-3.043186) [0.0025], but for the 250 smallest are -0.010431 (-0.815052) [0.4158] and for the 250 largest -0.006195 (-0.463631) [0.6433]. These piece-wise linear results in Table 4 might be interpreted to show a “threshold” size before the relationship applies, and a “satiation” point beyond which increases in size have no effect. The creative researcher may provide explanations for threshold and satiation effects—effects that are spurious and arise because of using the unbalanced regression (10).

Comparative Statics: Using the data in the simulation illustrates a number of results. The initial data series, R_{jt} , S_{jt-1} were generated as $\alpha_{sm} = -\alpha_{la} = 0.20$, $\sigma_u = 1$, $G^{1/2} = 10$, $u_{jt} \sim \text{iid } \sigma_u N_u(0, 1)$ and $S_{jt-1} \sim \text{iid } G^{1/2} \sigma_u N_x(0, 1)$. By saving the set of realizations $\{u_{jt}, S_{jt-1}\}$, the precise effects of parametric changes can be found. Panel B shows results for three cases. (i) Double the initial series R_{1jt} , S_{1jt-1} to give the series R_{2jt} , S_{2jt-1} : this leaves $\hat{g}$ unaffected. (ii) Double S_{1jt-1} to give S_{2jt-1} , but hold constant the R_{1j} : the slope and t-value are approximately halved. (iii) Keep the S_{1jt} constant but double R_{1jt} to R_{3jt} : the slope and t-value double.

Monthly versus Annual Data: Whether data are monthly or say annual does not affect results. Let all data be in logs or differences of logs. Monthly and annual measures for variance and firm life are $\sigma_{u,mth}^2 = \sigma_{u,yr}^2 / 12$, $G_{mth} = 12 G_{yr}$, and hence $N(0, G_{yr} \sigma_{u,yr}^2) = N(0, G_{mth} \sigma_{u,mth}^2)$. In (7) implicitly $\alpha_{sm,mth} = (12)^{-1/2} \alpha_{sm,yr}$, giving $ER_{jt,mth} / \alpha_{sm,mth} = ER_{jt,yr} / \alpha_{sm,yr}$.

6. SUMMARY AND CONCLUSIONS

Virtually no cross-section finance study reports the order of integration of its variables, nor do those panel studies where the number of time periods T is small and the cross-section N is large. This paper shows that severe problems can arise in cross-section or panel studies that regress $I(0)$ dependent variables on $I(1)$ explanatory variables; these "unbalanced regression" problems are different from those in unbalanced time-series regressions, but are comparably severe. This paper also provides step-by-step procedures to deal with unbalanced regressions: It discusses how to avoid problems, to diagnose problems and to remedy problems.

Many regressors in cross-section finance studies appear to be $I(1)$, as Appendix II shows in a brief survey focused on articles within 2005 in the most prestigious finance and econometrics Journals. For example, firm size is likely $I(1)$ whether measured by market or accounting data. Further, many ratios *may* be $I(1)$, as suggested by the finding that the return on equity and the rate of return on invested capital are $I(1)$, and many micro-structure variables may be $I(1)$, as suggested by the finding that volume for Dow Jones 30 stocks cannot reject the unit root null.

This paper focuses on cases where the $I(0)$ dependent variable is importantly associated with an $I(0)$ explanatory variable. The researcher, however, omits the true $I(0)$ regressor and as a proxy for it uses an $I(1)$ explanatory variable that shows sample correlation with the true $I(0)$ regressor. This paper explores in detail theoretical results for this case, whether the true $I(0)$ variable is continuous (i.e., can take on any real value) like the change in earnings or is categorical (i.e., can represent only a limited number states) like large versus small firms, and also provides empirical and simulation examples. If the cross section is large, the researcher is likely to find a spurious relationship between the $I(0)$ dependent variable and the $I(1)$ regressor.

In these cross-section unbalanced regressions, the researcher may find no relationship, abandon this path and thus fail to uncover the true $I(0)$ explanatory variable. Or, the regression may show a

small but statistically significant relationship. This estimated relationship is spurious, but arises because the cross-section is large. The researcher may accept the spurious relationship with the $I(1)$ regressor, and fail to explore farther to find the true relationship with an $I(0)$ regressor that shows sample correlation with the $I(1)$ variable. In an unbalanced regression, a significant slope on an $I(1)$ regressor is a diagnostic, however: It strongly suggests the data contain a valid, stronger relationship to an $I(0)$ variable.

In designing a cross-section or panel study, unit-root tests provide guidance on how to treat variables econometrically, rather than "proving" variables are $I(1)$ or not (Campbell and Perron, 1991). The researcher may have cogent arguments that a variable is $I(0)$, but if the data cannot reject the unit-root null, it is often sound econometric practice is to treat the variable as $I(1)$.

The researcher may argue T for his/her data is too small to allow unit-root tests; some approaches, however, require modest T , e.g., see Breitling and Meyer (1994), Levin and Lin (1991) and Im, Pesaran and Shin (2003). Further, the literature provides much time-series evidence on some of the variables used in cross section studies, for example, firm size, assets, etc. In addition, in doubtful cases the researcher might replace the suspect variable with a related stationary variable, for example, the first difference or a related categorical variable.

This paper's discussion—of unbalanced-regression problems and their solutions—applies directly to the many finance studies that use a single cross-section or use panel techniques where N is large but T is small (Wooldridge, 2002; Hsiao, 2003; Mátyás and Sevestre, 2006; Baltagi, 2005). Most often these studies take T as fixed and asymptotic results depend on N growing large.

When both T and N are large, unbalanced-regression problems arise that are a combination of the cross-section problems discussed here and the more familiar time-series problems, but this paper does not analyze such problems. The large- N , large- T case seldom applies to corporate finance panels where typically (a) N is large N and T is small, and (b) the researcher takes T as fixed and

relies on asymptotics from N growing large. In contrast, as Phillips and Moon (1999, p. 1059) point out, results for panels with non-stationary variables generally require that the rate condition $N/T \rightarrow 0$ hold, or “the limit distribution theory ... is most likely to be useful in practice when N is moderate and T is large.” (see also surveys by Baltagi and Kao, 2000 and Phillips and Moon, 2000).

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Table 1. Representative Articles Using Cross-Section, Panel Methods

Independent Variable	Author(s)	Topic/Short Title	Dependent Variable
<i>Size, Price, Scale Measures</i>			
log of firm size (market value)	Datta, Iskandar-Datta, Raman	Managerial Stock Own.	Percentage of debt maturing in 3 years
log(market capitalization)	Fernando, Gatchev, Spindt	Underwriting	measure of rep., lead SEO underwriter
log(total assets)	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
log(total assets)	Molina	Firms' leverage	S&P ratings
log(total assets)	Malmendier and Tate	CEO overconfidence	Capex relative to capital stock
log(fund's total net assets)	Cooper, Gulen and Rau	Mutual-fund names	0-1 variable as firm changes name
log(GDP)	Beck, et al.	Firm size and growth	3-yr percentage change in sales
average of log proceeds	Keloharju, Nyborg, Rydqvist	Underpricing in auctions	percentages of face value
log of SEO proceeds	Keloharju, Nyborg, Rydqvist	Underpricing in auctions	percentages of face value
Total assets	Haslem	Managerial opportunism	cum. ab. returns, <i>given</i> window sizes
log total assets	Jenter	Managers' portfolios	Net pur. of company stock rel. to past
GDP per capita	Beck, et al.	Firm size and growth	3-yr percentage change in sales
GDP per capita	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
GDP per capita	Chan, Covrig and Ng	Domestic, foreign biases	Domestic bias in share holdings
GDP per capita	Gelos and Wei	Transparency	residuals from an ICAPM regression
GDP per capita	Stulz	Financial globalization	percent of portfolio family-held
Stock market turnover	Stulz	Financial globalization	percent of portfolio family-held
Principal amount	Longstaff, Mithal, Neis	Corporate yield structure	Av. non-default component of spread
log of amount of bond issue	Yasuda	Bank relationships	Underwriting fees (gross spread)
log IPO filing size	Lungqvist and Wilhelm	Prospect Theory, IPOs	zero-one variable
Demand per bidder in auction	Keloharju, Nyborg, Rydqvist	Underpricing in auctions	percentages of face value
Aggregate demand in auction	Keloharju, Nyborg, Rydqvist	Underpricing in auctions	percentages of face value
Auction size	Keloharju, Nyborg, Rydqvist	Underpricing in auctions	percentages of face value
Value of shares owned	Jenter	Managers' portfolios	Net pur. of company stock rel. to past
Total pay	Jenter	Managers' portfolios	Net pur. of company stock rel. to past
Free cash flows	Haslem	Managerial opportunism	cum. ab. returns, <i>given</i> window sizes
Total assets	Haslem	Managerial opportunism	cum. ab. returns, <i>given</i> window sizes
CEO salary	Haslem	Managerial opportunism	cum. ab. returns, <i>given</i> window sizes
log sales	Perry and Peyer	Board seat accumulation	cum. abnormal return, days -1 to +1
log sales	Grinstein and Michaely	Institutional holdings	chg. in inst. holdings/rel. to total
log sales	Beck, et al.	Firm size and growth	3-yr percentage change in sales

Independent Variable	Author(s)	Topic/Short Title	Dependent Variable
<i>Size, Price, Scale Measures</i> (cont.)			
turnover (maximum during quarter)	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
log(issue proceeds)	Fang	Investment bank reps.	0-1 variables in probit regression, <i>or</i> gross spread on issue
1/(principal amt. of SEO offering)	Drucker and Puri	Lending, underwriting	Underwriters' spreads (percentage), <i>or</i> 0-1 variables
Value of SEO issuance by all firms	Drucker and Puri	Lending, underwriting	"
Value of loan facility	Drucker and Puri	Lending, underwriting	"
log(principal amount of offering)	Drucker and Puri	Lending, underwriting	"
Flow to mutual fund	Cooper, Gulen and Rau	Mutual-fund names	0-1 variable as firm changes name
Loan size	Degryse and Ongena	Distance, lending	loan rate until next revision
log(no. of stocks in HH port.)	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
log(value of stocks, HH port.)	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
log(household income)	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
log(market value)	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
log(number of employees)	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
log(expected proceeds of IPO)	Corwin and Schultz	IPO syndicates	zero-one in probit regression
Proceeds	Corwin and Schultz	IPO syndicates	zero-one in probit regression
Transaction size (IPO)	Derrien	IPO pricing, hot markets	oversubscription (a ratio)
log(float)	Derrien	IPO pricing, hot markets	oversubscription (a ratio)
volume	Derrien	IPO pricing, hot markets	oversubscription (a ratio)
Ratios			
ROA	Perry and Peyer	Board seat accumulation	cum. abnormal return, days -1 to +1
No. r of shares traded/benchmark	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
Market share of sales	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
Market share of sales	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
Market to book	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
Market to book	Grinstein and Michaely	Institutional holdings	chg. in inst. holdings/rel. to total
Market to book	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
Net working capital/total assets	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
(CAPEX+R&D)/(total assets)	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
(cash+tradable secs.)/(total assets)	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model

Independent Variable	Author(s)	Topic/Short Title	Dependent Variable
Ratios (cont.)			
(size of offer)/(offer+capitalization)	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
Leverage	Faccio and Masulis	Payment method, M&A	Per. of cash financing, tobit model
Leverage	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
Leverage	Fang	Investment bank reps.	0-1 variables in probit regression <i>or</i> gross spread on issue
Leverage	Drucker and Puri	Lending, underwriting	Underwriters' spreads (percentage), <i>or</i> 0-1 variables
Leverage	Ivković and Weisbenner	Local information	Av. miles to portfolio stock
Repurchases/book	Grinstein and Michaely	Institutional holdings	chg. in inst. holdings/rel. to total
Book-value leverage	Molina	Firms' leverage	S&P ratings
Market-value leverages	Molina	Firms' leverage	S&P ratings
(sales expenses)/sales	Molina	Firms' leverage	S&P ratings
(sales expenses)/sales	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
EBITDA/total assets	Molina	Firms' leverage	S&P ratings
PPE/(total assets)	Molina	Firms' leverage	S&P ratings
(imports+exports)/(2xGDP)	Chan, Covrig and Ng	Domestic, foreign biases	Domestic bias in share holdings
(FDI stock)/GDP	Chan, Covrig and Ng	Domestic, foreign biases	Domestic bias in share holdings
(Stock market capitalization)/GDP	Chan, Covrig and Ng	Domestic, foreign biases	Domestic bias in share holdings
log(Val. stocks traded)/(mkt. cap.)	Chan, Covrig and Ng	Domestic, foreign biases	Domestic bias in share holdings
Stock market capitalization to GDP	Stulz	Financial Globalization	percent of portfolio family-held
(market assets)/(book assets)	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
(CAPEX)/(total assets)	Leary and Roberts	Capital structure	"
(amt. SEO offering)/(market cap.)	Drucker and Puri	Lending, underwriting	Underwriters' spreads (percentage), <i>or</i> 0-1 variables
(net operating income)/(total assets)	Leary and Roberts	Capital structure	Changes in leverage, <i>or</i> probability of four discrete outcomes
(cash flow)/(capital stock)	Malmendier and Tate	CEO overconfidence	Capex relative to capital stock
(issue proceeds)/(firm's equity cap.)	Fang	Investment bank reps.	0-1 variables in probit regression, <i>or</i> gross spread on issue
(budget balance)/GDP	Gelos and Wei	Transparency	residuals from an ICAPM regression
(current account balance)/GDP	Gelos and Wei	Transparency	residuals from an ICAPM regression

Independent Variable	Author(s)	Topic/Short Title	Dependent Variable
<i>Ratios</i> (cont.)			
Turnover/(market capitalization)	Gelos and Wei	Transparency	residuals from an ICAPM regression
CAPEX/(total assets)	Bates	Asset sales	0, 1 or 2 in logistic regression
(operating inc.)/(interest expense)	Bates	Asset sales	"
(operating income)/(total assets)	Bates	Asset sales	"
(firm debt) - (median industry debt)	Bates	Asset sales	"
(total debt)/(total assets)	Bates	Asset sales	Bates: 0, 1 or 2 in logistic regression
(cash flow)/(total assets)	Bates	Asset sales	"
cash/(total assets)	Bates	Asset sales	"
Turnover (first day)/float	Derrien	IPO pricing	oversubscription (a ratio)
<i>Trends</i>			
log of days from IPO to SEO	Fernando, Gatchev, Spindt	Underwriting	measure of rep., lead SEO
underwriter			
Age of bond issue	Longstaff, Mithal, Neis	Corporate structure	Av. non-default component of spread
log(1 + age at IPO)	Lungqvist and Wilhelm	Prospect Theory, IPOs	zero-one variable
log(days from IPO to SEO)	Lungqvist and Wilhelm	Prospect Theory, IPOs	zero-one variable
log(number of issues + 1)	Yasuda	Bank relationships	underwriting fees (gross spread)
log(maturity)	Yasuda	Bank relationships	underwriting fees (gross spread)
years tenure of CEO	Malmendier and Tate	CEO overconfidence	Capex rel. to capital stock
time trend	Fang	Investment bank reps.	0-1 variables in probit regression, <i>or</i> gross spread on issue
log(maturity)	Fang	Investment bank reps	0-1 variables in probit regression <i>or</i> gross spread on issue
Age of firm	Drucker and Puri	Lending, underwriting	Underwriters' spreads (percentage), <i>or</i> 0-1 variables
Length of loan	Drucker and Puri	Lending, underwriting	"
log(fund's age)	Cooper, Gulen and Rau	Mutual-fund names	0-1 variable as firm changes name
log(1+duration of relationship)	Degryse and Ongena	Distance, lending	loan rate until next revision
number of years' experience	Clement and Tse	Analysts and herding	forecast error scaled by price

TABLE 2. EMPIRICAL DISTRIBUTION OF ESTIMATED SLOPES AND THEIR *t*-VALUES**Estimating equation: $y_{j,t} = a + g x_{jt} + z_{jt}$**

<i>N</i> =50	<i>Mean</i>	<i>Std Dev</i>	<i>Minimum</i>	<i>Maximum</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>G</i>
$\hat{g}$	0.017925	0.030134	-0.11968	0.15968	-0.049688	0.23034	10
t_g	0.62424	1.03709	-3.71335	4.87423	-0.010632	0.15004	10
$\hat{g}$	0.009021	0.020962	-0.071487	0.086828	-0.006906	0.030994	20
t_g	0.44205	1.02082	-3.36093	5.10835	0.019129	0.058199	20
<i>N</i> =100							
$\hat{g}$	0.017817	0.020508	-0.055274	0.088754	-0.002693	-0.047553	10
t_g	0.87551	1.00638	-2.68733	4.66734	0.025024	-0.031547	10
$\hat{g}$	0.008600	0.014566	-0.045487	0.061779	0.016414	0.068855	20
t_g	0.59697	1.00403	-3.03260	4.16482	0.026325	0.013089	20
<i>N</i> =250							
$\hat{g}$	0.017872	0.012940	-0.027533	0.066889	-0.000553	0.0089420	10
t_g	1.38726	1.00537	-2.13323	5.35359	0.015865	0.013174	10
$\hat{g}$	0.008912	0.009057	-0.022711	0.043285	0.001831	0.030121	20
t_g	0.97780	0.99604	-2.54748	4.74834	0.013053	0.073351	20
<i>N</i> =500							
$\hat{g}$	0.017812	0.009127	-0.017902	0.055412	-0.025020	0.076102	10
t_g	1.95686	1.00448	-1.97598	5.82773	-0.017759	0.065315	10
$\hat{g}$	0.008981	0.006512	-0.019099	0.032961	-0.031441	-0.022414	20
t_g	1.39288	1.01339	-3.03511	5.21713	-0.008332	0.0021424	20
<i>N</i> =1000							
$\hat{g}$	0.017840	0.006494	-0.006220	0.052009	0.001267	0.082935	10
t_g	2.77075	1.01009	-0.94148	8.29562	0.009061	0.11612	10
$\hat{g}$	0.008907	0.004597	-0.010962	0.026877	-0.026178	0.038270	20
t_g	1.95214	1.00952	-2.63105	5.80972	-0.021985	0.034229	20

Notes: *N* is the number of firms in the cross section, and *G* is the number of periods for which the DGP has run. The DGP is $y_{j,t} = \alpha + \beta \Delta x_{jt} + u_{jt}$. The data are simulated with the error u_{jt} normal and the increment Δx_{jt} normal. The variances of $y_{j,t}$, Δx_{jt} are set equal to those in the data for 1996 discussed in section 4, β is set equal to 0.17, the estimate of β for 1996, and the variance of $u_{j,t}$ is then found from the equation for the DGP. Results are for various values of *N* and for *G*= 10 or *G* = 20. For each pair of *N*, *G*, the simulations have 10,000 replications.

TABLE 3. PERCENTAGE PRICE APPRECIATION, EXPLAINED BY CHANGE IN EARNINGS OR LEVEL OF EARNINGS

Panel 3.A. $\Delta \ln P_{j,T} = a + b \Delta \ln X_{j,T} + u_{j,T}$

Variable ^a	Coefficient	Std. Err.	t-Statistic	Prob.	Variable ^b	Coefficient	Std. Err.	t-Statistic	Prob.
a_{1996}	0.065778	0.009657	6.811125***	0.0000	a_{1995}	0.146954	0.010198	14.41022***	0.0000
$\Delta \ln X_{j,1996}$	0.177701	0.012964	13.70773***	0.0000	$\Delta \ln X_{j,1995}$	0.156725	0.013740	11.40638***	0.0000
R^2	0.159658				R^2	0.120456			

Panel 3.B. ^a $\Delta \ln P_{j,T} = a + b \ln X_{j,T} + u_{j,T}$

Variable	Coefficient	Std. Err.	t-Statistic	Prob.	Variable ^b	Coefficient	Std. Err.	t-Statistic	Prob.
a_{1996}	0.067386	0.011217	6.007421***	0.0000	a_{1995}	0.144483	0.011580	12.47730***	0.0000
$\ln X_{j,1996}$	0.020529	0.010405	1.972933**	0.0488	$\ln X_{j,1995}$	0.035769	0.010974	3.259458***	0.0012
R^2	0.003920				R^2	0.011060			

Panel 3.C. Pooled Regression

$$\Delta \ln P_{j,t} = a_{1995} D_{1995} + a_{1996} D_{1996} + b \Delta \ln X_{j,t} + u_{j,t}$$

Variable	Coefficient	Std. Err.	t-Statistic	Prob.
a_{1996}	0.067363	0.010966	6.143153***	0.0000
a_{1995}	0.146223	0.010014	14.60209***	0.0000
$\Delta \ln X_{j,t}$	0.166718	0.009456	17.63162***	0.0000
R^2_{1996}	0.156602	R^2_{1995}	0.119967	

$$\Delta \ln P_{j,t} = a_{1995} D_{1995} + a_{1996} D_{1996} + b \ln X_{j,t} + u_{j,t} \quad t=1995,1996$$

Variable	Coefficient	Std. Err.	t-Statistic	Prob.
a_{1996}	0.066489	0.009831	6.763507***	0.0000
a_{1995}	0.166273	0.011081	15.00528***	0.0000
$\ln X_{j,t}$	0.020588	0.007390	2.786022***	0.0054
R^2_{1996}	0.003920	R^2_{1995}	0.004017	

Notes: $\ln X_{j,T}$ is log earnings, $X_{j,T} > 0$, for firm j in the cross-section at time T .

^a 991 observations. Sample restrictions: $X_{j,1996}, X_{1995} > 0, B_{j,1995}, B_{j,1994} > 0$.

^b 1023 observations. Sample restrictions: $X_{j,1995}, X_{j,1994} > 0, B_{j,1994}, B_{j,1993} > 0$.

*, **, ***: Significant at the 10, 5 and 1 percent levels.

TABLE 4. RATES OF RETURN, REGRESSED ON LOG SIZE: SIMULATION STUDY**Panel 4.A. Effects of Alternative Sub-Samples**

$$R_{jt} = a + g S_{jt-1} + z_{jt}$$

Sample	Slope	Std. Error	t-Statistic	Prob.	R ²
Whole	-0.016406	0.003409	-4.812320	0.0000	0.022679
Smaller 500	0.000971	0.007964	0.121879	0.9030	0.000030
Larger 500	-0.003759	0.007692	-0.488689	0.6253	0.000479
Largest 250	-0.006195	0.013363	-0.463631	0.6433	0.000866
Middle 500	-0.041741	0.013716	-3.043186	0.0025	0.018257
Smallest 250	-0.010431	0.012798	-0.815052	0.4158	0.002672

Panel 4.B. Comparative Statics Effects of Changes in Parameters

DGP: $R_{1t} = \alpha + \alpha_{1996}$
 $D_{jt} = \{1, 0\}$ as $S_{1jt-1} < S_{1mt-1}$ or $S_{1jt-1} \geq S_{1mt-1}$,
 $S_{1mt-1} = \text{median of } S_{1mt-1}$
 $\alpha = 0.00$
 $u_{1jt} \sim N(0, 1)$,
 $S_{1jt-1} \sim 10^{1/2} * N(0, 1)$,

$R_{2jt} = 2 * R_{1jt}$
 $S_{2jt-1} = 2 * S_{1jt-1}$

	Slope	Std. Error	t-Statistic	Prob.	R ²
R_2, S_2	-0.016406	0.003409	-4.812320	0.0000	0.022679
R_1, S_2	-0.008317	0.003394	-2.450865	0.0144	0.005983
R_2, S_1	-0.032584	0.003475	-9.378051	0.0000	0.080987

Notes: The simulated data for R_{1jt} and S_{1jt-1} are the same used in Figure 1. Data are generated for 1,000 firms. The smaller-half firms' R_{1jt} are generated as $0.20 + \text{IIN}(0, 1)$, the larger-half firms' as $-0.20 + \text{IIN}(0, 1)$. The R_{1jt} , S_{1jt} were saved and used to construct the R_{2jt} , S_{2jt} by multiplying each value by 2.

FIGURE 1. RATES OF RETURN AND LOG SIZE

Notes: 1,000 rates of return are generated. For the 500 smallest firms, the DGP is $0.20 + N(0, 1)$; for the 500 largest firms, the DGP is $-0.20 + N(0, 1)$. For each firm, its initial log size is zero. Each firm has existed for twenty periods, and its growth rate for each period is iid $N(0, 1)$. Log firm size is normalized by subtracting the median value.