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Transmit Precoding and Receive Power Splitting for Harvested Power Maximization in MIMO SWIPT Systems

Deepak Mishra, *Member, IEEE* and George C. Alexandropoulos, *Senior Member, IEEE*

Abstract—We consider the problem of maximizing the harvested power in Multiple Input Multiple Output (MIMO) Simultaneous Wireless Information and Power Transfer (SWIPT) systems with power splitting reception. Different from recently proposed designs, with our optimization problem formulation we target for the jointly optimal transmit precoding and receive uniform power splitting (UPS) ratio maximizing the harvested power, while ensuring that the quality-of-service requirement of the MIMO link is satisfied. We assume practical Radio-Frequency (RF) energy harvesting (EH) receive operation that results in a non-convex optimization problem for the design parameters, which we first formulate in an equivalent generalized convex problem that we then solve optimally. We also derive the globally optimal transmit precoding design for ideal reception. Furthermore, we present analytical bounds for the key variables of both considered problems along with tight high signal-to-noise ratio approximations for their optimal solutions. Two algorithms for the efficient computation of the globally optimal designs are outlined. The first requires solving a small number of non-linear equations, while the second is based on a two-dimensional search having linear complexity. Computer simulation results are presented validating the proposed analysis, providing key insights on various system parameters, and investigating the achievable EH gains over benchmark schemes.

Index Terms—RF energy harvesting, MIMO, precoding, power splitting, rate-energy trade off, SWIPT, nonconvex optimization.

I. INTRODUCTION

There has been recently increasing interest [2]–[4] in utilizing Radio Frequency (RF) signals for transferring simultaneously energy and data, also known as Simultaneous Wireless Information and Power Transfer (SWIPT). This technology can play a major role in the practical ubiquitous deployment of low power wireless devices in fifth generation (5G) wireless networks and beyond [4]–[7]. Particularly, it can be one of the promising candidates for enabling the perpetual operation of small cells, Internet-of-Things (IoT) [2], Machine-to-Machine (M2M) communications and cognitive radio networks [5]–[7].

Despite these merits, SWIPT suffers from some fundamental bottlenecks. First and foremost, the signal processing

and resource allocation strategies for wireless information and energy transfer differ significantly for achieving their respective goals [8], [9]. In fact, there exists a non-trivial trade off between information and energy transfer that necessitates thorough investigation for optimizing the SWIPT performance. In addition, this performance is impacted by the low energy sensitivity and RF-to-Direct Current (DC) rectification efficiency [3]. Another practical concern is that the existing RF EH circuits cannot decode the information directly and vice-versa [10], [11]. Lastly, the available solutions [12], [13] for realizing practical SWIPT gains require high complexity and are still far from providing analytical insights. To confront with these bottlenecks, Multiple-Input-Multiple-Output (MIMO) technology and resource allocation schemes as well as cooperative relaying strategies have been recently considered [3], [10]–[22]. In this paper, we are interested in optimizing the efficacy of MIMO systems for efficient SWIPT.

A. State-of-the-Art

The non-trivial trade off between information capacity and average received power was firstly investigated in [8], [9] for a Single-Input-Single-Output (SISO) link. Then, the authors in [11] discussed why the SWIPT theoretical gains are difficult to realize in practice and proposed some practical Receiver (RX) architectures. Among them belong the Time Switching (TS), Power Splitting (PS), and Antenna Switching (AS) [14] architectures that use one portion of the received signal (in time, power, or space) for EH and another one for Information Decoding (ID). In [12], Transmitter (TX) precoding techniques for efficient MIMO SWIPT systems were presented. Recently, Spatial Switching (SS) was proposed [16] that first decomposes MIMO channel to its spatial eigenchannels and then assigns some for energy and some for information transfer [10].

The aforementioned SWIPT RX architectures have been lately considered in various MIMO systems [16]–[22]. For example, the transmit power minimization satisfying both energy and rate requirements was investigated in [16] for MIMO SWIPT with SS. In [17], a Semi-Definite Programming (SDP) relaxation technique for a multi-user multiple-input single-output system was used to study the joint TX precoding and PS optimization. A second-order cone programming relaxation solution for the latter problem with significantly reduced computational complexity than SDP was proposed in [18]. In [19] and [20], more general MIMO interference channels were investigated adopting the interference alignment

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technique. Authors in [21] considered a multi-antenna full duplex access point and a single-antenna full duplex user, and investigated the joint design of TX precoding and RX PS ratio for minimizing the weighted sum transmit power. However, these MIMO SWIPT works presented suboptimal iterative algorithms based on convex relaxation which that are unable to provide key insights on the joint optimal design.

B. Motivation and Key Contributions

A major goal of RF EH systems is the optimization of the end-to-end EH efficiency [2] by maximizing the rate-constrained harvested energy for a given TX power budget. This is in principle challenging with the available EH circuitry implementations, where the RF-to-DC rectification is a non-linear function of the received RF power [22]–[25]. This fact leads naturally to the necessity of optimizing the harvested power rather than the receiver power treated in the existing literature [12]–[21]; therein, constant RF-to-DC rectification efficiency has been assumed. In this paper, we study the problem of maximizing the harvested power in MIMO SWIPT systems with practical PS reception [12], while ensuring that the quality-of-service requirement of the MIMO link is met. We note that, although the PS architecture involves higher RX complexity, it is more efficient than TS since the received signal is used for both EH and ID. In addition, PS is more suitable for delay-constraint applications. We are interested in finding the jointly optimal TX precoding scheme and the RX Uniform PS (UPS) ratio for the considered optimization problem, and in gaining analytical insights on the interplay among various system parameters. To our best of knowledge, this joint optimization problem for maximizing the harvested DC power has not been considered in the past, and available designs for practical MIMO SWIPT are suboptimal. The key contributions of this paper are summarized below.

- We present an equivalent generalized convex formulation for the considered non-convex harvested power maximization problem that helps us in deriving the global jointly optimal TX precoding and RX UPS ratio design. We also present the globally optimal TX precoding design for ideal reception. For both designs there exists a rate requirement value determining whether the TX precoding operation is energy beamforming or information spatial multiplexing. This *novel* feature stems from our novel formulation involving rate constrained EH optimization and does not appear in available designs [12], [16]–[22].
- We investigate the trade off between the harvested power and achievable information rate for both globally optimal designs. Practically motivated asymptotic analysis for obtaining computationally efficient optimal solution in the high Signal-to-Noise-Ratio (SNR) regime is provided.
- We detail a computationally efficient algorithm for the global optimal design and present a low complexity alternative algorithm based on a two-dimensional (2-D) linear search. The complexity of the latter algorithm is *linear* in the number of MIMO spatial eigenchannels.
- We carry out a detailed numerical investigation of the presented joint optimal solutions to provide insights on

Fig. 1. Adopted MIMO SWIPT system model with UPS ρ reception.

the impact of key system parameters on the trade off between harvested power and achievable information rate.

The key challenges with our problem formulation addressed in this paper include its generalized convexity proof given the non-linear rectification property and the analytical exploration of non-trivial insights on its controlling variables, which helped us in designing a low complexity global optimization algorithm. Additionally, we would like to emphasize that our performance results are valid for any practical RF EH circuit model [22]–[24], and our key system design insights can be extended to investigate multiuser MIMO SWIPT systems.

Notations: Vectors and matrices are denoted by boldface lowercase and boldface capital letters, respectively. The transpose and Hermitian transpose of $\mathbf{A}$ are denoted by $\mathbf{A}^T$ and $\mathbf{A}^H$, respectively, and $\det(\mathbf{A})$ is the determinant of $\mathbf{A}$, while $\mathbf{I}_n$ ($n \geq 2$) is the $n \times n$ identity matrix and $\mathbf{0}_n$ ($n \geq 2$) is the n -element zero vector. The trace of $\mathbf{A}$ is denoted by $\text{tr}(\mathbf{A})$, $[\mathbf{A}]_{i,j}$ stands for $\mathbf{A}$'s (i,j) -th element, $\lambda_{\max}(\mathbf{A})$ represents the largest eigenvalue of $\mathbf{A}$, and $\text{diag}\{\cdot\}$ denotes a square diagonal matrix with $\mathbf{a}$'s elements in its main diagonal. $\mathbf{A} \succeq 0$ and $\mathbf{A} \succ 0$ mean that $\mathbf{A}$ is positive semi definite and positive definite, respectively. $\mathbb{C}$ represents the complex number set, $(x)^+ \triangleq \max\{0, x\}$, $\lceil x \rceil$ denotes the smallest integer larger than or equal to x , $\mathbb{E}\{\cdot\}$ denotes the expectation operator, and $\mathcal{O}(\cdot)$ is the Big O notation denoting order of complexity.

II. SYSTEM AND CHANNEL MODELS

We consider the MIMO SWIPT system of Fig. 1, where the TX is equipped with N_T antenna elements and wishes to simultaneously transmit information and energy to the RF-powered RX having N_R antenna elements. We assume a frequency flat MIMO fading channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ that remains constant during one transmission time slot and changes independently from one slot to the next. The channel is assumed to be perfectly known at both TX and RX. The entries of $\mathbf{H}$ are assumed to include independent, zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. So, the rank of $\mathbf{H}$ is $r = \min(N_R, N_T)$. The baseband received signal $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$ at RX is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^{N_T \times 1}$ denotes the transmitted signal with covariance matrix $\mathbf{S} \triangleq \mathbb{E}\{\mathbf{x}\mathbf{x}^H\}$ and $\mathbf{n} \in \mathbb{C}^{N_R \times 1}$ represents the AWGN vector having ZMCSCG entries each with variance σ^2 . The elements of $\mathbf{x}$ are assumed to be statistically independent,

(a) P1110 EVB characteristics [24]. (b) RF EH circuit designed in [26].

Fig. 2. Variation of harvested DC power and RF-to-DC efficiency with received RF power for practical two EH circuits [24], [26].

the same is assumed for the elements of $\mathbf{n}$. For the transmitted signal we finally assume that there exists an average power constraint across all TX antennas denoted by $\text{tr}(\mathbf{S}) \leq P_T$.

Capitalizing on the signal model in (1), the average received power P_R across all RX antennas can be obtained as $P_R \triangleq \mathbb{E}\{\mathbf{y}^H \mathbf{y}\}$. Note that the averaging is performed over the transmitted symbols during each coherent channel block. As the noise strength (generally lower than -80dBm) is much below than the received energy sensitivity of practical RF EH circuits (which is around -20dBm) [2], we next neglect the contribution of $\mathbf{n}$ to the harvested power. Note, however, that the analysis and optimization results of this paper can be easily extended for non-negligible noise power scenarios. We therefore rewrite P_R as the following function of $\mathbf{H}$ and $\mathbf{x}$

$$P_R \triangleq \mathbb{E}\{\mathbf{x}^H \mathbf{H}^H \mathbf{H} \mathbf{x}\} = \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H). \quad (2)$$

As demonstrated in Fig. 1, we consider UPS ratio $\rho \in [0, 1]$ at each RX antenna element. This ratio reveals that ρ fraction of the received signal power at each antenna is used for RF EH, while the remaining $1 - \rho$ fraction is used for ID. With this setting together with the previous noise assumption, the average total received power $P_{R,E}$ available for RF EH is given by $P_{R,E} \triangleq \rho P_R = \rho \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H)$. This definition for the average received power is the most widely used definition [12], [13] for investigating the performance lower bound with the PS RX architectures. Supposing that $\eta(\cdot)$ denotes the RF-to-DC rectification efficiency function, which is in general a non-linear positive function of the received RF power $P_{R,E}$ available for EH [22]–[24], the total harvested DC power is obtained as $P_H \triangleq \eta(\rho P_R) \rho P_R$. Despite this circuit dependent non-linear relationship between η and $P_{R,E}$, we note that P_H is monotonically non-decreasing in $P_{R,E} = \rho P_R$ for any practical RF EH circuit [22]–[24] due to the law of energy conservation. For instance, to give more insights, we plot both η and $P_H \triangleq \eta(P_{R,E}) P_{R,E}$ as a function of the received RF power $P_{R,E}$ variable at the input of two real-world RF EH circuits, namely, (i) the commercially available Powercast P1110 evaluation board (EVB) [24] and (ii) the circuit designed in [26] for low power far field RF EH in Figs. 2(a) and 2(b), respectively. So, using $P_H = \mathcal{F}(P_{R,E})$, where $\mathcal{F}(\cdot)$ represents a *non-linear non-decreasing* function, we are able to obtain the jointly global optimal design.

III. JOINT TX AND RX OPTIMIZATION FRAMEWORK

Here we present the mathematical formulation of the optimization problem. Then in Section III-A, we consider the practical case of UPS reception and prove an interesting property of the underlying optimization problem that will be further exploited in Section IV for deriving the globally optimal design. Aiming at comparing with the ideal reception case, we present its mathematical formulation in Section III-B.

A. UPS Reception

Focusing on the MIMO SWIPT model of Section II, we consider the problem of designing the covariance matrix $\mathbf{S}$ at the multi-antenna TX and UPS ratio ρ at the multi-antenna EH RX for maximizing the total harvested DC power, while satisfying a minimum instantaneous rate requirement R in bits per second (bps) per Hz for information transmission. So, the proposed design framework is mathematically expressed as:

$$\mathcal{OP} : \max_{\rho, \mathbf{S}} P_H = \eta(\rho \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H)) \rho \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H)$$

$$\text{s.t. (C1): } \log_2(\det(\mathbf{I}_{N_R} + (1 - \rho)\sigma^{-2}\mathbf{H} \mathbf{S} \mathbf{H}^H)) \geq R,$$

$$(C2): \text{tr}(\mathbf{S}) \leq P_T, (C3): \mathbf{S} \succeq 0, (C4): 0 \leq \rho \leq 1,$$

where constraint (C1) represents the minimum instantaneous rate requirement, (C2) is the average transmit power constraint, while (C3) and (C4) are the boundary conditions for $\mathbf{S}$ and ρ . It can be easily concluded that P_H is jointly non-concave in regards to the unknown variables $\mathbf{S}$ and ρ . However, in the following Lemma 1 we show that the received RF power $P_{R,E}$ available for EH is jointly pseudoconcave in $\mathbf{S}$ and ρ .

Lemma 1: $P_{R,E}$ is jointly pseudoconcave in $\mathbf{S}$ and ρ .

Proof: With $\text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H)$ being linear in $\mathbf{S}$, we deduce that the total average received RF power $P_{R,E} = \rho \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H)$ available for EH is the product of two positive linear functions of ρ and $\mathbf{S}$. Since the product of two positive linear (or concave) functions is log-concave [27, Chapter 3.5.2] and a positive log-concave function is also pseudoconcave [13, Lemma 5], $P_{R,E}$ is jointly pseudoconcave in $\mathbf{S}$ and ρ . ■

We now show that solving $\mathcal{OP}$ is equivalent to problem $\mathcal{OP1}$:

$$\mathcal{OP1}: \max_{\rho, \mathbf{S}} P_{R,E} = \rho \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H), \text{ s.t. (C1), (C2), (C3), (C4).}$$

Proposition 1: Solution pair $(\mathbf{S}^*, \rho^*)$ of $\mathcal{OP1}$ solves $\mathcal{OP}$.

Proof: Irrespective of the circuit-dependent non-linear relationship between η and $P_{R,E}$, P_H is monotonically non-decreasing in $P_{R,E}$ [22]–[24]. It can be concluded from [27], [28] that the monotonic non-decreasing transformation P_H of the pseudoconcave function $P_{R,E}$ is also pseudoconcave and possesses the unique global optimality property [28, Props. 3.8 and 3.27]. This reveals that $\mathcal{OP}$ and $\mathcal{OP1}$ are equivalent [29], sharing the same solution pair $(\mathbf{S}^*, \rho^*)$. ■

It can be deduced from Proposition 1 that one may solve $\mathcal{OP1}$ and then use the resulting maximum received power $P_{R,E}^* = \rho^* \text{tr}(\mathbf{H} \mathbf{S}^* \mathbf{H}^H)$ to compute the maximum harvested power as $P_H^* = \eta(P_{R,E}^*) P_{R,E}^*$. Although $\mathcal{OP1}$ is nonconvex, we prove in the following theorem a specific property for it that will be used in Section IV to derive its optimal solution.

Theorem 1: $\mathcal{OP1}$ is a generalized convex problem and its optimal solution can be obtained by solving KKT conditions.

Proof: As shown in Lemma 1, $P_{R,E}$ is jointly pseudo-concave in $\mathbf{S}$ and ρ . It follows from constraint (C1) that the function $R - \log_2(\det(\mathbf{I}_{N_R} + (1 - \rho)\sigma^{-2}\mathbf{H}\mathbf{S}\mathbf{H}^H))$ is jointly convex on ρ and $\mathbf{S}$; this ensues from the fact that the matrix inside the determinant is a positive definite matrix [12], [17]–[19]. In addition, constraints (C2) and (C3) are linear with respect to $\mathbf{S}$ and independent of ρ , and constraint (C4) depends only on ρ and is convex. The proof completes by combining the latter findings and using them in [29, Theorem 4.3.8]. ■

Capitalizing on the findings of Proposition 1 and Theorem 1, we henceforth focus on the maximization of the received RF power $P_{R,E}$ for EH. The jointly optimal TX precoding and UPS design for this problem will also result in the maximization of the harvested DC power P_H^* for any practical RF EH circuitry. Also, the proposed joint transceiver design in this paper is different from the ones in existing works [12]–[21] considering the received RF power for EH as a constraint and using a trivial linear RF EH model for their investigation.

B. Ideal Reception

To investigate the theoretical upper bound for $P_{R,E}$, we now consider an ideal RX architecture capable of using all received RF power for both EH and ID. In particular, we remove ρ from $\mathcal{OP}1$ and (C1) and consider the following optimization:

$$\begin{aligned} \mathcal{OP}2 : \max_{\mathbf{S}} P_R &= \text{tr}(\mathbf{H}\mathbf{S}\mathbf{H}^H), \quad \text{s.t. (C2), (C3),} \\ \text{(C5)} : \log_2(\det(\mathbf{I}_{N_R} + \sigma^{-2}\mathbf{H}\mathbf{S}\mathbf{H}^H)) &\geq R. \end{aligned}$$

From the findings in the proof of Theorem 1, the objective function P_R of $\mathcal{OP}2$ along with constraints (C2) and (C3) are linear in $\mathbf{S}$. In addition, (C5) is convex due to the concavity of logarithm with respect to $\mathbf{S}$. Combining the latter facts yields that $\mathcal{OP}2$ is a convex problem, and hence, its optimal solution can be found using the Lagrangian dual method [27], [29].

IV. OPTIMAL TX PRECODING AND RX POWER SPLITTING

Here, we first investigate trade off between energy beamforming and information spatial multiplexing in $\mathcal{OP}1$. Then, we present the global optimal solutions for $\mathcal{OP}1$ and $\mathcal{OP}2$.

A. Energy Beamforming versus Spatial Multiplexing

Let us consider the reduced Singular Value Decomposition (SVD) of the MIMO channel matrix $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{V} \in \mathbb{C}^{N_T \times r}$ and $\mathbf{U} \in \mathbb{C}^{N_R \times r}$ are unitary matrices and $\mathbf{\Lambda} \in \mathbb{C}^{r \times r}$ is the diagonal matrix consisting of the r non-zero eigenvalues of $\mathbf{H}$ in decreasing order of magnitude. Ignoring the rate constraint (C1) in $\mathcal{OP}1$ (or equivalently in $\mathcal{OP}$) leads to the rank-1 optimal TX covariance matrix $\mathbf{S}^* = \mathbf{S}_{\text{EB}} \triangleq P_T \mathbf{v}_1 \mathbf{v}_1^H$ [12], [30], where $\mathbf{v}_1 \in \mathbb{C}^{N_T \times 1}$ is the first column of $\mathbf{V}$ that corresponds to the eigenvalue $[\mathbf{\Lambda}]_{1,1} \triangleq \sqrt{\lambda_{\max}(\mathbf{H}^H \mathbf{H})}$. This TX precoding, also known as *transmit energy beamforming*, allocates P_T to the strongest eigenmode of $\mathbf{H}^H \mathbf{H}$ and is known to maximize the harvested or received power. On the other hand, it is also well known [31] that one may profit from the existence of multiple antennas and channel estimation techniques to realize spatial multiplexing of multiple data streams. Spatial multiplexing adopts the

Fig. 3. Trade off between received power for EH and achievable information rate. R_{th} is the switching point between TX precoding modes: energy beamforming and information spatial multiplexing.

waterfilling technique to perform optimal allocation of P_T over all the available eigenchannels of MIMO channel matrix. Evidently, for our problem formulation $\mathcal{OP}1$ including the rate constraint (C1) and PS reception, we need to investigate the underlying fundamental trade off between TX energy beamforming and information spatial multiplexing. As previously described, these two transmission schemes have contradictory objectives, and thus provide different TX designs.

Suppose we adopt energy beamforming in $\mathcal{OP}1$, resulting in the received RF power $P_{R,\text{EB}} \triangleq \rho_{\text{EB}} P_T [\mathbf{\Lambda}]_{1,1}^2$ where ρ_{EB} represents the unknown UPS parameter. To find the optimal UPS parameter ρ_{EB}^* , we need to seek for the best power allocation $(1 - \rho_{\text{EB}}^*)$ for ID meeting the rate requirement R . To do so, we solve (C1) at equality over UPS parameter yielding

$$\rho_{\text{EB}}^* \triangleq \max \left\{ 0, 1 - (2^R - 1) \sigma^2 [P_T [\mathbf{\Lambda}]_{1,1}^2]^{-1} \right\}. \quad (3)$$

It can be concluded that both ρ_{EB}^* and the maximum received RF power given by $\rho_{\text{EB}}^* P_T [\mathbf{\Lambda}]_{1,1}^2$ are decreasing functions of R . This reveals that there exists a rate threshold R_{th} such that, when $R > R_{\text{th}}$, one should allocate P_T over to at least two eigenchannels instead of performing energy beamforming, i.e., instead of assigning P_T solely to the strongest eigenchannel. We are henceforth interested in finding this R_{th} value. Consider the optimum power allocation p_1^* and p_2^* for the two highest gained eigenchannels with eigenmodes $[\mathbf{\Lambda}]_{1,1}$ and $[\mathbf{\Lambda}]_{2,2}$, respectively, with $[\mathbf{\Lambda}]_{1,1} > [\mathbf{\Lambda}]_{2,2}$. By substituting these values into (C1), defining $\mathcal{Z}_1 \triangleq \sqrt{([\mathbf{\Lambda}]_{1,1}^2 p_1^* - [\mathbf{\Lambda}]_{2,2}^2 p_2^*)^2 + 2^{R+2} [\mathbf{\Lambda}]_{1,1}^2 [\mathbf{\Lambda}]_{2,2}^2 p_1^* p_2^*}$, and solving at equality for the optimum UPS parameter $\rho_{\text{SM}_2}^*$ for spatial multiplexing over two eigenchannels deduces to

$$\rho_{\text{SM}_2}^* \triangleq 1 + \frac{\sigma^2}{2} \left(\frac{1}{[\mathbf{\Lambda}]_{1,1}^2 p_1^*} + \frac{1}{[\mathbf{\Lambda}]_{2,2}^2 p_2^*} + \frac{\mathcal{Z}_1}{[\mathbf{\Lambda}]_{1,1}^2 [\mathbf{\Lambda}]_{2,2}^2 p_1^* p_2^*} \right), \quad (4)$$

resulting in the maximum received RF power for EH given by $\rho_{\text{SM}_2}^* (p_1^* [\mathbf{\Lambda}]_{1,1}^2 + p_2^* [\mathbf{\Lambda}]_{2,2}^2)$. We now combine the latterly obtained maximum received RF power with spatial multiplexing and that of energy beamforming to compute R_{th} . The rate threshold value that renders energy beamforming more

beneficial than spatial multiplexing in terms of received RF power can be obtained from solution of following inequality

$$\rho_{\text{EB}}^* P_T [\mathbf{\Lambda}]_{1,1}^2 > \rho_{\text{SM}_2}^* (p_1^* [\mathbf{\Lambda}]_{1,1}^2 + p_2^* [\mathbf{\Lambda}]_{2,2}^2). \quad (5)$$

Substituting (3) and (4) into (5), defining $\mathcal{Z}_2 \triangleq ([\mathbf{\Lambda}]_{1,1}^2 - [\mathbf{\Lambda}]_{2,2}^2)([\mathbf{\Lambda}]_{1,1}^2 p_1^* + p_2^* [\mathbf{\Lambda}]_{2,2}^2)^2$, and applying some algebraic manipulations yields the desired threshold in closed-form:

$$R_{\text{th}} \triangleq \log_2 \left(1 + \frac{p_2^* ([\mathbf{\Lambda}]_{1,1}^2 - [\mathbf{\Lambda}]_{2,2}^2)}{\sigma^2} + \sqrt{\frac{\mathcal{Z}_2}{[\mathbf{\Lambda}]_{1,1}^2 [\mathbf{\Lambda}]_{2,2}^2 \sigma^2 p_1^*}} \right). \quad (6)$$

Remark 1: The rate threshold R_{th} given by (6) evinces a switching point on the desired TX precoding operation, which is graphically presented in Fig. 3. When the rate requirement R is less or equal to R_{th} , energy beamforming is sufficient to meet R , and hence, can be used for maximizing the received RF power. For cases where $R > R_{\text{th}}$, statistical multiplexing needs to be adopted for maximizing the received RF power for EH while satisfying R . This explicit non-trivial switching point R_{th} for the TX precoding mode is unique to the problem formulation considered in this paper, and has not been explored or investigated in the relevant literature [12], [16]–[22] for the complementary problem formulations therein. We next use R_{th} in (6) to obtain solutions for $\mathcal{OP}1$ and $\mathcal{OP}2$.

B. Globally Optimal Solution of $\mathcal{OP}1$

Associating Lagrange multipliers μ and ν with constraints (C1) and (C2), respectively, while keeping (C3) and (C4) implicit, the Lagrangian function of $\mathcal{OP}1$ is defined as

$$\mathcal{L} = \rho P_R - \nu (\text{tr}(\mathbf{S}) - P_T) - \mu (R - \log_2[\det(\mathcal{Z}_3)]). \quad (7)$$

where $\mathcal{Z}_3 \triangleq (\mathbf{I}_{N_R} + (1 - \rho) \sigma^{-2} \mathbf{H} \mathbf{S} \mathbf{H}^H)$. Using Theorem 1, the global optimal solution $(\mathbf{S}^*, \rho^*)$ for $\mathcal{OP}1$ is obtained from the following four Karush-Kuhn-Tucker (KKT) conditions (the subgradient and complimentary slackness conditions are defined, whereas the primal feasibility (C1)–(C4) and dual feasibility constraints $\mu, \nu \geq 0$ are kept implicit):

$$\frac{\partial \mathcal{L}}{\partial \mathbf{S}} = \frac{\mu (1 - \rho)}{\sigma^2 \ln 2} \mathbf{H}^H \mathcal{Z}_3^{-1} \mathbf{H} + \rho \mathbf{H}^H \mathbf{H} - \nu \mathbf{I}_{N_T} = 0, \quad (8a)$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = -\mu \text{tr} \left(\frac{\mathbf{H} \mathbf{S} \mathbf{H}^H}{\sigma^2 \ln 2} \mathcal{Z}_3^{-1} \right) + \text{tr}(\mathbf{H} \mathbf{S} \mathbf{H}^H) = 0, \quad (8b)$$

$$\mu (R - \log_2[\det \mathcal{Z}_3]) = 0, \quad (8c)$$

$$\nu (\text{tr}(\mathbf{S}) - P_T) = 0. \quad (8d)$$

Solving (8) yields the KKT point [27], [29] defined by the optimal solution $(\mathbf{S}^*, \rho^*, \mu^*, \nu^*)$. It is noted that it must hold $\nu \neq 0$, because the total available transmit power P_T is always fully utilized due to the monotonically increasing nature of the objective function $P_{R,E}$ in $\mathbf{S}$. This implies that $\text{tr}(\mathbf{S}) = P_T$, which means that constraint (C2) is always satisfied at equality. Similarly, it must hold $\mu \neq 0$, because the received RF power is strictly increasing in ρ and, as such, the fraction $1 - \rho$ allocated for ID needs to be sufficient in meeting (C1).

Recalling the trade off discussion in Section IV-A, when $R \leq R_{\text{th}}$, the optimal TX covariance matrix is given as $\mathbf{S}^* =$

$\mathbf{S}_{\text{EB}}$. For this case the optimum TX precoding operation is energy beamforming, i.e., $\mathbf{F} \triangleq \mathbf{V}(\mathbf{P}^*)^{1/2} \in \mathbb{C}^{N_T \times r}$ where the $r \times r$ matrix $\mathbf{P}^*$ is defined as $\mathbf{P}^* = \text{diag}\{[P_T \ 0 \ \cdots \ 0]\}$, and the optimal UPS ratio is ρ_{EB} given by (3). Substituting $\mathbf{S}_{\text{EB}}$ and ρ_{EB} into (8a) and (8b), yields corresponding multipliers:

$$\mu_{\text{EB}} \triangleq \sigma^2 \ln 2 \left(1 + \frac{(1 - \rho_{\text{EB}}) P_T [\mathbf{\Lambda}]_{1,1}^2}{\sigma^2} \right), \quad (9a)$$

$$\nu_{\text{EB}} \triangleq \frac{\mu_{\text{EB}} (1 - \rho_{\text{EB}}) [\mathbf{\Lambda}]_{1,1}^2}{\ln(2) (\sigma^2 + (1 - \rho_{\text{EB}}) P_T [\mathbf{\Lambda}]_{1,1}^2)} + [\mathbf{\Lambda}]_{1,1}^2 \rho_{\text{EB}}. \quad (9b)$$

We therefore conclude that $(\mathbf{S}^*, \rho^*, \mu^*, \nu^*)$ is given as $(\mathbf{S}_{\text{EB}}, \rho_{\text{EB}}, \mu_{\text{EB}}, \nu_{\text{EB}})$ for $R \leq R_{\text{th}}$. When $R > R_{\text{th}}$, the optimum TX precoding operation is spatial multiplexing and we thus to obtain the TX covariance matrix, rewrite (8a) in the following form after applying algebraic simplifications.

$$\mathbf{\Lambda} \mathbf{V}^H \mathbf{S} \mathbf{V} \mathbf{\Lambda} = \frac{\mu}{\ln 2} (\nu \mathbf{I}_r - \rho \mathbf{\Lambda}^H \mathbf{\Lambda})^{-1} \mathbf{\Lambda}^2 - \frac{\sigma^2}{1 - \rho} \mathbf{I}_r. \quad (10)$$

By performing the necessary left and right multiplications of (10) with $\mathbf{\Lambda}^{-1}$, $\mathbf{V}$, and $\mathbf{V}^H$ and setting ρ , μ , and ν to their optimal values ρ_{SM} , μ_{SM} , and ν_{SM} for spatial multiplexing, the optimal TX covariance matrix for $R > R_{\text{th}}$ can be derived as:

$$\mathbf{S}_{\text{SM}} = \mathbf{V} \left(\frac{\mu_{\text{SM}} (\nu_{\text{SM}} \mathbf{I}_r - \rho_{\text{SM}} \mathbf{\Lambda}^H \mathbf{\Lambda})^{-1}}{\ln 2} - \frac{\sigma^2 \mathbf{\Lambda}^{-2}}{1 - \rho_{\text{SM}}} \right) \mathbf{V}^H. \quad (11)$$

On simplifying (8b) to solve for the optimal μ_{SM} yields

$$\mu_{\text{SM}} = \frac{\text{tr}(\mathbf{H} \mathbf{S}_{\text{SM}} \mathbf{H}^H)}{\text{tr} \left(\frac{\mathbf{H} \mathbf{S}_{\text{SM}} \mathbf{H}^H}{\sigma^2 \ln 2} (\mathbf{I}_{N_R} + (1 - \rho_{\text{SM}}) \sigma^{-2} \mathbf{H} \mathbf{S}_{\text{SM}} \mathbf{H}^H)^{-1} \right)}. \quad (12)$$

Evidently from (11), $\mathbf{S}_{\text{SM}} \in \mathbb{C}^{N_T \times N_T}$ can be expressed as $\mathbf{S}_{\text{SM}} \triangleq \mathbf{V} \mathbf{P}^* \mathbf{V}^H$ with the $r \times r$ matrix $\mathbf{P}^* \triangleq \text{diag}\{[p_1^* \ p_2^* \ \cdots \ p_r^*]\}$ representing the optimal power allocation matrix among $\mathbf{H}$'s eigenchannels, whose entries are given by

$$p_k^* = \left(\frac{\mu^*}{\ln 2 (\nu_{\text{SM}} - \rho_{\text{SM}} [\mathbf{\Lambda}]_{k,k}^2)} - \frac{\sigma^2}{(1 - \rho_{\text{SM}}) [\mathbf{\Lambda}]_{k,k}^2} \right)^+. \quad (13)$$

The optimal ρ_{SM} , μ_{SM} , and ν_{SM} is the solution of the system with the three equations (8c), (8d), and (12) after setting $\mathbf{S} = \mathbf{S}_{\text{SM}}$ and satisfying $\mu, \nu > 0$ and $0 \leq \rho < 1$. Later in Section VI we first reduce this system of equations to two, and then by exploiting the tight bounds on ν^* derived in Section V-A2, we present how it can be efficiently implemented.

Remark 2: Observing (11) and (13) leads to the conclusion that the optimum TX precoding for $R > R_{\text{th}}$ is $\mathbf{F} = \mathbf{V}(\mathbf{P}^*)^{1/2}$ with the r diagonal elements of $\mathbf{P}^*$ given by (13). This precoding results in r parallel eigenchannel transmissions with power allocation obtained from a modified waterfilling algorithm, where water levels depend on R , P_T , $\mathbf{H}$, and σ^2 .

By combining these results for energy beamforming and spatial multiplexing, the optimal solution of $\mathcal{OP}1$ is given by

$$\mathbf{S}^* = \begin{cases} P_T \mathbf{v}_1 \mathbf{v}_1^H, & R \leq R_{\text{th}} \leq R_{\text{max}}, \\ \mathbf{V} \left(\frac{\mu^* \mathcal{Z}_4}{\ln 2} - \frac{\sigma^2 \mathbf{\Lambda}^{-2}}{1 - \rho} \right) \mathbf{V}^H, & R_{\text{th}} < R \leq R_{\text{max}}, \\ \text{Infeasible}, & R > R_{\text{max}}, \end{cases} \quad (14)$$

where $\mathcal{Z}_4 \triangleq (\nu^* \mathbf{I}_r - \rho \mathbf{\Lambda}^H \mathbf{\Lambda})^{-1}$, $\rho^* = \rho_{\text{EB}}$, $\mu^* = \mu_{\text{EB}}$, and $\nu^* = \nu_{\text{EB}}$ for $R \leq R_{\text{th}}$, and for $R > R_{\text{th}}$, $\rho^* = \rho_{\text{SM}}$, $\mu^* = \mu_{\text{SM}}$, and $\nu^* = \nu_{\text{SM}}$ are obtained from the solution of the system of equations described below (13). The feasibility of $\mathcal{OP}1$ depends on $R_{\text{max}} \triangleq \log_2 (\det (\mathbf{I}_{N_R} + \sigma^{-2} \mathbf{H} \mathbf{S}_{\text{WF}} \mathbf{H}^H))$, which represents the maximum achievable rate for UPS ratio $\rho = 0$ and $\mathbf{S}_{\text{WF}} \triangleq \mathbf{V} \mathbf{P}_{\text{WF}} \mathbf{V}^H$. In the latter expression, $\mathbf{P}_{\text{WF}} \triangleq \text{diag}\{[p_{\text{WF},1} \ p_{\text{WF},2} \ \dots \ p_{\text{WF},r}]\}$ is the $r \times r$ power allocation matrix whose rank r_w (non-zero diagonal entries) is [32]

$$r_w \triangleq \max \left\{ k \left| P_T - \sum_{i=1}^{k-1} \left(\frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2} - \frac{\sigma^2}{[\mathbf{\Lambda}]_{i,i}^2} \right) > 0, k \leq r \right\} \quad (15)$$

and its non-zero elements with index $k = 1, 2, \dots, r_w$ are obtained from the standard waterfilling algorithm as

$$p_{\text{WF},k} = \frac{P_T - \sum_{i=1}^{r_w-1} \left(\frac{\sigma^2}{[\mathbf{\Lambda}]_{i,i}^2} - \frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2} \right)}{r_w} + \frac{\sigma^2}{[\mathbf{\Lambda}]_{r_w,r_w}^2} - \frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2}. \quad (16)$$

Here we would like to add that based on (14) deciding whether the optimal TX precoding matrix $\mathbf{S}^*$ is denoted $\mathbf{S}_{\text{EB}}$ or $\mathbf{S}_{\text{SM}}$, the corresponding optimal TX signal vector $\mathbf{x}^* \in \mathbb{C}^{N_T \times 1}$ can be obtained as $\mathbf{x}_{\text{EB}} \triangleq \sqrt{P_T} \mathbf{v}_1 \tilde{\mathbf{x}}$ and $\mathbf{x}_{\text{SM}} \triangleq \mathbf{V} (\mathbf{P}^*)^{1/2} \tilde{\mathbf{x}}$. Here $\tilde{\mathbf{x}}$ is an arbitrary ZMCSCG signal and $\tilde{\mathbf{x}} \in \mathbb{C}^{r \times 1}$ is a ZMCSCG random vector, both having unit variance entries.

C. Globally Optimal Solution of $\mathcal{OP}2$

Like $\mathcal{OP}1$, the proposed rate threshold value in $\mathcal{OP}2$ determining the optimal TX precoding operation, is given by

$$R_{\text{th}}^{\text{id}} \triangleq \log_2 (1 + \sigma^{-2} P_T [\mathbf{\Lambda}]_{1,1}^2), \quad (17)$$

represents rate achieved by energy beamforming for ideal RX.

Lemma 2: Global optimal solution $\mathbf{S}_{\text{id}}^*$ of $\mathcal{OP}2$ is given by

$$\mathbf{S}_{\text{id}}^* = \begin{cases} P_T \mathbf{v}_1 \mathbf{v}_1^H, & R \leq R_{\text{th}}^{\text{id}} \leq R_{\text{max}}, \\ \mathbf{V} \text{diag}\{[p_1^{(\text{id})} \ p_2^{(\text{id})} \ \dots \ p_r^{(\text{id})}]\} \mathbf{V}^H, & R_{\text{th}}^{\text{id}} < R \leq R_{\text{max}}, \\ \text{Infeasible}, & R > R_{\text{max}}, \end{cases} \quad (18)$$

with $p_k^{(\text{id})}$ denoting power assignment to k -th eigenchannel is

$$p_k^{(\text{id})} = \left(\frac{\mu_2^*}{\ln 2 (\nu_2^* - [\mathbf{\Lambda}]_{k,k}^2)} - \frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2} \right)^+, \quad \forall k = 1, 2, \dots, r. \quad (19)$$

In the latter expression, $\nu_2^* \geq 0$ and $\mu_2^* \geq 0$ represent the Lagrange multipliers corresponding to constraints (C2) and (C5), respectively. These can be obtained using a sub-gradient method as described in [12, App. A] such that $\log_2 (\det (\mathbf{I}_{N_R} + \sigma^{-2} \mathbf{H} \mathbf{S}_{\text{id}}^* \mathbf{H}^H)) = R$ and $\text{tr} (\mathbf{S}_{\text{id}}^*) = P_T$.

Proof: The proof is provided in Appendix A. ■

Remark 3: It can be observed from the solution of $\mathcal{OP}2$ that our proposed TX precoding design significantly differs from that obtained from the solution of optimization problem (P3) in [12]. This reveals that the design maximizing the total received RF power for EH, while satisfying a minimum instantaneous rate requirement, is very different from the design that maximizes the instantaneous rate subject to a minimum constraint on the total received RF power.

V. ANALYTICAL BOUNDS AND ASYMPTOTIC RELAXATION

Here we first present analytical bounds for the UPS ratio ρ and Lagrange multipliers μ, ν defined in Section IV. Then tight asymptotic approximations for the joint design are presented.

A. Analytical Bounds

1) **UPS Ratio ρ :** The information rate is given by $\log_2 (\det (\mathbf{I}_{N_R} + (1 - \rho) \sigma^{-2} \mathbf{H} \mathbf{S} \mathbf{H}^H))$, which is a monotonically decreasing function of ρ . The upper bound ρ_{UB} on the feasible ρ value satisfying (C1) is given by the UPS ratio corresponding to the maximum achievable rate value R_{max} as achieved with statistical multiplexing over all available eigenchannels. Mathematically, ρ_{UB} as obtained by setting $\mathbf{S} = \mathbf{V} \mathbf{P}_{\text{WF}} \mathbf{V}^H$ with the entries of $\mathbf{P}_{\text{WF}}$ defined in (16), is

$$\rho_{\text{UB}} \triangleq \left\{ \rho \left| \det \left(\mathbf{I}_{N_R} + \frac{(1 - \rho) \mathbf{H} \mathbf{S}_{\text{WF}} \mathbf{H}^H}{\sigma^2} \right) = 2^R, \rho \leq 1 \right. \right\}. \quad (20)$$

Likewise, the lower bound on the feasible ρ meeting (C1) is given by the UPS ratio ρ_{EB} defined in (3). This lower bound happens with energy beamforming, where entire TX power is allocated to the best gain eigenchannel and the achievable rate is minimum. Combining these results, yields $\rho_{\text{EB}} \leq \rho \leq \rho_{\text{UB}}$.

2) **Lagrange Multipliers μ and ν for $R > R_{\text{th}}$:** To have non-negative power allocation p_1 over the best gain eigenchannel having eigenmode $[\mathbf{\Lambda}]_{1,1}$, it must hold from (13) that $\nu \geq \nu_{\text{LB}} \triangleq \rho [\mathbf{\Lambda}]_{1,1}^2$. Also, using the definition $p_1 = \alpha P_T$ with $\alpha \leq 1$ in (13) for $k = 1$ yields $\mu = (\nu - \rho [\mathbf{\Lambda}]_{1,1}^2) \left(\alpha P_T + \frac{\sigma^2}{(1 - \rho) [\mathbf{\Lambda}]_{1,1}^2} \right) \ln 2$. Since for the total received power holds $\text{tr} (\mathbf{H} \mathbf{S} \mathbf{H}^H) \leq P_T$ and also (12) holds, the upper bound for μ , denoted by μ_{UB} , can be obtained as

$$\mu < \frac{(a) (1 - \rho) \ln 2 \text{tr} (\mathbf{H} \mathbf{S} \mathbf{H}^H)}{r} < \mu_{\text{UB}} \triangleq \frac{(1 - \rho) P_T \ln 2}{r} \quad (21)$$

where (a) results from the high SNR approximation. Combining (21) with $\frac{\sigma^2}{(1 - \rho) [\mathbf{\Lambda}]_{1,1}^2} > 0$, leads to $\nu < \frac{(1 - \rho)}{\alpha r} + \rho [\mathbf{\Lambda}]_{1,1}^2$. Due to the highest power allocation over the best gain eigenchannel, it must hold $\alpha \geq \frac{1}{r}$, yielding $\nu_{\text{UB}} \triangleq 1 + \rho ([\mathbf{\Lambda}]_{1,1}^2 - 1)$. However as shown later, $\nu \ll \nu_{\text{UB}}$ because the total received power $P_R \ll P_T$. These bounds will be used in Section VI-B for efficiently implementing the global optimization algorithm.

B. Asymptotic Analysis

The received RF power for EH in SWIPT systems needs to be greater than energy reception sensitivity [2], [3], which is in the order of -10dBm to -30dBm , for having non-zero harvested DC power after rectification. Since, the AWGN power spectral density is around -175dBm/Hz leading to an average received noise power of around -100dBm for SWIPT at 915 MHz, the received SNR in practical SWIPT systems is very high, i.e., around 70dB, even for very high frequency transmissions. Based on this practical observation for SWIPT, we next investigate the joint design for high SNR scenarios.

1) *Globally Optimal Solution of OP1 for High SNR:* The optimal solution of OP1 for high SNR values defined as $(\mathbf{S}_a^*, \rho_a^*, \mu_a^*, \nu_a^*)$ can be obtained similarly to Section IV-B as

$$\mathbf{S}_a^* = \begin{cases} P_T \mathbf{v}_1 \mathbf{v}_1^H, & R \leq R_{th} \leq R_{max}, \\ \mathbf{V} \left(\frac{\mu_a^* (\nu_a^* \mathbf{I}_r - \rho_a^* \mathbf{A}^H \mathbf{A})^{-1}}{\ln 2} \right) \mathbf{V}^H, & R_{th} < R \leq R_{max}, \\ \text{Infeasible}, & R > R_{max}, \end{cases} \quad (22)$$

$\mu_a^* = r^{-1} (1 - \rho_a^*) \text{tr}(\mathbf{H} \mathbf{S}_a^* \mathbf{H}^H) \ln 2$, and the remaining two unknowns ρ_a^* and ν_a^* are given from the solutions of the equations $\text{tr}(\mathbf{S}_a^*) = P_T$ and $\log_2 \left(\det \left(\sigma^{-2} (1 - \rho) \mathbf{H} \mathbf{S}_a^* \mathbf{H}^H \right) \right) = R$. After some simplifications with (22), the power allocation is obtained as $p_{a,k}^* = \frac{\mu_a^*}{(\nu_a^* - \rho_a^* [\mathbf{A}]_{k,k}^2) \ln 2}, \forall k = 1, 2, \dots, r$. Hence, under high SNR, the optimal power allocation over available eigenchannels for $R > R_{th}$ is always greater than zero regardless of the relative strengths of the eigenmodes.

2) *Globally Optimal Solution of OP2 for High SNR:* Using the derived analytical bounds for ρ and ν along with Lemma 2, the asymptotic approximation $\mathbf{S}_{id,a}^*$ for $\mathbf{S}_{id}^*$ can be obtained as

$$\mathbf{S}_{id,a}^* = \begin{cases} P_T \mathbf{v}_1 \mathbf{v}_1^H, & R \leq R_{th}^{id} \leq R_{max}, \\ \mathbf{V} \text{diag}\{[p_1^{(id,a)} \dots p_r^{(id,a)}]\} \mathbf{V}^H, & R_{th}^{id} < R \leq R_{max}, \\ \text{Infeasible}, & R > R_{max}, \end{cases} \quad (23)$$

where each $p_k^{(id,a)}$ with $k = 1, 2, \dots, r$ is given by

$$p_k^{(id,a)} = \frac{\mu_{2a}^*}{(\nu_{2a}^* - [\mathbf{A}]_{k,k}^2) \ln 2}. \quad (24)$$

With $[\mathbf{A}]_{1,1}^2 < \nu_{2a}^* < [\mathbf{A}]_{1,1}^2 + 1$, solving $\sum_{k=1}^r p_k^{(id,a)} = P_T$ yields $\mu_{2a}^* = P_T \left(\sum_{k=1}^r \frac{1}{(\nu_{2a}^* - [\mathbf{A}]_{k,k}^2) \ln 2} \right)^{-1}$. Setting $\beta \triangleq \nu_{2a}^* - [\mathbf{A}]_{1,1}^2$ and substituting in (24), we can rewrite $p_k^{(id,a)}$ as

$$p_k^{(id,a)} = \beta p_1^{(id,a)} [\beta + [\mathbf{A}]_{1,1}^2 - [\mathbf{A}]_{k,k}^2]^{-1}. \quad (25)$$

To solve for $\beta \in (0, 1)$, we need to replace into the rate constraint expression leading to $\prod_{k=1}^r \left(\frac{p_k^{(id,a)} [\mathbf{A}]_{k,k}^2}{\sigma^2} \right) = 2^R$, which after some simplifications results in the expression

$$\prod_{k=1}^r \left(\frac{\beta [\mathbf{A}]_{k,k}^2}{\beta + [\mathbf{A}]_{1,1}^2 - [\mathbf{A}]_{k,k}^2} \right) = 2^R \left(\frac{\sigma^2}{p_1^{(id,a)}} \right)^r. \quad (26)$$

The $p_1^{(id,a)}$ in (26) can be obtained in closed-form as a function of β by solving $\sum_{k=1}^r p_k^{(id,a)} = P_T$ and using (25), yielding

$$p_1^{(id,a)} = P_T \left(1 + \sum_{k=2}^r \left(\frac{\beta}{\beta + [\mathbf{A}]_{1,1}^2 - [\mathbf{A}]_{k,k}^2} \right) \right)^{-1}. \quad (27)$$

Using these developments in Section VI-B3 we show that the asymptotically optimal TX precoding for OP2 can be obtained using a 1-D linear search over very short range $(0, 1)$ of β .

Remark 4: With the expressions (26) and (27) resulted from our derived asymptotic analysis, we have managed to replace the problem of finding the positive real values of μ_2 and ν_2 in OP2 along with the required waterfilling-based decision making process (involving a discontinuous function $(x)^+$ due to underlying (C3)) by a simple linear search over $\beta \in (0, 1)$.

VI. EFFICIENT GLOBAL OPTIMIZATION ALGORITHM

The goal here is to first present a global optimization algorithm to obtain the previously derived optimal solutions for OP1 and OP2 by effectively solving the KKT conditions. After that we present an alternate low complexity algorithm based on a simple 2-D linear search to practically implement the former algorithm in a computational efficient manner.

A. Solving the KKT Conditions

As discussed in Section IV, $\mathbf{S}^*$ and ρ^* of OP1 for $R > R_{th}$ are obtained by solving (8c), (8d), and (12) for ρ^*, μ^* , and ν^* after setting $\mathbf{S} = \mathbf{S}_{SM}$. Likewise, from Lemma 2, $\mathbf{S}_{id}^*$ of OP2 for $R > R_{th}^{id}$ is derived by solving $\log_2 \left(\det \left(\mathbf{I}_{N_R} + \frac{\mathbf{H} \mathbf{S}_{id}^* \mathbf{H}^H}{\sigma^2} \right) \right) = R$ and $\text{tr}(\mathbf{S}_{id}^*) = P_T$ in μ_2^* and ν_2^* .

1) *Reduction of the System of Non-linear Equations:* It is in general very difficult to efficiently solve a large system of non-linear equations. Hereinafter, we discuss the reduction of the number of the non-linear equations to be solved from three to two in OP1 and from two to one in OP2.

Let us denote the rank of the optimal TX covariance matrix by r_s . It represents the number of eigenchannels that have non-zero power allocation, i.e., $p_k > 0$ with $k = 1, 2, \dots, r_s$. Substituting this definition into (8d) and (13) with $\nu > 0$, we can express μ^* in terms of ν^* and ρ^* as

$$\mu^* = \frac{P_T + \sum_{k=1}^{r_s} \sigma^2 \left((1 - \rho^*) [\mathbf{A}]_{k,k}^2 \right)^{-1}}{r_s \sum_{k=1}^{r_s} \left((\nu^* - \rho^* [\mathbf{A}]_{k,k}^2) \ln 2 \right)^{-1}}. \quad (28)$$

Using definition of r_s in (8c) and (13) with $\mu > 0$, we obtain

$$\mu^* = 2^{\frac{R}{r_s}} \sigma^2 \left((1 - \rho^*) \left(\prod_{k=1}^{r_s} \frac{[\mathbf{A}]_{k,k}^2}{\nu^* - \rho^* [\mathbf{A}]_{k,k}^2} \right)^{\frac{1}{r_s}} \right)^{-1} \ln 2. \quad (29)$$

By combining (12), (28), and (29), the reduced system of two non-linear equations to be solved for ρ^* and ν^* as included in KKT point $(\mathbf{S}^*, \rho^*, \mu^*, \nu^*)$ for $R > R_{th}$ in OP1 is given by

$$\frac{P_T (1 - \rho^*) \sigma^{-2} + \sum_{k=1}^{r_s} [\mathbf{A}]_{k,k}^{-2}}{2^{\frac{R}{r_s}} r_s \sum_{k=1}^{r_s} (\nu^* - \rho^* [\mathbf{A}]_{k,k}^2)^{-1}} = \left(\prod_{k=1}^{r_s} \frac{[\mathbf{A}]_{k,k}^2}{\nu^* - \rho^* [\mathbf{A}]_{k,k}^2} \right)^{-\frac{1}{r_s}} \quad (30a)$$

$$\sum_{k=1}^{r_s} \frac{p_k^* [\mathbf{A}]_{k,k}^2 (1 - \rho^*)^{-1}}{1 + (1 - \rho^*) p_k^* [\mathbf{A}]_{k,k}^2} = \frac{\sum_{k=1}^{r_s} p_k^* [\mathbf{A}]_{k,k}^2 \left(\prod_{j=1}^{r_s} \frac{[\mathbf{A}]_{j,j}^2}{\nu^* - \rho^* [\mathbf{A}]_{j,j}^2} \right)^{\frac{1}{r_s}}}{2^{\frac{R}{r_s}} \sigma^2 \ln 2} \quad (30b)$$

where $p_k^* = \frac{\sigma^2}{(1 - \rho^*)} \left(\frac{2^{\frac{R}{r_s}} (\nu^* - \rho^* [\mathbf{A}]_{k,k}^2)^{-1}}{\left(\prod_{j=1}^{r_s} \frac{[\mathbf{A}]_{j,j}^2}{\nu^* - \rho^* [\mathbf{A}]_{j,j}^2} \right)^{\frac{1}{r_s}}} - \frac{1}{[\mathbf{A}]_{k,k}^2} \right)^+$, $\forall k =$

$1, 2, \dots, r_s$. In a similar manner, the single non-linear equation that needs to be solved for computing ν_2^* included in the KKT point $(\mathbf{S}_{id}^*, \mu_2^*, \nu_2^*)$ for $R > R_{th}^{id}$ in OP2 is given by

$$\left(\frac{P_T}{\sigma^2} + \sum_{j=1}^{r_s} \frac{1}{[\mathbf{A}]_{j,j}^2} \right) \left(\prod_{k=1}^{r_s} \frac{[\mathbf{A}]_{k,k}^2}{\nu_2^* - [\mathbf{A}]_{k,k}^2} \right)^{\frac{1}{r_s}} = \sum_{k=1}^{r_s} \frac{2^{\frac{R}{r_s}} r_s}{\nu_2^* - [\mathbf{A}]_{k,k}^2}. \quad (31)$$

Algorithm 1 Efficient Solution of the KKT Conditions

Input: $\mathbf{H}$, σ^2 , P_T , R , and $P_\delta = 10^{-3}$
Output: Maximized power $P_{R,E}^*$ for EH along with optimal $\mathbf{S}^*$, ρ^*

- 1: Obtain SVD, $\mathbf{H} = \mathbf{U} \text{diag}\{[\mathbf{\Lambda}]_{1,1} [\mathbf{\Lambda}]_{2,2} \dots [\mathbf{\Lambda}]_{r,r}\} \mathbf{V}^H$, along with r_w and $\{p_{\text{WF},k}\}_{k=1}^r$ using (15) and (16)
 - 2: Set $R_{\max} = \log_2(\det(\mathbf{I}_{N_R} + \frac{1}{\sigma^2} \mathbf{H} \mathbf{V} \text{diag}\{p_{\text{WF},k}\}_{k=1}^r \mathbf{V}^H \mathbf{H}^H))$
 - 3: Set $p_1^* = P_T - P_\delta$, $p_2^* = P_\delta$, and obtain R_{th} using (6)
 - 4: **if** $R > R_{\max}$ **then** ▷ Infeasible case
 - 5: **return** with an error message – $\mathcal{OP}1$ (or $\mathcal{OP}2$) is infeasible
 - 6: **else if** $R \leq R_{\text{th}} \leq R_{\max}$ **then** ▷ Energy Beamforming mode
 - 7: Set $\mathbf{S}^* = P_T \mathbf{v}_1 \mathbf{v}_1^H$, $\rho^* = \rho_{\text{EB}}$, $\mu^* = \mu_{\text{EB}}$, and $\nu^* = \nu_{\text{EB}}$
 - 8: **else** ▷ Modified Statistical Multiplexing mode
 - 9: Set $r_s = r_w + 1$
 - 10: **repeat** (Recursion)
 - 11: Set $r_s = r_s - 1$
 - 12: **if** $r_s = 1$ **then**
 - 13: Set $\mathbf{S}^* = P_T \mathbf{v}_1 \mathbf{v}_1^H$, $\rho^* = \rho_{\text{EB}}$, $\mu^* = \mu_{\text{EB}}$, $\nu^* = \nu_{\text{EB}}$
 - 14: **else**
 - 15: Solve equations (30a) and (30b) to obtain ρ^* and ν^*
 - 16: Obtain μ^* by substituting ρ^* and ν^* in (28)
 - 17: Set $\mathbf{S}^* = \mathbf{V} \text{diag}\{[p_1^* p_2^* \dots p_r^*]\} \mathbf{V}^H$ with $p_k^* = \begin{cases} \frac{\mu^*}{\ln 2 (\nu^* - \rho^* [\mathbf{\Lambda}]_{k,k}^2)} - \frac{\sigma^2}{(1-\rho^*) [\mathbf{\Lambda}]_{k,k}^2}, & k = 1, 2, \dots, r_s \\ 0, & r_s + 1 \leq k \leq r. \end{cases}$
 - 18: **until** $(p_k^* < 0 \ \forall k = 1, 2, \dots, r_s)$
-

Lemma 3: The rank r_s of the optimal TX covariance matrix $\mathbf{S}^*$ of $\mathcal{OP}1$ (or $\mathbf{S}_{\text{id}}^*$ of $\mathcal{OP}2$) is always lower or equal to the rank r_w of $\mathbf{S}_{\text{WF}}$ providing the maximum achievable rate $R_{\max}$.

Proof: The proof follows from Section IV-B, where $P_{R,E}$ for EH is given by the rank-1 covariance matrix $\mathbf{S}_{\text{EB}}$ implying TX energy beamforming. With increasing rate requirement $R > R_{\text{th}}$, the optimal TX precoding switches from energy beamforming to statistical multiplexing. In this case, the power allocated over the best gain eigenchannel monotonically decreases due to the power allocation among the other available eigenchannels, thus increasing r_s and decreasing $P_{R,E}$. The rate $R_{\max}$ is achieved by $\mathbf{S}_{\text{WF}}$ having rank $r_w \leq r$, which also results in the minimum $P_{R,E}$ for both $\mathcal{OP}1$ and $\mathcal{OP}2$. Therefore, r_w represents the maximum rank of the TX covariance matrix, hence it must hold $r_s \leq r_w$. ■

2) *Implementation Details and Challenges:* Here we first present the detailed steps involved in the implementation of solving the reduced system of non-linear equations to obtain the optimal design for both $\mathcal{OP}1$ and $\mathcal{OP}2$ via Algorithm 1. After that we discuss the practical challenges involved in implementing it directly using the commercial numerical solvers which may suffer from slow convergence issues as faced by the subgradient methods [12], [14], [16], [22] and semidefinite relaxations [17]–[21] used in the MIMO SWIPT literature.

From Algorithm 1, obtaining $\mathbf{S}^*$ and ρ^* involves solving (30a) and (30b) for at most r times, while considering positive power allocation over the k best gain eigenchannels with $k = 1, 2, \dots, r$. Since (C3) and (C4) had been kept implicit, we repeatedly solve the latter system of equations for at most $r_w \leq r$ times till we obtain a feasible solution $\mathbf{S}^*$ and ρ^* .

Algorithm 1 can be slightly modified to provide the optimal solution of $\mathcal{OP}2$. In particular, steps 7, 13, 15, 16, and 17 need to be updated for $\mathcal{OP}2$. Starting with steps 7 and 13, we need to remove ρ^* since $\mathcal{OP}2$ involves ideal reception and the

optimal values of Lagrange multipliers μ_2 and ν_2 for $R \leq R_{\text{th}}$ are given by $\mu_2^* = 0$ and $\nu_2^* = [\mathbf{\Lambda}]_{1,1}^2$. In addition, to find ν_2^* for $R > R_{\text{th}}$ in step 15 we need to solve (31). The solution ν_2^* of (31) needs then to update steps 16 and 17 in Algorithm 1, and the optimal μ_2^* and $p_k^{(\text{id})}$'s can be derived as

$$\mu_2^* = \left(P_T + \sum_{k=1}^{r_s} \frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2} \right) \left(r_s \sum_{k=1}^{r_s} \frac{1}{(\nu_2^* - [\mathbf{\Lambda}]_{k,k}^2) \ln 2} \right)^{-1} \quad (32a)$$

$$p_k^{(\text{id})} = \begin{cases} \frac{\mu_2^*}{(\nu_2^* - [\mathbf{\Lambda}]_{k,k}^2) \ln 2} - \frac{\sigma^2}{[\mathbf{\Lambda}]_{k,k}^2}, & k = 1, 2, \dots, r_s \\ 0, & r_s + 1 \leq k \leq r \end{cases} \quad (32b)$$

The convergence of Algorithm 1 to its globally optimal solution is guaranteed due to its generalized convexity property [28], [29], as proved in Theorem 1. However, its speed of convergence depends on the efficiency of deployed numerical methods for solving (30a) and (30b). Commercial mathematical packages like Matlab provide very efficient solvers for such non-linear systems having unique solution. But convergence speed of those solvers or conventional subgradient methods [33] depends on the starting point and step sizes.

To characterize the exact number of computations required in achieving a desired level of accuracy with the derived globally optimal solutions, regardless of the starting point and step-sizes fed to the numerical solvers, we next present a simple, yet efficient, 2-D linear search algorithm based on the Golden Section Search (GSS) method [34] that provides an effective way of practically implementing Algorithm 1. We would like to mention that the main steps involved in the global optimization algorithm implemented using Algorithm 2 remain the same as in Algorithm 1. Except that it presents an efficient way of implementing step 15 of Algorithm 1.

B. Two-Dimensional (2-D) Linear Search

As discussed in Section III, for a known ρ , $\mathcal{OP}1$ is a convex optimization problem having a linear objective and convex constraints. Using this property and the small feasible range of ρ given by $0 \leq \rho_{\text{LB}} \leq \rho \leq \rho_{\text{UB}} \leq 1$ as derived in Section V-A1, we propose to iteratively solve $\mathcal{OP}1$ for a given ρ value till the globally optimal $(\mathbf{S}^*, \rho^*)$ pair is obtained providing the unique maximum received power $P_{R,E}^*$. To traverse over the short value space of ρ we use the GSS method [34] that provides fast convergence to the unique root of an equation or a globally optimal solution of a unimodal function. For each feasible ρ value, we substitute into (30a) and then solve it for the optimal ν^* . As shown in Section V-A2, $\nu_{\text{UB}} - \nu_{\text{LB}} = 1 - \rho \leq 1$ implying that the search space for the optimal ν^* is very small. Thus, (30a) can be solved very efficiently for ν^* for a given ρ^* value by using the standard one-dimensional (1-D) GSS method or conventional root finding techniques available in commercial softwares.

1) *Implementation Details:* The algorithmic steps for the proposed 2-D GSS solution are summarized in Algorithm 2 which includes two linear searches. An outer search aiming at finding ρ^* and an inner one to seek for ν^* for each given ρ value. Due to the implicit consideration of (C3), obtaining ν^* for a given ρ^* involves solving (30a) using 1-D GSS

Algorithm 2 Global Optimization Algorithm based on 2-D GSS

Input: $\mathbf{H}$, σ^2 , P_T , R , and acceptable tolerance $\xi \ll 1$
Output: Maximized $P_{R,E}^*$ for EH along with optimal $\mathbf{S}^*$ and ρ^*

```

1: Follow steps 1, 2, and 3 of Algorithm 1 for initialization
2: if  $R > R_{\max}$  then step 5 of Algorithm 1      ▷ Infeasible case
3: else if  $R \leq R_{\text{th}} \leq R_{\max}$  then step 7 of Algorithm 1
4: else                                          ▷ Modified Statistical Multiplexing mode
5:   Obtain  $\rho_{\text{LB}}$  and  $\rho_{\text{UB}}$  by respectively using (3) and (20). Then,
     set  $\rho_p = \rho_{\text{UB}} - 0.618(\rho_{\text{UB}} - \rho_{\text{LB}})$ 
6:    $r_s = r_w + 1$ ,  $\nu_{\text{LB}} = \rho_p[\mathbf{A}]_{1,1}^2$ , and  $\nu_{\text{UB}} = \rho_p[\mathbf{A}]_{1,1}^2 + 1 - \rho_p$ 
7:   repeat (Inner loop: Recursion over feasible  $\nu$  range)
8:     Set  $r_s = r_s - 1$ , and check if  $r_s = 1$  to implement
       step 13 of Algorithm 1
9:     Substitute  $\rho^* = \rho_p$  in (30a), and solve it to obtain  $\nu^* \in$ 
        $(\nu_{\text{LB}}, \nu_{\text{UB}})$  using 1-D GSS
10:    Obtain  $\mu^*$  and  $p_k^*$  by using steps 16 and 17 of Algorithm 1
11:  until ( $p_k^* < 0 \forall k = 1, 2, \dots, r_s$ )
12:  Set  $P_{R,E}^* = \rho^* \sum_{k=1}^{r_s} p_k^* [\mathbf{A}]_{k,k}^2$ 
13:   $P_{R,p} = P_{R,E}^*$ ,  $\mu_p = \mu^*$ ,  $\nu_p = \nu^*$ ,  $\rho_q = \rho_{\text{LB}} + 0.618(\rho_{\text{UB}} - \rho_{\text{LB}})$ 
14:  Repeat steps 6 to 12 with  $\rho_p$  being replaced by  $\rho_q$  in these
    steps to obtain  $P_{R,E}^*$ ,  $p_k^*$ ,  $\mu^*$ , and  $\nu^*$  for  $\rho^* = \rho_q$ 
15:  Set  $P_{R,q} = P_{R,E}^*$ ,  $\mu_q = \mu^*$ ,  $\nu_q = \nu^*$ ,  $\Delta_\rho = \rho_{\text{UB}} - \rho_{\text{LB}}$ 
16:  while  $\Delta_\rho > \xi$  do (Outer loop: Recursion over feasible  $\rho$ )
17:    if  $P_{R,p} \geq P_{R,q}$  then
18:      Set  $\rho_{\text{UB}} = \rho_q$ ,  $\rho_q = \rho_p$ ,  $P_{R,q} = P_{R,p}$ , and  $\rho_p =$ 
         $\rho_{\text{UB}} - 0.618(\rho_{\text{UB}} - \rho_{\text{LB}})$ 
19:      Repeat steps 6 to 12 with  $\rho^* = \rho_p$  and set the results
        as  $P_{R,p} = P_{R,E}^*$ ,  $\mu_p = \mu^*$ ,  $\nu_p = \nu^*$ 
20:    else
21:      Set  $\rho_{\text{LB}} = \rho_p$ ,  $\rho_p = \rho_q$ ,  $P_{R,p} = P_{R,q}$ , and  $\rho_q =$ 
         $\rho_{\text{LB}} + 0.618(\rho_{\text{UB}} - \rho_{\text{LB}})$ 
22:      Repeat steps 6 to 12 with  $\rho_p$  replaced by  $\rho_q$  and to
        obtain  $P_{R,q} = P_{R,E}^*$ ,  $\mu_q = \mu^*$ ,  $\nu_q = \nu^*$ 
23:    Set  $c = c + 1$  and  $\Delta_\rho = \rho_{\text{UB}} - \rho_{\text{LB}}$ 

```

for at most $r_w \leq r$ times, while considering positive power allocation over the $k = 1, 2, \dots, r$ best gain eigenchannels.

Algorithm 2 can also be slightly modified to be used for obtaining the solution for $\mathcal{OP}2$, where due to ideal reception, the outer GSS over the feasible ρ values has to be removed and we only need to perform a 1-D GSS for ν_2^* over its feasible value range $[\mathbf{A}]_{1,1}^2 \leq \nu_2^* \leq [\mathbf{A}]_{1,1}^2 + 1$. So, for $\mathcal{OP}2$ we need to consider steps 1–12 of Algorithm 2, excluding the initialization step 5, and updating steps 6, 9, 10, and 12. Particularly, the bounds are given by $\nu_{2\text{LB}} = [\mathbf{A}]_{1,1}^2$ and $\nu_{2\text{UB}} = [\mathbf{A}]_{1,1}^2 + 1$ in step 6. In step 9, we need to solve (31) to find optimal ν_2^* for $R > R_{\text{th}}$. This ν_2^* value will then be used in step 10 to obtain the optimal μ_2^* and $p_k^{(\text{id})}$'s by substituting ν_2^* in (32a) and (32b). Lastly, we need to set $\rho^* = 1$ in step 12.

2) *Complexity Analysis:* Suppose that we want to calculate ρ^* and ν^* of $\mathcal{OP}1$ or ν_2^* of $\mathcal{OP}2$ through Algorithm 2 so as to be close up to an acceptable tolerance $\xi \ll 1$ to their globally optimal solutions. As seen from Algorithm 2, the search space interval after each GSS iteration reduces by a factor of 0.618 [34, Chap. 2.5]. This value combined with the unity maximum search length for ρ^* and ν^* gives the number of iterations $c^* = \left\lceil \frac{\ln(\xi)}{\ln(0.618)} \right\rceil + 1$ that are required to ensure that the numerical error is less than ξ . For example, $\xi = 10^{-3}$ results in $c^* = 16$. Note that c^* is a logarithmic function of ξ and is independent of N_T , N_R , and r . As each computation in GSS iteration for finding ρ^* involves an

inner GSS for computing ν^* , which is repeated for at most r_w runs, the total number of iterations required for finding the globally optimal solution of $\mathcal{OP}1$ within an acceptable tolerance ξ is given by $c_1^* \leq r_w c^* (c^* + 1)$. Since the number of function computations in GSS is one more than the number of iterations and $r_s \leq r_w \leq r$ from Lemma 3, the total number of computations involved in solving $\mathcal{OP}1$ are bounded by the value $r \left(\left\lceil \frac{\ln(\xi)}{\ln(0.618)} \right\rceil + 2 \right)^2$. Hence, the computational complexity of Algorithm 2 is $\mathcal{O}(r)$, i.e., linear in r . This complexity witnesses the significance of Algorithm 2 over Algorithm 1. Instead of directly implementing commercial numerical solvers or subgradient methods [33] for Algorithm 1, we use the 2-D GSS method as outlined in Algorithm 2.

Regarding the required number of iterations c_2^* for finding the solution of $\mathcal{OP}2$ it must hold $c_2^* \leq r_w c^* \leq r c^*$. As a result, the computational complexity of the modified Algorithm 2 for $\mathcal{OP}2$ is $\mathcal{O} \left(r \left(\left\lceil \frac{\ln(\xi)}{\ln(0.618)} \right\rceil + 1 \right) \right) = \mathcal{O}(r)$.

3) *High SNR Approximation:* Recalling Remark 4 in Section V-B2 holding for high SNR values and focusing on equations (26) and (27) for $R > R_{\text{th}}$, it becomes apparent that, since $p_k^{(\text{id},a)} > 0 \forall k = 1, 2, \dots, r$, then even with the implicit consideration of (C3) one does not need to repeatedly solve the 1-D GSS over ν_2^* . Thus, we only need to find $\beta \in (0, 1)$ from the following equation using the 1-D GSS method:

$$\frac{P_T}{2^r \sigma^2} \left(\prod_{k=1}^r \frac{\beta [\mathbf{A}]_{k,k}^2}{\beta + [\mathbf{A}]_{1,1}^2 - [\mathbf{A}]_{k,k}^2} \right)^{\frac{1}{r}} = \sum_{k=1}^r \left(\frac{\beta}{\beta + [\mathbf{A}]_{1,1}^2 - [\mathbf{A}]_{k,k}^2} \right). \quad (33)$$

The computational complexity of finding the globally optimal solution of $\mathcal{OP}2$ for high SNR values is therefore $\mathcal{O} \left(\left\lceil \frac{\ln(\xi)}{\ln(0.618)} \right\rceil + 1 \right) = \mathcal{O}(1)$, i.e., independent of r .

VII. NUMERICAL RESULTS AND DISCUSSION

In this section, we numerically evaluate the performance of the proposed joint TX precoding and RX UPS splitting design, and investigate the impact of various system parameters on its achievable rate-energy trade off. Unless otherwise stated, we set $\sigma^2 = \{-100, -70\}$ dBm by considering noise spectral density of -175 dBm/Hz as well as $P_T = 10$ W, and $\xi = 10^{-4}$. Furthermore, we model $\mathbf{H}$ as $\mathbf{H} = \{\theta h_{ij} \mid 1 \leq i, j \leq N\}$ with $N \triangleq N_R = N_T = \{2, 4\}$, where $\theta = \{0.1, 0.05\}$ models the distance dependent propagation losses and h_{ij} 's are ZMCSCG random variables with unit variance. With this definition, the average channel power gain is given by $\theta^2 = a d^{-n}$, where a is the propagation loss constant, n is the path loss exponent, and d is the TX-to-RX distance. So, for $a = 0.1$ and $n = 2$, $\theta = 0.1$ represents that $d = 3.16$ m. Whereas this separation becomes twice, i.e., $d = 6.32$ m, for $\theta = 0.05$. We assume unit transmission block duration, thus, we use the terms ‘received energy’ and ‘received power’ interchangeably. All performance results have been generated after averaging over 10^3 independent channel realizations. For obtaining $\mathbf{S}^*$ and ρ^* with the proposed design we have simulated Algorithm 2.

We consider 2×2 and 4×4 MIMO systems in Fig. 4 with both ideal and UPS reception and illustrate the rate-energy trade off for our proposed designs for different values for the propagation losses and noise variance parameters.

Fig. 4. Variation of the rate-energy trade off for varying parameters.

Fig. 5. Variation of η and P_H for Powercast RF EH circuit [24] with rate R near $R_{\max}$ for $P_T = 10\text{W}$, $\theta = 0.05$, and different values for N and σ^2 . Also, $R_{\max}$ is mentioned over the bars in (b).

As expected, our solution for $\mathcal{OP}2$ with ideal reception outperforms that of $\mathcal{OP}1$ that considers practical UPS reception. It is also obvious that increasing N improves the rate-energy trade off. This happens because both beamforming and multiplexing gains improve as N gets larger. Lesser noisy systems, when σ^2 decreases, and better channel conditions with increasing θ result in better trade off and enable higher achievable rates. The maximum achievable rate $R_{\max}$ in bps/Hz for the considered four cases $(\theta, \sigma^2) = \{(0.05, -70\text{dBm}), (0.1, -70\text{dBm}), (0.05, -100\text{dBm}), (0.1, -100\text{dBm})\}$ is given by $\{53.42, 57.77, 73.35, 77.7\}$ for $N = 2$ and $\{106.60, 114.42, 146.47, 154.28\}$ for $N = 4$. In addition, the average value of R_{th} in bps/Hz for these cases is given by $\{17.27, 18.72, 26.02, 27.98\}$ for $N = 2$ and $\{17.46, 19.05, 26.92, 28.73\}$ for $N = 4$. When $R < R_{\text{th}}$, the maximum received RF power $P_{R,E}^*$ for EH is achieved with TX energy beamforming. However, as R increases and becomes substantially larger than R_{th} , $P_{R,E}^*$ decreases till reaching a minimum value. For the latter cases, TX spatial multiplexing is adopted to achieve R and any remaining received power is used for EH. Further, with θ decreasing as $\{0.1, 0.01, 0.001\}$, the corresponding $R_{\max}$ varies as $\{77.70, 64.06, 50.76\}$ bps/Hz for $N = 2$ and $\{154.28, 127.89, 101.33\}$ bps/Hz for $N = 4$. Whereas, the corresponding $P_{R,E}^*$ for EH with $R = 0$ bps/Hz varies as $\{0.36\text{ W}, 3.52\text{ mW}, 35.74\text{ }\mu\text{W}\}$ for $N = 2$ and $\{0.93\text{ W}, 10.09\text{ mW}, 97.22\text{ }\mu\text{W}\}$ for $N = 4$ MIMO SWIPT systems.

Now considering Powercast RF EH circuit [24], we investigate the impact of η on P_H with varying rate requirements close to $R_{\max}$ because in this regime the corresponding $P_{R,E}^*$ decreases sharply as shown in Fig. 4. For each of the four cases

Fig. 6. Variation of the optimal power allocation p_1^* of the best gain eigenchannel of a 2×2 MIMO system as a function of rate R .

Fig. 7. Variation of optimal power allocation in a 4×4 system as a function of normalized rate $\frac{R}{R_{\max}}$ for $P_T = 10\text{W}$, $\sigma^2 = -70\text{dBm}$.

of varying N and σ^2 as plotted in Fig. 5, though η does not follow any trend (increasing for first two cases and decreasing then increasing for the next two), P_H is monotonically decreasing with increasing R from $R = R_{\max} - 9$ bps/Hz to $R = R_{\max} - 3$ bps/Hz, because this increase in rate R results in a lower $P_{R,E}^*$. So, this monotonic trend of optimized P_H in $P_{R,E}^*$ as depicted via Fig. 5 numerically corroborates the discussion with respect to the claim made in Proposition 1.

The variation of optimal power allocation with the proposed joint design for $\mathcal{OP}1$ is depicted in Figs. 6 and 7 for 2×2 and 4×4 MIMO systems, respectively, as a function of R . Particularly, Fig. 6 illustrates the optimal power allocation p_1^* over the best gain eigenchannel for 2×2 system, while the optimal power allocation p_1^*, p_2^*, p_3^* , and p_4^* over the $r = 4$ available eigenchannels is demonstrated in Fig. 7. As shown, p_1^* monotonically decreases from $p_1^* \approx P_T$ (this happens for $R \leq R_{\text{th}}$ where TX energy beamforming is adopted) to the equal power allocation $p_1^* \approx p_2^* \approx \frac{P_T}{2}$ (for large $R = R_{\max}$, TX spatial multiplexing is used). As from (13), $p_1^* \geq p_2^*$, we note that with $P_T = 10\text{W}$ for $N = 2$, $p_1^* \geq 5\text{ W}$ in Fig. 6. A similar trend is observed in Fig. 7. For the plotted normalized rate constraint range, most of P_T is allocated to the best gain eigenchannel to perform TX energy beamforming, while the remaining power is allocated to the rest eigenchannels for meeting rate requirement R with spatial multiplexing.

In Fig. 8, the optimal UPS ratio ρ^* is plotted versus R for 2×2 and 4×4 systems. It is shown that ρ^* monotonically decreases with increasing R in order to ensure that sufficient fraction of the received RF power is used for ID, thus, to satisfy the rate requirement. Lower σ^2 , larger N or equivalently r , and higher θ result in meeting R with lower fraction $1 - \rho$ of the received RF power dedicated for ID. Thus, for these

Fig. 8. Variation of the optimal UPS ratio ρ^* for $\mathcal{OP}1$ versus rate R for $P_T = 10W$ and different values for N , θ , and σ^2 .

Fig. 9. Comparison against numerically optimized DPS (plotted using 'x' markers) for $P_T = 10W$, $\theta = 0.05$, and varying N , σ^2 .

Fig. 10. Variation of optimal Lagrange multipliers for $\mathcal{OP}1$ for $R > R_{th}$, $P_T = 10W$, and varying N , θ , and σ^2 .

cases, a larger portion of received power can be used for EH.

We next compare the considered UPS RX operation against the more generic Dynamic PS (DPS) design, according to which each antenna has a different PS value. Since replacing DPS in our formulation results in a non-convex problem, we obtain the optimal PS ratios for the N RX antennas from a N -dimensional linear search over the N PS ratios $\rho_1, \rho_2, \dots, \rho_N$ to select the best possible N -tuple. In Fig. 9, we plot $P_{R,E}^*$ for both UPS and DPS RX designs for 2×2 and 4×4 systems with $\theta = 0.05$ and varying σ^2 . In all cases, the performance of optimized DPS is closely followed by the optimized UPS with an average performance degradation of less than 0.9mW for $N = 2$ and 2.1mW for $N = 4$. This happens because the average deviation of all PS ratios in the DPS design from the UPS ratio ρ is less than 0.001. A similar observation regarding the near-optimal UPS performance was also reported in [14] for $N_T = 1$ at TX. This study corroborates the adoption of UPS instead of DPS that incurs very high implementation complexity without yielding relatively large gains.

Fig. 11. Validating the accuracy of the proposed high SNR approximation for the globally optimal power allocation p_1^* for $\mathcal{OP}2$.

Fig. 12. Comparison of the rate-energy trade off between the proposed joint TX and RX design and the benchmark semi-adaptive schemes OPS and OTCM for $P_T = 10W$, $\theta = 0.05$, and different N and σ^2 .

The Lagrange multipliers μ^* and ν^* in $\mathcal{OP}1$ are available in closed-form as (9a) and (9b), respectively, for $R \leq R_{th}$. However, one needs to solve a system of non-linear equation for these multipliers, as described in Section VI-A1, for $R > R_{th}$. In Fig. 10, we plot the variation of μ^* and ν^* in $\mathcal{OP}1$ for $R > R_{th}$. As shown, μ^* and ν^* monotonically increase and decrease, respectively, with increasing R . The average value for $[\mathbf{A}]_{1,1}^2$ for the considered pair values $(\theta, N) = \{(0.1, 2), (0.05, 2), (0.1, 4), (0.05, 4)\}$ is $\{0.036, 0.009, 0.093, 0.025\}$, and it is evident from Fig. 10(b) that ν^* is very close to its lower bound given by $\nu_{LB} = \rho^*[\mathbf{A}]_{1,1}^2$. Also, Fig. 10(a) showcases that the range of μ^* is similarly small to ν^* . These findings corroborate the fast convergence of Algorithm 2 that exploits the short search space of ν^* in the solution of $\mathcal{OP}1$ or ν_2^* in $\mathcal{OP}2$. Fig. 11 includes results with the derived tight asymptotic approximation $\mathbf{S}_{id,a}^*$ for the globally optimal solution $\mathbf{S}_{id}^*$ of $\mathcal{OP}2$ in Section V-B2 using the efficient implementation of Section VI-B3. As shown, the results with TX precoding design $\mathbf{S}_{id,a}^*$ (or $\mathbf{P}_{id,a}^*$), which have been obtained from the solution of (33) in β , match very closely with the results for the globally optimal design $\mathbf{S}_{id}^*$ (or $\mathbf{P}_{id}^*$) for $\mathcal{OP}2$ implemented using Algorithm 2.

We finally present in Fig. 12 performance comparison results between the proposed joint design, as obtained from the solution of $\mathcal{OP}1$, and two benchmark schemes. The first scheme, termed as Optimal TX Covariance Matrix (OTCM), performs optimization of $\mathbf{S}$ for a fixed UPS ratio $\rho = 0.5$, and the second scheme, termed as Optimal UPS Ratio (OPS), optimizes ρ for given $\mathbf{S} = \mathbf{S}_{WF}$. It is observed that for

2×2 MIMO systems, OPS performs better than OTCM, while for 4×4 system, the converse is true. This happens because OTCM performance improves with increasing N (or r). For both N value, the proposed joint design provides significant energy gains over OTCM and OPS. Particularly, the performance enhancement for $N = 2$ is 71.15% and 87.4%, respectively, over OPS and OTCM schemes, while for $N = 4$ this enhancement becomes 127.0% and 77.4%, respectively.

VIII. CONCLUSIONS

In this paper, we investigated EH as an add-on feature in conventional MIMO systems that only requires incorporating UPS functionality at reception side. We particularly considered the problem of jointly designing TX precoding operation and UPS ratio to maximize harvested power, while ensuring that the quality-of-service requirement of the MIMO link is satisfied. By proving the generalized convexity property for a specific reformulation of the harvested power maximization problem, we derived the global jointly optimal TX precoding and RX UPS ratio design. We also presented the globally optimal TX precoding design for ideal reception. Different from recently proposed designs, the solutions of both considered optimization problems with UPS and ideal RXs unveiled that there exists a rate requirement value that determines whether the TX precoding operation is energy beamforming or information spatial multiplexing. We also presented analytical bounds for the key variables of our optimization problem formulation along with tight practically-motivated high SNR approximations for their optimal solutions. We presented an algorithm for efficiently solving the KKT conditions for the considered problem for which we designed a linear complexity implementation that is based on 2-D GSS. Its complexity was shown to be independent of the number of transceiver antennas, a fact that renders the proposed algorithm suitable for energy sustainable *massive* MIMO systems considered in 5G applications. Our detailed numerical investigation of the proposed joint TX and RX design validated the presented analysis and provided insights on the variation of the rate-energy trade off and the role of various system parameters. It was shown that our design results in nearly doubling the harvested power compared to benchmark schemes, thus enabling efficient MIMO SWIPT communication. This trend holds true for any practical non-linear RF EH model. We intend to extend our optimization framework in multiuser MIMO communication systems and consider the more general non-uniform PS reception in future works.

APPENDIX A PROOF OF LEMMA 2

We associate the Lagrange multipliers $\nu_2 \geq 0$ and $\mu_2 \geq 0$ with (C2) and (C5) in $\mathcal{OP}2$ while keeping (C3) implicit. The Lagrangian function for $\mathcal{OP}2$ can be written as

$$\mathcal{L}_2 \triangleq P_R - \nu_2 (\text{tr}(\mathbf{S}) - P_T) - \mu_2 \times (R - \log_2 (\det(\mathbf{I}_{N_R} + \sigma^{-2} \mathbf{H} \mathbf{S} \mathbf{H}^H))). \quad (\text{A.1})$$

Let us first investigate the $R > R_{\text{th}}^{\text{id}}$ scenario, where for fixed $\mu_2 > 0$ and $\nu_2 > 0$, the problem of finding $\mathbf{S}$ that maximizes the Lagrangian $\mathcal{L}_2(\mathbf{S}, \mu_2, \nu_2)$ is expressed using (A.1) as

$$\mathcal{OP}3 : \max_{\mathbf{S}} \log_2 \left(\det \left(\mathbf{I}_{N_R} + \frac{\mathbf{H} \mathbf{S} \mathbf{H}^H}{\sigma^2} \right) \right) - \text{tr}(\mathbf{Q} \mathbf{S}) \quad \text{s.t. (C3)},$$

where matrix $\mathbf{Q} \in \mathbb{C}^{N_T \times N_T}$ is defined as $\mathbf{Q} \triangleq \frac{\ln 2}{\mu_2} (\nu_2 \mathbf{I}_{N_T} - \mathbf{H}^H \mathbf{H})$. $\mathcal{OP}3$ has a structure similar to the problem in [12, eq. (16)] and its bounded optimal value can be obtained for any $\mathbf{Q} \succ 0$, $\mu_2 \geq 0$, $\nu_2 > \lambda_{\max}(\mathbf{H}^H \mathbf{H})$ as

$$\mathbf{S}_{\text{id}}^* \triangleq \mathbf{Q}^{-\frac{1}{2}} \tilde{\mathbf{V}} \tilde{\Lambda}_o \tilde{\mathbf{V}}^H \mathbf{Q}^{-\frac{1}{2}} \\ = \tilde{\mathbf{V}} \left(\ln 2 \mu_2^{-1} \left(\nu_2 \mathbf{I}_r - \Lambda^H \Lambda \right) \right)^{-1} \tilde{\Lambda}_o \tilde{\mathbf{V}}^H, \quad (\text{A.2})$$

where unitary matrix $\tilde{\mathbf{V}} \in \mathbb{C}^{N_T \times r}$ is obtained from the reduced SVD of $\sqrt{\sigma^{-2}} \mathbf{H} \mathbf{Q}^{-\frac{1}{2}} = \tilde{\mathbf{U}} \tilde{\Lambda} \tilde{\mathbf{V}}^H$ with unitary matrix $\tilde{\mathbf{U}} \in \mathbb{C}^{N_R \times r}$ and diagonal matrix $\tilde{\Lambda} \in \mathbb{C}^{r \times r}$ containing the r eigenvalues of $\sqrt{\sigma^{-2}} \mathbf{H} \mathbf{Q}^{-\frac{1}{2}}$ in decreasing order. The entries of diagonal matrix $\tilde{\Lambda}_o \in \mathbb{C}^{r \times r}$, obtained by using waterfilling solution [30], are related with the diagonal entries of $\tilde{\Lambda}$ as

$$[\tilde{\Lambda}_o]_{i,i} = \left(1 - [\tilde{\Lambda}]_{i,i}^{-2} \right)^+, \quad \forall i = 1, 2, \dots, r. \quad (\text{A.3})$$

The right-hand side of the equality in (A.2) results from rewriting $\mathbf{Q}$ as $\mathbf{Q} = \mathbf{V} \left(\frac{\ln 2}{\mu_2} (\nu_2 \mathbf{I}_r - \Lambda^H \Lambda) \right) \mathbf{V}^H$, yielding

$$\frac{1}{\sqrt{\sigma^2}} \mathbf{H} \mathbf{Q}^{-\frac{1}{2}} = \mathbf{U} \frac{\Lambda}{\sqrt{\sigma^2}} \left(\frac{\ln 2}{\mu_2} (\nu_2 \mathbf{I}_r - \Lambda^H \Lambda) \right)^{-\frac{1}{2}} \mathbf{V}^H. \quad (\text{A.4})$$

Clearly, (A.4) is the reduced SVD of matrix $\sqrt{\sigma^{-2}} \mathbf{H} \mathbf{Q}^{-\frac{1}{2}}$. Thus, we set $\tilde{\mathbf{V}} = \mathbf{V}$, $\tilde{\mathbf{U}} = \mathbf{U}$, and

$$\tilde{\Lambda} = \frac{\Lambda}{\sqrt{\sigma^2}} \left(\frac{\ln 2}{\mu_2} (\nu_2 \mathbf{I}_r - \Lambda^H \Lambda) \right)^{-\frac{1}{2}}. \quad (\text{A.5})$$

Finally, $\mathbf{S}_{\text{id}}^* = \mathbf{F}_{\text{id}} \mathbf{F}_{\text{id}}^H$ where $\mathbf{F}_{\text{id}} \triangleq \mathbf{V} \mathbf{P}_{\text{id}}^{1/2}$ with the diagonal matrix $\mathbf{P}_{\text{id}}$ defined as $\mathbf{P}_{\text{id}} \triangleq \left(\frac{\ln 2}{\mu_2} (\nu_2 \mathbf{I}_r - \Lambda^H \Lambda) \right)^{-1} \tilde{\Lambda}_o$. Combining (A.3) and (A.5), the diagonal entries of $\mathbf{P}_{\text{id}}$ are

$$p_k^{(\text{id})} = \left(\frac{\mu_2^*}{\ln 2 (\nu_2^* - [\Lambda]_{k,k}^2)} - \frac{\sigma^2}{[\Lambda]_{k,k}^2} \right)^+, \quad (\text{A.6})$$

$\forall k = 1, 2, \dots, r$. For $R \leq R_{\text{th}}^{\text{id}}$, $\mathbf{S}_{\text{id}}^* = \mathbf{S}_{\text{EB}} = P_T \mathbf{v}_1 \mathbf{v}_1^H$ is deduced from the discussion in Section IV-A. Here (C5) is satisfied at strict inequality and holds $\mu_2^* = 0$ and $\nu_2^* = [\Lambda]_{1,1}^2$.

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