Induction in Printed Circuit Boards using Magnetic Near-Field Transmissions

Simon Arkeholt

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Supervisors: Roger Magnusson, Claes Vahlberg
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To Julia, the love of my life
Abstract

In 1865 Maxwell outlined the theoretical framework for electromagnetic field propagation. Since then many important developments have been made in the field, with an emphasis on systems using high frequencies for long-range interactions. It was not until recent years that applications based on short-range inductive coupling demonstrated the advantages of using low frequency transmissions with magnetic fields to transfer power and information. This thesis investigates magnetic transmissions in the near-field and the possibility of producing induced voltages in printed circuit boards. A near-field magnetic induction system is designed to generate a magnetic flux in the very low frequency region, and used experimentally to evaluate circuit board induction in several interesting environments. The resulting voltages are measured with digital signal processing techniques, using Welch’s method to estimate the spectrum of the received voltage signal. The results show that the amount of induced voltage is proportional to the inverse cube of the transmission distance, and that the system is able to achieve a maximum induced voltage of 65 μV at a distance of 2.5 m and under line-of-sight conditions. It is also concluded that conductive obstructions, electromagnetic shielding and background noise all have a large impact on the obtained voltage, either cancelling the signal or causing it to fluctuate.
Acknowledgments

This thesis concludes five years of studies at Linköping University, which have been filled with many memorable experiences and moments. The completion of this work would not have been possible without some dedicated persons, to whom I would like to express my gratitude.

First of all, I would like to thank the staff at FOI for the opportunity to work with a very interesting and rewarding topic. A special thanks goes to my supervisor Claes Vahlberg, for providing immense support and useful suggestions throughout this thesis and for always being available for questions. I would also like to thank Roger Magnusson, my supervisor at the Department of Physics, Chemistry and Biology, for his invaluable advice and comments regarding my report. I am also very grateful for the involvement of my examiner Peter Münger, whose encouragement has been very inspirational and helpful.

I would like to take this opportunity to thank all my friends, who have been a tremendous support throughout the years. It is thanks to you that my five years here in Linköping have been among the best of my life. Finally, I would like to thank my immediate family for always believing in me and supporting me in times of need. You have all played a huge part in the making of this thesis, and for this I will always be most grateful.

Linköping, June 2018
Simon Arkeholt
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## Notation

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<th>Quantity</th>
<th>Unit</th>
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<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
<td>[rad s(^{-1})]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Attenuation constant</td>
<td>[Np m(^{-1})]</td>
</tr>
<tr>
<td>$C$</td>
<td>Capacitance</td>
<td>[F]</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>Charge density</td>
<td>[C m(^{-3})]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Conductivity</td>
<td>[S m(^{-1})]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coupling coefficient</td>
<td>[1]</td>
</tr>
<tr>
<td>$J$</td>
<td>Current density</td>
<td>[A m(^{-2})]</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field</td>
<td>[V m(^{-1})]</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$L$</td>
<td>Inductance</td>
<td>[H]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Intrinsic impedance</td>
<td>[\Omega]</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic field</td>
<td>[T]</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Magnetic flux</td>
<td>[Wb]</td>
</tr>
<tr>
<td>$A$</td>
<td>Magnetic vector potential</td>
<td>[Wb m(^{-1})]</td>
</tr>
<tr>
<td>$M$</td>
<td>Mutual inductance</td>
<td>[H]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Permeability</td>
<td>[H m(^{-1})]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Permittivity</td>
<td>[F m(^{-1})]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phase constant</td>
<td>[rad m(^{-1})]</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance</td>
<td>[\Omega]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Resistivity</td>
<td>[\Omega \ m]</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
<td>[m s(^{-1})]</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number</td>
<td>[m(^{-1})]</td>
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## Mathematical notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tr>
<td>$j$</td>
<td>Imaginary unit</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Nabla operator</td>
</tr>
<tr>
<td>$\nabla^2$</td>
<td>Vector Laplace operator</td>
</tr>
<tr>
<td>$\oint_C$</td>
<td>Curve integral</td>
</tr>
<tr>
<td>$\oint_S$</td>
<td>Surface integral</td>
</tr>
<tr>
<td>$\oint_V$</td>
<td>Volume integral</td>
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## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>ADC</td>
<td>A/D Converter</td>
</tr>
<tr>
<td>DAC</td>
<td>D/A Converter</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive Force</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>NFMI</td>
<td>Near-Field Magnetic Induction</td>
</tr>
<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Squared</td>
</tr>
<tr>
<td>VLF</td>
<td>Very Low Frequency</td>
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</table>
This thesis is about the effects of electromagnetic (EM) transmissions on printed circuit boards (PCB) in close proximity to the transmitter. This chapter starts with a brief introduction and background to the field, followed by the purpose and novel contributions of this work. Afterwards the problem statement of the thesis is presented, and necessary delimitations in the work are discussed. The chapter then concludes with a description of the thesis outline.

1.1 Background

In 1865 Maxwell published his famous paper in which the equations describing EM wave propagation were outlined [1]. The ensuing research has led to many important applications such as radio communications and radar positioning. However, most of the emphasis has so far been on long range systems that utilize high transmission frequencies. It was not until the development of near-field magnetic induction (NFMI) systems that the advantages of low frequency technologies were first demonstrated. These devices rely on alternating magnetic fields for transfer of power and information, and has the advantage of lower power consumption, more available bandwidth and more secure channels compared to corresponding radio technology [2, 3].

The potential benefits have initiated a lot of research in recent years, and many important near-field applications have been developed. Indoor positioning [4, 5], underwater communication and navigation [6], wireless power transfer [7], underground sensor networks [8–11] and integrated body systems [12] are only a few examples of areas where NFMI systems are of interest. An interesting question that arises is whether magnetic transmissions can affect surrounding electronics.
Is it possible that an induced voltage arises due to closed loops on a PCB surface, and can that interfere with the functionality of the device? If so, at what distances and under which conditions must these effects be taken into account?

As of today there are no reports on PCB effects caused by time-varying EM fields in the near-field propagation region. A dipole model for PCB near-field emission was proposed by Liu et al. [13], but the paper does not cover the case of induction. The closest any paper has come to a complete physical model of PCB near-field induction caused by magnetic transmissions is Smith et al. [14] in 2016. However, the transmitter and receiver were located at the same position and the effects of obstacles and noise sources were not investigated. Ma et al. [10] presented an experimental study of magnetic transmissions and compared it with theoretical results, but did not cover any other case than open air propagation. In order to fully understand EM field interactions with electronics more research is therefore necessary.

1.2 Purpose

The aim of this thesis is to investigate the possibility of EM induction in a PCB, using magnetic transmissions in the very low frequency (VLF) band. Unlike previous work in the field, this thesis contributes with an investigation of PCB induction in non-ideal environments and at different distances between the PCB and transmitter. Different transmission frequencies are investigated along with the effect of obstacles in the line-of-sight (LOS) propagation path. Also included is a discussion regarding common EM noise sources and their possible impact on measurement results.

1.3 Problem statement

The thesis will address the following questions:

1. How do magnetic transmissions with low frequency work, and how does the EM field propagate in the near-field region?

2. How are printed circuit boards affected by alternating magnetic fields in a near-field environment?

   (a) Is it possible to detect induced voltages in a PCB?

   (b) How large voltages can be achieved at various distances?

   (c) How does the frequency affect the amount of induced voltage?

   (d) How do common EM noise sources and obstacles affect induced voltage strength in a PCB?
1.4 Delimitations

Electromagnetic field propagation in air has been investigated exhaustively since the 1800s, and is today well modelled mathematically. Therefore the emphasis of this thesis will be on PCB induction and not the very complicated process of making a general model for EM propagation in every conceivable environment. The experimental studies will be limited to static setups, so neither transmitter nor PCB receiver will be moving during measurements.

Furthermore, the effects of different obstacles and noise sources will only be investigated in environments that are interesting for applications. To save time only a few transmission distances will be tested in each case, and transmission frequencies will be limited to the VLF region.

1.5 Thesis outline

In Chapter 2 the necessary theoretical background in the field of electrodynamics is presented. Models and equations for induction and EM field propagation in various media are derived, along with a theoretical NFMI system model.

In Chapter 3 important results from previous investigations are presented. The chapter contains simulation results of common EM noise sources in a realistic environment, along with measurement results from similar NFMI systems.

In Chapter 4 the experimental methods used to answer the questions formulated in section 1.3 are described. This includes design and setup of transmission- and measurement equipment, test environments and experimental studies.

In Chapter 5 the results obtained in the field studies are presented in a way that provides a solid basis for the discussion and conclusions in subsequent chapters.

In Chapter 6 the results are analyzed in a systematic way to find answers to the questions stated in section 1.3. This includes a discussion on sources of error and the applicability of the method.

In Chapter 7 the thesis is concluded by providing an answer to each of the questions formulated in section 1.3. This is complemented with suggestions for further research that can be made in the same research area.
In this chapter the theoretical framework of EM wave propagation, PCB induction and near-field magnetic transmissions is presented. Starting with the fundamentals of EM theory, subsequent parts cover wave propagation in air and loss effects in solid materials. Next, the relations describing magnetic field radiation from a loop antenna are derived, and the concept of EM induction is presented. A theoretical NFMI system model is used to derive an expression for induced current at a given transmission distance and frequency. The chapter finishes with an overview of the PCB and a description of the signal processing techniques used to measure induced voltage experimentally.

2.1 Electromagnetic waves

The EM field is a vector field containing the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, meaning that each point $(x, y, z)$ in space can be associated with a time-varying vector function $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$ [15]. EM fields propagate through all types of media as waves, described by two wave equations for the electric and magnetic field components respectively. The foundation for these relations, and the entire field of electrodynamics, are Maxwell’s equations [16]:

\begin{align*}
\nabla \cdot \mathbf{E} &= \frac{\rho_q}{\epsilon}, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{B} &= \mu \mathbf{J} + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t}.
\end{align*}

(2.1a) (2.1b) (2.1c) (2.1d)
Here \( \mathbf{J} \) denotes the current density, \( \mu \) the permeability, \( \rho_q \) the charge density and \( \epsilon \) the permittivity of the propagation medium. Equations (2.1b) and (2.1d) illustrate an important consequence of Maxwell’s equations: a time-varying electric field results in an orthogonal and time-varying magnetic field and the other way around, leading to self-sustaining propagation [15]. These equations are Faraday’s and Ampère’s law respectively, and hold for all propagation media and all points in time.

To illustrate the impact of permittivity and permeability on EM field strength, focus will be on plane waves in this section. This is a far-field approximation that in general does not hold in close vicinity to a transmitter, the area of interest in this thesis [17]. However, the analysis is considerably easier for planar waves and the results provide a better understanding of near-field propagation as well. Throughout this section the spherical coordinate system will be used, illustrated in figure 2.1 below. This will provide a radial dependence of EM field strength and induced current, which will simplify experimental setups.

\[ \nabla \times \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial \mathbf{B}}{\partial t}, \]

\[ \nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}, \]

Figure 2.1: Illustration of the spherical coordinate system. Each point in space is defined by an azimuthal angle \( \phi \), a polar angle \( \theta \) and a distance \( r \).

2.1.1 Wave propagation in air

Under LOS conditions in air there are no charges or currents present that need to be taken into account. The difference in permittivity and permeability of air compared to vacuum is negligible, so Maxwell’s equations (2.1) reduces to

\[ \nabla \cdot \mathbf{E} = 0, \]

\[ \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}, \]

\[ \nabla \cdot \mathbf{B} = 0, \]

\[ \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \]
where $\mu_0$ and $\epsilon_0$ are the permittivity and permeability of vacuum. On this form Faraday’s and Ampère’s laws are coupled, first-order differential equations that depend on two unknown vector fields. By taking the curl of both sides and using the definition of the Laplace operator, the fact that the divergence of the electric and magnetic fields are both zero can be utilized to get second-order differential equations that are entirely decoupled. These are the homogeneous Helmholtz’s equations in three dimensions:

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2},$$  

(2.3a)

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}.$$  

(2.3b)

Both equations satisfy the three-dimensional wave equation, and solving them results in an expression for electric and magnetic field propagation on complex form as

$$E(r, t) = E_0 e^{j(k\cdot r - \omega t)} \hat{n},$$  

(2.4a)

$$B(r, t) = B_0 e^{j(k\cdot r - \omega t)} (\hat{k} \times \hat{n}).$$  

(2.4b)

Here $k$ is the wave vector, the magnitude of which is given by the wave number $k = \omega \sqrt{\mu \epsilon}$. The propagation vector $\hat{k}$ points in the direction of propagation, and the polarization vector $\hat{n}$ is always perpendicular to it [16]. The $E$- and $B$-fields are therefore mutually perpendicular to each other and the propagation direction at the same time, and maintain a constant peak amplitude in air. The propagation speed is equal to $c = 1/\sqrt{\mu_0 \epsilon_0}$, which is the speed of light in vacuum. An illustration of the propagating $E$- and $B$-fields in air is shown in figure 2.2.

![Figure 2.2: Propagation of electromagnetic waves in vacuum. The electric and magnetic fields are perpendicular to each other and the propagation direction. The wave amplitude remains constant over time.](image)

### 2.1.2 Wave propagation in solid materials

In this thesis the effects of EM waves that propagate in non-LOS conditions and through other media than air are investigated. Therefore, the relative permittivity and permeability of the medium in question have to be taken into account. If the obstructing object is a conductor the free current density may be non-zero, so the
conductivity $\sigma$ must also be considered. In a general anisotropic medium all these parameters are represented by second rank tensors, but can for linear and isotropic media be written as scalars. Maxwell’s equations (2.1) are modified to

\begin{align*}
\nabla \cdot \mathbf{E} &= \frac{\rho_0}{\epsilon}, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{B} &= \mu \sigma \mathbf{E} + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t},
\end{align*}

(2.5a) (2.5b) (2.5c) (2.5d)

where $\sigma$, $\mu$ and $\epsilon$ all are material-dependent with relative permeability and permittivity $\mu_r$ and $\epsilon_r$ respectively. It can be shown that the free charge density $\rho_0$ dissipates with time, so it can be approximated with zero after a transient time period [16]. As in the previous section Helmholtz’s equations can be found by taking the curl of equations (2.5b) and (2.5d) to get the following $\mathbf{E}$- and $\mathbf{B}$-field wave equations in linear media:

\begin{align*}
\tilde{\mathbf{E}}(\mathbf{r}, t) &= E_0 e^{j(\hat{k} \cdot \mathbf{r} - \omega t)} \mathbf{n}, \\
\tilde{\mathbf{B}}(\mathbf{r}, t) &= B_0 e^{j(\hat{k} \cdot \mathbf{r} - \omega t)} (\hat{k} \times \hat{n}).
\end{align*}

(2.6a) (2.6b)

The only difference compared to propagation in air, as in equations (2.4), is that the wave number $k = |\mathbf{k}|$ is complex in linear media. This is due to the polarization and magnetization of the material, which varies with the EM field. For time-harmonic waves the inertia of the particles results in an out-of-phase response, represented by a complex permittivity and permeability. For a certain angular frequency $\omega$ the complex permittivity can be written as

\[ \tilde{\epsilon} = \epsilon' - j\epsilon'' = \epsilon' - j\frac{\sigma}{\omega}, \]

(2.7)

where the real part accounts for polarization and the imaginary part for conduction currents and energy dissipation. The energy dissipation is caused by the force necessary to overcome the inertia, combined with the occurrence of eddy currents. The eddy currents create a new EM field, distorting the first one and resulting in a decrease in amplitude [11]. In section 2.1.5 the frequency dependence of $\epsilon$ is discussed further. The permeability on the other hand can be approximated as a purely real quantity in the VLF region, so here the same notation will be used as before [18].

### 2.1.3 Wave attenuation

The complex wave number has consequences for the EM wave propagating through the material. Using the permittivity from equation (2.7) the wave number can be rewritten as a complex quantity

\[ \tilde{k} = \omega \sqrt{\mu \epsilon'} \left( 1 - j \frac{\sigma}{\epsilon' \omega} \right) = \beta + j\alpha, \]

(2.8)
where \( \alpha \) is the attenuation constant and \( \beta \) is the phase constant, as described by the following relations:

\[
\alpha = \omega \sqrt{\frac{\mu \varepsilon'}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\varepsilon' \omega} \right)^2} - 1 \right)}, \quad (2.9a)
\]

\[
\beta = \omega \sqrt{\frac{\mu \varepsilon'}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\varepsilon' \omega} \right)^2} + 1 \right)}.
\] (2.9b)

Inserting (2.8) into the wave propagation equations in (2.6) results in

\[
\hat{E}(r, t) = E_0 e^{-\alpha \hat{k} \cdot r} e^{j(\beta \hat{k} \cdot r - \omega t)} \hat{n}, \quad (2.10a)
\]

\[
\hat{B}(r, t) = B_0 e^{-\alpha \hat{k} \cdot r} e^{j(\beta \hat{k} \cdot r - \omega t)} (\hat{k} \times \hat{n}). \]

(2.10b)

Since the attenuation constant is real-valued it causes the electric and magnetic fields to decrease in amplitude as they travel through other media than vacuum or air. At the same time the phase constant determines how much the phase of the waves shift as they propagate. Unlike vacuum or air, the electric and magnetic fields are thus not in phase in linear media. An illustration of the propagating \( E \)- and \( B \)-fields in lossy media is shown in figure 2.3 below. [11, 16]

![Figure 2.3](image)

**Figure 2.3:** Propagation of electromagnetic waves in lossy media. The amplitude is no longer constant as it attenuates over time.

The constants in equations (2.9) illustrate that not only is the material of importance, but also the wave frequency \( \omega \). For good conductors \( \sigma \) is very high, and \( \sigma \gg \varepsilon' \omega \). The expression in equation (2.9a) can then be approximated with

\[
\alpha \approx \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\pi f \mu \sigma}, \quad (2.11)
\]

where the frequency \( f \) is in Hz. In this work the distance EM waves can propagate in a material before they are attenuated by a significant factor is of interest. The skin depth \( \delta \), or depth of penetration, is the distance EM waves propagate before their amplitudes are attenuated by a factor of \( e^{-1} \) [11]. It is given by

\[
\delta = \frac{1}{\alpha} = \left[ \omega \sqrt{\frac{\mu \varepsilon'}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\varepsilon' \omega} \right)^2} - 1 \right)} \right]^{-1} \approx \frac{1}{\sqrt{\pi f \mu \sigma}}, \quad (2.12)
\]
This motivates the use of low frequencies when attempting to reach a receiver behind solid obstructions. An EM wave with low frequency will not loose as much amplitude when propagating through the object, and might still be discernible from background noise when it reaches the receiver on the other side.

2.1.4 Refraction losses

In addition to losses that occur inside a material, losses in the interface between two media also have to be considered. When an EM wave is incident on a phase boundary $S$, as shown in figure 2.4, only a certain fraction of it will transmit across the boundary while the rest will reflect back.

\[ T_\parallel = \frac{\overline{B}_t}{\overline{B}_i} = \frac{2\eta_1 \cos \theta_i}{\eta_0 \cos \theta_i + \eta_1 \cos \theta_t}, \quad (2.13a) \]
\[ T_\perp = \frac{\overline{B}_t}{\overline{B}_i} = \frac{2\eta_1 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_0 \cos \theta_t}, \quad (2.13b) \]

for p-polarized ($\parallel$) and s-polarized ($\perp$) EM waves respectively. $T$ is the transmission coefficient, and has a corresponding reflection coefficient $R = 1 - T$ for the reflected wave amplitude. The intrinsic impedance $\eta$ is the ratio between the electric and magnetic field in a medium, and is given by

\[ \eta = \sqrt{\frac{j\mu \omega}{\sigma + j\varepsilon \omega}}. \quad (2.14) \]

Both attenuation and refraction losses are a major concern when trying to induce currents in a PCB located behind obstructing objects. Not only do transmitted waves loose amplitude inside the object, but going in and out of it as well. [19]
2.1.5 Frequency dependence of permittivity

When discussing wave attenuation at different frequencies (see section 2.1.2) it was noted that the permittivity is frequency-dependent. In linear and isotropic media where the permeability is low, i.e. excluding any ferromagnetic materials, the permittivity can be modelled using the Debye equation

\[ \hat{\epsilon}(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau_0}, \]

where \( \epsilon_s \) is the static permittivity at frequency zero and \( \epsilon_\infty \) permittivity when the frequency goes to infinity. The relaxation time \( \tau_0 \) is characteristic of the material, originating from the modelling of an electron as a harmonic oscillator. The Debye equation performs poorly when the frequency range is large (more than two orders of magnitude), but as this work focuses on the narrow vlf range it holds well in this case. For low frequencies \( j\omega\tau_0 \ll 1 \), so the static permittivity can be used in all calculations. [20]

In reality other aspects such as temperature also affects the permittivity and skin depth, making it very complicated to theoretically model EM losses. This work will therefore rely on previous experimental research, presented in chapter 3.

2.2 Radiation in the near-field

So far wave propagation have been described in general terms, without taking into account the distance to the transmitter or how the waves were generated. In this section an expression for magnetic field radiation as a function of distance from an antenna is derived. Radiation is the transfer of EM energy outwards from a source, and is the result of charges accelerating in a time-harmonic current [16]. The transmitter is modelled as a loop antenna with radius \( b \), shown in figure 2.5.

![Figure 2.5: Model of a loop antenna with radius b. A sinusoidal current I flows through the loop, radiating electromagnetic energy outwards to a point R1 from the loop edge and R from its centre point.](image)
Usually the finite propagation speed of EM waves has to be taken into account, since it takes a certain amount of time to transfer information from a transmitter to a receiver. In the VLF band however, when the fields are varying slowly, a quasi-static approximation can be made that the response is instant throughout space. Since the divergence of B is zero it can be rewritten as the curl of a magnetic vector potential A [21]. To simplify derivations the vector phasor for time-harmonic fields is introduced

\[ \mathbf{J}(R, t) = \text{Re}\{\mathbf{J}(R)e^{-j\omega t}\} = \text{Re}\{\mathbf{J}(R)e^{-jkR_1}\}, \]  

(2.16)

and the vector phasor magnetic potential in open air, depending only on space and not time, can be written as:

\[ \mathbf{A}(R) = \frac{\mu_0}{4\pi} \oint_{\mathcal{V}} \frac{\mathbf{J}(R)e^{-jkR_1}}{R_1} d\mathbf{V}. \]  

(2.17)

A small element of thin wire with cross-section S is equal to the volume element as \(d\mathbf{V} = Sd\mathbf{l}\). Since the current is given by \(I = J(R)S\), this enables us to rewrite the potential in terms of current and a length element of the conducting wire in the antenna as

\[ \mathbf{A}(R) \approx \frac{\mu_0 I}{4\pi} \oint_{C} \frac{e^{-jkR_1}}{R_1} d\mathbf{l}. \]  

(2.18)

For a relatively small loop \(R \approx R_1\), and the exponential can be simplified using Taylor expansion to the first order term,

\[ e^{-jkR_1} = e^{-jkR_1}e^{-jk(R_1-R)} \approx e^{-jkR(1 - jk(R_1 - R))}. \]  

(2.19)

Combining equations (2.18) and (2.19) results in an approximate expression for the potential as

\[ \mathbf{A}(R) = \frac{\mu_0 I}{4\pi} e^{-jkR} \left[ (1 + jkR) \oint_{C} \frac{d\mathbf{l}}{R_1} - jk \oint_{C} d\mathbf{l} \right]. \]  

(2.20)

Switching from the Cartesian coordinate system to spherical coordinates yields, for a fixed radius and polar angle

\[ d\mathbf{l} = b\sin\phi \, d\phi \, \hat{\phi}. \]  

(2.21)

The law of cosines and the assumption that \(R^2 \gg b^2\) result in

\[ \frac{1}{R_1} = \frac{1}{R} \left( 1 + \frac{b^2}{R^2} - \frac{2b}{R} \sin \theta \sin \phi \right)^{-1/2} \approx \frac{1}{R} \left( 1 - \frac{2b}{R} \sin \theta \sin \phi \right)^{-1/2} \approx \frac{1}{R} \left( 1 + \frac{b}{R} \sin \theta \sin \phi \right), \]  

(2.22)
and using the result of equations (2.21) and (2.22) directly in combination with equation (2.20) yields the expression

\[ A(R) = \frac{\mu I b}{4\pi} e^{-j\tilde{k}R} \left[ (1 + j\tilde{k}R) \int_C \sin \phi \, d\phi - j\tilde{k} \int_C \sin \phi \, d\phi \right] \hat{\phi} \]

\[ \approx \frac{\mu I b}{4\pi} e^{-j\tilde{k}R} (1 + j\tilde{k}R) \int_C \left( 1 + \frac{bR}{R} \sin \phi \sin \theta \right) \frac{\sin \phi}{R} \, d\phi \hat{\phi} \]

\[ = \frac{\mu I b^2}{4\pi R^2} e^{-j\tilde{k}R} (1 + j\tilde{k}R) \sin \theta \int_C \sin^2 \phi \, d\phi \hat{\phi} \]

\[ = \frac{\mu I \pi b^2}{4\pi R^2} e^{-j\tilde{k}R} (1 + j\tilde{k}R) \sin \theta \hat{\phi}. \]

This expression for the phasor magnetic vector potential depends only on the transmission distance, material parameters, antenna design and applied current. What remains is to determine the magnetic field by taking the curl of the phasor magnetic potential \( A(R) \) in the spherical coordinate system, which results in

\[ B_r = 2j \omega I \pi b^2 k^2 \cos \theta \left[ 1 \left( \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right) \right] e^{-jkR}, \]

\[ B_\theta = j \frac{\omega I \pi b^2}{4\pi \eta_0} k^2 \sin \theta \left[ \frac{1}{(kR)^2} + \frac{1}{(kR)^3} - \frac{j}{(kR)^3} \right] e^{-jkR}, \]

\[ E_\phi = -j \frac{\omega \mu_0 I \pi b^2}{4\pi} k^2 \sin \theta \left[ -\frac{1}{(jkR)} + \frac{1}{(kR)^2} \right] e^{-jkR}, \]

where the electric field component is a result of Ampère’s law in free space [22]. It is important to note that the magnetic field is always a real quantity. The real part in equation (2.16) was omitted in the derivation of the complex representation but is still necessary to get the actual magnetic field strength.

From the magnetic field equations the near-field region can be defined by looking at which term is dominating at different distances. If the denominator \( kR \ll 1 \) the last terms in (2.24) are large compared to the other terms, which is equivalent with a radiation distance \( R \ll \frac{\lambda}{2\pi} \). In the VLF region frequencies range from 3 to 30 kHz [21]. As illustrated in table 2.1 below, this means that the near-field limit stretches very far from the transmitter.

**Table 2.1:** Extent of the near-field region for frequencies in the VLF domain, calculated using the definition above.

<table>
<thead>
<tr>
<th>Frequency [kHz]</th>
<th>Wavelength [km]</th>
<th>Near-field limit [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>99.9</td>
<td>15.9</td>
</tr>
<tr>
<td>10</td>
<td>30.0</td>
<td>4.8</td>
</tr>
<tr>
<td>20</td>
<td>15.0</td>
<td>2.4</td>
</tr>
<tr>
<td>30</td>
<td>10.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>
For all practical experimental setups the quasi-static approximation can therefore be used to simplify the expressions for radiating magnetic and electric fields in air. Assuming that the transmitter coil has \( N \) turns the results are

\[
\mathbf{B} = \mu_0 N I b^2 \left( 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right), \tag{2.25a}
\]

\[
\mathbf{E} = -j \omega \mu_0 N I b^2 \frac{1}{4 R^3} \sin \theta \hat{\phi}. \tag{2.25b}
\]

### 2.3 Electromagnetic induction

When a stationary circuit is located in a time-varying \( \mathbf{B} \)-field an induced current will appear in it. The fundamental postulate of this EM induction is Faraday’s law in equation (2.1),

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{2.26}
\]

Integrating both sides of equation (2.26) and applying Stoke’s theorem for a stationary current yields

\[
\oint_{C} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint_{S} \mathbf{B} \cdot d\mathbf{S} \iff \varepsilon = -\frac{d\Phi}{dt}, \tag{2.27}
\]

which is Faraday’s law of EM induction [16]. It states that the induced voltage \( \varepsilon \), or electromotive force (EMF), is equal to the negative time derivative of the magnetic flux \( \Phi \) though a closed loop. The induced EMF can be increased by either increasing the flux itself or the rate at which it changes. The minus sign accounts for the direction of the induced current, which according to Lenz’s law is such that it produces a flux that cancels out the change [21]. One way of increasing the magnetic flux is by increasing the number of loop turns, i.e. replacing the closed loop with a coil. For a coil with \( N \) turns, equation (2.27) is modified as

\[
\varepsilon = -N \frac{d\Phi}{dt}. \tag{2.28}
\]

Suppose that a transmitter-receiver setup consists of two coils 1 and 2, and that a magnetic flux \( \Phi_1 \) is generated by running a current \( I_1 \) through coil 1. The total magnetic flux that passes through coil 2 to produce an EMF is proportional to the current in coil 1 as

\[
\Phi_{12} = \frac{L_{12} I_1}{N_2}, \tag{2.29}
\]

where \( N_2 \) is the number of turns in the second coil. The so-called mutual inductance \( L_{12} \) depends on the relative positions, shapes and orientations of the two coils. According to (2.28) and Lenz’s law, a current will be produced in coil 2 which creates another magnetic flux that passes back through coil 1. As a result, coil 1 induces an EMF in itself which is also proportional to the current \( I_1 \) as

\[
L_1 = \frac{N_1 \Phi_1}{I_1}. \tag{2.30}
\]
where $L$ is the self-inductance of the coil. A more convenient way of expressing the magnetic flux linkage between two coils is by introducing the coupling coefficient $\kappa$ as the fraction of the total flux $\Phi_1$ that passes through coil 2. Equation (2.29) is then modified to

$$\kappa \Phi_1 = \frac{L_{12} I_1}{N_2}. \quad (2.31)$$

In linear and homogeneous materials $L_{12} = L_{21}$, so both parameters can be denoted as $M$. Combining equations (2.30) and (2.31) for both coils results in the following relationship for the coupling coefficient, expressed in terms of mutual- and self-inductance:

$$\kappa = \frac{M}{\sqrt{L_1 L_2}}. \quad (2.32)$$

The coupling coefficient is a measure of strength for the magnetic flux that is transferred from one coil to another. It can vary between 0 for two uncoupled coils and 1 for perfectly coupled coils. Tightly coupled coils have values of $\kappa$ above 0.5, otherwise they are said to be loosely coupled. [23]

### 2.4 NFMI system

The system model used for analysis of near-field magnetic induction is shown in figure 2.6 below. The system consists of a transmitter coil $T_X$ with radius $a_t$ and a receiver coil $R_X$ with radius $a_r$, aligned coaxially at a distance $d$ from each other.

![Figure 2.6: Circuit diagram of a NFMI system. Voltage is generated in the primary circuit and transferred inductively to the secondary circuit.](image)

A sinusoidal current $i_s(t) = I_0 \sin \omega t$ is supplied to the transmitter coil from a signal generator, resulting in a voltage $u_s(t)$ over the transmitter. The coils are modelled as ideal inductors with self-inductance $L_t$ and $L_r$ respectively, connected in series with resistors $R_t$ and $R_r$ that represent the ohmic resistance of the wires. The transmitter and receiver are coupled with mutual inductance $M$, resulting in a coupling coefficient $\kappa$ as described by equation (2.32). The load resistor $R_L$ represents PCB components and $C$ the transmission cable capacitance. The coil wire capacitances are small and can be neglected in the VLF range.
Since alternating currents and voltages are used the model has to be transformed into its equivalent circuit to analyze it theoretically, replacing each component with their respective complex impedance as shown in figure 2.7 \[8\]. The resistor impedances are equal to their respective resistance \(R_t\) and \(R_r\), while the impedance of the inductors are given by \(j\omega L_t\) and \(j\omega L_r\). The induced EMF over the receiver coil is denoted \(U_r\). An expression for voltage \(U_{load}\) over a load impedance \(Z_{load}\) is required, as this is what will be measured during experiments.

\[
Z_{load} = \frac{R_L}{1 + j\omega CR_L}, \quad (2.33a)
\]
\[
U_{load} = \frac{j\omega MI_t}{Z_{load} + j\omega L_r + R_r} \cdot Z_{load}. \quad (2.33b)
\]

This is the desired analytical expression for induced voltage across the load, depending on known input parameters \(I_t\) and \(\omega\), physical characteristics of the transmitter coil in \(R_t\) and \(L_t\) and the geometry of the system setup in \(M\). The task ahead is to determine these unknown system quantities.

### 2.4.1 Coil resistance

Electrical resistance is a measure of how hard it is for a current to pass through a component or wire. For the transmitter and receiver coils the resistance depends on the material and the geometry of the wire. For a homogeneous wire this is described by the following relation [21]:

\[
R = \frac{L}{\sigma S}. \quad (2.34)
\]

High conductivity and large cross-section area \(S\) make it easier for current to pass, lowering the resistance, while a large wire length \(L\) increases resistance. As shown in section 2.1.5 however, the frequency-dependent skin depth also affects the current density in the wire. For high frequencies the current density is large close to the wire surface and almost zero inside, lowering the effective area used for conduction and increasing the resistance.
Assuming that the wire is made of a good conductive material, for example copper, equation (2.12) can be used to calculate the skin depth. If the skin depth is lower than the wire radius \( r \), equation (2.34) has to be modified with a new effective area which can be shown to be equal to \( \pi(2\delta r - \delta^2) \). If the skin depth is greater than the radius the full cross section area given by \( \pi r^2 \) is used, resulting in

\[
R = \begin{cases} 
\frac{L}{\sigma \pi r^2}, & r \leq \delta, \\
\frac{L}{\sigma \pi (2\delta r - \delta^2)}, & r > \delta.
\end{cases} \tag{2.35}
\]

Another effect which has to be taken into account in closely wound coils is the so-called proximity effect [24]. It arises whenever two or more conductors close to each other conducts an alternating current. The magnetic field produced by each wire induces eddy currents in the neighbours, the magnitude of which are greater in the cross-section area close to the wire. The result is a current density as shown in figure 2.8. The effective conductive area decreases further, in addition to the skin effect, and resistance is even greater.

![Proximity effect](image)

**Figure 2.8:** Proximity effect in two conductors where current flows in the same direction. The current density (grey) is shifted towards the far edges of the conductors due to cancelling eddy currents.

In inductors, where many conductive wires are closely wound, the proximity effect can become dominating over the skin effect. For this reason Litz wires are often used in applications [24]. A Litz wire consists of multiple insulated strands wired in a bundle, ensuring that the skin depth is smaller than each individual strand radius while minimizing proximity effect losses. The result is a wire with smaller resistance than a corresponding solid wire, but otherwise equal properties.

### 2.4.2 Mutual inductance

The mutual inductance between two coaxial coils, aligned as in figure 2.6, can be expressed in terms of the elliptic integrals [25] as

\[
M = N_t N_r \mu \sqrt{a_t a_r} \left[ \left( \frac{2}{\sqrt{m}} - \sqrt{m} \right) K(m) - \frac{2}{\sqrt{m}} E(m) \right], \tag{2.36a}
\]

\[
m = \frac{4a_t a_r}{(a_t + a_r)^2 + d^2}, \tag{2.36b}
\]

where the coil radii and distance is the same as in figure 2.6. The factors \( K(m) \) and \( E(m) \) are the complete elliptic integrals of the first and second kind respectively.
This equation does not take into account that each wire is separated by a small distance. A more correct model would be to sum up the contributions made by each turn in the transmitter on each and every turn in the receiver, resulting in

\[ M = \sum_i \sum_j \mu \sqrt{a_t a_r} \left( \frac{2}{\sqrt{m}} - \sqrt{m} \right) K(m) - \frac{2}{\sqrt{m}} E(m) \]  

(2.37a)

where \( m = \frac{4a_t a_r}{(a_t + a_r)^2 + d_{ij}^2} \),

(2.37b)

where \( i = 1, \ldots, N_t, \ j = 1, \ldots, N_r \) and the distance between loop \( i \) in the transmitter and loop \( j \) in the receiver is \( d_{ij} \).

An alternative expression can be found using the definition and magnetic vector potential in section 2.2. The flux from the transmitter through the receiver is given by the line integral of the magnetic vector potential as

\[ \Phi_{tr} = \oint_C A_{tr} \cdot dl \approx \frac{\mu I_t N_t \pi a_t^2 a_r^2}{2(a_r^2 + d^2)^{3/2}}. \]  

(2.38)

Finally, the definition of mutual inductance in equation (2.29) and the assumption that coil radii are small compared to the distance yields that

\[ M = \frac{N_r \Phi_{tr}}{I_t} \approx \frac{\mu N_t N_r \pi a_t^2 a_r^2}{2d^3}. \]  

(2.39)

It is easy to see that the number of turns and radii of the coils are of great importance for obtaining a large value of \( M \), and in turn induced voltage, in this model system.

### 2.4.3 Coil inductance

The transformer and receiver coils can be modelled as magnetic dipoles assuming that their height is low compared to their radii. Thus the self-inductance for respective coil can be obtained from the expression in equation (2.30) and the magnetic vector potential as in the previous section. If the wire radius is small compared to the coil radius, and the number of turns in the receiver coil is 1, the results are the following approximations:

\[ L_t \approx N_t^2 \mu_0 a_t \left( \log \left( \frac{8a_t}{h_t} \right) - \frac{1}{2} \right), \]  

(2.40a)

\[ L_r \approx \mu_0 a_r \left( \frac{\mu_r}{4} + \log \left( \frac{8a_r}{r} \right) - 2 \right), \]  

(2.40b)

where \( \mu_r \) is the relative permeability of the coil core and \( h_t \) the coil height [26]. The expression for self-inductance depends greatly on coil geometry; the number of turns, winding height, coil radius, and number of layers being the parameters with the most influence. This is investigated in great detail by Agbinya [22], but for the scope of this thesis equations (2.40a) and (2.40b) are sufficient.
2.4.4 Impedance matching

The transmitter antenna presents a load on the signal generator, described by the impedances $R_t$ and $j\omega L_t$. The signal generator also contains an internal impedance $Z_G$, which will affect how much effect is transferred from the generator to the transmitter. When the two impedances are not matched, part of the signal will reflect back to the generator and lower the overall efficiency of the system. The fraction of reflected power is given by the reflection coefficient [17]

$$\gamma = \frac{Z_{load} - Z_G}{Z_{load} + Z_G} = \frac{(Z_t + Z_{rt}) - Z_G}{(Z_t + Z_{rt}) + Z_G}.$$  \hspace{1cm} (2.41)

This equation is fully equivalent with Fresnel’s laws for transmission and reflection in section 2.1.4. The only difference is that the impedances are a result of electrical components and not intrinsic impedances in the propagation media.

Another common way of describing the power loss is in terms of the voltage standing wave ratio. If there is a cable between the generator and transmitter, the reflections will give rise to a standing wave inside the cable. By measuring the amplitude changes over the cable the amount of reflection, and thus power loss, can be determined. The voltage standing wave ratio can be expressed in terms of the reflection coefficient as

$$VSWR = \frac{1 + |\gamma|}{1 - |\gamma|}. \hspace{1cm} (2.42)$$

It is clear from equation (2.41) that in order to reduce the reflection to zero and maximize the transmitter efficiency, the reflection coefficient must be as close to zero as possible. This is achieved by matching the total transmitter impedance with the complex conjugate of the generator impedance, which for a purely resistive generator corresponds to matching resistances [9]. In general the same impedance matching can be done for the receiver load as well. This work investigates the effects on an arbitrary receiver however, so the main focus is to ensure that the magnetic transmissions are as effective as possible.

2.5 Printed circuit boards

Most electronic devices manufactured today rely on integrated circuit technology for their functionality. Integrated circuits are small chips containing electrical components, designed to perform a specific task in a system. The PCB is used to mount the integrated circuits and connect them to each other in an organized and space-efficient way. Each board is unique in its design, but in general they all consist of the same types of layer: a substrate, conductive copper films, prepreg layers, a solder mask and a silkscreen.

The substrate is usually made of sturdy FR4 composite fiberglass, which is covered with a sheet of conductive copper acting as an internal ground plane. The ground plane is covered with an insulating prepreg layer and attached to another copper layer. This conductive signal layer contains all component connections, manufac-
tured by photo-lithography etching. The connections are covered in a solder metal such as tin, which in turn is protected by the (usually green) solder mask. The silkscreen is an indicator with letters and symbols used for assembly. An example of a double-sided PCB is shown in figure 2.9 below. Typical layer specifications for this design are summarized in table 2.2.

![Image of a standard four-layer PCB without components, with the solder mask and silkscreen clearly visible. The copper connections in the signal layer are noticeable in darker color.](image)

**Figure 2.9:** Image of a standard four-layer PCB without components, with the solder mask and silkscreen clearly visible. The copper connections in the signal layer are noticeable in darker color.

The main goal of this thesis is to utilize the conductive connections on the PCB as receivers in the NFMI system. Assuming that there are components on the PCB, the copper connections can be modelled as flat coils in which the induced current can arise. Finding an exact model is very hard, as the interaction distance between connections is very small and the fact that there exist no standard PCB pattern. Agbinya [22] gives an approximate expression for the inductance of a flat rectangular coil which might be used for analysis, but for the purpose of this work it is sufficient to use a model PCB for theoretical analysis instead.

**Table 2.2:** Typical thicknesses in a double-sided PCB with four conductive layers. The total thickness of the PCB is approximately 1.6 mm.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Function</th>
<th>Material</th>
<th>Thickness [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Signal</td>
<td>Copper</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Prepreg</td>
<td>Fiberglass</td>
<td>231</td>
</tr>
<tr>
<td>2</td>
<td>Ground</td>
<td>Copper</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Substrate</td>
<td>FR4</td>
<td>940</td>
</tr>
<tr>
<td>3</td>
<td>Ground</td>
<td>Copper</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Prepreg</td>
<td>Fiberglass</td>
<td>231</td>
</tr>
<tr>
<td>4</td>
<td>Signal</td>
<td>Copper</td>
<td>36</td>
</tr>
</tbody>
</table>
2.6 Voltage measurements

A convenient method of measuring induced voltage is through the power spectral density (PSD) of the received signal. The main advantage of this approach is that the transmitted signal easily can be differentiated from the background noise without the use of complex hardware. However, since the signal is sampled at discrete points in time the exact PSD can not be calculated directly. Instead a spectral estimate must be used, obtained using for example Welch’s method [27].

In Welch’s spectrum estimate the sampled signal \( y[k] \) is divided into \( R \) windows of size \( M \) and with an overlap of \( L \) samples. By applying the discrete Fourier transform the periodogram for each individual window can be computed. The spectral estimate \( \hat{\Phi}(\omega) \) is defined as the average of all periodograms, given by

\[
\hat{\Phi}(\omega) = \frac{1}{R} \sum_{k=1}^{R} \frac{T}{M} |Y[n]|^2.
\]  

(2.43)

Here \( T \) denotes the sampling period and \( Y[n] \) the Fourier transform of the sampled signal. If the sampling frequency \( f_s = 1/T \) and window size are both known, the induced voltage \( U \) can then be calculated using the following expression:

\[
U = \sqrt{\frac{\hat{\Phi}(\omega) f_s}{M}}.
\]

(2.44)

By changing the window size and number of overlapping samples the resolution and variance of the spectrum, and in turn the obtained voltage signal, can be tuned. Increasing \( M \) results in better resolution but more noise, while an increase in \( L \) yields a smoother signal.
In this chapter previous research related to the subject of this thesis is presented. The purpose is to give a more in-depth view of the field and its applications, and to provide useful data for experimental setups and procedures. The purpose of most NFMI systems is to transfer power or information in situations where conventional radio systems exhibit poor performance. A lot of progress in recent years has for example been made in the field of underground and underwater communications, where attenuation is low in the VLF range.

### 3.1 Air propagation

The main issue with all types of NFMI signals is that their amplitude is proportional to the inverse cube of the transmission distance. The weak signals necessitate several amplifiers to detect induced voltages in the receiver. The transmitter and receiver coils also have to be designed as to maximize the magnetic flux that reaches the receiver. One such NFMI system was designed and tested by Ma et al. [10] with the purpose of investigating VLF communication. The system, shown in figure 3.1, included three amplifiers: one at the transmitter to increase transmission power and two at the receiver to detect the signal.

To increase system efficiency an impedance-matching circuit was added between the power amplifier and transmitter to maximize the transmitted power. The low signal-to-noise ratio between induced voltage and power line interference prompted the use of a BP and HP filter, connected in series to the receiver, to remove the utility frequency and its harmonics. Both transmitter and receiver were rectangular with a diagonal of 1 m. The coils consisted of 15 and 49 turns of copper wire respectively.
Figure 3.1: NFMI system used for wireless communication. The extra capacitors in the primary and secondary coils were used to tune the system to a specific resonance frequency. Adapted from Ma et al. [10].

The system was tested at a distance of 5, 6, 7 and 9 m for various angles, ranging from coaxial alignment at $0^\circ$ to perpendicular alignment at $90^\circ$. The simulated and measured magnetic field strength for a transmission frequency of 5 kHz is shown in figure 3.2 below. Ma et al. [10] calculated the voltage to be about seven orders of magnitude greater than the magnetic field, resulting in voltages in the range of tens of mV at the receiver. The maximum distance at which communication was successfully established was 28.5 m.

Figure 3.2: Theoretical and measured magnetic field strength for a frequency of 5 kHz at various transmission distances. The signal profiles agree very well, but the magnitude is slightly lower. Adapted from Ma et al. [10].

3.2 Water propagation

Attenuation losses in water is much greater than in air, owing to the salinity which increases conductivity and the polarity of water molecules which increases
the dielectric constant. For loop antennas the attenuation constant at 14 MHz is 2.6 dB m\(^{-1}\), which outperforms conventional antennas where losses are as high as 80 dB m\(^{-1}\). In an experiment by Shaw et al. [28] wave propagation through sea water was examined for MHz frequencies. The conductivity for salt water was approximated to 4 S m\(^{-1}\) and the transmitted power was 5 W. In the near-field region \((R < 10 \text{ m})\) signal strength decreased rapidly, about 60 dB for the first few meters, before settling into far-field propagation. The signal strength at 90 m was still higher than the noise level (measured at -140 dB), meaning that data transfer would be theoretically possible at this distance.

A problem with EM waves in water is that the propagation path is not always trivial. This issue was investigated by Tyler and Sanford [6] for a NFMI low-frequency system in the ocean coastal region. The high attenuation in water resulted in the waves propagating through the more penetrable seabed and air in certain regions. Compared to sea water, where conductivity is between 2 and 6 S m\(^{-1}\), the conductivity is only 0.1–1 S m\(^{-1}\) in seabed and zero in open air. The conclusion is that there are three main propagation modes possible for water propagation: direct mode, down-over-up mode and up-over-down mode, each taking the path of lowest attenuation. The different modes are illustrated in figure 3.3 below.

![Figure 3.3: Different propagation modes in a coastal NFMI system for underwater communication. 1) Direct mode. 2) Up-over-down mode. 3) Down-under-up mode. Adapted from Tyler and Sanford [6].](image)

Another issue with water propagation is the transmission losses that occur in the interface with air. Even if the conductivity is lower, as is the case with fresh water compared to sea water, transmission losses can still have a large impact on the signal. Jiang and Georgakopoulos [19] investigated transmission losses in water with a conductivity of 0.01 S m\(^{-1}\). For plane waves with frequency 23 kHz transmission losses were calculated to about 15 dB, while propagation losses ranged between 0.25 and 25 dB depending on propagation depth. Adding transmission and propagation losses together, results showed that the VLF region is optimal for air-water transmission at greater depths while the MHz region is optimal in shallow waters.
3.3 Underground propagation

The underground propagation medium consists primarily of inorganic materials such as rock, soil, water and different minerals. Most of these materials are non-magnetic with permeability close to that of free space, meaning that it is mainly the permittivity and conductivity that cause EM waves to attenuate. These parameters in turn depend heavily on the water content, chemical composition and structure of the ground. In ordinary soil conductivity is very low, but in porous rocks a lot of water can assemble and increase conductivity by a factor of up to 80 times that of dry soil. Igneous rock types for example have very low conductivity when compared to sedimentary rock, though variations due to age and location may also affect the conductivity.

Modelling ground attenuation is usually very difficult as ground usually contains several layers and regions of different materials. Because of this a homogeneous model is often used, with a set of effective parameters that describe the ground with good accuracy. The skin depth for a variety of rocks and minerals were compiled by Abrudan et al. \[11\] for 1 kHz, 10 kHz and 10 MHz, a selection of which is shown in table 3.1. The table also includes the skin depth in different types of water media.

**Table 3.1**: Skin depths for various common propagation materials at 1 kHz, 100 kHz and 10 MHz. Adapted from Abrudan et al. \[11\].

<table>
<thead>
<tr>
<th>Ground material</th>
<th>f=1 kHz</th>
<th>f=100 kHz</th>
<th>f=10 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gneiss</td>
<td>≥ 1000</td>
<td>≥ 1000</td>
<td>≥ 1000</td>
</tr>
<tr>
<td>Feldspar</td>
<td>≥ 1000</td>
<td>≥ 105.23</td>
<td>≥ 42.54</td>
</tr>
<tr>
<td>Clay (dry)</td>
<td>≥ 159.17</td>
<td>≥ 16.11</td>
<td>≥ 3.55</td>
</tr>
<tr>
<td>Marble</td>
<td>≥ 159.16</td>
<td>≥ 15.94</td>
<td>≥ 1.88</td>
</tr>
<tr>
<td>Limestone</td>
<td>≥ 112.54</td>
<td>≥ 11.27</td>
<td>≥ 1.25</td>
</tr>
<tr>
<td>Diabase</td>
<td>≥ 69.3</td>
<td>≥ 6.93</td>
<td>≥ 0.73</td>
</tr>
<tr>
<td>Basalt</td>
<td>≥ 48.66</td>
<td>≥ 4.87</td>
<td>≥ 0.49</td>
</tr>
<tr>
<td>Sandstone</td>
<td>≥ 15.92</td>
<td>≥ 1.59</td>
<td>≥ 0.16</td>
</tr>
<tr>
<td>Clay (moist)</td>
<td>≥ 15.92</td>
<td>≥ 1.59</td>
<td>≥ 0.16</td>
</tr>
<tr>
<td>Ice</td>
<td>≥ 1000</td>
<td>≥ 1000</td>
<td>≥ 1000</td>
</tr>
<tr>
<td>Drinking water</td>
<td>71.18 – 225.18</td>
<td>7.15 – 23.53</td>
<td>1.06 – 9.53</td>
</tr>
<tr>
<td>Sea water</td>
<td>6.37 – 15.92</td>
<td>6.37 – 15.92</td>
<td>64 × 10⁻³ – 0.16</td>
</tr>
<tr>
<td>Saline water (3–20 %)</td>
<td>3.56 – 6.16</td>
<td>0.36 – 0.62</td>
<td>32 – 36 × 10⁻³</td>
</tr>
</tbody>
</table>

A waveguide system for underground communication was suggested by Sun and Akyildiz \[9\] in 2009. The system consisted of several relay coils between the transmitter and receiver, each coil forwarding the magnetic flux by inducing a current in the neighbouring relay. Such a system was shown to have lower attenuation than with a single transmitter-receiver system, both at low and high frequencies and for different water content in the ground.
Regardless of ground material, a phenomenon which also has to be taken into account is multi-path fading. When both transmitter and receiver are buried underground there are not one but two different propagation paths between them. The first is the direct path between transmitter and receiver coils, but an additional signal that reflects of the air-ground interface may also reach the receiver. This causes multi-path fading due to destructive interference, and signal power decreases. If the coils are buried deep enough however, the reflection effects can be neglected and the channel is modelled as having a single propagation path. Akyildiz et al. [8] investigated the attenuation losses for different frequencies in the MHz range and determined this depth to be around 2 m.

### 3.4 Electromagnetic noise

All electrical cables carrying a current will produce a magnetic field, and all magnetic fields can in turn induce currents in closed wire loops. The electric power grid, the geomagnetic field and electronic devices represent the greatest sources of EM noise in a NFMI system, and may have to be taken into account during measurements. Deltuva and Lukočius [29] used FEM simulations to calculate magnetic field strength originating from six types of overhead power lines with a voltage of 400 kV. The magnetic field strength were simulated at 1.5 m above ground, in close vicinity to power lines suspended 54 m above ground. The maximum field strengths varied between 35 and 50 A m$^{-1}$ for the six power line types, with a maximum occurring at 10 m distance from the power line.

Power lines buried underground have also been examined using FEM simulations. In a paper by Machado [30] a standard coaxial cable was simulated, buried at a depth of 1.5 m, and the theoretical magnetic field strength at the surface was calculated. For a frequency of 50 Hz and soil conductivity 0.01 S m$^{-1}$ the magnetic field did not exceed 0.3 T, but is still quite significant 7.5 m from the cable.
In this chapter the experimental setups and equipment used to investigate NFMI transmissions and PCB induction under various conditions are presented. The practical work was divided into several parts. First, a NFMI system was designed based on the theory in chapter 2, including software for instrument communication and signal processing. Following this a transmitter was manufactured, with design parameters optimized in MATLAB to produce as much magnetic flux and induced PCB voltage as possible. Finally, a number of field experiments were planned and executed in order to answer the questions posed in the problem statement.

4.1 System layout

The NFMI system consisted of several components, as illustrated in figure 4.1. The MATLAB software in the PC generated a discrete frequency sweep at 28 evenly distributed frequencies in the VLF region, starting at 3 kHz and finishing at 30 kHz. The signal passed through a D/A converter (DAC) to a VLF amplifier, where the signal power was increased. The signal was then transmitted by a coil antenna $T_X$ to a receiver $R_X$ placed some distance apart.

The received signal was amplified by a preamplifier, which included a BP filter for filtering out low- and high frequency noise. The signal was subsequently converted back to digital representation in the A/D converter (ADC) before being acquired by the PC software. A multimeter measured the actual RMS voltage and current over the transmitter in order to normalize the induced voltage with respect to the RMS current. The oscilloscope provided real-time auxiliary monitoring of the signals during experiments.
Figure 4.1: Illustration of the NFMI system used in this thesis.

4.1.1 Computer

A standard PC was used to generate signals and process acquired data. All results in this thesis were obtained using the same computer, the specifications of which are listed in table 4.1 below. The software was written in MATLAB and relies on functionality from the Data Acquisition Toolbox to produce and acquire signals.

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating system</td>
<td>Windows 10 Home</td>
</tr>
<tr>
<td>Processor</td>
<td>Intel Core i7-3632QM CPU @ 2.20 GHz</td>
</tr>
<tr>
<td>Memory</td>
<td>8 GB RAM</td>
</tr>
<tr>
<td>USB port</td>
<td>Intel USB 3.0</td>
</tr>
</tbody>
</table>

Table 4.1: Computer specifications.

A total of three programs were implemented to carry out the measurements, the pseudocode of which are presented in appendix B. In order to align the transmitter and receiver coaxially Calibration.m plotted a continuous PSD for the received signal, which reaches its maximum when the coils are aligned. The transmissions and data acquisition were managed by DataAcquisition.m, which generated the frequency sweep and collected the receiver signal at each frequency in the sweep. DataProcessing.m used digital signal processing techniques to calculate induced voltage from the acquired signal. A Welch spectrum estimate was obtained using the MATLAB-function pwelch, which returned a PSD in units of V^2 Hz^{-1}. The Welch window size and sample overlap were selected to give a good trade-off between resolution and noise. Since the sampling frequency and window size were known the induced voltage could then be calculated using equation 2.44.

4.1.2 DAQ module

The National Instruments USB-6251 BNC is a data acquisition (DAQ) module capable of operating as ADC and DAC at the same time. It is optimized for state-of-the-art performance at fast sample rates, while providing a simple USB connection with the PC and compatibility with the Data Acquisition Toolbox. It has a maximum sample rate of 1.25 MS s^{-1} with a resolution of 16 bits. During measurements the sampling frequency was 100 kHz. The I/O voltage range was set to ±5 V, resulting in a sensitivity of 56 μV with accuracy 1.01 mV.
4.1.3 VLF Amplifier

The Samson SX 3200 is a stereo power amplifier designed for audio signals, with low distortion and wide dynamic range in the vLF region. To ensure optimal amplifier operation at all frequencies the load DC impedance must be at least 8 Ω for mono applications such as this system. The instrument is capable of delivering 2200 W into the load, though this was intentionally limited to 1000 W during experiments. The actual obtained power also varied due to the frequency-dependent transmitter impedance, resulting in lower output power at higher frequencies.

4.1.4 Multimeter

To measure the voltage and current delivered to the transmitter a Fluke 289 True-RMS industrial logging multimeter was connected to four 0.15 Ω parallel coupled resistors, located between the amplifier output and transmitter. The multimeter has a voltage resolution of 0.01 V in the used 500 V operating range, with an accuracy between 0.4 and 0.7 % in the vLF region. The current resolution in the used 10 A operating range is 0.001 A, with an accuracy of 3 % for vLF frequencies.

4.1.5 Preamplifier

The Stanford Research Systems SR560 low-noise preamplifier provides DC-coupled amplification with a gain ranging from 1 to 50000, and is capable to output a maximum of 10 Vpp. The noise is less than 4 nV Hz$^{-1/2}$, which makes it ideal to measure weak induced voltages. In addition to amplifying the signal the preamplifier also contains a variable BP filter with 6 dB/octave roll-off and 10 % accuracy, enabling it to filter out both low-frequency noise from the electric power grid and high frequency noise from radio transmitters in the area. During measurements the amplifier gain was set to 50000, with filter passband between 1 and 100 kHz.

4.1.6 Oscilloscope

Rigol DS1054Z is a digital oscilloscope with 50 MHz bandwidth, four analog channels, real-time sample rate 1 GS s$^{-1}$ and 4 ns peak detection. The instrument was set up in continuous mode and connected to both the input and output channels of the DAQ module to monitor the generated and acquired signals in real-time.

4.2 Receiver designs

Three different receivers were designed and used during measurements, as illustrated in figure 4.2. The first receiver was a simple model coil, consisting of a single copper wire loop with enclosed area 12.5 cm$^2$. The dimensions were chosen to provide a loop area close to the ones found on a PCB and, since the parameters of this receiver were known, enable a numerical analysis and comparison with theoretical induced voltages. The second and third receiver were both designed by soldering a closed loop on a PCB, of the same type as described in table 2.2, representing the enclosed area created when integrated circuits are attached to the board. The small PCB receiver had a loop area of approximately 12.5 cm$^2$ and the
large around 5.5 cm$^2$. All receivers were connected to the preamplifier via a load resistor of 100 kΩ and a 1 m coaxial cable with capacitance $75 \times 10^{-12}$ F m$^{-1}$, which together make up the receiver load impedance in figure 2.7.

**Figure 4.2:** The receivers designed for experiments. **Left:** Model coil with the copper wire loop visible. **Middle:** Small PCB receiver. **Right:** Large PCB receiver. The closed loops in both circuit boards are indicated in red.

### 4.3 Transmitter design

The transmitter coil was designed to maximize the magnetic flux and induce as much voltage as possible in the receiver. As previously seen in section 2.4.2, this is achieved by having a large coil radius with many turns of wire. To avoid efficiency losses however, the DC impedance of the coil had to match the 8 Ω output impedance of the vlf amplifier. Another constraint was the access to coil wire, which limited the material to copper and the total length of the winding to 1000 m. The specifications of the wire used are presented in table 4.2.

**Table 4.2:** Physical properties for the copper wire used in the coils.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire diameter</td>
<td>0.85 mm</td>
</tr>
<tr>
<td>Resistivity</td>
<td>$1.67 \times 10^{-8}$ Ω m</td>
</tr>
<tr>
<td>Conductivity</td>
<td>$5.99 \times 10^7$ S m$^{-1}$</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>$\approx 1.0$</td>
</tr>
</tbody>
</table>

The optimization was performed using MATLAB software. A grid with $N_t$ ranging between 1 and 150 turns and $a_t$ between 0.01 and 0.4 m was created and used in equations (2.35) and (2.40) to yield the total transmitter impedance for all practical combinations of $N_t$ and $a_t$. The combinations resulting in a DC impedance of $8 \pm 0.5$ Ω was subsequently used in equations (2.33) and (2.39) to compute induced voltage as a function of $N_t$ and $a_t$. The result is shown in figure 4.3 below. The maximum induced voltage was achieved with a transmitter radius of 0.4 m and 112 wire turns. Consequently these parameters were chosen when manufacturing the transmitter, which is shown mounted on a tripod in figure 4.4.
4.4 Experimental setups

The experimental work was performed on FOI grounds. The test areas were selected as to maximize the distance to interfering objects, while providing a genuine test environment for each scenario. The locations, labeled 1 to 5, are shown in figure 4.5. All studies were performed according to safety regulations concerning time-varying magnetic fields. The reference level for public exposure in the VLF region is $2.7 \times 10^{-5}$ T, which is well above the fields produced by this system [31].

**Figure 4.3:** Optimization of transmitter coil parameters. **Left:** Transmitter DC impedance as a function of coil radius and number of turns, with the $8 \pm 0.5 \ \Omega$ limits marked in red. **Right:** Induced voltage in a theoretical receiver as a function of coil radius and number of turns.

**Figure 4.4:** The finished transmitter used in the thesis. The copper wire was winded on a Divinycell board and attached to a tripod stand for stability.
The experimental procedure varied slightly depending on the different setups and goals, but can in general be described by the following steps:

i. The transmitter and receiver were placed coaxially facing each other, using the calibration software to find the optimal lineup.

ii. The background PSD was collected with the transmitter switched off.

iii. The received spectra from a transmitted frequency sweep between 3 and 30 kHz was acquired.

iv. The receiver type was interchanged and the sweep repeated.

v. The transmitter/receiver position was changed and steps iii–v were repeated.

4.4.1 Study 1 - LoS transmissions

In the first study transmissions under LoS conditions were investigated. The objective was to verify the theory and get a relationship between the three receivers at multiple transmission distances. This also enabled extrapolation of results obtained from the model coil in following experiments. The experiment was performed in test area 1, which is far away from any buildings or electrical grids. The transmitter remained fixed throughout the study while the distance to the receiver was increased gradually, as illustrated in figure 4.6 below.

**Figure 4.6:** Illustration of the setup used in study 1.
4.4 Experimental setups

4.4.2 Study 2 - Indoor environments

In the second study only the model coil and small PCB were used. The purpose was to investigate the effects of placing the receiver indoors while the transmitter is located outdoors, obstructing the transmission with a solid wall. The test was conducted in area 5, with the transmitter fixed 1 m outside a 40 cm thick wall of brick and concrete. The induced voltage was measured at three distances: 1.8 m, 5 m and 10 m from the transmitter. The model coil was used for all distances and the small PCB receiver was used at 5 m only. The setup is illustrated in figure 4.7.

![Figure 4.7: Illustration of the setup used in study 2.](image)

4.4.3 Study 3 - Car measurements

This study aimed at investigating the disturbance caused by cars, which have conductive and possibly ferromagnetic metals in the chassis. The transmitter was placed at a fixed distance of 5 m from the car, as illustrated in figure 4.8. The model coil was placed at four different positions on the car: underneath on the chassis, inside the engine hood, on the passenger seat and inside the trunk. The small PCB was also used for measuring underneath the car. The measurements were performed in test area 2 and repeated twice, the second time as a reference with the car removed and the receiver at the same positions.

![Figure 4.8: Illustration of the setup used in study 3.](image)

4.4.4 Study 4 - Ground obstruction

In the fourth study the model coil and small PCB were buried beneath 0.2 m of soil and obstructed by an additional 5 m of elevated soil. The objective was to investigate how ground obstructions affect transmissions. The experiment was conducted in test area 4 where a wooden box, $5 \times 3 \times 0.8$ m in size and containing representative soil, was located. The transmitter was fixed at the edge of the box, 5 m from the receivers, and the receivers buried right next to the opposite edge. A reference measurement 0.5 m above ground was also carried out. An illustration is shown in figure 4.9 below.
4.4.5 Study 5 - Metal obstacles

The objective of the fifth study was to investigate the effect of large metal obstructions. The test was performed in area 3 where a shipping container, $5.5 \times 2.4 \times 2.5$ m in size and made of iron, was situated. Measurements were carried out with the container obstructing the path between the model coil and transmitter, followed by a reference measurement without any obstructions. The transmission distance remained fixed at 5 m throughout. The setup is illustrated in figure 4.10 below.

4.4.6 Study 6 - Electromagnetic shielding

In the last study the goal was to investigate the effects of EM shielding. In the first part a closed circular loop with diameter 7 cm was solded together from a standard jump cable. It was then fixed coaxially around the model coil in order to produce a cancelling magnetic flux in the opposite direction of the transmission. In the second part the receiver was covered with varying layers of aluminum foil. 1, 3, 5 and 10 layers were tested, with a thickness of 14 μm per layer. The receiver and transmitter were fixed 5 m from each other in test area 3, as illustrated in figure 4.11. A comparative reference measurement without any shielding was also performed at the same location and distance.
In this chapter the results of the NFMI investigations are presented. The experimental setup and induced RMS voltage as a function of frequency are featured in a separate section for each study, starting with study 1 and finishing with study 6. The chapter concludes with a comparison across all studies to assess under which conditions the voltage were cancelled out the most. All experiments were performed using the NFMI system described in chapter 4, with aforementioned measurement parameters summarized in table 5.1 below.

**Table 5.1: Summary of system parameters used during measurements.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency sweep range</td>
<td>3–30 kHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>100 kHz</td>
</tr>
<tr>
<td>DAQ voltage range</td>
<td>±5 V</td>
</tr>
<tr>
<td>Preamplifier gain</td>
<td>50000x</td>
</tr>
<tr>
<td>BP filter passband</td>
<td>1–100 kHz</td>
</tr>
</tbody>
</table>

Since the transmitter power, and in extension the RMS current in the coil, varied depending on the transmission frequency, all obtained voltages were multiplied by a normalization factor $k(f)$ given by the following expression:

$$k(f) = \frac{I_{\text{theoretical}}}{I_{\text{actual}}(f)}.$$  \hfill (5.1)

Here $I_{\text{theoretical}}$ was set to 1 A while $I_{\text{actual}}(f)$ was measured with the multimeter at each frequency. The results can thus be interpreted as induced RMS voltage achieved with transmitter current 1 A, which can easily be scaled to other currents.
5.1 LoS transmissions

In this section results from the free space measurement is presented. The results were obtained using the experimental setup shown in figure 5.1 below. Data was acquired during 2 seconds for each frequency, resulting in a total of 200000 × 28 samples per sweep. The Welch window size was set to 8192 during evaluation, with a 50 % window overlap of 4096 samples.

![Experimental setup used in study 1. The transmitter and receiver were placed in an open area without any obstacles. The other system components can be seen in the bottom left of the picture.](image)

Figure 5.1: Experimental setup used in study 1. The transmitter and receiver were placed in an open area without any obstacles. The other system components can be seen in the bottom left of the picture.

Figure 5.2 shows induced voltage as a function of frequency for all three receiver types, plotted relative to each other at the four different transmission distances. The results were compared with the theoretical voltage for the model coil, calculated using equation (2.33). It can be seen that the theoretical voltage agrees very well with the obtained voltage at 2.5, 5 and 10 m, showing a distinct linear trend with increasing frequencies. At 15 m the measured voltage deviates more from the theoretical curve, maintaining a linear frequency dependence but with a magnitude several times higher than predicted.

An induced voltage is observed in both PCB receivers, albeit lacking a clear linear frequency dependence at shorter distances. The small PCB receiver, which had approximately the same loop area as the model coil, demonstrates an almost identical voltage curve at low frequencies before starting to deviate at 6 kHz and eventually levelling out above 20 kHz. The large PCB receiver, which had a smaller loop area, exhibits lower induced voltage than the model coil and small PCB at 2.5 m and 5 m, but had the same signal magnitude at 10 m and 15 m.

Comparisons of induced voltage at different transmission distances are shown in figures 5.3 and 5.4 for each individual receiver type. The measured voltage was plotted against the theoretical voltage for the model coil at each distance. Again it is evident that the induced voltage measured at 10 and 15 m has the same order of magnitude in all three receivers, although it is expected to be significantly lower. As the distance is doubled, first from 2.5 to 5 m and then to 10 m, all receivers experience an eight-fold decrease in induced voltage.
5.1 LoS transmissions

Figure 5.2: Comparison of induced RMS voltage at 2.5 m (top left), 5 m (top right), 10 m (bottom left) and 15 m (bottom right) for all receiver types.

Figure 5.3: Comparison of induced RMS voltage in the model coil at different transmission distances.
Figure 5.4: Comparison of induced RMS voltage in the small PCB (left) and large PCB (right) at different transmission distances.

5.2 Indoor environments

The second study was performed using the setup in figure 5.5. The data acquisition time was increased to 10 seconds per frequency, yielding $1000000 \times 28$ samples for each sweep. The Welch window size was 250000, with a 125000 sample overlap.

Figure 5.5: Experimental setup used in study 2. The transmitter was located outside the building (left) with the receiver on the inside (right).

The indoor environment influenced the magnitude substantially, as shown in figure 5.6. The induced voltage is several times lower than the predicted voltage levels at both 1.8 m and 5 m. At 10 m the measured voltage surpasses the theoretical at frequencies above 8 kHz, and ultimately even exceeds the voltage obtained at 5 m. Comparing the results to the free space measurements in figure 5.3 it is noticeable that the signal is more noisy indoors, in particular at 5 and 10 m. A comparison of induced voltage in all receiver types can be seen in figure 5.7, plotted against the theoretical voltage for the model coil. It is evident when comparing to the
corresponding graph in figure 5.2 that the PCB receivers achieves higher induced voltage in the indoor environment than outdoors. There is a considerable amount of noise present in all three measurements, especially above 18 kHz. The noise causes fluctuations in measured voltage of more than 10 µV.

**Figure 5.6:** Comparison of induced RMS voltage in the model coil in an indoor environment at different transmission distances.

**Figure 5.7:** Comparison of induced RMS voltage for all receiver types in an indoor environment. The transmission distance was fixed at 5 m.
5.3 Car measurements

Results from the third study is presented in this section. During measurements the data acquisition time was kept at 10 seconds per frequency, with the Welch window size and overlap remaining at 250000 and 125000 samples respectively. The setup is shown in figure 5.8 for the measurement under the engine hood.

![Experimental setup used in study 3. The car and transmitter were placed at a fixed distance from each other during all measurements.](image)

Figure 5.8: Experimental setup used in study 3. The car and transmitter were placed at a fixed distance from each other during all measurements.

Figure 5.9 shows induced voltages inside the engine hood and underneath the chassi for the model coil, compared with the reference measurements and theoretical voltages at the same positions. Placing the receiver inside the engine hood does not result in any significant increase or attenuation of the induced voltage, remaining at the same level as the reference for all frequencies. Underneath the chassi a slight increase in measured voltage is visible, while the reference level is somewhat lower than the theoretical curve.

![Induced RMS voltage inside the engine hood (left) and underneath the car chassi (right), compared with a reference measurement at the same position. The transmission distances were 5.9 and 5 m respectively.](image)

Figure 5.9: Induced RMS voltage inside the engine hood (left) and underneath the car chassi (right), compared with a reference measurement at the same position. The transmission distances were 5.9 and 5 m respectively.

Placing the receiver inside the trunk or interior of the car results in considerable attenuation of induced voltage, as shown in figure 5.10. The linear frequency
dependence almost disappears, and the signal level is several times lower than both the theoretical curve and the reference measurement. A distinct peak can be seen at 27 kHz in both the reference- and measured voltage curve.

![Graph showing induced RMS voltage in the interior (left) and trunk (right) of the car, compared with a reference measurement at the same position. The receiver was located 6.2 and 6.15 m from the transmitter respectively.](image)

**Figure 5.10:** Induced RMS voltage in the interior (left) and trunk (right) of the car, compared with a reference measurement at the same position. The receiver was located 6.2 and 6.15 m from the transmitter respectively.

### 5.4 Ground obstruction

In this section results from the fourth study is presented. Again the data acquisition time was set to 10 seconds per frequency, and the Welch window size was also maintained at 250000 samples with a 50% window overlap. The experimental setup is shown in figure 5.11 below.

![Experimental setup used in study 4. The receivers were located behind the wooden box and buried beneath 2 dm of soil.](image)

**Figure 5.11:** Experimental setup used in study 4. The receivers were located behind the wooden box and buried beneath 2 dm of soil.

A comparison between induced voltage underground and above ground is shown in figure 5.12 below, along with the theoretical voltage curve. It is apparent that noise effects increase appreciably when the receivers are buried below ground. Compared with the reference measurement both model coil and PCB have a very
unstable signal, in particular for frequencies above 15 kHz. Neither of the receivers experience any attenuation of the induced voltage as a result of being buried underground. The reference measurement closely follows the theoretical curve and remains similar to the free space measurement in figure 5.2.

![Graph](image.png)

**Figure 5.12:** Induced RMS voltage in the model coil (left) and small PCB (right) when buried underground, compared with a reference measurement above ground at the same distance.

### 5.5 Metal obstacles

The fifth study was carried out using the setup shown in figure 5.13. The data acquisition time was 10 seconds per frequency, and the Welch window size was set to 250000 samples with a 50% window overlap.

![Image](image.png)

**Figure 5.13:** Experimental setup used in study 5. Both receiver and transmitter were located in the middle of the container at opposite sides.

The resulting induced voltage is shown in figure 5.14, where it is compared with the theoretical voltage and a reference measurement at the same distance. The linear relationship between voltage and frequency is still visible, but the voltage is attenuated substantially compared with the reference measurement.
5.6 Electromagnetic shielding

The experimental setup of the sixth study is shown in figure 5.15. Once again a 10 second data acquisition time was used, with a Welch window size of 250000 samples and a 50% window overlap used during evaluation.

Adding additional layers of aluminum foil eventually cancels out the induced voltage completely, as shown in figure 5.16. The voltage signal increasingly flattens out at higher frequencies as the thickness increases. At 70 μm and above the signal is almost completely independent of frequency. At lower frequencies the flattening is not quite as appreciable, and even with a 70 μm aluminum layer a frequency dependence is visible up to 10 kHz.
Figure 5.17 shows that the opposite magnetic flux created by the conductive wire loop almost entirely cancels out induced voltage in the receiver. The frequency dependence is still visible, but the magnitude of the signal is several times lower than in the reference measurement.

![Graph showing induced voltage with and without flux-cancelling wire loop](image)

**Figure 5.16:** Induced RMS voltage in the model coil with various layers of aluminum foil, compared with a reference measurement at the same distance.

![Graph showing theoretical and induced voltage](image)

**Figure 5.17:** Induced RMS voltage in the model coil with a flux-cancelling wire loop, compared with a reference measurement at the same distance.
5.7 Comparison of results

As a final comparison all measurements performed at 5 m were compared with each other to investigate under which conditions induced voltage is cancelled the most. The results are shown in figure 5.18. It can be seen that neither placing the receiver indoors or burying it underground has a substantial effect on induced voltage magnitude. On the other hand the signal becomes very noisy in both cases. Inside the trunk or interior of a car the cancellation effects are much more tangible, with induced voltage decreasing by a full order of magnitude in both measurements. Using an obstructing metal container, a cancelling wire loop or 140 µm of aluminum foil all result in approximately the same level of attenuation, clearly visible but not quite as large as placing the receiver inside a car.

![Figure 5.18: Comparison of induced RMS voltage cancellation in measurements performed with 5 m transmission distance.](image-url)
In this chapter the results are analyzed and put into a broader context. The applicability of the model NFMI system to predict induced PCB voltage under different conditions is discussed, along with any issues or sources of error that might have affected the results. Limitations surrounding the method and equipment are also accounted for in order to propose further research in the field.

6.1 Results

Under LOS conditions the NFMI model and quasi-static approximation for radiating magnetic fields accurately predict induced voltage in a receiver coil. When the distance is doubled the signal decreases by a factor of approximately 8, which agrees with the inverse cube relationship for radiating EM fields in the near-field, given by equation (2.25). The PCB receivers demonstrate a nonlinear relationship with the transmission frequency, with a decline in voltage at higher frequencies. This is most likely due to the intermediary copper sheets inside the PCB and the eddy currents that arise within these layers. In equation (2.11) it can be seen that the attenuation constant is expected to increase with the square-root of frequency, which explains the observations.

Placing the receivers indoors resulted in a more unpredictable situation, reflected by the graphs in figure 5.6. The EM field propagation indoors is more complicated due to the large amount of conductive objects commonly found in a building. It is possible that materials with high relative permeability act as waveguides, similar to the seafloor during underwater propagation as shown in figure 3.3. This may be one of the reasons why the induced voltage obtained at 10 m was higher than at 5 m, and also why the signal levels are lower than predicted by equation 2.33.
Cars are very effective in cancelling out induced voltage, but only at certain positions. The difference between the graphs in figures 5.9 and 5.10 could be explained by the fact that both the trunk and interior was completely surrounded by conductive sheet metal, acting as a shielding layer, while the engine hood and undercarriage positions were more exposed to the transmitted EM field. Since the engine hood has no undercarriage the waves may have been able to bend around the edges of the sheet metal on the sides to reach the receiver. The peak at 27 kHz is present in both the measurement curve and reference, indicating that the results are influenced by external factors. Underground power cables or resonating metal objects might be possible explanations, but both theories are hard to verify.

The induced voltage magnitude is not affected significantly by placing the receivers underground. This is not surprising considering that VLF frequencies are used in underground communications systems due to their ability to penetrate ground. A single box with 5 m of soil is simply not enough to cause a significant attenuation of EM waves with a wavelength several kilometers long. The skin depth was shown in table 3.1 to be affected by many different factors, for example water and mineral content in the soil, which is the most probable reason why the signal is unstable compared to the LOS situation.

There are several ways of cancelling out induced voltage in a PCB. For the case of the container there are some fringe effects present, where the EM field manages to bend around the container edges and cause a weak signal in the receiver. The graphs in figure 5.16 illustrates how conductive materials in general are very effective in attenuating EM fields. Covering the receiver with aluminum foil seems to prevent the fringe effects observed with the container, resulting in a very effective signal cancellation. The conductive wire loop results in good cancellation, but might have yielded even better results with a smaller loop area. The extra area most likely leads to flux leakage and less effective cancellation.

Overall, the results obtained from measurements are in line with the theoretical framework for NFMI transmissions. The theory holds well in an ideal LOS setup as well as in more realistic situations with noise and obstacles. The VLF band demonstrates a good trade-off between propagation ability and induced voltage strength. At lower frequencies the signal is weaker due to the linear frequency dependence, and at higher frequencies the eddy currents attenuates the EM field to counteract any increase in voltage. Based on the graphs it can be concluded that an induced RMS voltage strength of 65 µV can be achieved in a PCB at a distance of 2.5 m and under ideal conditions. Increasing the distance to 5 m yields a maximum voltage of 9 µV, and going as far as 10 or 15 m results in approximately 2 µV. However, it is questionable if the curve at 15 m is a result of the weak transmission signal or caused primarily by the background noise.

6.2 Sources of error

Due to the experimental nature of this thesis there are many sources of error that might have affected the results. The most significant source of error is the
background noise, which adds to the received signal spectrum and in extension also the calculated induced voltage. At a distance of 15 m the background spectrum and received signal spectrum have approximately the same magnitude, leading to a calculated voltage virtually independent of frequency. The greatest source of background noise is the electric power grid, which produces very strong signals at multiple frequencies and is present in almost all urban environments. The background spectra collected during experiments (see appendix C) reveals that indoor environments are affected in particular, with multiple strong peaks in the VLF region.

Another phenomenon which has to be taken into account in spectrum estimates such as Welch’s method is aliasing. With a sampling frequency of 100 kHz the Nyquist frequency is just slightly above the VLF band at 50 kHz. Transmitting at certain frequencies can therefore result in aliasing and stronger spectrum peaks than actually received. Getting rid of aliasing effects would require an anti-aliasing filter, which was not available for this work.

6.3 Method

The experimental nature of this thesis had both advantages and disadvantages. The main advantage is that the simulation environments that would have been required to accurately predict induced voltages are very complicated and time-consuming to set up. Even with a very complex model, incorporating factors such as surrounding buildings and water content in the ground, the simulations would still have to be verified experimentally to be useful. However, simulations could also have provided insights into EM field propagation that the experimental results cannot answer. Visualization of the fringe effects around conductive objects, induced voltage as a function of both frequency and distance and the possible advantages of waveguides are all subjects that are well suited for future work using simulations.

There are also some issues regarding the equipment used in the NFMl system. The VLF amplifier was optimized to operate with audio signals, i.e. in the region 2–20 kHz. It is not known whether this affected the measurements when transmitting at a frequency above 20 kHz, but it might have contributed to the noisy behaviour in some of the graphs. The transmitter coil was designed with the best materials available, but is far from the ideal coil described in the theory. An improved transmitter coil could for example utilize Litz wires to increase the number of wire turns, and in extension the transmitted magnetic flux. The wiring was also far from ideal, with some unintentional separations that yields magnetic flux leakage.

Some practical issues also arose during the execution of the experimental studies. First of all the desired coaxial transmitter-receiver alignment was not always easy to achieve, despite using the calibration software. This was in particular the case during the indoor and container studies in which there were no unobstructed view of the setup. A general non-coaxial model could be an interesting topic for future research, but in this thesis the setups were to remain as similar as possible.
To achieve good resolution in the spectrum a large Welch window size had to be used, which in turn required a large amount of samples. In studies 2-6 a total of 28000000 samples were obtained per sweep, each having a size of 330 MB. As the device used for data storage had a capacity of only 16 GB, this was the maximum amount of samples that was practical to acquire. This is also part of the reason why the data acquisition time in study 1 is lower than in the rest. Having an acquisition time of 10 seconds per frequency would have resulted in several hours of extra time necessary to complete the tests, and would have required more storage space than available.
In this chapter the thesis is concluded. The questions posed in the problem statement are answered using the conclusions drawn from the results and ensuing discussion. Suggestions are given for further research in the field, which could provide more insights into EM wave propagation in general and PCB induction in particular.

7.1 Concluding remarks

In this thesis the possibility of EM induction in a PCB has been investigated. A NFMI system for low frequency magnetic transmissions has been designed, based on inductively coupled transmitter and receiver circuits. Using experimental studies in a number of realistic environments, answers to the questions in the problem statement have subsequently been found:

1. How do magnetic transmissions with low frequency work, and how does the EM field propagate in the near-field region?

   Time-harmonic currents in a closed wire loop give rise to radiating EM waves, explained by Maxwell’s equations. Faraday’s law for EM induction results in magnetic coupling between a transmitter and receiver coil, which is utilized to transfer information or power in a NFMI system.

   The plane wave model is sufficient to explain near-field propagation in most cases. The quasi-static approximation results in a EM field that attenuates with the inverse cube of distance, which could be observed experimentally. In linear and isotropic media EM waves attenuate due to the complex wave number and intrinsic properties of the material. Losses are also caused by scattering and reflection in media interfaces.
2. How are printed circuit boards affected by alternating magnetic fields in a near-field environment?

(a) Is it possible to detect induced voltages in a PCB?

A clear induced voltage can be detected at distances up to 10 m in LOS conditions, with the frequency domain providing an effective way of measuring the signal. The conductive connections on a PCB are large enough to obtain a signal at even greater distances, but it is harder to differentiate induced voltage from noise.

(b) How large voltages can be achieved at various distances?

In a LOS situation the maximum obtained voltage was 65 μV, at a transmission distance of 2.5 m. At 5 m the voltage dropped to 9 μV, and at 10 m it decreased to around 2 μV. In environments with a lot of conductive and magnetic materials present the voltages are generally lower, but waveguide effects can result in higher voltages as well.

(c) How does the frequency affect the amount of induced voltage?

There is a nonlinear relationship between induced voltage and frequency in the VLF region. At lower frequencies the PCB voltage follows the expected linear curve, but at higher frequencies eddy currents in the embedded copper sheets counteract the increasing voltage to yield a frequency-independent signal. The penetration ability of EM waves also affects the amount of induced voltage. In the VLF region the waves were able to penetrate both solid walls and several meters of soil, demonstrating a good trade-off between propagation and inductive properties.

(d) How do common EM noise sources and obstacles affect induced voltage strength in a PCB?

- Indoor environments result in a noisy and unpredictable voltage due to the large amount of electronics and metal objects present. Voltage strength can increase significantly depending on the surroundings, but is generally lower than under LOS conditions.

- Cars are very effective in cancelling out induced PCB voltage when placed in the trunk or the interior, due to the shielding metal in the chassis and sheet plates. In the engine hood and undercarriage the induced voltage remains unaffected.

- Solid ground do not affect voltage strength, but increases the amount of noise in the signal.

- Obstacles made of metal cancel out most of the induced voltage, but due to fringe effects the voltage is not entirely lost.

- Shielding the PCB with a flux-cancelling wire loop reduces the voltage significantly. A covering 42 μm thick metal foil is sufficient to yield a negligible voltage under the conditions tested in this thesis.
7.2 Future work

During the course of this work several interesting topics to research further in the field were discovered. The following list contains suggestions for future work, including some interesting NFMI scenarios and setups.

- **Non-coaxial coil alignments:** In this thesis all receivers were aligned coaxially with the transmitter. In most applications however such a setup can not be guaranteed, which is an incentive to investigate a wider range of NFMI setups and coil angles.

- **Simulations of magnetic field propagation:** In order to properly understand EM wave propagation in the near-field, and to visualize the fringe effects that might occur for certain obstacles, simulations are necessary. It would also be interesting to setup the studies again in a simulation environment and compare the results.

- **Wider frequency range:** Limitations on time and instrumentation meant that frequencies outside the VLF range were not investigated. Effects that appear in a PCB outside this region might be of interest though, so it would be natural to expand the frequency range and look into more scenarios.

- **Investigation of waveguides:** The use of certain materials as waveguides has been investigated in previous works, and was believed to affect the results obtained from the second study. It would be interesting to design and evaluate such a waveguide with a NFMI system, and investigate if the range and induced voltage strength can be improved.

- **Information extraction:** An interesting question arose when it was concluded that NFMI transmissions can affect a PCB: Is it possible to detect the EM field produced by a PCB to extract information about the components on the board or their operations? The demand for safe electronics make this a very interesting topic for the future.

- **Effects on electronic devices:** In this work no PCB had actual components attached to them. What would happen if this was the case, and could the function of electronic devices be affected if the PCB was subjected to EM induction? This is a very interesting question, and one which can be investigated for a variety of devices and possible countermeasures.
Appendices
In this chapter of the appendix the equations used to theoretically calculate induced voltage over PCB components are derived. The expressions describing the NFMI system are derived analytically for the model in section 2.4, using Kirchhoff’s and Ohm’s laws and the relations in section 2.3. The circuit diagram for the model is illustrated in figure A.1. The receiver in the secondary circuit has a load consisting of a theoretical component resistance $R_L$ and a cable capacitance $C$ connected in parallel.

![Circuit Diagram](image)

**Figure A.1: Circuit diagram of the NFMI system model.**

Since the signals involved are all sinusoidal, the circuit can be transformed into its equivalent circuit by applying the relations

\[
Z_L = j\omega L, \quad \text{(A.1a)}
\]

\[
Z_C = \frac{1}{j\omega C}, \quad \text{(A.1b)}
\]

from which the circuit in figure A.2 is obtained.
A Circuit analysis

The parallel load impedances in the secondary circuit can be replaced with a single equivalent load impedance $Z_{\text{load}}$, given by the relation

$$\frac{1}{Z_{\text{load}}} = \frac{1}{R_L} + \frac{1}{1/j\omega C} \iff Z_{\text{load}} = \frac{R_L}{1 + j\omega CR_L},$$

(A.2)

which results in the modified circuit in figure A.3. Here the voltage $U_{\text{load}}$ over the load impedance has been introduced, along with the generated voltage $U_t$ over the transmitter inductor and the induced emf $U_r$ over the receiver inductor.

The coil potentials can be derived from the definitions of mutual inductance and self-inductance in section 2.3, which results in the expression

$$U_r = j\omega MI_t - j\omega L_r I_r,$$

(A.3)

where the minus sign appears due to Lenz’s law. Ohm’s law can now be used in combination with Kirchhoff’s voltage law in the secondary circuit to rewrite the load potential and receiver current as

$$U_{\text{load}} = j\omega MI_t - j\omega L_r I_r - R_r I_r,$$

(A.4a)

$$I_r = U_{\text{load}}/Z_{\text{load}}.$$  

(A.4b)

When combining these two equations the result is an expression for load voltage which only depends on known component parameters and transmitter current:

$$U_{\text{load}} = \frac{j\omega MI_t}{Z_{\text{load}} + j\omega L_r + R_r Z_{\text{load}}}.$$  

(A.5)
In this chapter of the appendix pseudocode for the MATLAB scripts are presented, as an insight into the algorithms used to acquire measurement data. The functionality of the system relied primarily on DataAcquisition.m and DataProcessing.m, with Calibration.m in an auxiliary function during field experiments. The programs are presented in algorithms 1 to 3 below.

**Algorithm 1: Calibration.m**

**Output:** real-time plot of Welch’s spectrum estimate  
**Input:** plot duration, Welch window size, sampling frequency, signal amplitude, frequency sweep vector

```
begin
  Initialize parameters;
  Setup DAQ module;
  for all frequencies in sweep vector do
    generate continuous sinusoid in transmitter;
    while elapsed time < plot duration do
      collect data during a short time interval;
      calculate Welch’s spectrum estimate;
      update spectrum plot;
    end
  terminate transmission;
  reset elapsed time;
end
```
Algorithm 2: DataAcquisition.m

**Output:** signal matrix containing measured data for each frequency in sweep  
**Input:** data acquisition time, Welch window size, sampling frequency,  
signal amplitude, frequency sweep vector, file name

```
begin
    Initialize parameters;
    Setup DAQ module;
    for all frequencies in sweep vector do
        generate continuous sinusoid in transmitter;
        collect data during specified acquisition time;
        add acquired signal to matrix;
    end
    Save workspace including signal matrix with given file name;
end
```

Algorithm 3: DataProcessing.m

**Output:** plots with measured and theoretical load voltage versus frequency  
**Input:** file name, distance, plot range

```
begin
    Load signal matrix from file;
    Initialize parameters and physical constants;
    for all frequencies in plot range do
        Plot acquired signal;
        Calculate Welch’s spectrum estimate;
        Plot acquired spectrum;
        Calculate theoretical voltage with transmitter current 1 A;
        Calculate measured voltage using pwelch;
        Normalize result with respect to measured transmitter current;
    end
    Plot measured and theoretical voltage versus frequency;
    Save plot with given file name;
end
```
In this chapter of the appendix plots that were omitted in chapter 5 are presented. In figure C.1 an example of aliasing is shown, obtained during the LOS measurements with the small PCB receiver. The transmission frequency was 30 kHz during acquisition. Additional peaks appear at multiples of 10 kHz and adds to the actual received signal peak at 30 kHz, resulting in the sudden spike in voltage in figure 5.2. In figure C.2 background spectra obtained during all studies are shown.

**Figure C.1:** Aliasing in the received signal spectrum obtained during study 1, using the small PCB receiver and a transmission frequency of 30 kHz.
Figure C.2: Background spectrum of each individual study, plotted from 0 kHz to the Nyquist frequency 50 kHz. **Top left:** Study 1. **Top right:** Study 2. **Middle left:** Study 3. **Middle right:** Study 4. **Bottom left:** Study 5. **Bottom right:** Study 6.
Bibliography


