

# Reduced Fuel Consumption of Heavy-Duty Vehicles using Pulse and Glide

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Master of Science Thesis in Electrical Engineering

**Reduced Fuel Consumption of Heavy-Duty Vehicles using Pulse and Glide:**

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## **Abstract**

The transport sector always strive towards reduced fuel consumption for heavy-duty vehicles. One promising control strategy is to use Pulse and Glide. The method works by acceleration to a high speed and then glide in neutral gear to a low speed.

Two different control strategies and four different glide options were investigated. The two strategies were either to follow the optimal BSFC-line or using optimal control. For each strategy, different velocity spans between the upper and lower velocity were tested.

The results show that the fuel consumption can be reduced up to 8.1 % compared to a constant speed driving strategy. The fuel consumption was reduced the most for lower velocities and if the difference between the upper and lower velocity for the Pulse and Glide strategy was kept small. The fuel saving can be explained due to increased engine efficiency during the pulse. The results also show that the difference between the rule-based and optimization based control strategy is small. It can be concluded that a near-optimal strategy for a heavy-duty vehicle utilizing Pulse and Glide is to always pulse on the optimal BSFC-line.



## **Acknowledgments**

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David Forsberg och Marcus Hall*



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# Notation

## ABBREVIATIONS

Abbreviation	Meaning
PnG	Pulse and Glide
ICE	Internal combustion engine
HDV	Heavy-duty vehicle
BSFC	Brake specific fuel consumption
OCP	Optimal control problem



# 1

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## Introduction

In 2015, 75 % of all freight transportation in the European Union was done by road [17]. Heavy-duty vehicles stand for 25 % of all greenhouse gas emissions from the transport sector in the EU and is projected to grow up to 33 % in 2030 [23]. To reduce the fuel consumption for heavy-duty vehicles is therefore very important.

Human eco-driving have been tested and can reduce fuel consumption by 5 % to 15 % when using taught techniques, i.e., maintaining most economic speed, avoid unnecessary braking, and use the most optimal gear [15]. It is also found that after being educated the behaviour would slowly degrade because of human complacency. If these techniques could be translated into algorithms they would not degrade.

One promising strategy to reduce the fuel consumption is to use Pulse and Glide (PnG). This works by accelerating and then gliding in neutral gear instead of keeping a constant speed. By doing this the engine can work in operating points with better efficiency compared to constant speed driving. The concept of gliding in neutral gear have been around for a couple of years under the name Eco-roll. Scania introduced it to their heavy-duty vehicles in 2013 [21]. This works by engaging neutral gear and glide during the downhill sections of the road.

### 1.1 Motivation

Scania says that around 30 % the life cycle cost of a diesel heavy-duty truck is fuel consumption [20]. By finding methods to reduce consumption even the slightest will in turn reduce cost and CO<sub>2</sub> emissions. Pulse and Glide has showed

promising results on passenger cars [9] and will therefore be studied further on heavy-duty vehicles.

## 1.2 Purpose

The purpose of this thesis is to investigate how Pulse and Glide functions and why the method works. The Pulse and Glide method should also be compared with constant speed driving and a strategy should be developed for when Pulse and Glide can be used.

## 1.3 Problem Description

The problem description for this thesis work is:

- How does Pulse and Glide work for a rule-based method and optimization based method?
- When should Pulse and Glide be used?
- What is the optimal way to Pulse and Glide?

When answering these questions, the focus is mainly on the fuel consumption.

## 1.4 Method

First, to get an understanding of the subject, the related research in the area were examined. Then the following work plan was established:

- Create a model of the truck and a model for the engine and driveline.
- Develop a way to test the average fuel consumption of a truck in constant speed versus a truck in PnG.
- Formulate an optimal control problem and solve it by minimizing the fuel consumption with PnG.

## 1.5 Delimitations

To help with calculations and focusing on the main topic some simplifications have been made. The driveline is considered stiff for simplification of engine torque transfer to the wheels and the pump losses are not considered.

The traffic will not be considered in any scenario. Since there is no traffic, braking will not be considered either.

The thesis focuses on simulations and no live vehicle experiments are done.

The code developed in this thesis is not intended for live vehicle implementations and computational complexity is not considered.

## **1.6 Thesis Outline**

The thesis is divided into 7 chapters. Chapter 2 shows related work and research of Pulse and Glide. Chapter 3 goes through all the models used in detail. Chapter 4 focus on the different strategies of Pulse and Glide and the method is set up as an optimal control problem. Chapter 5 is results and Chapter 6 is discussion of the results. Chapter 7 is conclusions and future work.



# 2

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## Related Research

In this chapter the related research is presented. The relevant areas are: modeling and simulation, longitudinal control, Pulse and Glide, and optimal control.

### 2.1 Modeling and Simulation

Modeling and simulation are very important tools for the automotive industry. By having high quality models, new technologies can be tested in simulation. This speed up development time and thus saves the industries money.

Sivertsson and Eriksson have developed an engine model in [24]. The model is of a diesel-electric powertrain. It is a mean value engine model that consist of four states and three control signals. The four states are engine speed, intake manifold pressure, exhaust manifold pressure, and turbocharger speed. The control signals are fuel injection per cycle, wastegate position, and generator power. The model captures the non-linearity of a diesel engine and is stated to be well suited for optimal control.

For more information about engine modeling, see Eriksson and Nielsen [6].

### 2.2 Longitudinal Control

This section covers methods that can improve the fuel consumption by only changing the way the vehicle is controlled. First the simple cruise control is presented

and then more advanced methods like look-ahead control, eco-roll, and platooning.

The most common longitudinal control system is the cruise control and is described by Rajamani [19]. The system works by holding a desired constant speed regardless of the circumstances. This is an improvement in driving comfort as the driver can keep the foot off the pedal but if a preceding vehicle is travelling at a lower speed the driver must manually brake. From a fuel consumption point of view the cruise controller is best suited for flat terrain. Since the controller always tries to keep the desired speed, if the vehicle is driving in a steep hill the controller increases the throttle and thus the fuel consumption. The same problem goes for driving downhill, as the controller must apply brakes to slow down the vehicle to the desired speed, a lot of energy is dissipated.

A more advanced system is the look-ahead controller [7]. It uses information about the road topography ahead of the vehicle and a navigation system to see if an uphill or downhill section of the road is approaching. The controller is then matching the speed to avoid unnecessary gear shifts and braking. For example, if a vehicle is approaching a slope it starts to glide to avoid braking in the slope. In the article by Hellström et al. [7] they are investigating a fuel optimal look-ahead control for a single heavy-duty vehicle without increasing travel time. Hellström et al. designs an efficient algorithm that uses dynamic programming to find the optimal solution to the optimal control problem. From the results, the authors conclude that the look-ahead controller saves up to 3.5 % fuel, with no increase in trip time, and with 42 % less gear shifts, compared to a conventional cruise controller.

More recent studies are combining the look-ahead control with eco-roll [14], [5]. By putting the gear in neutral during downhill sections of the road the fuel consumption can be reduced. When the gear is in neutral the engine is not connected to the wheels and fuel is only needed to keep the engine idling and the vehicle can roll for a longer distance compared to when a gear is engaged. The master thesis by Mancino [14] and Chen [5] have investigated this and put together an algorithm on a Scania truck that manages to reduce fuel consumption by almost 1 % on an example road.

Platooning is another hot research topic. Platooning uses the reduction in air resistance when following close to another vehicle using the slipstream it creates. It increases the risk of rear ending the leading car and since the distance between cars are reduced a robust control system is needed. In Alam's doctoral thesis [1], a framework for platooning with Eco-rolling and communication between the trucks are presented. The results show that the fuel consumption can be reduced by 3.9-6.5 % in a real-life experiment. It is also demonstrated how the fuel saving potential depends on the position of the vehicle in a platoon. The vehicles that is behind the first in the platoon had more fuel reduction than the first vehicle, but even the first vehicle saved fuel by driving in a platoon. Ohlsén and Sten [18] introduced a more sophisticated engine model than Alam. By using dynamic programming to optimize the fuel consumption for platooning with a



look ahead controller together with Pulse and Glide, the fuel consumption could be reduced, and Pulse and Glide was responsible for around 1-3 % of the fuel reduction.

For more information on these methods see Vehicle Dynamics and Control by Rajamani [19].

## 2.3 Pulse and Glide

Pulse and Glide is a driving method that works by accelerating in a very efficient engine operating points and then glide, i.e., rolling without input from the gas pedal. In simulations this have showed to decrease fuel consumption by almost 9 % [12]. In no traffic and flat ground, the Pulse and Glide will achieve a cycle of accelerating to a certain speed and then coast to a lower speed.

In the simulations made by Li et al. [12], they use two cars where one is following the other. The leading car is going in constant speed and the following car is using a Pulse and Glide strategy. This way the relative distance and speed error between the cars can be used to control the following vehicle that will find a motion cycle that it will repeat. Li at al. introduces a graphic explanation of the Pulse and Glide strategy and why the fuel consumption can be reduced compared to driving in a constant speed. They also show how the fuel consumption for different speed depends on which gear they use for pulsing and gliding. A part of what makes Pulse and Glide possible is the non-linearity of the fuel rate and engine power along the optimal brake specific fuel consumption line (BSFC). The optimal BSFC-line is where the engine is most efficient for each engine speed and engine torque. [12].

Li and Peng [11] show that with an ideal engine using Pulse and Glide is not the most optimal driving strategy. This is because with an ideal engine the main contributory in power demand is air resistance that increases proportional to the square of speed. They also showed that when using a real engine, the Pulse and Glide strategy is no longer optimal when the minimum BSFC-point have been met. This is not a surprising thing since it is the minimal BSFC-point that is being exploited in this strategy.

Xu et al. [25] show how the Pulse and Glide method works for a passenger car with a step-gear transmission and why the consumption can be lowered. The authors claim that the PnG strategy is useful in the speed range of 30-120 km/h and reduces the fuel consumption compared to driving in a constant speed. They develop an optimization strategy based on numerical optimal control. From the results they show that the fuel consumption can be reduced up to 32.6 % compared to driving at a constant speed of 60 km/h during ideal conditions. The most fuel-efficient strategy for the gliding part is to glide in neutral gear and shut off the engine. The other strategies for the gliding part in descending order are to glide in neutral gear with engine idling, glide in different gear than the

pulse gear and glide with the same gear as used for the pulsing part. They also calculated for which gear and speed, the best efficiency is.

As the automotive industry always strives for reduced fuel consumption hybridization is a hot topic. Xu et al. have investigated the usage of Pulse and Glide in hybrid electric vehicle in [26], where the vehicle is charge-sustaining. For fuel economy the best strategy is still to pulse and glide with only the internal combustion engine. This is because the vehicle inertia has less conversion losses than the battery. The benefit of PnG in hybrid vehicles is that it offers an additional degree of freedom as you can choose either to glide or charge the battery. This can be useful for driving comfort and in dense traffic situations. With this method the ICE can operate at an efficient point without compromising the driving comfort as the speed fluctuates.

In the article by Li et al. [13], a Pulse and Glide strategy for an entire platoon is investigated with both the preceding and the following vehicle using PnG. They design a switching logic for PnG that takes the intervehicle distance into consideration. The authors claim that the fuel reduction is up to 21.8 % compared to a LQ-based controlled platoon.

## 2.4 Optimal Control

To reduce the fuel consumption in heavy-duty vehicles, the problem is set up as an optimal control problem. There are several different ways to solve the optimal control problem. One often used method is dynamic programming [3]. This is an optimization method that can solve complex problems by dividing it into sub-problems. Dynamic programming is best suited for low dimension problems. For higher order dimensions the complexity increases exponentially. This is referred to as the "curse of dimensionality" [4].

Hellström et al. [7] uses a dynamic programming algorithm to minimize the fuel consumption for a look-ahead controller. To speed up the computations they use an approach developed by Monastyrsky and Golownykh [16]. By transferring the trip time into the objective function with a weight-parameter  $\beta$ , that is a trade-off between fuel consumption and trip time, the dimension is reduced. With this formulation trip time is no longer a state and the complexity is reduced. The problem is instead how to set the parameter  $\beta$ .

Another approach introduced to speed up computations by Hellström et al. is by only considering a truncated horizon in each optimization. Since the driving conditions may change during a mission, the optimization must be computed during the drive mission. By introducing the truncated horizon, an approximate solution is found and the accuracy depends of the length of the horizon. The same optimization approach is used in Ohlsén and Sten [18] and Mancino [14].

A strategy to solve the optimal control problem is presented in Xu et al. [25] and in [26]. The authors convert the non-smooth optimal control problem to non-

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linear programming problem by using the multi-phase knotting pseudospectral method which gives more accuracy.



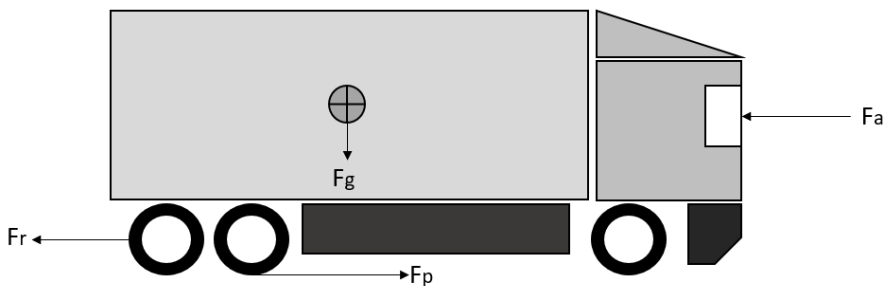
# 3

## Modeling

In this chapters the models used in this work are presented. The chapter is divided into Vehicle model, Engine model, driveline model, constant speed model, and gear model.

### 3.1 Vehicle Model

The forces acting on a truck driving on a flat road is air resistance  $F_a$ , rolling resistance  $F_r$ , propulsion force  $F_p$ , and gravitational force  $F_g$ , which can be seen in Figure 3.1. The air resistance is based on air density  $\rho_a$ , vehicle-to-air-relative velocity  $v$ , the cross-sectional area  $A$ , and the drag coefficient  $C_D$ , in the following equation



**Figure 3.1:** Forces acting on a truck driving on flat road.

$$F_a = \frac{1}{2} \rho_a C_D A v^2 \quad (3.1)$$

The rolling resistance is based on the mass, the rolling resistance coefficient, and the gravitational constant and is thus a constant force calculated as

$$F_r = C_r m_v g \quad (3.2)$$

The gravitational force is not considered since the vehicle is driving on a flat road.

The force that overcomes the resistance forces and drives the vehicle forward is called the propulsion force. The propulsion force is generated from the engine torque and is then transferred to the wheels through the driveline. The propulsion force is further explained in Section 3.2.

The dynamics of the vehicle can then be explained with Newton's 2nd law of motion according to

$$m_v \frac{dv}{dt} = F_p - F_a - F_r \quad (3.3)$$

The used symbols and their descriptions can be found in Table 3.1.

**Table 3.1:** Description of symbols and their units

Symbol	Description	Unit
$F_a$	Air resistance	[N]
$F_r$	Rolling resistance	[N]
$F_p$	Propulsion force	[N]
$\rho_a$	Air density	[kg/m <sup>3</sup> ]
$v$	Vehicle-to-air relative velocity	[m/s]
$C_D$	Drag coefficient	[-]
$A$	Cross-sectional area	[m <sup>2</sup> ]
$C_r$	Rolling resistance coefficient	[-]
$m_v$	Vehicle mass	[kg]
$g$	Gravitational constant	[m/s <sup>2</sup> ]

## 3.2 Engine Model

The used engine model is developed by Sivertsson and Eriksson [24]. The engine model is a mean value engine model, (MVEM), and the torque is obtained by calculating the indicated engine torque and subtract with the engine friction. The formulas used to calculate the engine torque is described by

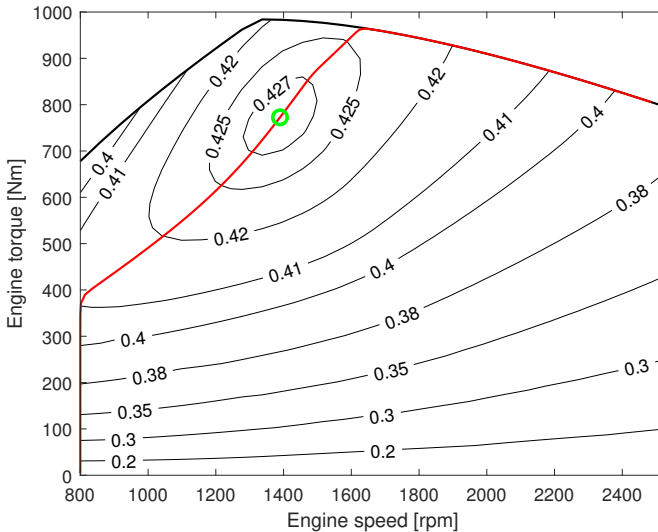
$$T_e = \frac{W_{ig} - W_{fric}}{2\pi \cdot n_r} \quad (3.4)$$

$$W_{ig} = n_{cyl} \cdot q_{LHV} \cdot u_f \cdot \eta_{ig} \quad (3.5)$$

$$W_{fric} = V_D \cdot (\alpha_1 \cdot \omega_e^2 + \alpha_2 \cdot \omega_e + \alpha_3) \quad (3.6)$$

where  $u_f$  is the fuel injection,  $\eta_{ig}$  is the indicated efficiency,  $q_{LHV}$  is the lower heating value of the fuel,  $n_{cyl}$  is the number of cylinders,  $V_D$  is the displaced volume,  $\omega_e$  is the engine speed,  $\alpha$  are friction coefficients, and  $n_r$  is the number of crank revolutions per cycle. The symbols are explained in Table 3.2.

From [24], a typical engine map is also derived. The engine map is obtained by measuring engine torque, engine speed and injected fuel on an engine. From the measurement the efficiency in every operating point, the maximum torque, and engine speed can be obtained and the engine map can be modeled depending on the torque model. The used engine map can be seen in Figure 3.2. For every engine speed there is an engine torque that gives the most efficient operating point. This is called the optimal break specific fuel consumption line, also called optimal BSFC-line, shown as a red line in Figure 3.2. The most efficient point in the map is shown as a green dot in the figure.



**Figure 3.2:** Shows the engine map. The red line is the optimal BSFC-line and the green dot is the most efficient point in the map.

The fuel consumption for the engine can be obtained from the fuel mass flow that is given by

$$\dot{m}_f = \frac{n_{cyl}}{2\pi \cdot n_r} \omega_e \cdot u_f \quad (3.7)$$

When the engine is idling, the fuel mass flow is calculated according to

$$\dot{m}_{f,idle} = \frac{n_{cyl}}{2\pi \cdot n_r} \omega_{e,idle} \cdot u_{f,idle} \quad (3.8)$$

When the vehicle is gliding with a gear engaged, the fuel mass flow is zero. The engine is connected to the driveline and the vehicle is slowed down by the engine drag torque which propagates down the driveline and translates to a friction force according to

$$F_{fr} = \frac{T_{e,fr} \cdot i_g \cdot i_f}{r_w} \quad (3.9)$$

where  $T_{e,fr}$  is the engine drag torque that is calculated as

$$T_{e,fr} = \frac{W_{fric}}{2\pi \cdot n_r} \quad (3.10)$$

where  $W_{fric}$  is calculated with Equation (3.6).

The used parameters in Equation (3.7) - (3.10) can be found in Table 3.2.

### 3.3 Driveline Model

The driveline consists of all the component that translates the torque from the engine to a force at the wheels that propels the vehicle. The parts are the clutch, transmission, propeller shaft, final drive, drive shaft, and the wheels. The engine and the driveline can be seen in Figure 3.3. In this model the driveline is assumed to be stiff with a no slip relation. All the components in the driveline are also assumed to be ideal which means that there are no losses from the engine to the wheels.

With a stiff driveline and with no slip, the relation between vehicle speed and engine speed can be found. The rotational speed of the wheel is given by

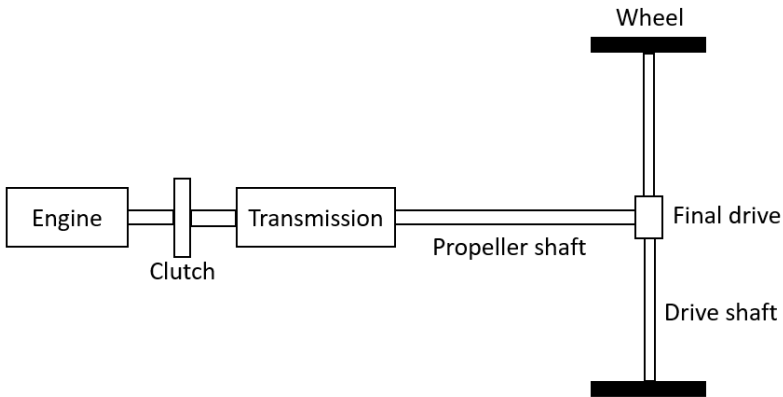
$$\omega_w = \frac{v}{r_w} \quad (3.11)$$

The engine speed can then be obtained by considering the gear ratio of the transmission and the final drive ratio, which is given by



**Table 3.2:** Description of symbols and their units

Symbol	Description	Unit
$T_e$	Engine torque	[Nm]
$T_{e,ig}$	Engine indicated torque	[Nm]
$\dot{m}_f$	Fuel mass flow	[kg/s]
$\omega_e$	Engine speed	[rad/s]
$\dot{\omega}_e$	Engine acceleration	[rad/s <sup>2</sup> ]
$\omega_w$	Wheel speed	[rad/s]
$u_f$	Fuel injection	[mg/cycle-cylinder]
$\eta_e$	Engine efficiency	[-]
$\eta_{ig}$	Indicated efficiency	[-]
$q_{LHV}$	Lower heating value	[J/kg]
$F_{fr}$	Friction force	[N]
$T_{e,fr}$	Friction torque	[Nm]
$i_g$	Gear ratio	[-]
$i_f$	Final gear ratio	[-]
$r_w$	Radius of the wheels	[m]
$J_e$	Engine inertia	[kgm <sup>2</sup> ]
$W_{fric}$	Friction work	[J]
$n_r$	Crank revolutions per cycle	[-]
$V_D$	Displaced volume	[m <sup>3</sup> ]
$\alpha_1$	Quadratic engine friction coefficient	[-]
$\alpha_2$	Linear engine friction coefficient	[-]
$\alpha_3$	Static engine friction coefficient	[-]

**Figure 3.3:** A sketch of the engine and the driveline

$$\omega_e = \omega_w \cdot i_g \cdot i_f \quad (3.12)$$

The engine speed in radians per second can then be converted to revolutions per minute with the following equation

$$N_e = \frac{30}{\pi} \cdot \omega_e \quad (3.13)$$

The produced torque from the engine is transferred through the driveline to the wheels. As the driveline is assumed to be stiff, the relation between engine torque and vehicle propulsion force is given by

$$F_p = T_e \frac{i_g \cdot i_f}{r_w} \quad (3.14)$$

where the used parameters can be found in Table 3.2.

### 3.4 Constant Speed Model

To compare Pulse and Glide, a model for the constant speed driving is developed. Since the speed is constant, the acceleration is zero. The propulsion force is then equal to the resistance forces, according to

$$F_p = F_a + F_r \quad (3.15)$$

With the propulsion force given, the engine torque can be calculated by rewriting Equation (3.14)

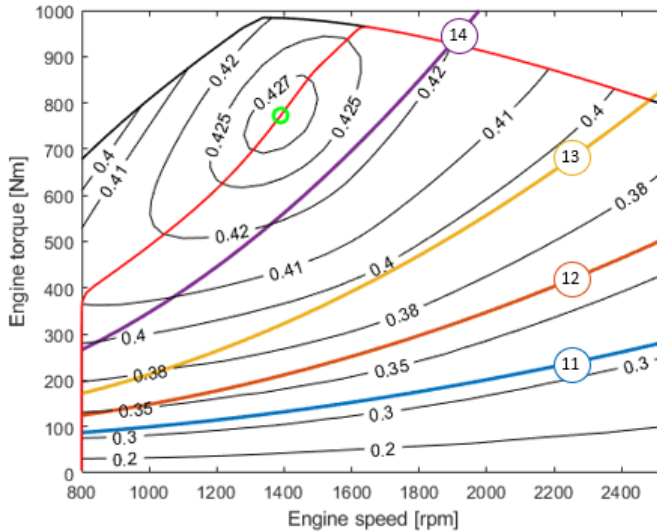
$$T_e = \frac{F_p \cdot r_w}{i_g \cdot i_f} \quad (3.16)$$

For a given vehicle speed, the engine speed can be calculated from Equation (3.11) and (3.12). With the engine torque and engine speed, the fuel injection can be calculated from Equation (3.4)-(3.6). The fuel mass flow for constant speed driving is then given by

$$\dot{m}_{f,const} = \frac{n_{cyl}}{2\pi \cdot n_r} \omega_{e,const} \cdot u_{f,const} \quad (3.17)$$

During constant speed driving, the load on the engine is reduced compared to acceleration phase. In Figure 3.4, the engine map with gear lines for constant speed is showed. For a given speed, the engine is always operating on a point that lies on one of the gear lines. In the figure none of the lines goes through the best efficiency area. To get the operation points on the optimal BSFC-line the vehicle

needs to be accelerating. In order to always be in high efficiency operation points the vehicle will have to shift between accelerating and decelerating i.e. Pulse-and-Glide.



**Figure 3.4:** Shows the engine map with gear lines for constant speed driving. The red line with a green circle is the optimal BSFC-line. The blue, orange, yellow and purple lines show the efficiency when driving in constant speed for gear 11, 12, 13, and 14.

## 3.5 Gear Shift Model

When the PnG strategy is used, more gear shifts are done compared to a constant speed driving strategy. In the start of the glide phase the engine does not instantly go down to idle engine speed and this needs to be considered for a more accurate fuel consumption model. For the gear shifting process, the same approach is used as in Hellström et al. [7] and Ivarsson et al. [8].

### 3.5.1 Gear Shift

The most common transmission for heavy duty vehicles is the automated manual transmission. The transmission does not use a regular clutch and gear shifts are done by engine control. First the transmission is controlled to a state where no torque is transmitted, in order to easily disengage and engage a gear. The engine

torque is then controlled such as the input and output rotational speeds of the transmission is matched. The dynamics of the gear change can be explained with the following equation

$$J_e \dot{\omega}_e = T_e = T_{e,ig} - T_{e,fr} \quad (3.18)$$

where  $J_e$  is the engine inertia,  $\dot{\omega}_e$  is the engine acceleration, and  $T_e$  is engine torque which is obtained from the indicated engine torque and the engine friction.

To synchronize the current engine speed with the new engine speed, fuel has to be injected if the gear is shifted down. In case of an up-shift, the engine is slowed down by the engine drag torque and no fuel is needed. The synchronization of engine speed is equivalent to changing the rotational energy, calculated as

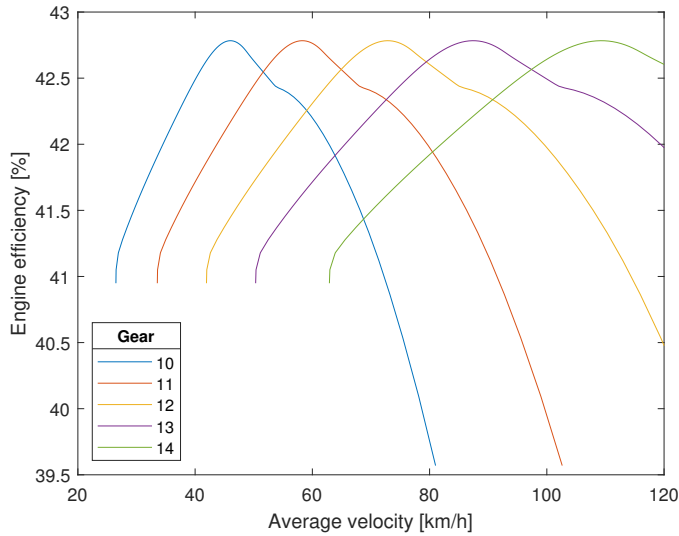
$$\Delta e = \frac{1}{2} J_e (\omega_1^2 - \omega_0^2) \quad (3.19)$$

where  $\omega_1$  is the final engine speed and  $\omega_0$  is the initial engine speed. The change of rotational energy is then added to the fuel consumption.

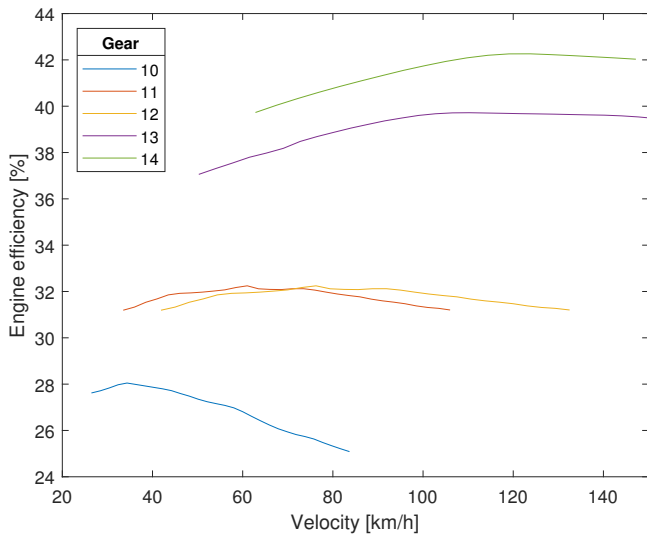
### 3.5.2 Gear Selection

The number of gears and what their gear ratios are is of vital importance since the right gear gives better efficiency. When moving along the optimal BSFC-line the efficiency of all the gears are known for each speed. To be able to get an overview of the efficiency, the relevant gears are plotted at different speeds in Figure 3.5.

For constant speed driving, the chosen gear should always be as high as possible without stalling the engine. By choosing a high gear, the engine speed is reduced and thus also the engine friction, which gives a higher efficiency. The engine efficiency for different gears and velocities for constant speed driving can be seen in Figure 3.6.



**Figure 3.5:** Engine efficiency for different gears and average velocities when moving along the optimal BSFC-line.



**Figure 3.6:** Engine efficiency for different gears and velocities during constant driving.



# 4

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## Pulse and Glide

In this chapter the Pulse and Glide strategy is explained and analyzed. First the background is presented, then a strategy for following the optimal BSFC-line is developed. After that, the Pulse and Glide method is formulated as an optimal control problem. Lastly, the energy requirements are analyzed.

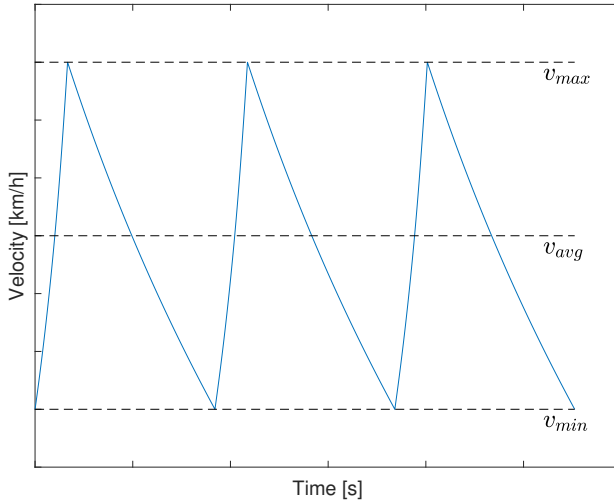
### 4.1 Background

Pulse and Glide is a driving strategy that works by shifting between acceleration and deceleration. Instead of keeping a constant speed, PnG accelerates up (pulse) to a high velocity and then coast (glide) to a low velocity. The main idea of PnG is to put kinetic energy into vehicle during the acceleration and then use that energy to glide. An internal combustion engine is operating at its most efficient point during high loads. By pulsing, the engine can work in those operating points compared to constant speed driving, where the load is lower and efficiency worse. During the pulse phase, the fuel consumption is higher, and the vehicle is accelerating forward until the chosen upper velocity is reached. The vehicle is then decelerating until the chosen lower velocity is reached and the cycle starts over again. An example is showed in Figure 4.1

The PnG strategy shows promising results in Li et al. [13] and Xu et al. [25]. The authors focused on cars with step-gear transmissions. The simulations showed fuel saving for every different PnG strategy compared to constant speed driving. Scania has showed that it works for heavy-duty vehicles already in production [22].

Although research has been done on the topic, the focus has been on passenger

cars and not on heavy-duty vehicles.



**Figure 4.1:** Plot showing the difference in velocity when using Pulse and Glide between  $v_{min}$  and  $v_{max}$  and holding the average velocity  $v_{avg}$

## 4.2 Optimal BSFC-line Method

The optimal BSFC-line method is a PnG strategy to always operate on the optimal BSFC-line when pulsing. In this section the calculation of the PnG phases are described as well as the switching from pulse to glide. Lastly, the gear shift is presented.

### 4.2.1 Pulse Phase

During the acceleration phase the engine is assumed to always operate on the optimal BSFC-line. By doing this assumption, the pulse trajectory is known. Since a starting velocity is set, the engine speed for the chosen gear is known and the engine torque can be obtained from the engine map. The gear will be chosen such that the chosen gear has the highest efficiency for the average velocity, see Figure 3.5. The following acceleration formula is then achieved

$$a_{pulse} = \frac{F_p - F_a - F_r}{m_v} \quad (4.1)$$



with  $F_p$  defined as

$$F_p = \frac{T_{e,BSFC} \cdot i_g \cdot i_f}{r_w} \quad (4.2)$$

With the acceleration known, the velocity can then be calculated by integrating the acceleration over the time spent in pulse mode.

$$v_{pulse} = \int_0^{t_p} a_{pulse} dt \quad (4.3)$$

With the velocity known, it can be integrated again to get the distance travelled.

$$s_{pulse} = \int_0^{t_p} v_{pulse} dt \quad (4.4)$$

The description of the symbols can be seen in Table 4.1.

**Table 4.1:** Description of symbols and their units

Symbol	Description	Unit
$a_{pulse}$	Pulse acceleration	[m/s <sup>2</sup> ]
$T_{e,BSFC}$	Torque at the optimal BSFC-line	[Nm]
$v_{pulse}$	Pulse velocity	[m/s]
$t_p$	Time in pulse	[s]
$s_{pulse}$	Pulse distance	[m]
$a_{glide}$	Glide acceleration	[m/s <sup>2</sup> ]
$F_{fr}$	Engine friction force when gliding with gear engaged	[N]
$v_{glide}$	Glide velocity	[m/s]
$t_g$	Time in glide	[s]
$s_{glide}$	Glide distance	[m]
$\dot{m}_{f,pulse}$	Pulse fuel mass flow	[kg/s]
$\dot{m}_{f,idle}$	Idle fuel mass flow	[kg/s]
$\bar{v}$	Average velocity	[m/s]

## 4.2.2 Glide Phase

During the glide phase, there are a few different options available. The engine can either be turned on or turned off and the gearbox can be in neutral or with a gear engaged. For the glide phase, the following options have been examined.

- Glide in neutral with the engine off
- Glide in neutral with the engine on
- Glide in gear with the engine on

For the case when the gearbox is in neutral, the engine is decoupled from the driveline which reduces the engine drag torque. The only remaining resistance forces are aerodynamic drag and rolling resistance. With this approach a large amount of the stored kinetic energy can be used, allowing the vehicle to glide for a longer distance. The deceleration for the glide phase can be calculated using the following formula

$$a_{glide} = \frac{-F_r - F_a}{m_v} \quad (4.5)$$

The other case is to glide with a gear engaged. By doing this the engine drag torque is included and the deceleration is therefore higher. No fuel is needed since the fuel injection is cut off and the engine uses the kinetic energy from the vehicle to keep the engine running. When gliding in a higher gear the friction force is lower. With a gear engaged, the deceleration for the glide phase is calculated as

$$a_{glide} = \frac{-F_{fr} - F_r - F_a}{m_v} \quad (4.6)$$

where  $F_{fr}$  is the friction force defined as

$$F_{fr} = \frac{T_{e,fr} \cdot i_g \cdot i_f}{r_w} \quad (4.7)$$

where  $T_{e,fr}$  is the engine friction torque that is calculated with Equation (3.9) and (3.6). From the acceleration, the velocity can be obtained by integrating over the time in glide.

$$v_{glide} = \int_{t_p}^{t_p+t_g} a_{glide} dt \quad (4.8)$$

And again, for the distance travelled.

$$s_{glide} = \int_{t_p}^{t_p+t_g} v_{glide} dt \quad (4.9)$$

### 4.2.3 Switching Map

One important factor to consider when the PnG method is used is that the average velocity can change compared to constant speed driving. To avoid this a switching logic for the method is developed. This is done by creating a phase curve where the velocity and the distance of the vehicle is compared with the average velocity and the average distance for a vehicle traveling at a constant speed.

When the vehicle is using the optimal BSFC-line method, the torque trajectory is already known. By using Equation (4.1) - (4.4), the velocity trajectory and the distance trajectory can be obtained. The glide phase is also known and depend on which glide strategy that is used. For a given strategy, the velocity and distance trajectory can be calculated with Equation (4.5) - (4.9). The relative difference for the distance and velocity for both the pulse and glide curves can be calculated according to

$$\Delta s_{pulse} = s_{pulse} - \bar{v}t_p \quad (4.10)$$

$$\Delta v_{pulse} = v_{pulse} - \bar{v} \quad (4.11)$$

$$\Delta s_{glide} = s_{glide} - \bar{v}t_g \quad (4.12)$$

$$\Delta v_{glide} = v_{glide} - \bar{v} \quad (4.13)$$

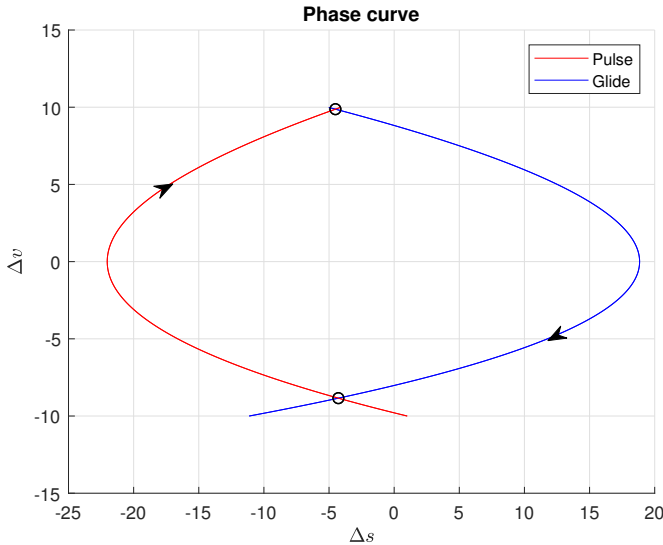
where the symbols are explained in Table 4.1.

Equation (4.10) and (4.11) are then put together to create the pulse curve, and Equation (4.12) and (4.13), to create the glide curve. The two curves can then be plotted together and where they intersect, the switching from pulse to glide or glide to pulse is done. The vehicle is always moving on the line and the PnG operation is moving clockwise in the phase curve and creates a switching logic for PnG, see Figure 4.2.

When the phase curve is calculated, different upper and lower velocities can be tested by introducing a constant to the relative distance for the pulse phase as

$$\Delta s_{pulse} = (s_{pulse} - \bar{v}t_p) + k \quad (4.14)$$

The relative distance for the pulse curve is therefor moved to the right and a new pair of intersection points is received. With the new intersections the vehicle is using the PnG method in a smaller range between the upper and lower velocity and the ellipse of the switching logic is smaller. The constant  $k$  can be used as a parameter for testing different velocity spans while still making sure that the average velocity for the PnG operation is kept the same as for the constant

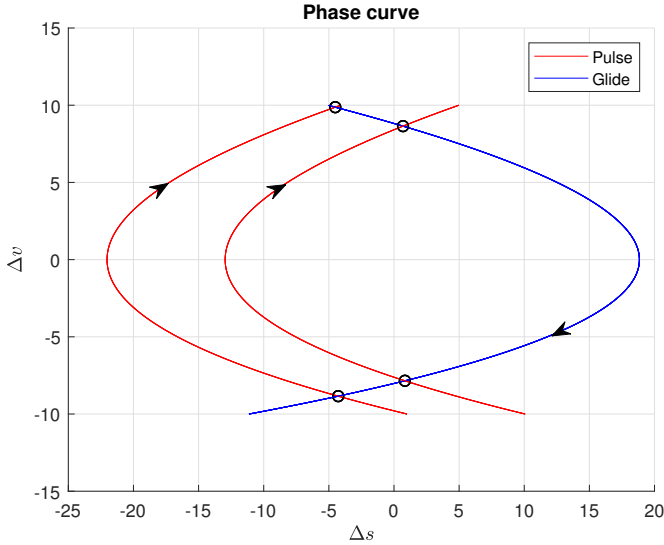


**Figure 4.2:** Phase curve for the PnG operation. The red curve shows the difference in distance and velocity for the pulse phase, compared to constant speed driving. The blue curve shows the difference in distance and velocity for the glide phase, compared to constant speed driving. The intersections of the blue and red curves show the upper and lower velocity for the PnG operation. The arrows show the how the difference from the average velocity and distance is moving.

speed driving. The phase curve for two different  $k$ -values can be seen in Figure 4.3.

The complete strategy for using Pulse and Glide, with the information from the phase curve, can be stated as

1. Set a desired average velocity, a maximum velocity, and a minimum velocity.
2. Use Figure 3.5 and choose the most efficient gear for the average velocity.
3. Choose a glide strategy and use Equation (4.1) - (4.9) to calculate the velocity trajectory and the distance trajectory for the pulse and glide phases.
4. Use Equation (4.10) - (4.14), and set a  $k$ -value, to calculate the phase curve.
5. Try different  $k$ -values until the wanted velocity span is reached.
6. Pick the values of the the upper and lower velocity, i.e., where the pulse and glide curves intersect.
7. Set the upper and lower velocity for PnG to those velocities and calculate the corresponding engine speeds. Calculate the engine torque on the op-



**Figure 4.3:** Phase curve for two different  $k$ -values. The red curves show the difference in distance and velocity for the pulse phase, compared to constant speed driving. The blue curve shows the difference in distance and velocity for the glide phase, compared to constant speed driving. The intersections of the blue and red curves show the upper and lower velocity for the PnG operation. The arrows show the how the difference from the average velocity and distance is moving.

timal BSFC-line for the engine speeds and pulse between those speeds by following the optimal BSFC-line.

The same strategy can be used for different Pulse and Glide strategies. The appearance of the phase curve is going to be different, but the approach is the same. By using this strategy to calculate the upper and lower velocity, the average velocity for the PnG operation is kept the same as for constant speed driving.

#### 4.2.4 Fuel Consumption

The fuel consumption for the PnG operation can be calculated by first determine the fuel mass flow. For the first assumption when the engine is turned off during the glide phase, fuel is only spent during the pulse. The fuel mass flow is calculated as

$$\dot{m}_{f,pulse} = \frac{n_{cyl}}{2\pi \cdot n_r} \omega_{e,pulse} \cdot u_{f,pulse} \quad (4.15)$$

For the optimal BSFC-line, with a known engine speed, the fuel injection can be obtained from the engine map.

Next case is the assumption that the engine is idling during the glide phase. The fuel mass flow can be calculated by first determined the fuel mass flow for the pulse and then for idle. The idle fuel flow is constant and does not change. The fuel mass flow for the glide phase when the engine is idling is calculated with Equation (3.8).

The total fuel mass is then calculated by integrating the fuel mass flow for the pulse over time spent pulsing and the fuel mass flow for idling over the time spent gliding.

$$m_{f,tot} = \int_0^{t_p} \dot{m}_{f,pulse} dt + \int_{t_p}^{t_p+t_g} \dot{m}_{f,idle} dt \quad (4.16)$$

The total distance travelled is first calculated and then the fuel consumption can be determined.

$$s_{tot} = s_{glide} + s_{pulse} \quad (4.17)$$

$$fuel/meter = \frac{m_{f,tot}}{s_{tot}} \quad (4.18)$$

## 4.2.5 Gear Shift

The gear shifts from neutral to a pulse gear and vice versa are assumed to occur instantly with no delay. When gliding it is assumed to be enough time to prepare to switch gear into gear at the right time. If there is no time to prepare for a gear shift, there is a short moment when there is no torque from the engine. It is also assumed that the engine can be turned on instantly when enough preparation time is given as well as no fuel cost for turning it on and off.

It is assumed that the demanded engine torque is achieved directly, however, in reality it takes time to reach the wanted torque due to turbo lag. When the fuel injection is high and the charging pressure is low it creates black smoke and to avoid this the fuel injection is limited for short period until the turbocharger have speed up [8]. With the assumption that the gear shift occurs instantly, the engine is always switching between operating on the optimal BSFC-line and idling.

## 4.2.6 Engine and Driveline Synchronization

When a glide starts the engine flywheel still has a lot of energy from the high engine speed in the end of a pulse. With a flywheel the engine speed cannot drop

instantly. During the time it takes for the engine speed to reach the idle engine speed, no fuel is injected into the engine. When this is not accounted for the engine speed drops instantly to idle engine speed and the energy in the flywheel is dissipated.

In Section 3.5.1, the energy from a gear shift is presented. The energy, demanded or gained, from the gear shift is calculated with Equation (3.19). When shifting into neutral for a glide, the energy saved is calculated and then converted to fuel. The fuel is then subtracted from the fuel needed to keep the engine at idle speed for the whole duration of the glide. The fuel is calculated as

$$m_{save} = \Delta e \cdot q_{LHV} \quad (4.19)$$

$$m_{f,idle} = \dot{m}_{f,tot} \cdot t_g \quad (4.20)$$

$$m_{f,idle,tot} = m_{f,idle} - m_{save} \quad (4.21)$$

## 4.3 Optimal Control

The Pulse and Glide strategy can be set up as an optimal control problem where the objective is to minimize the fuel consumption. For this problem the YOP toolbox is used [10]. The toolbox is based on the CasADi optimal control software [2]. The toolbox provides two different ways to discretize optimal control problems using either direct multiple shooting or direct collocation.

The direct methods work by transforming the optimal control problem to a nonlinear programming problem. The direct multiple shooting conversion is done by first discretizing the independent variable into a finite number of intervals. The independent variable is the time. In each time interval the control signal is parametrized, often as a constant. This makes the control signal constant in each of the intervals. Then the continuous time dynamics is discretized one interval at the time using a numerical integrator. The control signal in these segments are the constant assigned values for the segments. Since the system have been integrated separately on each time interval it is no longer continuous. To fix this a constraint binding these together exists in the nonlinear programming problem and thus it has been converted.

In the case of direct collocation, it follows almost the same idea as the direct multiple shooting. However, in the direct collocation the numerical integration methods is in the nonlinear programming problem. The used integration method is an implicit Runge-Kutta method based on collocation. This method approximates the state trajectory using polynomial interpolation. Starting with a few of predetermined points the polynomial is differentiated according to time. At the start points the polynomial is equal to the state dynamics. Following this both the pronominal and the state trajectory can be integrated. For this problem the direct collocation method was used.

For the optimal control problem formulation, the Pulse and Glide cycle start at the glide phase at the upper velocity. For a given upper and lower velocity the glide trajectory is already known which means that the mass fuel and the distance travelled for the glide phase is known, using the equations in Section 4.2.2. With this information only the pulse phase is directly optimized but it is also the optimal solution for the complete Pulse and Glide cycle.

### 4.3.1 Objective

For the optimal control problem, the goal is to minimize the fuel consumption. The fuel consumption is expressed as fuel mass per meter. This is because when testing different velocities, the pulse and glide time will be different and thus the distance travelled will as well. The objective is formulated as the sum of the mass fuel used in pulse and in glide over the sum of the distance travelled in pulse and in glide. The objective can be written as

$$J = \frac{m_p + m_g}{s_p + s_g}$$

### 4.3.2 System Dynamics

The model in the optimal control problem has to follow the engine and driveline dynamics shown in shown in Chapter 3. The system dynamics can be explained with the following equations

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)) \\ \dot{x} &= [\dot{s}, \dot{v}, \dot{m}_f] \\ \dot{s} &= v \\ \dot{v} &= \frac{F_p - F_a - F_r}{m_v} \\ \dot{m}_f &= \frac{n_{cyl}}{2\pi \cdot n_r} \omega_e \cdot u_f\end{aligned}$$

### 4.3.3 Initial and Terminal Conditions

The optimal control problem must fulfill a set of constrains for the first and last step of the optimization. The Pulse and Glide cycle begins at the highest velocity and then start to glide. During the glide phase the vehicle cannot be controlled. The consumed fuel during the glide phase can be determined prior to the optimization depending on which glide strategy that is used. During the time when the vehicle is gliding from the upper velocity to the lower velocity, the distance



travelled, and the consumed fuel can be obtained. The initial condition for the states is therefor the distance travelled, consumed fuel and that the velocity must be at the lower velocity limit. The start time for optimization must be equal the the total glide time. The initial conditions can be expressed as

$$\begin{aligned}x(t_g) &= x_{start} \\t_{start} &= t_{glide}\end{aligned}$$

In the last step, the velocity of the vehicle must be the desired upper velocity. To keep the desired average velocity of the vehicle, the travelled distance must be equal to the average distance which is calculated as the average velocity multiplied with the time. The terminal conditions can be expressed as

$$\begin{aligned}v(t_p) &= v_2 \\s(t_p) &= \bar{v}t_p\end{aligned}$$

#### 4.3.4 Path Constraints

For this problem there are a set of path constraints that have to be fulfilled. The engine speed must be in the permitted speed range of the engine. The states also have to be in the allowed range. For distance and mass fuel, the minimum and maximum value is between zero and infinity. For the velocity the minimum and maximum values are the desired lower and upper velocity. Finally, the engine power has to not exceed the maximum power of the chosen engine. The path constraints can be summarized as

$$\begin{aligned}\omega_{min} &\leq \omega \leq \omega_{max} \\x_{min} &\leq x \leq x_{max} \\0 &\leq P_e \leq P_{e,max}\end{aligned}$$

#### 4.3.5 Control Constraints

For this optimal control problem, the control signal is fuel injection. The control signal must stay within the allowed range for the fuel injection for the chosen engine. The control constraint can be formulated as

$$0 \leq u_f \leq u_{f,max}$$

### 4.3.6 Complete OCP

The complete optimal control problem for the PnG strategy is shown below. The objective is to minimize the fuel consumption and the control signal is fuel injection.

$$\begin{aligned}
 & \underset{u_f(t)}{\text{minimize}} && J = \frac{m_p + m_g}{s_p + s_g} \\
 & \text{subject to} && \dot{x}(t) = f(x(t), u(t)), \\
 & && x(0) = x_{start}, \\
 & && t_{start} = t_{glide}, \\
 & && v(t_p) = v_2, \\
 & && s = \bar{v}t, \\
 & && 0 \leq u_f \leq u_{f,max}, \\
 & && \omega_{min} \leq \omega \leq \omega_{max}, \\
 & && x_{min} \leq x \leq x_{max}, \\
 & && 0 \leq P_{ice} \leq P_{max}
 \end{aligned} \tag{4.22}$$

## 4.4 Energy Analysis

To get a better understanding of the Pulse and Glide method, an analyze of the energy needed by and the energy supplied to the engine is performed. The PnG method take advantage of the increased engine efficiency during the pulse as the engine can work near the most optimal BSFC-point. During the pulse phase, the velocity of the vehicle is both under and over the average velocity.

At higher velocity, as the resistance forces are higher, the constant driving method is working at better operating points. This is because at higher velocities, the highest gear is often used. The efficiency for the highest gear for constant driving is almost as good as the optimal BSFC-line efficiency, see Figure 3.6.

In Figure 3.5 and 3.6, the different engine efficiencies are shown for different velocities when using PnG and constant speed driving. For constant speed the efficiencies come in steps of when a new gear can be achieved presented in Table 4.2.

To calculate the energy needed, first the forces are considered. For constant speed driving, the propulsion force is equal to the resistance forces, as in Equation (3.15). By choosing the most efficient gear at the chosen velocity, the torque and engine speed can be calculated with Equation (3.11), (3.12) and (3.16). The description for the following symbols are found in Table 4.3. The power needed to travel at constant speed is calculated with

**Table 4.2:** Showing the speed and efficiency spans for gear 12, 13, and 14, for constant speed driving.

Gear [-]	Speed span [km/h]	Efficiency span [-]
12	33.51 - 50.27	31.19 - 31.97 %
13	50.27 - 62.83	37.06 - 37.85 %
14	62.83 - 70.00	39.73 - 40.33 %

**Table 4.3:** Symbol description for Chapter 4.4

Symbol	Description	Unit
$P_{const}$	Engine power needed to keep constant speed	[W]
$T_{e,const}$	Torque at constant speed	[N/m]
$\omega_{e,const}$	Constant engine speed	[N/m]
$t_{PnG}$	Time for a PnG cycle	[s]
$E_{const,needed}$	Total energy needed	[J]
$E_{const,in}$	Total energy supplied	[J]
$\eta_{const}$	Engine efficiency in constant speed	[-]
$P_{pulse}$	Engine power needed when pulsing	[W]
$T_{e,BSFC}$	Torque at the BSFC-line	[N/m]
$t_p$	Pulse time	[s]
$E_{PnG,needed}$	Total energy needed for a Pulse	[J]
$E_{const,in}$	Total energy supplied for a pulse	[J]
$\eta_{PnG}$	Engine efficiency in pulse	[-]

$$P_{const} = T_{e,const} \cdot \omega_{e,const} \quad (4.23)$$

The energy needed is then obtained by multiplying with the time for complete PnG cycle and the total energy supplied to the engine can be obtain by considering the engine efficiency according to

$$E_{const,needed} = P_{const} \cdot t_{PnG} \quad (4.24)$$

$$E_{const,in} = \frac{E_{const,needed}}{\eta_{const}} \quad (4.25)$$

For the chosen velocities, the engine speeds can be obtained with Equation (3.11), (3.12), and the most efficient pulse gear. As the engine is following the BSFC-line, the torque for the engine speeds is known. The power can then be calculated as

$$P_{pulse} = T_{e,BSFC} \cdot \omega_e \quad (4.26)$$

The total energy needed for PnG is then obtained by multiplying with the pulse time as

$$E_{PnG,needed} = P_{pulse} \cdot t_p \quad (4.27)$$

With the total energy demand calculated, the energy that needs to be supplied to the engine can be calculated by consider the engine efficiency with the following equation

$$E_{PnG,in} = \frac{E_{PnG,needed}}{\eta_{PnG}} \quad (4.28)$$

# 5

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## Results

In this chapter, the results from the simulations are presented and the parameters from Table A.1 are used. First, the results are presented for BSFC-line method and then for the optimal control. Lastly, the result from the energy analyze is presented.

### 5.1 Optimal BSFC-line Method

For the simple model the average velocities 50, 60 and 70 km/h were tested. For each average velocity 10 different k-values were tested which gave 10 different velocity spans. For this model 4 different configurations were tested for the glide phase: engine turned off with gear in neutral, engine turned on with gear in neutral, engine turned on with gear in neutral and with synchronization energy included, and engine turned on with a gear engaged. The fuel consumption for the different strategies were then compared to the fuel consumption for travelling at a constant speed.

#### 5.1.1 Fuel Consumption

The fuel consumption compared to constant speed driving can be seen in Table 5.1, 5.2, and 5.3. The fuel consumption for constant speed driving corresponds to 100 %.

**Table 5.1:** Optimal BSFC-line results for 50 km/h with gear 11.

Fuel consumption compared to constant speed driving at 50 km/h					
K-value	Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine on with sync [%]	Engine on glide in gear [%]
-1	41.2-59.8	93.20	99.60	98.96	108.88
-5.4	41.7-59.3	93.05	99.45	98.81	108.70
-9.8	42.2-58.6	92.91	99.31	98.67	108.52
-14.2	42.7-58.0	92.76	99.16	98.52	108.34
-18.6	43.3-57.3	92.62	99.01	98.38	108.16
-23	44.0-56.5	92.47	98.87	98.23	108.00
-27.4	44.8-55.6	92.33	98.73	98.09	107.81
-31.8	45.7-54.6	92.18	98.58	97.95	107.63
-36.2	46.9-53.2	92.04	98.44	97.81	107.46
-40.6	49.8-50.2	91.90	98.29	97.66	107.29

**Table 5.2:** Optimal BSFC-line results for 60 km/h with gear 12.

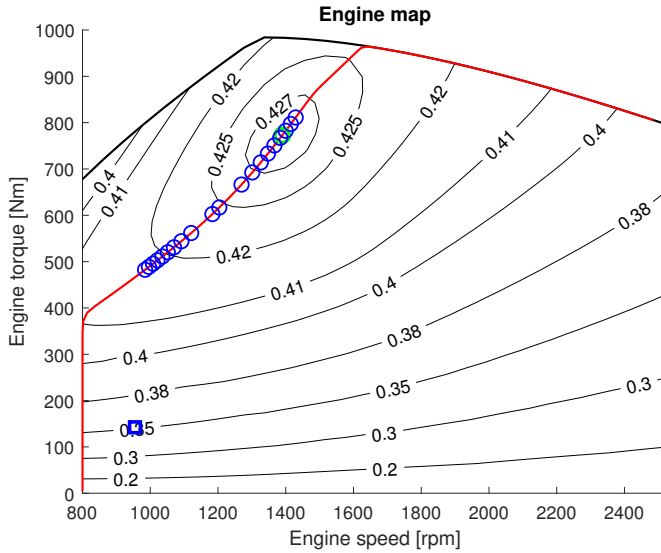
Fuel consumption compared to constant speed driving when at 60 km/h					
K-value	Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine on with sync [%]	Engine on glide in gear [%]
-1	51.2-69.8	96.67	100.76	100.39	107.90
-5.28	51.7-69.3	96.53	100.62	100.25	107.76
-9.56	52.2-68.6	96.39	100.48	100.11	107.61
-13.83	52.7-68.0	96.25	100.34	99.97	107.46
-18.11	53.4-67.2	96.11	100.20	99.83	107.31
-22.39	54.0-66.4	95.97	100.07	99.70	107.17
-26.67	54.8-65.6	95.84	99.93	98.56	107.02
-30.94	55.7-64.5	95.70	99.79	98.42	106.87
-35.22	57.0-63.2	95.56	99.65	98.29	106.73
-39.5	59.8-60.2	95.43	99.52	98.15	106.58

**Table 5.3:** Optimal BSFC-line results for 70 km/h with gear 13.

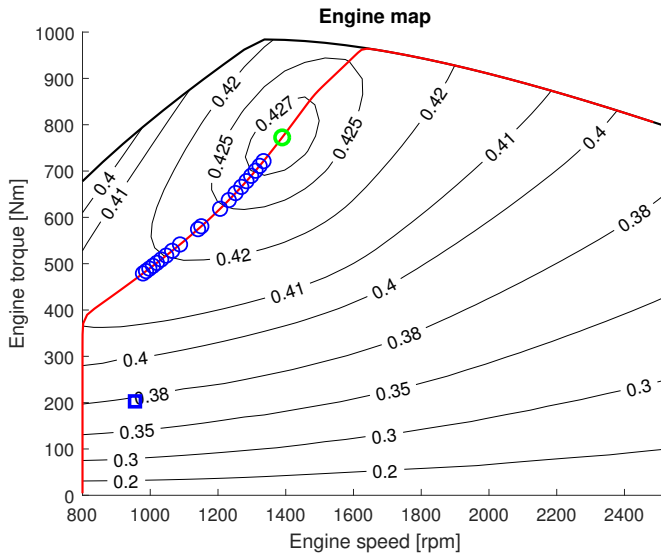
Fuel consumption compared to constant speed driving at 70 km/h					
K-value	Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine on with sync [%]	Engine on glide in gear [%]
-1	61.3-79.7	100.56	103.05	102.84	108.00
-5.53	61.7-79.2	100.44	102.93	102.71	107.87
-10.07	62.2-78.5	100.32	102.81	102.59	107.75
-14.6	62.8-77.9	100.20	102.69	102.47	107.63
-19.13	63.4-77.2	100.08	102.56	102.35	107.51
-23.67	64.1-76.4	99.96	102.44	102.23	107.38
-28.2	64.8-75.5	99.84	102.32	102.11	107.26
-32.73	65.8-74.5	99.71	102.20	101.99	107.14
-37.27	66.9-73.2	99.59	102.08	101.87	107.02
-41.8	69.6-70.4	99.47	101.96	101.75	106.69

### 5.1.2 Engine Map

The points used in the engine map shown in Figure 5.1. The points are the minimum and maximum values and are paired together. The velocity span with the highest velocity difference have the maximum point that have the highest rpm and the minimum point with the lowest rpm. The smallest velocity span has the maximum and minimum points next to each other close to the 42.5% efficiency line. In Figure 5.2 and 5.3 the pairings work in the same way. The blue square in the figures is the operating point for constant speed driving.

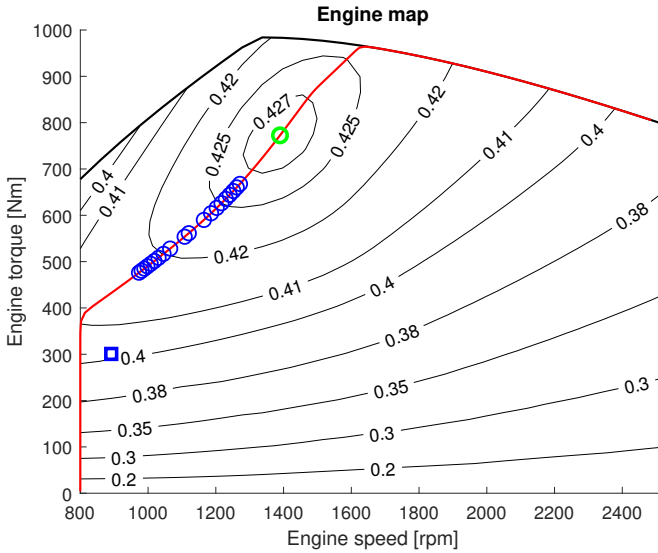


**Figure 5.1:** Showing the minimum and maximum points used for each different velocity span with gear 11. The blue square is showing the operating point for constant speed driving.



**Figure 5.2:** Showing the minimum and maximum points used for each different velocity span with gear 12. The blue square is showing the operating point for constant speed driving.

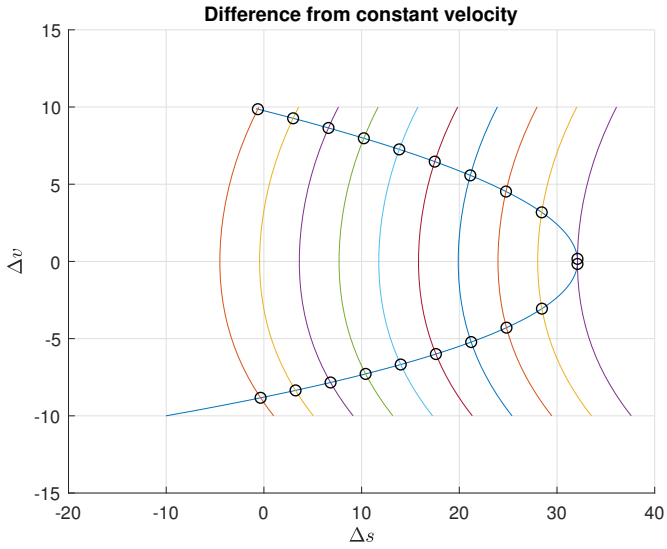




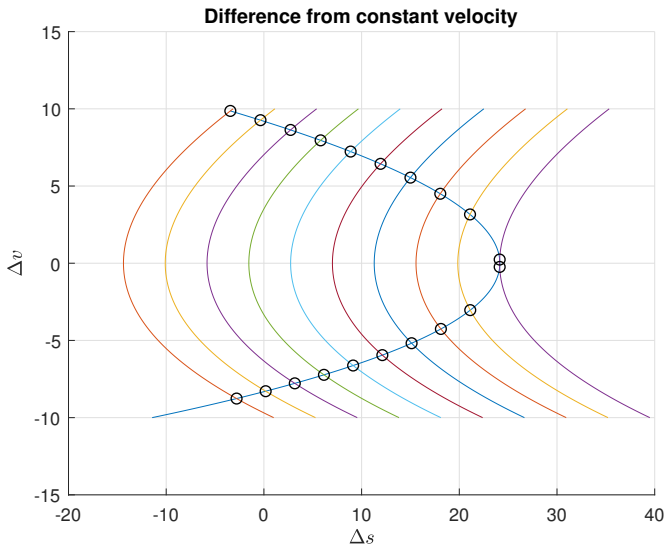
**Figure 5.3:** Showing the minimum and maximum points used for each different velocity span with gear 13. The blue square is showing the operating point for constant speed driving.

### 5.1.3 Phase Curve

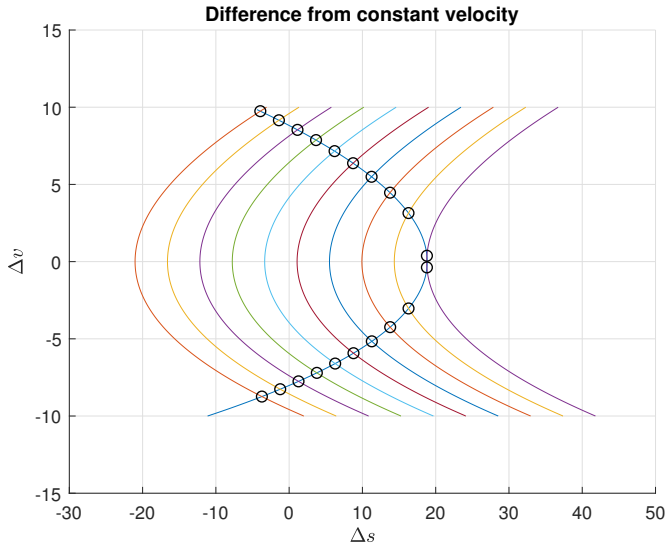
The phase curves for the velocities 50km/h, 60km/h and 70km/h and the velocity span for each velocity are shown in Figure 5.4, 5.5 and 5.6. The k-values and velocity span values for 50km/h, 60km/h, and 70km/h are found in Table 5.1, 5.2, and 5.3.



**Figure 5.4:** Showing the difference from constant for different velocity spans when the desired velocity is 50 km/h.



**Figure 5.5:** Showing the difference from constant speed driving for different velocity span when the average velocity is 60 km/h.



*Figure 5.6: Showing the difference from constant for different velocity span when the desired velocity is 70 km/h.*

## 5.2 Optimal Control

In this section the results from the optimization is presented. The average velocities were set to 50, 60, and 70 km/h. For the OCP, three different configurations for the glide phase were tested: engine turned off with gear in neutral, engine turned on with gear in neutral, engine turned on with gear in neutral and with synchronization energy included. The option to glide in gear is disregarded due to the poor results in Section 5.1. Figures of the phase curve and control signal is found in Appendix A

### 5.2.1 Fuel Consumption

The fuel consumption compared to constant speed driving can be seen in Table 5.4, 5.5, and 5.6. The upper and lower velocity was set to the same as for the optimal BSFC-line method.

**Table 5.4:** Optimization results for 50 km/h with gear 11.

Fuel consumption compared to constant speed at 50 km/h						
Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine on with [%]	Engine on sync		
41.2-59.8	93.20	99.52	98.90			
41.7-59.3	93.05	99.38	98.75			
42.2-58.6	92.91	99.24	98.61			
42.7-58.0	92.76	99.10	98.47			
43.3-57.3	92.62	98.96	98.33			
44.0-56.5	92.47	98.81	98.19			
44.8-55.6	92.33	98.67	98.05			
45.7-54.5	92.18	98.53	97.91			
46.9-53.2	92.04	98.39	97.77			
49.8-50.2	91.90	98.25	97.63			

**Table 5.5:** Optimization results for 60 km/h for gear 12.

Fuel consumption compared to constant speed at 60 km/h						
Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine on with [%]	Engine on sync		
51.2-69.8	96.67	100.70	100.34			
51.7-69.3	96.53	100.56	100.20			
52.2-68.6	96.39	100.43	100.06			
52.7-68.0	96.25	100.29	99.93			
53.4-67.2	96.11	100.15	99.79			
54.0-66.4	95.97	100.02	99.66			
54.8-65.5	95.84	99.88	99.52			
55.7-64.5	95.70	99.75	99.39			
57.0-63.2	95.56	99.61	99.25			
59.8-60.2	95.42	99.48	99.12			

**Table 5.6:** Optimization results for 70 km/h for gear 13.

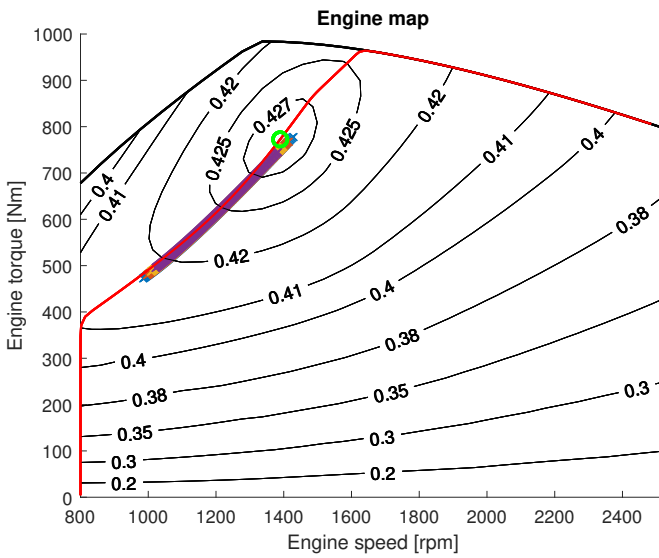
Fuel consumption compared to constant speed at 70 km/h					
Velocity span [km/h]	Engine off [%]	Engine on [%]	Engine with [%]	on sync	
61.3-79.7	100.56	102.99	102.78		
61.7-79.2	100.44	102.87	102.66		
62.2-78.5	100.31	102.75	102.54		
62.8-77.9	100.19	102.63	102.43		
63.4-77.2	100.07	102.51	102.31		
64.1-76.4	99.95	102.39	102.19		
64.8-75.5	99.83	102.27	102.07		
65.8-74.5	99.71	102.15	101.95		
66.9-73.2	99.59	102.04	101.83		
69.6-70.4	99.47	101.92	101.71		

### 5.2.2 Engine Map

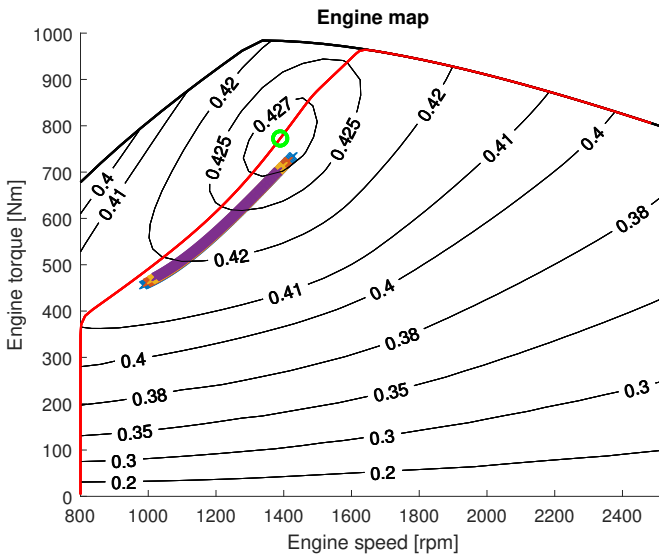
In this section the engine maps for the different velocities are shown. In Figure 5.7, the engine map for when the engine is turned off during the glide phase for the velocity of 50km/h. The engine map when the engine is turned on during the glide is shown in In Figure 5.8

The same are done for the velocity of 60 km/h and 70 km/h. First the engine map for when the engine is off during the glide, shown in figure 5.9 for 60km/h and 5.11 for 70km/h. For the engine map when the engine is on is found in Figure 5.10 for 60km/h and 5.12 for 70km/h

## Average Velocity 50 km/h

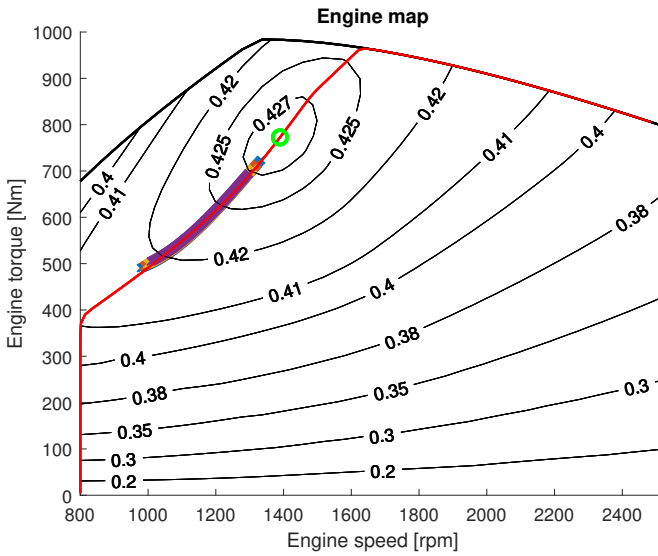


**Figure 5.7:** The engine map when optimizing for the average velocity 50 km/h with engine off in glide phase.

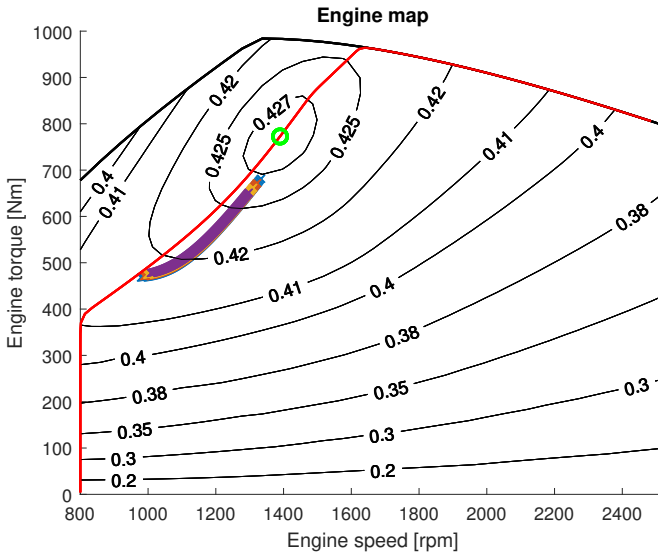


**Figure 5.8:** The engine map when optimizing for the average velocity 50 km/h with engine on in the glide phase.

## Average Velocity 60 km/h

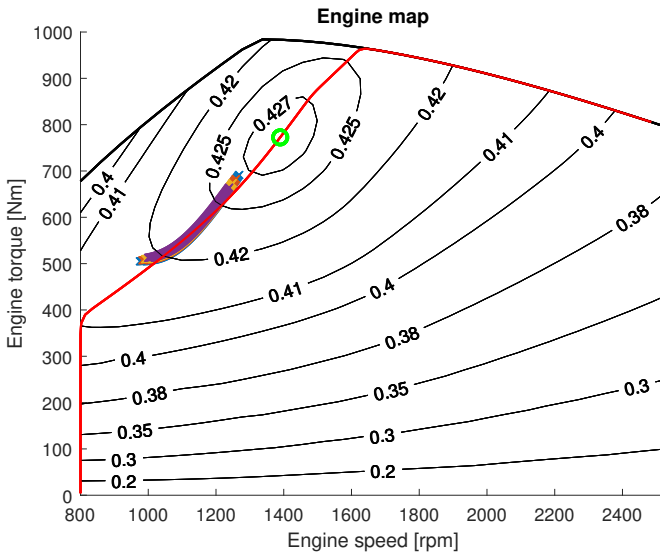


**Figure 5.9:** The engine map when optimizing for the average velocity 60 km/h with engine off in the glide phase.

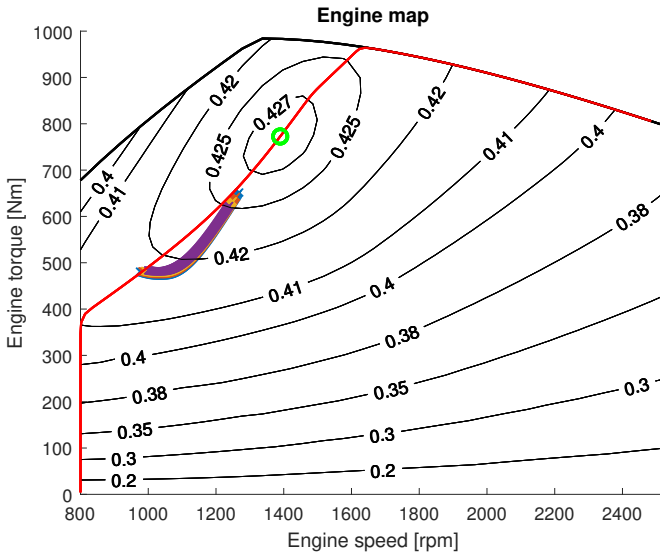


**Figure 5.10:** The engine map when optimizing for the average velocity 60 km/h with engine on in the glide phase.

## Average Velocity 70 km/h



**Figure 5.11:** The engine map when optimizing for the average velocity 70 km/h with engine off in glide the phase.



**Figure 5.12:** The engine map when optimizing for the average velocity 70 km/h with engine on in the glide phase.



## 5.3 Energy Analysis

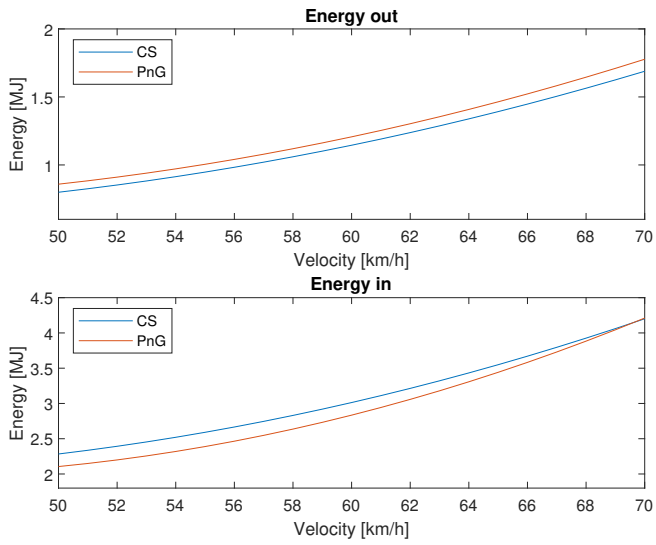
The energy demand and energy supplied when holding the average speed during a Pulse and Glide cycle is presented in Table 5.7 and Table 5.8. The PnG case is running with the engine off during the glide phase and only the energy to accelerate the vehicle from a minimum- to a maximum velocity is calculated. A graphic explanation of the energy in and out from the engine can be seen in Figure 5.13.

**Table 5.7:** Energy out from the engine for constant speed driving and PnG.

Avg. velocity [km/h]	Energy out, Con- stant Speed [MJ]	Energy out, PnG [MJ]
50	0.7993	0.8586
60	1.1449	1.2066
70	1.6887	1.771

**Table 5.8:** Energy supplied to the engine for constant speed driving and PnG.

Avg. velocity [km/h]	Energy in, Con- stant Speed [MJ]	Energy in, PnG [MJ]
50	2.2836	2.1054
60	3.0129	2.8343
70	4.2007	4.2110



**Figure 5.13:** The energy demand and energy supplied to the engine from 50-70 km/h for constant speed driving and PnG.

# 6

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## Discussion

### 6.1 Models

In this work some assumption for the models have been made. In reality the driveline is not stiff and not 100 % efficient. When comparing constant speed driving to Pulse and Glide, it would affect the Pulse and Glide method more than the constant speed. This is because of the shifting loads from the engine when using Pulse and Glide. In constant speed driving, the load is constant and thus would the non-stiff driveline have constant properties.

The engine maps are specified for constant operating points and moving between these points causes a drop in efficiency. For the Pulse and Glide this mean that the efficiency will be lower than the efficiency used, while for the constant speed case there would be no difference.

Another thing that has not been accounted for is the fuel injection for when the engine is started. For the engine off configuration, the results would be affected. In the smaller velocity ranges, the additional fuel to start the engine is not negligible and is a large amount of the total fuel consumed for the pulse. In the larger velocity spans the results are not affected as much.

### 6.2 Results

From the results in Table 5.1, 5.2, and 5.3, it can be seen that PnG has its most fuel reduction for lower velocities. The average velocity of 50 km/h was the most

fuel efficient and decreased as the velocity increased. The fuel saving potential reaches its upper limit at 70 km/h.

The best strategy for the glide phase is to turn off the engine, the second is engine on in neutral with synchronization energy included, third is engine on with neutral gear, and the worst is engine on with a gear engaged. This result is in line with the previous research done on PnG except for the case with a gear engaged. From the results, the gear engaged consumes more fuel than the constant speed driving. This can be explained due to the increased engine friction which reduces the glide time and distance. The fuel that is saved in the glide phase when the fuel injection is cut off is not big enough to compensate for the friction. However, it should be noted that the strategy to glide with the engine turned off is not always possible. The engine often drives other components in the vehicle which cannot be turned off. To overcome this an electric motor could be installed but that increases the cost.

At 50 km/h the fuel saving compared to constant speed driving is up to 8.1 %, seen in the last row in Table 5.1. This can be explained due to the increased engine efficiency when using PnG. In Figure 5.1, the engine efficiency for PnG is between 41.7-42.7 % while for the constant speed driving it is only 35 %. This part load driving is bad in a fuel consumption point of view.

For 70 km/h, the PnG method is no longer useful with this vehicle setup. As the velocity increases, the load on the engine increases and a higher gear with a better efficiency can be used for constant speed. Air resistance increases square to the velocity and becomes significant larger. The increased engine efficiency can no longer compensate for the increased air resistance.

By studying Table 5.1, 5.2, and 5.3, it can be seen that the lowest velocity span between the maximum and minimum velocity gives the best fuel consumption. When the velocity span it kept small, the engine can work in an operating point with an efficiency of 42.3 %. With a larger span, the efficiency is varying between 41.5-42.7 %. The time spent in the region under 42.3 % increases the fuel consumption more than what is saved in the higher efficiency area. Also, as the velocity span is larger, more time is spent in a velocity above the average. This increases the air resistance as it is proportional to the square of velocity.

From Figure 5.13 it can be seen that the energy demand for PnG is always higher than for constant speed driving but the energy that needs to be supplied to the engine is lower. The difference in energy demand for PnG and constant speed slightly increases as the velocity increases. For the energy supplied to the engine, the difference is decreasing faster when the velocity is increasing. This means that the conversion from input chemical energy to the output mechanical energy is more efficient for PnG than for constant speed driving. From this it can be concluded that the fuel reduction mainly depends on the engine efficiency and partly due to the air resistance.

The configuration with the engine on with synchronization taken account for is more effective at lower velocities than at higher shown in Table 5.1, 5.2, and 5.3.

This is cause of the difference in engine speed from the start of the glide to the start of the pulse. In lower speeds a higher difference in engine speed is achieved because of the lower gear and thus saves more fuel.

When comparing the optimal BSFC-line and optimal control, it shows that the best improvement is 0.08 %, which can be seen in Table 5.1 and 5.4 for the engine off configuration. The optimizer was always at least as good as the BSFC-line approach. When running the optimizer with the engine off during glide at 50 and 60 km/h, the engine operating points was essentially on the optimal BSFC-line, shown in Figure 5.1 and Figure 5.2. At 70 km/h the operating points followed an arc a little above the BSFC-line but had the same results. It can be explained that the optimizer weighs fuel/distance and the fraction was better when using less fuel but does not travel as far.

The most interesting results from the optimal control comes when the engine is turned on during the glide phase. Shown in Figure 5.8 and 5.10, the operating points are slightly below the optimal BSFC-line. The objective was to minimize fuel/distance and when the idle fuel mass is included in the initial condition for the fuel mass, the optimal strategy is to travel for a longer distance compared to following the optimal BSFC-line. At 70 km/h the operating points are in an arc under the optimal BSFC-line shown in Figure 5.12.

When comparing the two different methods it can be concluded that the optimal BSFC-line method is almost as good as the optimal control method. From this a framework for Pulse and Glide can be created. If the optimal BSFC-line is known for a selected vehicle configuration, a near-optimal strategy is to always pulse on the optimal BSFC-line. This can easily be implemented on a real vehicle as the engine torque already is calculated and no real-time optimization is needed.

Some of the speed intervals are not possible to achieve due to limitations that not have been modeled. In Section 4.2.4, gear changing time and the turbo lag can take around one second each. These can be prepared for in the glide phase, but this require that the phase is at least two seconds long. The shortest of our velocity intervals have a glide time of around 1.5 seconds. This makes the results from that interval invalid since they are not possible to do.

Although the PnG strategy reduces the fuel consumption, there are some drawbacks as well. By using a PnG strategy the wear on the mechanical parts is higher. The increases gear shift could affect the life span of the gearbox. It is possible that the fuel saved is not worth the decreased lifetime of the gearbox.

Another disadvantage is the driver comfort because of the many acceleration and deceleration. Traffic is also not taken into consideration. Usually traffic goes at a constant speed and will make the PnG strategy hard to execute since other vehicles will probably interfere.



# 7

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## Conclusions and Future Work

### 7.1 Conclusions

This thesis work has investigated the fuel saving potential for Pulse and Glide. Two different methods of different complexity have been developed and analyzed. The first method was to follow the optimal BSFC-line in the engine map. The second method was based on optimal control to find the best fuel consumption. Three different strategies for the glide phase were investigated. Glide in neutral gear with the engine turned off, glide in neutral gear with the engine turn on, and glide in neutral gear with the engine turned on with the gear synchronization energy included. Glide in gear was also investigated for the optimal BSFC-line method but was later rejected since no fuel saving could be found.

It can be concluded that the best strategy is to glide in neutral gear with the engine turned off. The highest fuel reduction can be seen for lower velocities and with a short range between the maximum and minimum velocity. For 50 km/h the fuel consumption is reduced 7-8 %, for 60 km/h 3-4 % and for 70 km/h up to 0.5 %. For our vehicle, the fuel saving potential reaches its upper limit at 70 km/h. The upper limit can be explained due to the air resistance is squared to the relative to air velocity as well as the engine operating points become more efficient when driving in constant speed for higher velocity.

The optimal control based method showed an improvement of 0.08 % compared the optimal BSFC-line method. The engine was working in almost the same operating points as before. It can be concluded that the BSFC-line method is near-optimal and shows almost the same results as the optimal control method. From that a benchmark for Pulse and Glide can be distinguished. Instead of using time consuming and computational heavy optimization, a near-optimal strategy is to

always follow the optimal BSFC-line during the pulse phase.

## 7.2 Future Work

There are several different aspects open for further investigation. As the trend in the industry is towards hybridization and electrification it would be interesting to study the possibilities for Pulse and Glide in that area. One scenario could be to investigate the usage of energy storage devices such as battery, flywheel and super-capacitor.

Furthermore, the sophistication degree of engine and driveline models could be studied how it affect the fuel consumption. Another thing to consider is the assumption where the driveline is not stiff and the turbocharger dynamics are included.

During this work computational complexity was not considered. One interesting improvement would be to implement the code on an ECU and perform real-life experiments. With this, the model could be validated against the real-life data.

The increased gear shifts when using Pulse and Glide could be studied further. It would be interesting to see how the frequency of the gear shift increases wear and the life span of the gear box.



# Appendix



# A

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## Appendix

## A.0.1 Parameters

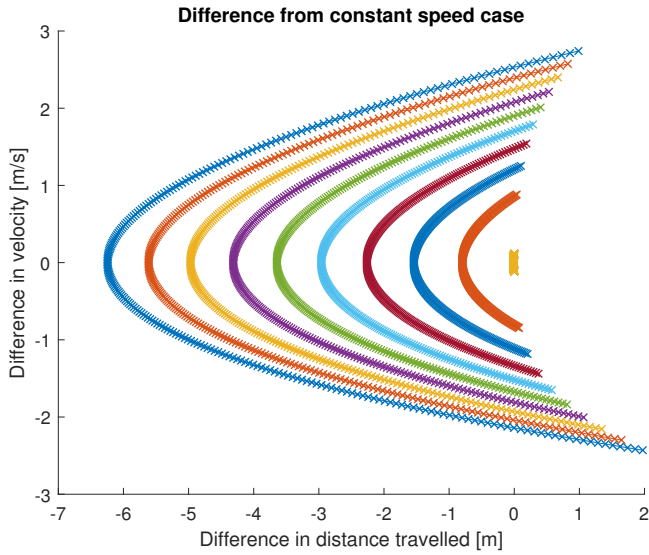
The parameters used in the simulations can be seen in Table A.1.

**Table A.1:** Parameters used in simulations

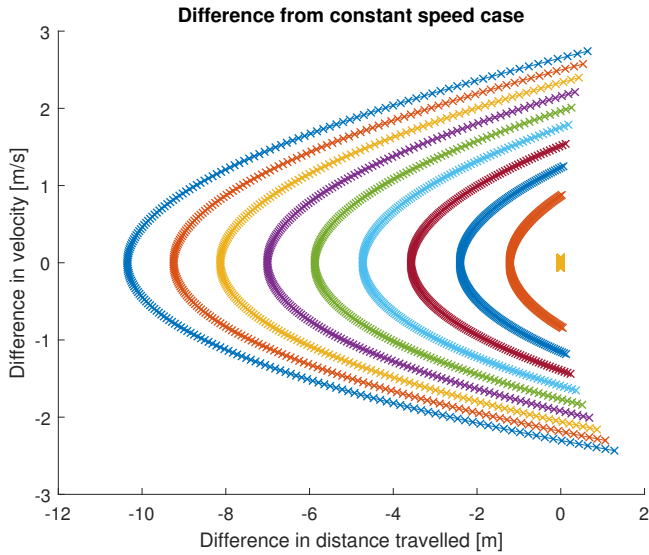
Used parameters		
Symbol	Symbol Description	Value
$g$	Earths gravitation constant [m/s <sup>2</sup> ]	9.81
$q_{LHV}$	Lower heating value [J/kg]	$42.9 \cdot 10^6$
$n_r$	Crank revolutions per cycle [-]	2
$V_D$	Engine volume [m <sup>3</sup> ]	$6.7 \cdot 10^{-3}$
$c_d$	Aerodynamic drag coefficient [-]	0.5
$c_r$	Rolling friction coefficient [-]	$6 \cdot 10^{-3}$
$A_f$	Cross-sectional area [m <sup>2</sup> ]	7
$m_v$	Vehicle mass [kg]	$1 \cdot 10^4$
$n_{cyl}$	Engine cylinders [-]	6
$r_w$	Wheel radius [m]	0.5
$J_e$	Rotational inertia [kg/m <sup>2</sup> ]	2.5
$i_{g,11}$	Gear 11 ratio [-]	1.5
$i_{g,12}$	Gear 12 ratio [-]	1.2
$i_{g,13}$	Gear 13 ratio [-]	1
$i_{g,14}$	Gear 14 ratio [-]	0.8
$i_f$	Final drive ratio []	3
$\alpha_1$	Quadratic engine friction coefficient [-]	$3.7591 \cdot 10^{-5}$
$\alpha_2$	Linear engine friction coefficient [-]	$-5.233 \cdot 10^{-3}$
$\alpha_3$	Static engine friction coefficient [-]	0.7196
$\omega_{e,idle}$	Idle engine speed [rad/s]	83.7758
$u_{f,idle}$	Idle fuel injection [kg/cycle-cylinder]	$3.863 \cdot 10^{-6}$
$r_c$	Compression ratio [-]	17.2

## A.0.2 Phase Curves

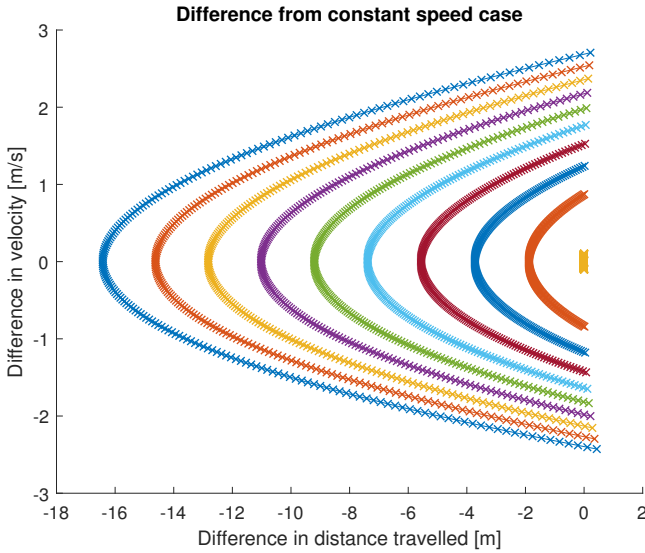
The phase curves from the optimal control strategy can be seen in Figure A.1, A.2, and A.3.



**Figure A.1:** Difference from constant speed case in velocity and distance for 50 km/h



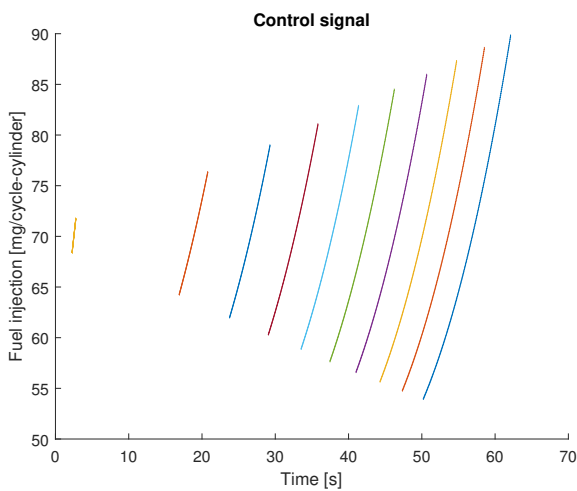
**Figure A.2:** Difference from constant speed case in velocity and distance for 60 km/h



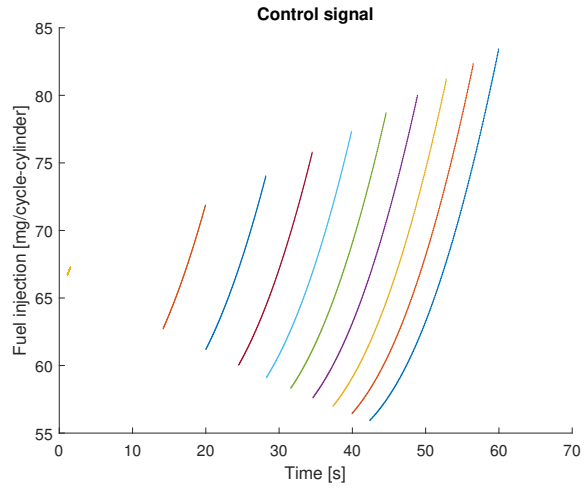
**Figure A.3:** Difference from constant speed case in velocity and distance for 70 km/h.

### A.0.3 Control Signal

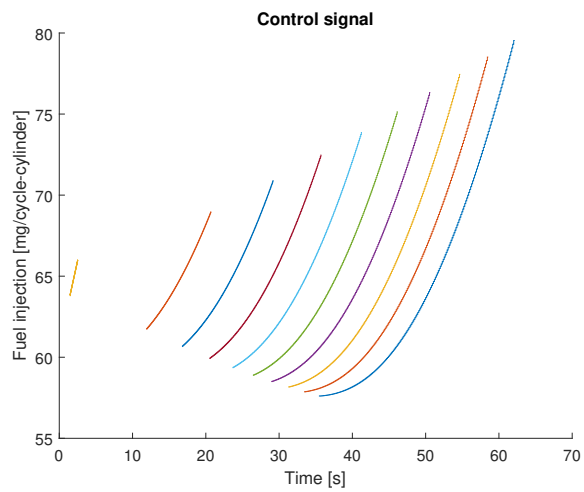
The control signals for 50, 60, and 70 km/h are shown in Figure A.4, A.5, and A.6.



**Figure A.4:** Shows the fuel injection for an average velocity of 50 km/h.



**Figure A.5:** Shows the fuel injection for an average velocity of 60 km/h.



**Figure A.6:** Shows the fuel injection for an average velocity of 70 km/h.





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