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Meta-Analysis of Public Transport Demand

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1. Introduction

The study of public transport demand has over the years resulted in a large amount of published and unpublished reports, papers and articles. For excellent reviews see Webster and Bly (1980), Goodwin (1992), Oum et al (1992) and Balcombe et al (2004). Despite all this knowledge, new studies are still performed in this area. The new studies often use the same methods that have been used for a long time, and apply them to new (perhaps better) data. This is of course interesting and necessary; preferences might vary between countries and over time. Nevertheless, the large number of previous studies hopefully has something to say about the demand elasticities in regions and cities not yet investigated.

Another reason for doing new studies is the wide variation in the estimated elasticities. This contributes to uncertainty as to whether or not the “true” relationship between travel demand and different variables has been found. Despite this variation a price-elasticity of –0.3 is often mentioned as a rule of thumb (Nelson 1972, Goodwin, 1992, Oum et al, 1992, Bresson et al 2003). This result originates from calculations of means using several studies. Such calculations do not take the different conditions under which the estimates have been obtained into consideration. Equivalent rules of thumb have not been produced for other demand elasticities. This is strange since price is not the only important policy variable.

The resulting estimates in different studies vary not only between regions and over time. Results differ depending on the models used and the quality of the data. It should hopefully be possible to learn something from the large number of previous studies, which could be used to guide the choice of model and estimation method in new studies.

The purpose of this paper is to present summaries of elasticities found in previous studies, and to account for the large variation between the estimates. Relevant summary measures are
of great importance for policy makers while a good explanation of the variation will contribute to future demand modelling.

A systematic approach to investigation of existing public transport study results is provided by meta-analysis. Nijkamp and Pepping (1998) and Kremers et al (2002) use meta-analysis to investigate the differences in price-elasticities estimated in studies both of public transport and of transport in general. The former study investigated own-price elasticities of public transport estimated in Europe. The analysis was performed through rough set analysis (see van den Bergh et al. (1997) for more on rough set analysis). They included disaggregated choice models as well as direct demand models estimated using aggregated data. Not surprisingly, they found that the elasticities estimated using individual choice-models were different from those obtained using aggregated demand models. Nijkamp and Pepping (1998) also concluded that the results obtained were region-specific.

The latter meta-analysis used a larger amount of studies made in many countries. They also included both individual choice-models and models using aggregated data. In their material, studies of many different modes of transport (both passenger and freight) were included. Kramers et al. (2002) used a general equilibrium framework of transportation containing the different modes of transport and models.

An important finding was that microeconomic models produced estimates different from micro-econometric, and mode choice-models.² Estimation of microeconomic models resulted in higher price-elasticities. It was also concluded that models using only data generated in urban areas produced elasticities different from those using national aggregates.

Mode-choice models based on disaggregate (individual) data are often used for value of time studies (access time, waiting time, riding time). Wardman (1998, 2001) uses meta-

² They define microeconomic models as being derived through utility-maximising, while the relation of micro-econometric models to microeconomic theory is less clear. In mode choice-models a given number of trips are allocated between different modes of transport.
analysis to summarise and explain the variation in travel time and service quality valuations. The findings are interesting, and useful also for understanding public transport demand. The format of mode-choice models is, however, different in kind and will not be included in the present study.

The present study differs from Nijkamp and Pepping (1998) and Kremers et al. (2002) in some important ways; (1) demand elasticities with respect to price, vehicle-kilometres, income, car ownership and price of petrol is analysed whereas the previous studies concerns price elasticities only. (2) In this study the effect on the estimates of treating potentially endogenous variables as exogenous is investigated. (3) This meta-analysis is based on more estimates of direct demand models for of public transport then both Nijkamp and Pepping (1998) and Kremers et al (2002) which strengthens the results regarding such models. (4) Only direct demand models of bus patronage are included, whereas the two aforementioned studies include other modes of passenger transport, as well as freight transport. When the results from different studies are compared and perhaps combined it is important that they concern the same thing (Rosenthal, 1991). Although there certainly are reasons for assuming that there is a common component in the demand for different modes of transport the risk of obtaining irrelevant results due to comparing apples and oranges is reduced when only one mode of transport is investigated. (5) The present study focus is on local transport demand; intercity travel is not included but some of the studies include both local and regional travel.

In this meta-analysis, five different demand-elasticities are examined by separate models where different factors such as model specification as well as type and origin of data are used as explanatory variables. These are elasticities with respect to price, vehicle-kilometres, income, the price of petrol, and the rate of car ownership.

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3 In some studies a small amount of rail travel is included. This is however a very small part of the total amount of trips made in those cases.
In the next section the concept of meta-analysis will be discussed, thereafter the empirical data in the form of elasticities estimated in different studies will be described. Section four introduces the variables used to explain the variation in elasticities while the empirical results are presented in section five. The conclusions are divided in two parts, in section six the expected elasticities are summarised while section seven contains recommendations concerning future demand modelling.

2. General discussion of meta-analysis

Meta-analysis can be defined as the study of studies. It refers to the statistical analysis of a large collection of results from individual studies for the purpose of integrating the findings (Glass (1976). In meta-analysis results from several studies are combined in order to draw some general conclusion (for several examples of definitions of meta-analysis see van den Bergh et al, 1997).

Initially meta-analysis was applied to studies performed in medicine and psychology, the use thereafter spread to other natural sciences. Meta-studies in other social sciences than psychology have become more and more common since the seventies. In economics this development has been slower but it has gained pace since the nineties. (van den Bergh et al, 1997)

The questions asked in meta-analysis vary, van den Bergh et al (1997) lists seven alternative insights sought in a meta-study:

1) Summarising over a collection of similar studies, relationships, indicators and so on.
2) Averaging, possibly using weights, for collections of values obtained in similar studies.
3) Comparing, evaluating and ranking studies on the basis of well-defined criteria or goal functions.

4) Aggregating studies, by taking complementary results or perspectives.

5) Apprehending common elements in different studies.

6) Comparing outcomes of different methods applied to similar questions.

7) Tracing factors that are responsible for differing results across similar studies.

The methods used to answer these questions vary and a single study usually focuses on one or two of them. For textbooks on meta-analytical methods see Hunter et al (1982), Hedges and Olkin (1985), Rosenthal (1991) and van den Bergh et al (1997).

The approach originates from the need to systematize results that differ in magnitude and sometimes in direction. The problem of different studies resulting in different answers is particularly problematic for decision-makers actually trying to use existing research as a basis for decisions. It is even suggested that additional studies performed within a specific area only contribution is to add to the confusion regarding the state of the world. (Light and Pillemer, 1984, van den Bergh et al, 1997) Another aspect of the same phenomenon is that a decision-maker picks a study presenting results corresponding to his or her views and uses it as an argument in the discussion. In most cases in the social sciences it is possible for the opponents to find another single study with an apposing view. In this case there is obviously a need for methods to combine results.

The other area where meta-analysis is useful is when new research is conducted. A common disposition of a paper is to first present previous results discussing pros and cons in using different approaches. Thereafter the results from ones own study is presented. Ideally the review makes it possible to use prior knowledge and therefore improve the method used in the new study. Worst-case scenario is that the new study just adds to the confusion by
presenting new results using in principle the same methods as before on a new but similar material. The problem is of course to determine what is already known and which methods previously used should be used again and which should not. One approach to solving this problem is meta-analysis. It provides a systematic way to summarise previous knowledge.

A general model for a meta-study could be stated as (van den Bergh 1997):

\[ Y = f(X, R, T, L) + e \]  

where \( Y \) is the result under investigation (for example, the effect of a drug, or the effect of price changes on travel demand). The vector \( X \) represents explanatory variables affecting the dependent variable \( Y \); in the case of price elasticities, \( X \) would stand for level demand and price respectively. \( R \) represents characteristics of the research method used when obtaining \( Y \), \( T \) the time-period in which the studies were made and \( L \) the geographical area in which the studies were made.

3. The object of the present study: different public transport demand-elasticities

To facilitate interpretation and comparison the effect of a variable \( x_i \) on demand \( (Q) \) is often described in terms of elasticities \( (E) \) which are defined as:

\[ E = \frac{\Delta Q / Q}{\Delta x_i / x_i} \quad \text{or} \quad E = \frac{\Delta Q}{\Delta x_i} \frac{x_i}{Q} \]

In practice, several measures of elasticity exists in the literature but the most common approach at least in studies presented in academic journals is to observe the change in demand
when the change in the $x$ variable is vanishingly small. This measure is called the point elasticity due to the fact that it refers to a specific point on the demand function. The point elasticity is therefore:

$$E^{Pi} = \lim_{\Delta x \to 0} \frac{\Delta Q / Q}{\Delta x / x} = \frac{\partial Q}{\partial x} \frac{x}{Q}$$

The elasticity measure is therefore interpreted as the percentage change in demand due to a one percent change in the $x$ variable. It is important to remember that this elasticity differs depending on where the point in question is located on the demand function.

Most of the demand-elasticities referred to in this study are from articles published in academic journals and all are in the form of point elasticities. This means that they have been through the process of peer-review. Although this guarantees a certain level of quality, it also raises the question of “publication bias”. In meta-analysis there is often concern that only studies that have obtained significant and expected results (at least the right sign) are being published. It is possible that studies using sound methods and good data will not be reported, because the results are not as expected. If this is true the outcome of a meta-analysis using only published studies as input will be a reflection of this, and therefore biased. (Hunter et al. 1982, Light and Pillemer 1984, Rosenthal 1991) In most cases discussed in the literature the effect of one variable (the use of a drug for example) on a phenomenon (a person’s health) were studied. In economics however the effect of several variables are often considered simultaneously which might reduce the risk of (or at least the size of) publication bias. Studies where one, two or more estimates are non-significant or show the wrong (according to expectations) sign might still be published as long as the overall picture seems all right.

For other definitions of elasticity used see Webster and Bly (1980) or Balcombe et al (2004).
The number of estimates included\(^5\) (N), and the variations of the study results in each of the five cases are given in table 1, and are illustrated diagrammatically in figure 1–5 below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std. Error</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{price}}$</td>
<td>81</td>
<td>-0.38</td>
<td>0.23</td>
<td>-1.32</td>
<td>-0.009</td>
</tr>
<tr>
<td>$E_{\text{vkm}}$</td>
<td>58</td>
<td>0.72</td>
<td>0.37</td>
<td>0.075</td>
<td>1.88</td>
</tr>
<tr>
<td>$E_Y$</td>
<td>22</td>
<td>0.17</td>
<td>0.63</td>
<td>-0.82</td>
<td>1.18</td>
</tr>
<tr>
<td>$E_{\text{car}}$</td>
<td>8</td>
<td>-0.86</td>
<td>1.17</td>
<td>-0.37</td>
<td>0</td>
</tr>
<tr>
<td>$E_{\text{petrol}}$</td>
<td>17</td>
<td>0.38</td>
<td>0.31</td>
<td>0</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 1. Summary of the elasticity values from the studies included in the present meta-analysis

There are 81 estimated price-elasticities ranging from -0.009 to -1.32 with a mean value of -0.38. The mean value is in line with the often reported rule of thumb of -0.3 but the variation is obviously large as can be seen in figure 1.

The 58 obtained observations of elasticities with respect to vehicle-kilometres supplied range from 0.075 to 1.88 with a mean value of 0.72. As usual, this elasticity is higher (absolute value) than the price elasticity. This is in line with the results reported by Webster.

\(^5\) In some studies, more than one model is estimated which mean that there are more estimates than studies.
and Bly (1980), one of the few (perhaps the only one) summaries of elasticities of demand with respect to vehicle kilometres made. The distribution of the observations is shown in figure 2.

![Figure 2. The distribution of elasticities with respect to supply of vehicle-kilometres](image)

Income-elasticity of public transport demand is often debated. Economic theory suggests that demand for a specific good will rise with higher income unless the good in question is an inferior good. The 22 obtained income elasticities range from –0,82 to 1,18 with a mean of 0,17. This illustrates the problem, the variation is great and not even the expected sign is certain. The question of income-elasticity is crucial for the future of public transport. If patronage is declining with increases in income the situation is bound to get worse. Income will probably continue to rise and patronage will then continue to fall.

It is probable that much of the confusion originates from changes in modal choice. In general, a person with higher income travel more but people owning a car does not increase their demand for public transport when their income increase. Since the probability of owning a car increases with income this might cloud the effect of income on public transport demand. The variation is shown in figure 3.
17 estimates of elasticity of demand with respect to price of petrol were obtained. These range from 0 to 1.04 with a mean of 0.38. The variation is shown in figure 4.

Only eight estimates of elasticities with respect to car possession were obtained. They range from 0 to –3.37, the last one indicating that public transport demand is highly sensitive to the level of car possession. The mean is –0.86 also a fairly high elasticity. The variation and the small number of estimates make the effect of car possession on public transport demand an interesting area of future research. The variation is shown in figure 5.
4. Study characteristics used as explanatory variables

The variables thought to influence the estimated elasticities can be divided into three categories: type of model, data characteristics and environmental factors. (compare to Espey 1998 and Kremers et al. 2002) The first category concerns differences in model-specification such as functional form and variables included in the model. Data used in estimation can be of different origin. They could for example come from time-series or cross-sectional observations. Such differences are included in the second category. These correspond to the R variables in model (1) Environmental factors refer to both where and when the data is collected, that is T and L in model (1).

4.1. Functional form

A central issue in modelling demand is the choice of functional form for the relationship. The wrong choice might result in biased estimates (Green 2000). Therefore the functional form of the model used for obtaining a specific elasticity might have influenced the result.

The two most common direct demand models used are the log-linear and the linear model. These models are not derived from microeconomic foundations and one might therefore say that they strictly speaking are not economic models of travel demand. Kremers et al. (2002)
refer to such models as micro-econometric models as apposed to microeconomic models. In
the material used for this study 55 models were estimated using a log-linear model while 24
were estimated using a linear model. One possible explanation for the popularity of the log-
linear model is that the estimated parameters represent constant elasticities, which makes
interpretation straightforward. It is interesting to note that in most cases the choice of
functional form is not discussed; instead a specific functional form is assumed. One of the
exceptions is Wang and Skinner (1984) who uses Box-Cox transformations to determine if a
linear or a log-linear model should be used.

Frankena (1978) as well as Dargay and Hanly (2002) estimates models were all variables
except the price of a public transport journey enters the model in logarithms. The own-price is
included in levels so that the estimated price elasticity will not be constant. None of the
studies presents a motivation to why price-elasticity but not the other elasticities varies.
Nelson (1972) estimates a model where vehicle-kilometres affects demand through a non-
linear function while price enters the model linearly and the rest of the variables are in
logarithms. He on the other hand uses theoretical arguments (although not utility maximising
consumers) as motif for his choice of model.

Selvanathan and Selvanathan (1994) estimate complete demand systems for Australia and
the U.K. including public transport as one of the consumption categories. They estimate a
Rotterdam model, which is derived from microeconomic theory. It should be said that the
Rotterdam model through its functional form is bound to get income elasticities close to one
(Phlips, 1983).

On the basis of the discussion above variables that distinguishes between elasticities
obtained using a log-linear model, a linear model, a semi-logarithmic model and a complete
demand system will be included in the model used in this study.
4.2. Included variables

If a variable that affects public transport demand is excluded from the model the estimates obtained might be biased. This occurs if the excluded variable is correlated with one or more of the included variables (Greene 2000). If, for instance, car ownership is correlated with the supply of vehicle-kilometres or income the estimated effects of these variables on travel demand will be biased if car ownership is not included in the model.

Income is used as one of the explanatory variables in 22 of the models included in this study. This might seem like a small number considering that income is central to demand according to economic theory. One possible explanation to this is that income often is highly correlated with other variables, which might make the effect of income on travel demand hard to separate from other effects. This also contributes to the confusion regarding income-elasticity that exists. In this material the estimates range from −0.82 to 1.18 so it seems like not even the sign is known.

Car ownership seems to be significant in explaining number of trips made in most studies where it is included in the model. The availability of alternative modes of transportation is crucial to the probability of choosing public transport when making a trip. It is therefore likely that the estimated elasticities (income as well as others) are affected by the inclusion or exclusion of car possession as one of the explanatory variables. One exception is Nelson (1972) who includes car possession in the model but arrives at non-significant estimates.

Economic theory postulates that given the consumers preferences it is the relative price of a good together with disposable income that determines the demand for that good. Some measure of the price of other commodities might therefore be included in a demand model for public transport. In 17 of the models studied the price of petrol is included as a measure of the
price of using the car. Since the car is one of the main competitors to public transport the exclusion of such measure might affect the estimates.

It is also possible that changes in other prices could have an influence on demand for public transport. It is rare that people travel with public transport for the mere fun of it. They need to travel in order to consume other goods. Demand is therefore said to be a derived demand, which makes it likely that a rise in other prices than the price of public transport will result in less trips made. Public transport might therefore be regarded as a complement good to most other goods.

In studies where it is included, the supply of vehicle-kilometres is a key factor in explaining demand for public transport. 58 of the models studied include some measure of supply based on vehicle-kilometres. The inclusion of vehicle-kilometres as one of the explanatory variables is motivated by its connection to the time needed for a specific trip with public transport. In most cases an increase in vehicle-kilometres supplied implies that the time needed to make a trip has decreased which means that generalised cost of making a trip within that system has fallen. This is because the increase in supply of vehicle-kilometres often originates from an increased frequency or higher route density (Webster and Bly 1980).

Therefore the inclusion or exclusion of vehicle-kilometres or some other measure of time cost might influence the estimated elasticities.

In order to account for the variation among estimates due to exclusion of variables that might affect public transport demand dummies indicating whether or not the estimated models included income, car ownership, price of petrol and a supply-measure among the explanatory variables will be included in the model of this study.

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6 Sohn (1975) uses an index measuring the total cost of private car use. This study is treated in the same way as those including price of petrol.
If one or more of the variables in the demand model are treated as exogenous when they are endogenous the estimated results will be biased (Greene 2000). It is likely that the number of trips made influence the supply of vehicle-kilometres and that the later therefore should be treated endogenously. For example, Holmgren (2005) showed that there is a two-way relationship between number of trips made and vehicle-kilometres supplied in Swedish counties. Skinner (1984) argues that the use of monthly data instead of yearly data reduces the problem of simultaneity. However, Nelson (1972), Veatch (1973) Garbade and Soss (1976), Frankena (1978) and Fitzroy and Smith (1999) estimate models where number of trips and vehicle-kilometres are determined simultaneously. In the present study, a variable indicating if the model used treated vehicle-kilometres as exogenous or endogenous will therefore be included.

4.3. Type of data

The origin of the data might influence the result of the estimation and should therefore be represented in the present study. Most estimates included in this study originate from urban data although some were obtained using data from rural as well as urban areas. Elasticities might be different in urban and rural areas. If that is true elasticities estimated with a mixture of data will differ from those using only urban data. Therefore a variable indicating if the study in question used urban data or a mixture will be included in the present study.

The estimates might also differ between studies using yearly or monthly data. If Skinner (1984) is right and using monthly data results in less simultaneous bias such a difference will certainly be present. In the present study a variable showing if the model was estimated using monthly or yearly data will be included.

Veatch (1973) estimated a linear model where trips made, vehicle-kilometres provided and car possession where treated endogenously. Since he does not present elasticities or the levels of the data used his results will not be included in the quantitative part of this study.
Most of the studies use per capita data while others use data in levels. There should be no difference between them if population is included as one of the explanatory variables. However, except for Nelson (1972) population is not included in any of the models studied. A dummy indicating that data in levels were used will therefore be included in this study.

### 4.4. Environmental factors

It is also possible that elasticities differ between countries and regions. This might be due to differences in preferences or due to other factors. The level of the variables observed might vary substantially between countries, which will result in different elasticities of demand. Regional dummies will therefore be included in (1). In this study difference will be made between Europe and USA, Canada and Australia (in the future called Am/Aus). The city structure in Am/Aus is often adapted to a high level of car use. This might result in lower (absolute values) elasticities since a greater part of the passengers are captive riders. Unfortunately no studies of public transport demand in Africa, Asia or South America were found.

Preferences or the levels of the observed variables might have changed over time. This would cause elasticities to be different depending on when the observations were made. In this study the estimates will be divided into three groups depending on in which period most of the observations were made. Studies in the first group used data primarily generated 1950 to 1969, the second, 1970 to 1989 and the last, 1990 or later.

### 5. Estimation and results

Two of the most common reasons for performing a meta-study are to provide a compact summary of the known results, and to explain the variation in results with variations in the
research-method used. A linear model capable of doing this is provided by Stanley and Jarrell (1989), it is used in the present study and could be stated as:

\[ E^j = a^j + b^j X + c^j R + d^j T + g^j L + u^j \]  

(2)

Where E is a vector of elasticities of type j, a is the constant connected to elasticity j, b, c d, and g are parameter vectors for each type of explanatory variable. X, R, T and L are vectors of explanatory variables of different kind.

All explanatory variables are dummies and the constant a, should therefore be interpreted as the expected elasticity in a reference-model while the other parameters show the effect on the estimated relationship if a specific method is used. The reference model is a benchmark where all the dummies are zero. The choice of reference-model does not affect the results. The expected value of any other model could be calculated using the estimates of a and the other estimated parameters obtained from (2). In this study the reference-model is a log-linear static model where car possession, price of petrol, income and supply of vehicle kilometres are included among the explanatory variables. The reference-model is estimated with yearly data obtained in urban areas only and vehicle-kilometres is treated endogenous. The other estimated coefficients show deviations from the reference-model. The reference-model used is chosen to facilitate interpretation. It is made as broad as possible by including all explanatory variables considered in the study and treating vehicle-kilometres endogenous.

For examples of models used in meta-analysis see Espey (1998), Wardman (2001) and Kremers et al. (2002).

The complete set of variables thought to influence estimates (described above) and therefore tested is shown in Appendix 1.
The final models were obtained using stepwise regression (Neter et al. 1996). When possible due to degrees of freedom, backward selection was used in order to go from the most general model to a specific one. The results as to each of the aforementioned elasticities will be shown and discussed in turn.

### 5.1. Price elasticity

The model explaining price-elasticity is:

\[
E_{P_i} = \alpha_0 + \alpha_1 d_{LR,i} + \alpha_2 d_{Nsim,i} + \alpha_3 d_{Lin,i} + \alpha_4 d_{New,i} + \alpha_5 d_{Near,i} + \alpha_6 d_{Sys,i} + \\
\alpha_7 d_{M,i} + \alpha_8 d_{Cross,i} + \alpha_9 d_{Mix,i} + \alpha_{10} d_{Lev,i} + \varepsilon_i
\]

Table 2 below show the results from estimation of (3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.75***</td>
<td>0.098</td>
<td>-7.63</td>
</tr>
<tr>
<td>LR</td>
<td>-0.16***</td>
<td>0.043</td>
<td>-3.83</td>
</tr>
<tr>
<td>Nsim</td>
<td>0.29***</td>
<td>0.064</td>
<td>4.53</td>
</tr>
<tr>
<td>Lin</td>
<td>-0.24***</td>
<td>0.051</td>
<td>-4.66</td>
</tr>
<tr>
<td>New</td>
<td>0.16***</td>
<td>0.054</td>
<td>3</td>
</tr>
<tr>
<td>Ncar</td>
<td>0.26***</td>
<td>0.072</td>
<td>3.59</td>
</tr>
<tr>
<td>Sys</td>
<td>-0.23*</td>
<td>0.12</td>
<td>-1.84</td>
</tr>
<tr>
<td>M</td>
<td>-0.26***</td>
<td>0.053</td>
<td>-4.98</td>
</tr>
<tr>
<td>Cross</td>
<td>-0.31*</td>
<td>0.16</td>
<td>-1.99</td>
</tr>
<tr>
<td>Mix</td>
<td>-0.22***</td>
<td>0.067</td>
<td>-3.33</td>
</tr>
<tr>
<td>Lev</td>
<td>0.16**</td>
<td>0.068</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Table 2. Estimates of (3) where price elasticity is the dependent variable

The estimated price-elasticity for the reference-model is –0.75. It seems obvious that the elasticity of the reference-model (-0.75) differs from the stated rule of thumb of -0.3. A
formal test of this where $H_0: \alpha_0 = -0.3$ against $H_1: \alpha_0 \neq -0.3$ resulted in an F-value of 20.96. $H_0$ could therefore be rejected. The conclusion is therefore that the expected elasticity differs from -0.3. This is not strange since most models estimated differ from the reference model. A test to see if this is also the case in Am/Aus is stated $H_0: \alpha_0 + \alpha_4 = -0.3$ which is tested against $H_1: \alpha_0 + \alpha_4 \neq -0.3$. Again, $H_0$ is rejected (F-value=9.14).

If instead $H_0: \alpha_0 + \alpha_2 + \alpha_4 = -0.3$ is tested against $H_1: \alpha_0 + \alpha_2 + \alpha_4 \neq -0.3$ the F-value is 0.003. This indicates that the rule of thumb applies to results from models where vehicle-kilometres is treated exogenous but not when vehicle-kilometres is treated endogenous. This is true for Europe as well as Am/Aus.

The simultaneous models included in this study where estimated using 2SLS which gives unbiased (but inefficient) results even if vehicle-kilometres in reality is exogenous. Therefore the results points to the importance of treating the supply of vehicle-kilometres endogenously when modelling public transport demand.

It is also interesting to note that linear models seem to generate higher (absolute value) elasticities than log-linear models. This is an indication of the importance of model-specification.

Another interesting result is that exclusion of car ownership from the demand model results in a lower (absolute value) price-elasticity. This implies that exclusion of the variable results in a downward bias of the estimated elasticity.

The type of data used seems to influence the estimated elasticity. Monthly time-series data instead of yearly, data used in levels as well as data from both urban and rural areas instead of urban only are factors that according to (3) make a difference.

There is also a regional difference between the price-elasticities. As can be seen in table 1 above, price-sensitivity is smaller (ceteris paribus) in Am/Aus (USA, Canada, Australia) than
in Europe. The expected price-elasticity in these areas is –0.59. This might be due to different preferences or different levels of the variables observed.

The long-run price-elasticity is higher than the static or short run elasticity, which is no surprise. More interesting is the fact that the dummy for short-run elasticities does not enter (3). This means that there is no difference between the elasticities obtained using a static model and short-run elasticities.

5.2. Elasticity with respect to vehicle-kilometres

The model, found through stepwise regression, for elasticity with respect to vehicle-kilometres supplied is:

\[ E_{vkm,i} = \beta_0 + \beta_1 d_{LR,i} + \beta_2 d_{Cross,i} + \beta_3 d_{Mix,i} + \beta_4 d_{Lev,i} + \delta_i \]  

(4)

Table 3 below show the estimates from (4).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.05***</td>
<td>0.17</td>
<td>6.29</td>
</tr>
<tr>
<td>LR</td>
<td>0.33**</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td>Cross</td>
<td>0.48*</td>
<td>0.24</td>
<td>2.05</td>
</tr>
<tr>
<td>Adj-R2:</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td>-0.58**</td>
<td>0.19</td>
<td>-3.08</td>
</tr>
<tr>
<td>Lev</td>
<td>-0.4*</td>
<td>0.17</td>
<td>-2.33</td>
</tr>
</tbody>
</table>

Table 3. Estimates of (4). Elasticities with respect to vehicle-kilometres is the dependent variable

The estimated elasticity for the reference-model is 1.05 while the long-run elasticity is 1.38. This show that demand at least in the long run is elastic with respect to changes in vehicle-kilometres supplied which is important to have in mind when policy is discussed.
While it is no difference between estimates of $E_{vkm}$ obtained using a linear instead of a log-linear model, including both urban and rural areas have a significant effect on the estimated elasticity.

Cross-sectional data yields higher expected elasticity than time-series. Results from cross-sectional data is often interpreted as long run results which might be one explanation.

Treating vehicle-kilometres exogenously instead of endogenously do not affect the elasticity obtained. This suggests that estimates of $E_{vkm}$ are less sensitive to model-specification than $E_p$.

5.3. **Income elasticity**

The final model chosen when explaining $E_y$ is:

$$E_{y,i} = \gamma_0 + \gamma_1 d_{Sys,i} + \gamma_2 d_{Lev,i} + \gamma_3 d_{Npetoli} + \phi_i$$ (5)

Table 4 show the estimates of (5).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>T-stat</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.62***</td>
<td>0.15</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>Sys</td>
<td>1.09***</td>
<td>0.28</td>
<td>3.85</td>
<td></td>
</tr>
<tr>
<td>Lev</td>
<td>0.83***</td>
<td>0.16</td>
<td>5.15</td>
<td>Adj-R2: 0.68</td>
</tr>
<tr>
<td>NPetrol</td>
<td>0.42**</td>
<td>0.18</td>
<td>2.36</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Estimates from (5). Income-elasticities is the dependent variable

The estimated income-elasticity is –0.62. This indicates that the income-effect is indeed negative and public transport therefore is an inferior good. However, if one considers estimates derived from a complete demand system (derived from economic theory) instead of those from single equation models the picture is different. The point estimate is then 0.47.
Testing $H_0: \gamma_0 + \gamma_1 = 0$ against $H_1: \gamma_0 + \gamma_1 \neq 0$ results in an F-value of 2.38. Hence, $H_0$ could not be rejected. In the system based models the expected income parameter is positive but not significantly different from zero. The same is true if variables are entered in levels or the price of gasoline is excluded from the model (F-values of 1.46 and 2.27 respectively).

Estimates of $E_x$ seem to be highly sensitive to the specification of the model, which accounts for the confusion around its sign and size. Further investigation of how income affect public transport patronage is called for.

### 5.4. Elasticity with respect to price of petrol

The final model explaining elasticities of demand with respect to price of petrol is:

$$E_{Gas} = \lambda_0 + \lambda_1 d_{LR,i} + \lambda_2 d_{New,i} + \lambda_3 d_{Lev,i} + \lambda_4 d_{Nsup,i} + \eta_i$$

(6)

Estimates from (6) are shown below in table 5.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.4***</td>
<td>0.11</td>
<td>3.7</td>
</tr>
<tr>
<td>LR</td>
<td>0.33*</td>
<td>0.18</td>
<td>1.8</td>
</tr>
<tr>
<td>New</td>
<td>0.42***</td>
<td>0.14</td>
<td>3.06</td>
</tr>
<tr>
<td>Adj-R2:</td>
<td>0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lev</td>
<td>-0.62***</td>
<td>0.13</td>
<td>-4.82</td>
</tr>
<tr>
<td>Nsup</td>
<td>0.16</td>
<td>0.11</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Table 5. Results from estimation of (6)

The expected elasticity of demand with respect to price of petrol is 0.4 in the reference model. Estimates obtained using data from Am/Aus are significantly higher than those from the reference model (obtained in Europe). Hence, public transport patronage in Europe is less sensitive to changes in price of petrol than it is in Am/Aus.
Meta-analysis of public transport demand

Estimating a model where the variables are used in levels results in significantly lower elasticity of demand. Testing $H_0: \lambda_0 + \lambda_3 = 0$ against $H_1: \lambda_0 + \lambda_3 \neq 0$ results in an F-value of 2.14 which means that $H_0$ could not be rejected. Using levels seem to result in expected elasticity not significantly separated from zero. This again shows the importance of model specification.

5.5. Elasticity with respect to car ownership

The final model explaining elasticities of demand with respect to car ownership is:

$$E_{Car,i} = \chi_0 + \chi_1 d_{Lev,i} + \kappa_i$$

(7)

The results estimates of (7) are shown in table 6.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>T-stat</th>
<th>Adj-R2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.48**</td>
<td>0.52</td>
<td>-2.85</td>
<td>0.21</td>
</tr>
<tr>
<td>Lev</td>
<td>1.25</td>
<td>0.74</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Estimates of (7). Elasticity with respect to car ownership is the dependent variable

Table 5 shows that the expected elasticity with respect to car possession is $-1.48$. Patronage seems to be highly affected by car possession, which is no surprise. The results seem to be unaffected by the choice of model or origin of data tested for. One should be aware of the small amount of estimates included in the study (eight), which makes the results somewhat unreliable. It is possible that the results would be different if more estimates were included in the study. There obviously is a need for more knowledge in this area.
6. Summary of expected elasticities

Table 7 show a summary of the expected value of different elasticities.

<table>
<thead>
<tr>
<th>Elasticity with respect to</th>
<th>SR Europe</th>
<th>LR Europe</th>
<th>SR Am/Aus</th>
<th>LR Am/Aus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own Price</td>
<td>-0.75</td>
<td>-0.91</td>
<td>-0.59</td>
<td>-0.75</td>
</tr>
<tr>
<td>95% CI</td>
<td>(-0.55 to -0.95)</td>
<td>(-0.71 to -1.11)</td>
<td>(-0.4 to -0.78)</td>
<td>(-0.55 to -0.95)</td>
</tr>
<tr>
<td>Vehicle-Kilometres</td>
<td>1.05</td>
<td>1.38</td>
<td>1.05</td>
<td>1.38</td>
</tr>
<tr>
<td>95% CI</td>
<td>(0.71 to 1.39)</td>
<td>(1.01 to 1.75)</td>
<td>(0.71 to 1.39)</td>
<td>(1.01 to 1.75)</td>
</tr>
<tr>
<td>Income</td>
<td>-0.62</td>
<td>-0.62</td>
<td>-0.62</td>
<td>-0.62</td>
</tr>
<tr>
<td>95% CI</td>
<td>(-0.3 to -0.94)</td>
<td>(-0.3 to -0.94)</td>
<td>(-0.3 to -0.94)</td>
<td>(-0.3 to -0.94)</td>
</tr>
<tr>
<td>Price of Petrol</td>
<td>0.4</td>
<td>0.73</td>
<td>0.82</td>
<td>1.15</td>
</tr>
<tr>
<td>95% CI</td>
<td>(0.16 to 0.64)</td>
<td>(0.38 to 1.08)</td>
<td>(0.56 to 1.08)</td>
<td>(0.65 to 1.65)</td>
</tr>
<tr>
<td>Car Ownership</td>
<td>-1.48</td>
<td>-1.48</td>
<td>-1.48</td>
<td>-1.48</td>
</tr>
<tr>
<td>95% CI</td>
<td>(-0.21 to -2.75)</td>
<td>(-0.21 to -2.75)</td>
<td>(-0.21 to -2.75)</td>
<td>(-0.21 to -2.75)</td>
</tr>
</tbody>
</table>

Table 7. Summary of the elasticities from the reference model

The expected short-run (static) own-price elasticity is -0.75 in Europe and -0.59 in Am/Aus. Formal testing shows that they both are significantly different from the often stated rule of thumb. Interestingly the expected value of the price-elasticity obtained using a model where vehicle-kilometres is treated exogenous is not significantly different from -0.3 (the famous rule of thumb). That is to say that the rule of thumb originates from models treating vehicle-kilometres exogenous. The long-run price-elasticities are somewhat higher as could be expected, -0.91 and -0.75 in Europe and Am/Aus respectively.

The expected value of the short-run (static) elasticity with respect to vehicle-kilometres is 1.05 and the long run elasticity is 1.38. This elasticity is higher (absolute values) than the price elasticity which is in line with previous findings. This suggests that with the present levels of fare and service, demand is more sensitive to changes in the latter.
When it comes to income elasticity the picture is still unclear. The expected value is -0.62. If this is true the future for public transport seems dark indeed. Continuing increases in income will then cause patronage to keep decreasing. Fortunately the expected value of income elasticity obtained from complete demand systems derived from economic theory is 0.47. A formal test shows that this elasticity is not significantly different from zero. Hence, the question of income elasticity needs more attention in the future.

Short-run (static) elasticity with respect to the price of gasoline is 0.4 in Europe and 0.82 in Am/Aus. The long run elasticities are 0.73 and 1.15 respectively.

The number of studies of the effect of car ownership on public transport demand is small (eight) and the results should therefore be interpreted carefully. The expected elasticity of demand with respect to car ownership is -1.48.

It is obvious that more research into the effects of car ownership and income on public transport demand is needed.

7. Lessons for future demand modelling

One should be careful when using the results from a meta-analysis to draw conclusions of what constitutes a good model. If all studies included in the meta-analysis suffer from specification error, it is not possible to obtain unbiased estimates through the meta-analysis. However, to some extent meta-analysis could be used to determine the characteristics of a good model.

Exclusion of a relevant explanatory variable will result in biased estimates if the excluded variable is correlated with the variables included in the model. Therefore if the exclusion of a specific variable result in significantly different estimates than if it is included one can draw

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8 No systematic differences between long- and short-run estimates or between regions are found. Therefore the expected elasticity with respect to income and car ownership is the same in all contexts.
the conclusion that it should be included in a correct model. If, for instance, models where the level of car ownership is left out give different results from those where it is included and it is correlated with included variables (possibly income) the conclusion is that it should be included in the correct model. If there is no significant difference between including and excluding a variable it is still possible that that variable has explanatory value and should be included. A similar case could be made for using the variables in per capita form instead of levels. If there is a difference between them per capita should be used unless population is explicitly included among the explanatory variables. (Greene 2000)

The question of treating vehicle-kilometres endogenous is harder but indications of an answer could be found under specific circumstances. If single equation methods (like two-stages least squares) are used for estimating the simultaneous models and there is a significant difference between models treating vehicle-kilometres endogenous and those who do not, it indicates that it should be treated endogenous. This conclusion is due to the fact that single equation methods provide consistent (but inefficient) estimates if a variable is treated endogenously even though it is exogenous. Hence, if there is a difference it is probably due to the fact that treating a variable as exogenous when it is endogenous leads to bias. If instead system methods (like three-stages least squares) are used for estimation the simultaneous models, no conclusion could be drawn even if there is a difference between them and those treating vehicle-kilometres exogenously. This is due to the fact that system methods are sensitive to specification error not only in the demand equation. A difference could therefore be due to the fact that the variable should be treated endogenously or misspecification. (Greene 2000)

When it comes to functional form of the model, meta-analysis could not provide any guidance towards the correct model. However, if the results differ between models with different functional form it shows that the choice is important. As an economist one is
inclined to advocate models consistent with economic theory. If other models are used the reasons for it should be clearly stated.

The results show that a good model includes car ownership, price of petrol, some measure of public transport supply as well as own price and income. It also seems like vehicle-kilometres should be treated endogenously. It is most interesting to see that the rule of thumb of a price elasticity of -0.3 only seems to hold if vehicle kilometres is treated as an exogenous variable.

One could also conclude that the variables should be in per capita form if population is not included among the explanatory variables. It is also clear that the choice of functional form affects the estimates which show the importance of model selection.

To reach reliable results, future models of public transport demand should be formulated with the above presented results in mind. Some of these results might seem obvious but the point is that many studies are still made that do not follow these guidelines and this affects the results which are used in policy recommendations.

References


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Nelson, Gary (1972), *An econometric model of urban bus transit operations*, PhD-thesis, Rice University


Wardman Mark (2001), A review of British evidence on time and service quality valuations, *Transportation Research Part E*, 37, pp. 107-128

## Appendix 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{SR})</td>
<td>1 if the dependent variable is a short run estimate. 0 otherwise</td>
</tr>
<tr>
<td>(d_{LR})</td>
<td>1 if the dependent variable is a long run estimate. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Nsim})</td>
<td>1 if all variables are treated as exogenous. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Lin})</td>
<td>1 if the estimate is obtained using a linear model. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Sys})</td>
<td>1 if the estimate is obtained from a complete demand system. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Sem})</td>
<td>1 if the dependent variable is obtained using a semi-logarithmic model. 0 otherwise</td>
</tr>
<tr>
<td>(d_{M})</td>
<td>1 if the model is estimated using time-series with monthly observations. 0 otherwise</td>
</tr>
<tr>
<td>(d_{cross})</td>
<td>1 if the model is estimated using cross-section data. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Pan})</td>
<td>1 if the model is estimated using panel-data. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Mix})</td>
<td>1 if the estimate is obtained using data from both urban and rural areas. 0 otherwise</td>
</tr>
<tr>
<td>(d_{New})</td>
<td>1 if the estimate is obtained using data from Canada, USA and/or Australia. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Lev})</td>
<td>1 if the model is estimated using the variables in levels. 0 if they are used in per capita form</td>
</tr>
<tr>
<td>(d_{Nsup})</td>
<td>1 if the model has no supply-variable among the explanatory variables. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Ncar})</td>
<td>1 if the estimated model does not include car ownership among the explanatory variables. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Ninc})</td>
<td>1 if income is not among the explanatory variables. 0 otherwise</td>
</tr>
<tr>
<td>(d_{Npet})</td>
<td>1 if the price of petrol is not included among the explanatory variables. 0 otherwise</td>
</tr>
<tr>
<td>(d_{P1})</td>
<td>1 if the main part of the data used in estimation is from before 1970. 0 otherwise</td>
</tr>
<tr>
<td>(d_{P2})</td>
<td>1 if the main part of the data used in estimation is from the period 1970-1989. 0 otherwise</td>
</tr>
</tbody>
</table>

Table 8. Explanatory variables tested when explaining the different elasticities.