A System Study on Ultrasonic Transceivers for Haptic Application

Ishan Arya and Viswanaath Sundaram
Abstract

We are investigating the use of ultrasound in Haptic applications. Initially a brief background of ultrasonic transducers and its characteristics were presented. Then a theoretical research was documented to understand the concepts that govern haptics. This section also discusses the algorithm adopted by various researches to implement haptics in the professional world. Then investigations were made to understand the behavior of ultrasonic transducers and conduct software simulations to obtain various results. At first simulations were conducted on Field II software. This simulations involved the creation of elements in transducers, transducer’s spatial impulse responses, transducer’s impulse response in time and frequency domain, effect of adding apodization to the transducers, pulse echo response of the transducers, beam profile variation along the focal length of the transducers. Then a Matlab based GUI was used to study the relationship between number of elements in transducers, the frequency of the input signal and duty cycle variation of the input wave. A concept of phase shift, which explains the time delay generation was also coded in Matlab.
Acknowledgments

We would like to express our great appreciation to our examiner, Professor Atila Alvandpour for his valuable suggestions, advice and guidance for the planning and development of this thesis work. He has shown us how to perform the research in a professional and scientific manner. We thank him for his continuous guidance and encouragement from initial steps to the end of the thesis. Without his generous feedback, it would have been not possible for us to complete the thesis work.

We also would like to thank research engineer, Mr. Arta Alvandpour for his support in setting up the office and system environment for working in thesis.

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Ishan Arya and Viswanaath Sundaram
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<td>Resistor, Inductor and Capacitor</td>
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<td>BD</td>
<td>Beam Diameter</td>
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<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
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<td>Programmable Gain Amplifier</td>
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<td>AAF</td>
<td>Anti Aliasing Filter</td>
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<td>T/R</td>
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Technology has been increasing exponentially with time in the last decade. These revolutions in technology has brought major updates in every field such as engineering, medical applications, financial sectors, etc. The improvements in the medical treatment and surgical procedures have contributed largely towards the better health of humans. On contrary, different challenges in terms of treatment have been on rise; for e.g. surgery performed on extremely sensitive organs of human body, delivery of antibodies to a particular core organ of the body. Due to the intricacies of human body, these treatments require extreme caution and perfection. In this scenario, technology has been of great importance and help. The new techniques of medical imaging have brought great revolution in this sector. Haptic Feedback Technology and 3D imaging of internal organs provides the doctors and analysts with a clear vision of the internal anatomy of humans, which in turn helps them to plan better in advance to tackle the issues in a much cautious and effective way.

1.1 Thesis Outline

The outline of the thesis is presented as follows: Initially the basic buildup, the core fields and and intended goals have been discussed in the introductory chapter. Chapter 2 describes the basics behind ultrasonic principles, its applications in medical imaging in accordance with haptics. In the thesis, ultrasonic transducers are chosen as the primary source for the generation of ultrasound, which is later configured to obtain tactile sensations. So Chapter 2 also discusses the basics of ultrasonic transducers. Chapter 3 describes the basic research on haptic technology and its underlying principles for creating a tactile sensation. Chapter 4 provides the introduction to the tools, softwares and concepts used in the generation of haptic in the most basic way. Chapter 5 displays the results and
inferences obtained from the simulations conducted using the tools, softwares and concepts presented in Chapter 4. Chapter 6 describes the conclusion and future work in the presented research in this thesis. This is followed tool license agreement copy for using TAC GUI and bibliography in the respective order.

1.2 Introduction to Medical Imaging

Medical Imaging is a technologically advanced technique that consists of different processes to obtain the images of the various internal organs of human anatomy for the purpose of treatment and surgery of the patients. This technique not only provides images but also provides the plots depicting the behavior of various internal organs in the human body. The most common imaging techniques are X-ray radiography, Magnetic Resonance Imaging (MRI) and ultrasonography.

1.2.1 X-Ray Radiography

X-ray is the common and one of the oldest techniques incorporated in the medical imaging field. It consists of the uses of ionized radiation to produce images of the internal organs of the human body. It is used to detect bone fractures, tendon and muscle tears and pinpoint the presence of any stones or blockages in the tissues. Different organs of the body absorb varying amount of X-ray. For e.g. bones absorb most and hence they appear white in color whereas muscles and skin absorb less and pass more, so they are gray in the x-ray images. A test object is rotated and simultaneously x-rays are projected onto it from different angles and 2D images are obtained. An algorithm then reconstructs the internal organs and combines them to give a 3D image.

1.2.2 Ultrasonography

Ultrasonography is a medical imaging technique based on application of ultrasound. It is also referred as sonography. This method is used to obtain images of internal body organs, detection of any disease, narrowing down of a pathogen in the body. Its unique applications include the detection of pregnancy related and infertility related issues in women. It can also be used for the detection and diagnosis of heart diseases, prostrate glands in men.

In spite of the intricate tasks and applications covered by sonography, it stands on the basic scientific principle of using a simple sound wave having a particular frequency. The simplicity of the generation of ultrasound makes sonography as the most commonly used imaging technique in today’s world. It also provides the advantage of dealing with live images. Due to this, user can select the particular area for diagnosis in real time. Its safety and portability alleviates the need of patients going to the lab every time for tests. The equipment for the sonography can be easily carried to the patient’s home for conducting various tests. Sonography is performed with the use of ultrasonic transducers that transmits the sound into the body. Some of these sound pulses are reflected (from the
tissue or muscle depending on the impedance of the material) as an echo. 2D images are obtained by phased array of transducers by sweeping the beam electronically. A combination of these 2D images results into the construction of 3D images.

1.3 SONAR

One of the most common and major application of ultrasound is underwater range finding. This use of ultrasound is referred to as SONAR (Sound Navigation And Ranging). Sound travels faster in water than in air. Due to this property of sound, ultrasound can be used to navigate and locate obstacles underwater by emitting the ultrasonic pulse in a particular direction. If an obstacle is present in the path of the pulse, it gets reflected back as an echo to the transmitter and is detected through the receiver. By calculating the time difference between the emitted pulse and the received echo, it is possible to calculate the distance between the source and obstacle. This technique is used by ships and submarines to detect underwater obstacles.

1.4 Haptic Relation

According to (M. Mihelj [17]), the word haptic is derived from a Greek word hap-tikos which means ability to touch or feel objects. The concept of haptic constitute of two crucial sensations namely, tactile and kinesthetic. The tactile sensations provide the information about any impetus on the human skin whereas the kinesthetic senses enlightens the humans about the then current body motion and posture. When any device induces the kinesthetic or tactile sensations to the user, the concept of haptic display is introduced. The concept of haptic application can be seen in Fig. 1.1.

Like medical imaging, haptics using ultrasound is a great potential application. Although the application of haptics using ultrasound have still not been incorporated into our daily life, but its inherent quality promises an exponential growth in the upcoming future. Some of the possible application which could come into use in the near future are: levitation of small particles using ultrasound, delivery of drugs and antidotes to different internal organs of the body, controlling the interface of stereo systems in automobiles without actual touch.

Our aim is to perform a detailed analysis on the fundamental principles and concepts on which haptic feedback is based, and implement it using ultrasonic transducer.
1.5 Goal of the Thesis

The main goal of the thesis is to conduct a system study on the principles of haptic, implementing electronic beamformation using ultrasound and creating a sense of touch by constantly varying the acoustic pressure at the target point.

This is currently an upcoming research field and has tremendous potential for creating technical advancements in medical field. Since this field is still in its nascent stage, no particular software or tool available for accurate modeling and accurate validation of haptics. In order to accomplish the main goal of the thesis the following tasks needs to be completed:

1. Studying the characteristics and transfer function of a piezoelectric transducer to establish the mathematical relation between the frequency of the input signal of the transducer and its corresponding output.

2. Understanding the principles underlying haptics.

3. Performing simulations on various characteristics of transducers using Field II software.

4. Implementing the concept of phase delay to focus ultrasound beam at the target point.

5. Achieving a constant variation in pressure by using a pulse width modulated wave as input.

The credibility of this work depends on the implementation, application and the evaluation methods.
2 Theory

2.1 Basic of Ultrasonic Principles

Human ear is capable of perceiving sounds whose frequencies lie between 20Hz to 20 KHz. Any sound with the frequency greater than 20 KHz is termed as ultrasound. The frequency of ultrasound is the number of cycles in one second and is inverse of the time period. According to (NDT [18]), the velocity of ultrasound is related to its wavelength as in Eq. (2.1)

\[ \lambda = \frac{c}{f} \]  

(2.1)

where ‘c’ is the velocity of ultrasound, ‘f’ is the frequency of ultrasound and ‘\( \lambda \)’ is its wavelength.

The waves representing ultrasound are either longitudinal (the particles motion is along the line of propagation) or shear waves (the particle motion is perpendicular to propagation direction). Ultrasound can be used to obtain information about an object. One such example is to calculate thickness of an object, according to (NDT [18]) is given by Eq. (2.2)

\[ T = \frac{ct}{2} \]  

(2.2)

where ‘T’ is the thickness of material, ‘c’ is sound velocity, ‘t’ is the time of flight.

Every ultrasonic system has the potential to detect an errors in any obstacle along its path of propagation. This property is described as sensitivity.

2.2 Piezoelectric Transducer

As per (Center [6]), a transducer is a simple device which converts one form of energy to another form of energy. At the elementary level, a transducer is classified
into two categories namely input and output transducers. They differ with each other on the basis of the type of input signal given to them. The transducers that convert electrical energy into another forms of energy, or vice versa with the use of material such as quartz crystal, are termed as piezoelectric transducers. This conversion of energy into various different forms inside piezoelectric transducers forms the base for ultrasound testing. When mechanical stress is applied to certain piezoelectric material, they produce an electrical charge in response to that input. This process is termed as piezoelectric effect. The piezoelectric transducer produces ultrasound when it is subjected to an electrical signal of a particular frequency, thus acting as a transmitter. It can also produce electrical signal as an output when subjected to ultrasound, thus acting as a receiver. Piezoelectric Transducers are also classified based on their application: contact transducers and non contact transducers.

2.2.1 Contact Transducers

These type of transducers are deployed in direct contact with the surface of testing product. They are encased so as to bear the frictional contact with any material. The cut section of contact transducers can be seen in Fig 2.1. These transducers possess replaceable wear plate, thus providing a longer durability. According to (Center [6]), the most important parts of these transducers are backing material, a piezo(active) element and a wear plate. The piezo element is adjusted to 1/2 the targeted wavelength. The impedance matching layer is squeezed in between active element and the the surface of the transducer. The width of the matching layer is 1/4 of the target wavelength so that perfect impedance matching is attained. This provides an advantage of keeping the waves in phase. The use of wear plate is to prevent the piezo element being scratched by the matching layer.

Figure 2.1: Cut Section of Contact Transducer. Adapted From Center [6]

2.2.2 Immerse Transducers

Immersion transducers do not have any direct contact with the component that is inspected. They have been specifically designed to operate in a liquid medium. This type of transducer has a built in matching impedance layer, to produce more
sound energy in the liquid medium to inspect the component. It can be used in a water tank or bubbler system in scanning applications.

2.3 Characteristics of Piezoelectric Transducers

According to (Center [6]), the characteristics of piezoelectric transducers such as efficiency, resonance and damping are described briefly in this section.

2.3.1 Efficiency and Bandwidth

Efficiency of a transducer depends on its sensitivity and resolution. Sensitivity is directly proportional to the product of the both transmitter and receivers efficiency. The capacity of a transducer to detect any errors near the surface of material is termed as resolution.

Every transducer has a frequency mentioned on it. This frequency is known as the center frequency and is dependent on the backing material. The most efficient damping is produced when the impedance of backing material is analogous to that of the piezo element. The bandwidth and sensitivity of the transducers is directly proportional to the impedance matching. Bandwidth of transducers is inversely proportional to penetration of ultrasound in a material.

2.3.2 Resonance

Every transducer has a tendency to vibrate at its natural frequency with maximum amplitude at its lowest harmonic frequency. The natural frequency of a transducer is dependent on its shape, size and material. Natural frequency is also known as resonant frequency. When we provide an energy source possessing a frequency which is approximately equal to its natural frequency, the phenomenon known as resonance occurs. If the frequency of the energy source is greater than the natural frequency, then the output frequency has a low amplitude.

2.3.3 Damping

The ability of a transducer to decrease the number of vibrations or noise is termed as damping. The process through which the system decreases the vibrations involves the absorption of some part of mechanical or electrical energy. One advantage of damping phenomenon is that it helps alleviate the excess higher frequency unwanted noise.

2.3.4 Frequency Response

The transducer produces an output frequency in response to the frequency of the input given to it. This inherent property of the transducers to produce the required frequencies is termed as frequency response.
2.4 RLC Model of a Piezoelectric Transducer

The RLC circuit of a piezoelectric device represents its electromechanical behavior. This model is valid for only one resonance. The RLC circuit can be seen in Fig 2.2. In the Fig 2.2, $C_0$ is dielectric capacitance $R_1$, $C_1$, $L_1$ are the resistor, capacitor and inductor connected in series. According to (as in Asztalos [2]), pattern of piezo element’s response can be seen in Fig 2.3. It depicts that increase in cycling frequency is proportional to piezo’s first approximation of frequency where the impedance observed is minimal ($f_m$) i.e. at this point admittance is maximum. The $f_m$ is approximately equal to series resonance frequency, $f_s$, where impedance of piezo element is zero. With the further increase in cycling frequency, the rise in impedance is maximum. The frequency at which this happens is symbolized as $f_n$, which is approximately equal to the parallel resonance frequency $f_p$. The maximum impedance frequency is also termed as anti resonance frequency, $f_a$, the piezo element depicts capacitive behavior for frequencies $f<f_s$ and $f>f_p$, whereas it depicts inductive behavior for frequency $f_m<f<f_n$. The maximum response from the piezo element is observed between $f_m$ and $f_n$.

According to (Asztalos [2]), the series resonance frequency $f_s$, is given by the Eq. (2.3)

$$f_s = \frac{1}{2\pi} \sqrt{\frac{1}{L_1 C_1}}$$  \hspace{1cm} (2.3)

where $f_s$ is the series resonance frequency [Hz], $L_1$ is the inductance of mechanical circuit [H], and $C_1$ is the capacitance of mechanical circuit [F]. According to (Asztalos [2]), the parallel resonance frequency $f_p$, is given by Eq. (2.4)

$$f_p = \frac{1}{2\pi} \sqrt{\frac{C_0 + C_m}{L_mC_0C_m}}$$  \hspace{1cm} (2.4)
2.5 Transfer Function of a Transducer

Figure 2.3: Impedance as a function of frequency. Adapted From Asztalos [2]

where \( f_p \) is the parallel resonance frequency [Hz], \( L_m \) is the inductance of mechanical circuit [H], \( C_o \) is the capacitance of transducer below resonance frequency - \( C_m \), \( C_m \) is the capacitance of the mechanical circuit[H].

2.5 Transfer Function of a Transducer

The transfer function of a transducer can be obtained through several equivalent circuit methods such as Mason model and KLM model. The KLM model is much more efficient than Mason model, as it uses acoustic transmission lines and avoids the negative capacitance which appears in the Mason model. The simplified view of a transducer can be seen in Fig. 2.4 and the KLM model of a transducer is shown in Fig. 2.5

Figure 2.4: Simplified View of a Transducer. Adapted From Chunyan Gao [7]

where \( Z_0 \), \( d_0 \), \( c_0 \), \( k_t \), \( K_0 \), \( \varepsilon_0 \) represent the acoustic impedance, thickness, ultrasonic speed, electromechanical coupling factor, sound wave number and relative dielectric constant of piezoelectric chip respectively. \( Z_{p1} \), \( d_{p1} \), \( c_{p1} \), \( K_{p1} \) represent the acoustic impedance, thickness, ultrasonic speed and sound wave number of matching layer respectively. \( Z_l \) represents the acoustic impedance of the load. \( Z_b \)
represents the acoustic impedance of the backing and Rs represents the internal resistance of the power. The transfer function of a transducer described by KLM model according to (Chunyan Gao [7], S J H. Kervel [22], M. Castillo [16]) is given by Eq. (2.5), if the excitation signal to the transducer is a unit impulse, i.e. $U(\omega) = 1$

$$T(\omega) = F_{\omega} = \frac{2Z_t}{-Z_{in}Z_tN_{t21} + Z_{in}N_{t11} - N_{t12} + Z_tN_{t22}}$$  \hspace{1cm} (2.5)


$$N_t = \begin{bmatrix} N_{t11} & N_{t12} \\ N_{t21} & N_{t22} \end{bmatrix}$$

and

$$Z_{in} = \frac{N_{t22}Z_t - N_{t12}}{N_{t11} - N_{t21}Z_t}$$ \hspace{1cm} (2.6)

$N_2$ is the matrix defining capacitance and reactance of the piezoelectric chip, $N_3$ is the transform coefficient matrix, $N_4$ is the matching matrix between backing and the chip, $N_6$ is the chip connected to the backing layer, $N_7$ is the matrix of matching layer.

### 2.6 Single Element Transducer

The transducers that consists of only one active element are termed as single element transducers. These can be efficiently administered as transmitters, receivers or transceivers. Since they contain only one active element, they are a very economical solution in any application. The single element transducers are categorized in two shapes: plane transducers and focused transducers. The plane transducers have the limitation in terms of lateral resolution and sound intensity. The inherent focusing property of focused transducers helps in improving the extent of lateral resolution and sound intensity. The method of constructing focused transducers involves usage of a lens or modeling of the piezo element at various angles.
2.6 Single Element Transducer

2.6.1 Quality Factor, Focusing and Resolution Principles of Single Element Transducers

Quality Factor (Q) determines the rate at which a transducer loses the energy. According to (William D [27]), the Q is directly proportional to the energy stored in the transducer and inversely proportional to the energy lost from the transducer and is expressed as in Eq. (2.7)

\[ Q = \frac{f_r}{\text{Bandwidth}} \]  

(2.7)

The resolution and depth of penetration into any object are dependent on ultrasonic frequency. The frequency is directly proportional to the resolution and inversely proportional to the penetration. The frequency is also directly proportional to the attenuation.

Every transducer has the potential to distinguish between discrete structures which are encountered in front of them. This capacity for every transducer is termed as its resolution and according to (William D [27]), its is dependent upon the transducer type, beam pattern, bandwidth, frequency, sound speed, object attenuation and other processing electronics. There are two types of resolution namely: axial (the potential to distinguish between discrete structures along beam axis) and lateral (the potential to distinguish between discrete structures perpendicular or lateral to the beam axis).

2.6.2 Radiation Field of a Single Element Transducer

The radiation field of a single element ultrasonic can be seen in Fig. 2.6. In this the intensity of sound is inversely proportional to the color darkness. Lighter the color, higher the intensity of sound. The field from the transducer does not emerge from one single point on the surface, but it emerges from a number of points on the transducer surface. The waves emerging from the transducers interferes with each other and this becomes a crucial factor affecting the ultrasound intensity. This continuous wave interactions causes disruptions in the sound field in proximity of the source and gives rise to near field. The ultrasonic beam depicts a much higher uniformity in its pattern once it surpasses the near field region. This area is known as far field. According to (University [26]), the transition between near field and far field occurs at a distance N, which is known as natural focus of the transducer. As per (William D [27]), the range of near field is dependent on dimension of the transducer (a is the radius) and the wavelength and is expressed mathematically as in Eq. (2.8)

\[ \text{Near field range} = \frac{a^2}{\lambda} \]  

(2.8)

The near field distance N is also represented by Eq.(2.9)

\[ \text{Near field distance} = \frac{D^2 f}{4c} \]  

(2.9)

where D is the element diameter, f is the frequency and c is the material sound velocity.
Figure 2.6: Radiation Field of Ultrasonic. Adapted From Center [6]

The sensitivity of the transducer is dependent on the beam diameter at the point of interest. The amount of energy reflected by the flaw is more when beam diameter is small. According to (NDT [18]), the -6 dB pulse-echo beam diameter at the focus can be calculated with Eq.(2.10) and for a flat transducer use Eq. (2.11)

\[ BD(-6dB) = \frac{1.02Fc}{fD} \] (2.10)

\[ BD(-6dB) = \frac{0.2568DS_F}{fD} \] (2.11)

where BD is the beam diameter, F is the focal length, c is the material sound velocity, f is the frequency, D is the diameter and \( S_F \) is the normalized focal length. According to (NDT [18]), the focal zone is located on the axis, when the amplitude of the pulse echo signal amplitude drops to -6dB at the focal point and the length of the focal zone is given by the Eq.(2.12)

\[ F_Z = N \times S_F^2 [2/(1 + 0.5S_F)] \] (2.12)

2.6.3 Transmitter and Receiver circuit

Ultrasonic transducers are found extremely useful in many industrial applications such as non-destructive testing, human machine interaction and even estimating the distance of an object. In order to perform these tasks both accurately and efficiently, the ultrasonic transducer is linked to both the transmitter and receiver circuit.

Fig. 2.7 shows the block diagram of transmitter and receiver circuit of ultrasound, for a single element transducer. The transducer is connected to both high voltage transmitter and front end receiver through a coaxial cable. As mentioned in (Taehoon Kim and Kim [25]), when the circuit is in transmit mode, an electrical pulse from the high voltage transmitter is given as input to the transducer. The transducer then emits ultrasonic waves towards the focal point. If the waves hit an object/obstacle in its path, it gets reflected back to the transducer. When the reflected wave reaches the transducer, the T/R switches to receive mode thus disconnecting the high voltage transmitter. The reflected echo signal is then converted into electrical signal and further processed in the front end receiver.
The front end receiver circuit is made of a low-noise amplifier (LNA), which acts as a preamplifier; a programmable-gain amplifier (PGA), which provides time-gain compensation to account for the attenuation of echo signal in body tissues as a function of the distance traveled; an antialiasing filter (AAF), which restricts the bandwidth of the signal to satisfy the Nyquist-Shannon sampling theorem; and an ADC, which digitizes the electrical echo signal for subsequent image processing.

### 2.7 Multiple-Element Transducer Array

Multiple Element transducer contains group of transducer, commonly known as arrays. There are many different types of arrays and their names clearly state the type and the sequence of operation, for example, linear sequenced array, describes both how the array is constructed (linear) and how it is operated (sequenced). However, in most cases the names are incomplete and just mention the type of the array alone. As mentioned in (Kremkau [15]) transducers elements can be arranged in a straight line (linear array), or curved line (convex array), or phased array (square, circular etc. based on their application). According to (Stephen W. Smith [24]) there are three different criteria which determine the size and shape of the transducer. (1) The elements must have sufficient angular sensitivity to steer the phased array over a +/- 45° sector angle. (2) The arrays must have enough inter element spacing to avoid grating lobe artifact (3) the width of each rectangular element must be small compared to the transducer thickness to remove parasitic lateral mode vibrations from the desired transducer pass band.

However, these transducer arrays have certain drawbacks in the fabrication process, as it is difficult to achieve the required sensitivity and bandwidth from such small elements. The 2-D array transducers are mainly used for three dimensional imaging systems, development of high speed C-scans, levitation of a small particle, which could potentially be used in medical field and Haptic applications. We will be discussing more about Haptics and its related concepts in the next chapter.

*Figure 2.7: Transmitter and Receiver Circuit. Adapted From Taehoon Kim and Kim [25]*
3.1 Basics about Haptics

For haptics, an artificial ambiance is created where virtual objects are generated through the means of some processing unit and then humans interact with these virtual objects through the means of motor sensations. Generally, any haptic based unit comprises of a display, which depicts images and sound on the basis of human interaction with the computer. The generation of any virtual object or of the haptic sensations is dependent on human’s capability to decipher the objects haptic parameters, the technical ability to generate objects in real time and the precision of haptic device for providing the right stimulus.

Any haptic system that constitute only visual and audio sensations of the user limits its application. However the haptic system that involves the audio-visual sensations and most importantly comprises of providing the opportunity to humans for manipulating the objects such as grasping, moving, etc is considered to be a highly efficient haptic unit.

According to (M. Mihelj [17]), a haptic interface is the device that is used for manipulation of virtual objects. It records positions/impetus force and displays the same after manipulation, to the user. This device acts as a substitute for hand related tasks in real world. It receives motor inputs from the user and in response to that, it displays haptic image to the user.

According to (M. Mihelj [17]), the typical block diagram of any haptic system can be seen in Fig. 3.1 The human interacts with the haptic interface via the means of any movement. This interface gauges and records the stimulus provided by the human. The recored input value is used as a reference input to the teleoperation system or to the virtual environment. The teleoperation system consists of a slave robot, whose job is to perform the task in reality, that the human instructs to the interface. The slave system and the object affected by it ,
both comprise of programmed virtual environment. The controlling part of slave system is independent of the environment and forms the basis for comparing the output of haptic interface and the output readings of the slave system. After the operation of slave system, the haptic interface’s task is display the result movement/force back to the user. There can be many probable causes of forces i.e. they can be due to interactions between different objects or interaction between an object or a slave system. Due to this reason, collision detection is a crucial part of the loop. In reality, collision detection is due to the interaction between robot and its environment, whereas in virtual surrounding, it involves interactions between virtual objects which can me modeled using various ways. The method of computing force in teleoperation system involves usage of force sensor attached on the slave robot. However, in case virtual environment, the method of computing force involves measuring contact force based on physical model of the object. The measured force is then sent to the user through haptic interface. The local feedback loop is responsible for synchronizing the movement of haptic interface with the measured force.

Haptic devices are required in variety of places depending upon the task. In the field of entertainment, simple haptic devices are used in video games. They provide number of stimuli to the user thus inducing tactile sensations in the user. Haptic devices reduces the need of visual feedback and improves the efficiency of the task. Haptic devices do not create commotion in the surrounding i.e. only necessary information is provided to the user at right moment whereas, in audio or visual feedback there are lot of unwanted feedbacks also.

There are number of ways in which haptics can be done: Majority of the haptic feedback which we get is through vibration, which can be induced by using a band or ring which the user wears while interacting with the system, another method of using haptic feedback is through air vortices. This is been conceptualized in a technology named Aireal, which will be discussed briefly in the next section, the last and final way of creating mid air haptic feedback without using any external actuators is by using ultrasound.
3.2 Haptics using Air Vortices

AIREAL is a technology that produces tactile sensations in the air without the use of any equipments to be worn while experiencing it. It is a huge boost in this field as not wearing any physical equipments will not hamper natural user interaction. The concept described in this paragraph about AIREAL is derived from (Rajinder Sodhi [21]). AIREAL induces tactile sensation in mechanoreceptors by the use of air pressure fields, which are compressed in nature. It involves the usage of air vortices to impart the haptic feel. Air vortices are the rings of air, which induce a force that the user feels while interacting with it. An air vortex is formed when air is squeezed out of a circular opening. The air particles at the center of the opening moves with a higher speed as compared to the particles which are present at the edges of the ring. This difference in speed is due to the frictional pull between particles of air. When the air ejects from the opening, the speed anomalies between different molecules results into the circular formation of air in the form of a ring. As this ring increases its diameter and passes a nominal value, it kicks off from the opening of AIREAL and travels through the air by using its rotatory momentum. This rotatory motion inhibits the amount of energy loss and hence helps in maintaining the stability of vortex. The advantages of using these air vortices are: Tactile sensations can be felt over large distances, the equipment for generation of vortices in comparatively cheap and the air vortices can be dynamically directed to a particular location by controlling the configuration of the nozzle in AIREAL. According to (Rajinder Sodhi [21]), air vortices are described as a field where behavior of air has been noticed in a whirlpool motion circumventing a translational axis. The air vortices exert a considerable amount of force when they collide any obstacle in their field of propagation and can traverse upto long distances without getting degraded in speed and form. An AIREAL device transmitting air vortex can be seen in Fig. 3.2

3.3 Haptics using Ultrasound

Haptic touch generation using ultrasound is one of the most interesting technique. In this approach we use the principle of acoustic radiation pressure by focusing ultrasound. The ultrasonic transducers emit Ultrasound which is made to converge and create a primary focal point by the concept of beamforming and phase computation. (Beamforming will be discussed in detail later in this chapter). This acoustic force when reflected from the focal point, creates a displacement in the skin tissue, which in turn induces a feeling of touch in the mechanoreceptors of our skin. However a single ultrasonic transducer doesn’t have enough potential to create the force required to activate the mechanoreceptors in the skin. As a result, an array of transducers is needed to create required force to create a sense of touch and this force can be varied by changing the size of the array of transducers. By this approach, we can create the feel of 3 dimensional shapes in mid air without any external actuators. Biologically, the mechanoreceptors are densely populated at the fingertips as compared to the palm. This is the reason
why the haptic feedback is more strong at our fingertips as compared to rest of the palm. One factor which makes ultrasound better when compared to other techniques is its increased accuracy and the range at which the tactile sensation can be felt. By the use of ultrasonic transducer array, a single feedback point could be formed at finger to provide a feel of mid air haptics. Later, this single point of feedback can be altered in position continuously so as to provide an illusion of continuous movement at human hand.

### 3.4 Principles used in Haptics

Acoustic waves are longitudinal in nature and inherit the concept of reflection, interference and diffraction. According to (Beyer [4]) the wave equation is represented by Eq.(3.1)

\[
\frac{\delta^2 p}{\delta x^2} - \frac{1}{c^2} \frac{\delta^2 p}{\delta t^2} = 0
\]

(3.1)

where ‘\(p\)’ is the acoustic pressure and ‘\(c\)’ is the speed of sound and ‘\(x\)’ is its position.

#### 3.4.1 Acoustic Radiation Force

Whenever an acoustic wave strikes any object along its line of propagation, it results into the creation of a force, which is then exerted on that inhibiting object. This force is termed as acoustic radiation force. This force is represented by negative gradient of Gor’kov potential in Eq.(3.2). We can understand the main mechanism underlying the acoustic radiation force by using a simple analysis.
We assume that the particle is spherical and rigid with dimensions smaller than the wavelength of the incident wave, but much larger than the viscous and thermal skin depth. By looking into this idea, we realize that no force will appear on the particle if it has the same acoustic properties as the surrounding medium. The factors which govern acoustic radiation force are size of particle, amplitude of ultrasound and acoustic contrast. Hence, the field incident on the particle will be reflected from its surface.

As a result, the radiation force acting on the particle will be a combination of the incident and reflected wave. According to (Asier Marzo and Drinkwater [1]) to calculate the force exerted on a sphere due to a complex pressure field, the negative gradient of the Gor’kov potential $U$ is used as in (3.2)

$$ F = -\nabla U $$

(3.2)

and the complex acoustic pressure $P$ at point $r$ due to a piston source emitting at a single frequency is shown in Eq. (3.3)

$$ P(r) = P_0 A \frac{D_f(\theta)}{d} \exp(i(\phi + kd)) $$

(3.3)

where $P_0$ represents transducer amplitude and is a constant, $A$ is the peak to peak amplitude of the excitation signal, $d$ is the propagation distance in free space. $D_f$ is the far-field directivity function, which depends on the angle between the transducer normal and $r$. Directivity is the measure of degree to which the sound emitted from a source is concentrated in a particular direction. The directivity function of a circular piston source as in (Asier Marzo and Drinkwater [1]) is represented by Eq. (3.4)

$$ D_f = 2J_1(kasin\theta)/kasin\theta $$

(3.4)

where $J_1$ is the Bessel function, $a$ is the piston radius, $k$ is the wave number, and is given as $k = 2\pi/\lambda$, where $\lambda$ is the wavelength and $\phi$ is the initial phase of the piston. The directivity plot of a transducer is shown in Fig. 3.3.

**Figure 3.3: A typical directivity plot of transducer. Adapted From Park [19]**

As per, (Asier Marzo and Drinkwater [1]), the potential function is expressed using the acoustic pressure and velocity as shown in Eq.(3.5).
3.4.2 Maximum Displacement

When acoustic radiation force interacts with our tissue, it creates a shear wave, which in turn causes a displacement in skin thereby triggering mechanoreceptors in skin. It is due to this displacement that we are able to feel the tactile sensation. According to (Benjamin Long [3]), the maximum displacement of a medium, induced by acoustic radiation force from focused ultrasound is given by Eq. (3.8)

\[
U_{\text{max}} = \begin{cases} 
\frac{\alpha a}{\rho c_1 c_t} I t_0, & \text{where } t_0 << \frac{a}{c_t} \\
\frac{a}{\rho c_1 c_t} a^2 I = \frac{a}{\mu c_1} a^2 I = kW, & \text{where } t_0 >> \frac{a}{c_t} 
\end{cases}
\]

where ‘a’ is the radius of focal region, ‘\(t_0\)’ is the duration of the pulse, ‘\(c_1\)’ is the speed of shear waves propagation, ‘\(c_t\)’ is the speed of sound, ‘\(\mu\)’ is the shear elastic modulus, ‘\(\alpha\)’ is the absorption coefficient, ‘I’ intensity of pulse, ‘W’ is the acoustical power and ‘k’ is amalgamated constant.

3.4.3 Acoustic Pressure Function for a Transducer

According to (Park [19]), the acoustic pressure at a point above the surface of the transducers is given by Eq. (3.9)

\[
p(x, \omega) = \frac{-i \omega \rho v_0}{2\pi} \int_s \exp\left(\frac{i k r}{r}\right) ds
\]

Where ‘\(p\)’ is complex pressure amplitude at point \(x\), ‘\(\omega\)’ in angular frequency of ultrasonic wave, ‘\(\rho\)’ is the distance between point source and center of transducer plane, ‘\(r\)’ is the distance point source and point \(x\).
3.5 Beamforming

**For Far Field**

For far field, the complex pressure is divided into two parts: on axis and off axis. The on axis pressure decays exponentially with the increase in distance from the simulation plane as described in (Park [19]) is given by Eq. (3.10)

\[
p(z, \omega) = \rho c v_0 \left( \exp(ikz) - \exp(i k \sqrt{z^2 - a^2}) \right)
\]

(3.10)

Where ‘z’ is the distance between on axis point and transducer surface, ‘a’ is the radius of the transducer, ‘\(\rho\)’ is the distance between the point source and center of transducer plane, ‘k’ is the wave-number, ‘c’ is the speed of sound, ‘\(v_0\)’ is the constant velocity of sound in z direction.

The off axis pressure function as per (Park [19]) is given by Eq. (3.11)

\[
p(x, \omega) = (-i \omega \rho v_0 a^2) \frac{\exp(i k R) J_1(k \sin \theta)}{k \sin \theta}
\]

(3.11)

Where ‘R’ is the distance from center of transducer plane to point x, ‘\(\theta\)’ is the angle made by R and z-axis and ‘\(J_1\)’ is the first order Bessel function. The geometry for the same can be seen in Fig. 3.4

**Figure 3.4:** Geometry for off axis calculation of complex pressure function.  
*Adapted From Park [19]*

![Geometry for off axis calculation of complex pressure function.](image)

3.5 Beamforming

Beamforming is a signal processing technique which is applicable in the case of phased arrays to achieve directional transmission and reception of signals. For instance, when a stone is thrown into a pond, we see circular waves emerging away from the impact point. When multiple stones are thrown, we tend to observe an interference pattern. When the maxima of two stones interfere, it gets amplified. Beamforming is about controlling this interference pattern, and forming a beam-like interference pattern where the amplification occurs predominantly in one particular direction.
In our case, we use small piezoelectric transducers instead of stones to create the interference pattern. As mentioned in the previous chapter, when an alternating voltage signal is applied to the piezoelectric transducer, it starts to vibrate and emits sound. If the spacing between the transducer elements and the delay in the element’s signals is just right, we can create an interference pattern according to our needs, where in majority of the signal energy is focused in one angular direction. The same principle is applied when the transducer is used to receive the sound. Just by adjusting the amplitude and delay of the received signal on each transducer element, it is possible to receive the echo from a particular angular direction. According to (Biegert and 2018 [5]), beamforming can be formulated mathematically using Eq. (3.12)

\[ \zeta(\theta) = \sum_{k=0}^{k=N} a_k(\theta) . x_k \]  

(3.12)

where N is the number of elements, k is the index variable, \( a_k \) is the complex coefficient of the \( k^{th} \) element, \( x_k \) is the voltage response from the \( k^{th} \) element, \( \zeta \) is the beam response, \( \theta \) is the angle of the beam main lobe.

### 3.5.1 Analog Beamforming

In this beamforming, the input analog signal at the transmitter is altered due to the modifications in amplitude or phase. After transmission, the signals are added before ADC conversion at receiver. A traditional analog filter comprises of transducer array for transmission and receiving of signals and analog filter for the purpose of filtration of the received signal. This analog filter consists of a delay line. The role of this delay line is to add delay to each of the received signal. The two advantages of using analog filter in an ultrasound system is that it reduces the power consumption of the system and decreases the number of components in system. Analog beamforming can be performed in variety of ways in different implementation setups.

- When pressure waves are the signals at the receiver and transducer array convert these signals to voltage or current.

- When different types of filter such as narrow filter, finite impulse response filter or infinite impulse response filter are used in the analog filter section before the receiver.

- Two types of modules can be used for reducing the side lobes; either summation module or apodization circuit.

The typical transmitter and receiver module displaying analog beamforming can be seen in Fig. 3.5 and Fig. 3.6

### 3.5.2 Digital Beamforming

In digital beamforming, the amplitude and phase modifications are made to the digital signal before the Digital to analog converter in transmission side. In
the receiving end each transducer element is connected to an analog to digital converter (ADC) followed by the signal summation. The block diagram of digital beamforming is shown in Fig. 3.7.

Figure 3.7: Transmitter and Receiver of Digital Beamformer. Adapted from Kerem Karadayi and Kim [13]
Once, the received signal passes through analog signal conditioning, the signals are digitized using ADCs, so that digital beamforming can be performed. After beamformation, the signal goes into demodulator, to get rid of the carrier frequency of and extract the complex baseband data. This data is then used for signal and image processing.

**Figure 3.8:** Schematic diagram illustrating the principle of digital beamforming. Adapted from Kerem Karadayi and Kim [13]

![Schematic diagram](image)

Figure 3.8 is a schematic diagram showing the principle of digital beamforming. Each transducer element receive the reflected echo signal from the target point at different timings. According to (Kerem Karadayi and Kim [13]) in Fig. 3.8 the element in the center receives the signal first when compared to other elements. The transducers at both the ends will be the last to receive the echo. The received echoes are then aligned properly by introducing appropriate delay to each channel, before summation.

Time delay quantization errors are minimized in digital beamforming. The delay accuracy in analog beamformers is in the range of 20ns. For operations at high frequencies (above 10MHz) the quantization noise will appear due to the increase in the side lobe level, thus reducing the contrast resolution. The digital beamformers can be used in high frequency operations due to its greatly improved delay accuracy.

### 3.6 Computation of Amplitude and Phase

In this section we discuss the computation of phase and amplitude at different control points. To get haptic feedback, amplitude at each control point needs to be controlled. We can do this by changing the phase to make the waves constructively interfere in points of high amplitude and destructively interfere in points of low amplitude. So, to obtain the desired amplitudes at the given points, we must determine the phase values relative to each other. According to (Benjamin Long [3]), control points with same amplitude, tend to amplify each other
if they move closer and their phase difference is zero, or cancel out each other if
the phase difference is half a period and they move closer.

### 3.6.1 Model of Acoustic Field

To understand the calculation of phase and amplitude, it is of paramount impor-
tance to understand the acoustic field, $\psi$ produced by n ultrasonic transducers.
According to (Emad S. Ebbini [8]), the wavefunction at far field in 3D can be
mathematically stated as in Eq. (3.13)

$$f(\Delta x, \Delta y, \Delta z) = \frac{e^{ik\frac{2}{3}\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}}{[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^\frac{3}{4}}$$

(3.13)

where $\Delta x = x-x'$, $\Delta y = y-y'$, $\Delta z = z-z'$ and x, y, z are the coordinates with re-
spect to the aperture and $x'$, $y'$, $z'$ provide absolute positions in far field. The
sound wave emitting from a transducer q can be divided into 4 parts: product
of emission amplitude $A_q^{emit}$, a phase offset $e^{i\phi_q}$, amplitude attenuation function
$A_q^{attn}(x',y',z')$ and a phase difference function $e^{ik_q(x',y',z')}$. So for n transducers,
the field $\psi_\Omega$ is expressed mathematically by Eq. (3.14)

$$\psi_\Omega(x',y',z') = \sum_{q=1}^{n} A_q^{emit} e^{i\phi_q}.\psi_q(x',y',z')$$

(3.14)

where $\psi_q(x',y',z')$ is the product of $A_q^{attn}(x',y',z')$ and $e^{ik_q(x',y',z')}$. Now a set
of m control points were selected in $x'$, $y'$, $z'$ and are represented by $(\chi_1,...,\chi_m)$,
where each elements gives information about phase and amplitude and are com-
ponent of field $\phi_\Omega$. The phase and amplitude can be obtained by equation $Ax = b$, where A is given by

$$A = \begin{bmatrix}
\psi_1(\chi_1) & \cdots & \psi_n(\chi_1) \\
\vdots & \ddots & \vdots \\
\psi_1(\chi_m) & \cdots & \psi_n(\chi_m)
\end{bmatrix}$$

and vector x is given by $[A_1^{emit} e^{\phi_1},...,A_n^{emit} e^{\phi_n}]^T$ and b is given by $[\psi'_\Omega(\chi_1),...\psi'_\Omega(\chi_1)]^T$
and these values are solved using the minimum solver algorithm as presented in
(Emad S. Ebbini [8]).

### 3.6.2 Mathematical Representation of Phase and Amplitude

A matrix R is used to represent the phase and amplitude effect of one control
point on other. This is shown by the Eq. (3.15)

$$Rx = \lambda x$$

(3.15)

According to (Benjamin Long [3]) we can quite simply find a solution for any one
control point, we then use symbolic algebra to algebraically generate a simplified
minimum-norm solution for each single control point case:

\[ A_q^{emit} e^{i\phi_q} = \frac{A_q^{attn}(\chi_C - \chi_q) e^{ik_q(\chi_C - \chi_q)} A_C}{\sum_{i=0}^{n} (A_i^{attn}(\chi_C - \chi_i))^2} \]  

(3.16)

where \( \chi_C \) is the position of the control point, with \( A_C \) its amplitude, while \( \chi_q \) is the transducer origin. Using these resulting complex values for the transducer emissions from Eq. (3.14), we generate hypothesized single control point fields \( \psi_{\Omega C}^{1\ldots m} \). From this matrix \( R \) is constructed

\[ R = \begin{bmatrix} \psi_1(\chi_{C1}) & \ldots & \psi_m(\chi_{C1}) \\ \vdots & \ddots & \vdots \\ \psi_1(\chi_{Cm}) & \ldots & \psi_m(\chi_{Cm}) \end{bmatrix} \]

which is the vector of amplitude and phases of the control point depending on the amplification eigenvalue given. The estimation of maximally amplified control points is generated by the assumption of weighted ‘mixing’ of each single control point solution. Therefore, finding a large eigenvalue \( \lambda \) leads to a large constructive amplification of phases in the eigenvector. This phase can then be used in any linear system to generate similar amplitudes and for more efficient transducer usage. A simple power method can be used to find the eigenvector with the largest eigenvalue. This method can be estimated and restarted from previous solution, thus providing a time-bound solution. By using this method, it is possible to make control points of a linear system coexist, thus reducing the exclusion caused by destructive interference and noise by constructive interference.

3.6.3 Optimization of Calculated Parameters

Now that we calculated the phases and amplitude for the array of transducers, its important that we set some algorithm to optimize the power and easily modify the matrix configuration. For this, the algorithm called tikhonov regularization was used according to (Benjamin Long [3]). In this the linear system \( Ax = b \) was enlarged and modified as

\[ \begin{bmatrix} A \\ \sigma_1 \gamma & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \sigma_n \gamma \end{bmatrix} x = \begin{bmatrix} b \\ 0 \\ \vdots \end{bmatrix} \]

and \( \sigma_q \) can be represented by Eq.

\[ \sigma_q = \sqrt{\sum_{i=0}^{m} \frac{A_q^{attn}(\chi_{Ci} - \chi_q) A_C}{m}} \]  

(3.17)

where \( \gamma \) is either 0 or 1 depending on whether the method for computing phase and amplitude involves the usage of minimum norm solver algorithm or if it
involves the optimization of amplitude to equal values so as to provide better power emission.

### 3.6.4 Haptic Efficiency

Once the computation of phase and amplitude is achieved, the next step is low frequency modulation. As described in (Asier Marzo and Drinkwater [1]), haptics is only perceivable by humans at 200 Hz and to attain this 200 Hz modulation (Benjamin Long [3]), the array is is excited for $(1/400)$th second period and then is not excited for next $(1/400)$th second period. However, as a result of this process, there is a considerable undesired loss in power.

So, a solution for outwitting this obstacle is achieved which involves using of both the halves of modulation cycle to attribute for the output. It involves careful generation and division of control points in two different parts and then emitting them in an alternating manner. The proximity of the control points determines the effectiveness of phase computation method. So based on this, usage of principal component analysis provides with probability of bifurcating the plane into two highly populated control point groups. Therefore, by choosing between either of the fields in a continuous manner, a powerful perception of tactile sensations is created.

### 3.6.5 Summary of the Wave Synthesis Algorithm

The procedure for wave generations involves the following steps: The acoustic field produced by a single transducer is being calculated and is then projected as a large modeled volume. Then that particular transducer undergoes offsetting for position, phase and amplitude for real values. This involves formations of constructive interferences and destructive interferences. After this a control point is defined in the plane. Then optimal values of phase is calculated. There will be more than one solutions (so there will be repeated calculations of the phases) for optimal value of phases but the value with maximum intensity is being sent to the transducer array. The flowchart for the same can be seen in Fig. 3.9

### 3.7 Alternate Method Used for Amplitude Modulation

The previous section describes the wave synthesis algorithm and amplitude computation in a generic way. Another way of implementing amplitude modulation of the input wave (for the purpose of achieving tactile sensation) is presented in this section. It was adapted from (Peter R. Smith and Freear [20]).

The concept which was used in implementation of this thesis work was derived from the method presented in this section and will be discussed in the results section of the thesis.
3.7.1 Input wave

The input wave given to the transducer can be of any type and in our case we use a half square wave. Despite using a half square wave, we get a sinusoidal output from the transducer due to its resonating nature. In order to get the tactile sensation, the output wave from the transducer needs to be modulated at 200Hz.
frequency. So, for the transducer to emit a sound wave modulated at a particular frequency, the input signal must be pulse width modulated.

Figure 3.10: Triangular (sawtooth) symmetrical pulse width modulation consisting of a carrier (dotted line), and a desired output level (gray solid line). Adapted From Peter R. Smith and Freear [20]

The conventional carrier based PWM generates pulses of varying width by comparing the carrier wave of known form to a desired output level or modulating wave. As seen in Fig. 3.10 a conventional triangular carrier wave with a specific output level can vary the pulse width of the output wave in a linear fashion. Therefore the width of the output pulse is directly proportional to the dc level.

A linear triangular carrier wave is defined as

\[
c(t) = A \cdot \left| \frac{2}{\pi} \arcsin\left( \sin(\omega t + \phi) \right) \right| + L
\]

(3.18)

where A is a scaling factor, t is time, \( \phi \) is phase, L is an arbitrary dc offset, and \( \omega = 2\pi f \), with f representing frequency. By using the Eq. (3.18) the width of the successive pulses can be modulated by comparing both the carrier c(t) and modulating wave m(t) as shown in Fig. 3.11.

Figure 3.11: Conventional carrier-based pulse-width modulation (PWM) featuring the carrier c(t) and modulating wave m(t). Adapted From Peter R. Smith and Freear [20]
This chapter describes the implementation of Matlab Based Graphical User Interface (GUI) describing the various elements involved in ultrasonic transducer configuration simulation. It involves validation and modifications made in this GUI, with the aim of incorporating the concept of haptics through it. After that the chapter describes the mathematical equation incorporated by us to implement haptics and how it is different from the generic application of haptics. The second part of this chapter describes the use of an ultrasonic transducer simulation platform named ‘Field II’. This platform allowed us to develop various transducers configurations and plot its impulse responses in different scenarios. The third part of this chapter states concept developed by us though Matlab code to represent the concepts of beamforming, transducer geometry, delay generation and phase generation for array of transducers.
4.1 Matlab Based GUI

The Fig. 4.1 depicts the graphical user interface named as Transducer array Calculation(TAC), which was taken from (Kohout [14]). This simulation platform alleviates the need for any hardware tools required to run trials for computing various parameters of ultrasonic transducers. It helps us to visualize directivity patterns, near field region, far field region of a transducer. It provides us with the possibility to configure transducer array size, selection of single element in the array, excitation of input signal and impedance and attenuation characteristics of a transducer. It helps us to view the input signal either in time or frequency domain, the relationship between angle of the transducer and its frequency, directivity pattern of a transducer, pressure variation in near field. it provides a user friendly feature of importing user input configuration of transducers, attenuation user input file, impedance user input file and user input excitation signal.

**Figure 4.1: Original GUI. Adapted From Kohout [14]**

4.1.1 Input Field

The input platform of the TAC GUI is shown in the Fig. 4.2. The GUI allows the user to load and save the transducer configuration, select the type of excitation signal given to the transducer, change the transducer geometry, spacing between the transducer and load attenuation and impedance configuration of the transducer.

A new transducer configuration can be given by the user by changing the following parameters like number of X and Y elements, spacing between the X and Y elements, phase shift of the excitation signal in both X and Y direction,
rectangular patch parameters of individual elements. In the transducer array, every single element can be configured separately. It is possible to activate and deactivate individual elements using the green and red buttons as shown in Fig. 4.3. A single element from the array can be selected and visualized to know the phase and amplitude of the element.

**Figure 4.3: Single Element Configuration. Adapted From Kohout [14]**

### 4.1.2 Excitation Signals

The excitation signal given to the transducer can also be selected and modified according to the user requirements. By clicking the ‘options’ button present on the right top of the array input parameters, the input wave can be selected as
shown in Fig. 4.4. The frequency and other parameters can be set by the user to achieve the desired input waveform. It is also possible to load any arbitrary signal by using the option ‘other wave’. The input signal can be represented in both time and frequency domain.

Figure 4.4: Excitation Signal Editor. Adapted From Kohout [14]

4.2 Modified GUI

The modified version of GUI can be seen in Fig. 4.5. These modifications were done by us by keeping in mind the following aim: adding and deleting features according to the requirements and specifications of our project. The inclusion of additional parts required the coding in Matlab. The Matlab codes for the updated components can be found in the Appendix section. The differences in original and modified GUI are summarized in Table. 4.1
Table 4.1: Comparison of the Features Between original and Modified GUI

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original GUI</th>
<th>Modified GUI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of Sound</td>
<td>Varies from (300-1500)m/s</td>
<td>Varies from (300-1500)m/s</td>
</tr>
<tr>
<td>Types of input wave</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Actual diagram of input waves</td>
<td>Not possible</td>
<td>Possible</td>
</tr>
<tr>
<td>Frequency slider change visible in input wave graph</td>
<td>Not possible</td>
<td>Possible</td>
</tr>
<tr>
<td>Domain</td>
<td>Time and Frequency</td>
<td>Time</td>
</tr>
<tr>
<td>Haptics</td>
<td>Not Possible</td>
<td>Possible</td>
</tr>
<tr>
<td>Pulse Width Modulation</td>
<td>Not Possible</td>
<td>Possible</td>
</tr>
<tr>
<td>Transducer array surface</td>
<td>Visible</td>
<td>Visible</td>
</tr>
<tr>
<td>Directivity Patterns</td>
<td>Visible</td>
<td>Visible</td>
</tr>
</tbody>
</table>

Figure 4.5: Overall View of Modified GUI
4.3 Field II

Field II is a simulation program which was developed with the aim of providing hands on experience with the concepts of ultrasound imaging; both theoretically and practically. It involves the concepts of acoustic physics, impulse responses, sound field information. The principle on which Field II is based on involves calculation of ultrasonic field for the wave. This is achieved through spatial response. The response represents the ultrasonic field at a point in space. The input to the transducer is modeled as direct delta function. Convolution of spatial impulse response(SIR) with the input signal results in the formation of the wave. The type of input signal used here is not limited as the process is based on linear system. For a spherical wave the received response is analogous to the spatial impulse response.

Different transducers have to be simulated using different algorithm for obtaining their spatial response. Also, apodization makes it is difficult to generalize a method for calculating their response. So, Field II alleviates this issue by partitioning the transducer surface into small sections, calculating their individual responses and then adding them up together. Usually, impulse responses are calculated for frequencies in GHz owing to the shrill irregularities in the responses. This issue is tackled in Field II, as it involves tracking of time responses and incorporates integrated spatial impulses as intermediate processes.

4.3.1 Spatial Impulse Response

According to (Jensen [12]), any linear electrical system is characterized by its impulse response. This can be seen in Fig. 4.6. The output $y(t)$ for any input $x(t)$ is given by Eq. (4.1)

$$y(t) = h(t) \ast x(t) = \int_{-\infty}^{\infty} h(\theta)x(t-\theta)d\theta$$  

(4.1)

where $h(t)$ is the impulse response of the liner system. The linear acoustic system is analogous to the electrical system and it works on same principle. Linear acoustic system can be seen in Fig. 4.7. The point $\vec{r}_1$ is the point where acoustic pressure from transducer is measured by small hydrophone and $\vec{r}_2$ denotes the position of the acoustic transducer. The output signal increases or diminishes based on the towards or away movement of the hydrophone w.r.t to the transducer. Thus according to (G.E.Tupholme [9],Stepanishen [23],Jensen [12]), the impulse responses are dependent on the relative positions of the receiver and the transmitter. Spatial Impulse Response (SIR) is the response we get at a particular point in space if the transducer vibrates as a delta pulse. Fig. 4.8 displays the concept of SIR.

The wave equation for the velocity potential $\psi$ is given as in Eq. (4.2)

$$\Delta^2 \psi - \frac{1}{c_0^2} \frac{\partial^2 \psi}{\partial^2 t} = 0$$  

(4.2)
The pressure according to Eq. (4.2), is given by Eq. (4.3)

$$p(\vec{r}, t) = \rho_0 \frac{\partial \psi(\vec{r}, t)}{\partial t}$$

(4.3)

According to (Jensen [10]), the solution to the wave equation is expressed as in Eq. (4.4)

$$\psi(\vec{r}_1, \vec{r}_2, t) = v_e(t) \ast h_a(\vec{r}_1, \vec{r}_2, t)$$

(4.4)

where $v_e(t)$ is the velocity wave form and $h_a(\vec{r}_1, \vec{r}_2, t)$ is the apodized spatial impulse response. So now based on Eq. (4.4), the pressure field is given as in Eq. (4.5)

$$p(\vec{r}_1, \vec{r}_2, t) = \rho_0 \frac{\partial v_e(t)}{\partial t} \ast h_a(\vec{r}_1, \vec{r}_2, t)$$

(4.5)

According to (Jensen [10]), the spatial impulse response is calculated by observing a point over time and integrating from each of the spherical waves. It is
Implementation

mathematically expressed with Huygens’ Principle as Eq. (4.6)

\[ h(\vec{r}_1, t) = \int \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi|\vec{r}_1 - \vec{r}_2|} dS \]  \hspace{1cm} (4.6)

where \( |\vec{r}_1 - \vec{r}_2| \) is the distance from the transducer at position \( \vec{r}_2 \) to the field point at \( \vec{r}_1 \), \( \delta(t) \) is the Dirac delta function, ‘\( S \)’ is the area and ‘\( c \)’ is the speed of sound.

### 4.3.2 Spatial Impulse Response Calculation According to Field II

According to Huygens’ Principle as described in [Jensen and Svendsen [11]], for calculating the SIR, an aperture will have to partitioned into many points which can become quite a cumbersome task. So to overcome this, Field II incorporates the concept of dividing the aperture into many small rectangles. As seen in Fig. 4.9, the one small rectangle is placed in xy plane with center at origin. It describes far field response and the location of field point by position vector, which is divided into unit vector \((x_e, y_e, z_e)\) and a distance \( l \). The far field spatial response represents a trapezoid shape as depicted in Fig. 4.10, \( t_1 \) is th time of flight(t.o.f) from the nearest corner of the piston to the field point. Similarly \( t_2, t_3, t_4 \) represents different t.o.f. from the corners with increasing distances from field point respectively. The trapezoid response is result of the convolution of the two input rectangular pulses. By projecting the dimensions of the small rectangle onto the line through center and field point, the width of the pulses are calculated.

![Figure 4.9: Rectangular Piston Orientation. Adapted From Jensen and Svendsen [11]](image)

\[ \Delta t_1 = \min \left( \frac{w_y y_e}{c_0}, \frac{w_x x_e}{c_0} \right) \] \hspace{1cm} (4.7)

\[ \Delta t_2 = \max \left( \frac{w_y y_e}{c_0}, \frac{w_x x_e}{c_0} \right) \] \hspace{1cm} (4.8)

\[ t_1 = \frac{l}{c_0} - \frac{\Delta t_1 + \Delta t_2}{2} \] \hspace{1cm} (4.9)
Figure 4.10: Far Field Response. Adapted From Jensen and Svendsen [11]

\[ t_2 = t_1 + \Delta t_1 \]  
\[ t_3 = t_1 + \Delta t_2 \]  
\[ t_4 = t_1 + \Delta t_1 + \Delta t_2 \]

where \( w_x \) and \( w_y \) are the side lengths of rectangle and \( t_1, t_2, t_3, t_4 \) are the arrival times and area of the trapezoid is given by Eq. (4.13)

\[ a_{\text{rec}}(l) = \frac{w_x \cdot w_y}{2\pi l} \]
This section elucidates the simulation results that were obtained while running the simulation tests on Field II and modified TAC GUI. The parameters which were simulated in Field II are as follows: single element array transducer and its corresponding parameters, an array of transducers and its parameters, SIR and pressure along the axis for array of transducers, pressure along radial line, pressure field in xz plane, apodization concept addition, sensitivity of the receiver array and dynamic focusing, pulse echo response and grating lobes for the array of transducers. The results comprises of Matlab coding and its corresponding simulated graphs for the parameters set by us.

The later part of the result section displays the simulation results on TAC GUI. It displays the directivity plots, near field region, far field region, haptics for various configurations of transducer parameters.
5.1 Field II Simulations

5.1.1 Creation of a Single Element Transducer

This simulation displays the Matlab plots for the single element transducer where the following parameters were set as: number of elements = 1, width of the element = 18.5mm , height of the element = 13mm, kerf = 0 (spacing between elements; 0 because there is only one element, number of subdivisions in x-direction of element) and number of subdivisions in y direction of the element are both equal to 1, the natural intrinsic focus irrespective of the number of elements is set at [0,0,60], the center frequency was set to 2.5 MHz and Sampling Frequency was set to 100MHz. In Fig. 5.1, a single element transducer can be seen and the measurement point (i.e. the measurement point is defined as a vector) displayed is present at [20,40,60].

Figure 5.1: Single Element Transducer

Figure 5.2: Excitation Pulse and Transducer Impulse Response
In Fig. 5.2, the excitation pulse given to the transducer and its impulse response can be seen, where the fractional bandwidth was set as 0.6. The impulse response in frequency domain is represented in Fig. 5.3.

**Figure 5.3: Transducer Impulse Response in Frequency Domain**

From Fig.5.3, we can conclude that the single element transducer acts as a band pass filter. The frequency values at -6db point from the maximum value on either sides of the peak, are measured and subtracted which gives the value of bandwidth. It can be also be calculated by the Eq. (5.1)

\[
\text{Fractional Bandwidth} = \frac{\text{Bandwidth}}{\text{Center Frequency}}
\] (5.1)

The spatial impulse response is shown in Fig. 5.4 and the transmitted pressure field is given in Fig. 5.5. The reason for the shape of spatial impulse response in Fig. 5.4 is as follows: When the rectangular pulses are directed towards the rectangular aperture but have not reached the first edge yet, the SIR is zero. As first wave from the series of waves hits the edge of aperture, the SIR begins to increase and with the passage of time as more pulses enter the aperture, the SIR increases linearly. However, due to the small size of a single aperture, the time at which the last wave enters the aperture, the first wave has already started to emerge out from the aperture edge. Hence the graph begins to fall. At this moment, the ratio of number of pulses emerging out from the aperture is less compared to the pulses entering the aperture. So the decline is slow. With the passage of time, as the ratio of number of pulses emerging out from the aperture increases, the graph begins to fall more steeply and once the wave has crossed the aperture completely, the graph attains zero.
5.1.2 Creation of Array of Elements

This simulation displays the Matlab plots for the array of element where the following parameters were set as: number of elements = 64, width of the element = pitch - kerf, height of the element = 13mm, kerf = 0.025mm, pitch = 0.290mm and number of subdivisions in y direction of the element is 30, number of subdivisions in x direction is 5, the natural intrinsic focus irrespective of the number of elements is set at [0,0,60], radius of curvature = 60mm, the center frequency
was set to 2.5 MHz and Sampling Frequency was set to 100MHz. In Fig. 5.6, an array of elements can be seen and the measurement point (i.e. the measurement point is defined as a vector) displayed is present at [20, 40, 60]. In Fig. 5.7, the excitation pulse given to the array of elements and its impulse response can be seen, where the fractional bandwidth was set as 0.6. The impulse response in frequency domain is represented in Fig. 5.8.

**Figure 5.6: Array of Elements**

![Array of Elements](image1)

**Figure 5.7: Excitation Pulse and Transducer Impulse Response**

![Excitation Pulse and Transducer Impulse Response](image2)
From Fig. 5.8, we can conclude that the array of elements acts as a band pass filter. The frequency values at -6db point from the maximum value on either sides of the peak, are measured and subtracted which gives the value of bandwidth. It can be also be calculated by the Eq. (5.1) The spatial impulse response is shown in Fig. 5.9 and transmitted pressure field is shown in Fig. 5.10.

The reason for the shape of spatial impulse response in Fig. 5.9 is as follows: When the rectangular pulses are directed towards the rectangular aperture but have not reached the first edge yet, the SIR is zero. As first wave from the series of waves hits the edge of aperture, the SIR begins to increase and with the passage
of time as more pulses enter the aperture, the SIR increases linearly. Due to
the increase in the number of elements in the array, it takes more time for the
wave to completely cross the array. Once the entire wave is inside the array, the
SIR attains a constant value till the first pulse hits the edge of the array. When
number of pulses leaving the aperture starts to increase, the SIR value begins to
drop with time and finally reaches zero once the wave has left the array.

Figure 5.10: Transmitted Pressure Field for Array of Transducer

5.1.3 Spatial Impulse Response and Pressure Measurement
Along a Line

Figure 5.11: Transducer Array with Measurement Points Spread from -20mm to 20mm
Here we used the same configuration of transducer as set in the previous section. However the measurement points are extended on a line from $x = -20\text{mm}$ to $x = 20\text{mm}$. The figure for the same is shown in Fig. 5.11.

Fig. 5.12 shows the beam profile plots of Fresnel and Fraunhofer and the simulated pressure. According to Fresnel and Fraunhofer approximation, the pressure field at the focal range is proportional to the Fourier transform of the aperture. In this case the Fourier transform is $\text{sinc}(x^aT_x^f0/(R^c))$ where $aT_x$ represents then number of individual point sources, $R$ is the focus depth, $f_0$ is the frequency and $c$ is the speed of sound. The beam pattern of the simulated plot shown in Fig. 5.12 is different from the Fresnel and Fraunhofer due to the following reasons,

1. In Fresnel and Fraunhofer the apertures are considered as point sources, whereas the simulated plot is for rectangular apertures.

2. For rectangular apertures, there can be off axis spatial impulse response, which leads to overlapping of impulse response of individual apertures without exact addition or cancellation happening.

3. The side lobes are affected by the broad band of the excitation signal, i.e as the bandwidth increases to 1.0, the side lobes are reduced. this can be seen in Fig. 5.13, and as the bandwidth decreases to 0.2 the side lobes are increased, as can be seen in Fig. 5.14.

**Figure 5.12: Beam Profile at 60mm depth (BW = 0.6 MHz)**
Figure 5.13: Beam Profile at 60mm depth (BW = 1.0 MHz)

Figure 5.14: Beam Profile at 60mm depth (BW = 0.2 MHz)

5.1.4 Beam Profile Variation with Focal Length

This simulation displays the Matlab plots for beam profile variation for array of transducer where normalized pressure was plotted against focal length. The following parameters were set as: number of elements = 64, width of the element = pitch - kerf, height of the element = 13mm, kerf = 0.025mm, pitch = 0.290mm and number of subdivisions in y direction of the element is 30, number of subdivisions in x direction is 5, the natural intrinsic focus irrespective of the number
of elements is set at [0,0,60], radius of curvature = 60mm, the center frequency was set to 2.5 MHz and Sampling Frequency was set to 100MHz. The calculation was made for 100 points lying between 5mm and 150mm depth. The transducer array can be seen in Fig. 5.15.

**Figure 5.15: Transducer Array plot along Radial line**

The radius of curvature is 60mm and if the focal depth is set to 60mm, the maximum amplitude of normalized pressure is not obtained exactly at the focal depth because the condition $R = 2F$ is not satisfied (where $R$ is ROC and $F$ is focal depth). This can be seen in Fig. 5.16, where the maximum value of normalized pressure is obtained at 51.4mm focal depth.

**Figure 5.16: Normalized Pressure vs Focal Depth for focus at 60mm**
Instead if we set the value of F to 30mm, the condition gets satisfied and maximum pressure amplitude is obtained at approximately 30mm focal depth. This is seen in Fig. 5.17

5.1.5 Apodization

Apodization is the technique used to reduce the strength of the side lobes, which are formed during the focusing of ultrasound. The amplitude across the aperture
of the transducer is varied in such a way, that the center element of the probe head is excited with a voltage of greater amplitude when compared to the elements in the edges of the probe. Side lobes are lobes which are formed by the sides of the main lobe. Even though these side lobes have low amplitude and intensity, if they get reflected back from an obstacle with high enough amplitude, the receiver will misinterpret it to be the response of the main lobe. Therefore, we use this technique to reduce the side lobes. The concept of hanning window is used in apodization to reduce the side lobes. The pressure field plots before apodization is shown in Fig. 5.18 and pressure field plot after apodization can be seen in Fig. 5.19. The concept of apodization was implemented in Matlab using predefined Matlab function of hanning window in Field II files. A vector was created to hold values of the weights of the transducer elements. Then ‘hanning()’ function was incorporated to implement apodization. Some other windowing techniques like rectangular window and tukey window functions were also implemented in the similar way. However, hanning was chosen as final choice as it reduced the side lobes quite significantly as compared to others.

**Figure 5.19: Transmit Beam Pressure Field in Whole XZ plane with apodization**

![Transmit beam pressure field](image)

**5.1.6 Pulse Echo Field**

The combination of transmit field and receive sensitivity yields pulse echo response. It depicts the area from which the echoes are are emerging from. It is basically the convolution of excitation function, transmit and receive impulse response, transmit and receive spatial impulse response.

The simulation displays the Matlab plot for pulse echo plot in Fig. 5.20 The following parameters were set as: number of elements = 64, width of the element = pitch - kerf, height of the element = 13mm, kerf = 0.025mm, pitch = 0.290mm
and number of subdivisions in y direction of the element is 15, number of subdivisions in x direction is 5, the natural intrinsic focus irrespective of the number of elements is set at [0,0,60], radius of curvature = 60mm, the center frequency was set to 2.5 MHz and Sampling Frequency was set to 100MHz. Hanning window was used for addition of apodization in transmit as well as receive side.

Figure 5.20: Pulse Echo Response
5.2 Transducer Array Calculation GUI

In this section we will display the graphical results of radiation pattern, pressure values obtained in far field for different values of frequency, input signal and transducer array configuration. The simulations are run with different parameters for comparison as shown below.

5.2.1 Single Element Configuration

The Fig. 5.21 shows the radiation pattern of 1 element in the transducer. The spacing parameters were kept same as in Fig. 5.22 and the speed of sound was 343 m/s.

![Figure 5.21: Single Element Array](image)

*Table 5.1: Pressure Variations of Single Element Array for Different Frequencies Without Duty Cycle Variation*

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Frequency</th>
<th>Pressure(Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*1</td>
<td>20kHz</td>
<td>5</td>
</tr>
<tr>
<td>1*1</td>
<td>40kHz</td>
<td>5.6</td>
</tr>
<tr>
<td>1*1</td>
<td>60kHz</td>
<td>6.1</td>
</tr>
</tbody>
</table>

The simulations results shown in the tabular columns 5.1 are run for 3 different frequencies 20kHz, 40kHz and 60kHz with different array size. The speed
of sound is kept constant (343 m/s) and the pressure and intensity values are obtained at a distance of 5 cm from the array. All the other spacing parameters were kept same as Fig. 5.22

5.2.2 Comparison of Multi Element Array

In this subsection the element array configuration that we used initially can be seen in Fig. 5.22. This configuration was given an input signal of 20 Khz square signal.

![Figure 5.22: Element Array Configuration](image)

The corresponding radiation pattern can be seen in Fig. 5.23.

![Figure 5.23: Radiation Pattern of 3*3 Matrix](image)

As seen in Fig. 5.23 the radiation pattern is for a 3*3 matrix array, which shows the maximum Z distance (cm) at which the pressure can be felt. The radiation pattern in Fig. 5.27 is obtained for a 6*6 matrix by changing the number of elements
in the transducer array configuration. From these two graphs it can be clearly seen that by increasing the number of elements in the matrix (keeping other parameters constant), it is possible to obtain the pressure at a much higher distance.

**Figure 5.24: Radiation Pattern of 6*6 Matrix**

![Radiation Pattern of 6x6 Matrix](image)

The far field regions of 3x3 and 6x6 matrix is seen in Fig. 5.29 and Fig. 6.1 respectively.

**Figure 5.25: Far Field Region of 3x3 Matrix**

![Far Field Region of 3x3 Matrix](image)
The far field plots of both the configurations show the pressure felt at a distance of 10cm from the transducer array. By looking into Fig. 5.29 and Fig. 6.1 we can see that the amount of pressure felt in a 6*6 matrix is 91 dB(SPL) which is 0.70 Pa, whereas the pressure felt in 3*3 is just 22.91 dB(SPL) which is 0.00025 Pa. This shows by increasing the number of element in an array, we can obtain more amount of pressure at a desired distance.

The simulations results shown in the tabular columns 5.2, 5.3,5.4 are run for 3 different frequencies 20kHz, 40kHz and 60kHz with different array size. The speed of sound is kept constant (343m/s) and the pressure and intensity values are obtained at a distance of 5cm from the array. All the other spacing parameters were kept same as Fig. 5.22

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Frequency</th>
<th>Pressure(Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3*6</td>
<td>20kHz</td>
<td>91.64</td>
</tr>
<tr>
<td>3*6</td>
<td>40kHz</td>
<td>100</td>
</tr>
<tr>
<td>3*6</td>
<td>60kHz</td>
<td>108.3</td>
</tr>
</tbody>
</table>
Table 5.3: Pressure Variations of 3*3 Element Array for Different Frequencies Without Duty Cycle Change

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Frequency</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3*3</td>
<td>20kHz</td>
<td>45.82</td>
</tr>
<tr>
<td>3*3</td>
<td>40kHz</td>
<td>50</td>
</tr>
<tr>
<td>3*3</td>
<td>60kHz</td>
<td>54.15</td>
</tr>
</tbody>
</table>

Table 5.4: Pressure Variations of 6*6 Element Array for Different Frequencies Without Duty Cycle Change

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Frequency</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6*6</td>
<td>20kHz</td>
<td>183.3</td>
</tr>
<tr>
<td>6*6</td>
<td>40kHz</td>
<td>200</td>
</tr>
<tr>
<td>6*6</td>
<td>60kHz</td>
<td>216.6</td>
</tr>
</tbody>
</table>

From the tables 5.2, 5.3, 5.4, it is inferred that for each combination of matrix of elements, the pressure and intensity increased with the increase in frequency. It is also inferred that pressure and intensity is directly proportional to the number of elements in the array.

5.2.3 Pulse Width Modulated Wave

The simulations displayed in the previous section were run using a normal square wave input (of ultrasonic frequency). But, in order to achieve tactile sensation the output wave needs to be modulated to a frequency of 200Hz so that there is a constant change in the output pressure value. This modulation is achieved by using a pulse width modulated square wave by constantly changing the duty cycle of each pulse. The pulse width modulated square wave with a duty cycle variation of 2% in each step is shown in Fig. 5.27. This concept of varying the output pressure amplitude by varying the duty cycle is derived from (Peter R. Smith and Freear [20]).
When the PWM signal is given as input to the transducer, the output pressure of the sound wave varies constantly, showing that the output pressure is proportional to the width of the pulse. But, the change in pressure is not directly or linearly proportional to the width of the pulse. According to (Peter R. Smith and Freear [20]) a linear relationship between pulse width and pressure output does not exist; a percentage increase in pulse width does not provide the same percentage increase in output pressure. On contrary when we consider a linear relationship between pulse width and amplitude, the variation in amplitude of the output wave with respect to input, can be seen in Fig. 5.28.
Table 5.5: Minimum and Maximum Pressure for 40KHz input wave with 20% duty cycle change in different configurations

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Minimum Pressure (Pa)</th>
<th>Maximum Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*1</td>
<td>5.57</td>
<td>7.86</td>
</tr>
<tr>
<td>3*3</td>
<td>50.12</td>
<td>70.82</td>
</tr>
<tr>
<td>3*6</td>
<td>100.84</td>
<td>141.6</td>
</tr>
<tr>
<td>6*6</td>
<td>200.6</td>
<td>283.3</td>
</tr>
</tbody>
</table>

Table 5.6: Minimum and Maximum Pressure for 40KHz input wave with 10% duty cycle change in different configurations

<table>
<thead>
<tr>
<th>Array Size</th>
<th>Minimum Pressure (Pa)</th>
<th>Maximum Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*1</td>
<td>4.5</td>
<td>7.86</td>
</tr>
<tr>
<td>3*3</td>
<td>40.32</td>
<td>70.82</td>
</tr>
<tr>
<td>3*6</td>
<td>88.96</td>
<td>141.6</td>
</tr>
<tr>
<td>6*6</td>
<td>165.34</td>
<td>283.3</td>
</tr>
</tbody>
</table>

The simulations conducted using both 10% and 20% duty cycle change are performed, until 80% pulse width is achieved. As a result the maximum pressure obtained in the tables 5.5 and 5.6 are the same, whereas the minimum pressure varies depending on minimum pulse width used, which is 20% and 10% in the tables 5.5 and 5.6. We infer from Tables 5.5 and 5.6, that the pressure at a chosen distance falls between the maximum and minimum pressure values and is never constant as compared with the values in Tables 5.1, 5.3 and 5.4 respectively. This helps us to conclude that constant pressure variation is observed for duty cycle change of the input wave for a given value of frequency.

5.2.4 Phase Shift

The x phase shift and y phase shift field options in TAC GUI does not provide the exact result as per the scope of our project. So to overcome this limitation, we adopted the method for calculating the phase shift for the elements from (Park [19]). The concept was implemented in Matlab. The distance between the center of each transducer and the focus point is calculated. From the distances we obtain the propagation time for every single transducer and calculate the delay time needed for every single element. The delay is calculated by reducing the propagation time of every individual element from the max propagation time. According to (Biegert and 2018 [5]), the phase shift given to the transducers is given by Eq.
\[ \phi = 2.\pi.f.\delta t \] (5.2)

where \( \phi \) is the phase shift, \( f \) is the transmit frequency and \( \delta t \) is the time delay of the signal for each element.

**Figure 5.29: 3*3 circular transducers**

![Diagram of 3*3 circular transducers]

A 3*3 square array of circular transducers with the point of focus represented by a blue point is shown in Fig.5.29. The coordinate points are [0.01,0.01,100e-3] m. The phase shift values obtained from the time delay using Eq. (5.2) are shown in the tabular column 5.7

**Table 5.7: Phase Shift Values for 3*3 Matrix**

<table>
<thead>
<tr>
<th>Transducer Number</th>
<th>Delay(in μs)</th>
<th>Phase Shift(in deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.9384</td>
<td>85.51</td>
</tr>
<tr>
<td>2</td>
<td>10.547</td>
<td>151.87</td>
</tr>
<tr>
<td>3</td>
<td>11.998</td>
<td>172.77</td>
</tr>
<tr>
<td>4</td>
<td>4.5174</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>9.1040</td>
<td>131</td>
</tr>
<tr>
<td>6</td>
<td>10.547</td>
<td>151.87</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>4.5174</td>
<td>65</td>
</tr>
<tr>
<td>9</td>
<td>5.9384</td>
<td>85.5</td>
</tr>
</tbody>
</table>

It can be seen from the table 5.7 that the transducer farthest(i.e 7) from the point of focus needs the least delay. This is because, it has the maximum propaga-
tion time for the sound wave to reach the point of focus. However, the transducer closest to the point of focus (i.e 3) has the maximum time delay.


6

Conclusion

6.1 Thesis Contribution

The initial part of the thesis constitutes the understanding of characteristics and working principles of an Ultrasonic Transducer. The next section involves the research on Haptic feedback using ultrasonic transducers. This provides us the insight of the principles that govern haptics.

For validating the characteristics of transducer we used Field II simulation tool. In this, we initially measured the impulse responses of single element and multi element transducer in both time as well as frequency domain. With their frequency domain analysis we concluded that both, the single and multi element transducer act as band pass filter. This can be seen from Fig. 5.3 and 5.8. Next, we simulated transducer beam profile for a multi element array and compared it with Fresnel and Fraunhofer approximation. The simulated beam profile was much wider then the Fresnel and Fraunhofer beam profile. It was also observed that with the increase in bandwidth of the input signal of a transducer, the side lobes in the beam profile plot can be reduced. The beam profile was then simulated with focal length to observe the variation in pressure at various focal depth. It gave a clear validation that the maximum pressure was obtained when ROC is equal to twice the focal length. The concept of apodization was validated using hanning window to reduce the number of side lobes in the pressure field. This comparison is shown in Fig. 5.18 and 5.19.

A Matlab based GUI was also used to perform simulations of transducer pressure field. The pressure field simulations were conducted for single element as well as multi element array. By observing the pressure plots and the comparing their pressure and intensity values, it is unambiguous to say that by increasing the number of elements and frequency of the input signal of the transducer, the pressure and intensity values increases. Furthermore, the far field pressure plots
for various configurations also proves that by increasing the distance of the focus from the array the pressure value decreases. The above mentioned analyses were simulated for both normal square wave input as well as pulse width modulated wave. The concept of phase shift and delay were implemented in Matlab which demonstrated that by changing these values the point of focus for beam formation can be altered by the user.

6.2 Future Work

The field of haptics is very interesting and is growing rapidly. The research presented in this paper can form the basis for the following areas in future:

1. Development of electronics hardware, which will involve the concepts of duty cycle variation and amplitude modulation presented by the research work. The guidance for the electronics can be derived from Asier Marzo and Drinkwater [1].

2. Merging of the phase shift concept presented in the thesis research work along with the TAC GUI. This could be done to obtain dynamic relationship between time and acoustic pressure variation.

3. The thesis research work could provide the requisite knowledge for any aspirant who wants to continue the topic as a PhD research.
6.3 License for using TAC GUI

The following license was obtained for using TAC GUI in our thesis work. The license copy can be found in https://se.mathworks.com/matlabcentral/fileexchange/35657-transducer-array-calculation-tac-gui.

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Bibliography


