# Modelling and Identification of a RUAV 

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Master of Science Thesis in Electrical Engineering Modelling and Identification of a RUAV

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#### Abstract

Modelling a linear mathematical model of a radio controlled (RC) helicopter in hover is the main goal of this thesis. The thesis introduces a general description about how RC-helicopters work and different phenomenons that effect the behaviour of a RC-helicopter. These phenomenons play an important role in the modelling part.

The model equations of the RC-helicopter are computed by deriving mathematical descriptions of different helicopter characteristics. The flapping motion of the main rotor and the flybar are modelled since they play major role in describing helicopter dynamics. The model is linearised by using stability and control derivatives and a model structure is presented. The method describes how the external forces and moments in the rigid body equations of motion can be expressed as continuous functions of the model states and inputs. The model is divided into multiple sub-models that describe the different dynamics of the RUAV. The parameters of the model are estimated using system identification methods. The prediction error method proved itself successful and the achieved models can accurately estimate the pitch, roll and yaw rate of the helicopter. These models could be used for further development of control designs.


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## Notation

## Nomenclature

| Name | Description |
| :---: | :---: |
| $a$ | Tilting angle of the TPP in the longitudinal direction |
| $A_{1}$ | Blade pitch when the blade is places along $x$-axis |
| $A_{b}$ | Cross coupling term of the main rotor |
| $b$ | Tilting angle of the TPP in the lateral direction |
| $B_{1}$ | Blade pitch when the blade is places along $y$-axis |
| $B_{a}$ | Cross coupling term of the main rotor |
| c | Tilting angle of the flybar in the longitudinal direction |
| $c_{l \alpha}$ | The aerofoil's lift curve slope |
| $d$ | Tilting angle of the flybar in the lateral direction |
| $F$ | Vector of the external forces acting on the fuselage |
| $F_{\text {aero }}$ | The aerodynamic lift force acting on the blade |
| $F_{\text {cent }}$ | The centrifugal force acting on the blade |
| $F_{\text {inertia }}$ | The inertia force acting on the blade |
| $h_{m}^{\text {b }}$ | Vector representing the position of the main rotor from the centre of gravity of the RUAV |
| I | Inertia matrix |
| $I_{b}$ | Moment inertia of the blade |
| $J$ | Orientation matrix |
| $k_{\beta}$ | Spring constant used to describe the attachment between the blade and the rotor shaft |
| $K_{\beta}$ | The compound stiffness constant |
| $L_{b}$ | Roll pitch derivative |
| $m$ | Total mass of the helicopter |
| $m_{b}$ | Blade mass per unit length |
| M | Vector of the external moments acting on the fuselage |
| $M_{a}$ | Pitch pitch derivative |
| $p$ | Roll rate |
| $q$ | Pitch rate |
| $r$ | Yaw rate |
| $r_{f b}$ | Yaw rate gyro feedback |
| $R_{b}^{g}$ | Rotation matrix from B-frame to G-frame |
| $T_{m}$ | Thrust vector |
| $u$ | Linear velocity along $x$-axis |
| $\boldsymbol{u}$ | The input vector |
| $\boldsymbol{u}_{\text {servo }}$ | The adjusted input vector |

Nomenclature

| Name | Description |
| :---: | :---: |
| $U$ | Air velocity |
| $U_{\infty}$ | Freestream velocity |
| $U_{T}$ | Tangential component of the air velocity |
| $U_{P}$ | Perpendicular component of the air velocity |
| $v$ | Linear velocity along $y$-axis |
| $v_{i}$ | Rotor inflow velocity |
| $w$ | Linear velocity along $z$-axis |
| $x$ | The state vector |
| $\boldsymbol{x}_{\text {servo }}$ | The state vector |
| $X_{a}$ | Longitudinal force derivative |
| $y_{b}$ | A section of the rotor blade |
| $Y_{b}$ | Lateral force derivative |
| $Y_{t r}$ | Tail rotor thrust |
| $\alpha_{D}$ | The angle between the hub and the air velocity vector |
| $\alpha$ | Blade aerodynamic angle of attack |
| $\beta$ | Flapping angle of the main rotor |
| $\beta_{f l y}$ | Flapping angle of the flybar |
| $\gamma$ | Lock number of the main rotor |
| $\gamma_{s}$ | Lock number of the flybar |
| $\delta_{l a t}$ | RC transmitter for lateral movements |
| $\delta_{l o n}$ | RC transmitter for longitudinal movements |
| $\delta_{\text {ped }}$ | RC transmitter for yaw movements |
| $\delta_{\text {col }}$ | RC transmitter for heave movements |
| $\bar{\delta}_{l a t}$ | The adjusted lateral input |
| $\bar{\delta}_{\text {lon }}$ | The adjusted longitudinal input |
| $\delta_{1}$ | S.Bus signal sent to servomotor 1 |
| $\delta_{2}$ | S.Bus signal sent to servomotor 2 |
| $\delta_{4}$ | S.Bus signal sent to servomotor 4 |
| $\delta_{6}$ | S.Bus signal sent to servomotor 6 |
| $\theta$ | Pitch angle |
| $\Theta$ | Pitch angle of the main blade |
| $\Theta_{0}$ | Average pitch angle of the main blade |
| $\tau_{f}$ | Rotor time constant |
| $\tau_{s}$ | Flybar time constant |
| $\tau_{M}^{b}$ | Torque vector created by the forces produced from the rotor thrust |
| $\tau_{\beta}^{b}$ | Vector of the torsional torques |
| $\phi$ | Roll angle |
| $\Phi$ | Aerodynamic inflow angle |
| $\psi$ | Yaw angle |
| $\Psi$ | Azimuth angle |
| $\rho$ | Air velocity |
| $\Omega$ | Rotation velocity of the main rotor |


| Abbreviations |  |
| :---: | :--- |
| Abbreviation | Description |
| ARX | Auto-regressive exogenous |
| ARMAX | Auto-regressive-moving-average model with exoge- |
|  | nous inputs model |
| ANN | Artificial neural network |
| BEC | Battery eliminator circuit |
| ECEF | Earth-centred, earth-fixed |
| ENU | East north up |
| ESC | Electronic speed control |
| IMU | Inertial measuring unit |
| LGF | Local geodetic frame |
| LTP | Local tangent plane |
| MSD | Mass-spring-damper |
| MIMO | Multiple-input multiple-output |
| NED | North east down |
| PEM | Prediction error methods |
| PWM | Pulse-width modulation |
| RC | Radio control |
| RUAV | Rotor unmanned aerial vehicle |
| SISO | Single-input single-output |
| TPP | Tip path plane |
| WGS | World geodetic system |

## 1

## Introduction

This master's thesis was carried out in collaboration with UAS Europe AB. This chapter introduces the main goal of the thesis together with background and the limitations. It then showcases some related work and finally presents the structure of the rest of the thesis.

### 1.1 Background

Recently, there has been an increased interest in Rotor Unmanned Aerial Vehicles (RUAV). Due to their capability of hovering, vertical takeoff and vertical landing, the usage of rotorcrafts has become attractive in different applications, both in civilian and military areas, e.g., search and rescue, surveillance and remote inspection.

The absence of a pilot on-board the aircraft has several advantages, for example the reduction of the aircraft size and the costs [10]. Rotorcrafts have a few disadvantages in comparison to fixed wing aircraft, e.g., they are slower and less fuel efficient. However, besides the advantages mentioned above, they present other attributes that make them valuable when operating in urban or other cluttered environments, e.g., slow speed cruise and manoeuvrability in smaller spaces.

It is important to achieve accurate positioning with RUAVs to guarantee that the desired missions can be accomplished safely and successfully. To ensure good performance while keeping a desired position or following a reference trajectory sent by the operator, well designed control structures are required [16]. Designing a good controller requires the development of a good mathematical dynamic model of the RUAV, that can be used to simulate its behaviour under different operating conditions. This can be a challenging task due to the nonlinearity and instability of the RUAVs dynamics.

### 1.2 Problem Formulation

The goal of this master's thesis is to present a method to derive a model of a RUAV in hover conditions. The method will then be applied to a rotorcraft and the results will be analysed. When designing a controller for a system, for both time and economic reasons, it is easier to work with a model in order to test the system in a simulation environment rather than test the system in real life. In this thesis, the following problems will be addressed:

- What model structure is able to deliver a good model of a hovering RUAV?
- What identification approaches and methods are suitable for the chosen model structure?
- Is the presented method suitable to model a hovering RUAV?


### 1.3 Limitations

The model will be linearised for a hovering RUAV and therefore it will not represent the RUAV if it is used outside of hover conditions.

The systems that are used to measure the input and output of the RUAV has some limitations themselves. The sample rate of these systems are a little bit slower and all the needed outputs are not measurable which implies that another system is used to measure the output of the RUAV. It is a motion capture system at Linköping University which needs to be booked and prepared in advance. In addition, the RUAV is quite complicated to control manually, and therefore a pilot is needed in order to fly the helicopter when doing tests. These two limitations strongly restrict the freedom of flying.

### 1.4 Related Work

As a result of the increased popularity of RUAVs, several papers dealing with problems similar to the ones introduced in the thesis have been encountered. However, this field of research is still young and therefore it still has a lot of room for improvement.

System identification is common when deriving a model for a RUAV. In [15, 23], the authors present approaches to derive a linear parameterised model. They also explain how the parameters can be obtained when working with frequencydomain data. Other articles, $[3,8,10]$, use the same model structure but estimate the parameters using different methods.

Some literature, such as $[2,11]$, present first principle approaches when modelling the RUAV. However these are for full size helicopter and are usually missing a flybar which is a component specific to model helicopters. In [6, 9], models where the flybar dynamics are described in detail are presented.

Detailed descriptions of a helicopter's components and motions can be found in [11, 20]. Many of the properties of a helicopter also apply to a RUAV. Some
basic knowledge of rigid body motion can be found in [17]. The approaches of system identification used during the project are based on [12, 14].

### 1.5 Thesis Outline

The thesis is divided into six different chapters which are organised as follows:

- Chapter 2 explains some basic theory regarding helicopters and presents the hardware used on the modelled RUAV.
- Chapter 3 describes the modelling of the RUAV. A linearised model structure around hover mode is presented.
- Chapter 4 explains the different approaches and methods that can be used in system identification.
- Chapter 5 presents the estimation of the model's parameters.
- Chapter 6 concludes the thesis and discusses the results. Future work is also proposed.


### 1.6 Divided Work

The project group consists of two members who worked together. At the beginning of the project, both members focused on getting a good understanding of the system. Afterwards, one of the members focused on deriving a model structure of the RUAV in hover mode. The other member concentrated on the system identification part of the project and creating scripts for the estimation and the validation of the model. The model was divided into several sub-models, see Chapter 5, and each member was responsible for some the sub-models. Finally, the different parts of the report were divided in a similar structure as described above.


## System Overview

In this chapter, basic terminologies and components of helicopters are defined. The chapter begins with a description of the system's input signals followed by the system's movements. Then some general components of helicopters are described and some phenomenons affecting the helicopter are explained. These are general for all radio controlled (RC) helicopters. Finally, the hardware for the helicopter used in this thesis is presented. Figure 2.1 shows the main components of the RC helicopter studied in this thesis.


Figure 2.1: A picture of the RC helicopter studied in this project.

### 2.1 Inputs

The helicopter is controlled with the help of four inputs sent from the pilot to the RC transmitter: $\delta_{l a t}$ for lateral movements, $\delta_{l o n}$ for longitudinal movements, $\delta_{p e d}$ for yaw movements and $\delta_{\text {col }}$ for heave movements, see Figure 2.2. The RC


Figure 2.2: Corresponding motion to each input.
transmitter mixes these inputs to generate signals to the necessary servomotors that correspond to the user inputs. These signals are used during later stages of system identification and they are denoted:

$$
\left[\begin{array}{llll}
\delta_{1} & \delta_{2} & \delta_{4} & \delta_{6}
\end{array}\right]
$$

Their number corresponds to which channel the signal is received and which servo they control, see Figure A. 1 in Appendix A. The relationship between the two types of inputs is described in Section 3.11. The speed of the rotor blade is constant when hovering and is therefore not regarded as an input. The input signal $\delta_{4}$ affects the tilting of the tail rotor blades while the three other signals, $\delta_{1}, \delta_{2}$ and $\delta_{6}$, control the three servomotors that affect the attitude of the swashplate. A swashplate is a device which controls the pitch of the rotor blades, see Figure 2.3. It can be divided into two parts: a stationary and a rotating part. The stationary part can be tilted in any direction and moved vertically along the rotor shaft by the servomotors, while the rotating part rotates with the blades and the rotor shaft and stays parallel to the stationary plate. The swashplate mechanism looks different in presence of a flybar, as will be presented in Section 2.3.

(a) A $3 D$ representation of the swashplate.

(b) A $2 D$ representation of the swashplate.

Figure 2.3: A simplified representation of the swashplate.

### 2.2 Basic Movements

### 2.2.1 Heave

For a helicopter to be able to fly, the lift force has to be greater than the force of gravity. Heave is made with the help of the rotor blades, which can change pitch angles together. As can be seen in Figure 2.4, the rotor blades are tilted together in the same direction by moving the swashplate vertically along the rotor shaft. This is called collective pitch [2]. The collective pitch introduces a vertical force along the rotor shaft. If the helicopter is tilted, the collective pitch will also generate a horizontal force, therefore, when in motion, the collective pitch has to be increased to maintain altitude.


Figure 2.4: An example of collective pitch. The rotor blades are represented twice with a $180^{\circ}$ phase difference with respect to the $z$-axis.

### 2.2.2 Roll and Pitch

A roll or a pitch motion makes the helicopter move in the horizontal plane and adjusts its attitude. By tilting the helicopter, the thrust generated by the rotor blades is not only generating lift, but some of the forces are translated horizontally. A tilting motion of the aircraft is generated by a change of the tip path plane (TPP), i.e., the plane that can be traced out from the blade tips. When the swashplate is at an angle, as can be seen in Figure 2.5, different lifts will be generated on different sides of the helicopter. The movement, often called cyclic [2], will move the TPP and therefore generate a roll or pitch movement.


Figure 2.5: An example of a cyclic movement. The rotor blades are represented twice with a $180^{\circ}$ phase difference with respect to the $z$-axis.

### 2.2.3 Yaw

The yaw of the rotorcraft is controlled by the tail rotor: by tilting the tail rotor blades, different yaw motions can be achieved. The tail rotor blades are also powered through the main motor but with a different ratio than the main rotor blades. The yawing moment is very sensitive and today almost all RC helicopters are equipped with a yaw rate feedback controller to improve yaw damping, which consists of a feedback controller and a gyro sensor [4].

### 2.3 Flybar

The flybar is a component mostly used on RC helicopters and it plays a major role in increasing the stability of the rotorcraft. There are mainly three different types of flybars.

According to [9], the first type of flybar system was the Bell system. It introduced a bar with weights at each end and it was connected to a mixing bar that took input from the flybar and the swashplate, and outputs to the main rotor blade, similarly to the mixing bar in Figure 2.6. When a change in pitch or roll was given by the pilot or disturbances tried to tilt the TPP, the flybar would adjust the cyclic pitch in order to counter the tilt. After a few seconds, the flybar


Figure 2.6: Structure of the flybar system as shown in [9]. Fixed joints are represented by a " " and ball joints are represented by a "O". The input to the main rotor blade is controlled by both the flybar and from the swashplate through the mixing bar.
would follow the movement and the effect would not persist. The Bell system augmented the stability but decreased the responsiveness of the system.

The second type of flybar system was a Hiller control system. The weights at the end of the bar were replaced by small aerofoil paddles. The cyclic pitch was only controlled by the flybar. When the pitch angle of the paddles changed after an input from the swashplate, the flybar would tilt to not be parallel to the TPP. The main rotor blades would receive a cyclic pitch and would try to follow the flybar's plane. This solution is often found on fixed pitch helicopters that do not have a collective pitch and can only control the heave by changing the rotor speed.

The last type is a combination of the aforementioned and is the Bell-Hiller control system. Similar to the Hiller system, it consists of a bar with aerofoils at each end where the tilt motion of the flybar is separated from the TPP. It is able to provide cyclic pitch to the main rotor blades. As can be seen in Figure 2.6, the
main rotor blade takes inputs directly from the swashplate and indirectly from the flybar through a mixing bar. The flybar will always remain $90^{\circ}$ out of phase with the main rotor blade due to the geometry of the assembly. Also due to the geometry of the arms around the slider, present in Figure 2.6, the slider is only able to move during collective pitch and not during cyclic movements. This system provides stability without loosing too much responsiveness. Increasing the weight of the paddles will reduce the response speed and augment the stability, while increasing the length of the flybar will augment the response speed but also decrease the stability.

For the past years, RC helicopters often comes with a 3-axis gyro which stabilises the helicopter without the usage of any flybar. These types of helicopters are popular for acrobatic flying.

### 2.4 Translating Tendency

As discussed earlier, the tail rotor controls the yaw motion of the rotorcraft. Part of the torque created by the main rotor makes the helicopter fuselage rotate in the opposite direction. The tail rotor produces a thrust to the side, which leads to a torque that counters this motion. However, this also leads to a sideways force that makes the aircraft drift sideways in the same direction of the tail rotor's thrust, which is called translating tendency, see Figure 2.7. Letting the helicopter tilt slightly to the opposite side produces a force that counters the drift.


Figure 2.7: The tail rotor produces a counter thrust opposite to the torque. The thrust produced by the tail rotor makes the helicopter move laterally.

### 2.5 Gyroscopic Precession

When looking at a stand still helicopter, the rotor blades and the components around it seem to be rotated $90^{\circ}$ along the $z$-axis. However, during motion, the helicopter works as intended due to gyroscopic precession. When an outer force is applied to a rotating body, the result of the force is manifested $90^{\circ}$ later in the direction of rotation, see Figure 2.8. The geometry and mechanisms of helicopters are usually designed with this in mind and therefore when the swashplate is tilted forward, it results in a forward motion [18].


Figure 2.8: Gyroscopic precession

### 2.6 Hardware

The helicopter studied during the thesis is an Align T-REX 600E PRO purchased and assembled in 2013. However, it never took off due to a lack of time and interest. A thorough verification of its assembly was made and some new components were purchased. The helicopter gets its input through a Futuba R617FS 7-Channel Receiver that sends the signals to the servo motors. The helicopter's motor is powered by two Gens Ace Li-Po 3700 mAh 6 s 60 c batteries in series going through an Electronic Speed Control (ESC). The receiver and the servos are powered by a Nano-Tech Li-Po 2000mAh 2s 20c battery going through a Battery Eliminator Circuit (BEC) to lower the voltage at around 6 V .

The RC transmitter sending the inputs is a Futaba 6EX-2.4GHz 6-channel, FASST Radio control system for Airplanes/Helicopters. A sketch of the transmitter with the inputs can be seen in Figure A.2, in Appendix A.

Previously, the helicopter could not record any data, either its inputs or outputs and therefore an autopilot was installed. Due to lack of space, it was mounted at the back of the helicopter. If more space is available, the autopilot should be mounted as close as possible to the centre of gravity of the aircraft. The installed hardware is an EasyPilot 3.0, which consists of autopilot hardware with embedded software and Vehicle Specific Module ground-side software. The autopilot hardware consists of an IMU, which is able to acquire sensor data from 3-axis accelerometers, 3-axis gyroscope and 3-axis magnetometer, a GPS and multiple
pressure sensors. A Futaba R6203SB 2.4GHz FASST Micro S.Bus HV Receiver is connected to the autopilot in order to intercept the signals sent from the RC transmitter to be able to log them. The logged signals are S.bus signals which are unitless. The EasyPilot 3.0 is a product made and developed by UAS Europe $A B$ and used on their fixed wing UAVs. The autopilot is equipped with a MicroSD Card module where the data is logged. The EasyPilot 3.0 is powered by a Bormatec Li-Po $2200 \mathrm{mAh} 3 s 25$ c. A schematic diagram of the system with the different components can be seen in Figure 2.9.

Helicopter


Figure 2.9: An overview of the system and its components. The solid lines represent channels from where the components are powered and the dotted lines represent signal channels.

Unfortunately, all the needed outputs can not be measured by the autopilot and therefore, due to lack of time and knowledge, a motion capture system was used in order to collect the output data needed for system identification. The plan was to use the capture motion system Qualisys available in Visionen Arena at Linköping University. By placing markers on the helicopter, the Qualisys system would be able to track the position and the attitude of the body, from which the needed values for the system identification can be derived. More information about the collection of data can be found in Section 4.1.

## 3

## Modelling

The main goal of this chapter is to present a method to derive a model for the helicopter dynamics. For this purpose, basic understanding of the rotor, fuselage and flybar dynamics is needed, and therefore introduced. Finally, the linearised model used to describe the RUAV behaviour in hovering is presented.

A description of the different coordinate systems that can be used for describing the rigid body equations, is first introduced. After a short introduction of the rigid body equations of motion, the rotor dynamics are modelled. The method used to linearise the system is also presented in this chapter. The fuselage dynamics are described and a short description of coupling the fuselage dynamics with the rotor dynamics is presented. The flybar dynamics and how they are coupled to the rotor dynamics are also demonstrated in this chapter. Afterwards, descriptions of the yaw and heave dynamics are presented. The chapter ends with a short summary of all the equations used in the linear model.

### 3.1 Coordinate System

The main aim of this section is to describe the theory about relevant coordinate systems that are used. The motion of the body of RUAV in the three dimensional space is explained by using two coordinate systems. One is a global coordinate system while the other one is local. The global coordinate system is the inertial frame that is fixed to the Earth, and is called $G$-frame (global frame) in this thesis. The local coordinate system is fixed to the body of the RUAV. The origin of this coordinate system is located to the centre of mass of the RUAV and its rotation follows the rotation of the RUAV's body. This frame is called B-frame (body frame). There are different ways to describe the G-frame, e.g., Earth Centred, Earth Fixed (ECEF) or Local Geodetic Frame (LGF). The orientation of the B-frame to the Gframe can also be described by different methods, for example Euler Angles or


Figure 3.1: An illustration of EFEC coordinate frame.
unit quaternions.

### 3.1.1 Coordinate Frames

## Earth Centred, Earth Fixed (ECEF)

The ECEF is a Cartesian coordinate system that has its origin at the Earth's centre of mass and rotates with the Earth around its spin axis. It can also be considered as an inertial frame, because the origin is fixed to the Earth's centre of mass and is not able to accelerate. The $z$-axis points towards the North Pole, along the Earth's rotation axis, the $x$-axis is parallel with the prime meridian (Greenwich meridian) of the Earth and the $y$-axis is orthogonal to the first mentioned axes to make the system right handed [3], see Figure 3.1.

A position in the Earth's atmosphere is often described using the World Geodetic System (WGS). WGS is an Earth-centred, Earth-fixed terrestrial reference system that uses a set of model parameters and constants that describe the shape, the size, the gravity and the geometric fields of the Earth. The origin of the frame is located to the Earth's centre of mass. Geographical coordinates of a point near the surface of the Earth are defined by the terms longitude $(\lambda)$, latitude $(\phi)$, and height ( $h$ ). The longitude angle $(\lambda)$ is the angle between the position of the point and the Greenwich meridian. The latitude angle $(\phi)$ represents the angle between the position of the point and the equatorial plane. The height $(h)$ is the distance between the position of the point and the surface of the Earth.


Figure 3.2: An illustration of The East North Up (ENU) frame. Two of the axes build a tangential plane to the earth surface, while the third one is orthogonal to this plane.

## Local Geodetic Frame

A Local Geodetic Frame (LGF), also known as Local Tangent Plane (LTP) or navigation frame, is a coordinate frame that is used to represent the attitude and the velocities of vehicles that are on or near the surface of the Earth.

A LGF is a coordinate frame where two of the axes build a tangential plane to the Earth surface, while the third axis is orthogonal to this plane. The origin is located near the surface of the Earth, which is the centre of gravity of the RUAV in this case, see Figure 3.2. This fixed point and the tangential plane do not rotate with the RUAV. LGF is defined by two configurations, the East North Up (ENU) frame and the North East Down (NED) frame. Both are right hand systems [3].

The $x$-axis is aligned with the Earth's rotation axis, while the $y$-axis points to the east, if the NED frame is used. If the ENU frame is used the axes will change the direction with each other. The third axis is orthogonal to the plane created by the first two axes, to make the coordinate system right handed, and is parallel to the direction of the Earth's gravitational field. The direction of the third axis depends on which one of the aforementioned configurations is used [3].

## Body Frame

The body frame or the body coordinate system (B-frame) is a coordinate frame fixed to the aircraft, with its origin in the centre of gravity of the aircraft. The origin and the axes of this frame rotate with the aircraft [3]. The directions of the
axes in B-frame are described below
$x$-axis The x -axis is pointing along the aircraft towards the nose.
$y$-axis The $y$-axis is pointing to the right of the aircraft.
$z$-axis The z-axis is pointing downwards through the bottom of the aircraft.

### 3.1.2 Attitude

For both autonomous and piloted flights, estimating the current orientation of the aircraft is important. The attitude of the RUAV is expressed as the orientation of the B-frame with respect to a local NED-frame. Euler angles and quaternions are the methods that are widely used to describe the orientation of the RUAV and convert the translational and rotational movements between two coordinate systems. These two methods are explained below.
Euler angles Euler angles describe the rotation of the rigid body with respect to a fixed frame through the composition of three successive rotations around the three axes. Euler angles are widely used to describe the attitude of the aircraft, but the rotation matrix, used to express the orientation of body coordinate system with respect to the fixed frame, becomes singular when the pitch angle $(\theta)$ or the roll angle $(\phi)$ reaches $90^{\circ}[22,25]$.

Quaternions Quaternions are used to explain the rotation between two coordinate frames by using four elements, offering a singularity free parameterisation. Using quaternions, the rotation between frames is performed as a single rotation around an imaginary vector. This method has no singularities and only four parameters [1].
As mentioned above, if the Euler angles method is used the rotation matrix will become singular when the pitch $(\theta)$ or roll angle $(\phi)$ reaches $90^{\circ}$. This does not occur when using quaternions. Since it is not expected for the RUAV to reach this phase, Euler angles are used in this thesis. Therefore, the attitude of the RUAV is expressed in Euler angles roll $(\phi)$, pitch $(\theta)$ and yaw $(\psi)$, whose description is presented below:
Roll ( $\phi$ ) The roll angle describes the rotation of the B-frame around the $x$-axis. It is defined so that the roll gets positive values when the RUAV turns to the right.

Pitch $(\theta)$ The pitch angle describes the rotation of the B-frame around the $y$ axis. It is defined so that the pitch gets positive values when the nose of the RUAV is moved upwards.

Yaw $(\psi)$ The yaw angle describes the rotation of the B-frame around the $z$-axis. It is defined so that the yaw gets positive values when the RUAV turns to the right in aerial viewpoint.
The rotation of the RUAV is defined as the rotation of the B-frame with respect to the NED-frame. The rotations are often expressed as roll rate ( $p$ ), pitch rate ( $q$ ) and yaw rate $(r)[15,22]$.


Figure 3.3: Illustration of transformation of Euler angles from B-frame to $G$-frame.

## Transformation matrices

The rotation of the RUAV, using Euler angles, is described as a product of three basic rotation matrices. The first one describes the rotation around the $x$-axis and is defined as

$$
C_{\phi}=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{3.1}\\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{array}\right] .
$$

The second one describes the rotation around the $y$-axis,

$$
\boldsymbol{C}_{\boldsymbol{\theta}}=\left[\begin{array}{ccc}
\cos \theta & 0 & \sin \theta  \tag{3.2}\\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{array}\right]
$$

The last one describes the rotation around the z -axis,

$$
\boldsymbol{C}_{\psi}=\left[\begin{array}{ccc}
\cos \psi & -\sin \psi & 0  \tag{3.3}\\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right] .
$$

The product of these three matrices, eqs. (3.1)-(3.3), gives the rotation matrix, which performs all the three rotations mentioned above, and describes the rotation of a vector from one coordinate system to another [9]. Therefore, the rotation from the B-frame to G-frame can be expressed as

$$
\begin{equation*}
R_{b}^{g}=C_{\psi} C_{\theta} C_{\phi} \tag{3.4}
\end{equation*}
$$

The rotation matrix has the following appearance

$$
\boldsymbol{R}_{\boldsymbol{b}}^{g}=\left[\begin{array}{ccc}
c \theta c \psi & s \phi s \theta c \psi-c \phi s \psi & s \phi s \psi+c \phi s \theta c \psi  \tag{3.5}\\
c \theta s \psi & c \phi c \psi+s \phi s \theta s \psi & c \phi s \theta s \psi-s \phi c \psi \\
-s \theta & s \phi c \theta & c \phi c \theta
\end{array}\right]
$$

with $c(\cdot)$ and $s(\cdot)$ shorthand notations for $\cos (\cdot)$ and $\sin (\cdot)$, respectively. Figure 3.3 illustrates the transformation of a vector between two coordinate systems using Euler angles. Therefore, a vector expressed in the B-frame, $\boldsymbol{a}^{\boldsymbol{b}}$, can be wrapped to G -frame by the equation below

$$
\begin{equation*}
a^{g}=R_{b}^{g} \cdot a^{b} \tag{3.6}
\end{equation*}
$$

The rotation matrix is orthogonal, i.e., $\left(\boldsymbol{R}_{b}^{g}\right)^{-1}=\left(\boldsymbol{R}_{b}^{g}\right)^{T}$ [22]; this also means that a vector expressed in G-frame, $\boldsymbol{a}^{g}$, can be transformed back to the B-frame through

$$
\begin{equation*}
\boldsymbol{a}^{\boldsymbol{b}}=\left(\boldsymbol{R}_{\boldsymbol{b}}^{\boldsymbol{g}}\right)^{T} \cdot \boldsymbol{a}^{\boldsymbol{g}} \tag{3.7}
\end{equation*}
$$

The rotation matrix is used to transform the position and linear velocities of a point expressed in B-frame to G-frame and vice versa. The relationship between the rotation of the RUAV expressed in B-frame, $\left[\begin{array}{lll}p & q & r\end{array}\right]^{T}$, and the angular change of the orientation angles, Euler angles, can be defined in the following way

$$
\left[\begin{array}{c}
\dot{\phi}  \tag{3.8}\\
\dot{\theta} \\
\dot{\psi}
\end{array}\right]=J\left[\begin{array}{l}
p \\
q \\
r
\end{array}\right],
$$

where $\boldsymbol{J}$ defined by

$$
J=\left[\begin{array}{ccc}
1 & s \phi t \theta & c \phi t \theta  \tag{3.9}\\
0 & c \phi & -s \phi \\
0 & \frac{s \phi}{c \theta} & \frac{c \phi}{c \theta}
\end{array}\right]
$$

and $t$ is an abbreviation of $\tan (\cdot)$ [9, 15, 22]. An illustration of the coordinate axes, rotation angles, linear and angular velocities is presented in Figure 3.4.


Figure 3.4: An illustration of the coordinate axes $x, y, z$; the body linear velocities $u, v, w$; the Euler angles $\phi, \theta, \psi$; and the body angular velocities $p, q, r$.

### 3.2 Rigid Body Equations

The rigid body equations are the starting point for the development of the model of the RUAV behaviours. The rotorcraft has six degrees of freedom and the rigid body dynamics of the fuselage can be represented by the following Newton-Euler equations of motion,

$$
\begin{align*}
& m \frac{d \boldsymbol{v}}{d t}=\boldsymbol{F}  \tag{3.10}\\
& \boldsymbol{I} \frac{d \boldsymbol{\omega}}{d t}=\boldsymbol{M} \tag{3.11}
\end{align*}
$$

where $\boldsymbol{F}=\left[\begin{array}{lll}X & Y & Z\end{array}\right]^{T}$ is the vector of external forces acting on the helicopter, $\boldsymbol{M}=\left[\begin{array}{lll}L M N\end{array}\right]^{T}$ is the vector of external moments acting on the helicopter, $\boldsymbol{v}=$ $\left[\begin{array}{lll}u & v & w\end{array}\right]^{T}$ represents the fuselage linear velocities, $\omega=\left[\begin{array}{lll}p & q & r\end{array}\right]^{T}$ represents the fuselage angular velocities, $m$ is the vehicle mass and $I$ is the inertial tensor. The inertia matrix $I$ can be represented by:

$$
\boldsymbol{I}=\left(\begin{array}{ccc}
I_{x x} & 0 & -I_{x z}  \tag{3.12}\\
0 & I_{y y} & 0 \\
-I_{x z} & 0 & I_{z z}
\end{array}\right)
$$

The components $I_{x y}$ and $I_{y z}$ are zero because of the symmetry of the helicopter. $I_{x z}$ is also neglected since it is much smaller than the other terms, according to [15, 19, 22].

The external forces are produced by the main rotor, the tail rotor, gravity and aerodynamic forces from the fuselage and the tail. With respect to the body coordinate system, the rigid body equation can be represented using the kinematic principles of moving coordinate frame of reference:

$$
\begin{align*}
& m \dot{\boldsymbol{v}}+m(\boldsymbol{\omega} \times \boldsymbol{v})=\boldsymbol{F}  \tag{3.13}\\
& \mathbf{I} \dot{\boldsymbol{\omega}}+(\boldsymbol{\omega} \times \boldsymbol{I} \boldsymbol{\omega})=\boldsymbol{M} \tag{3.14}
\end{align*}
$$

Eq. (3.13) gives the equations representing the translational motion,

$$
\begin{align*}
\dot{u} & =(-w q+v r)+\frac{X}{m}  \tag{3.15}\\
\dot{v} & =(-u r+w p)+\frac{Y}{m}  \tag{3.16}\\
\dot{w} & =(-v p+u q)+\frac{Z}{m} \tag{3.17}
\end{align*}
$$

while eq. (3.14) gives the equations representing the rotational motion of the
rotorcraft,

$$
\begin{align*}
& \dot{p}=\frac{-q r\left(I_{y y}-I_{z z}\right)+L}{I_{x x}}  \tag{3.18}\\
& \dot{q}=\frac{-p r\left(I_{z z}-I_{x x}\right)+M}{I_{y y}}  \tag{3.19}\\
& \dot{r}=\frac{-p q\left(I_{x x}-I_{y y}\right)+N}{I_{z z}} . \tag{3.20}
\end{align*}
$$

### 3.3 Gravitational Forces

Gravity is an inevitable force that is assumed to always act vertically. The gravity vector in the inertial reference frame can be described as:

$$
g^{g}=\left[\begin{array}{l}
0  \tag{3.21}\\
0 \\
g
\end{array}\right]
$$

With the help of eq. (3.5), gravity can be described in the body frame:

$$
\boldsymbol{g}^{\boldsymbol{b}}=\boldsymbol{R}_{b}^{g} \boldsymbol{g}^{g}=\left[\begin{array}{c}
-g \sin \theta  \tag{3.22}\\
g \cos \theta \sin \phi \\
g \cos \theta \cos \phi
\end{array}\right]
$$

### 3.4 Rotor Dynamics

To be able to compute the rotor dynamics, it is necessary to simplify the rotor equations of motion, which can be extremely complex. These simplifications help to express the rotor forces and moments as functions of the rotor states.

Generally, the rotor blades can flex and move under the effect of unsteady aerodynamic loads. These airloads are induced by the control inputs and the helicopter's motion itself. A description of the most important aspects of rotor dynamics and a summary of the tip path plane model are presented in the following sections.

### 3.4.1 Body Motion of the Rotor Blade

In general, the rotor blades of smaller helicopters have higher stiffness than the rotor blades of regular helicopters. Beside their rotation around the hub, the attachment point between the blades and the rotor shaft, the blades have three main movements: flapping, lead-lagging and feathering, see Figure 3.6.

Flapping is caused by the aerodynamics loads, for example lift force. This movement is defined as the deflection of the blades relative to the main rotor. The cyclic blade flapping is a primary source for helicopter manoeuvring, because it has indirect control of the direction of the rotor thrusts and torques. This makes it one of the most important mechanisms of the rotor behaviour. The flapping


Figure 3.5: An illustration of the blade flapping angle and Coriolis force, which causes lagging movement.


Figure 3.6: An illustration of the flapping, lag, and feathering motion of the rotor blade.
angle $\beta$ is described as the angle between the blade and the hub plane [22], see Figure 3.5.

Flapping causes a drag force-(Coriolis force-) build up, which affects the blade in the direction of the rotation plane, making the blades try to turn in the same direction as the drag force. The lead-lagging motion can be neglected for the rotors with two blades, specially for small helicopters like the one used in this thesis [22]. The blade can feather (rotate) around a third axis, causing change in the pitch angle of the blade. Feathering is affected by the command inputs sent from the pilot, i.e., the changes in the orientation of the swashplate. Feathering can be seen as a function of the collective pitch, longitudinal- and lateral cyclic pitch, and can also be affected by the elastic (torsional) deformations of the blades [11].

To estimate the position of the rotor blade, the flapping angle $\beta$ and the azimuth angle $\Psi$ can be used. The azimuth angle is defined as the angle between
the rotating blade and the $x$-axis in the B-frame, assuming the blades are rotating clockwise [15, 22]. The derivation of the azimuth angle gives the rotor speed $\Omega$.

### 3.4.2 The Mechanism of the Swashplate

The rotor speed is controlled and kept constant in most rotorcrafts. Changing the blade pitch angle leads to changes in the thrust and rotor moments. The swashplate mechanism has an important role in the control of the pitch angle of the blade. The main aim of this mechanism is to change the magnitude of the pitch angle of the blade and also express it as a function of the blade angular position around the hub, i.e., the azimuth angle $\Psi$. The blade pitch is described by

$$
\begin{equation*}
\Theta(\Psi)=\Theta_{0}-A_{1} \cos \Psi-B_{1} \sin \Psi \tag{3.23}
\end{equation*}
$$

where $\Theta_{0}$ is the average pitch angle of the blades, controlled by changing the collective control input $\delta_{c o l} . A_{1}$ is the amount of blade pitch the blade experiences when the blade is placed along the $x$-axis of the B-frame (above the tail), and $B_{1}$ is the amount of blade pitch the blade experiences when it is placed along the $y$-axis in the B-frame (on the right-hand side). These two terms are described as a function of the longitudinal and lateral cyclic inputs, $\delta_{l o n}$ and $\delta_{l a t}$. These functions contain linear coefficients that convert the input from the pilot stick into angular change of the pitch angle

$$
\begin{equation*}
A_{1}=B_{l a t} \delta_{l a t}, \quad B_{1}=A_{l o n} \delta_{l o n} \tag{3.24}
\end{equation*}
$$

$A_{l o n}$ and $B_{l a t}$ are linear coefficients that convert the input from the pilot stick, see Section 3.5.4.

### 3.4.3 The Aerodynamics of the Main Rotor

The air velocity at a blade element has an important role in determining the aerodynamic forces acting on the blades and therefore, its computation is of interest. The air velocity, $U$, is divided into two components, tangential $U_{T}$ and perpendicular $U_{P}$, which are used to represent and determine the total air velocity. These two terms, together with the angles that are used to determine the aerodynamics of the rotor, are presented in Figure 3.7, where the tangential $U_{T}$ and perpendicular $U_{P}$ components are illustrated with respect to the hub plane.

The main contribution affecting the air velocity at the blade is the main rotor rotation velocity $\Omega$. Other terms that can affect the air velocity are caused by the blade flapping, the rotor inflow, the rotational and translational motions of the RUAV. When the RUAV is flying in the air, the advancing blade experiences higher air velocity than the retreating blade. This, together with rotation of the main rotor, creates periodically variation of the air velocity at the blade [11, 15].

Figure 3.8 presents the main components used for the computation of the total air velocity at the rotor blade. Two of these components are used to represent the vehicle velocity relative the undisturbed air. The first one is the angle $\alpha_{D}$ between the air velocity vector and the hub plane, and the other one is a freestream


Figure 3.7: Part of blade aerofoil located at a section $y_{b}$. The figure also shows the blade velocity components, the angles that are used to determine the aerodynamic angles, the elemental lift force $d L$ and the drag force $d D$.


Figure 3.8: An illustration of the different parameters that affect the air velocity. This figure shows the angle $\alpha_{D}$ between the freestream velocity $U_{\infty}$ and the hub plane.
velocity $U_{\infty}$, assuming that the air-stream is parallel to to $x$-axis in the B-frame and there is no slide slip. The tangential velocity component of the air velocity at section $y_{b}$ at the rotating blade can be expressed as

$$
\begin{equation*}
U_{T}=\Omega y_{b}+\left(U_{\infty} \cos \alpha_{D}\right) \sin \Psi \tag{3.25}
\end{equation*}
$$

The perpendicular term of the air velocity is a function of the perpendicular component of the freestream velocity $U_{\infty}$, the flapping rate of the rotor blade $\dot{\beta}$, the rotations of the RUAV, $p$ and $q$, and finally the rotor inflow velocity $v_{i}$ [15]. The perpendicular component of the air velocity can be written as

$$
\begin{equation*}
U_{P}=U_{\infty} \sin \alpha_{D}+v_{i}-y_{b}(p \sin \Psi+q \cos \Psi)+y_{b} \dot{\beta} \tag{3.26}
\end{equation*}
$$

It follows that the total air velocity acting on the blade is

$$
\begin{equation*}
U=\sqrt{U_{T}^{2}+U_{p}^{2}} \tag{3.27}
\end{equation*}
$$

The angle of the aerofoil relative to the total air velocity vector is the blade aerodynamic angle of attack $\alpha$ and is defined as the angle between the blade pitch angle $\Theta$ and the aerodynamic inflow angle $\Phi[11,15]$

$$
\begin{equation*}
\alpha=\Theta-\Phi \tag{3.28}
\end{equation*}
$$

where $\Phi$ is a function of the perpendicular and tangential components of the air velocity

$$
\begin{equation*}
\Phi=\tan ^{-1}\left(\frac{U_{P}}{U_{T}}\right) \tag{3.29}
\end{equation*}
$$

The incremental lift $d L$ is a function of the air density $\rho$, the blade chord length $c$ and the aerofoil's lift curve slope $C_{l \alpha}$. This force is defined as the force produced by a blade element and acts perpendicularly to the total air velocity, see Figure 3.7.

$$
\begin{equation*}
d L=\frac{1}{2} \rho U^{2} c C_{l \alpha} \alpha d y_{b} \tag{3.30}
\end{equation*}
$$

The elemental blade $d D$ is defined as the force created by the blade and acts in line with the total air velocity $[11,15]$.

$$
\begin{equation*}
d D=\frac{1}{2} \rho U^{2} c C_{d \alpha} \tag{3.31}
\end{equation*}
$$

In conclusion, the lift and drag forces acting on a blade element can be expressed as

$$
\begin{gather*}
d F_{z}=d L \cos \Phi+d D \sin \Phi  \tag{3.32a}\\
d F_{x}=d L \sin \Phi+d D \cos \Phi \tag{3.32b}
\end{gather*}
$$

Assuming that the inflow angle is too small during hovering, one can get the following approximation

$$
\begin{equation*}
d F_{z} \approx d L, \quad d F_{x} \approx d D \tag{3.33}
\end{equation*}
$$

Therefore, integrating the elemental lift and drag forces along the blade total length gives the total lift and drag forces that affect one blade.

The aerodynamic forces acting on the rotating blades are periodic forces that make the blade experience periodic flapping motion, which creates a torque acting on the helicopter via the main rotor. More about flapping motion will be mentioned in future sections of this thesis.

The inflow velocity $v_{i}$ used in this section is steady and consistent. In reality, the rotor inflow is a function of the aerodynamic forces acting on the blades. The aerodynamics of the blade are a dynamic system, meaning that the inflow velocity changes with the vehicle's different motions [11, 15]. More about the airflow velocity and how it changes with the vehicle's motions can be found in [11, 22].

### 3.5 Simplified Rotor Equation of Motion

Before extending the rigid body model with the tip path plane (TPP) model, some simplifying assumptions are made regarding the TPP model.

### 3.5.1 Simplifying Assumptions

- Structural simplifications regarding the blades: the blades are rigid in bending, torsion and linear blade stress. The flapping angle $\beta$ is assumed to be small.
- Aerodynamics simplification: the inflow angle $\Phi$ is small, $U_{P} / U_{T} \ll 1$. Other terms, like the blade stall, the reversed rotor flow and blade tip losses, are all neglected [15].


Figure 3.9: Representation of the moment from the flapping spring and the forces acting on a blade element of length $d y_{b}$ and mass $m_{b} d y_{b}[15,19]$.

- The lead-lag motion caused by Coriolis forces is neglected, due to the fact that the forces produced by the lead-lag motion are much smaller than the forces caused by the flapping motion [15, 22].


### 3.5.2 Derivation of the Flapping Equations of Motion

The blade is represented as a rigid beam which is connected with the main rotor shaft by a flapping hinge. The flapping hinge and a linear torsional spring, with spring constant $k_{\beta}$, are used to represent the model that describes the attachment between the blade and the rotor shaft. This model is used in [15]. The rotor equations used to represent the blade flapping motion are computed by using the moment equilibrium about the flapping hinge. The flapping is assumed to happen at the point where the main rotor shaft and the hub plane intersect with each other. Figure 3.9 shows the torques and the forces acting on the rotor blade: the mass per unit length of the blade is $m_{b}$ and the forces acting on the blade are located at the radial distance $y_{b}$. This implies that the total mass of a specific section of the blade with radial distance $y_{b}$ is totally $m_{b} d y_{b}[15,19]$.

The forces acting on the rotor blade need to be identified in order to continue the analysis of the flapping motion. The first one is the periodic aerodynamic lift force $F_{\text {aero }}$, which is perpendicular to the blade and directed upwards. $F_{\text {aero }}$ is assumed to be equal to $F_{z}$, computed in eq. (3.33), according to [15]. The two remaining forces are the inertial forces, the inertia force $F_{\text {inertia }}$ and the centrifugal force, $F_{\text {cent }}$. The inertia force, $F_{\text {inertia }}$, is defined as the force acting perpendicular to the blade, but in the opposite direction of the periodic aerodynamic lift force and the flapping motion. The acceleration of the blade element created by the flapping motion is $\ddot{\beta} y_{b}$, thus the inertia force is expressed as

$$
\begin{equation*}
d F_{\text {inertia }}=m_{b} \ddot{\beta} y_{b} d y_{b} . \tag{3.34}
\end{equation*}
$$

The centrifugal force, $F_{\text {cent }}$, is created by the centripetal acceleration, parallel to
the hub plane and directed outwards. The full expression of this force is

$$
\begin{equation*}
d F_{\text {cent }}=m_{b} \Omega^{2} y_{b} \cos \beta d y_{b} . \tag{3.35}
\end{equation*}
$$

After deviating the forces acting on the blade, the moment equilibrium about the hinge, at point $O$, is

$$
\begin{equation*}
\int_{0}^{R} m_{b} \Omega^{2} y_{b}^{2} \cos \beta \sin \beta d y_{b}+\int_{0}^{R} m_{b} \ddot{\beta} y_{b}^{2} d y_{b}+k_{\beta} \beta=\int_{0}^{R} y_{b} d F_{\mathrm{aero}} d y_{b}, \tag{3.36}
\end{equation*}
$$

where $R$ is the total length of the blade. After assuming $\beta$ is small $(\sin \beta \approx$ $\beta, \cos \beta \approx 1$ ) and rearranging eq. (3.36), one gets

$$
\begin{equation*}
\left(\beta \Omega^{2}+\ddot{\beta}\right) \int_{0}^{R} m_{b} y_{b}^{2} d y_{b}+k_{\beta} \beta=\int_{0}^{R} y_{b} d F_{\mathrm{aero}} d y_{b} \tag{3.37}
\end{equation*}
$$

where the first integral in the equation above represents the moment inertia of the blade

$$
\begin{equation*}
I_{b}=\int_{0}^{R} m_{b} y_{b}^{2} d y_{b} \tag{3.38}
\end{equation*}
$$

Differentiating the flapping angle with respect to the position of the rotating blade, i.e., the azimuth angle $\Psi$ gives (3.37) another form. This transformation will be used to derive a model of the flapping angles, more details will be explained in the next two sections. The derivatives of the flapping angle with respect to the azimuth angle will be marked with the symbol ('). From the previous sections, it is known that the rotation velocity of the rotor blade $\Omega$ is the derivative (over time) of the azimuth angle, which yields $\Psi=\Omega t$. The derivatives of the flapping angles with respect to the azimuth angle become

$$
\begin{gather*}
\dot{\beta}=\frac{\delta \beta}{\delta \Psi} \frac{\delta \Psi}{\delta t}=\Omega \beta^{\prime}  \tag{3.39}\\
\ddot{\beta}=\frac{\delta \dot{\beta}}{\delta \Psi} \frac{\delta \Psi}{\delta t}=\Omega^{2} \beta^{\prime \prime} \tag{3.40}
\end{gather*}
$$

Using (3.38), (3.39) and (3.40) in (3.37) yields

$$
I_{b} \Omega^{2}\left(\beta^{\prime \prime}+\beta\right)+k_{\beta} \beta=\int_{0}^{R} y_{b} d F_{\mathrm{aero}} d y_{b}
$$

which is equivalent to

$$
\begin{equation*}
\beta^{\prime \prime}+\left(1+\frac{k_{\beta}}{I_{b} \Omega^{2}}\right) \beta=\frac{1}{I_{b} \Omega^{2}} \int_{0}^{R} y_{b} d F_{\mathrm{aero}} d y_{b} \tag{3.41}
\end{equation*}
$$



Figure 3.10: A description of eq. (3.44) where the first harmonic $\beta_{1 c}$ (or $\beta_{1 s}$ ) represents the longitudinal (or lateral) tilting of the tip path plane.

The dynamics in (3.41) are similar to the differential equation of a mass-springdamper (MSD) system ( $m \ddot{x}+c \dot{x}+k x=F$ ). Generally, the natural frequency of a MSD-system is $\omega_{n}=\sqrt{k / m}$ and is independent of the damping coefficient. Observing eq. (3.41) and comparing it with a MSD-system gives the natural flapping frequency $[15,19]$,

$$
\begin{equation*}
\lambda_{\beta}^{2}=1+\frac{k_{\beta}}{I_{b} \Omega^{2}} \tag{3.42}
\end{equation*}
$$

### 3.5.3 Tip Path Plane (TPP) Equation of the Main Rotor

Previous sections show that the flapping angle $\beta$ is a function of the azimuth angle $\Psi$. Thus, the flapping angle is a $2 \pi$-periodic function and all the periodic angles can be expressed as a Fourier series,

$$
\begin{align*}
\beta(\Psi) & =\beta_{0}-\beta_{1 c} \cos \Psi-\beta_{1 s} \sin \Psi-\beta_{2 c} \cos 2 \Psi-\beta_{2 s} \sin 2 \Psi-\cdots \\
& =\beta_{0}+\sum_{k=1}^{n}\left(-\beta_{k c} \cos k \Psi-\beta_{k s} \sin k \Psi\right) \tag{3.43}
\end{align*}
$$

where $\beta_{0}, \beta_{k c}$ and $\beta_{k s}$ are the coefficients of the Fourier series, $k=1,2, \ldots, n$. Previous works, such as $[15,19,22]$, have shown that the first harmonic of the infinite series has much bigger effect than the other harmonics, for example the effect of the second harmonic is less than $10 \%$ of the first harmonics [15]. The first harmonics are, therefore, sufficient to give an approximation of the blade flapping behaviour, which is

$$
\begin{equation*}
\beta(\Psi) \approx \beta_{0}(t)-\beta_{1 c}(t) \cos \Psi-\beta_{1 s}(t) \sin \Psi \tag{3.44}
\end{equation*}
$$

This equation describes the motion of the rotor TPP, and this type of motion gives the rotor motion a shape similar to a cone, see Figure 3.10. The coefficient $\beta_{0}$ is called the coning angle, which is the angle between the blades and the hub plane when the blades are not tilting. The periodic coefficient $\beta_{1 c}$ illustrates the tilting of the TPP in the longitudinal direction, and $\beta_{1 s}$ represents the tilting of the TPP in the lateral direction. In the following sections, the notations $a$ and $b$ are used instead of $\beta_{1 c}$ and $\beta_{1 s}$, respectively.

Substituting eq. (3.44) in (3.41) and matching all the non periodic terms with $\beta_{0}$, all the sine terms with $b$, and all the cosine terms with $a$, provides the dynamic
equations of the TPP $[15,19,22]$. The vector $\boldsymbol{e}=\left[\beta_{0} a b\right]^{T}$ represents the tip path plane state vector. All these operations and eq. (3.23) provide a second order differential equation describing the tip path plane dynamics

$$
\begin{equation*}
\ddot{e}+D \dot{e}+K e=F \tag{3.45}
\end{equation*}
$$

where $\boldsymbol{D}$ represents the damping matrix, $\boldsymbol{K}$ stands for the stiffness matrix, and $\boldsymbol{F}$ describes the forces terms. A detailed analysis of eq. (3.45) is introduced in [5]. To provide a practical model of TPP that can be used to extend the rigid body model, eq. (3.45) has to be simplified. Some of the important simplifications made in $[15,19]$ are used and presented in the next section.

### 3.5.4 First Order Tip Path Plane Equations

For computing a simplified TPP model, assumption and simplifications made in [15] are used. This model can be used for system identification. One of these assumptions is that the coning angle $\beta_{0}$ is assumed to be constant and therefore, the dynamics related to the coning angle are neglected. Other assumptions are that the effects of the hinge offset, the blade pitch-flap coupling coefficient and the forward speed are disregarded.

The tip path plane model will be complex and hard to use for control design purposes if the aforementioned assumptions are not used. Following the simplifications made in [15] and [19] leads to the simplified equations of the flapping dynamics

$$
\begin{align*}
& \tau_{f} \dot{a}=-a-\tau_{f} q+\frac{p}{\Omega}+A_{b} b+A_{l o n} \delta_{l o n}  \tag{3.46}\\
& \tau_{f} \dot{b}=-b-\tau_{f} p+\frac{q}{\Omega}+B_{a} a+B_{l a t} \delta_{l a t} . \tag{3.47}
\end{align*}
$$

Eq. (3.46) and (3.47) are approximations of the TPP dynamics affected by control inputs and motions of the RUAV. $\tau_{f}$ is the rotor time constant, an important parameter in the TPP dynamics $[15,22]$, and is given by

$$
\begin{equation*}
\tau_{f}=\frac{16}{\gamma \Omega} . \tag{3.48}
\end{equation*}
$$

Eq. (3.48) shows that the rotor time constant is dependent on the rotor velocity $\Omega$ and the Lock number $\gamma$, which can be written as

$$
\begin{equation*}
\gamma=\frac{\rho c C_{l \alpha} R^{4}}{16} \tag{3.49}
\end{equation*}
$$

where $\rho$ is the air density, $c$ is the length of the blade chord, $C_{l \alpha}$ is the lift curve slope, and $R$ is the radius of the rotor.
$A_{b}$ and $B_{a}$ are the main rotor cross coupling terms, which can be expressed as

$$
\begin{equation*}
A_{b}=-B_{a}=\frac{8}{\gamma}\left(\lambda_{\beta}^{2}-1\right) . \tag{3.50}
\end{equation*}
$$

$A_{l a t}$ and $B_{l o n}$ are linear coefficients that convert the inputs from the pilot stick to the change of the pitch angle of the blades in the lateral and longitudinal directions, respectively.

### 3.6 Stability and Control Derivatives

The stability derivatives model is a linearised model of the equations of motion where the forces and torques are expressed as functions of the system states and inputs. The following sections give a presentation of the derivatives and the linearisation of the rigid body equations of motion. For more detailed information about this method, one can check works such as [15, 19, 22].

### 3.6.1 Linearisation of the Rigid Body Equations of Motion

Equations (3.15)-(3.20) describe the rigid body motion of the RUAV as a function of the external forces and torques that act on the fuselage. These forces and torques need to be expressed as functions of the states and control inputs of the system. In general, eqs. (3.15)-(3.20) can be expressed as a system of nonlinear differential equations

$$
\begin{equation*}
\dot{x}=f(x, u) \tag{3.51}
\end{equation*}
$$

where $\boldsymbol{x}$ represents the state vector of the RUAV

$$
\boldsymbol{x}=\left[\begin{array}{llllllllll}
u & v & w & \phi & \theta & p & q & r & a & b \tag{3.52}
\end{array}\right]^{T}
$$

and $\boldsymbol{u}$ is the vector that consists of the control inputs of the system

$$
\boldsymbol{u}=\left[\begin{array}{llll}
\delta_{\text {lat }} & \delta_{l o n} & \delta_{p e d} & \delta_{\text {col }} \tag{3.53}
\end{array}\right]^{T}
$$

In the nonlinear model of the system, every physical effect and component should be modelled. This leads to a detailed nonlinear model that needs more states than the ones used for the rigid body equations, which results in (3.51) being a high order system. These additional states are used to describe, for example, the main rotor and the effects created by the rotor inflow dynamics. The parametric linearised model can be computed from the linearisation of the rigid body equations of motion, where the high order dynamics are presented by highly simplified expressions [15, 22].

The nonlinear differential equations of motion can be linearised around an operating point $\boldsymbol{x}_{\mathbf{0}}, \boldsymbol{u}_{\mathbf{0}}$

$$
\begin{equation*}
\dot{x}=\frac{\partial f}{\partial x}\left(x_{0}, u_{0}\right) \delta x(t)+\frac{\partial f}{\partial u}\left(x_{0}, u_{0}\right) \delta \boldsymbol{u}(t), \tag{3.54}
\end{equation*}
$$

which leads to a linear system model

$$
\begin{equation*}
\delta \dot{x}=\boldsymbol{A} \delta \boldsymbol{x}+\boldsymbol{B} \delta \boldsymbol{u} \tag{3.55}
\end{equation*}
$$

in state space form with state and control matrix $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. $\delta \boldsymbol{x}$ and $\delta \boldsymbol{u}$ are the linear deviations of the system states and the control inputs, respectively. Thus, the symbol " $\delta$ " is used to indicate "deviation". Therefore, the states and the control inputs around a chosen operating point can be described by

$$
\begin{equation*}
\boldsymbol{x}=\boldsymbol{x}_{0}+\delta \boldsymbol{x} \quad \text { and } \quad \boldsymbol{u}=\boldsymbol{u}_{0}+\delta \boldsymbol{u} \tag{3.56}
\end{equation*}
$$

Fixing the values of the states to values that describe the chosen operating point and solving $f(\boldsymbol{x}, \boldsymbol{u})=0$ to compute the values of the remaining states and control inputs gives the trim parameters for the chosen operating point. Since the system will be linearised around the hover condition, all the linear and angular velocities are set to zero

$$
\begin{equation*}
v_{0}=\omega_{0}=0 \tag{3.57}
\end{equation*}
$$

The linearised form of the rigid body equations of motion is now

$$
\begin{align*}
& \delta \dot{u}=\underbrace{\left(-w_{0} \delta q+\delta w q_{0}+v_{0} \delta r+\delta v r_{0}\right)}_{=0}+\Delta X / m=\frac{\Delta X}{m}  \tag{3.58}\\
& \delta \dot{v}=\underbrace{\left(-u_{0} \delta r+\delta u r_{0}+w_{0} \delta p+\delta w p_{0}\right)}_{=0}+\Delta Y / m=\frac{\Delta Y}{m}  \tag{3.59}\\
& \delta \dot{w}=\underbrace{\left(-v_{0} \delta p+\delta v p_{0}+u_{0} \delta q+\delta u q_{0}\right)}_{=0}+\Delta Z / m=\frac{\Delta Z}{m}  \tag{3.60}\\
& \delta \dot{p}=\underbrace{\left(q_{0} \delta r-\delta q r_{0}\right)\left(I_{y y}-Y_{z z}\right) / I_{x x}}_{=0}+\Delta L / I_{x x}=\frac{\Delta L}{I_{x x}}  \tag{3.61}\\
& \delta \dot{q}=\underbrace{\left(p_{0} \delta r-\delta p r_{0}\right)\left(I_{z z}-Y_{x x}\right) / I_{y y}}_{=0}+\Delta M / I_{y y}=\frac{\Delta M}{I_{y y}}  \tag{3.62}\\
& \delta \dot{r}=\underbrace{\left(p_{0} \delta q-\delta p q_{0}\right)\left(I_{x x}-Y_{y y}\right) / I_{z z}}_{=0}+\Delta N / I_{z z}=\frac{\Delta N}{I_{z z}} \tag{3.63}
\end{align*}
$$

Equations (3.58)-(3.63) describe the response of the RUAV around the hover mode as a function of the deviations in the external forces and torques acting on the fuselage.

### 3.6.2 Extending the External Forces and Torques by Using the Derivatives

The external forces and torques can be expressed as continuous functions of system states and control inputs. Therefore, changes in these forces and torques can be described using only the first order terms of Taylor series expansion, since the dependency on the system states and input needs to be linear. For example, the lateral force component can be written as

$$
\begin{align*}
\frac{\Delta Y}{m} & =\frac{\partial Y}{\partial u} \delta u+\frac{\partial Y}{\partial v} \delta v+\frac{\partial Y}{\partial w} \delta w+\ldots+\frac{\partial Y}{\partial \delta_{l a t}} \delta \delta_{l a t}+\frac{\partial Y}{\partial \delta_{l o n}} \delta \delta_{l o n}+\ldots \\
& =\sum_{k=1}^{10} \frac{\partial Y}{\partial \boldsymbol{x}_{k}} \delta \boldsymbol{x}_{k}+\sum_{k=1}^{4} \frac{\partial Y}{\partial \boldsymbol{u}_{k}} \delta \boldsymbol{u}_{k} \tag{3.64}
\end{align*}
$$

The terms that describe the partial derivatives of the forces or torques with respect to the system states are called stability derivatives. For example, the derivative of the lateral force with respect to the lateral velocity can be written as:

$$
\begin{equation*}
\frac{\partial Y}{\partial v}=Y_{v} \tag{3.65}
\end{equation*}
$$

The terms that describe the partial derivatives of the forces or torques with respect to the control inputs of the system are called control derivatives. The control derivatives have similar expression as the stability derivative. For example, the derivative of the lateral force as a function of the longitudinal input can be expressed as

$$
\begin{equation*}
\frac{\partial Y}{\partial \delta_{l o n}}=Y_{\delta_{l o n}}, \tag{3.66}
\end{equation*}
$$

which simplifies eq. (3.64) to the following expression

$$
\begin{align*}
\frac{\Delta Y}{m} & =Y_{u} \delta u+Y_{v} \delta v+Y_{w} \delta w+\ldots+Y_{\delta_{l a t}} \delta \delta_{l a t}+Y_{\delta_{l o n}} \delta \delta_{l o n}+\ldots \\
& =\sum_{k=1}^{10} Y_{x_{k}} \delta \boldsymbol{x}_{k}+\sum_{k=1}^{4} Y_{u_{k}} \delta x_{k} \tag{3.67}
\end{align*}
$$

The same method is used to express the other force and torque components as functions of the system states and inputs. Another example is the pitch torque, which can be written as:

$$
\begin{align*}
\frac{\Delta M}{I_{y y}} & =M_{u} \delta u+M_{v} \delta v+M_{w} \delta w+\ldots+M_{\delta_{l a t}} \delta \delta_{l a t}+M_{\delta_{l o n}} \delta \delta_{l o n}+\ldots \\
& =\sum_{k=1}^{10} M_{x_{k}} \delta \boldsymbol{x}_{k}+\sum_{k=1}^{4} M_{u_{k}} \delta \boldsymbol{u}_{k} \tag{3.68}
\end{align*}
$$

To ease the notation in what follows, $\delta$ and $\Delta$ will be dropped from all the variables. For example, eq. (3.67) will be written as

$$
\begin{equation*}
\frac{Y}{m}=Y_{u} u+Y_{v} v+Y_{w} w+\ldots+Y_{\delta_{l a t}} \delta_{l a t}+Y_{\delta_{l o n}} \delta_{l o n}+\ldots \tag{3.69}
\end{equation*}
$$

Each force and torque component does not depend on all the states and the inputs of the system. The task in the following sections is to find which states and inputs are relevant to express each force and moment components.

### 3.7 Coupling the Dynamics of the Main Rotor and the Fuselage

To couple the TPP equations with the rigid body equations of motions, the forces and torques created by the rotor should be expressed as functions of the rotor
flapping. Simplified expressions of the forces and torques, using the TPP model, will be presented.

Figure 3.11 illustrates the relation between the thrust vector of the main rotor $T_{M}$, the forces and torques created by the rotor and the rotor TPP angle. The tilting motion of the thrust vector produces the rotor hub forces. It is assumed that the thrust vector acts perpendicularly to the TPP, which means that the pilot indirectly controls the rotor hub forces. These assumptions are used for flight with low speed or hover, and this means that the hub forces are the projections of the thrust vector on the $x$ - and $y$-axis of the $B$-frame $[7,15,19]$. The components of the thrust vector alongside the axis in the B-frame is

$$
\boldsymbol{T}_{\boldsymbol{M}}=\left[\begin{array}{l}
T_{x}  \tag{3.70}\\
T_{y} \\
T_{z}
\end{array}\right]=\left[\begin{array}{c}
-T \sin a \cos b \\
T \sin b \cos a \\
-T \cos a \cos b
\end{array}\right] \approx\left[\begin{array}{c}
-T a \\
T b \\
-T
\end{array}\right] .
$$

where the flapping angles are assumed to be small $(\sin \xi \approx \xi, \cos \xi \approx 1$ with $\xi \rightarrow 0$ ) and $T$ is the magnitude of the thrust produced by the main rotor.

The total torque produced by the main rotor and acting on the fuselage is a result of the forces produced by the rotor thrust and the rotor's stiffness moments. The vector describing the position of the main rotor from the centre of gravity, expressed in B-frame, is

$$
\boldsymbol{h}_{M}^{\boldsymbol{b}}=\left[\begin{array}{l}
x_{m}  \tag{3.71}\\
y_{m} \\
z_{m}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
h
\end{array}\right] .
$$

According to (3.71), the main rotor is assumed to be aligned with the centre of gravity. The torques created by the forces produced by the rotor thrust are

$$
\boldsymbol{\tau}_{\boldsymbol{M}}^{\boldsymbol{b}}=\boldsymbol{h}_{\boldsymbol{M}}^{\boldsymbol{b}} \times \boldsymbol{T}_{\boldsymbol{M}}=\left[\begin{array}{c}
L_{T}  \tag{3.72}\\
M_{T} \\
N_{T}
\end{array}\right]=\left[\begin{array}{c}
h T_{y} \\
h\left(-T_{x}\right) \\
0
\end{array}\right] \approx\left[\begin{array}{c}
h T b \\
h T a \\
0
\end{array}\right],
$$

where $L_{T}$ and $M_{T}$ are the roll and pitch torques produced by rotor thrust. The vector of the torsional torques created by the flapping of the tip path plane and the flapping spring $k_{\beta}$ is denoted $\tau_{\beta}^{b}$. The longitudinal and lateral torsional torques are expressed as functions of the TPP longitudinal and lateral flapping angles $a$ and $b$, respectively

$$
\boldsymbol{\tau}_{\boldsymbol{\beta}}^{\boldsymbol{b}}=\left[\begin{array}{c}
L_{K}  \tag{3.73}\\
M_{K} \\
N_{K}
\end{array}\right]=\left[\begin{array}{l}
b \\
a \\
0
\end{array}\right] k_{\beta}=\left[\begin{array}{c}
k_{\beta} b \\
k_{\beta} a \\
0
\end{array}\right] .
$$

Summing the torques in (3.72) and (3.73), gives the total lateral and longitudinal torques acting on the fuselage

$$
\begin{align*}
& L_{R}=h T b+k_{\beta} b=\left(h T+k_{\beta}\right) b  \tag{3.74}\\
& M_{R}=h T a+k_{\beta} a=\left(h T+k_{\beta}\right) a . \tag{3.75}
\end{align*}
$$

These equations give the compound stiffness constant

$$
\begin{equation*}
K_{\beta}=h T+k_{\beta} . \tag{3.76}
\end{equation*}
$$



Figure 3.11: Pitch torques as a result of the blade flapping motion. $M_{K}$ is the torque caused by the flapping spring $k_{\beta} . M_{T}$ is the torque produced from the tilting of the thrust vector $T_{m}$.

It follows that the compound stiffness constant $K_{\beta}$ depends on the stiffness of the rotor and the rotor thrust, which consequently means that the total lateral and longitudinal torques acting on the centre of gravity also depend on these two terms. The ratio between the stiffness of the rotor and the rotor thrust varies depending on the design of the rotorcraft $[4,15,19,22]$.

### 3.7.1 Deriving Rotor Force and Torque Derivatives

The forces and the torques of the rotor can be described using the stability derivatives. The mass of the RUAV is used to normalise the forces in the translational equations, and the moments of inertia normalise the respective torques in the rotational equations. In hover and low speed movements, the magnitude of the rotor thrust $T$ is approximated to $T=m g$, which gives the longitudinal and lateral forces derivatives

$$
\begin{align*}
X_{a} & =-\frac{T}{m}=-g \longleftrightarrow  \tag{3.77}\\
Y_{b} & =\frac{T}{m}=g . \tag{3.78}
\end{align*}
$$

The roll and pitch torques derivatives can be computed by using eq. (3.76)

$$
\begin{align*}
L_{b} & =\frac{K_{\beta}}{I_{x x}}=\frac{h T+k_{\beta}}{I_{x x}}  \tag{3.79}\\
M_{a} & =\frac{K_{\beta}}{I_{y y}}=\frac{h T+k_{\beta}}{I_{y y}} \tag{3.80}
\end{align*}
$$

These four derivatives are also called flapping derivatives [15, 22].

### 3.7.2 Connecting the Rotor and the Fuselage Equations of Motion

The flapping derivatives are used to couple the rigid body equations of motion with the rotor TPP equations. In the translational longitudinal and lateral equations of motion, the longitudinal and lateral forces derivatives ( $X_{a}$ and $Y_{b}$ ) are used to replace the control derivatives $X_{l o n}$ and $Y_{l a t}$, respectively,

$$
\begin{align*}
& \dot{u}=\frac{\Delta X}{m}=(-g) \theta+X_{u} u+\cdots+X_{a} a+X_{b} b  \tag{3.81}\\
& \dot{v}=\frac{\Delta Y}{m}=g \phi+Y_{v} v+\cdots+Y_{a} a+Y_{b} b  \tag{3.82}\\
& \dot{w}=\frac{\Delta Z}{m}=Z_{u} u+Z_{v} v+\cdots+Y_{c o l} \delta_{c o l} . \tag{3.83}
\end{align*}
$$

The pitch and roll torques derivatives, computed in eq. (3.80) and (3.79), are also used to replace the control derivatives $L_{l a t}$ and $M_{l o n}$ in the rigid body equations of motion

$$
\begin{align*}
& \dot{p}=\frac{\Delta L}{I_{x x}}=L_{u} u+L_{v} v+\cdots+L_{a} a+L_{b} b  \tag{3.84}\\
& \dot{q}=\frac{\Delta M}{I_{y y}}=M_{u} u+M_{v} v+\cdots+M_{a} a+M_{b} b  \tag{3.85}\\
& \dot{r}=\frac{\Delta N}{I_{z z}}=N_{r} r+\cdots+N_{p e d} \delta_{p e d} . \tag{3.86}
\end{align*}
$$

Figure 3.12 presents a block diagram showing how the rotor is coupled with the fuselage for the pitch dynamics.

Some of the main differences between the coupled rotor-fuselage equations and the rigid body equations are mentioned below:

- The forces and the torques are expressed by the flapping derivatives $X_{a}$, $Y_{b}$, and $M_{a}, L_{b}$, respectively. This means that the cyclic inputs $\delta_{l a t}$ and $\delta_{\text {lon }}$ enter directly the rotor dynamics, instead of the fuselage dynamics, see eqs. (3.46)-(3.47) $[15,22]$.
- The stability derivatives $L_{p}$ and $M_{q}$ work as damping parameters, which are neglected because the damping of both roll and pitch is explained by the rotor dynamics ( $\tau_{f} q$ and $\tau_{f} p$ in (3.46) and (3.47), respectively). For


Figure 3.12: A simple block-diagram of the pitch dynamics describing the coupling between the rigid body dynamics and the flapping motions
example, for the pitch damping, the flapping response of the angular speed in pitch $q$ in eq. (3.47) is $-\tau_{f} q$. It follows that the damping torque created by the pitch motion is $-M_{a} \tau_{f}[15,21]$.

- The speed derivatives $M_{u}$ and $L_{v}$ are not neglected in the roll and pitch equations of motions, because separating the rotor and fuselage aerodynamics speed effect is difficult to achieve.

Based on the assumptions above and previous works, such as [15, 19, 22], $\dot{u}, \dot{v}, \dot{p}$ and $\dot{q}$ can be derived

$$
\begin{align*}
\dot{u} & =X_{u} u-g \theta+X_{a} a  \tag{3.87}\\
\dot{v} & =Y_{v} u+g \phi+Y_{b} b  \tag{3.88}\\
\dot{p} & =L_{u} u+L_{v} v+L_{b} b  \tag{3.89}\\
\dot{q} & =M_{u} u+M_{v} v+M_{a} a, \tag{3.90}
\end{align*}
$$

where

$$
\begin{align*}
\dot{\phi} & =p  \tag{3.91}\\
\dot{\theta} & =q \tag{3.92}
\end{align*}
$$

### 3.8 Flybar Dynamics

### 3.8.1 The Flybar Equations of Motion

The flybar has a big effect on the response of the system and, therefore, needs to be modelled. The flybar can be considered as a secondary rotor, which also receives cyclic inputs from the swashplate. The collective pitch input does not affect the flybar. The reason is that the flybar is not designed to create thrust and, therefore, does not experience coning.

The flapping motion of the flybar paddles can be expressed similarly to the main blades flapping motion using the tip path plane eq. (3.44) in Section 3.5.3 [15]

$$
\begin{equation*}
\beta_{f l y}(\Psi)=-\beta_{f l y, 1 c} \cos \psi-\beta_{f l y, 1 s} \sin \psi \tag{3.93}
\end{equation*}
$$

where the states of the flapping motion of the flybar are defined in the same way as the states of the flapping of the main blades. $\beta_{f l y, 1 c}$ and $\beta_{f l y, 1 s}$ are the first harmonic coefficients that represent the tilting of the flybar paddles in the longitudinal and lateral directions, respectively. The notations $c$ and $d$ will be used instead of $\beta_{f l y, 1 s}$ and $\beta_{f l y, 1 c}$, respectively.

The dynamics used to describe the states of the flapping motion of the flybar are similar to the equations of the main blades flapping equations, eq. (3.46) and (3.47). The lock number of the flybar flapping $\gamma_{s}$ is usually smaller than the lock number of the main blades, see Section 3.5.4. Smaller lock number means that the flapping time constant $\tau_{s}$ is bigger, see eq. (3.48), and this also means that the coupling between longitudinal and lateral flapping will be reduced [4, 15, 22]. Based on simplifications and assumptions made in these works, simplified equations describing the flapping of the flybar are presented below

$$
\begin{align*}
\tau_{s} \dot{c} & =-c-\tau_{s} q+C_{l a t} \delta_{l a t}  \tag{3.94}\\
\tau_{s} \dot{d} & =-d-\tau_{s} p+D_{l o n} \delta_{l o n} \tag{3.95}
\end{align*}
$$

where, $C_{l a t}$ and $D_{l o n}$ are linear coefficients that convert the inputs from the pilot stick to the flybar's cyclic pitch in the lateral and longitudinal direction, respectively.

### 3.8.2 Coupling the Flybar and the Main Rotor Dynamics

Since the flybar does not have collective pitch dynamics, it does not create any thrust and the paddles of the flybar swing freely around the main rotor. The only part of the RUAV dynamics affected by the flybar dynamics is the cyclic pitch dynamics to the main blades, see Section 2.3. The flybar adjusts the cyclic pitch inputs by using the mixing bar and this component is proportional to the flapping angle of the flybar. The resulting adjusted cyclic inputs sent to the blades, $\bar{\delta}_{l o n}$ and $\bar{\delta}_{l a t}$, are a sum of the direct cyclic inputs and the effect from the flybar

$$
\begin{align*}
\bar{\delta}_{l o n} & =\delta_{l o n}+K_{c} c  \tag{3.96}\\
\bar{\delta}_{l a t} & =\delta_{l a t}+K_{d} d . \tag{3.97}
\end{align*}
$$

Using the adjusted cyclic inputs in eq. (3.46) and (3.47), which describe the flapping dynamics, gives the following results

$$
\begin{align*}
& \tau_{f} \dot{a}=-a-\tau_{f} q+A_{b} b+A_{l o n}\left(\delta_{l o n}+K_{c} c\right)+A_{l a t} \delta_{l a t}  \tag{3.98}\\
& \tau_{f} \dot{b}=-b-\tau_{f} p+B_{a} a+B_{l a t}\left(\delta_{l a t}+K_{d} d\right)+B_{l o n} \delta_{l o n} \tag{3.99}
\end{align*}
$$

which can be rewritten as

$$
\begin{align*}
& \tau_{f} \dot{a}=-a-\tau_{f} q+A_{b} b+A_{l o n} \delta_{l o n}+A_{c} c+A_{l a t} \delta_{l a t}  \tag{3.100}\\
& \tau_{f} \dot{b}=-b-\tau_{f} p+B_{a} a+B_{l a t} \delta_{l a t}+B_{d} d+B_{l o n} \delta_{l o n} \tag{3.101}
\end{align*}
$$

where $A_{c}=A_{l o n} K_{c}$ and $B_{d}=B_{l a t} K_{d}$. The off-axis flapping response terms $\frac{p}{\Omega}$ and $\frac{q}{\Omega}$ from eq. (3.46) and (3.47), respectively, are neglected to simplify the flapping
model $[9,15,19]$. Cross-coupling terms $A_{l a t} \delta_{l a t}$ and $B_{l o n} \delta_{l o n}$ are added in these equations, which are used to capture off-axis effects that are not modelled, if they exist.

### 3.9 Yaw Dynamics

As mentioned in Section 2.2.3, the yaw motion is mostly controlled by the tail rotor thrust. Most of the RC helicopters are equipped with an active yaw-damping system that must be accounted for. The system contains a yaw gyro and amplifier which have integral characteristics [24]. The block diagram of this system is presented in Figure 3.13. The system can be represented by the following equations:

$$
\begin{gather*}
Y_{t r}=\left(\delta_{p e d}-r_{f b}\right) K_{r}  \tag{3.102}\\
\dot{r}=Y_{t r} l_{t r}  \tag{3.103}\\
\dot{r}_{f b}=-K_{r f b} r_{f b}+K_{r a} r \tag{3.104}
\end{gather*}
$$

where $Y_{t r}$ is the tail rotor thrust, $\delta_{p e d}$ is the control input, $r_{f b}, K_{r}, K_{r f b}$ and $K_{r a}$ are the yaw rate amplifier parameters and $l_{t r}$ is the distance between the tail rotor and the centre of mass of the helicopter.

Due to the translating tendency, the yaw dynamics are also affected by the roll $p$ and the lateral $v$ dynamics. The yaw and heave dynamics are strongly coupled, due to the fact that the changes in the collective pitch creates a reaction in both the main rotor and the tail rotor because of the changing thrust. It follows that the yaw dynamics becomes affected by changed forces and torques created by the main rotor [22]. These conclusions, together with (3.102) and (3.103), give

$$
\begin{equation*}
\dot{r}=N_{r} r+N_{r f b} r_{f b}+N_{w} w+N_{v} v+N_{p} p+N_{p e d} \delta_{p e d}+N_{c o l} \delta_{c o l}, \tag{3.105}
\end{equation*}
$$

where $N_{r f b}=N_{p e d}=l_{t r} K_{r}$.


Figure 3.13: Yaw-damping system.

### 3.10 Heave Dynamics

The rigid body equation for the heave dynamics in hover mode can be expressed as

$$
\begin{equation*}
\dot{w}=\underbrace{\left(-v_{0} p+u_{0} q\right)}_{=0}+\frac{\Delta Z}{m}=Z_{w} w+Z_{c o l} \delta_{c o l} \tag{3.106}
\end{equation*}
$$

where the control derivative $Z_{c o l}$ describes the changes of the thrust created by the main rotor when the collective pitch of the blades changes. The speed derivative $Z_{w}$ represents terms such as fuselage drag and rotor damping. As mentioned in the previous section, the yaw and heave dynamics are heavily coupled. According to $[15,16]$, the tip path plane dynamics can be used to express the effect of the centrifugal forces on the heave dynamics. Adding these conclusions will expand eq. (3.106) to

$$
\begin{equation*}
\dot{w}=Z_{a} a+Z_{b} b+Z_{w} w+Z_{r} r+Z_{c o l} \delta_{c o l} \tag{3.107}
\end{equation*}
$$

### 3.11 Linearised Model

Now that all the dynamics of the RUAV have been modelled and simplified, they can be combined to build a complete parameterised linear model. This model will be used for the system identification of the RUAV dynamics in hover mode. The general form of the state-space model is

$$
\begin{equation*}
\dot{x}=A x+B u, \tag{3.108}
\end{equation*}
$$

where the state vector $x$ is

$$
\boldsymbol{x}=\left[\begin{array}{lllllllllllll}
u & v & p & q & \phi & \theta & a & b & w & r & r_{f b} & c & d
\end{array}\right]^{T} .
$$

States that describe the augmented dynamics ( $r_{f b}, c$ and $d$ ) are added at the end of the state vector. The input vector is

$$
\boldsymbol{u}=\left[\begin{array}{llll}
\delta_{\text {lat }} & \delta_{\text {lon }} & \delta_{\text {ped }} & \delta_{\text {col }}
\end{array}\right]^{T}
$$

A summary of the eleven states that describe the dynamics of the RUAV is presented below

- Longitudinal and lateral dynamics describing the coupled fuselage-main rotor-flybar dynamics
- Longitudinal and lateral motions of the fuselage ( $\dot{u}, \dot{v}$ ) (Eqs. (3.87) and (3.88))

$$
\begin{aligned}
\dot{u} & =X_{u} u-g \theta+X_{a} a \\
\dot{v} & =Y_{v} u+g \phi+Y_{b} b .
\end{aligned}
$$

- The roll and pitch motions of the fuselage ( $\dot{p}, \dot{q}$ ) (Eqs. (3.89) and (3.90))

$$
\begin{aligned}
& \dot{p}=L_{u} u+L_{v} v+L_{b} b \\
& \dot{q}=M_{u} u+M_{v} v+M_{a} a .
\end{aligned}
$$

- Longitudinal and lateral flapping dynamics of the main bar ( $\dot{a}, \dot{b})$ (Eqs. (3.100) and (3.101))

$$
\begin{aligned}
& \tau_{f} \dot{a}=-a-\tau_{f} q+A_{b} b+A_{c} c+A_{l o n} \delta_{l o n}+A_{l a t} \delta_{l a t} \\
& \tau_{f} \dot{b}=-b-\tau_{f} p+B_{a} a+B_{d} d+B_{l a t} \delta_{l a t}+B_{l o n} \delta_{l o n} .
\end{aligned}
$$

- The flybar longitudinal and lateral dynamics ( $\dot{c}, \dot{d}$ ) (Eqs.(3.94) and (3.95))

$$
\begin{aligned}
\tau_{s} \dot{c} & =-c-\tau_{s} q+C_{l a t} \delta_{l a t} \\
\tau_{s} \dot{d} & =-d-\tau_{s} p+D_{l o n} \delta_{l o n} .
\end{aligned}
$$

- Heave dynamics
- Vertical motion of the fuselage ( $\dot{w}$ ) (Eq. (3.107))

$$
\dot{w}=Z_{a} a+A_{b} b+Z_{w} w+Z_{r} r+Z_{c o l} .
$$

- Yaw dynamics
- Yaw motion of the fuselage and yaw rate gyro feedback $\left(\dot{r}_{,} \dot{r}_{f b}\right)$ (Eqs. (3.104) and (3.105))

$$
\begin{aligned}
\dot{r} & =N_{r} r+N_{r f b} r_{f b}+N_{w} w+N_{v} v+N_{p} p+N_{p e d} \delta_{p e d}+N_{c o l} \delta_{c o l} \\
\dot{r}_{f b} & =K_{r} r+K_{r f b} r_{f b} .
\end{aligned}
$$

The dynamics mentioned above give 11 states. Two states describing the roll and pitch Euler angles, $\phi$ and $\theta$ respectively, are added to the system, which make the total order of the system being 13. The final form of the $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices can now be derived and both are presented below,

$$
\boldsymbol{A}=\left[\begin{array}{ccccccccccccc}
X_{u} & 0 & 0 & 0 & 0 & -g & X_{a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Y_{v} & 0 & 0 & g & 0 & 0 & Y_{b} & 0 & 0 & 0 & 0 & 0 \\
L_{u} & L_{v} & 0 & 0 & 0 & 0 & 0 & L_{b} & 0 & 0 & 0 & 0 & 0 \\
M_{u} & M_{v} & 0 & 0 & 0 & 0 & M_{a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{\tau_{f}} & \frac{A_{b}}{\tau_{f}} & 0 & 0 & 0 & \frac{A_{c}}{\tau_{f}} & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & \frac{B_{a}}{\tau_{f}} & -\frac{1}{\tau_{f}} & 0 & 0 & 0 & 0 & \frac{B_{d}}{\tau_{f}} \\
0 & 0 & 0 & 0 & 0 & 0 & Z_{a} & Z_{b} & Z_{w} & Z_{r} & 0 & 0 & 0 \\
0 & N_{v} & N_{p} & 0 & 0 & 0 & 0 & 0 & N_{w} & N_{r} & N_{r f b} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{r} & K_{r f b} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{s}} & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right],
$$

$$
\boldsymbol{B}=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{A_{l a t}}{\tau_{f}} & \frac{A_{l o n}}{\tau_{f}} & 0 & 0 \\
\frac{B_{l a t}}{\tau_{f}} & \frac{B_{l o n}}{\tau_{f}} & 0 & 0 \\
0 & 0 & 0 & Z_{\text {col }} \\
0 & 0 & N_{\text {ped }} & N_{\text {col }} \\
0 & 0 & 0 & 0 \\
0 & \frac{C_{l o n}}{\tau_{s}} & 0 & 0 \\
\frac{D_{l a t}}{\tau_{s}} & 0 & 0 & 0
\end{array}\right] .
$$

Since the RC transmitter mixes the inputs to a individual signal to each servomotor, these signals will instead be used as inputs. As mentioned in Section 2.1 and 2.6 , the direct inputs from the pilot joysticks are functions of the servo signals. $\delta_{1}, \delta_{2}$ and $\delta_{6}$ are the signals from the servos that change the attitude of the swashplate. This means that $\delta_{l a t}, \delta_{l o n}$ and $\delta_{c o l}$ are all functions of the three servo signals. $\delta_{\text {ped }}$ can be expressed as a function of $\delta_{4}$, since $\delta_{4}$ is the signal sent to the servo which changes the collective pitch angle of the tail blades. These new signals are S.Bus signals which are unitless. Finally, the linearised model considered in this work, is

$$
\begin{equation*}
\dot{\boldsymbol{x}}=\boldsymbol{A} \boldsymbol{x}+\boldsymbol{B}_{\text {servo }} \boldsymbol{u}_{\text {servo }}, \tag{3.109}
\end{equation*}
$$

where the input vector $\boldsymbol{u}_{\text {servo }}$ is

$$
\boldsymbol{u}_{\text {servo }}=\left[\begin{array}{llll}
\delta_{1} & \delta_{2} & \delta_{4} & \delta_{6} \tag{3.110}
\end{array}\right]^{T}
$$

And the $\boldsymbol{B}_{\text {servo }}$ matrix is

$$
\boldsymbol{B}_{\text {servo }}=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{A_{1}}{\tau_{f}} & \frac{A_{2}}{\tau_{f}} & 0 & \frac{A_{6}}{\tau_{f}} \\
\frac{B_{1}}{\tau_{f}} & \frac{B_{2}}{\tau_{f}} & 0 & \frac{B_{6}}{\tau_{f}} \\
Z_{1} & Z_{2} & 0 & Z_{6} \\
N_{1} & N_{2} & N_{4} & N_{6} \\
0 & 0 & 0 & 0 \\
\frac{C_{1}}{\tau_{s}} & \frac{C_{2}}{\tau_{s}} & 0 & \frac{C_{6}}{\tau_{s}} \\
\frac{D_{1}}{\tau_{s}} & \frac{D_{2}}{\tau_{s}} & 0 & \frac{D_{6}}{\tau_{s}}
\end{array}\right] .
$$

## 4

## System Identification

The goal of the identification procedure is to "obtain a good and reliable model with a reasonable amount of work", quoting [12]. In this case the procedure can be divided into four main steps:

- Collection of flight data
- Choice of a model structure
- Choice of identification method
- Validation

The upcoming sections of this chapter will dive deeper into the different steps.

### 4.1 Collection of Flight Data

Quality data is essential to successfully identifying the system and the choice of input signal is crucial as well since it needs to excite many of the system's dynamics. The choice of using a binary signal usually works with most simple systems, however, since the helicopter is an unstable and complex system, the choice of input signal needs to be well thought-out. An input signal often used in aircraft identification is the frequency sweeps signal, see Figure 4.1. It consists of continuous sinusoidal signals where the frequency increases over time. If a programmable RC transmitter is available, programming the frequency sweep would be the best option, however, executing it manually works as well. The sweeps should start slowly and the frequency of the sweeps should be increased until the pilot reaches the maximum frequency he can manually achieve and still maintain control of the aircraft. It is therefore important to make sure that the helicopter is in hover mode when beginning and ending the frequency sweep.


Figure 4.1: Example of a frequency sweep signal, where the pilot increases the frequency and the amplitude with time

The recorded data will be divided into validation and estimation data and the validation data will be of a different type. This is discussed further in Section 4.4.

Because of the instabilities of the rotorcraft and for economic and safety reasons, the helicopter is flown by a professional pilot. This makes the data collection a closed-loop experiment, with the pilot and the active yaw-damping, mentioned in Section 3.9, as controllers.

Before usage, the data is filtered using a moving average filter, the mean is removed and linear trends are removed. The data is recorded during the whole duration of the tests and it is trimmed to retrieve the interesting parts that are needed. Multiple Matlab scripts are developed to treat the data.

Multiple flight tests are recorded and processed for the identification process, see Chapter 5 for more details.

### 4.2 Choosing a Model Structure

The choice of a model structure is an essential step in system identification, and they can be categorised into three different categories:

Black-box model A model where the description between input and output does not rely on prior knowledge of the system. The only information available is the input and output data and therefore, the usage of various mathematical structures needs be used. Some familiar linear models are ARX and ARMAX, while ANN (Artificial Neural Networks) for nonlinear systems [13].

White-box model A model that fully relies on prior knowledge of the system and all of its subsystems. There is a lot of physical knowledge of the system
and no operation data is needed to generate a well working model.
Grey-box model This is a mix of the two previously mentioned models. It is a model where the structure is known, however, the parameters are unknown. Instead of measuring the parameters manually, which can be both time consuming, expensive and sometimes not possible, they can be estimated.

In this thesis, the structure is the grey-box model derived in Chapter 3 and presented in Section 3.11.

### 4.3 Identification Method

After choosing a model structure, its parameters need to be estimated. There are multiple approaches and methods which will be discussed in the upcoming sections.

### 4.3.1 Closed-Loop Identification

As previously mentioned, the flight data is collected in a closed-loop. The main problem with closed-loop data is the correlation between unmeasured noise in the output and the input. Consequently, some open-loop identification methods might not work as well in this environment. The different approaches to obtain a successful identification can be divided into three main groups:
The Direct Approach The simplest approach where the feedback is ignored and the system is identified as an open-loop system using measurements of the input, $u(t)$, and the output, $y(t)$. This implies that the feedback system does not need to be known.

$$
\begin{equation*}
y(t)=G_{s}(t) u(t)+H(t) e(t), \tag{4.1}
\end{equation*}
$$

where $G_{s}$ is the dynamics model to be identified and $H$ the noise model. According to [12], the direct approach should always be the approach to try first and the others should only be considered if it fails. This method does not require special algorithms and is comparable with the classical identification method, however, as previously mentioned, some identification methods can fail to deliver a result.

The Indirect Approach The approach only works if the regulator, $K(t)$, is known and when the external command signal, $r(t)$, is measurable. The closedloop system can be described as

$$
\begin{equation*}
y(t)=G_{0}^{c}(t) r(t)+v_{c}(t), \tag{4.2}
\end{equation*}
$$

where $G_{0}^{c}=\left(I+G_{0} K\right)^{-1} G_{0}, G_{0}$ is the open-loop system and $v_{c}(t)$ is the closed-loop noise. The closed-loop system can then be identified with the help of

$$
\begin{equation*}
y(t)=G^{c}(t) r(t)+H_{*} e(t), \tag{4.3}
\end{equation*}
$$

where $G^{c}$ is the closed-loop model and $H_{*}$ is a fixed noise model.

The Joint Input-Output Approach In this approach, the output, $y(t)$, and the input, $u(t)$, of the plant are regarded as the output of the system driven by the external command signal, $r(t)$, and the unmeasured noise, $e(t)$. The extended plant can be described by

$$
\left[\begin{array}{l}
y(t)  \tag{4.4}\\
u(t)
\end{array}\right]=\left[\begin{array}{c}
G^{c}(t) \\
S_{u}
\end{array}\right] r(t)+H_{0} e(t)
$$

where $S_{u}$ is the sensitivity of the closed-loop and $H_{0}$ is a complex prefilter.
The direct approaches will be tried to begin with and if it does not yield successful results, other methods will be tried.

### 4.3.2 Parameter Estimation

Consider a standard linear state space structure in discrete time:

$$
\begin{align*}
& x((k+1) T)=A(\theta) x(k T)+B(\theta) u(k T)+K(\theta) e(k T) \\
& y(k T)=C(\theta) x(k T)+e(k T) \tag{4.5}
\end{align*}
$$

where $\theta$ is the parameter vector and $K(\theta)$ is the matrix which will model the noise, $e(t) . K$ is the same size as $A$ and its parameters are set free and are able to obtain any value in order to model the noise. Working in discrete-time is more usual than working in continuous-time since the collected data is sampled. Defining:

$$
\begin{align*}
& G(q, \theta)=C(\theta)(q I-A(\theta))^{-1} B(\theta) \\
& H(q, \theta)=I+C(\theta)(q I-A(\theta))^{-1} K(\theta) \tag{4.6}
\end{align*}
$$

where $q$ is the shift operator, it follows that the model can be rewritten as

$$
\begin{equation*}
\mathcal{M}: y(t)=G(q, \theta) u(t)+H(q, \theta) e(t) . \tag{4.7}
\end{equation*}
$$

According to [14], the one-step-ahead predictor, denoted $\hat{y}(t \mid \theta)$, is given by:

$$
\begin{equation*}
\hat{y}(t \mid \theta)=\left[1-H^{-1}(q, \theta)\right] y(t)+H^{-1}(q, \theta) G(q, \theta) u(t) \tag{4.8}
\end{equation*}
$$

assuming that the filter $H(q, \theta)$ is stable. The prediction error is defined as:

$$
\begin{equation*}
\varepsilon(t, \theta)=y(t)-\hat{y}(t \mid \theta) . \tag{4.9}
\end{equation*}
$$

## Prediction Error Methods

The prediction error in eq. (4.9), can be extended by defining a stable linear filter, $L(q)$, which acts as a frequency weighting to enhance certain frequency regions,

$$
\begin{equation*}
\varepsilon_{F}(t, \theta)=L(t) \varepsilon(t, \theta) . \tag{4.10}
\end{equation*}
$$

Given the model, $\mathcal{M}$, and the collected data, $Z^{N}$, with $N$ number of data samples, the loss function can be calculated as

$$
\begin{equation*}
V_{N}\left(\theta, Z^{N}\right)=\frac{1}{N} \sum_{t=1}^{N} \ell\left(\varepsilon_{F}(t, \theta)\right) \tag{4.11}
\end{equation*}
$$

where $\ell(\cdot)$ is a scalar-valued function. While for single-input single-output (SISO) systems, the standard choice of the $\ell\left(\varepsilon_{F}\right)$ is quadratic,

$$
\begin{equation*}
\ell\left(\varepsilon_{F}(t, \theta)\right)=\frac{1}{2} \varepsilon_{F}^{2}(t, \theta) \tag{4.12}
\end{equation*}
$$

for multiple-input multiple-output (MIMO) systems, it is defined by

$$
\begin{equation*}
\ell\left(\varepsilon_{F}(t, \theta)\right)=\frac{1}{2} \varepsilon_{F}^{T}(t, \theta) \Lambda^{-1} \varepsilon_{F}(t, \theta), \tag{4.13}
\end{equation*}
$$

where $\Lambda$ is a symmetric, positive semidefinite $p \times p$ weighting matrix described in detail in [12]. The estimated parameter vector, $\hat{\theta}_{N}$, can the be obtained by minimising $V_{N}$

$$
\begin{equation*}
\hat{\theta}_{N}=\arg \min _{\theta} V_{N}\left(\theta, Z^{N}\right) \tag{4.14}
\end{equation*}
$$

The family of methods corresponding to eq. (4.14) is called prediction error methods (PEM). The different methods are defined by different choices of $\ell$, prefilters $L$ and choices of approaches to minimise the loss function. The prediction error estimate can be calculated with the help of the function pem already implemented in the Matlab System Identification Toolbox.

Multiple search methods are available as options within the toolbox in order to solve eq. (4.14), such as the subspace Gauss-Newton least squares search, the adaptive subspace Gauss-Newton search, the Levenberg-Marquardt least squares search, the steepest descent least squares search, etc. The search method option can be set to auto and the pem function will try different methods one after the other at each iteration. In this thesis, the Levenberg-Marquardt least squares method has proven itself to work well when comparing to other methods and will therefore be used. More information on the search methods and how they work can be found in [12].

As previously mentioned, the noise of the system is modelled with the help of the matrix $K$. In MATLAB, this is done by defining the variable option DisturbanceModel = 'estimate'. The noise model is then estimated by the pem function, but its study will not be further analysed in this work.

The pem function also allows the model to be estimated with the aim of achieving a good predictor model or a good fit for simulation of model response. In this project, the focus is set to prediction because it is shown to be more stable when estimating parameters.

When predicting the parameters, initial values have to be given to the algorithm. Unfortunately, the PEM is very sensitive to those initial values and can easily get stuck in local minimum. To obtain a good guess of the initial values,
the system is divided into multiple simplified subsystems. For example, from the linearised equation in Section 3.11, the attitude dynamics could be expressed as a smaller subsystem, i.e., the rolling dynamics:

$$
\left[\begin{array}{c}
\dot{p}  \tag{4.15}\\
\dot{b} \\
\dot{d}
\end{array}\right]=\left[\begin{array}{ccc}
0 & L_{b} & 0 \\
-1 & -\frac{1}{\tau_{f}} & \frac{B_{d}}{\tau_{f}} \\
-1 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right] \cdot\left[\begin{array}{l}
p \\
b \\
d
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
\frac{B_{1}}{\tau_{f}} & \frac{B_{2}}{\tau_{f}} & \frac{B_{6}}{\tau_{f}} \\
\frac{D_{1}}{\tau_{f}} & \frac{D_{2}}{\tau_{f}} & \frac{D_{6}}{\tau_{f}}
\end{array}\right] \cdot\left[\begin{array}{l}
\delta_{1} \\
\delta_{2} \\
\delta_{6}
\end{array}\right]
$$

In a similar fashion, this can be done for the pitching dynamics as well. When the parameters for the angular dynamics have been identified, the estimated parameters are used as initial parameters and the horizontal dynamics are identified with a similar approach for the linear velocities $u$ and $v$. The yaw and heave dynamics can easily be separated from the rest of the dynamics and once these systems are identified, the full model dynamics are identified with all the cross coupled terms. Multiple iterations of the process have to be made, and the initial values are tweaked and adjusted during the process. This process is presented in Chapter 5 in more details.

### 4.4 Validation

The parameter estimation process should in theory give the best model, however, it is unsure if this model could be good enough for the different and possible applications. There are multiple validation methods depending on the applications of the model. A way to validate the model is to predict the output of the predicted model, $\hat{y}(t, \theta)$, with input data and compare it against the measured output of the true system, $y(t)$. The fit of the predicted model can be estimated by

$$
\begin{equation*}
\text { fit }=100 \cdot\left(1-\frac{\left\|\hat{y}\left(t \mid \hat{\theta}_{N}\right)-y(t)\right\|_{2}}{\|y(t)-\bar{y}\|_{2}}\right) \tag{4.16}
\end{equation*}
$$

where $\|\cdot\|_{2}$ is the Euclidean norm and $\bar{y}$ is the mean value of $y(t)$. The best fit is obtained when $f$ it $=100$.

### 4.4.1 Residual Analysis

The residuals are the parts of the data the model could not emulate and are given by

$$
\begin{equation*}
\varepsilon(t)=\varepsilon\left(t, \hat{\theta}_{N}\right)=y(t)-\hat{y}\left(t \mid \hat{\theta}_{N}\right) . \tag{4.17}
\end{equation*}
$$

Preferably, the residuals should be independent from the input signals. Otherwise, it means that there are components in $\varepsilon(t)$ that are correlated to the input, $u(t)$, which indicates that the model $\hat{y}\left(t \mid \hat{\theta}_{N}\right)$ is missing some dynamics and could be improved. The covariance between the residuals and past inputs is typically
studied with:

$$
\begin{equation*}
\hat{R}_{\varepsilon u}^{N}(\tau)=\frac{1}{N} \sum_{t=1}^{N} \varepsilon(t+\tau) u(t) \tag{4.18}
\end{equation*}
$$

If the residuals, $\varepsilon(t)$, and the input signal, $u(t)$, are independent, eq. (4.18) is normally distributed with an average value of zero and with the variance:

$$
\begin{equation*}
P_{r}=\frac{1}{N} \sum_{k=-\infty}^{\infty} R_{\varepsilon}(k) R_{u}(k) \tag{4.19}
\end{equation*}
$$

Eq. (4.18) is often represented in a diagram together with the lines $\pm 3 \cdot \sqrt{P_{r}}$ [14]; if $\hat{R}_{\varepsilon u}^{N}(\tau)$ is outside of those lines, it is an indication that some dynamics are not modelled. This is called cross-correlation and usually all inputs are tested.

Similarly, the covariance between the residuals themselves is typically studied with the help of:

$$
\begin{equation*}
\hat{R}_{\varepsilon}^{N}(\tau)=\frac{1}{N} \sum_{t=1}^{N} \varepsilon(t) \varepsilon(t-\tau) . \tag{4.20}
\end{equation*}
$$

If theses values are not small, it means that there is a correlation between the residuals, and that some parts of $\varepsilon(t)$ could have been predicted with an improved model. This is usually called autocorrelation.

### 4.4.2 Cross-Validation

It is best to validate the models with fresh data not used during the estimation process. This is called cross-validation and is a straightforward and appealing method. If a model predicts the output better than another, it is considered to be a better model. Cross-validation means that an extra set of data has to be collected. The new collected data does not need to replicate the input data, which means that the input can be of another type. A 3-2-1-1 input is used for the validation, see Figure 4.2. The 3-2-1-1 signal is made by having a positive signal for three time units, a negative for two time units, then a positive for one time unit and finishing with a negative signal for one time unit. The amplitude of the signal can be adjusted in order to keep the helicopter stable in the air. Using a programmable transmitter would be ideal since this type of signal can be hard to reproduce safely, however, it still works well to validate the estimated model.


Figure 4.2: Example of a 3-2-1-1 signal used during the data collection


## Results

This chapter will present and comment the results of the system identification.
As mentioned in Chapter 4, the model was divided into multiple sub-models that represent different dynamics in the system. This was done in order to easier find the initial values of the full model's parameters and with some of the submodels, simple controllers could be implemented in order to control the attitude. The models are implemented in MATLAB and, with the help of the pem function, the parameters of each model can be estimated. The chosen initial parameters are inspired from the results of similar works [10, 15], however, every helicopter is different and thus the parameters need to be tweaked. A MATLAB script was created in order to obtain the best parameters as possible. The pem function is run a first time with chosen initial values to obtain a stable model. The resulting model's parameters are used as initial values when the function is run again. The model is compared to previous estimated model to find out if the latest model has a better fit. If it has, the model is saved and is now representing the best model that the other have to compare against. The function is run multiple times and with the best fitting model's parameters used as initial values. The parameters should not be identical since the pem function has already has achieved a local minimum. Therefore a reasonable random value is added to all parameters which might give a better model with a new local minimum. This can be put in an endless loop until a satisfactory result is achieved. The process allows the parameters to drift if the initial values were incorrect. However, when the initial system is not stable, the pem function crashes, therefore, some exception handling is present in order to make sure that the script does not crash and that some new initial values are used instead. The algorithm is explained in pseudocode in Appendix 1. Multiple good models are saved before achieving the best model because even though a model has a better fit to the estimation data, the validation process might show that the model is not representative of the true
system.
The input data used for the estimation of the sub-models is a frequency sweep while the input data used to validate the models is multiple 3-2-1-1 signals.

The chapter is divided into multiple sections. In each section, one of the submodels is presented and results regarding the model's fit, the validation data, and the residuals analysis are illustrated.

### 5.1 Roll Rate Identification

The sub-model describing the roll rate, is expressed by

$$
\left[\begin{array}{c}
\dot{p}  \tag{5.1}\\
\dot{b} \\
\dot{d}
\end{array}\right]=\left[\begin{array}{ccc}
0 & L_{b} & 0 \\
-1 & -\frac{1}{\tau_{f}} & \frac{B_{d}}{\tau_{f}} \\
-1 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right] \cdot\left[\begin{array}{l}
p \\
b \\
d
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
\frac{B_{1}}{\tau_{f}} & \frac{B_{2}}{\tau_{f}} & \frac{B_{6}}{\tau_{f}} \\
\frac{D_{1}}{\tau_{f}} & \frac{D_{2}}{\tau_{f}} & \frac{D_{6}}{\tau_{f}}
\end{array}\right] \cdot\left[\begin{array}{l}
\delta_{1} \\
\delta_{2} \\
\delta_{6}
\end{array}\right] .
$$

The model's fit to the estimation data is $87.58 \%$. Figure 5.1 a shows a 1 -step prediction of the estimated model against the validation data, the fit to the validation data is $85.78 \%$. Figure 5.1 b shows a simulation of the estimated model against the validation data, the fit to the validation data is $65.49 \%$. The residual analysis is presented in Figure 5.2. The first panel in the figure represents the autocorrelation of the residuals while the three other subplots represent the cross-correlation for the inputs $\delta_{1}, \delta_{2}$ and $\delta_{6}$ and the residual. The values of the estimated parameters are demonstrated in Table 5.1.


Figure 5.1: Validation of the roll rate model against validation data, where one step prediction and simulation are used.


Figure 5.2: Residual analysis of the roll rate model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

Table 5.1: Parameters of the roll rate model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $L_{b}$ | 291.9 | $\left[\frac{1}{s^{2}}\right]$ |
| $B_{d}$ | 0.007361 | $[-]$ |
| $B_{1}$ | 0.007614 | $\left[\frac{d e g}{s}\right]$ |
| $B_{2}$ | 0.01151 | $\left[\frac{d e g}{s}\right]$ |
| $B_{6}$ | -2.615 | $\left[\frac{d e g}{s}\right]$ |
| $D_{1}$ | -2.378 | $\left[\frac{d e g}{s}\right]$ |
| $D_{2}$ | -2.378 | $\left[\frac{d e g}{s}\right]$ |
| $D_{6}$ | -1.735 | $\left[\frac{d e g}{s}\right]$ |
| $\tau_{f}$ | 0.1131 | $s$ |
| $\tau_{s}$ | 2.639 | $s$ |

### 5.2 Pitch Rate Identification

The pitch rate dynamics is represented by

$$
\left[\begin{array}{l}
\dot{q}  \tag{5.2}\\
\dot{a} \\
\dot{c}
\end{array}\right]=\left[\begin{array}{ccc}
0 & M_{a} & 0 \\
-1 & -\frac{1}{\tau_{f}} & \frac{A_{c}}{\tau_{f}} \\
-1 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right] \cdot\left[\begin{array}{l}
q \\
a \\
c
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
\frac{A_{1}}{\tau_{f}} & \frac{A_{2}}{\tau_{f}} & \frac{A_{6}}{\tau_{f}} \\
\frac{C_{1}}{\tau_{f}} & \frac{C_{2}}{\tau_{f}} & \frac{C_{6}}{\tau_{f}}
\end{array}\right] \cdot\left[\begin{array}{c}
\delta_{1} \\
\delta_{2} \\
\delta_{6}
\end{array}\right] .
$$

The model's fit to the estimation data is $94.57 \%$. Figure 5.3 a shows a 1 -step prediction of the estimated model against the validation data, the fit to the validation data is $92.64 \%$. Figure 5.3 b shows a simulation of the estimated model against the validation data, the fit to the validation data is $82.12 \%$. The residual analysis
is presented in Figure 5.4. Table 5.2 shows the values of the estimated parameters.

(a) 1-Step Predicted response comparison.

(b) Simulated response comparison

Figure 5.3: Validation of the pitch rate model against validation data, where one step prediction and simulation are used.


Figure 5.4: Residual analysis of the pitch rate model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

Table 5.2: Parameters of the pitch rate model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $M_{a}$ | 383.5 | $\left[\frac{1}{s^{2}}\right]$ |
| $A_{c}$ | 0.03194 | $[-]$ |
| $A_{1}$ | -0.009377 | $\left[\frac{d e g}{s}\right]$ |
| $A_{2}$ | -0.01199 | $\left[\frac{d e g}{s}\right]$ |
| $A_{6}$ | 0.01126 | $\left[\frac{d e g}{s}\right]$ |
| $C_{1}$ | 0.2731 | $\left[\frac{d e g}{s}\right]$ |
| $C_{2}$ | -0.3056 | $\left[\frac{d e g}{d}\right]$ |
| $C_{6}$ | -0.1562 | $\left[\frac{d e g}{s}\right]$ |
| $\tau_{f}$ | 0.0876 | $s$ |
| $\tau_{s}$ | 1 | $s$ |

### 5.3 Yaw Rate Identification

The simplified model used to demonstrate the yaw dynamics is presented by

$$
\left[\begin{array}{c}
\dot{r}  \tag{5.3}\\
\dot{r}_{f b}
\end{array}\right]=\left[\begin{array}{cc}
N_{r} & N_{r f b} \\
K_{r} & K_{r f b}
\end{array}\right] \cdot\left[\begin{array}{c}
r \\
r_{f b}
\end{array}\right]+\left[\begin{array}{c}
N_{4} \\
0
\end{array}\right] \delta_{4} .
$$

According to [15], $K_{r f b}=2 \cdot N_{r}$ and $N_{4}=-N_{r f b}$, which are implemented in the model (5.3). The model's fit to the estimation data is $92.03 \%$. Figure 5.1a shows a 1 -step prediction of the estimated model against the validation data, the fit to the validation data is $92.43 \%$. Figure 5.1 b shows a simulation of the estimated model against the validation data, the fit to the validation data is $79.63 \%$. Figure 5.6 presents the residual analysis. Table 5.3 presents the estimated values of the parameters in (5.3).

Table 5.3: Parameters of the yaw rate model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $N_{r}$ | $-1.477 \cdot 10^{4}$ | $\left[\frac{1}{s}\right]$ |
| $N_{r f b}$ | -11.55 | $\left[\frac{d e g}{\mathrm{de} \cdot \mathrm{s}^{2}}\right]$ |
| $K_{r}$ | $-3.773 \cdot 10^{7}$ | $[-]$ |
| $K_{r f b}$ | $-2.954 \cdot 10^{4}$ | $[-]$ |
| $N_{4}$ | 11.55 | $\left[\frac{d e g}{s^{2}}\right]$ |



Figure 5.5: Validation of the yaw rate model against validation data, where one step prediction and simulation are used.


Figure 5.6: Residual analysis of the yaw rate model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

### 5.4 Roll Rate and Lateral Velocity Identification

This model describe the roll and lateral dynamics and is based on the model used in Section 5.1. Two states, $v$ and $\phi$, and three parameters, $Y_{v}, L_{b}$ and $L_{v}$, are added to this model. Similar to Section 5.5, the values in Table 5.1 are used as
initial values for the estimation process. The model is presented by

$$
\left[\begin{array}{c}
\dot{v}  \tag{5.4}\\
\dot{p} \\
\dot{\phi} \\
\dot{b} \\
\dot{d}
\end{array}\right]=\left[\begin{array}{ccccc}
Y_{v} & 0 & -g & Y_{b} & 0 \\
L_{v} & 0 & 0 & L_{b} & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -\frac{1}{\tau_{f}} & \frac{B_{d}}{\tau_{f}} \\
0 & -1 & 0 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right] \cdot\left[\begin{array}{c}
v \\
p \\
\phi \\
b \\
d
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{B_{1}}{\tau_{f}} & \frac{B_{2}}{\tau_{f}} & \frac{B_{6}}{\tau_{f}} \\
\frac{D_{1}}{\tau_{f}} & \frac{D_{2}}{\tau_{f}} & \frac{D_{6}}{\tau_{f}}
\end{array}\right] \cdot\left[\begin{array}{l}
\delta_{1} \\
\delta_{2} \\
\delta_{6}
\end{array}\right]
$$

The model's fit to the estimation data is $87.53 \%$. Figure 5.7 a shows a 1 -step prediction of the estimated model against the validation data, the fit to the validation data is $84.89 \%$. Figure 5.7 b shows a simulation of the estimated model against the validation data, the fit to the validation data is $72.08 \%$. Figure 5.8 illustrates


Figure 5.7: Validation of the combined roll rate and lateral velocity model against validation data, where one step prediction and simulation are used.
the residual analysis. Table 5.4 shows the estimated values of the parameters used in model (5.4).


Figure 5.8: Residual analysis of combined the roll rate and lateral velocity model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

Table 5.4: Parameters of the combined roll rate and lateral velocity model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $Y_{v}$ | -1.45 | $\left[\frac{1}{s}\right]$ |
| $Y_{b}$ | 9.81 | $\left[\frac{\mathrm{deg} \cdot \mathrm{s}^{2}}{}\right]$ |
| $L_{v}$ | -1.595 | $\left[\frac{\mathrm{rad}}{\mathrm{s} \cdot \mathrm{m}}\right]$ |
| $L_{b}$ | 646.4 | $\left[\frac{1}{s^{2}}\right]$ |
| $B_{d}$ | 0.0001531 | $[-]$ |
| $B_{1}$ | 0.0078 | $\left[\frac{\mathrm{deg}}{\mathrm{s}}\right]$ |
| $B_{2}$ | 0.003627 | $\left[\frac{\mathrm{deg}}{\mathrm{s}}\right]$ |
| $B_{6}$ | 0.009397 | $\left[\frac{\mathrm{deg}}{\mathrm{s}}\right]$ |
| $D_{1}$ | -12.6 | $\left[\frac{\mathrm{deg}}{\mathrm{s}}\right]$ |
| $D_{2}$ | 10.88 | $\left[\frac{\mathrm{deg}}{\mathrm{s}}\right]$ |
| $D_{6}$ | 2.247 | $\left[\frac{\mathrm{ed}}{\mathrm{s}}\right]$ |
| $\tau_{f}$ | 0.05671 | s |
| $\tau_{s}$ | 3 | $s$ |

### 5.5 Pitch Rate and Longitudinal Velocity Identification

This model is based on the model used in Section 5.2 and describes the pitch rate behaviour, extended with the longitudinal velocity. Two states, $u$ and $\theta$, and three parameters, $X_{u}, X_{a}$ and $M_{u}$, are added to the model in (5.2). The values from table 5.2 are used as initial values for the estimation process for this model.

The model is presented by

$$
\left[\begin{array}{c}
\dot{u}  \tag{5.5}\\
\dot{q} \\
\dot{\theta} \\
\dot{a} \\
\dot{c}
\end{array}\right]=\left[\begin{array}{ccccc}
X_{u} & 0 & -g & X_{a} & 0 \\
M_{u} & 0 & 0 & M_{a} & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -\frac{1}{\tau_{f}} & \frac{A_{c}}{\tau_{f}} \\
0 & -1 & 0 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right] \cdot\left[\begin{array}{l}
u \\
q \\
\theta \\
a \\
c
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{A_{1}}{\tau_{f}} & \frac{A_{2}}{\tau_{f}} & \frac{A_{6}}{\tau_{f}} \\
\frac{C_{1}}{\tau_{f}} & \frac{C_{2}}{\tau_{f}} & \frac{C_{6}}{\tau_{f}}
\end{array}\right] \cdot\left[\begin{array}{l}
\delta_{1} \\
\delta_{2} \\
\delta_{6}
\end{array}\right] .
$$

The model's fit to the estimation data is $94.88 \%$. Figure 5.9 a shows a 1 -step prediction of the estimated model against the validation data, the fit to the validation data is $92.60 \%$. Figure 5.9 b shows a simulation of the estimated model against the validation data, the fit to the validation data is $79.39 \%$. Figure 5.10 illustrates


Figure 5.9: Validation of the combined pitch rate and longitudinal velocity model against validation data, where one step prediction and simulation are used.
the residual analysis. Table 5.5 shows the estimated values of the parameters in the model.


Figure 5.10: Residual analysis of the combined pitch rate and longitudinal model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

Table 5.5: Parameters of the combined pitch rate and longitudinal model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $X_{u}$ | -12.71 | $\left[\frac{1}{s}\right]$ |
| $X_{a}$ | -9.81 | $\left[\frac{[d}{d e g \cdot s^{2}}\right]$ |
| $M_{u}$ | 205.5 | $\left[\frac{d e g}{s \cdot m}\right]$ |
| $M_{a}$ | 669.6 | $\left[\frac{1}{s}\right]$ |
| $A_{c}$ | 0.4687 | $[-]$ |
| $A_{1}$ | -0.0002098 | $\left[\frac{d e g}{s}\right]$ |
| $A_{2}$ | -0.001058 | $\left[\frac{d e g}{s}\right]$ |
| $A_{6}$ | 0.002435 | $\left[\frac{d e g}{s}\right]$ |
| $C_{1}$ | -0.4068 | $\left[\frac{d e g}{s}\right]$ |
| $C_{2}$ | -0.5533 | $\left[\frac{d e g}{s}\right]$ |
| $C_{6}$ | 0.4863 | $\left[\frac{d e g}{s}\right]$ |
| $\tau_{f}$ | 0.04649 | $s$ |
| $\tau_{s}$ | 2.402 | $s$ |

### 5.6 Combined Lateral and Longitudinal Velocity Identification

This model is a combination of the models introduced in Sections 5.5 and 5.4 in order to estimate the cross-coupled parameters. $\dot{x}_{\text {comb }}=A_{\text {comb }} x_{\text {comb }}+B_{\text {comb }} u_{c o m b}$ is a 10 th order system where

$$
x_{\text {comb }}=\left[\begin{array}{llllllllll}
u & v & p & q & \phi & \theta & a & b & c & d
\end{array}\right]^{T} .
$$

Only three inputs signals are used for this system, $\delta_{1}, \delta_{2}$ and $\delta_{6}$. Therefore, the input vector is

$$
\boldsymbol{u}_{\text {comb }}=\left[\begin{array}{lll}
\delta_{1} & \delta_{2} & \delta_{6}
\end{array}\right]^{T}
$$

The compound $\boldsymbol{A}_{\text {comb }}$ and $\boldsymbol{B}_{\text {comb }}$ matrices are presented below

$$
\begin{gathered}
\boldsymbol{A}_{\text {comb }}=\left[\begin{array}{cccccccccc}
X_{u} & 0 & 0 & 0 & 0 & -g & X_{a} & 0 & 0 & 0 \\
0 & Y_{v} & 0 & 0 & g & 0 & 0 & Y_{b} & 0 & 0 \\
L_{u} & L_{v} & 0 & 0 & 0 & 0 & 0 & L_{b} & 0 & 0 \\
M_{u} & M_{v} & 0 & 0 & 0 & 0 & M_{a} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{\tau_{f}} & \frac{A_{b}}{\tau_{f}} & \frac{A_{c}}{\tau_{f}} & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & \frac{B_{a}}{\tau_{f}} & -\frac{1}{\tau_{f}} & 0 & \frac{B_{d}}{\tau_{f}} \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{s}} & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{s}}
\end{array}\right], \\
\\
\boldsymbol{B}_{\text {comb }}=\left[\begin{array}{cccccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{A_{1}}{\tau_{f}} & \frac{A_{2}}{\tau_{f}} & \frac{A_{6}}{\tau_{f}} \\
\frac{B_{1}}{\tau_{f}} & \frac{B_{2}}{\tau_{f}} & \frac{B_{6}}{\tau_{f}} \\
\frac{C_{1}}{\tau_{f}} & \frac{C_{2}}{\tau_{f}} & \frac{C_{6}}{\tau_{f}} \\
\frac{D_{1}}{\tau_{f}} & \frac{D_{2}}{\tau_{f}} & \frac{D_{6}}{\tau_{f}}
\end{array}\right] .
\end{gathered}
$$

The initial values for the parameters used in this model are taken from Table 5.5 and 5.4. The combined model is extended with four parameters, $L_{u}, M_{v}, A_{b}$ and $B_{a}$. The input data comes from both pitch and roll movements and therefore two data sets were merged together in order to estimate the parameters.

The model's average fits to the estimation data are $77.83 \%$ and $86.96 \%$ for pitch and roll rates, respectively. Figure 5.11a shows a 1 -step prediction of the estimated model against the validation data with a roll movement as input while Figure 5.11 b shows a 1 -step prediction of the estimated model against the validation data with a pitch movement as input. Figure 5.12 a shows a simulation of the estimated model against the validation data with a roll movement as input while Figure 5.12a shows a simulation of the estimated model against the validation data with a pitch movement as input. Figure 5.13 shows the residual analysis of this model. Table 5.6 shows the values of the parameters the model.

(a) 1-Step Predicted response comparison when roll movements are sent as input.

(b) 1-Step Predicted response comparison when pitch movements are sent as input.

Figure 5.11: 1-Step Predicted response comparison of the lateral and longitudinal velocity model against validation data.


Figure 5.12: Simulation response comparison of the combined lateral and longitudinal velocity model against validation data.


Figure 5.13: The residual analysis of the compound model. In darker blue are the amplitudes of the different correlations and in light blue is the confidence interval.

Table 5.6: Parameters of the combined lateral and longitudinal velocity model.

| Parameter | Value | Unit |
| :---: | :---: | :---: |
| $X_{u}$ | -25.1 | $\left[\frac{1}{s}\right]$ |
| $X_{a}$ | -9.81 | $\left[\frac{m}{d e g \cdot s^{2}}\right]$ |
| $Y_{v}$ | -2.878 | $\left[\frac{1}{s}\right]$ |
| $Y_{b}$ | 9.81 | $\left[\frac{\frac{1}{m}}{d e g \cdot s^{2}}\right]$ |
| $L_{u}$ | 3.374 | $\left[\frac{d e g}{s \cdot m}\right]$ |
| $L_{v}$ | -6.132 | $\left[\frac{d e g}{s \cdot m}\right]$ |
| $L_{b}$ | 577.7 | $\left[\frac{1}{s^{2}}\right]$ |
| $M_{u}$ | 75.24 | $\left[\frac{d e g}{s \cdot m}\right]$ |
| $M_{v}$ | 4.215 | $\left[\frac{d e g}{s \cdot m}\right]$ |
| $M_{a}$ | 446.3 | $\left[\frac{1}{s^{2}}\right]$ |
| $A_{b}$ | 0.1019 | $[-]$ |
| $A_{c}$ | 0.01481 | $[-]$ |
| $A_{1}$ | -0.005656 | $\left[\frac{d e g}{s}\right]$ |
| $A_{2}$ | -0.01327 | $\left[\frac{d e g}{s}\right]$ |
| $A_{6}$ | 0.007841 | $\left[\frac{d e g}{s}\right]$ |
| $B_{a}$ | 0.03708 | $[-]$ |
| $B_{d}$ | $7.702 \cdot 10^{-5}$ | $[-]$ |
| $B_{1}$ | 0.01144 | $\left[\frac{d e g}{s}\right]$ |
| $B_{2}$ | 0.0007465 | $\left[\frac{d e g}{s}\right]$ |
| $B_{6}$ | 0.009988 | $\left[\frac{d e g}{s}\right]$ |
| $C_{1}$ | -1.856 | $\left[\frac{d e g}{s}\right]$ |
| $C_{2}$ | -3.098 | $\left[\frac{d e g}{s}\right]$ |
| $C_{6}$ | 1.144 | $\left[\frac{d e g}{s}\right]$ |
| $D_{1}$ | -9.261 | $\left[\frac{d e g}{s}\right]$ |
| $D_{2}$ | 7.743 | $\left[\frac{d e g}{s}\right]$ |
| $D_{6}$ | -3.379 | $\left[\frac{d e g}{s}\right]$ |
| $\tau_{f}$ | 0.07545 | $s$ |
| $\tau_{s}$ | 3 | $s$ |

### 5.7 Summary

The initial values for the simple pitch and roll model were taken from previous works. The two models have two parameters in common, $\tau_{f}$ and $\tau_{s}$, however, the models are estimated independently. The initial values of these two parameters were the same but after multiple iterations of finding the best model, the parameters have drifted to different values. This could be because some dynamics are not included in these simplified model structures and the parameters are trying to compensate for that. Another reason could be that the estimation of these two models is executed based on different data sets.

When the roll and pitch models are extended, all parameters that were present in the simplified models have drifted to different values. The same phenomena happens for the combined lateral and longitudinal velocity model. Because of two different values of $\tau_{f}$ and $\tau_{s}$ from the previous models, both were tested as initial value.

The yaw rate model's parameters has high values when comparing to the other parameters. This is due to the simplifications that were made to model, and it is now separated from all other models. The model could be scaled down, however, the best fit and residuals were achieved with these values.

The final values from table 5.6 are different than the initial values, however, they are deemed normal and the values are accepted as a final result.

## Conclusions

### 6.1 Discussion

Since the project members did not have any knowledge about helicopters, the first few weeks of the project were spent on studying helicopters in general. The autopilot used for data recording is a product made by UAS Europe AB and some time was spent on installing the autopilot and also understanding how it works.

Regarding the data recording, there were some problems at the beginning of the project; the autopilot used for the data recording is programmed to function with airplanes. Data regarding the linear velocities were neither estimated nor recorded. An alternative solution in order to estimate the linear velocities was to use the Qualysis system at Linköping University for collecting the data and getting better measurements of the position and the attitude of the RUAV.

Unfortunately, the project group only had one opportunity to use the Qualisys system for the data collection. However, due to technical difficulties, the data was unusable and because of the unavailability of the pilot and the capture motion system, no more tests were made. It follows that measurement data from the autopilot was used for the system identification. The data used from the autopilot was the angular velocities: roll rate $p$, pitch rate $q$ and yaw rate $r$.

As can be seen in Chapter 5, the fit of the different developed sub-models is satisfactory. All the models have a fit of more than $70 \%$ when prediction is used to compare the models with the validation data. Figures where the models are simulated and compared with the validation data show good fit values which means that the models can be used in a simulation environment. The autocorrelation and cross-correlation stay within the confidence intervals most of the time. The roll rate model and its extended models have a tendency to exceed those limits which might be due to translating tendencies seen in Section 2.4 or other disturbances and dynamics that are not modelled.

The reason behind not continuing estimating the whole linear system is that the data needed for describing and estimating the heave dynamics was unusable.

Dividing the model into smaller sub-models and then extending their dynamics was the used method. It turned out to give good initial values for the extended models and if the linear velocities were known, this would have worked for the full model as well. Only the angular velocities could be measured for the system identification which represent only three of the thirteen states. With more measurements, the linear velocities and the angles could also be measured which increases the number of measured states.

### 6.2 Conclusions

The derived model structure has proven itself to describe some the dynamics of the helicopter in hover mode accurately. The parameters estimated using the prediction error method are deemed to be accurate and are comparable with similar works in the same field. With more sensors and with the identification methods presented in this thesis, the parameters of the full model can be derived to accurately describe all the dynamics of the helicopter in hover mode.

### 6.3 Future Work

As mentioned in the previous section, measuring more data should be a priority in order to extend the model of the RUAV. Improving the hardware and the software will give the ability to get measurements of more states, and will consequently improve the system identification. The Qualysis system can be used to validate the quality of the measurements.

With the current model, a simulation environment of the pitch, roll and yaw could be implemented. As seen from the results, the developed models are able to simulate the dynamics quite well. With a simulation, the next step would be to develop controllers and control strategies to control the behaviour of the RUAV and keep it in hover. Since the system is linear, model based controllers such as LQR and MPC can be used. Here comes the important role of simulation. Having a simulation of the system to test and validate the system and the chosen control strategies saves time and is good for economic reasons.

After estimating a full linear model of the RUAV in hover mode, the next step could be to extend the model to describe the behaviour of the RUAV in other modes, e.g., forward motion.

## Appendix

## A

## Figures

In this section, figures that clarify aspects of the helicopter are presented.

## A. 1 Helicopter



Figure A.1: Placement of the different servos. Helicopter viewed from behind. Servos 1, 2, and 6 are controlling the attitude of the swashplate while servo 4 controls the pitch of the tail blades.


Figure A.2: RC transmitter

## A. 2 Scripts

```
Algorithm 1 Pseudo code of the MATLAB script used for the parameter estimation
    guess an initial best model
    while not cancelled by the user do
        parameters \(=\) best model's parameters + reasonable random values
        try
            current model \(=\) pem(parameters, chosen model structure)
            if current model is better than best model then
                best model \(=\) current model
        catch model unstable, pem not possible
            restart the while loop
            catch any other exception
            break
            end try
Verify best model
```


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