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Faces of Swedish research in mathematics education

Christer Bergsten
Linköpings universitet

Introduction
The recent appearance of international research reviews (Grouws, 1992; Bishop et al, 1996) as well as national (e.g. Arzarello & Bartolini Bussi, 1998; Blum et al, 1992; Douady & Mercier, 1992; Kieran & Dawson, 1992) in the field of mathematics education is a significant mark of the state of the art, showing that the field has become so vast and complex that it has become difficult to overlook. This is also underlined by the fact that its state as a scientific discipline has been the focus of a recent ICMI study (Sierpinska & Kilpatrick, 1998). Swedish research studies were reviewed already in the general overview by Werdelin (1973). More recently, information about the work of present Swedish researchers, based on answers to questionnaires, was given in Johansson (1991, 1994). A survey of recent Swedish research on mathematical disabilities is made in Sahlin (1997). See also Engström (1999).

This presentation is based on looking at Swedish research publications from the past, on the answers to questionnaires that were sent to Swedish researchers in the end of 1997 (reply from 30 persons) and to departments of teacher training, education, and mathematics in 1999 (reply from 10 departments), and comments from colleagues. By showing different faces of Swedish research in mathematics education the product set of their characteristics will display the varieties and potentials of the research efforts. After a look at history, a short review of research problems and methods, examples of research results, research communities, and future perspectives will be presented.

Beginning with a general outlook on the object of the research interest, i.e. the teaching and learning of mathematics in Sweden, it has been said that before the time of the "new math" there had been very little change in the way mathematics was taught at schools (Magne, 1986). The teacher showed the students, using explanations and visual tools (object lessons), how a mathematical technique worked (for example two digit multiplication), and the students worked, under the guidance and help from the teacher, through a number of tasks to practice this technique from a book of exercises, which was the only "textbook". This was called teaching by rules, and was considered the most efficient method for mathematics teaching in the upper grades of compulsory school. In the lower grades the heuristic method of learning¹ was considered to be more adapted to the level of thinking of young children (Bergsten, 1939). Both these methods

¹ The student should first get so simple tasks that he is able to solve them by himself, with his own methods, then by systematic training with more and more difficult tasks he will develop mathematical skills. The key word was learning by practice. (Bergsten, 1939)
were rooted in the strong Swedish tradition of elementary mathematics teaching to keep
the learning activities close to the reality of the young child, using concrete materials
and visual tools to make children develop their mathematical skills through their own
activities. (See e.g Wigforss, 1952) The ideas behind the new math were therefore
strongly criticised (Lindström, 1968) already before its curriculum came into play in 1969,
but nevertheless it put a stop to a promising development of mathematics teaching in
the country. Now, it did not take long before the poor results of the new programme,
mainly on computation skills, were highlighted in the PUMP project (Kilborn, 1979), and
the low Swedish results (in international comparison) from the second IEA study 1980
(Skolöverstyrelsen, 1983), started intense activities on a national level to continuing
education of mathematics teachers in compulsory school. The emergence of the
electronic calculator during this time "complicated" the situation, and the time since
then in mathematics education in Sweden has been, as I see it, "a search for identity".
The need for research in mathematics education has therefore only been increasing
since the new math changed the scene, even the curriculum from 1980 ("back to basics"),
and from 1994 (again with more emphasis on mathematical thinking and under-
standing) have tried to put mathematics education in Sweden on a positive
development.\footnote{2} The Swedish results in the international TIMSS study (Skolverket, 1996)
indicated that the efforts made had got a positive pay off.

\textbf{Face 1: History}

The development of Swedish research and developmental studies in mathematics
education could be termed as a move from content oriented and experience based
"guidance" to research based "didactics" (didaktik). The title of the last book from one
of the nationally most well known mathematics educators in Sweden during the 19th
century, Karl Petter Nordlund, is "Vägledning vid den första undervisningen i räkning"
("A guidance to early arithmetic teaching", my translation; Nordlund, 1910).

On the international level this move or shift parallels the paradigm shift in
mathematics education research around 1980, when positivistic models of hypothesis
testing gave way to methods more apt to the practitioner's perspective. To quote
Kilpatrick (1992, p. 31), "research in mathematics education was moving out of the
library and laboratory and into the classroom and school". This also explains the shift:
When scientific research put itself outside the (strong) guidance tradition it did not
have any effect on it, when moving inside, it did. It was also at this period when
research journals in mathematics education appeared\footnote{3} as well as research institutes\footnote{4}.

\footnote{2} See Wyndhamn (1997) for an analysis of the mathematics curriculum development in Sweden; see
\footnote{3} Journal for Research in Mathematics Education, Educational Studies in Mathematics, Zentralblatt
für Didaktik der Mathematik
\footnote{4} Shell Centre, Institut für Didaktik der Mathematik, Instituts de Recherche de l’Enseignement des
Mathématiques.
The book by Nordlund (1910) is a very detailed description of what lessons in elementary mathematics topics should look like, with extensive use of concrete material and student activity. It is interesting to note that the work by for example Bergsten (1939) and much later the textbook by Anderberg (1992) still belong to the same tradition. At the turn of the century (i.e. 1900) there was a strong movement to make mathematics teaching more ”åskådlig” (it is hard to find an appropriate English word for this Swedish word, which means making mathematics more clear or lucid, for example by using visual means for displaying mathematical meaning), by mathematics educators like Ehlin, Kruse and Setterberg, influenced by the Perry-movement in England (Wistedt et al, 1992). Nordlund had already used that principle for 40 years by then (his first mathematics textbook dates from 1867), possibly carrying on the tradition from Comenius’ work in Sweden 200 years earlier⁵.

Wigforss extended this tradition of ”åskådlighet” (lucidity) by the development of diagnostic testing materials of high quality that became widely used (e.g. Wigforss, 1946). The development of standardized tests on a national scale, by Wigforss, to support the marking system, was another early significant contribution (see Kilpatrick & Johansson, 1994). Other research studies during the period before the new math were few and individual products rather than long term results from mathematical education research groups or milieus, for example the ingenious early interview study by Jonsson (1919) and the powerful factor analytical studies by Werdelin (1958, 1961).

The shift in mathematics teaching with the new math initiated, by the problems it evoked, an increased interest in the nature of mathematical skills and knowledge, and also in teacher training. This is shown by the increased number of projects and studies in the field that appeared (e.g. Kilborn, 1979). Also in the US, there was an explosion in the number of articles that appeared in the field (Kilpatrick, 1992). Subject matter based didaktik was the focus of a conference in Marstrand (Marton, 1986), after which the term matematikdidaktik⁶ was beginning to be widely used. The first course at university level in Sweden by the name matematikdidaktik was organized by Wyndhamn and Unenge in Linköping in 1985 (10 credit points). During the 15 years that have followed more than 15 PhD works have been presented, the largest number within the phenomenographic approach, along with many other studies. Textbooks for teacher training in this new research based matematikdidaktik like Unenge et al (1994) or Bergsten et al (1997) look very different from those mentioned above in the ”guidance” tradition.

In some of the national research reviews that have been presented a ”national tradition” or ”trend” has been able to identify. In Germany, for example, there is the Stoffdidaktik tradition since the 1950:s (vom Hoffé, 1998), in Italy the two trends of

⁵ Nordlund’s introduction of the heuristic method for teaching, as an alternative to the mechanistic teaching methods ”by rules” that were common at that time, was inspired by his teacher Kjell Dahl in Uppsala (Bergsten, 1939).

⁶ English translation didactics of mathematics or mathematics education.
concept-based didactics and innovation in the classroom were identified (Arzarello & Bartolini Bussi), in the Netherlands there is the realistic approach (DeCorte & Verschaffel, 1986), in France there is the well known conception of didactic engineering (see the paper by Artigue in this volume). And so forth. Is it possible to find a typical "Swedish" tradition in mathematics education research, an identifiable trend that dominates, or has dominated, the national scene?

**Face 2: Research problems**

The range of problems that are studied in the field of mathematics education research can be listed along several dimensions – focus on different actors (students, teachers...), organisation of teaching (groupings, individualisation...), mental processes (reasoning, visualisation...), focus on topic areas (geometry, algebra...), and many more. Within each dimension the focus can be on a general level (how does understanding in geometry develop?) or on a more specific level (how do students in grade 9 understand the concept of similarity?). A look at 23 Swedish PhD works during the period 1919-1999 (of which 15 are from the last ten years) that could be classified as belonging to the field of mathematics education, showed an almost non-overlapping distribution of research problem areas: problem solving (Wyndhamn, 1993), arithmetic with school beginners (Neuman, 1987; Ahlberg, 1992; Ekeblad, 1996), structure of mathematical knowledge (Werdelin, 1958; Bergsten, 1990), computers in mathematics education (Hedrén, 1990; Dahland, 1998), long term development of mathematical ability (Pettersson, 1990), students ways of solving arithmetic tasks (Jonsson, 1919), the organisation of learning (Ekman, 1968; Dunkels, 1996), fractions and reflective thinking (Engström, 1997), effects of curriculum change (Håstad, 1978; Kristiansson, 1979; Hellström, 1985), individualisation (Larsson, 1973), mathematical modelling (Wikström, 1997), understanding graphs (Åberg-Bengtsson, 1998), teachers’ and students’ conceptions of mathematics/teaching and learning mathematics (Löthman, 1994, Sandahl, 1997), teachers’ different ways of handling content (Runesson, 1999), influence of social factors on mathematics achievement (Chen, 1996). Other studies focus, additional to the above mentioned areas, on gender, communication between students and between students and tasks, informal (“everyday”) knowledge, quality of children’s mathematical thinking, teacher students, mathematical disabilities, and undergraduate mathematics education.

From a quantitative point of view, a big proportion of the Swedish research has been made within a number of projects, sometimes on a large scale, often funded by the National Agency for Education. Examples of such projects are (in alphabetical order) ADM, ALM, ARK (including DIM and RIMM), BIM, DIS, DOS, GEM, GUMA, HÖJMA, MYT, PUMP, and "Matematik i en skola för alla" (Mathematics in a school for all), "Problemlösning som metafor och praktik" (Problem solving as metaphor and practice), and "Vardagskunskaper och skolmatematik" (Every day knowledge and school mathematics).7

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7 For explanations of acronyms and references please contact present author.
Problem areas in mathematics education that recently have attracted most interest and research attention in Sweden are (according to the questionnaire from 1997 as mentioned above) young children’s conceptions of mathematics and early number learning, teachers’ and teacher students’ conceptions of teaching and learning, assessment and evaluation of knowledge, technology in mathematics education, problem solving and communication in the classroom, mathematical disabilities, gender issues, and research approaches such as phenomenography and constructivism. Learning issues in undergraduate mathematics have recently come into focus in some studies. Topic oriented studies are very few, as well as theoretical studies of epistemological character, and there is no stoffdidaktik tradition in the German sense (exceptions are for example the work by Kilborn, 1979, and by Bergsten, 1990). Qualitative methods are dominating, in particular in combination with the phenomenographic approach, and more or less well structured methods of triangulation are often used to increase the validity of the studies.

**Face 3: Research methods and results**

Methods of research in mathematics education in Sweden have followed the international trend, i.e., from an early dominance of quantitative studies towards an increasing number of qualitative studies. Experimental designs, with experimental and control groups, using pre- and post-test techniques, have been used by for example in Werdelin (1968), Hedrén (1990) and Ahlberg (1992). Examples of studies using psychometric methods are Werdelin (1958, 1961), Bergsten (1990), Pettersson (1990), and Chen (1996). A number of national survey studies have been produced by the National School Board, for example on grades 5 and 9 in 1992 (Skolverket 1993a, 1993b). Some interesting longitudinal studies have been conducted, for example on early number conceptions (Neuman, 1987) and alternatives to standard algorithms (Hedrén, 2000). Today interview techniques are dominating the scene, but Jonsson’s early study (Jonsson, 1919) proves there is a long tradition in Sweden for qualitative methods. Modern techniques such as video recordings have been used in Löthman (1994) and Dunkels (1996).

Phenomenography (Marton, 1981) has a strong position in mathematics education in Sweden, as in the studies by Neuman (1987), Ahlberg (1992, 1997), Ekeblad (1996), and others. Piagetian constructivism has fewer exponents (e.g. Engström, 1997), and so has the Vygotskian perspective (Wyndham, 1993; the paper by Säljö in this volume). Studies on a more subject oriented theoretical level are also less frequent (e.g. Kilborn, 1979; Bergsten, 1990).

It is often said, by practitioners, that research is very interesting but not of so much use in the daily work in classroom reality. Now, research results are always of a theoretical nature, and even if they sometimes are on a very general level, they can nevertheless form a basis for designing curricula, as well as teaching or learning situations. For example, knowledge at different levels of the education system of the five examples of
major findings of research in mathematics education, given by Niss (1998), would be a
safe basis to avoid many mistakes in the design of teaching and learning activities.

It is not possible, in this limited format, to list the "Swedish results". Instead some
sharp results that seem to have obvious teaching implications will be shown, ordered
chronologically from 1919 to 2000.

Mental calculation methods
In a study by Jonsson (1919) a series of interviews on mental calculation methods was
conducted. His results are still relevant for the discussion today when the teaching of
formal calculation methods (‘standard’ algorithms for addition, multiplication and so on)
in elementary school is questioned, in favour of building on children’s own methods.
Jonsson found that despite the extensive training on only one formal method for doing
additions fourteen out of fifteen students interviewed in grade 2 used other methods
when they were free to choose. He also found that students chose the methods, in each
calculation, that needed as little thinking effort and time as possible.

Rules first or discovery learning?
The effects on learning of different organisations of learning situations have been much
studied in pedagogical research. Since mathematics is, among other things, a rule-using
discipline, it has been common to use two different methods of instruction, i.e. rule first
(given by teacher or book) – then practice, or let the students themselves discover the
rule from the material (discovery or heuristic learning). Studies on this problem were of
interest for example for the design of programmed teaching in the sixties. This was
studied for example in the BIM project. In an experimental study by Werdelin (1968) with
211 grade 6 students, one group (A) got the principle (law of distribution) before the
eamples, group B first some examples, then the principle, and more examples, and finally
group C only examples. To measure the effects of the different treatments one test was
given immediately after the experiment, one test two weeks later, and one test to measure
transfer effects. There was a significant difference at the immediate test to the advantage
of group A, a difference that however disappeared after two weeks. There was no
difference on the transfer test. The main conclusion (from this and other studies) was
that the advantages of the heuristic method become visible in a long term perspective,
when combined with other important more general aspects of learning, such as drawing
your own conclusions and make a synthesis of what you experience. One could
comment that this research provides scientific support to the ideas of Nordlund in the
19th century (cf above).

Doing mathematics without using understanding
In a study on the different steps involved in solving a complex mathematical task,
Ekenstam and Nilsson (1977) showed how students strongly depend on their familiarity
with certain standard patterns when solving mathematical tasks, rather than trying to
think what the problem was about, or consider the meaning of the mathematical
expressions as starting point for their reasoning. To construct their tests, Ekenstam and
Nilsson chose a ‘top task’, e.g., an equation like \( \frac{3x-2}{2} = \frac{x}{3} \) and broke it down to the small steps that were used in solving the task. This produced a series of tasks like \( 3(3x-2)=2x \), \( 9x-6=2x \), \( 7x-6=0 \) and \( 7x=6 \). To find out at which step the students had problems, each of these five tasks were included in the test items, along with some similar tasks that changed e.g., the \( x \) to a \( t \), the \( - \) to a \( + \), or the particular numbers involved. From 10 top tasks 130 items were thus constructed, distributed over 10 tests of mixed items. The tests were distributed to a sample of 2000 students beginning upper secondary school so that each item was solved by approximately 200 students. It was observed that the solution frequencies strongly depended on minor changes, from a mathematical point of view, of the task characteristics. As an example, by rotating the same right angled triangle, the solution frequency to an area calculation task changed from 39% to 64%, or (in a simplification task) inverting the expression \( \frac{(mv)^2}{mv^2} \) caused a change from 41% to 17%. Another remarkable observation was the big difference in difficulty between the simplification tasks \( \frac{a^2}{a} \) and \( \frac{a}{a^2} \), a difference that disappeared when visible coefficients were used: \( \frac{3a^2}{6a} \) and \( \frac{6a}{3a^2} \). The main conclusion from the study is that the mathematical skills of many students are based on the application of trained patterns rather than on understanding of what they are doing. A similar conclusion is made in recent interview studies by Lithner (1998, 1999) on undergraduate mathematics problem solving.

**The effect of using electronic calculators**

In Sweden there were some early studies on a broad basis investigating the effects of the use of the electronic calculator in elementary mathematics teaching. In a study during the years 1977-1983, called the RIMM project (Hedrén & Köhlin, 1983), 7 classes were studied during their years in the Swedish "middle grades" (grades 4-6). Using an experimental design with experiment and control groups, it was clearly shown that the experimental groups, where the calculator was consequently used, showed the same ability as the control groups to do arithmetic calculations, mentally as well as with pencil and paper, but showed significantly better results on number estimation, choosing correct operation, and to use relevant information when solving word problems. The explanation given for the results was the increased amount of training on problem solving that was possible when the calculator reduced the time needed for performing the necessary calculations. This idea also formed the basis of the longitudinal project ADM, investigating the use of computers and later graphic calculators in secondary school (Björk & Brolin, 1995).

**Connecting everyday knowledge to mathematics teaching**

There has been a long tradition in Sweden to relate children’s mathematical work in school to their every day world outside school, something that is also visible in Swedish
curriculum texts. However, this has caused a tension in school mathematics between every day mathematics (since ICME5 known as ethno-mathematics) and "academic math", i.e. mathematics as a scientific discipline. In a three years project "Everyday knowledge and school mathematics", the aspect of using everyday knowledge to learn mathematics was highlighted (Wistedt et al, 1992). The project was a co-work between a teacher, a pedagogue, and two mathematicians, the main material videotaped and audio-taped classroom activities, including group work, and discussions among teachers.

Using everyday knowledge in school mathematics means for students that their reference world (everyday experience) in some way must connect to the world of (school) mathematics. To do this connection the student has to create a reference domain that picks out those aspects from the reference world that come into play in the mathematics "game" in school. This reference domain is a kind of model world (the terms are from Schoenfeld, 1986). To use everyday knowledge for learning mathematics the reference domain is the crucial link between the existing intuitive knowledge of the student (his/her reference world) and the new knowledge (of mathematics) he/she is trying to construct. The different modes of thinking and usage of words in these worlds create conflicts in the learning process, and one main conclusion is that instead of being a process of induction from the reference world via the reference domain to the world of mathematics, learning mathematics by connecting everyday knowledge means that the student is using knowledge and experiences from two worlds – the everyday world and mathematics – when solving the problems. The results show that students use knowledge from both worlds, the reference world and mathematics, when constructing their reference domain.

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LEARNING TASK
  /
REFERENCE DOMAIN
  |
REFERENCE WORLD   MATHEMATICS
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This means that the learning paradox is coming into play, since it seems as the student needs an existing conception of mathematical abstractions to move from reference world to reference domain, a domain which is supposed to be the link to understanding the mathematical abstractions. To use some kinds of manipulatives for learning, for example, presupposes that the manipulatives be interpreted in a way that presupposes knowledge of the mathematics they are supposed to learn by the material. To come out of this paradox, and to learn mathematics by connecting to everyday knowledge, a dialectic view of learning in a cultural perspective is needed. Connecting everyday knowledge can work as an instrument in a mutual communication between a personal world of
experience and a cultural tradition such as mathematical thinking. The main result from the study is the opening of a way to bridge the gap between everyday knowledge and mathematics, by showing the way students construct reference domains that build both from contexts of practice and of theory.

**Young children’s mathematical thinking (pre-school and first school years)**

This problem area has attracted several Swedish researchers, with the most extensive studies by Ann Ahlberg and Dagmar Neuman. An increased interest in pre-school mathematical experience can be noticed the last years, with a number of recent Swedish publications.

As an example, Ahlberg’s study "Children’s ways of handling and experiencing numbers" (1997) is showing the complexity of early number experience. The study is part of the project "Numerosity and the development of arithmetic skills among visual impaired children, hearing impaired children and children without these impairments". It is an interview study within the phenomenographic framework, where the interview is treated as a conversation with a structure and a purpose. 38 children from 3 different pre-schools (average age 6.7 years) were interviewed on 3 different kinds of tasks, *every day problems, decomposition problems* (cf Neuman, 1987), and *contextual problems* (cf Ekeblad, 1996). Children were not allowed to use any manipulatives. Interview outcomes were classified into a number of main categories, in line with the phenomenographic approach, and were analysed under the main headings *Ways of handling numbers* and *Ways of experiencing numbers*. One of the main results is that there was not a one-to-one correspondence between the way children handle numbers and the way they experience them, as shown in the matrix on next page.

Five categories of handling numbers were identified, of which *Saying numbers* and *Counting* were the most frequent. The four identified categories of experiencing numbers all come into play, more or less, when children handle numbers by counting, structuring, and using known facts. This pairing of different dimensions deepens the picture of the complexity of early numerosity development, and Ahlberg concludes:

> When trying to grasp numerosity children handle numbers in a vast array of ways, and thereby experience different aspects of numbers. However, in spite of using different ways of handling numbers, the numbers may appear in the same way to them and they may experience the same meaning. Consequently, there is not only one pathway, but many pathways to numbers.”

/.../

Understanding numbers and learning arithmetic skills is not only a question of the quantification of objects or fingers. Neither is it a matter of learning how to count on the number sequence or developing logical thinking. It is instead, a question of being able to explore and discern different aspects and possible qualities of the numbers - of experiencing numbers in the sense of sensuously and simultaneously perceiving different aspects of numbers. (Ahlberg 1997, p. 109)
<table>
<thead>
<tr>
<th>Ways of Handling Numbers</th>
<th>Number Words</th>
<th>Extents</th>
<th>Positions in Sequence</th>
<th>Composite Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAYING NUMBERS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random Number Words</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Successive Numbers</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>ESTIMATING</td>
<td>•</td>
<td>•</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COUNTING</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double Counting</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Counting and Tapping</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Counting and Looking</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Counting and Listening</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Finger Counting</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using Fingers; Counting All</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Using Fingers; Touching</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Using Fingers; Looking</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>STRUCTURING</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Seeing</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Using Derived Facts</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>USING KNOWN FACTS</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Table from Ahlberg, 1997, p 85

The results have clear implications for teaching.

**Problem solving**

Jan Wyndhamn, together with Roger Säljö, has done extensive work within the socio-cultural and situated cognition framework. Wyndhamn, with Riesbeck and Scholtz, has recently finished a project called "Problem solving as metaphor and practice", where problem solving activities in classrooms and teachers’ views on problem solving were scrutinized using qualitative data techniques (Wyndhamn, Riesbeck & Scholtz, 2000). Some of the results indicate that problem solving in practice most often reduces to solving word problems in class, and that group work activities become just another version of ordinary work with mathematical tasks. No transfer effects were found between everyday mathematics and academic math (cf Wistedt et al, 1992). Problem solving in school mathematics seems to reduce itself to a metaphor for a practice, related more to the organization of teaching than to mathematical content.
One critical question is the dissemination of research results. It is not always easy to pick up a "result" from a study and ask practitioners (e.g. teachers) to use it. It depends, among other things, strongly on the origin of the research question and the level of generality of the result. Bishop (1998) argues that researchers should become more aware of the fact that practitioners are the only actors for change:

"The research site should be the practitioners’ work situation, and the language, epistemologies, and theories of practitioners should help to shape the research questions, goals and approaches." (p. 43).

Many of the persons in Sweden doing research and developmental studies in mathematics education have a background as teachers, keeping their research close to the teaching and learning practice. Maybe this is the "Swedish tradition" in mathematics education research. The question of information and a common discourse still remains, however. The only Nordic research journal in mathematics education, Nomad (Nordic Studies in Mathematics Education) is still young and has not yet succeeded to reach a broader audience among practitioners. The journal Nämaren for teachers of mathematics has been the most important source in this respect in Sweden for the last 25 years (though not a research journal), as well as the mathematics teacher congress Matematikbiennalen (every second year since 1980) and its regional follow-up meetings, and meetings arranged by mathematics teacher associations. In some recent publications Swedish research has been made available for a broader audience, primarily for use in teacher training (e.g. Ahlberg, 1995; Gran, 1998; Neuman, 1989). Hopefully, the present conference will contribute to an increased awareness and interest also among practitioners what research can offer.

**Face 4: Research communities**

Before the "shift" after the new math most Swedish research in mathematics education took place at departments of education. At the end of the seventies some PhD programmes also involved departments of mathematics, but it is not until recently that departments of mathematics have begun to create research milieus in matematikdidaktik, as in Umeå. The first PhD thesis with a mathematics education content at a department of mathematics in Sweden was Dunkels (1996) in Luleå. Other sites in Sweden for research activities in the field of mathematics education are the national testing institutions in Stockholm (PRIM) and in Umeå (Nationella provgruppen). These institutions do test constructions and research on assessment in mathematics, providing long term descriptions of mathematical skills and attitudes of Swedish school children. In fact, results of such measurements often produce the strongest direct influence on practice. As an example, the studies by Lindblad (1978), followed up by Ljung (1987), caused a change of the prerequisites for entering teacher training colleges, and the second IEA study started huge efforts on a national level to educate Swedish teachers of mathematics (see Utbildningsdepartementet, 1986)
There are also networks and organisations outside universities that play an important role for research in mathematics education. The network *Women and mathematics* under the leadership of Barbro Grevholm has organised a number of international conferences in Sweden, with proceedings of research papers, one of which was the ICI study conference on gender issues in mathematics education (Grevholm & Hanna, 1995; Hanna, 1996). The present conference was organised by the new and independent *Swedish society for research in mathematics education* (SMDF).

**Face 5: Looking ahead**

With this history behind, what might the future of research in math education look like in a small country like Sweden? In fact, some opposite trends can be identified at present. On the national level teacher education seems to move towards establishing a more generalized educator profile, with less emphasis on teaching subject matter towards a teacher as an administrator and supervisor of learning. On the local level, at departments of mathematics and didactics, new research milieus for mathematics education are being established, and teacher training programmes include courses of a scientifically oriented *matematikdidaktik*. Again, on the government level, resources have been given for researching and educating teachers of mathematics in subject matter and *didaktik*.

The increased interest among teachers as researchers (as mentioned above), and the increased emphasis of a research based teacher training, are important backups for the future development of Swedish *matematikdidaktik*. For this we need people that can inspire the way Andrejs Dunkels did, and Gudrun Malmer is doing. I also believe that the expansion of research in *matematikdidaktik* into the departments of mathematics will be a necessary and important factor for a promising development of the range and quality, and in the search for an identity, of Swedish research in mathematics education. Today, measured in number of publications, Swedish research in mathematics education is hardly visible on the international scene in the increasing stream of articles and books. No doubt a change is on the way. Using a metaphor one could say that research in mathematics education in Sweden is a kettle of water heated up so that it, maybe, soon will start boiling.

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*See the web page of SMDF at www.mai.liu.se/SMDF/*
References


Plenary talks


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