Probabilistic Analysis of Brake Noise

A Hierarchical Multi-fidelity Statistical Approach

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Master of Science Thesis in Mechanical Engineering


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Abstract

Computer Aided Engineering driven analysis is gaining grounds in automotive industry. Prediction of brake noise using CAE techniques has become popular due to its overall low cost as compared to physical testing. However, the presence of several uncertain parameters which affect brake noise and also the lack of basic understanding about brake noise, makes it difficult to make reliable decisions based on CAE analysis. Therefore, the confidence level in CAE techniques has to be increased to ensure reliability and robustness in the CAE solutions which support design work. One such way to achieve reliability in the CAE analysis is investigated in this thesis by incorporating the effects of different sources of uncertainty and variability in the analysis and estimating the probability of design failure (probability of brake noise above a certain threshold). While incorporating the uncertainties in the CAE analysis ensures robustness, it is computationally intensive. This thesis work aims to gain an understanding about a brake noise - creep groan, and to bring robustness into the CAE analysis along with reduction in computational time.

A probabilistic analysis technique called hierarchical multi-fidelity statistical approach is explored in this thesis work, to estimate the probability of design failure or design robustness at a faster rate. It incorporates the stochasticity in the input parameters while running simulations. The method involves application of a hierarchy of approximations to the system response computed with variations in mesh resolution or variations in number of modes or changing solver time step, etc. And finally it uses the probability theory, to relate the information provided by approximate solutions to get the target failure estimation. Through this method, reliable data regarding the probability of design failure was approximated for every simulation and at a reduced computational time. Additionally, it provided information about critical parameters that influenced brake noise which was meritorious for design management. Estimation of probability of design failure by this method has been proved to be reliable in the case of brake noise according to the simulation results and the method can be considered robust.

KEYWORDS: Uncertainty and variability, probabilistic analysis, hierarchical multi-fidelity statistical approach, probability of design failure, creep groan.
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### ABBREVIATIONS

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<th>Abbreviation</th>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>NVH</td>
<td>Noise Vibration and Harshness</td>
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<td>CAE</td>
<td>Computer-Aided Engineering</td>
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<td>CEA</td>
<td>Complex Eigenvalue Analysis</td>
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<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
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<td>ADAMS $^{TM}$</td>
<td>Automated Dynamic Analysis of Mechanical Systems</td>
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<tr>
<td>NTF</td>
<td>Noise Transfer Function</td>
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<td>ANOVA</td>
<td>Analysis of Variance</td>
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<tr>
<td>CCDF</td>
<td>Complimentary Cumulative Distribution Function</td>
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<td>PDF</td>
<td>Probability Distribution Function</td>
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<td>LFM</td>
<td>Lower Fidelity Model</td>
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Introduction

1.1 Brake Noise - Background

Brakes are one of the most important parts in vehicles regarding the safety and performance attributes. They have undergone several changes in their design and development from simple block brakes in the nineteenth century to the present era of high performance brakes, while in the past century, the major focus was on increasing braking characteristics and reliability. In the recent decades focus has been towards customer comfort referring to vibrations and acoustics in general. Unwanted noise emerging from brakes often cause acoustic discomfort for the passengers in the vehicle and this results in major unexpected warranty claims from customers. This is because unattended brake noise often creates the assumption of defective brakes although the functionality of the brake is in proper conditions. Due to these constant developments in the overall vehicle design and improvements in acoustics, focus about noise and vibrations have increasingly gained importance [46].

Often brakes tend to produce noise induced by frictional instabilities. Such brake noise events are triggered by both forced and self-excited vibrations, induced by frictional forces. During the process of braking, the friction pads are brought into contact with a disc surface. On contact, most of the kinetic energy is converted into heat by friction and a part of it is converted into energy to the system which leads to noise [11].
1.2 CAE driven product development

Substantial developments in the analysis of brake noise have been achieved in the past decades through deterministic analysis of dynamical behaviour of disc and brake pad using simulation tools (e.g., Finite Element Analysis) and experimental methods. Application of Computer Aided Engineering (CAE) tools have helped to simulate the dynamics of the system, analyzing the existing issues to obtain an understanding of interaction and develop counter measures to solve identified issues. CAE has facilitated in verifying the design by performing most of the analysis on virtual models thereby lowering the costs of physical prototypes involved [15]. Deterministic analysis methods such as Finite Element Methods, can be applied in different design stages in several iterations of the process. Introduction of CAE tools early in the design process makes the design cycle a bit longer, but assists in identification of errors at a faster rate and also allows more exploration of the design space. This facilitates in reducing the number of physical prototypes involved in testing [1]. Therefore, there is a big push in automotive industry to finalize engineering decisions for analysis of brake noises, based on CAE solutions applied at an early design and development phase.

The process of CAE driven product development is shown in the figure 1.1.

**Product Development – Traditional way**

- **Concept Design**
- **Detailed Design**
- **CAE for virtual verification and Testing**
- **Experimental Verification and Validation**
- **Production**

*Design time is relatively small*
*Validation is time and cost consuming*

**Overall Time saving**

**CAE Driven Product Development**

- **Concept Design**
- **CAE for virtual verification**
- **Detailed Design**
- **Experimental Validation and Testing**
- **Production**

*Design time increases to add FEA & simulation*
*Validation time reduces with fewer prototypes*

**Cost savings**

**Repetitive Iterations**

*Figure 1.1: CAE Driven Product Development at Volvo Cars [1]*
1.3 Towards risk/uncertainty informed virtual verification

A number of probabilistic analysis tools have been developed during the last decades for quantification of uncertainty and its propagation. However many of the complex systems are still designed/verified based on deterministic analysis. Applying deterministic analysis enables us to calculate noise events in a brake system, with known nominal values of system/environmental parameters, but it does not consider the effect of stochasticity/variability in the brake/chassis system. The construction of CAE methods that produce accurate analysis of system response is a fundamental element in science and has great potential in applications for improving reliability-based structural designs. However, such deterministic analyses cannot be relied upon completely since they do not explicitly address the uncertainty in system, input and environmental conditions, and therefore they often lead either to an overly robust design, or in rare cases, a design solution that is too prone to failure [35].

In view of these issues, the use of probabilistic analyses for early design/verification is becoming more widespread as many industries are currently working on improving the conditions for reliability in their products/solutions. The variability in component material properties and environmental conditions along with the lack of knowledge about the underlying physics of complex systems make it difficult to generate reliable analysis based on deterministic CAE models. Therefore, the level of confidence in CAE models needs to be increased to ensure reliability and to secure robustness in CAE solutions [35].

There is a growing interest amongst premium automotive companies on development of CAE solution-tools for a systematic way of treating uncertainty analysis/quantification in robust analysis of brake noise for early verification. For more detail regarding the tool and the method, the significant work from Jaguar Land Rover [31], Diamler AG [45]; [43], Robert Bosch GmbH [44] are available in the reference list. The main challenge in probabilistic analysis of a complex dynamical system such as the brake/chassis system, as studied by the above mentioned work, is the robustness and the computational efficiency of the method. This brings into the scope for the thesis to adopt a hierarchical multi-fidelity statistical approach, in order to balance the computational effort while fulfilling the robustness of design failure risk estimation. The method employs a hierarchy of approximations to the system response computed with different resolutions (e.g. different mesh resolution, different number of modes, different solver time step, etc.). This approach enables us to use low-fidelity solutions to accelerate the uncertainty quantification of brake/chassis systems, while granting unbiased estimation of system failure risk (reliability). It also helps in managing changes, during fast iterations of the design process, by incorporating the information from previous analyses to accelerate the probabilistic analysis of different design proposals. See [36], for details regarding the method development and implementation of the probabilistic analysis tool.
1.4 Thesis scope

The main objective of this thesis is to gain an understanding of the fundamental mechanism of a particular brake noise, creep groan and apply suitable probabilistic method for failure (noise event) analysis in a faster way.

To be specific, the goals of this thesis are:

- To define suitable metrics for the detection and evaluation of groan noise.
- To robustly estimate likelihood of noise-event occurrence.
- To adopt a hierarchical multi-fidelity statistical approach to open up the possibility to speed up the failure analysis of noise events in complex systems (e.g. Brake noise analysis).
- To incorporate uncertainty / CAE modelling errors systematically at fast iterations of the design/verification process.
- Investigation of root cause events for creep groan noise by probabilistic failure analysis. Additionally, gaining insight about noise events and the stochastic parameters relations/correlations that results in design failure (i.e. noise event).

Figure 1.2: Probabilistic Analysis of brake noise event [47]
1.5 Tasks Involved

The organization of this thesis in as follows:

- Literature study on different methods for analysis of brake noise.
- Understanding the physics behind groan/squeal noise by solving illustrative benchmark examples of brake noise problems in general.
- Simulation of brake noise event through deterministic analysis.
- Explore possible metrics for detection and evaluation of groan noise.
- Performing uncertainty analysis on the given ADAMS (Multi-body dynamics simulation software) brake model.
- Estimation of failure through a hierarchical multi-fidelity approach.
- Probabilistic failure analysis to investigate the root cause events and possible noise reduction proposals.

1.6 Limitations and Assumptions

The main focus of the thesis involves analysis of brake noise by use of CAE models and calculation of probability of failure; hence physical (experimental) tests will not be carried out in this thesis. There are a number of reasons leading to groan noise both at the production stage and during vehicle life. It could be because of the manufacturing variations, geometric variations and variations due to diverse loading and braking conditions and aging. But, the scope lies in studying the variation at the origin and propagation of groan brake noise.
2.1 Brakes

2.1.1 Background

Brake system converts kinetic and/or potential energy raised due to friction into heat in order to reduce speed and eventually stop the vehicle.

There are two types of brake used in vehicles: Drum brakes and disc brakes. Drum brakes consist of a pair of brake shoes which are pressed towards the inside of a drum located on the wheel hub by hydraulic/mechanical actuators [19]. In this thesis work, the brake system under study is of a disc brake type.

2.1.2 Disc Brakes

Disc Brakes consist of a brake disc, a caliper assembly, brake pads and hardware essential to mount the components on the vehicle. Disc brakes generate the braking forces at the surface of a brake disc that rotates with the wheel. The U-shaped brake caliper is attached to non-rotating suspension components. There are two types of caliper designs, as well as brake disc designs, which are commonly used in wheel brakes:

- **Fixed Caliper** – A fixed caliper is rigidly mounted to the vehicle for transmitting the braking torque and it consists of, at least, two pistons located on each side of caliper that press the brake pads onto the brake disc. The two pistons, or more, are hydraulically linked to equalize the actuating force between the assemblies. Figure 2.1 represents a fixed caliper design disc brake. It should be noted that the total number of pistons is a multiple of two, i.e. 2, 4 or 6, and depends on the brake size.
Literature Study

Figure 2.1: Representation of Fixed caliper Disc Brake [26]

- **Floating Caliper** - A rigidly mounted caliper bracket holds a movable (“floating”) caliper. Due to the hydraulic pressure from the brake fluid on one side, a single or two pistons present inside the caliper forces the inner brake pad against the brake disc while the caliper body is simultaneously forced in the opposite direction. This moves the caliper and assists by indirectly pressing the outer brake pad against the disc. Figure 2.2 represents a fixed caliper design disc brake.

Figure 2.2: Representation of Floating caliper Disc Brake [26]
2.2 Noise and Vibration issues in vehicle brakes

Though effective functionality of wheel brakes ensure accidental safety for the passengers, they are often prone to other dynamic instabilities in the form of noise and vibrations. ‘Brake noise’ may be defined as the audible noise evolving from a disc brake (or a drum brake) during vehicle use, which occurs at definite frequencies that are independent of usage conditions such as rotor speed [12]. Brake noises can be classified in two different ways: Frequency based classification also called 'Traditional Classification' and Physical Origin based classification called 'Phenomenological Classification' [22].

2.2.1 Traditional Classification

Conventionally, brake noise and vibrations are classified according to their dominant frequencies as high frequency domain (>1kHz) and low frequency domain (<1kHz).

In the low frequency domain, there are two major types of structural vibrations namely, judder and groan, and their associated airborne noise called hum and moan respectively. Judder is caused due to the geometrical variations or geometrical irregularities of the brake disc while groan is a phenomenon appearing as a result of certain specific friction-velocity characteristics or stick/slip.

The high frequency domain consists of disc brake squeal which is the result of energy exchange between different vibration modes within the brake system [22]. An important drawback of frequency based classification is that the phenomena of one and the same physical origin with different frequencies might be split into two different classes, and at the same time, fundamentally different phenomena will be included in the same class.

A schematic representation of frequency based classification of brake noises is displayed in figure 2.3 below.
2.2.2 Phenomenological classification

The physical origin of the phenomenon exciting the vibro-acoustic activity of the brake joints can be used to classify brake noise in proposed by Jacobson [22] as,

- **Forced Vibrations** - These are represented by judder and its related structural brake noise called hum.

- **Vibrations due to friction characteristics** - These are represented by creep groan and dynamic groan with their associated moan noise. Creep groan is related to stick-slip motion of the disc-pad interface. Dynamic groan is an instability phenomenon appearing as a result of friction velocity characteristic called ‘negative damping’ [22].

- **Vibrations due to resonances of brake components** - These vibrations appear in the form of noise propagating through air rather than the car structure. They are represented by squeal noise and wire brush noises. It is generally accepted that mode-coupling, resulting in the appearance of an unstable mode is significant for the squeal noise. Squeal can be considered to be the result of two modes on the complete brake system interacting with each other.

Phenomenological classification offers the main advantage of distinguishing two fundamentally different brake vibrations judder and groan from each other which overlap in the frequency range of 400-500Hz. Also another advantage is that it is coupled to the way of modelling the vibrations [22].

*Figure 2.3: Frequency based classification of Brake noises [17]*
2.3 **CAE approaches in simulation of brake noise**

Brake noise and vibrations have been extensively researched over many years by notable researchers. Researchers have taken approaches towards understanding the nature and characteristic features of dynamic instabilities associated with brake noise through lumped parameter models, mathematical models, stability criteria, modelling and simulations [12]. Deterministic Analysis and Probabilistic Analysis are two ways of tackling brake noise issue and they are discussed below.

### 2.3.1 Deterministic Analysis

Deterministic analysis involves analytical or numerical modelling (based on finite element models) which can simulate different structures, material compositions and operating conditions of a disc brake. The methods used to carry out a deterministic analysis of a brake may be classified into: Complex Eigenvalue Analysis in the frequency domain and Transient Analysis in the time domain [32].

- **Complex Eigenvalue Analysis (CEA)**

  CEA is a linearized approach to simulate the natural oscillation behaviour of a mechanical system. CEA involves computing the eigenvalues/vectors to determine the system stability along with the inclusion of damping effects [21]. In fact this approach finds the eigenvalues of a system in a complex form like

  \[
  \lambda_j = \alpha_j + i \omega_j
  \]

  (2.1)

  The real part \(\alpha_j\) of the eigenvalue indicates the rate of growth/decay of the system oscillations and the imaginary part \(\omega_j\) shows the oscillation frequency. Positive real parts of the system eigenvalues indicate instability. A positive real part tends to enlarge the amplitude of oscillations without bound in a linearized system if no damping is considered in the system [31].

  The limitation of the complex eigenvalue analysis is that it linearizes the equation of motion about a non linear static equilibrium point and hence the effects of non-linearities are only accurate close to steady sliding point [31]. The non-stationary features, such as time-dependent material properties, cannot be included. Another drawback is that while one could identify the existence of unstable modes in the system, it does not indicate level of groan noise produced. This might lead to overestimation of noise events. It is worth mentioning that underestimation of the noise events by the CEA is also possible, since this linear analysis cannot take into account the non-linear effects of contact/friction between the pad and disk. Despite these drawbacks, CEA is a common approach in brake squeal analysis [29].
• **Time-Domain/Transient analysis**

Transient analysis computes the time domain response of a system with respect to an initial condition (free vibration) or an excited source (forced vibration), using time integration solvers (e.g. ODE solvers). The time domain response consists of useful data about amplitude of non-linear oscillations, the frequency of oscillations, limit cycle of oscillations, the time delay between the input force and the resulting system response. Effects of non-linearities [40], effect of time dependent loads and information regarding the level of noise and limit cycle mode in systems can be determined only via transient analysis [31].

The major shortcoming of the transient analysis is its high computational time [32]. Time-domain analysis is the common practical approach in analysis of groan noise, but not a common approach for squeal noise detection, in industry. This is because, creep groan as a low frequency phenomenon can be modelled making few parts rigid or having a low number of degrees of freedom.

Illustrative numerical examples for deterministic analysis are presented and studied in the next chapter.

### 2.3.2 Probabilistic Analysis

While deterministic analysis with Finite Element Methods have been used for numerical modelling and simulation of brake noise, they do not consider the effect of uncertainties in the system and hence the result might not be very useful. Probabilistic analysis is a systematic way to quantify the performance of a system when it is subjected to uncertainties, which may arise during modeling, assignment of model parameters or stochasticity of environmental conditions [9].

Briefly stated, the uncertainties in a system have to be considered and they can be classified into two groups:

• **Aleatory uncertainty or statistical uncertainty**
  
  Aleatory uncertainty, also called as variability, indicates the random nature in the system properties which originates from manufacturing processes. Variations in material properties, component geometries and assemblies are common examples of this category of uncertainty.

• **Epistemic uncertainty or systematic uncertainty**

  This type of uncertainty is mainly due to lack of information It is also called reducible uncertainty since future investigations and experimental results can give insights about these uncertainties [31]. Our lack of knowledge in the friction model is a common example of this category of uncertainty.

Ignoring the effect of variability and uncertainty in CEA can result in underestimation and/or overestimation of the number of unstable modes. It is found out that in many cases variability and uncertainty play a significant role in generation/propagation of creep groan noise [31].
Evaluating the effects of uncertainties and their mitigation in the design-making process allows one to make risk-informed decisions [42] and it is gaining greater importance in design implications.

**Failure/reliability analysis**

Failure risk assessment/robustness assessment is used to investigate how the variation of an input parameter affects the response variation in the system output critical response [47].

A failure event can be represented as a scalar output response \( g \) exceeding a specified threshold \( b \). \( g \) is determined by a set of set of input parameters/variables \( X \) which is given by [42],

\[
g = h(X)
\]

where, ‘\( h \)’ is a function that represents a computational process (it can be a finite element model or a mathematical formula). In order to make decisions about \( g \) one needs to have information about its probability distribution. The probability of failure is given by,

\[
P_F = P(g > b)
\]

Assuming the prior knowledge about the uncertain input parameter \( x \in \mathbb{R}^n \) given by PDF is \( P(x) \) [35], the probability \( P_F \) can be determined as,

\[
P_F = \int_F P(x)dx = \int I_F(x)P(x)dx
\]

where, \( I_F \) is the indicator function, \( I_F(x) = 1 \), when its parameter are in the failure region \( x \in F \), otherwise it is 0. The parameter space region that corresponds to failure is denoted by \( F = g > b \) [35].

The decision can be made simply based on Probability Distribution Function or through finding \( P_F \) in Cumulative Distribution Function as in equation 2.4 which is the integral of failure samples in Probability Distribution Function and gives the probability of random variables below a particular limit [4] or Complementary Cumulative Distribution Function (CCDF) that gives the probability of random variable above a particular limit, which is just the opposite of Cumulative Distribution Function. Finding probability of failure in itself is equivalent to finding CCDF. Estimating the Cumulative Distribution Function is more difficult and computationally expensive, so, probability distribution analysis will be carried out on CCDF [42].

Reliability is used as a measure for system design evaluation and it represents the probability that a system will perform its intended functionality and fulfill the system requirements/attributes [42].
Mathematically, it is said to be the probability that a system for a given reference period does not reach a defined limit state [2]. Or simply, it is a complementary to the failure probability.

\[ R = 1 - P_F \]  

(2.5)

Figure 2.4 shows the steps involved in CAE assessment by probability method.

![Figure 2.4: Risk assessment steps in CAE probability method](image)

Simulation based probabilistic methods such as Monte-Carlo Simulations, Importance sampling etc have been developed to evaluate the system design reliability/robustness (i.e. to estimate failure probability, \( P_f \)), and use it as a proper measure for system design robustness/quality [28]. A brief description of common simulation based probabilistic methods is discussed below.

- **Monte-Carlo**
  The most common, yet practical probabilistic simulation based uncertainty analysis method is Monte-Carlo simulation. There are two advantages for this approach in comparison with other techniques. Firstly, the method is very simple as Monte-Carlo simulation spreads a large number of design samples over the design space and collects the results of the sampled points through a mapping function one by one. This mapping function is nothing but the relation between the output and input variables. Then the statistics
of the outputs are computed by the use of output scatter. Secondly, when the output or mapping function is a non-linear function of input variables, the most reliable method for uncertainty analysis is Monte-Carlo simulation. A major disadvantage with this method is it becomes very expensive in practice, especially when the mapping function is an expensive simulation code or an experiment [31].

- **Importance Sampling**
  Importance Sampling is one among the classical variance reduction techniques for improving the efficiency of Monte-Carlo methods. The general idea is that certain values of input stochastic variables have more influence on the measured output parameters during simulation than others. If these stochastic parameter values are emphasized and sampled more often, then the distribution contains only these values thereby reducing the estimator variance. Hence, the basic methodology in Importance Sampling is to choose a distribution which encourages the stochastic parameter values that have more contribution to the failure events [49].

- **Latin Hyper Cube**
  Latin Hyper Cube is a yet another statistical method for generating a near-random sample of parameter values. It involves stratification of input stochastic parameters. Latin hyper cube is helpful as it gives a better coverage of the stochastic parameter space when uncertainty analysis has to be carried on complex model that consume large amount of time [23].

There is no standard approach for analysis of uncertainty that can meet all needs or requirements. The approach mostly depends on the type of needs. Evaluating a multi-dimensional integral in equation 2.4 is one main concern in reliability problems. Direct numerical integration schemes or other analytical methods can be used only for simple reliability problems and not when the parameter space is of high dimension or when the failure region has a complex geometry in the parameter space. Monte-Carlo is one robust simulation method that can handle this problem. The calculation of probability of failure according to the Monte-Carlo method is shown in equation,

\[
P_F = E(I_F(x))
\]  

(2.6)

In the Monte-Carlo approach, the integral is viewed as an expectation \(E\) that leads logically to an estimation by means of “statistical averaging” based on independent identically distributed samples [35].

**Hierarchical multi-fidelity statistical approach**

Finite Element Method based probabilistic simulation methods are criticized for lack of computational efficiency and requiring immense computational effort to achieve small probability of failure levels, say \(P_f < (10^{-3}\) (or) \(10^{-6}\)), which is often the target requirement. It becomes more complex when multiple design solution have to be assessed. The necessity for reducing the computational effort has resulted in adopting approximate solutions for evaluating system response.
But due to the approximate nature, the reliability estimate can be not ‘consistent’ [9] [42]. To address this, hierarchical multi-fidelity approaches can drastically reduce the computational effort while providing a ‘consistent’ design failure estimation. It employs a hierarchy of approximations to the output response computed with different resolutions [45].

### 2.3.3 Surrogate Modelling

Surrogate modelling (e.g. Polynomial Chaos, see [31] for more details) is an efficient approach usually applied in practice along with Monte-Carlo simulation in constructing a surrogate model in place of the high-fidelity FE model. In other words, this method replaces the expensive finite element code with a cheap-to-evaluate mathematical function. This is done to reduce the computational work load and keep the advantages of Monte-Carlo Simulation.

It is important to consider that surrogate modelling is not a method of uncertainty analysis of the actual high-fidelity model, rather all methods of uncertainty analysis can be applied to the generated surrogate model. In fact, surrogate model can be assumed as a simplified mapping function, from input stochastic parameter space to the system output response space [31].

### 2.4 Discussion

A brief study on brake types, different noise problem in brakes and their classification has been studied. Two different approaches for analysis of noise problem - deterministic and non-deterministic approaches has been studied. Different non-deterministic approaches has been discussed which could be used based on the requirements. Risk assessment by using probability of failure as a metric is observed to be important when one is interested in knowing the confidence level of the system design. Hierarchical multi-fidelity statistical approach is something that is needed and useful when a complex model has to be assessed. Implementation of these important findings and understandings will be found in upcoming chapters.
3.1 Illustrative benchmark examples

3.1.1 Benchmark 1: CEA analysis and mode coupling in brake squeal noise

The First Benchmark is taken from Nobari [31] to study the friction induced vibration with asymmetric stiffness. The instability in eigenvalues can be analyzed in the 4-DOF model shown in figure 3.1

![Figure 3.1: Friction induced vibration with asymmetric stiffness [31]](image-url)
The generalised form of equation of motion for a system with \( N \) degrees of freedom can be written as:

\[
\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}_{\text{ext}}
\]  

(3.1)

where, \( \mathbf{M}, \mathbf{C}, \mathbf{K} \) represents Mass, Damping and Stiffness Matrix respectively. If we consider the solution for the expression \( \mathbf{u}(t) = \mathbf{U}e^{\lambda t} \), where, \( \lambda \) represents Eigenvalue and \( \mathbf{U} \) represents eigenvector. On substituting the solutions to the equation 3.1

\[
[\lambda^2 \mathbf{M} + \lambda \mathbf{C} + \mathbf{K}]\mathbf{U} = 0
\]  

(3.2)

The eigenvalues and eigenvectors are calculated by using 'polyeig' command in MATLAB as shown in Appendix A.1. But, It should be noted that in order to carry out complex eigenvalue analysis for an asymmetric system with non-proportional damping, the state-space equations of motion should be used in place of the second-order equations otherwise it does not capture the effect of asymmetric system and non-proportional damping together [31].

Some of the assumptions are made for this example which is shown in the figure 3.1 such that it becomes a lower order brake model. \( m_1 \) is taken as mass of the abutment, \( m_2 \) is the brake pad mass and \( m_3 \) is the pad back plate mass, the system has springs of stiffness \( k_1, k_2, k_3, k_4, k_5 \) connecting components, the contact stiffness, damping in the pad-disc interface are \( k_c, c_c \) respectively. The matrices of mass, damping and stiffness corresponding to the vector \( \mathbf{u} = \{x_1, y_3, x_2, y_2\}^T \), where \( x_1, y_3, x_2, y_2 \) are the displacement of corresponding mass \( (m_1, m_3, m_2) \) in \( x \) and \( y \) - directions.

\[
\mathbf{M} = \begin{bmatrix}
m_1 & 0 & 0 & 0 \\
0 & m_3 & 0 & 0 \\
0 & 0 & m_2 & 0 \\
0 & 0 & 0 & m_2 \\
\end{bmatrix}
\]

(3.3)

\[
\mathbf{C} = \begin{bmatrix}
c_1 & 0 & -c_1 & 0 \\
0 & 0 & 0 & 0 \\
-c_1 & 0 & c_1 & \mu c_c \\
0 & 0 & 0 & c_0 + c_c \\
\end{bmatrix}
\]

(3.4)

\[
\mathbf{K} = \begin{bmatrix}
k_1 + k_2 & 0 & -k_2 & 0 \\
0 & k_4 + k_5 & 0 & -k_4 \\
-k_2 & 0 & k_2 + 0.5k_3 & \mu k_5 - 0.5k_3 \\
0 & -k_4 & -0.5k_3 & k_4 + 0.5k_3 + k_c \\
\end{bmatrix}
\]

(3.5)

where \( m_i = 1 \text{kg} (i = 1, 2, 3) \), \( k_i = 100 \text{N/m} (i = 1, 2, 3, 4, 5) \), \( k_c = 2k_1 \), \( c_1 = c_0 = 0.5 \text{Ns/m} \) and \( c_c = 0.1 \text{Ns/m} \).
3.1 Illustrative benchmark examples

Figure 3.2: CEA results for varying $\mu$ with dampers

Figure 3.3: CEA results for varying $\mu$ without dampers
The values used are the same as in Nobari [31]. However, on removing the dampers completely, it is observed that eigenvalues coalesce with each other which is not the same case as in the actual model with dampers as shown in figure 3.3. Hence, it becomes important to consider dampers for friction modelling to have a ‘smoothing effect’ [20]. In figure 3.2, figure 3.3, it is observed that the real part remain close to each other and bifurcates after certain value of co-efficient of friction. It is vice versa for the imaginary part. The point at which this happens is called ‘bifurcation point’. It should be noted that the positive real part causes the amplitude to grow exponentially and tends the mode to couple, creating ‘mode coupling’ effect. In this example, squeal could not be predicted, since the system was linear.

3.1.2 Benchmark 2: Time domain and CEA analysis in brake squeal noise

In this benchmark, a study is made on a 4-DOF friction model with cubic non-linear contact forces in time and frequency domain to predict squeal.

Uncertainty analysis has been carried out in Zhang et al, from where this problem is referred. The equation of motion for a non-linear friction induced self-excited 4-DOF model shown in figure 3.4 can be written as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) + \mathbf{F}_{nl}\mathbf{u}(t) = \mathbf{0}$$

(3.6)

The matrices of mass, damping and stiffness corresponding to the vector, $\mathbf{u} =$

![Figure 3.4: 4-DOF friction model with cubic non-linear contact forces][50]
3.1 Illustrative benchmark examples

\[ \{x_1, y_1, x_2, y_2\}^T \]

\[
M = \begin{bmatrix}
m_1 & 0 & 0 & 0 \\
0 & m_1 & 0 & 0 \\
0 & 0 & m_2 & 0 \\
0 & 0 & 0 & m_2
\end{bmatrix} \qquad (3.7)
\]

\[
C = \begin{bmatrix}
c_2 & 0 & 0 & 0 \\
0 & c_1 & 0 & 0 \\
0 & 0 & c_3 & 0 \\
0 & 0 & 0 & c_4
\end{bmatrix} \qquad (3.8)
\]

\[
F_{nl} = (y_1 - y_2)^3 \begin{bmatrix}
-k_{53} \mu \\
k_{53} \\
k_{53} \mu \\
k_{53}
\end{bmatrix} \qquad (3.9)
\]

\[
K = \begin{bmatrix}
\sum_{i=1}^{2} k_i \cos^2 \alpha_i & \sum_{i=1}^{2} k_i \sin \alpha_i \cos \alpha_i - k_{51} \mu & 0 & k_{51} \mu \\
\sum_{i=1}^{2} k_i \sin \alpha_i \cos \alpha_i & \sum_{i=1}^{2} k_i \sin^2 \alpha_i + k_{51} & k_3 & -k_{51} \\
0 & k_{51} \mu & k_{51} \mu & -k_{51} \\
0 & 0 & -k_{51} & k_4 + k_{51}
\end{bmatrix} \qquad (3.10)
\]

where \(m_1\) is the mass of the block, \(m_2\) is the mass of the belt. \(k_1, k_2, k_3, k_4\) are stiffness of springs. \(c_1, c_2, c_3, c_4\) are viscous dampers, \(\mu\) is the coefficient of friction. A spring and a cubic spring with a stiffness of \(k_{51}\) and \(k_{53}\) in the contact surface is used to model the contact between the belt and the block which are coupled via a constant friction coefficient using the Anton–Coulomb's law [50].

The nonlinear contact spring force is characterized as shown in eq 3.9. All the input variables are identical to the reference. But, when the input variables are varied considering uncertainties, the observation is that, it squeals for some parameter combinations and it does not squeal for some. For example, when the stiffness of springs are increased no squeal occurs and vice versa.

Let us consider the input initial condition 0.1 m/s, the following displacement vs time plot shown in figure 3.5. It is observed that the signal is not decaying and it enters the repeating limit cycle where it repeats itself. One can come to the conclusion that the mode that follows the limit cycle is feeding the energy to the system [50]. These are some important findings from transient analysis which could not be found in CEA. Also, in some cases, when CEA analysis is carried out for the same set of input parameters that generated squeal, negative eigenvalues/stable modes were observed [50]. Hence, it becomes important to follow time domain analysis for accurately evaluating the noise problem even though it takes more computational time. Later the time domain is shifted to frequency domain by Fast Fourier Transform (FFT) and audible sound is calculated from FFT [6]. The matlab code for the problem is available in the Appendix A.2. Discussion on FFT and audible noise calculation will be found in Chapter 4.
3.1.3 Benchmark 3: Time domain analysis in creep groan noise

Creep Groan is a problem that arises due to a stick-slip phenomenon. Stick-slip phenomenon is the result of a difference between static and kinetic friction coefficient. This motion can easily be analyzed by both CEA and Transient analysis. In the following benchmark, the most widely considered 1-DOF model of frictional oscillator is represented as shown in 3.6. The input parameters for the model are chosen as in Ashraf et al [8].

The equation of motion for the model can be written as:

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = \mu_s N(\text{sign}(\dot{u}(t) - V_b)) - \sigma(\dot{u}(t) - V_b) \]  \hspace{1cm} (3.11)

where, \(M\) is the mass, \(K\) is the stiffness, \(C\) is the damping, \(V_b\) is the belt velocity, \(\mu_s\) is the static coefficient of friction, \(u\) is the position of the body with mass \(M\) at \(X\) at time \(t\) and \(\sigma\) is the friction coefficient versus relative velocity slope, \(N\) is normal force and the signum function \(\text{sign}(\dot{u}(t) - V_b)\) is: +1 for \(v_r > 0\) and -1 for \(v_r < 0\). where \(v_r = \dot{u}(t) - V_b\) is the relative velocity. The equation 3.11 is non-linear since the frictional force changes according to the changes in relative velocity. Also, the frictional force has the major influence on damping. It is
observed that when the mass was at rest \( v_r = 0 \), it is in stick phase and when the mass starts sliding \( v_r > 0 \) then slip occurs and groan noise is heard. This is evident from the results of transient analysis shown in figure 3.7. The MATLAB code for running this simulation is provided in Appendix A.3.

**Figure 3.6**: Brake Groan 1-DOF [8]

**Figure 3.7**: Brake Groan 1-DOF results in time and frequency domain
The frictional force applied on the system tends to push the mass move first until the spring force equals the frictional force. Meanwhile both the spring and friction forces keeps increasing. Now, the mass will try to slip in the same direction of motion. The mass tend to continue the motion as the static friction changes to kinetic. Again at certain moment both the forces equals again and the mass stops instantaneously [8]. This is observed in the limit cycle plot shown in figure 3.7. The stick appears as horizontal lines in the beginning and later when the mass starts to slip the slope begins.

3.2 Discussion

Thus three separate benchmarks were used to study the occurrence of brake squeal and groan by CEA, time domain and frequency domain analysis. Merits and demerits of each method are understood. The reason for noise is identified i.e. a non-linearity in frictional surface can be concluded to be a major reason for squeal by making a study on one model with linearity and one with non-linearity. A stick-slip phenomenon is the reason identified to be behind groan noise. So far no uncertainties in the parameters have been considered. Thus, through simple illustrative examples, the physics behind these noise events is studied. In the next chapter, a brake model designed in ADAMS at Volvo Cars Corporation is taken for study and different criteria for noise detection in the the model will be discussed.
Definition of failure state for groan noise problem

4.1 Creep Groan

Creep groan is a low frequency brake noise. It is a self-excited and friction-induced vibration which results in an unwanted noise in the frequency range up to 500 Hz. It is caused due to a stick-slip phenomenon. Creep groan events can occur as multiple fractions in a second [33].

Creep groan phenomenon might appear when there is simultaneous application of torque to the wheel and gradual release of brake pressure. Eventually the wheel torque load breaks down the friction between the rotor and the pad, causing the slip phenomenon and thereby releasing energy. If the torque load is not sufficient to maintain the slippage, then stick-slip vibrations can occur, which transmit a low frequency noise to the vehicle interior (suspension strut and chassis) that is groan [18].

Creep groan is a structure-borne noise that is related to dynamic characteristic of the vehicle. However, it has been primarily improved through friction material modifications in the past decades. The path of the creep groan noise is the chassis system, composed of strut, lower control arms and sub frame. Finally, the noise is transmitted to the driver ear through the vehicle body structure [48]. Groan noise generation, propagation path and noise through body frame is displayed in figure 4.1.
4.2 Brake Model

The current studies on Creep Groan noise are carried out on a MacPherson strut suspension modelled in ADAMS/View software. Automated Dynamic Analysis of Mechanical Systems (ADAMS) is a Multi-body simulation software and is used widely at Volvo Cars Corporation to study the dynamics of moving parts, forces and load distributions of the car model and sub-systems.

4.2.1 Component details

The brake model consists of components from the brake, suspension, and steering system. This is because groan originates in the brake system and propagates via suspension and steering systems to the interior cabin. As part of the brake system - a brake disc, brake pad, a floating caliper with a piston, caliper carrier are modeled. As part of the suspension system - a MacPherson strut independent suspension with telescopic damper, damper housing and accessory components with coil springs, wheel and tire are modeled. As part of the steering system - a knuckle, tie rod and lower control arm are modeled. The MacPherson strut is simple in design, does not require many components and uses less space compared to double wish-bone type suspension and is used in the front axle in smaller passenger cars by most of the manufacturers. The details of the model is shown in figure 4.2 and the details along with their connector/joint types and forces are provided in table 4.1 and 4.3. The different joints and the force provided acts as the boundary condition for individual components.

4.2.2 Modelling

The friction model chosen for the system follows Anton-coulomb law of friction. As creep groan is a structure-borne noise, it becomes important to consider the
4.2 Brake Model

subsidiary as well in the model [30]. This makes the model quite complex and
modelling the entire system using finite elements is merely impossible. Since
creep groan is a low frequency NVH problem, several components in the brake
system such as disc, caliper and brake pads behave as rigid bodies and the other
components which have resonances in the groan frequency range are modelled as
flexible bodies. Hence finite element modelling is not very suitable for modelling
creep groan phenomenon. Multi-body simulation through ADAMS allows to use
both rigid body and flexible parts (meshed parts) in the same model. In this
case, we will be modelling the vibration transmitting components (knuckle, tie
rod and lower control arm) as flexible bodies, and rest as rigid. And the flexible
bodies are made by meshing them in ANSA and importing them as .mnf files
into ADAMS. The procedure is discussed in Appendix B. In the dynamics model,
inertia, stiffness, damping are calculated based on structures of suspension, drive-
line, tire and vehicle body similar to [30]. Finding their values is not in the scope
of the work.

Table 4.2 shows the details of bushing forces applied in different joints for the
model, it is also highlighted in figure 4.2. Bushing forces between wheels and
tire, damping rod and damping house are not of interest, as the former is not in
the propagation path and the latter is affiliated to the suspension system. The
rest of the bushings are taken for study. In upcoming chapters, the stiffness and
damping of these bushings will be considered for uncertainty analysis.

**Figure 4.2:** Individual components and bushings (marked in yellow) detail
excluding wheel and tire for clarity
## Definition of failure state for groan noise problem

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Joint type</th>
<th>Part 1</th>
<th>Part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fixed</td>
<td>Piston</td>
<td>Brake Pad inner</td>
</tr>
<tr>
<td>2</td>
<td>Fixed</td>
<td>Brake pad inner</td>
<td>Lining inner</td>
</tr>
<tr>
<td>3</td>
<td>Fixed</td>
<td>Lining outer</td>
<td>Brake pad outer</td>
</tr>
<tr>
<td>4</td>
<td>Fixed</td>
<td>Brake pad outer</td>
<td>Caliper</td>
</tr>
<tr>
<td>5</td>
<td>Translation</td>
<td>Piston</td>
<td>Caliper</td>
</tr>
<tr>
<td>6</td>
<td>Fixed</td>
<td>Guide pin lower</td>
<td>Carrier</td>
</tr>
<tr>
<td>7</td>
<td>Fixed</td>
<td>Guide pin upper</td>
<td>Carrier</td>
</tr>
<tr>
<td>8</td>
<td>Translation</td>
<td>Guide pin upper</td>
<td>caliper</td>
</tr>
<tr>
<td>9</td>
<td>Fixed</td>
<td>Carrier</td>
<td>Knuckle</td>
</tr>
<tr>
<td>10</td>
<td>Fixed</td>
<td>Carrier</td>
<td>Knuckle</td>
</tr>
<tr>
<td>11</td>
<td>Spherical</td>
<td>Lower control arm</td>
<td>Knuckle</td>
</tr>
<tr>
<td>12</td>
<td>Spherical</td>
<td>Tie rod</td>
<td>Knuckle</td>
</tr>
<tr>
<td>13</td>
<td>Fixed</td>
<td>Damper house</td>
<td>Knuckle</td>
</tr>
<tr>
<td>14</td>
<td>Revolute</td>
<td>Knuckle</td>
<td>Disc Hub</td>
</tr>
<tr>
<td>15</td>
<td>Fixed</td>
<td>Disc</td>
<td>Disc Hub</td>
</tr>
<tr>
<td>16</td>
<td>Fixed</td>
<td>Disc</td>
<td>Wheel</td>
</tr>
<tr>
<td>17</td>
<td>Revolute</td>
<td>Wheel</td>
<td>Tyre</td>
</tr>
<tr>
<td>18</td>
<td>Fixed</td>
<td>Damping House accessor</td>
<td>Damping House</td>
</tr>
<tr>
<td>19</td>
<td>Spherical</td>
<td>Ground</td>
<td>Tie rod</td>
</tr>
<tr>
<td>20</td>
<td>Revolute</td>
<td>Ground</td>
<td>Tyre</td>
</tr>
</tbody>
</table>

*Table 4.1: Brake model - Joints*

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Forces</th>
<th>Part 1</th>
<th>Part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bushing_1</td>
<td>Damping rod</td>
<td>Damping House</td>
</tr>
<tr>
<td>2</td>
<td>Bushing_2</td>
<td>Damping rod</td>
<td>Damping House</td>
</tr>
<tr>
<td>3</td>
<td>Bushing_3</td>
<td>Ground</td>
<td>Damping rod</td>
</tr>
<tr>
<td>4</td>
<td>Bushing_4</td>
<td>Ground</td>
<td>Lower control arm front</td>
</tr>
<tr>
<td>5</td>
<td>Bushing_5</td>
<td>Ground</td>
<td>Lower control arm rear</td>
</tr>
<tr>
<td>6</td>
<td>Bushing_6</td>
<td>Ground</td>
<td>Tie rod</td>
</tr>
<tr>
<td>7</td>
<td>Bushing_7</td>
<td>Wheel</td>
<td>Tyre</td>
</tr>
<tr>
<td>8</td>
<td>Spring</td>
<td>Caliper</td>
<td>Piston</td>
</tr>
</tbody>
</table>

*Table 4.2: Brake model - Forces applied*

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Motion</th>
<th>Part 1</th>
<th>Part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rotational motion to revolute motion</td>
<td>Ground</td>
<td>Tyre</td>
</tr>
</tbody>
</table>

*Table 4.3: Brake model - Input motion*
4.3 Detection criterion and objective target setting for computation

Detection and evaluation of a creep groan event has been a challenging task due to the lack of well-defined metrics or criteria for quantifying a creep groan event [25]. The metrics chosen for the study of groan noise should lead to better accuracy of detection and evaluation. In this section, some metrics to detect and evaluate a creep groan event are proposed. Finally, a suitable metric from the list is chosen and applied for stochastic/probabilistic analysis.

No Groan condition
In the event of no groan as shown in figure 4.3, only initial disturbances of caliper acceleration or the bushing force is observed and they eventually come to a steady state since the value of static coefficient of friction is closer to the value of dynamic coefficient of friction.

For a no groan condition, the deceleration as a result of the brake pressure applied would indicate a sharp fall for both caliper acceleration and bushing force with no dynamics due to steady state condition. While in the case of groan condition, the deceleration would show dynamics with indication of stick-slip cycles.

4.3.1 Caliper acceleration
Caliper acceleration as a feature can be used to study creep groan since the friction induced vibrations due to the stick-slip phenomenon at the disc-pad interface causes a movement of the caliper in the tangential direction.

Figure 4.3: Time response for no groan condition—caliper acceleration and bushing force (for clarity, bushing force of strut top-mount bushing in x-direction alone is shown)
The absolute maximum value of time response and the frequency response of caliper acceleration can be used to evaluate groan behaviour similar to the results obtained in [13]. The response of caliper acceleration with time and frequency domain is described below:

- **Maximum of Time Response** - The time response plot of a caliper acceleration indicates repetitive stick-slip cycles along a certain frequency range and the frequency range repeats for every stick-slip cycle. A maximum of the absolute value of the caliper acceleration plotted can be used as a metric to evaluate the groan behaviour.

However, an appropriate threshold can not be chosen for the amplitude of caliper acceleration since the threshold is just a reference/approximate value obtained from multiple tests. In [34], the cut off magnitude of a significant acceleration spike has been set to 12.5 percent of the peak to peak range to quantify groan event. Similarly in [14], the value of the caliper accelerations greater than 1g of acceleration is considered to detect groan behaviour.

The time response of caliper acceleration and bushing force is plotted in figure 4.4. The threshold value of acceleration is approximated to be 1g similar to [14]. The repetitive frequency order for one complete stick-slip cycle which is quite similar for both caliper acceleration and bushing force is displayed in figure 4.5 which is a zoomed in version of figure 4.4 for clarity.

![Figure 4.4: Time response for caliper acceleration and bushing force (for clarity, response of strut top-mount bushing in x-direction alone is shown)](image-url)
4.3 Detection criterion and objective target setting for computation

Figure 4.5: Indication of repetitive stick-slip frequency in zoomed in time response plot for caliper acceleration and strut top-mount bushing force in x-direction

- **Maximum of Fast Fourier Transform (FFT)** - The analysis of characteristic patterns of acceleration signals in the frequency domain can be used to determine the critical frequencies resulting in groan behaviour [13]. One way to determine the critical frequency in the FFT plot is to calculate the peak of the FFT plot which is the fundamental frequency or the stick-slip frequency resulting in groan behaviour as shown in figure 4.6. The peaks of frequencies in an FFT plot is calculated by considering the absolute maximum value of the highest peak frequency in every iteration/simulation. This can be used as a metric for evaluating groan behaviour.

Figure 4.6: FFT plot for Caliper acceleration
4.3.2 Sound Pressure Level analysis

The sound pressure levels obtained from numerical analysis can be compared to a set of reference standards for brake noise to quantify groan behavior. The reference standards were assumed based on experimental conclusions from previous experiments and noise level standards for various regions in the world. For Europe the value was assumed to be $93\, dB$ while in US and Asia it was $85\, dB$ [36].

To calculate the value of sound pressure level in decibels, the time response of the bushing force, $f(t)$ is converted to frequency response, the frequency response of bushing forces, $f(\omega)$ and the corresponding Noise Transfer Functions in frequency domain are later multiplied.

The Noise Transfer Functions (NTF) corresponding to bushing forces is a way to analyze the transfer path of noise into the cabin interior. NTF are the relations between input and output signals based on the medium in which it gets transmitted. It is a calculation of the amount of an output noise perceived by the driver/passenger inside the car cabin, for a unit input force at the bushings.

The absolute value of sound pressure is the product of FFT of bushing forces, $f(\omega)$ and NTF in frequency domain, $NTF(\omega)$ [7]. This is shown in equation 4.1,
\[
Sound pressure = |NTF(\omega)\ast f(\omega) |
\]  
(4.1)

Sound pressure level is the logarithmic measure of the effective pressure of a sound relative to a reference value. The reference value is $20\mu Pa$, which is the threshold of human hearing [5]. The sound pressure level value is calculated by using the equation 4.2 [5].
\[
Sound pressure level = 20\log_{10}(p/p_0)\, dB
\]  
(4.2)

where, $p$ is the absolute square to square root of 2 of sound pressure and $p_0$ is the reference pressure.

The sound pressure level contribution by various paths of groan noise are shown in the figure 4.7 and a threshold for audible groan limit is set to $70\, dB$ based on experimental investigations for audible noise inside the car and making suitable assumptions. The absolute value of audible noise is indicated in the figure 4.7.

The absolute value of sound pressure level was found out to be $86\, dB$ in the model as shown in figure 4.8 and they were compared with reference standards assumed.
4.3 Detection criterion and objective target setting for computation

Figure 4.7: Sound pressure level (SPL) due to each path and audible noise limit. Transfer path are in the order of bushing_4, bushing_5, bushing_3, bushing_6 in x, y, z directions respectively. Refer table 4.2 for the bushing number details.

Figure 4.8: Overall Sound Pressure Level (SPL)
4.4 Discussion

The investigations performed to evaluate groan behaviour enabled a characterization of the dynamics of the brake system during the groan phase. Some of the objective parameters which had a significant effect on groan noise were caliper acceleration, bushing forces and subjective parameters like sound pressure levels were useful to quantify groan behaviour.

The responses for strut fore/aft acceleration and caliper acceleration have similar dynamic content and are closely correlated. This correlation is a result of coupling by knuckle rotation from friction force. It is believed that groan is a structure borne noise caused by a stick-slip phenomenon and tends to be amplified by cabin acoustics. The operating shape of the suspension has a vertical movement of the caliper which in turn rotates the knuckle and moves the strut in the fore/aft direction. Therefore, it is assumed that the primary noise path is through the strut mount to the interior body. Hence the measurement of the bushing force/strut mount force is a significant way to study groan behaviour [13]. The frequency response from strut fore/aft acceleration and bushing force indicate a large contribution at the stick-slip frequency and other responses occur at orders of the stick-slip frequency as shown in the figure 4.9 and figure 4.10.

As it is common for non-linear systems, the spectral responses occur in the orders of stick-slip frequency and it is possible for the stick-slip frequency to correspond to a higher frequency order similar to the results discussed in [13]. In this case, the stick-slip frequency peak is at the 3rd order for strut acceleration and 1st order for bushing force.

![FFT plot for Bushing Force](image)

**Figure 4.9: FFT plot for Bushing Force**
4.4 Discussion

Figure 4.10: FFT plot for Strut acceleration

A NTF plot is displayed for a bushing on the strut mount in figure 4.11, it indicates the magnitude of NTF in decibels in the plot.

Figure 4.11: Noise Transfer Function (NTF) plot for bushing on strut mount

The peaks of reference frequencies in the NTF plot refer to the sensitive range of frequencies regarding the body acoustics. As the peak of the desired frequency from FFT of bushing forces gets closer to the peak of the reference NTF frequencies, the probability of groan noise occurrence is higher.
This threshold distance is expressed by normalizing the desired frequency with respect to the reference frequency, as shown in the equation 4.3.

\[ |(f_r - f_d)/f_d| \geq a \]  \hspace{1cm} (4.3)

where \( f_r \) is the reference frequency considered in the NTF plot, \( f_d \) is the desired frequency from FFT of bushing force and the variable \( a \) is the ratio of the difference between the reference value and desired value with respect to the desired value of frequency. The value of \( a \) is usually expressed in percentage ratio and in this case, it was assumed to be 10 percent or simply the value 0.1 based on the proposal by Volvo Car Corporation. It is the minimum threshold distance that the desired frequency peak is supposed to be away from reference NTF peak frequency. This is suitable as a theoretical solution but in practice this approach towards noise reduction might be difficult due to variations while manufacturing.

There are no uncertainties considered in the calculation of NTF due to the large computational time. The consideration of uncertainties in the calculation of NTF itself would need a separate study. The difficulties in the measurement of the bushing force makes it useful to measure strut acceleration to evaluate groan performance as stated by [13]. Considering the ease of applying the metrics to evaluate groan behavior, in this case Sound Pressure Level is used for further studies.
5.1 Selection of stochastic input parameters

There are several different sources of uncertainty/variation that affect the groan noise problem. One main source of parameter variation is assumed to be the stochasticity in frictional coefficient of the pad. These stochastic parameters can directly affect the generation of groan noise. Furthermore, since creep groan is a structure-borne noise, the stochasticity in parameter values along the propagation path are taken into account. This section first deals with identifying the major influencing parameters in different propagation paths. Secondly, a multi-fidelity probabilistic approach is adopted to estimate the probability of failure in a robust and efficient way. Finally, a probabilistic failure analysis is done, using the obtained failure samples, for failure root cause analysis and giving proposals for groan noise reduction.

5.1.1 Significant sources of variation in brake groan problem

In order to track down the uncertain input parameters, it is important to understand the root causes of the problem. A fish bone diagram is a good tool to qualitatively present the root cause analysis and significant sources of variations that affect system response. The figure 5.1 covers most of the parameters that affect the creep groan based on a previous study at Volvo Cars. The input stochastic parameters taken for study are damping, translational and rotational stiffness of all the bushings and contacts used in all directions and coefficient of friction in static and dynamic conditions.
Probabilistic analysis of groan noise

**Figure 5.1: Fish bone diagram - Creep Groan**

**Figure 5.2: Percentage contribution of each parameter for different exit passage of noise propagation (LCA- Lower Control Arm)**
All together there are several parameters and it might become inefficient to study the influence of each parameter individually, when simulating complex dynamical systems such as a brake. Therefore, excluding the parameters that are not affecting the brake noise considerably can make the probabilistic failure analysis more efficient. Taguchi’s orthogonal array is one good way to reduce the number of simulations required, which is used in conjunction with Analysis of Variance (ANOVA). ANOVA is a method to identify the parameter with highest contribution to output response. This method is a good candidate as a first step parameter study to compare different stochastic parameters contribution into system response. However, one needs to be aware that the computational efficiency and the accuracy of this method is restricted by the number of stochastic parameters considered and the number of levels required to grid the parameter space. Moreover, it assumes the output response distribution to be a normal distribution (i.e. Gaussian), which is not the case for complex dynamical systems. The implemented procedure is shown in Appendix D, step by step. The percentage contribution for each factor considered is shown in figure 5.2.

Amongst all the input parameters considered, it is found that variation in bushing stiffness of the top strut mount has a major contribution to groan noise, as compared to other parameters, shown in figure 5.2. The error percentage indicates that, there is considerable contribution from other parameters that influence the objective or that there are some uncontrollable parameters present in the investigation. Note that, $\mu_s$ and $\mu_d$ are not considered here in the parameter analysis (for the propagation path stochastic parameter study) and only two levels of input parameters have to be considered. The obtained result is consistent with the modal analysis of the linearized brake model. The mode shape that is excited by the variation in brake torque due to stick-slip shows considerable deflection in the top-mount bushing. Modal analysis shows that bushing stiffness on the top strut mount deforms considerably which is consistent with the finding. Therefore, as indicated by this analysis, it is reasonable that the top-mount bushing stiffness has significant contribution in brake noise levels. So, the bushing stiffness in different directions can be considered as important stochastic parameters for probabilistic analysis in the following chapter, along with the coefficient of friction in stick-slip conditions.

5.2 Statistical properties of input stochastic parameters

As the input parameters are fixed, the variation range has to be set for sample distribution of parameters. The values used in Chapter 4 for input parameters are the nominal values. To consider the effect of variation in stochastic parameters, it is assumed that co-efficient of friction ($\mu_s, \mu_d$) can be varied up to 15 percent in automotive applications [31] and the stiffness variation in top mount bushing to be varied up to 15 percent from the nominal value, considering manufacturing variations and loading conditions. Here, the stiffness variation is taken the
same as 15 percent but for variation in $\mu_s$, $\mu_d$ - 11.11 percent is considered. Note that variation of $\mu_s$, $\mu_d$ is considered such that the static coefficient of friction is always higher than the dynamic coefficient of friction (motivated by the physics).

Table 5.1 shows the statistical properties of the input stochastic parameters, (i.e. nominal value, minimum and maximum values and PDF).

In reliability analysis, it is common to choose standard probability distributions, like, normal, log-normal, and uniform etc. Uniform distribution is one in which samples all through the interval have equal probability of occurring, whereas normal and log normal distribution are the one in which probabilities of samples closer to the mean are more likely to be occur.

For the model that was discussed in Chapter 4, in which knuckle, lower control arm and tie rod are designed flexible, simulation time is approximately 50-55 minutes for 50,000 time steps for every 0.1s with nominal groan value of 85.8 dB. The stiffness in top mount in three directions, $\mu_s$, $\mu_d$, in total 5 parameters, are considered as stochastic, as motivated by the analysis performed in above section.

### 5.3 Failure estimation by hierarchical multi-fidelity statistical approach

Considering the difficulties in the prediction and assessment of brake noise, if a practical way is employed to quantify the uncertainty in a brake system, it is possible to find out what percentage of a brake design will fail (in terms of noise). In this sense brake noise (groan noise for this thesis) can be considered as design failure problem [31] and failure analysis has to be carried out. Brake failure can be quantified by probability. In that case, probabilistic analysis has to be carried out on a model designed in ADAMS that takes 50-55 minutes and occupies 2.62 giga byte of storage space for each run. Carrying out several simulations on this model to confirm the probability obtained during the design phase is merely impossible. The interest for any CAE engineer would lie in creating a simpler model to reduce the time for the simulation and reducing the storage space for each simulation (if one wants to go for non-deterministic time-domain analysis) so that one can go for increased number of simulations. Hierarchical multi-fidelity is a good approach to handle this problem. It combines a lower-fidelity, cheap-
to-evaluate model with lower accuracy to accelerate the failure risk prediction of the true higher-fidelity model (the expensive-to-evaluate, but more accurate one) while granting a consistent (i.e. in a probability sense) estimation [16]. This method has been used in [42], [35], [31], [10], [45] for a similar purpose but in other applications. This approach has been adopted to manage model complexity and to manage design change. The method is explained in the subsection 5.3.1 and a detailed discussion regarding the approach with automatic test case generation in a Python environment can be found in [42], [36]. This method helps to reduce the number of simulations required for the higher fidelity model. It is done by fusing the information of the lower-fidelity model into target analysis of the higher-fidelity model.

Before beginning the analysis a threshold has to be set for noise level. If the design produces more noise than this threshold then it is considered to be a failed design. Based on the input provided by Volvo Cars, the target threshold for groan is 93dB for Europe and 85dB for Asia and United States. Based on the target proposal provided by Volvo Cars, 90dB is assumed as the target threshold in this case.

5.3.1 Method work-flow

- A LFM required is defined (explained in subsection 5.3.2). $N_p$ number of simulations are run on the LFM with the given uncertain input parameter set. This analysis carried out on LFM is called preliminary analysis.
- $N_p$ number of output response ($g_p$), i.e. noise in dB, is obtained as the result from preliminary analysis, which is arranged in ascending order of noise.
- The uncertain input parameters are ordered according to $g_p$.
- The ordered input parameters are divided into groups corresponding to $N_t$ number of simulations required in target analysis (analysis carried out on HFM or true model).
- A subset of the parameters in preliminary analysis chosen for target analysis is taken and $N_t$ number of simulations are run and the output response for target analysis ($g_t$) is obtained.
- The output response $g_t$ of target analysis is arranged in ascending order.
- Probability of failure of both high/low fidelity models are obtained and the corresponding CCDF are computed.
5.3.2 On choice of Lower Fidelity Models (LFM)

There are several ways to obtain a lower fidelity model. One could change the flexibility and rigidity in the model, vary time steps, and/or create a surrogate model. One option could be changing the temporal and spatial discretization resolution (i.e. equivalent to reducing spatial mesh size or degrees of freedom/time step) is always a straightforward approach to reduce model/solver complexity. This leads to a model with lesser degrees of freedom simulated with a coarser time step size. Thus, one can achieve accurate results without losing the design details by reducing the number of modes that one use for making knuckle/lower control arm/tie rod. Another option is to make reasonable parts flexible/rigid, without considering vibrational modes. In this subsection, an attempt is made to select some lower fidelity model that could be used as an alternate to the most accurate, true expensive model, in the preliminary failure analysis. Amongst several options flexibility/rigidity of the component and time step are chosen to vary to make lower-fidelity models. The models mentioned below are exploited to accelerate the failure analysis of the higher fidelity model (the true model discussed in Chapter 4 with 50,000 time step) for just 80 simulation runs.

**Failure analysis with LFM-1**

All components are modelled as rigid. It is run for 400 simulations (as it gives smoother CCDF plot for comparison) with the number of time steps being reduced from 50,000 to 500 (an assumption made to make the model cheaper) for each simulation. Each simulation takes only about a minute for this LFM as compared to 50-55 minutes with the true model. This LFM is taken for preliminary analysis and the true model is taken as HFM for target analysis. The hierarchical multi-fidelity statistical analysis is carried out, following the procedure explained in subsection 5.3.1. The simulations are faster, but the accuracy drops drastically since we get a considerable bias in failure risk estimation, i.e. probability of failure is estimated to be 100 percent. So, this model may not be a fast-to-evaluate model. Figure 5.3 shows that making all the components rigid results in the generation of higher groan noise (>90 db), compared to the true high-fidelity model.

**Failure analysis with LFM-2**

All components are modelled as rigid except knuckle which is made flexible. It is just an assumption made, as the vibration caused by stick-slip has to pass through the knuckle to reach other subsystems, so the knuckle alone is made flexible and rest of the components are made rigid. Similar to LFM-1, LFM-2 is run for 400 simulations with the number of time steps being reduced from 50,000 to 500 for each simulation. This LFM is taken for preliminary analysis and the true model is taken as HFM for target analysis. Each LFM simulation takes only 20 - 25 minutes, when the hierarchical multi-fidelity statistical analysis is carried out, following the procedure explained in subsection 5.3.1. The variation in output is comparatively low, and the probability of failure is .0425. Figure 5.3, 5.4 shows that this model generates a slightly higher noise for the same input than the higher fidelity model or simply over predicts the groan noise.
5.3 Failure estimation by hierarchical multi-fidelity statistical approach

Figure 5.3: $g$ plot for all rigid vs true model and knuckle flexible vs true model (in dB)

Figure 5.4: CCDF comparison plot for HFM with LFM 1 and LFM 2

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Model</th>
<th>Description</th>
<th>$P_f$ for LFM</th>
<th>$P_f$ for HFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LFM 1 + HFM</td>
<td>All rigid + True model</td>
<td>1</td>
<td>0.0225</td>
</tr>
<tr>
<td>2</td>
<td>LFM 2 + HFM</td>
<td>Knuckle flexible + True model</td>
<td>0.0425</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 5.2: Probability of Failure of lower-fidelity method
5.3.3 Results

The CCDF plot in figure 5.4, and table 5.2 is a comparison of results of failure analysis from the two different LFM taken for study. The CCDF plot comparison is a good way to study the response trend and correlation of responses between two designs. Here, probability of failure is found using a fixed threshold, whereas it should be noted that the noise threshold for a required failure probability can also be chosen for a design based on the CCDF plot. It is seen that $P_F$ of LFM 2 is in better correlation with its HFM as compared to correlation between LFM 1 and its corresponding HFM. So the $P_F$ of HFM corresponding to LFM 2 can be interpreted as a reliable solution. It clearly shows the variation in results based on the LFM considered and the necessity for the LFM response to be in better correlation with HFM. It is necessary to verify the method as well before concluding the results.

A probability analysis was carried out on the true model for the same set of input parameters by a standard Monte-Carlo method (reference method for verification) and 400 simulations were run for different inputs, in order to validate the method. Noise response in dB is found for varied input. CCDF is plotted as shown in figure 5.5. The Probability of failure of the model considered is found to be 0.0425. The difference between the methods is found to be only 6.25 percent. This shows that the Hierarchical multi-fidelity statistical method is robust and produces reliable results for prediction of failure events and has reduced the simulation time and storage memory drastically which is shown in table 5.3.

![Figure 5.5: CCDF plot for the true model](image)

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Method</th>
<th>Runs</th>
<th>Storage (GB)</th>
<th>Time (mins.)</th>
<th>$P_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Multi-fidelity method</td>
<td>480 (400+80)</td>
<td>241.8 (35.8+206)</td>
<td>11600 - 15000</td>
<td>.04</td>
</tr>
<tr>
<td>2</td>
<td>Reference method</td>
<td>400</td>
<td>1030</td>
<td>20000 - 22000</td>
<td>.0425</td>
</tr>
</tbody>
</table>

Table 5.3: Probability of Failure-Method comparison


5.4 Root cause analysis through probabilistic failure analysis

Apart from probabilistic analysis, the study of distribution of input parameter provides significant information about output profile. The true model and the second LFM-2 (knuckle alone designed flexible) is taken for study on input parameters. LFM-2 is considered, as it is observed to be more consistent with the true model, compared to the first LFM. The scatter between two input parameters is shown in figure 5.6, 5.7, 5.8. The plot is provided both in physical and in standard normal domain. Physical domain compares the values of input parameters directly irrespective of the units and standard normal domain shows the probability that a response exceeds or falls behind a threshold on a normal distribution. It should be noted that parameters of any combination and in any dimension can be used. Some of the key combinations are discussed below.

It is observed from figure 5.6 5.7 and 5.8 that there is an even spread of samples in both LFM and HFM. Also, the failure happens in similar region for both the models. So, the LFM can also be trusted, as long as there is a correlation between LFM and HFM. The plots between output responses of two models as shown in 5.3 provides this insight.

![Figure 5.6: μ_s vs μ_d plot](image-url)

*Figure 5.6: μ_s vs μ_d plot [*’* - LFM, *o* - HFM, blue - prior distribution, red - failure distribution]*
Figure 5.6 shows that, when the difference between $\mu_s$ and $\mu_d$ becomes higher or simply, when the $\mu_s$ value is increased and $\mu_d$ value is decreased from the nominal value, it leads to failure, i.e. brake groans. However, it should be noted that, changing the nominal values of $\mu_s$ and $\mu_d$ might affect performance of the brake. So, it is necessary to have a balance between performance requirements and noise requirements. Also, the variation of $\mu_s$ and $\mu_d$ is something that relies on the brake pressure input from the car driver, friction material wear, and varying dynamic conditions which could not be easily controlled. It should be noted that the failure region has the difference between $\mu_s$ and $\mu_d$ to be around 0.123. So, it is suggested to select the material for brake pad such that $\mu_s$ and $\mu_d$ difference does not touch this range in extreme conditions to meet the target threshold for groan noise (like Non-Asbestos Organic pads or comfort pads that minimizes the difference between $\mu_s$ and $\mu_d$).

Figure 5.7 and 5.8 shows that the brake groans for lower values of bushing stiffness of top mount in x-direction ($k_x$), whereas it groans for both higher and lower values of $k_y$ and $k_z$. This means reducing $k_x$ leads to failure, whereas it is not much dependent on y or z direction stiffness of the bushing. This shows the importance of $k_x$ for the groan problem, and the fact that for this model, one can decrease the noise levels by increasing the stiffness of the top-mount bushing in x direction. Increasing stiffness of rubber bushing in one direction can be achieved either by making changes in manufacturing or even while assembly, the later is simpler. It should, however, be noted that the bushing properties largely depends on an aging factor. So, the aging factor should also be considered while modifying the properties of rubber material.
5.5 Discussion

A probabilistic analysis was carried out for a brake model by hierarchical multifidelity statistical approach which is shown to be robust. To improve the noise performance of the brake model designed in ADAMS improvements can be made. Suggestions were provided for a design modification, based on probabilistic failure analysis and the noise root cause study. Changing the coefficient of friction, or using pad component materials with less variation in friction coefficients, can be one proposal. This leads to less difference between static and dynamic coefficients of friction, which results in a lower noise levels, but it is a major design proposal. As another proposal, increasing the top-mount stiffness, the parameters in propagation path of groan provided, can be made in order to decrease groan noise level in the cabin interior.

Figure 5.8: $k_x$ vs $k_z$ plot ['.' - LFM, 'o' - HFM, blue - prior distribution, red - failure distribution]
6

Conclusion and Future work

6.1 Conclusion

Investigation of the effects of uncertainty needs an appropriate metric and probability acts as a good uncertainty quantification tool. Probability of failure is used as a measure to quantify the uncertainty that arises due to the variation in input stochastic parameters. The work done in this thesis is continuation to the work carried out by Jaguar Land Rover [31]. With essential knowledge of the reliability and design failure risk, it becomes easier to take engineering decisions, and to avoid failure and overdesign (time consuming/higher DOF model) that might happen due to overestimation of the effect of uncertainties. This is achievable only by using non-deterministic analysis that lead us to reliable predictions of failure events (here groan noise). However, this is not achieved without computational cost. To tackle this, a hierarchical multi-fidelity statistical approach is adopted in this study, for probabilistic analysis of brake model. It reduces the simulation time and memory space while granting unbiased estimation of failure (i.e. probability of having groan higher than a threshold level). Applying this approach in the brake noise problem which is considered as the main novelty of the work, opens up the possibility to manage model complexity and design change during different design stages. Different sampling methods were discussed and Monte-Carlo is chosen over other methods for the first level (LFM), while in the second-level (HFM) a stratified sampling approach is performed, based the simulation results of the LFM. The probabilistic failure analysis, instead, helps in the root cause study and provides knowledge about the parameters (or parameter combination) which mostly affect the system output and how they affect the noise. It provides valuable information about the reason for groan noise and provides a chance to analyze parameters and take appropriate engineering decisions.
6.2 Future work

- Analysis can be carried out considering all the uncertain parameters/particular parameter of interest. The study could be on other uncertain parameters such as geometrical parameters (e.g. size, manufacturing tolerance, etc.) or more environmental conditions. Since the used statistical methods (i.e. Monte-Carlo, multi-fidelity simulation, etc.) are robust, with respect to the dimension of the parameter space, one can easily increase the number of stochastic parameters.

- The probability of failure of current model is found to meet requirements of European market. The noise level can be brought down for further improvements. Based on the probabilistic failure analysis and root cause study, the possible changes can be made in the input parameters to improve the chassis/brake noise performance. If it does not affect performance of the system. Increasing the stiffness of the top-mount bushing in $x$ direction can be one of such examples.

- The Hierarchical multi-fidelity statistical method used in this thesis, can be used for risk assessment of complex systems in other automotive applications, such, treating uncertainty in calculation of NTF, uncertainty in analysis of brake squeal problem.
A.1 Assymetric stiffness

% Title: Brake Squeal 4DOF Assymetric stiffness
% Reference: https://livrepository.liverpool.ac.uk/3002470/
% Page: Nobari Ami Pg:89
% Defining parameters
  clear all;
  m1=1;m2=m1;m3=m2;
  k1=100;k2=k1;k3=k2;k4=k3;k5=k4;
  kc=2*k1;
  c1=0.5;c0=c1;
  cc = 0.1;
  nu = .4:.1:1.3;
% Finding Frequency
  for i=1:length(nu)
    M=[m1,0,0,0,m3,0,0,0,m2,0,0,0,m2];
    K=[k1+k2,0,-k2,0,0,k4+k5,0,-k4,-k2,0,k2+ (.5*k3), (nu(i)*kc) - ...
       (0.5*k3);0,-k4,-.5*k3,k4+ (.5*k3)+kc];
    C=[c1,0,-c1,0,0,0,0,-c1,0,c1,nu(i)*cc;0,0,0,c0+cc];

    [u,v]=polyeig(K,C,M)
    for j=1:8
      freq(j,i)=v(j);
    end
  end
% plot for Eigen Value and Coefficient of Friction
A Appendix - Matlab codes

figure (1)
title ('Plot of Eign Value and Coefficient of Friction')
subplot (2,2,1)
plot (nu, real(freq (1,:)),nu, real(freq (3,:)));
ylabel ('Real part of Eigen Value $\lambda_1, \lambda_2$')
xlabel ('coefficient of friction ($\mu$)')
legend ('$\lambda_1$', '$\lambda_2$')

subplot (2,2,2)
plot (nu, real(freq (5,:)),nu, real(freq (7,:)));
ylabel ('Real part of Eigen Value $\lambda_3, \lambda_4$')
xlabel ('coefficient of friction ($\mu$)')
legend ('$\lambda_3$', '$\lambda_4$')

subplot (2,2,3)
plot (nu, imag(freq (1,:)),nu, imag(freq (3,:)));
ylabel ('Imaginary part of Eigen Value $\lambda_1, \lambda_2$')
xlabel ('coefficient of friction ($\mu$)')
legend ('$\lambda_1$', '$\lambda_2$')

subplot (2,2,4)
plot (nu, imag(freq (5,:)),nu, imag(freq (7,:)));
ylabel ('Imaginary part of Eigen Value $\lambda_3, \lambda_4$')
xlabel ('coefficient of friction ($\mu$)')
legend ('$\lambda_3$', '$\lambda_4$')

A.2 Brake Squeal 4-DOF model

% Topic : Mode coupling Brake squeal
% Input data
m1=1;m2=100;
k1 =2.488;k3=1;k4=1;k51 =1.333;k2 =1.203;k53=1;
u=.504;
c1 =0.05; c2 =0.05; c3 =0.05; c4 =0.05;
alp ha1 =30*$pi /180; alpha2=150*$pi /180;
k11=(k1*$cos (alpha1)*cos (alpha1)) + . . .
   (k2*$cos (alpha2)*cos (alpha2));
k12=((k1*$sin (alpha1)*cos (alpha1)) + . . .
     (k2*$sin (alpha2)*cos (alpha2)))-nu*k51;
k21=(k1*$sin (alpha1)*cos (alpha1)) + . . .
     (k2*$sin (alpha2)*cos (alpha2));
k22=((k1*$sin (alpha1)*sin (alpha1)) + . . .
     (k2*$sin (alpha2)*sin (alpha2)))+k51;
K=[k11,k12,0,k51*nu;k21,k22,0,-k51; . . 
   0,nu+k51,k3,-nu+k51;0,-k51,0,k4+k51];
M=[m1,0,0,0,m1,0,0,0,m2,0,0,0,0,0,m2];
C=[c2,0,0,0,c1,0,0,0,c3,0,0,0,0,0,c4];
% Eigen value and Eigen vectors
[u,v]=polyeig (K,C,M);
% Initial conditions
y0 = 0.1 * [0 0 0 0 1 1 1 1];

% State space form
A = [zeros(size(K)), eye(size(K)); -inv(M)*K, -inv(M)*C];

% Transient analysis
[t, y1] = ode45(@(t, y) (A*y) - ([zeros(4); inv(M)]*...
((y(2) - y(4))^3)*[-k53*nu, k53, k53*nu, -k53]'), [0, 200], y0);

figure (1)
subplot(2,2,1)
plot(t, y1(:,1), t, y1(:,2), t, y1(:,3), t, y1(:,4));

figure (1)
subplot(2,2,2)
plot(fgrid, Xabs);

figure (1)
subplot(2,2,3)
plot(y1(end-200:end,1), y1(end-200:end,5))

fs = 200;
X = fft(y1);
Xabs = abs(X);
N = length(Xabs);
fgrid = fs*(0:(N-1))/(N);
Xabs = Xabs(1:floor(N/4));
fgrid = fgrid(1:floor(N/4));

sp = sum(X,2);
P_eff = abs(sp)/sqrt(2);
P_eff_2 = P_eff.^2;
Lp = 20*log10(P_eff/2E-5);
Lp_tot = 10*log10(sum(P_eff_2)/4E-10);

bar(Lp_tot); hold on;
title({'Sound in dB'});
ylabel('[dB re. 2E-5 Pa]');
title('Overall SPL')
A Appendix - Matlab codes

legend([ ' Overall Lp (', num2str(round(Lp_tot)), ', dB) ' ]); 

A.3 Creep Groan 1-DOF

% Topic: Creep Groan 1DOF
% Input data
K=5;
M=1;
C=1.5;
sigma = 0.5;
nu_s = .5;
V_b = .5;
N=10;
x_dot = .5;

B=C+(nu_s*N*sigma);

[u,v]=polyeig(K,C,M);
x0=[0 .5];

A=[0,1; K/M, -C/M];

[t,y]=ode45(@(t,y) [(A*y)-([0;1/M]*...(nu_s*N*(sign(y(2)-V_b)-(sigma*(y(2)-V_b))))), [0 20],x0);

figure (1)

subplot(2,2,1)

plot(t,y);

hold all; plot(t(end-5000:end),y(end-5000:end,:), 'r ');

legend('Disp.', 'vel. ')

title({ ' Initial condition .1 and t=200 ', ...
for clarity x1, y1, x2, y2 alone is plotted '})

xlabel('time (s)' )

ylabel('Velocity (m/s)' )

fs = 200;

X = 2*ffft(y)/length(y);

Xabs = abs(X);

N = length(Xabs);

fgrid = fs*(0:(N-1))/N;

Xabs = Xabs(1:floor(N/4),:);

fgrid = fgrid(1:floor(N/4));

subplot(2,2,2)

plot(fgrid,Xabs);

title({ 'FFT plot '})

xlabel('Freq Hz')

ylabel('Amplitude')

subplot(2,2,3)

plot(y(end-5000:end,1),y(end-5000:end,2));

title({ 'Limit Cycle for las 5000 values of signal '})

xlabel('x1')

ylabel('x1 dot ')
A.4 Audible Noise for Creep Groan Model

\[
sp = \text{sum}(X, 2); \\
P_{\text{eff}} = \text{abs}(sp) / \sqrt{2}; \\
P_{\text{eff}}^2 = P_{\text{eff}}^2; \\
L_p = 20 \times \log_{10}(P_{\text{eff}} / 2E^{-5}); \\
L_{\text{p tot}} = 10 \times \log_{10}(\text{sum}(P_{\text{eff}}^2) / 4E^{-10}); \\
\]

\[
\text{subplot}(2, 2, 4) \\
\text{bar}(L_{\text{p tot}}); \text{hold on}; \\
\text{title}('\text{Sound in dB}') \\
\text{ylim}([50, 150]) \\
\text{ylabel}('[\text{dB re. } 2E^{-5} \text{ Pa}]') \\
\text{title}('\text{Overall SPL}') \\
\text{legend}(['\text{Overall } L_p (', \text{num2str(round}(L_{\text{p tot}})), ') \text{ dB}'])
\]

A.4 Audible Noise for Creep Groan Model

\[
\text{reaction}\_\text{forces}=\text{load}(\text{file}\_\text{location}); \\
\text{existing}\_\text{time}=\text{reaction}\_\text{forces}(:, 1); \\
\text{force}=\text{reaction}\_\text{forces}( :, 2: \text{end}); \\
\]

% plot of current time vs force
\[
\text{figure}(1) \\
\text{subplot}(2, 2, 1) \\
\text{plot}(\text{existing}\_\text{time}, \text{force}) \\
\text{title}('\text{Bushing forces vs time}') \\
\text{xlabel}('\text{time (s)}') \\
\text{ylabel}('\text{Bushing forces (N)}')
\]

% Resampling time and force
\[
[\text{existing}\_\text{time}\_\text{unique}, \text{ind}\_\text{etu}, \text{ind}\_\text{et}] = \text{unique}(\text{existing}\_\text{time}); \\
\text{force}\_\text{unique} = \text{force}(\text{ind}\_\text{etu}, :); \\
\text{new}\_\text{time} = \text{linspace}(\text{min}(\text{existing}\_\text{time}\_\text{unique}), \ldots \\
\text{max}(\text{existing}\_\text{time}\_\text{unique}), \text{length}(\text{existing}\_\text{time}\_\text{unique})));
\]

\[
\text{resampled}\_\text{force}=\text{interp1}(\text{existing}\_\text{time}\_\text{unique} , \ldots \\
\text{force}\_\text{unique}, \text{new}\_\text{time}); \\
\]

\[
\text{ind1} = \text{find}(0.4 <= \text{new}\_\text{time}); \\
\text{ind2} = \text{find}(1.2 >= \text{new}\_\text{time}); \\
\text{ind} = \text{intersect}(\text{ind1}, \text{ind2}); \\
\text{new}\_\text{time} = \text{new}\_\text{time}(\text{ind}); \\
\text{resampled}\_\text{force} = \text{resampled}\_\text{force}(\text{ind}, :);
\]

% Frequency and FFT
\[
\text{L} = \text{length}(\text{new}\_\text{time}); \\
\text{Ts} = \text{mean}(\text{diff}(\text{new}\_\text{time})); \\
\text{Fs} = 1/\text{Ts}; \\
\]
Fn = Fs/2;
Freq = linspace(0, 1, fix(L/2)+1)*Fn;
FFT = fft(resampled_force)/L;
Iv = 1:length(Freq);
FFT = (FFT(Iv,:))*2;

% Sound Pressure
Forces_order = [[4,7,1,10],[5,8,2,11],[6,9,3,12]];
FFT=FFT(1:length(NTFs20180226REIM(:,1)),:);
FFT=FFT(:,Forces_order);
NTF=10^6*(NTFs20180226REIM(:,2:2:end-1)+...
(i*NTFs20180226REIM(:,3:2:end)));
sp=FFT.*NTF;
subplot(2,2,2)
plot(0:1:500, abs(FFT))
title({'FFT plot'})
xlabel('Freq Hz')
ylabel('Amplitude')
subplot(2,2,3)
plot(0:1:500,abs(NTF))
title({'NTF plot'})
xlabel('Freq Hz')
ylabel('NTF')

% Sound pressure level
p_eff=abs(sp)/sqrt(2);
p_eff_2=p_eff.^2;
p_eff_2=sum(p_eff_2,1);
Lp=20*log10(p_eff_2/2E-5);
subplot(2,2,4)
bar(Lp)
xlabel('Transfer path #')
ylabel('[dB re. 2E-5 Pa]')
title('SPL due to each path')
grid on
Appendix - Flexible bodies in ADAMS

Ansa to .nas

- In ANSA, after meshing, and applying boundary conditions save the file in .nas format.

.nas to .ecd

- Save an existing .ecd file from previous work in the current folder that has the .nas file.
- Open .ecd file in Notepad / Notepad ++. Inside .ecd file, change line in which the file name with old .nas extension specified to the current .nas file name.
  - Make sure that the file name of .nas and .ecd are same.
.ecd to .mnf

- Open Shell terminal, 
  \textit{Start > All Programs > VCC CAE > Utilities > Bash shell}.

- Type \texttt{thinlinc} and press Enter on the keyboard to open thinlinc client window.

- Inside thinlinc window click on \textit{Computer} icon in bottom left corner. Click on \textit{Konsole} application to open it in a separate window.

- Open the current folder in which you have saved the .ecd file. 
  \textit{(Use \texttt{cd folder\_name} command to open the folder)}
• Use this command to coarse mesh the existing model
  \texttt{export MDI\_MNFWRITE\_OPTIONS="single,fast\_invar,strip\_face,coarsen(0.2,15,1)"}

• Use \texttt{nastran \_z \_q l file\_name.ecd} command to convert .ecd file to .mnf file

\textbf{Import .mnf in MSC ADAMS}

• Open Adams from Bash shell using the code \texttt{acar \_mdi \"-c aviewr ru-standand i exit\"}.

• After the ADAMS application is opened, click on \texttt{File>Import}. And then in the \textit{File Import} window, right click on \textit{File to Read} dialog box and select \textit{Browse} option to choose the .adm file you wish to open.
• Once you have imported the necessary model to ADAMS, right click on the body that needs to be made flexible. One will get Make Flexible option in the drop down list box.

Click on it to open Make Flexible dialog box. Choose one among the two options, Import MNF or Create New. Since we have already created .mnf file, Click Import MNF.
• **Swap a rigid body for a flexible body** window opens. In Alignment Tab, import the necessary MNF file by browsing from the empty space located.
• And click on *Align Flex Body CM with CM of Current Part* icon and click ok.
# Appendix - Shell Script

<table>
<thead>
<tr>
<th>USAGE</th>
<th>SCRIPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open a folder</td>
<td>cd folder_name/folder_name</td>
</tr>
<tr>
<td>One folder back</td>
<td>cd..</td>
</tr>
<tr>
<td>Open Thinlinc (UNIX/LINUX Environment)</td>
<td>thinlinc</td>
</tr>
<tr>
<td>Check number of users on all nodes</td>
<td>thinlinc -licall</td>
</tr>
<tr>
<td>Check which nodes you can log into</td>
<td>thinlinc -servers</td>
</tr>
<tr>
<td>Log in to a specific node</td>
<td>thinlinc -serv csX-n</td>
</tr>
<tr>
<td>Coarse mesh the ANSA model</td>
<td>export MDI_MNFWRITE_OPTIONS=&quot;single,fast_invar,strip_face,coarsen (0.2,15,1)&quot;</td>
</tr>
<tr>
<td>Convert .nas file to .ecd</td>
<td>nastran -z -q l file_name.ecd</td>
</tr>
<tr>
<td>Open ADAMS</td>
<td>acar -mdi &quot;-c aviewr ru-standand i exit&quot;</td>
</tr>
<tr>
<td>Run ADAMS Simulation</td>
<td>acar -submit l -a run.acf</td>
</tr>
<tr>
<td>Convert .res to .csv</td>
<td>vccpython xml_res_to_csv.py</td>
</tr>
<tr>
<td>Copy file from one folder to another</td>
<td>cae_data_sync_dirs /old_dir /new_dir</td>
</tr>
<tr>
<td>show all lines in a file</td>
<td>vis filename.extension</td>
</tr>
<tr>
<td>show all files in a folder</td>
<td>ll -rt</td>
</tr>
<tr>
<td>Open Mode frontier</td>
<td>frontier</td>
</tr>
<tr>
<td>Open Spyder (Python)</td>
<td>spyder</td>
</tr>
<tr>
<td>Open Matlab</td>
<td>matlab</td>
</tr>
<tr>
<td>Open ANSA</td>
<td>ansa</td>
</tr>
</tbody>
</table>
Appendix - Taguchi and ANOVA results

D.1 Taguchi Approach to parameter design

Taguchi Approach is a systematic and efficient method for determining near optimum design parameters for performance and cost [24]. It involves the following steps as shown in figure D.1 [41].

The following assumptions are made for the selection of input parameters, as several parameters affects the noise:

- As it is understood that generating factor definitely influences the noise. All parameters other than them are taken.
- Only the parameters that are present in transfer paths (end of tie rod, bushing and lower control arm) and the immediate contributor (contact stiffness and contact damping) is considered.
- Each parameter considered varies in all the three direction for simplicity, equal variation is considered in all direction.
- Mass of individual components are not considered as it will make the problem more complex

We end up with 12 input parameter. For 12 input parameters, two levels of input parameters (20 percent lower and twenty percent higher than nominal level) are taken which should be sufficient to study the behaviour on increase and decrease of each factor. For 12 input parameter and 2 levels, one ends up with L16 orthogonal array [24], i.e. the number of simulation needed reduces from $12^{12}$ to just 16 runs. The figure D.2 shows the input parameters and their levels in orthogonal array, It is observed that columns are mutually orthogonal. That is, for any
column, all combinations of levels occur and they occur an equal number of time [37].

![Flow chart](image)

**Figure D.1**: Design of Experiments procedure - flow chart

### D.2 Analysis of result

Once the levels are assigned to input parameters the simulation is run and the decibel sound obtained for each run at different level is analyzed by the following steps to get the percentage influence of each parameter.

#### D.2.1 Loss function

It is the difference between the target value of the output and the measured value of the output [39]. Different principle can be adopted as per maximization or minimization of the output [38]. Here, larger the better (here the worse) principle is adopted, since, if the noise level is more than 93 dB it is considered a failed design. For larger the worse,

\[
L_{ij} = \frac{1}{n} \sum_{k=1}^{n} \frac{1}{Y_{ijk}^2}
\]

Where, \( n \) is the number of repeated experiments, \( L_{ij} \) is the loss function of the \( i^{th} \) output in the \( j^{th} \) experiment and \( Y_{ijk} \) is the experimental value of the \( i^{th} \) output.
in the $j^{th}$ experiment at the $k^{th}$ test.

![Figure D.2: L16 Orthogonal Array (1-20 percent lower, 2-20 percent higher than nominal value)](image)
D.2.2 Normalizing the loss function:

As the measured units of the sound is different, the loss function is normalized in the range between zero and one [39].

\[ S_{ij} = \frac{L_{ij}}{\max(L_{ij})} \]  

(D.2)

Where, \( S_{ij} \) is the normalized loss function for the output in \( j^{th} \) experiment.

D.2.3 Signal to noise ratio

It is the ratio of strength of the signal to the proportion of the entire signal. Here, it is done to maximize the response signal by using the formula [38]:

\[ SN = -10\log(T_{L_{ij}}) \]  

(D.3)

where, SN is the signal to noise ratio.

D.3 ANOVA

ANOVA is a statistical method used to test differences between two or more means [27]. This method is used to interpret experimental data and make necessary decisions and it establishes the relative significance of input parameters in terms of percentage contribution to the output [41]. The method followed here is a proven method in the automotive industry for reducing the number of experiments and is referred from [38] [39] [41].

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>1</th>
<th>2</th>
<th>SSF</th>
<th>% contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact stiffness</td>
<td>2.345903</td>
<td>1.794364</td>
<td>0.019012</td>
<td>4.110954389</td>
</tr>
<tr>
<td>contact damping</td>
<td>2.406144</td>
<td>1.734123</td>
<td>0.028225</td>
<td>6.103165384</td>
</tr>
<tr>
<td>Top mout bushing stiffness</td>
<td>2.775296</td>
<td>1.36497</td>
<td>0.124314</td>
<td>26.87997924</td>
</tr>
<tr>
<td>Top mout bushing damping</td>
<td>1.785753</td>
<td>2.348557</td>
<td>0.01911</td>
<td>4.129879521</td>
</tr>
<tr>
<td>Top mout bushing Torsional stiffness</td>
<td>2.145625</td>
<td>1.994529</td>
<td>0.01425</td>
<td>0.080727776</td>
</tr>
<tr>
<td>LCA front Bushing Stiffness</td>
<td>2.153971</td>
<td>1.986295</td>
<td>0.00757</td>
<td>0.37995619</td>
</tr>
<tr>
<td>LCA front Bushing Damping</td>
<td>1.821729</td>
<td>2.318357</td>
<td>0.01542</td>
<td>3.33554437</td>
</tr>
<tr>
<td>LCA front Bushing Torsional stiffness</td>
<td>2.231689</td>
<td>1.908578</td>
<td>0.06525</td>
<td>1.410895679</td>
</tr>
<tr>
<td>LCA rear Bushing Stiffness</td>
<td>2.355381</td>
<td>1.784986</td>
<td>0.020327</td>
<td>4.395304754</td>
</tr>
<tr>
<td>LCA rear Bushing Damping</td>
<td>2.278412</td>
<td>1.861854</td>
<td>0.010845</td>
<td>2.344987977</td>
</tr>
<tr>
<td>LCA rear Bushing Torsional stiffness</td>
<td>2.212009</td>
<td>1.928258</td>
<td>0.005321</td>
<td>1.088093079</td>
</tr>
<tr>
<td>Tierod bushing Torsional Stiffness</td>
<td>1.433302</td>
<td>2.706965</td>
<td>0.01389</td>
<td>21.92294546</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td>0.109099</td>
<td>23.59022435</td>
</tr>
<tr>
<td>Overall percentage</td>
<td></td>
<td></td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Figure D.3: ANOVA table
This analysis is carried out on signal to noise ratios [39]. Total variability is measured by sums of squares of signal to noise ratio,

\[
SS_T = \left[ \sum_{i=1}^{N} y_i^2 \right] - T^2/N
\]  \hspace{1cm} \text{(D.4)}

where \( N \) is the total number of experiments, \( T \) is the sum of all experiments response variable and \( y_i \) is the \( i^{th} \) output. The total sum of squares includes the sum of squares due to each factor (\( SS_f \)) and the sum of squares of errors (\( SS_e \)). The ratio of (\( SS_f \)) to (\( SS_T \)) is the percentage contribution (\( P \)) by the factor [39].


[38] Philip J. Ross. *Taguchi techniques for quality engineering*. McGraw-


[47] Johannes Will. Cae-based robustness evaluation as part of the quality management. 05 2015.

