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Abstract

With origins in post-war development thinking, the core-periphery concept has spread across the social and, increasingly, the natural sciences. Initially reflecting divergent socioeconomic properties of geographical regions, its relational connotations rapidly led to more topological interpretations. In today's network science, the standard core-periphery model consists of a cohesive set of core actors and a peripheral set of internally disconnected actors.

Exploring the classical core-periphery literature, this paper finds conceptual support for the characteristic intra-categorical density differential. However, this literature also lends support to the notions of peripheral dependency and core dominance, power-relational aspects that existing approaches do not capture.

To capture such power-relations, this paper suggests extensions to the correlation-based core-periphery metric of Borgatti-Everett (2000). Capturing peripheral dependency and, optionally, core dominance, these extensions allow for either measuring the degree of such power-relational features in given core-periphery partitions, or as part of a criteria function to search for power-relational core-periphery structures.

Applied to the binary and valued citation data in Borgatti and Everett (2000), the proposed extensions seemingly capture dependency and dominance features of core-periphery structures. This is particularly evident when, circling back to the original domains of the concept, examining the network of European commodity trade in 2010.

Keywords

Core-periphery, History of ideas, Economic history, Peripheral dependency, Core dominance

Introduction

The origin of “center/core”,¹ “periphery” and their coupling into the conjoint concept of *repute*, stems from the work of Raul Prebisch (1950) (Janvry, 1975; Love, 1980). Originally applied to highlight divergent socioeconomic properties of regions, the concept became integral to the heterodox strands of postwar development thinking (Chase-Dunn & Hall, 1991; Frank, 1967; Galtung, 1971; Santos, 1970; Wallerstein, 1974). Prior to its recent entry into mainstream economics (e.g. Hojman & Szeidl, 2008; Krugman, 1990, 1991, 1998), the core-periphery concept was anything but dormant: whether used as a descriptive, explanatory, or analytical device, as a model, structure, or process, or as something spatial, metaphorical, or something in-between, the remarkable dissemination of the core-periphery concept took it from its origins in political economy and international relations to virtually all of the social, and increasingly also the natural, sciences.

Within network science, core-periphery is a structural template whose relevance as an analytical device, similar to its *raison d'être* in social science at large (McKenzie, 1977, p. 55), rests on the idea that the general relationship between core and periphery is important for understanding the system as a whole. In the core-periphery metric of Borgatti & Everett (2000), echoing the corresponding block image template in the blockmodeling tradition (Wasserman & Faust, 1994, p. 419ff; White, Boorman, & Breiger, 1976, pp. 742, 744), core and periphery are specified in terms of an intra-categorical density differential², where a high frequency of ties among core actors contrasts an absence of ties among peripheral actors. With overall connectivity typically viewed as an overarching criterion for core-periphery structures (Borgatti & Everett, 2000, p. 382; Borgatti, Everett, & Johnson, 2013, p. 225), the ties between the core and periphery subsets are either modeled as a density mid-way between the intra-categorical extremes or, as often recommended (e.g. Borgatti & Everett, 2000, p. 383; Boyd, Fitzgerald, & Beck, 2006, p. 167ff), ignored.

This article has two intertwined objectives. Whereas Borgatti and Everett view their seminal paper “as a starting point in a methodological debate on what constitutes a core/periphery structure” (2000, p. 376), the current article continues this debate by exploring topological core-periphery specifications that followed Prebisch’s original formulation. At odds with the claim “that the notion of a core/periphery structure has never been formally defined” (Borgatti & Everett, 2000, p. 375), several topological core-

¹ Whereas “center” was used in the original formulation of Prebisch and pre-world system scholars, Wallerstein instead preferred the categorical label of “core”. A similar terminological transition occurred in network analysis, where the early blockmodeling scholars preferred “center” (e.g. White, Boorman, & Breiger, 1976, p. 742) and later scholars used “core” (e.g. Borgatti & Everett, 2000). As I have found no functional distinction between “center” and “core” in neither of these traditions, I use these labels interchangeably.

² Node- and edge-level centrality-type metrics has also been suggested for identifying core-periphery structures, (Della Rossa, Dercole, & Piccardi, 2013; Lee, Cucuringu, & Porter, 2014). However, as pointed out by Borgatti and Everett, whereas core actors “are necessarily highly central as measured by virtually any measure [...] the converse is not true” (2000, p. 393; see also Lee et al., 2014, p. 4).

periphery specifications are found in this literature. Whereas the notions of sparse peripheries and dense cores find support in this literature, support is also found for characteristic features at the inter-categorical level, as patterns of *peripheral dependency* and *core dominance*. Such power-relational aspects of classical core-periphery thinking are not captured by existing models and metrics in network science.

Building on these findings, this paper proposes core-periphery models where the intra-categorical density differential is supplemented with criteria for dependent peripheries and dominating cores. Extending the core-periphery heuristic of Borgatti and Everett (2000), a correlation-based approach to capture peripheral dependency and, optionally, core dominance is proposed. Applicable to both binary and valued networks, the proposed extension can either be used to measure the degree of such power-relations for given core-periphery partitions, or as part of the criteria function for finding optimal partitions.

The proposed metric for power-relational core-periphery structures is subsequently applied on example networks. Beginning with three simple networks, the example section revisits the binary and valued Baker citation data analyzed by Borgatti & Everett (2000; Baker, 1992). Circling back to the political economy genesis of core-periphery thinking, a final example network of European commodity trade concludes this section.

A summary of the conceptual findings, suggested operationalization, and problematic areas identified from the examples concludes this article.

Intra-categorical density differential: dense core, sparse periphery

In the classical center-periphery (and centralized) block images (Breiger, Boorman, & Arabie, 1975; Mullins, Hargens, Hecht, & Kick, 1977; White et al., 1976), the proposed metrics of Borgatti and Everett (2000; Everett & Borgatti, 2000) and the subsequent heuristics and algorithms for finding core-periphery structures (Boyd et al., 2006; Boyd, Fitzgerald, Mahutga, & Smith, 2010; Muñiz & Carvajal, 2006; Rombach, Porter, Fowler, & Mucha, 2014), the intra-categorical density differential is the defining feature of core-periphery structures. This topological specification finds ample support in the “non-network” literature on core-periphery structures, particularly in the fields of international relations and political economy (Berman, 1974, p. 4; Chan, 1982, p. 315; Dominguez, 1971, p. 176; Galtung, 1966a, p. 146, 1971, p. 89; Gleditsch, 1967, p. 369; Mullins et al., 1977, pp. 49–56). Among these, the studies of Galtung (1966a) and Gleditsch (1967) are particularly noteworthy, looking at, respectively, cold war international relations (see also Gochman & Ray, 1979) and global air routes. In addition to the notions of a dense core and a sparse periphery, these two studies also specify the density of inter-categorical ties, modeled as mid-way between the two intra-categorical extremes (Galtung, 1966a, p. 146; Gleditsch, 1967, p. 369). Galtung and Gleditsch both find a distinct intra-categorical density differential in their

respective datasets, as well as inter-categorical densities that are in-between the densities of intra-categorical ties.³

However, whether *inter*-categorical ties are modeled as an in-between density or simply ignored, we would in either case be unable to identify notions of peripheral dependency and core dominance, features that were integral to how core-periphery structures were perceived in much of the literature following Prebisch.

Dependency and dominance: core-periphery power-relations

The center-periphery concept is foundational for the strand of development theory known as the dependency school (e.g. Amin, 1976; Cardoso & Faletto, 1967; Frank, 1967; Santos, 1970). Combining Latin American structuralism with neo-Marxism, the dependency scholars differed from the former by depicting underdevelopment in the periphery as the direct result of its relationships with the center. Interaction between developed and underdeveloped regions was characteristically described as a hierarchical series of monopolistic metropole-satellite relations, in which each satellite was confined to dealing only with their respective metropole (Frank, 1967, pp. 7, 15; Santos, 1970, p. 235). Channeling profits from the many third world peasants to the few European industrialists (Frank, 1967), such monopolistic-oligopsonistic dendritic structures were perceived as the root cause for the development of underdevelopment (see also Bauer, 1954, p. 103; Condliffe, 1951, p. 816; Meier & Baldwin, 1957, p. 332).

The core-periphery concept was an equally defining feature in the subsequent world-system perspective⁴ (Chase-Dunn, 1998; Wallerstein, 1974; see Oman & Wignaraja, 1991; So, 1990). Even though the categorization of societies into respective world-system strata often is based on the internal characteristics of respective society, particularly how the international division of labor is manifested at the regional levels (e.g. Bousquet, 2012, p. 124), the notions of core dominance and peripheral

³ What Galtung and Gleditsch do in these studies – categorical sorting of actors, calculation of intra- and inter-categorical densities, and interpreting by comparing these densities to an ideal model (Galtung, 1966a, p. 163; Gleditsch, 1967, p. 377) – is in essence blockmodeling, predating the studies that formally labeled the approach as such.

⁴ Evolving from dependency thinking, the world-system perspective differs from its predecessor in significant ways. First, supplementing the core and peripheral categories, the world-system perspective included a third category – the semiperiphery – reflecting a less deterministic and somewhat more dynamic world than the one typically described by dependency scholars (see Wallerstein, 1974, p. 403, 1979, p. 69). Secondly, surpassing the peripheral focus of the dependency school, the world-system perspective views the whole system of interconnected societies as the basic unit of analysis. Similarly, inspired by the “total history” of Braudel, Wallerstein also broadened the temporal horizon, extending world-system analysis back to the late 15th century (cf. Chase-Dunn & Hall, 1991; Gills & Frank, 2014). In the macro-sociological world-system tradition, Chase-Dunn defines world-systems “as intersocietal networks in which the interactions (e.g., trade, warfare, intermarriage, information) are important for the reproduction of the internal structures of the composite units and importantly affect changes that occur in these local structures.” (Chase-Dunn & Hall, 1991, p. 28).

dependency are integral aspects of the world-system perspective (e.g. Chase-Dunn & Grimes, 1995, p. 389; Gills & Frank, 2014, p. 7; Rokkan & Urwin, 1983; Wallerstein, 1974). With such an explicit emphasis on intersocietal relations in world-system analysis, the attribute-based definitions of world-system strata have been contested (e.g. Duvall, 1978, p. 59; Vanolo, 2010, p. 30), particularly in the series of blockmodeling studies of the modern world-system (e.g. Snyder & Kick, 1979; Breiger, 1981; Nemeth & Smith, 1985, p. 521; Smith & White, 1992, p. 859; see Lloyd, Mahutga, & Leeuw, 2015). Proclaiming a “natural wedding” between multi-relational blockmodeling and world-system analysis (Snyder & Kick, 1979, p. 1123), Snyder and Kick argue that although correlations might exist between regional/country attributes and world-system strata, they “do not represent such position any more than an individual’s income or education measures his or her (discrete) class position” (1979, p. 1102). Similarly, “[w]hen dependency is viewed as a referential context or in terms of structural position in the world-economy, the focus of the analysis is no longer on characteristics of individual countries, but on the relationships between countries” (Nemeth & Smith, 1985, p. 522).

A formalization of the topology of peripheral dependency and core dominance as found in the dependency and world-system traditions is provided by Galtung (1971). In his structural theory of imperialism, Galtung views imperialism as a specific system of dominance, primarily but not exclusively between nations, “that splits up collectivities and relates some of the parts to each other in relations of harmony of interest, and other parts in relations of disharmony of interests, or conflict of interest.” (1971, p. 81). He identifies two underlying mechanisms for imperialism – vertical interaction (e.g. unequal exchange, asymmetric interaction) and the so-called “feudal interaction structure” of core-periphery relations, the latter facilitating occurrences of the former. Mentioning this interaction structure in earlier writings (Galtung, 1966b, 1968), his 1971 study provides a set of rules for identifying center-periphery structures on the basis of interaction patterns between core and peripheral actors (1971, p. 89):

1. Interaction between Center and Periphery is *vertical*
2. Interaction between Periphery and Periphery is *missing*
3. Multilateral interaction involving all three is *missing*
4. Interaction with the outside world is *monopolized* by the Center, with two implications:
 - a. Periphery interaction with other Center nations is *missing*
 - b. Center as well as Periphery interaction with Periphery nations belonging to other Center nations is *missing*. (Galtung, 1971, p. 89; original italics)

Peripheral dependency – that each peripheral actor is connected to exactly one core actor – is given by the 4th rule and sub-rules. Together with the 2nd rule stating an internally disconnected periphery,⁵ the

⁵ In his 1966 article, Galtung notes that a periphery gone cohesive is no longer part of a center-periphery structure: rather, with Marxian-Engelsian undertones, a center-periphery system “can be destroyed if the underdogs unite”,

peripheral dependency rule also constitutes the criterion for overall center-periphery connectivity. These rules translate into characteristic patterns of core-periphery relations as given in the visual example that Galtung provides, reproduced in *Figure 1*.

Figure 1: Center-periphery structure according to Galtung (1971, p. 89)

Galtung's rules do not specify any topological characteristics of core actors, i.e. whether it is intra-core cohesion, dominance of peripheral actors, or both, that characterize core actors. The core actors in his visual example are dominating, each having ties with unique sets of peripheries, but the rules do not explicitly rule out the existence of core actors without ties to peripheral actors. The core actors in Galtung's example are connected, but it is noteworthy that the intra-core block in his example is less than complete (density of 0.667). Although the default Borgatti-Everett core-periphery heuristic, i.e. ignoring non-diagonal blocks, applied to the Galtung example reaches the absolute maximum at the intuitive partition, the coefficient for this solution is however less than ideal (0.795). This raises the question whether k-plexes, k-cores or similarly imperfect cliques are more appropriate to describe and capture intra-core connectivity (see Everett & Borgatti, 2000).

The notion of an intra-categorical density differential as a characteristic of core-periphery structures thus finds significant support in the 1960's literature and onwards. Concurrently, as the concept became integral to the dependency school and the subsequent world-system perspective, core-periphery relations were increasingly equated with peripheral dependency on the core and core dominance over the periphery. Despite this emphasis on such power-relations of core-periphery structures, peripheral dependency and core dominance are not part of the contemporary network-scientific conceptualization of core-periphery structures.

The next section will introduce extensions to the correlation-based core-periphery metric of Borgatti and Everett that capture patterns of peripheral dependency and, optionally, core dominance. As with the original metric of Borgatti and Everett, the extensions can be used either to measure the degree of power-relational patterns in pre-given core-periphery partitions or as part of a criteria function for finding such

as such transforming the system into a "class system" (1966a, p. 146). In blockmodeling terms, this would then correspond to a cohesive subgroup block image.

structures. Contrasting existing core-periphery metrics and heuristics, it is suggested, and demonstrated in the subsequent example section, that the proposed extensions are more suitable for analyzing core-periphery structures where such power-relations are substantively relevant, within as well as outside its original international relations context, than what the standard core-periphery model allows for.

Power-relational core-periphery models: peripheral dependency and core dominance

This section proposes how the criteria for peripheral dependency and dominating cores can be operationalized as extensions to the intra-categorical density differential criterion.⁶ Starting with the standard core-periphery block image consisting of a dense core (a so-called ‘complete block’ in the intra-core section) and a sparse periphery (a ‘null’ block in the intra-periphery section) – see Figure 2 – the criteria for peripheral dependency and core dominance are thus concerned with the patterns of ties in the off-diagonal blocks in Figure 2 below.

	C	P
Core (C)	com	-
Periphery (P)	-	nul

Figure 2: Ideal core-periphery blockimage (ignoring off-diagonal blocks) suggested by Borgatti and Everett (2000, p. 383)

The core-periphery metric of Borgatti and Everett (2000) corresponds to a correlation between observed and ideal values, where values in the intra-core and intra-periphery blocks are correlated with, respectively, ones and zeros, optionally also correlating values in each of the inter-categorical sections with a pre-specified “density”.⁷ The herein proposed extensions⁸ to this metric supplement the sets of observed and ideal values for the intra-categorical blocks with additional value-pairs for observed and ideal tie patterns in the inter-categorical blocks.

For directional networks, dependency and dominance are directional concepts. Outbound peripheral dependency and inbound core dominance are identified by examining periphery-to-core ties, whereas inbound dependency and outbound dominance are found in the core-to-periphery block in Figure 2. Which of these power-relational features to include depends on the theoretical significance and interpretational meaning of dependency and dominance in the particular context of the analyzed network and, of course, whether the network is directional or not.

⁶ A demonstrational Windows client that implements (using local optimization) the proposed extensions to the Borgatti-Everett metric can be found at <http://www.carlnordlund.net/>

⁷ As implemented in Ucinet (version 6.598), the inter-categorical “density” parameters in the Borgatti-Everett metric are not densities in the traditional binary blockmodeling sense; rather, similar to the case for the intra-categorical correlations, the values in the inter-categorical blocks are correlated to the specified “density” parameters. For instance, in the case of an inter-categorical block with a chequered binary patterns with alternating 1- and 0-cells, i.e. a block density of 0.5, setting the Ucinet parameters to 0.5 implies that each of the empirical ones and zeros are correlated to 0.5, vastly reducing the overall core-periphery fit in this case.

⁸ As with the original Borgatti-Everett metric, the specifications that follow are restricted to single-layer, one-mode networks.

Peripheral dependency

In the dependency and world-system perspective, the notion of peripheral dependency is part and parcel of the core-periphery concept. As reflected in Galtung's specification, the defining feature of a peripheral actor is being monopolized by a singular core actor and lacking ties to other actors. Expressed as an ideal block type as used in generalized blockmodeling, outbound peripheral dependency translates into a so-called 'row-functional' block in the periphery-to-core block (see, e.g., Doreian, Batagelj, & Ferligoj, 2005, p. 213), i.e. where there is exactly one tie in each row of the block. For inbound peripheral dependency, this procedure is done with respect to the columns in the core-to-periphery block (i.e. a so-called 'column-functional' block in generalized blockmodeling terminology).

In order to capture peripheral dependency, the total lists of observed and ideal values for the intra-categorical blocks are supplemented with value-pairs for the inter-categorical blocks.⁹ For outbound peripheral dependency, each row in the periphery-to-core block (see Figure 2) is first sorted. The largest value in each of these rows are correlated with unity whereas the remaining values are correlated with zero.¹⁰ If a block row contains two or more ties with the same maximum tie value, a likely scenario in binary networks, only one of these values are correlated to unity and remaining with zero.¹¹ For inbound peripheral dependency, the corresponding procedure is done with respect to block columns in the core-to-periphery block.

Core dominance

Mirroring peripheral dependency, the dependency and world-system traditions characteristically depict the core as dominating the periphery. This is reflected in the visual example provided by Galtung (1971), where each of the core actors are dominating exclusive sets of peripheral actors. However, as reflected in the overall peripheral focus of the dependency school, it is conceivable that core actors could be non-dominating, i.e. where the relatively high intra-core density constitutes the sole defining topological feature of core actors. Inclusion of core dominance as a criterion for power-relational core-periphery could thus be deemed optional to peripheral dependency, depending on the specifics of the research question that motivates a power-relational core-periphery analysis.

As core dominance implies having a variable number of ties with peripheral actors, translating this criterion to the correlation-based Borgatti-Everett metric is not as straight-forward as is the case for

⁹ Although not explored here, an alternative is to calculate separate correlations for, respectively, inter- and intra-categorical sections, using the latter as an outgoing stat rather than part of the criteria function.

¹⁰ An example of the calculation procedure for peripheral dependency and core dominance is given in the toy examples below.

¹¹ In cases where a presumed peripheral actor lacks ties to any of the presumed core actors, this means that one of these missing ties (0) is correlated to unity, whereas the remaining missing ties are correlated to zero. Although not explored here, a more 'penalizing' alternative is to correlate all these missing ties to unity.

peripheral dependency. To check whether the criterion for inbound core dominance is fulfilled¹², it is only necessary to examine the largest value in respective block column and, similar to the peripheral dependency criterion, correlate these values with unity. However, whereas the criteria for dense cores, sparse peripheries and peripheral dependency stipulates that all values in respective block are included in the correlation, the criterion for core dominance would have a much lower influence on the final correlation measure than what these other criteria would have. Addressing this, the suggested extension to the Borgatti-Everett metric for capturing core dominance is designed somewhat differently. For inbound core dominance, the highest value in each block column of the periphery-to-core block is obtained, a value that is correlated to unity not only once, but repeatedly for as many rows (i.e. peripheries) as there are in the particular partition.

A composite metric of power-relational core-periphery structures: combining intra- and inter-categorical criteria

As the default Borgatti-Everett metric is a correlation of value-pairs from the two diagonal block sections (see Figure 2), it depends on the relative contribution of value-pairs from respective block. In the default version of the metric, the total number of value-pairs and the relative contributions from the two intra-categorical sections depend on the relative sizes of respective category. The amount and distribution of value-pairs for a directional 20-actor network is given in *Figure 3* below.

Figure 3: Amount and distribution of value-pair correlations for a directional 20-actor network (without self-ties)

If we were to extend this metric with two power-relational criteria, the number of total value-pair correlations becomes equal to the number of possible ties in the network, irrespective of the relative sizes of respective category. For a directional 20-actor network, this corresponds to the top black line in

¹² Expressed as ideal block types in generalized blockmodeling, inbound and outbound core dominance corresponds to, respectively, a column-regular block in the periphery-to-core ties and a row-regular block in the core-to-periphery ties (see (Doreian, Batagelj, & Ferligoj, 2005, p. 213)

Figure 3 above. However, when including one, three or four criteria for dependency and dominance, the relative influences of intra- and inter-categorical correlations change and once again depend on relative cluster sizes. For instance, when only including outbound peripheral dependency in a directional network, only value-pairs for the periphery-to-core block is included. If we instead were to include all four power-relational criteria, each cell in both off-diagonal blocks are counted twice, biasing the final correlation in favor of power-relational patterns over the density differential criteria.

To keep the relative influence of block sections independent from the number of chosen power-relational criteria, the standard correlation coefficient formula used in the default Borgatti-Everett metric is replaced by its weighted version¹³. Thus, in addition to the two vectors with observed and ideal patterns, a third vector with weights for respective value-pair is included, using the formula for weighted correlation coefficient as follows:

$$r_w = \frac{\sum w_i (x_i - \bar{x})(y_i - \bar{y})}{\sum w_i \sqrt{\sum w_i (x_i - \bar{x})^2 \cdot \sum w_i (y_i - \bar{y})^2}}$$

where x_i and y_i are, respectively, the observed and ideal values, w_i is the weight for that value pair, and where the weighted means are $\bar{x} = \sum w_i x_i / \sum w_i$ and $\bar{y} = \sum w_i y_i / \sum w_i$.

For the value-pairs representing the observed and ideal ties within the core and periphery, respectively, their weights (w_{intra}) are always set to unity. For value-pairs representing ties between core and periphery, i.e. the inter-categorical ties checked by the dependency and dominance criteria, their weights (w_{inter}) depend on the number of criteria included in the analysis. For directional networks, setting w_{inter} to 2 divided by the number of power-relational criteria means that the relative influence of each potential tie on the final correlation remains the same, irrespective of the number of criteria. For symmetric networks, a single power-relational criterion should be weighted with unity whereas the value-pair weights should be set to 0.5 when both dependency and dominance are included. Suggested settings for w_{inter} for different number of power-relational criteria are given in Table 1 below.

¹³ Other options exist for combining the intra- and inter-categorical criteria. One such option, tentatively explored in the scope of this research project, is to calculate separate correlations for respective set of criteria, subsequently combining them using a Cobb-Douglas-type of utility function. A would-be theoretical advantage with separate correlations for intra- and inter-categorical criteria is that each have their own distinct mean values. This seemed to be particularly useful for valued networks where the values of core-periphery ties often seem to be lower than intra-core ties. An additional advantage is that a criteria function can be constructed with marginal rates of substitution (such as the Cobb-Douglas function), i.e. where a bad fit with respect to dependency and dominance cannot be fully compensated by a high-scoring intra-categorical density differential.

Symmetric networks		<i>W_{inter}</i>	<i>W_{intra}</i>
Dependency OR dominance		1	1
Dependency AND dominance		0.5	
Directional networks			
Number of power-relational criteria	1 (e.g. only outbound dependency)	2	
	2 (e.g. in- and outbound dependency)	1	
	3	2/3	
	4 (in- and outbound dependency and dominance)	0.5	

Table 1: Suggested weights for power-relational (inter-categorical) criteria

Examples

In this section, the proposed extensions to the Borgatti-Everett metric are applied to a set of example networks, comparing obtained partitions and criteria scores with those resulting from the default Borgatti-Everett heuristic. Beginning with three smaller examples, one which demonstrates the details of the calculation of dependency and dominance, this is followed by an analysis of the binary and valued journal citation data in (Borgatti & Everett, 2000; from Baker, 1992). Circling back to the political economy genesis of core-periphery thinking, the example section is rounded off by analyzing the network of European commodity trade in 2010 (Nordlund, 2016)

Toy examples: BEfig1, Galtung, Intercontinental trade

The first of the three smaller networks is the 10-actor network used by Borgatti and Everett (2000, p. 377; Figure 1) to exemplify an ideal core-periphery structure. With its intra-core clique and disconnected peripheries, all cores have ties to peripheral actors (i.e. core dominance) but two out of the six peripheral actors have ties to two core actors (i.e. not ideal peripheral dependency). The second example is the one provided by Galtung (1971). With ideal inter-categorical patterns of dependency and dominance and a lack of intra-peripheral ties, intra-core relations constitute an imperfect clique. Expanding into valued and directional networks, the third example is a 7-actor directional network of aggregate commodity trade in the 1995-1999 period between seven world regions: North and Latin America, Asia, Africa, Australasia, and Western and Eastern Europe. This example also serves to demonstrate how the correlation coefficient is calculated for dependency and dominance. For each of these examples, the optimal solution found by the default Borgatti-Everett metric is compared with those obtained when including, respectively, dependency, and both dependency and dominance.

In the core-periphery example provided by Borgatti and Everett (2000, p. 377) – see Table 2 below – the intuitive core consists of actors 1-4. With these actors constituting an ideal clique and remaining actors disconnected from each other, the default Borgatti-Everett finds this partition to be the optimal, and ideal, solution. Adding peripheral dependency, the same core is still found, though the non-dependency of actors 5 and 8 yields a slightly less ideal score. When including both dependency and dominance,

switching to the weighted correlation coefficient formula with a w_{inter} set to 0.5 (see *Table 1* above), the observed dominance for actors 1-4 is rewarded by an increased score.

BEfig1 [Binary, Symmetric]	Correlation	Core actors
Default BE	1	1,2,3,4
Default BE + dependency	0.897	
Default BE + dependency + dominance	0.956	

Table 2: Core-periphery example provided by Borgatti and Everett (2000, p. 377)

As previously observed, Galtung's so-called "feudal interaction structure" has an intuitive core that corresponds to a less-than-ideal clique. Although the default Borgatti-Everett metric finds this intuitive core, the correlation is less-than-ideal – see *Table 3* below. Adding peripheral dependency, subsequently also core dominance, the same intuitive core is found, with incrementally increasing scores.

Galtung [Binary, Symmetric]	Correlation	Core actors
Default BE	0.795	c1,c2,c3,c4
Default BE + dependency	0.917	
Default BE + dependency + dominance	0.945	

Table 3: Galtung's core-periphery example (Galtung, 1971, p. 89)

The final toy network consists of commodity trade between seven world regions, aggregated using Comtrade data for the 1995-1999 period (see Nordlund, 2010, p. 98). In *Table 4* below, the data matrix is blocked into a hypothetical partition where Asia, North America and Western Europe constitute a hypothetical core. Optimal correlations and their corresponding partitions for the default Borgatti-Everett metric and the two power-relational extensions are found in *Table 5*.

		Core			Periphery			
		ASI	NAM	WEU	AFR	AUS	EEU	LAT
Core	ASI	358	263		16	27	16	31
	NAM	226	185		10	17	9	133
	WEU	239	201		38	19	109	50
Periphery	AFR	15	13	38		0	1	3
	AUS	40	8	10	1		1	1
	EEU	20	10	100	2	0		2
	LAT	26	141	41	2	1	3	

Table 4: Aggregate intercontinental commodity trade 1995-1999 (billion USD)

Intercont. trade [Valued, Directional]	Correlation	Core actors
Default BE	0.975	NAM, WEU
Default BE + dependency	0.754	ASI, NAM, WEU
Default BE + dependency + dominance	0.801	NAM, WEU

Table 5: Core-periphery results for the intercontinental trade example (the dependency and dominance criteria are both in- as well as outbound)

At the optimal correlation of 0.975, the default Borgatti-Everett metric finds the optimal core to consist of North America and Western Europe. This is despite the fact that the flows between Asia and, respectively, North America and Western Europe represent the four largest dyads in the network. This is primarily due to intra-core variance: by placing Asia in the periphery, only the two relatively similar bilateral flows between North America and Western Europe are correlated with unity. Additionally, as Asia has relatively weak ties with the identified peripheral regions, i.e. a seemingly low degree of dominance, placing Asia in the periphery has only a marginal effect on intra-peripheral variance.

Adding in- and outbound peripheral dependency, the two significant Asian trading ties to North America and Western Europe imply non-dependence: keeping Asia in the periphery with the dependency criteria results in the low correlation of 0.536. Rather, the optimal solution (0.754) with the in- and outbound dependency criteria is found when Asia joins the core. However, when also adding the core dominance criteria, the relatively weak ties between Asia and, respectively, Africa, Australasia, Eastern Europe and Latin America (see Table 4 above) once again places Asia in the periphery, this time at a correlation of 0.801.

To exemplify the calculation procedure, correlations for the various criteria are calculated for the hypothetical partition in Table 4. Although this particular partition was only found when extending the default metric with peripheral dependency only, the correlations for all three varieties of the metric are calculated¹⁴. The three sections in Table 6 below contain the value-pairs to be correlated for the different criteria, where the weights of the value-pairs for peripheral dependency and core dominance depend on the number of criteria included (see Table 1 above). Results for the various metrics for this particular partition is given in Table 7.

¹⁴ In a local optimization search algorithm (such as implemented in the demonstrational software client accompanying this article), the algorithm calculates the correlations for neighboring partitions, i.e. partitions where either one actor is moved between clusters or two actors in different clusters are swapped with each other, repeating the exploration for the partition(s) that results in a higher correlation. For this small network, an exhaustive search was instead performed, i.e. examining all possible core-periphery partitions.

Default Borgatti-Everett				Peripheral dependency				Core dominance						
Value-pair weights: 1				Value-pair weights: 2/C				Value-pair weights: 2/C						
		Dyad	Value	Model			Dyad	Value	Model			Dyad	Value	Model
Dense core		ASI-NAM	358	1	Outbound dependency		AFR-ASI	15	0	Inbound dominance		AUS-ASI	40	1
		ASI-WEU	263	1			AFR-NAM	13	0			(AFR-ASI)	40	1
		NAM-ASI	226	1			AFR-WEU	38	1			(EEU-ASI)	40	1
		NAM-WEU	185	1			AUS-ASI	40	1			(LAT-ASI)	40	1
		WEU-ASI	239	1			AUS-NAM	8	0			LAT-NAM	141	1
		WEU-NAM	201	1			AUS-WEU	10	0			(AFR-NAM)	141	1
Sparse periphery		AFR-AUS	0	0			EEU-ASI	20	0			(AUS-NAM)	141	1
		AFR-EEU	1	0			EEU-NAM	10	0			(EEU-NAM)	141	1
		AFR-LAT	3	0			EEU-WEU	100	1			EEU-WEU	100	1
		AUS-AFR	1	0		LAT-ASI	26	0		(AFR-WEU)	100	1		
		AUS-EEU	1	0		LAT-NAM	141	1		(AUS-WEU)	100	1		
		AUS-LAT	1	0		LAT-WEU	41	0		(LAT-WEU)	100	1		
		EEU-AFR	2	0	Inbound dependency		ASI-AFR	16	0	Outbound dominance		ASI-LAT	31	1
		EEU-AUS	0	0			NAM-AFR	10	0			(ASI-AFR)	31	1
		EEU-LAT	2	0			WEU-AFR	38	1			(ASI-AUS)	31	1
		LAT-AFR	2	0			ASI-AUS	27	1			(ASI-EEU)	31	1
		LAT-AUS	1	0			NAM-AUS	17	0			NAM-LAT	133	1
		LAT-EEU	3	0			WEU-AUS	19	0			(NAM-AFR)	133	1
							ASI-EEU	16	0			(NAM-AUS)	133	1
							NAM-EEU	9	0			(NAM-EEU)	133	1
							WEU-EEU	109	1			WEU-EEU	109	1
						ASI-LAT	31	0		(WEU-AFR)	109	1		
						NAM-LAT	133	1		(WEU-AUS)	109	1		
						WEU-LAT	50	0		(WEU-LAT)	109	1		

Table 6: Value-pair correlations for the directional intercontinental trade example (C =Number of included power-relational criteria)

Intercont. trade [Valued, Directional]	Winter	Correlation	Core actors
Default BE	n/a	0.962*	ASI, NAM, WEU
Default BE + dependency (in/out)	1	0.754	
Default BE + dependency + dominance (both in/out)	0.5	0.706*	

Table 7: Core-periphery metrics for the intercontinental trade example with Asia, North America and Western Europe as the hypothetical core (*: non-optimal solutions for this partition)

The table with value-pairs for the peripheral dependency criteria demonstrates how only the largest tie a presumed peripheral actor has with the presumed core actors is correlated with unity, whereas the remaining ties to the other core actors are correlated with zero. For core dominance, the largest tie a presumed core has with the presumed peripheral actors is correlated with unity, repeating this correlation for the number of peripheries that exist. As can be seen in the right-hand part in Table 6, the dominance of Asia with respect to the peripheries in this partition is weaker than the dominance of North America (with respect to Latin America) and Western Europe (with respect to Eastern Europe).

The ideal power-relational core-periphery model is thus not static, but the specific values that are correlated with unity depend on the pattern and strength of core-periphery ties. This allows for identifying the specific ties that are the potential ties of dependency and dominance. For the partition given in Table 4, Table 6 and Table 7 above, i.e. the optimal solution when extending the default metric

with peripheral dependency, the ties of dependency are between Africa and Western Europe, East and West Europe, and North and Latin America, and, to a lesser extent, Asia's ties to Australasia and Latin America.

Example: Baker journal citation data

In his study of citation clusters among social work journals, Baker (1992) collected data on number of citations within and between 20 journals for the years 1985-86. This and subsequent analyses (Borgatti & Everett, 2000; Nordlund, 2016) found this network to correspond to a core-periphery structure. In Borgatti and Everett (2000), the original Baker data was (max-)symmetrized and self-ties excluded before demonstrating their suggested correlation-based metric on both the binary and valued versions of the data. For comparative purpose, the analyses below uses the identical binary and valued data¹⁵ as that used by Borgatti and Everett (2000, p. 386).

Binary citation data

Applying the default Borgatti-Everett heuristic on the binary citation data, the optimal solution (at a correlation of 0.860) consists of a 7-journal core – see *Figure 4* (solid-line partition). Adding the peripheral dependency criteria, the optimal solution places ASW in the core: although ASW lacks ties with two presumed core journals, its remaining five core ties makes it a weak peripheral candidate. In this solution, the non-dependence of all but three peripheral journals do however bring the correlation down to 0.686 when including the peripheral dependency criteria.

¹⁵ As noted in Nordlund (2016, p. 168), the symmetrization in Borgatti & Everett (2000, p. 386) of the original data (Baker, 1992, p. 159) contains a few errors. For comparative reasons, the current paper nevertheless uses the exact same (yet imperfectly max-symmetrized) data as used by Borgatti & Everett (2000).

	CW	CYSR	JSWE	SSR	SCW	SWRA	SW	ASW	CAN	FR	CSWJ	AMH	BJSW	PW	CCQ	JGSW	JSP	SWG	SWHC	IJSW
CW	1	1	1	1	1	1	1	1	1					1	1					
CYSR	1	1		1	1	1	1	1	1	1				1	1					
JSWE	1		1	1	1	1	1	1			1						1	1		
SSR	1	1	1	1	1	1	1	1		1							1	1		
SCW	1	1	1	1	1	1	1	1	1	1	1				1	1		1	1	
SWRA	1	1	1	1	1	1	1	1												
SW	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ASW			1	1	1	1	1	1						1						
CAN	1	1			1	1	1	1	1					1						
FR		1			1	1	1	1												
CSWJ				1	1	1	1	1												
AMH						1	1	1												
BJSW			1			1	1	1										1		
PW	1	1				1	1	1	1	1										
CCQ	1	1			1		1	1												
JGSW					1	1	1	1												
JSP				1																
SWG			1	1	1	1	1	1				1						1		
SWHC			1		1	1	1	1										1		
IJSW							1	1												

Figure 4: Optimal core-periphery solution for (binary) Baker citation data (dashed partition lines: peripheral dependency; solid partition lines: default Borgatti-Everett, and with both peripheral dependency and core dominance)

When also adding core dominance, the optimal solution (0.838) is the same as that for the default Borgatti-Everett metric, i.e. with ASW placed in the periphery. With ASW in the periphery, SWRA is provided with a peripheral journal to dominate. Although the multiple core ties of ASW deviates from the ideal patterns of peripheral dependency, keeping ASW in the core would yield a significant reduction in the correlation (0.771) due to the non-dominance of SWRA.

Substantively, it can be questioned whether dependency on singular core journals constitute a characteristic feature of peripheral journals. Although peripheral journals indeed might have preferences regarding the core journals it refers to, distinctions that are lost when dichotomizing the valued network, there are no theoretical grounds for a peripheral journal to *only* cite articles in one core journal. Inbound core dominance, however, is arguably a more relevant criterion in the journal citation context, i.e. where core journals are not only characterized by citing each other, but also being cited by peripheral journals. Running the correlation-based heuristic with the core dominance¹⁶ criteria, i.e. excluding peripheral dependency, the same 7-journal core as in Figure 4 is found, at the very high correlation of 0.946.

Valued citation data

This section analyzes the valued Baker citation network (Borgatti & Everett, 2000, p. 386; see also Nordlund, 2016, p. 167), using the identical dataset as used by Borgatti and Everett (see footnote 15).

¹⁶ As the data is symmetrized, both inbound and outbound core dominance is included. With non-symmetrized citation data, inbound core dominance is arguably a more relevant criterion for core journals than outbound core dominance (see also Doreian et al., 2005, p. 265ff)

Applying the default Borgatti-Everett metric, the optimal solution (0.815) corresponds to a core of SSR, SW and SCW – see *Figure 5* below.

Adding peripheral dependency, the core is supplemented with CSWJ and CW at the optimal correlation of 0.673 (dashed partition in *Figure 5* below). For this solution, the largest core tie of a peripheral journal represents on average 70 percent of its total core journal citations, ties that on average are 145 percent higher than their second-largest core journal ties. The peripheral dependency criteria for the valued citation data thus seem to capture a preference of peripheral journals to cite specific core journals. Although most peripheral journals in this partition have their strongest connection with the core journal SW, the peripheral journals CAN (Child Abuse and Neglect), CCQ (Child Care Quarterly), and CYSR (Children and Youth Services Review) have their most prominent ties with the core journal CW (Child Welfare).

	Cores																			
	+dependency					CAN	BJSW	AMH	ASW	FR	IJSW	JGSW	JSP	JSWE	PW	CCQ	CYSR	SWG	SWHC	SWRA
	+dep+dom																			
	default BE																			
	SW	SSR	SCW	CW	CSWJ															
SW		106	124	58	45	8	9	3	73	9	3	18		58	19		28	40	43	44
SSR	106		36	17	20				21				7	16			14	7		39
SCW	124	36		32	47	6			8	18		16		21		3	8	9	20	18
CW	58	17	32			9								11	7	12	70			8
CSWJ	45	20	47																	
CAN	8		6	9											7		12			
BJSW	19													13				2		
AMH	3																			
ASW	73	21	8											8	13					20
FR	9		18														4			
IJSW	3																			
JGSW	18		16																	
JSP		7																		
JSWE	58	16	21	11			13		18									9	7	24
PW	19			7		7			13								6			
CCQ			3	12													5			
CYSR	28	14	8	70		12				4					6	5				5
SWG	40	7	9				2							9					9	
SWHC	43		20											7				9		
SWRA	44	39	18	8					20					24			5			

Figure 5: Optimal solutions for valued citation data using default and extended criteria (shaded cells: correlated with unity for dependency criteria; bold/underline values: correlated with unity for dominance criteria)

Adding core dominance, the optimal solution (0.863) consists of the 4-journal core of SSR, SW, SCW and CW – see dash-line partition in *Figure 5* above. Due to its lack of ties with peripheral journals in this partition, CSWJ is placed in the periphery.

Contrary to what was the case for the binary citation data, using core dominance as the only power-relational criteria works less well for the valued citation data. Doing this results in an optimal solution with only SSR and SCW in the core: although this solution has a very high correlation (0.954), this is primarily due to the peripheral placement of SW where its very large ties with SSR and SCW are each correlated with unity 18 times for the core dominance criteria.

For the binary and valued Baker citation examples above, the suggested extensions to the Borgatti-Everett metric seem to capture power-relational core-periphery models. The suitability of different criteria and combinations thereof do however seem to differ between the binary and valued versions of the data. Whereas different specializations among journals within a particular discipline could imply that peripheral journals more often cite particular core journals, the loss of such distinctions in the binary data makes peripheral dependency a less useful criterion. The core dominance criterion does however work well in the binary citation data. The opposite seems to be the case for the valued citation data. Whereas the criteria for peripheral dependency captures the patterns of peripheral preference for specific core journals, using the criteria for core dominance *without* peripheral dependency results in an arguably non-intuitive partition, where the seemingly most corelike journal SW is surprisingly placed in the periphery.

International trade: EU/EFTA, 2010

This example captures the total commodity trade (in billion USD) between 30 countries within EU and EFTA in 2010 (Nordlund, 2016, pp. 172, 177). Using the default Borgatti-Everett heuristic, the optimal solution (0.867) results in a core consisting of Germany, France, the Netherlands, Great Britain and Belgium.

Adding in- and outbound peripheral dependency, Italy and Spain join the “default” core in the optimal solution at a correlation of 0.686. With both dependency and dominance, in- as well as outbound, the optimal coefficient (0.850) is obtained for a core consisting of Germany, France, and the Netherlands. All three solutions are indicated in *Figure 6* below.

	Cores																													
	+dependency																													
	default BE																													
	+dep+dom																													
	DEU	FRA	NLD	GBR	BEL	ITA	ESP	CHE	AUT	POL	SWE	CZE	NOR	IRL	HUN	DNK	PRT	SVK	FIN	ROM	GRC	SVN	LUX	BGR	LTU	EST	LVA	CYP	MLT	ISL
Germany (DEU)		103	78	71	63	78	37	56	59	38	27	32	10	5	21	17	10	10	9	10	7	4	5	3	3	1	1	1	0	0
France (FRA)	82		19	36	43	43	34	15	4	8	7	4	3	2	3	3	5	2	3	4	3	1	3	1	1	0	0	0	0	0
Netherlands (NLD)	91	25		41	73	26	14	8	4	6	9	4	3	3	4	6	4	1	4	2	3	1	1	1	1	0	0	0	0	0
Great Britain (GBR)	51	26	29		22	13	14	7	2	5	8	3	5	19	2	5	3	1	2	1	2	0	0	0	0	0	0	1	0	0
Belgium (BEL)	45	47	42	26		18	8	5	2	4	6	2	1	1	2	3	2	1	2	1	2	0	5	0	1	0	0	0	0	0
Italy (ITA)	58	45	9	22	12		22	18	10	10	4	5	2	1	4	3	4	2	2	7	6	4	1	2	1	0	0	1	1	0
Spain (ESP)	29	37	9	15	8	22		5	2	3	2	2	1	1	1	1	24	1	1	1	2	1	0	0	0	0	0	0	0	0
Switzerland (CHE)	44	15	3	9	4	14	4		8	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0
Austria (AUT)	45	6	2	4	2	11	2	8		3	2	4	1	0	5	1	0	2	1	3	1	2	0	1	0	0	0	0	0	0
Poland (POL)	38	9	6	9	4	10	4	1	3		4	8	2	0	5	2	0	3	1	2	0	1	0	1	2	1	1	0	0	0
Sweden (SWE)	18	7	7	10	7	5	3	1	2	3		1	11	0	1	11	1	0	7	0	0	0	0	0	1	1	0	0	0	0
Czech rep (CZE)	39	7	6	6	3	6	3	2	6	6	2		1	0	3	1	0	7	1	1	0	1	0	0	0	0	0	0	0	0
Norway (NOR)	23	6	11	30	5	2	2	0	1	3	13	1		2	0	3	1	0	2	0	0	0	0	0	0	0	0	0	0	0
Ireland (IRL)	19	8	5	20	20	4	4	6	1	1	2	1	1		0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0
Hungary (HUN)	22	4	3	5	1	5	2	1	4	3	1	3	0	0		1	0	3	0	5	0	1	0	1	0	0	0	0	0	0
Denmark (DNK)	15	4	4	6	1	3	2	1	1	2	12	1	5	1	1		0	0	2	0	1	0	0	0	0	0	0	0	0	0
Portugal (PRT)	6	6	2	3	1	2	11	0	1	0	1	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0
Slovakia (SVK)	12	4	2	2	1	3	2	0	3	4	1	7	0	0	4	0	0		0	1	0	0	0	0	0	0	0	0	0	0
Finland (FIN)	8	3	4	3	2	2	1	1	1	2	8	0	2	0	0	1	0	0		0	0	0	0	0	0	2	1	0	0	0
Romania (ROM)	9	4	1	2	1	6	1	0	1	1	0	1	0	0	2	0	0	0	0		1	0	0	2	0	0	0	0	0	0
Greece (GRC)	3	1	1	1	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	1		0	0	2	0	0	0	2	0	0
Slovenia (SVN)	5	2	0	1	0	3	0	0	2	1	0	1	0	0	1	0	0	0	0	0	0		0	0	0	0	0	0	0	0
Luxembourg (LUX)	4	2	1	1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0
Bulgaria (BGR)	2	1	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0		0	0	0	0	0	0
Lithuania (LTU)	2	1	1	1	0	0	0	0	0	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0		1	2	0	0	0
Estonia (EST)	1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	1		1	0	0	0
Latvia (LVA)	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1		0	0	0
Cyprus (CYP)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0		0	0
Malta (MLT)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
Iceland (ISL)	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 6: Optimal solutions for EU/EFTA trade data using default and extended criteria (shaded cells: correlated with unity for dependency criteria; bold/underline values: correlated with unity for dominance criteria)

For the majority of peripheral countries, Germany constitute the premier partner, both with respect to their inbound and outbound flows. Germany constitute the largest source for Norwegian imports, but the largest Norwegian export flow goes to Great Britain. For the obtained solution when only dependency is included, Portugal is tightly knit to Spain, Malta primary imports from the core are Italian while its exports to the core primarily goes to France, and Ireland obtains most of its imports from Great Britain. Looking at the distribution of peripheral exports to the 7 core countries in this solution (dashed line in Figure 6), the largest export flows from a periphery to the core countries represents on average 39 percent of all exports to the core. However, the second largest core export flows from the peripheries constitute on average 21 percent of total core exports, a lower degree of peripheral dependency that is captured by the somewhat low correlation coefficient for outbound peripheral dependency (i.e. 0.677).

A more obvious reason for the relatively low correlation for peripheral dependency for this dataset is, of course, the skewed valued degree distribution of the actors in the dataset. Although the largest import flow to Iceland is lower than the smallest import flow to Germany, neither the default nor the extended Borgatti-Everett correlation-based metric takes such unequal relational capacities into account in their correlations. While a suitable transformation of the valued data could alleviate such

differences prior to the identification of core-periphery structures, with or without power-relational criteria, such methodological endeavors are here left for future studies.

Conclusion

“Conceptions”, Wallerstein argues (1974, p. 36), “precede and govern measurements”. Exploring the history of the core-periphery concept and, particularly, its topological interpretations that followed in the post-war period, this paper found historical precedents for how core-periphery structures in contemporary network science is perceived, i.e. in terms of dense cores and sparse peripheries. However, this paper also found significant support for the power-relational notions of peripheral dependency and core dominance as integral aspects of the classical core-periphery concept.

Building on the literature overview findings, this paper proposed how peripheral dependency and core dominance can be operationalized as extensions to the correlation-based metric of Borgatti and Everett (2000). Testing the extended metrics on a handful of example datasets, comparing how obtained partitions differ from those obtained when excluding power-relational patterns between core and peripheral actors, the proposed criteria and their operationalizations seem apt at capturing dependency and dominance. For the binary and valued Baker citation networks as well as the intra-European trade data, the criteria for dependency and dominance integrate well with the intra-categorical density differential criteria. However, when only including the core dominance criterion, the obtained core-periphery partitions for the valued citation and trade networks seem non-intuitive. This could indicate that the herein suggested operationalization of core dominance is problematic, particularly with respect to valued networks.

The conventional network-scientific perception of what constitute a core-periphery structure is indeed a well-established concept that has proven, and continues to prove, its usefulness in a wide variety of network studies. However, for identifying peripheral dependency and core dominance, power-relational features that are intrinsically tied to the original core-periphery concept, this paper proposes alternative, power-relational core-periphery models that capture such patterns of core-periphery relations. Similar to how the original core-periphery concept disseminated across various disciplines, the applicability and theoretical significance of power-relational core-periphery models are not necessarily constrained within the context of international relations and political economy, but could equally have theoretical significance and applicability in vastly different types of networks and fields of study.

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