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Abstract

In the paper we propose a game-theoretic model of the mobile network market. The market is presented by three sides: primary mobile network operators (MNO), mobile virtual network operators (MVNO) and consumers of the services. MVNO are mobile operators without their own infrastructure. They buy resources from MNO and compete with other MVNO for the consumers selling a service in the mobile network market. We construct a two-stage game. On the first stage MVNO (players) select the MNO, one or several, and then announce the price for their service for the consumers in this MNO. After the profile of prices is determined, the consumers are distributed among MVNOs following the logistic function. The equilibrium in this two-stage game is constructed. For identical consumers the analytic formulas for the solution are derived.

1 Introduction

Technological maturation leads to market's formation, expansion and segmentation. Moreover, it results in enhancements of services. Indeed, it is true for many modern applications, the few of them are cloud computing, mobile technologies, personal computer architectures. For all these cases, we historically observed how big players create new markets and later small and medium sized enterprises (SMEs) populate it with own unique services and find specific market niche. For example, lately, there are many internet of things (IoT) vendors and various third-party cloud provider services. Currently, 4G mobile networks are widely deployed, and the 5th generation is coming at year 2020.

This paper constructs a two-level market model for mobile operators and study their behavior in market organization. Namely, we talk about mobile network operators (MNOs) and mobile virtual network operators (MVNO). For these two types of participants we construct a two-level market model and analyze it. Having said that, we emphasize that our model is general-purpose

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and applicable to many other cases including the aforementioned examples of cloud-computing, hardware (IoT and PC) manufacture and many others.

Worldwide, there are about one thousand MVNOs and three hundred MNOs. One of the leading countries in the number of MVNOs is Germany with over 120 MVNOs. In some countries such as Finland, the number of operators is much smaller, two-three MNOs (Sonera, Elisa, DNA) and a few MVNOs (Saunalahti, Cubio) [3].

The model that we construct is a two-level market model, however we intrinsically consider the third level of final consumers of the market products. In our model, the consumers are incorporated using a special construction – a logistic function – which properties we explain in upcoming section.

The main contributions in this paper are:

1. Formulation of two-level competitive market for MNOs and MVNOs.
2. Finding market organization for identical players.
3. General analysis of the pricing game.
4. Finding equilibrium for allocation games in two-player cases

The rest of the paper is organized as following. In Section 2, we describe the concept of mobile operator markets. In Section 3, we formulate a game-theoretic model of two-stage competition. In Section 4, we start by developing a simple model for identical players. In Section 5, the pricing game is developed and in Section 6 the allocation game. Section 7 summarizes the related work. Finally, Section 8 concludes the paper.

2 Mobile Network Operator Market

Mobile network operator (MNO) is an operator of an infrastructure that provides many mobile network services. Normally, it consists of many network components, including cables, mobile stations or base stations, billing, and etc. Nowadays, MNO starts to enrich infrastructure with powerful Cloudlet services². The advance of technology leads to the fact that more sophisticated services are provided by MNOs for end customers and business. The technologies that play the most important role today in these advancements are cloud computing, virtualization, network function virtualization and mobile service offloading.

Flexibility of MNO services result in formation of mobile virtual network operators (MVNOs) - service providers that as MNO provide mobile network services to the end users, but do not own any physical infrastructure (or their infrastructure plays minor role in the business). In a sense, MVNOs are brokers that buy services from MNOs, enhance them and sell to own customers as shown in Figure 1. From customer point of view, the difference between MNOs and MVNOs is negligible. From MNO's point of view, MVNOs are large customers that buy crude services as a wholesale commodity. From MVNO's side, they buy services from MNOs, integrate own product (services) into MNO services and sell the enrich and unique product to customers.

²<https://networks.nokia.com/solutions/multi-access-edge-computing>

Figure 1: MNO own the infrastructure, while MVNO can choose which MNO to use.

At a first glance, it may appear that MNOs will always dominate over MVNOs, but it turns out that MVNO could efficiently create unique services and increase market revenue. Moreover, MNO normally cannot push MVNOs out of the market due to competition legislation. If MNO wants to create a competitive service to that of MVNO, then the MNO compete on the same level as the MVNO, independently of physical infrastructure. Due to this argumentation, in this paper, we do not consider competition between MNOs and MVNOs, as they occupy different levels of the market organization. Instead, if a MNO wants to start to compete with some MVNO, then this MNO “creates” artificial MVNO on the level of other MVNOs and compete with them on the same level. If real scenario cannot be reduced to this MNO split, then that indicates that MNO is violating competition laws.

Each user selects MVNO services based on a set of factors, such as brand name, flexibility, price, availability, uniqueness and locality — the list can be long. In order to deal with user preference we incorporate a logistic function of the form

$$\gamma_i = \frac{\exp\{-\sum_{f \in F} k_{if} r_{if}\}}{\sum_{j \in I} \exp\{-\sum_{f \in F} k_{jf} r_{jf}\}}, \forall i \in I,$$

where I is the set of all MVNOs and γ_i is the fraction of users that is attracted by MVNO i ; the interest of customers to each MVNO is measured as a scalar weight value, which is computed as a production of weights – exponents of each influence factor, namely $\exp\{-k_{if} r_{if}\}$ for factor f in set F . Each γ_i is normalized so that $\sum_i \gamma_i = 1$. The logistic function is a natural construction that defines a portion of market attracted by each agent. As we can see, all

factors are incorporated into the logistic function allowing to deal with as many factors as we want, for each factor we need to provide its “coefficient of influence” — k_{if} , which in many cases is independent of MVNO (i.e., $k_{if} = k_f \forall i$, and “amount of resource” — r_{if} — a value indicating how much of particular factor parameter is developed for particular MVNO. For example, if we consider a brand-name as a factor, then high k_f compared to other coefficients may indicate that users consider brand recognition to be very important, and low r_{if} that MVNO i did not put a lot of money into brand development and is not so well recognized on the market.

In many cases, some factors are dominating and others (minor parameters) can be omitted. In that case we can skip unnecessary parameters. Additionally, in this paper we study a game where price is a controllable parameter, while brand or some other resource is not controllable, thus instead of writing factors that do not change during the game as a constant multiplier, i.e., $\exp\{-\sum_{f \in F} k_{if} r_{if}\} = K_i \exp\{-\alpha_{ij} q_i\}$, where K_i is some coefficient for player i and q_i is the price for the resources.

Finally, let's discuss market geographical distribution. We know that the market geographical distribution is defined solely by MNO's presence. If there are no MNO in some areas, then there is no possibility to get customers from this area. Moreover, some MNO could operate in one area while other in another, e.g. *operator A* is only in *city X*, and *operator B* is only in *city Y*. Each MVNO attracts customers from different regions and areas and if MVNO wants customers in *city Y*, then it has to work with *operator B*. In the most generic and complex case, we need to construct coverage maps of each MNO for each area and construct a graph-based model for the game. However, for simplicity (and we believe without loss of generality), we consider that each MNO has a fixed set of end-users that it covers and MVNOs are competing for the users on these markets defined by MNO coverage. For each resource that MVNOs occupy at market defined by MNO, they pay amount p .

3 Two-Stage Competition Model

Now let's build a MNO–MVNO market model. There are m mobile network operators (MNO) $\{M_1, M_2, \dots, M_m\}$. M_j is characterized by the parameters (p_j, m_j, r_j, c_j) , $j = 1, \dots, m$, where p_j - price for resource, m_j is the number of consumers, r_j is amount of a resource and c_j is a fee to join to this market. Denote $M = \{M_1, M_2, \dots, M_m\}$ the set of operators.

There are n mobile virtual network operators (MVNO) $\{V_1, V_2, \dots, V_n\}$ who compete in the market for the resources. We suppose that n is much larger than m . Each MVNO V_i has some private resource v_i , $i = 1, \dots, n$, represented as a scalar number. MVNO are the players in the following game. Competition consists of two stages. On the first stage the players (MVNO) choose some MNO to compete for the consumers. So, the strategy of player V_i is a subset $s_i \subset S_i$ from the feasible set $S_i \subseteq M$.

After the profile $s = (s_1, \dots, s_n)$ is formed the players announce the prices for their service in each market j : q_i^j , $i = 1, \dots, n$; $j \in s_i$. The profile of the prices we denote $q = (q_1, \dots, q_n)$. To avoid the monopoly we suppose that if the market M_j is occupied only by one player V_i there is a restriction for the price $q_i \leq Q_j$. So, the payoff of the player V_i on the non-competitive market $M_j \in s_i$ is equal

to

$$u_i^j(q) = (Q_j - p_j)m_j - c_j.$$

The payoff of player V_i on the competitive market $M_j \in s_i$ is equal to

$$u_i^j(q) = (q_i^j - p_j)m_j\gamma_i^j - c_j,$$

where γ_i^j is a proportion of consumers m_j who are interested in the service V_i . Here we will use the logistic function [6]

$$\gamma_i^j = \frac{\exp\{-\alpha_{ij}q_i + \beta_{ij}v_i\}}{\sum_{l:j \in s_l} \exp\{-\alpha_{lj}q_l + \beta_{lj}v_l\}} = \frac{k_{ij} \exp\{-\alpha_{ij}q_i\}}{\sum_{l:j \in s_l} k_{lj} \exp\{-\alpha_{lj}q_l\}}, \quad i = 1, \dots, n, j \in s_i, \quad (1)$$

where α_{ij} and β_{ij} reflect the influence of prices and attractiveness of service i on the number of customers for this service on market M_j , and

$$k_{ij} = \exp\{\beta_{ij}v_i\}, \quad i = 1, \dots, n, j \in s_i.$$

The general payoff of player V_i is the sum of payoffs in all used markets:

$$u_i(q) = \sum_{j \in s_i} u_i^j(q) = \sum_{j \in s'_i} \left((q_i^j - p_j)m_j\gamma_i^j - c_j \right) + \sum_{j \in s''_i} ((Q_j - p_j)m_j - c_j),$$

$$i = 1, \dots, n,$$

where s'_i and s''_i are competitive and non-competitive markets, respectively.

The objective of this paper is to find an equilibrium in the pricing model and then in the allocation problem.

4 Model with Identical Players

At the beginning consider a simple case where all MVNOs are identical, so all parameters $v_i, \alpha_{ij}, \beta_{ij}$ do not depend on the player i , and all markets are competitive. In this case let us assume $\alpha_{ij} = \alpha_j, j = 1, \dots, m$ and $\beta_{ij} = \beta_j, j = 1, \dots, m$. For simplicity, assume that $k_{ij} = 1$. Consider the pricing game on the first market. Let $n_1 \geq 2$ players compete in this market. They announce the prices $q = (q_1, \dots, q_{n_1})$. The payoff of player V_i is

$$u_i^1(q) = (q_i - p_1)m_1 \frac{\exp\{-\alpha_1 q_i\}}{\sum_{l=1}^{n_1} \exp\{-\alpha_1 q_l\}} - c_1, i = 1, \dots, n_1.$$

The first order condition for the equilibrium $\partial u_i(q)/\partial q_i = 0$ gives

$$\sum_{l=1}^{n_1} \exp\{-\alpha_1 q_l\} = (q_i - p_1) \sum_{l \neq i} \exp\{-\alpha_1 q_l\} \alpha_1.$$

By symmetry all prices in the equilibrium must be equal, for example q_1 . It yields

$$q_1 = p_1 + \frac{1}{\alpha_1} \frac{n_1}{n_1 - 1}.$$

Hence, the optimal payoff of player V_i on the first market is

$$u_i^1 = \frac{m_1}{\alpha_1} \frac{1}{n_1 - 1} - c_1, i = 1, \dots, n_1. \quad (2)$$

We see that the payoff of any player is a decreasing function of the number of players acted in this market. So, the allocation game presented here is a congestion game [10] which has a potential.

If the player V_i uses the allocation strategy s_i then its general payoff is

$$u_i = \sum_{j \in s_i} \left(\frac{m_j}{\alpha_j} \frac{1}{n_j(s) - 1} - c_j \right), i = 1, \dots, n, \quad (3)$$

where $n_j(s)$ is the number of players choosing the market M_j (so called congestion vector). To find the equilibrium in the congestion game we maximize the potential function which has the following form

$$P(s_1, \dots, s_n) = \sum_{j=1}^m \sum_{i=1}^{n_j(s)} \left(\frac{m_j/\alpha_j}{i-1} - c_j \right).$$

Consequently, the optimal allocation of the players among $\{M_1, \dots, M_m\}$ can be found as a solution of the optimization problem

$$\sum_{j=1}^m \left(\frac{m_j}{\alpha_j} \sum_{i=1}^{n_j} \frac{1}{i-1} - c_j n_j \right) \rightarrow \max$$

in condition

$$\sum_{j=1}^m n_j = \sum_{i=1}^n |s_i|.$$

For example if the players can choose only one market then $\sum_{i=1}^n |s_i| = n$. For large n and small fee ($c_j \approx 0$) the optimization problem becomes

$$\sum_{j=1}^m \frac{m_j}{\alpha_j} \log n_j \rightarrow \max$$

the solution of which is

$$n_j \approx \frac{\frac{m_j}{\alpha_j}}{\sum_{l=1}^m \frac{m_l}{\alpha_l}} n, j = 1, \dots, m,$$

Theorem 1. For large n the players are allocated proportionally to the ratio of numbers of consumers and the weight of the player in the market.

Notice that we do not know the precise location of the player but we know how many players will be in each market.

The result we have obtained here is non-trivial. As all the players are identical the properties of each market (MNO) become the dominating criteria. For

example, coefficients of price attractiveness for $MVNO_j$ toward market MNO_i (α_{ij}) reduces to attractiveness of market and as a result all players will be distributed over markets proportional to ratio of number of consumers on the market to the price attractiveness of the market.

5 Pricing Game

Now we consider a general case. Assume that the players are distributed among M_j in correspondence with the allocation rule $s = (s_1, \dots, s_n)$. Consider a market M_j . In this market we have $n_j = n_j(s)$ players. For convenience let us re-enumerate players inside the market from 1 to n_j . Pricing game in the market M_j in fact is a non-cooperative game on n_j players with payoff functions

$$u_i^j(q) = (q_i^j - p_j)m_j \frac{k_{ij} \exp\{-\alpha_{ij}q_i\}}{\sum_{l:j \in s_l} k_{lj} \exp\{-\alpha_{lj}q_l\}} - c_j, i = 1, \dots, n_j. \quad (4)$$

In the paper [6] we show that the game with these payoffs is a potential game ([8, 5]) and the equilibrium $q^* = (q_1^*, \dots, q_{n_j}^*)$ in this game can be found as a maximum of potential function

$$P_j(q) = \prod_{i=1}^{n_j} (q_i^j - p_j) \frac{\exp\{-\sum_{l=1}^{n_j} \alpha_{lj}q_l\}}{\sum_{l=1}^{n_j} k_{lj} \exp\{-\alpha_{lj}q_l\}}. \quad (5)$$

The equilibrium q^* can be found from the first order condition $\partial P_j(q)/\partial q_i = 0, i = 1, \dots, n_j$. It yields

$$\sum_{l=1}^{n_j} k_{lj} \exp\{-\alpha_{lj}q_l^*\} = \alpha_{ij}(q_i^* - p_j) \sum_{l=1(l \neq i)}^{n_j} k_{lj} \exp\{-\alpha_{lj}q_l^*\}, i = 1, \dots, n_j. \quad (6)$$

Suppose that a new player $n_j + 1$ joins the market M_j . New equilibrium prices $q' = (q'_1, \dots, q'_{n_j}, q'_{n_j+1})$ satisfy the system of equations

$$\sum_{l=1}^{n_j+1} k_{lj} \exp\{-\alpha_{lj}q'_l\} = \alpha_{ij}(q'_i - p_j) \sum_{l=1(l \neq i)}^{n_j+1} k_{lj} \exp\{-\alpha_{lj}q'_l\}, i = 1, \dots, n_j + 1. \quad (7)$$

The equations (6) for $i = 1, \dots, n_j$ can be rewritten as

$$\begin{aligned} \sum_{l=1}^{n_j} k_{lj} \exp\{-\alpha_{lj}q'_l\} &= \alpha_{ij}(q'_i - p_j) \sum_{l=1(l \neq i)}^{n_j} k_{lj} \exp\{-\alpha_{lj}q'_l\} \\ &+ k_{n_j+1,j} (\alpha_{n_j+1,j}(q'_{n_j+1} - p) - 1) \exp\{-\alpha_{n_j+1,j}q'_{n_j+1}\}, i = 1, \dots, n_j. \end{aligned} \quad (8)$$

Assume that we consider only the prices for the service satisfying the conditions

$$q_i \geq p_j + \frac{1}{\alpha_{ij}}, \forall i, j.$$

Compare the equations (6) and (8). The left part of the equations is a decreasing function of q_i but the right part of it is an increasing function. The difference is only in the additional term in the second line in (8) which is positive. From here it follows that

$$q'_i \leq q_i^*, i = 1, \dots, n_j.$$

Hence, if a new player joins the market the equilibrium prices of the players who compete in the market before it become less.

Moreover, q^* is the maximal point of the potential function (5), so

$$P(q') \leq P(q^*).$$

Consequently, $\partial P(q)/\partial q_i \geq 0, \forall i$ for $q = q'$. Because the game is potential, the signs of the derivatives of the payoff functions and the potential are the same. So, $\partial u_i(q)/\partial q_i \geq 0, \forall i$ for $q = q'$. It yields the following proposition.

Theorem 2. If a new player joins the market the payoffs of the players who competed in the market before become less.

6 Allocation Game

Note that the optimal payoffs of the players in the equilibrium in market M_j depend not only on the number of players n_j , but also depend on the characteristic of the players (parameters α_{ij}, k_{ij}). Thus, in a general case the allocation game is not potential. It was shown that if the payoffs are player specific in each market and depend only on the number of players in it then the game admits an equilibrium in pure strategies [7]. But here we have a more complicated allocation game.

6.1 Allocation game of two players in two markets

Depending on the parameters of the model the equilibrium in the allocation game can be achieved in pure and mixed strategies. Consider the simplest case of two players V_1, V_2 and two markets M_1, M_2 . For simplicity suppose that the registration fee is $c_j = 0, j = 1, 2$.

In case when V_1, V_2 choose M_1 their payoffs are

$$u_i(q_1, q_2) = (q_i - p_1) \frac{k_{i1} \exp\{-\alpha_{i1}q_1\}}{k_{11} \exp\{-\alpha_{11}q_1\} + k_{21} \exp\{-\alpha_{21}q_2\}}, i = 1, 2.$$

Finding the equilibrium (q_1^*, q_2^*) we obtain that this point lies on the hyperbola

$$((q_1^* - p_1)\alpha_{11} - 1)((q_2^* - p_2)\alpha_{21} - 1) = 1,$$

Figure 2: Numeric example of a game for 8 strong and 14 weak MVNOs

and the equilibrium prices satisfy the condition

$$q_i^* \geq p_1 + \frac{1}{\alpha_{i1}}, i = 1, 2.$$

For symmetric case we showed in Section 2 that the equilibrium prices are equal to

$$q_1^* = q_2^* = p_1 + \frac{2}{\alpha_1}$$

and the optimal payoffs are

$$u_1^{11} = u_2^{11} = \frac{m_1}{\alpha_1}.$$

In case when V_1, V_2 choose M_2 their payoffs are

$$u_1^{22} = u_2^{22} = \frac{m_2}{\alpha_2}.$$

In case when V_1, V_2 choose M_1, M_2 respectively then their payoffs are

$$u_i^{12} = (Q_i - p_i)m_i, i = 1, 2.$$

Finally, if V_1, V_2 choose M_2, M_1 respectively then their payoffs are

$$u_i^{21} = (Q_j - p_j)m_j, i = 1, 2, i \neq j.$$

So, we obtain the bimatrix game with the payoff matrix

$$\begin{bmatrix} \left(\frac{m_1}{\alpha_1}, \frac{m_1}{\alpha_1} \right) & ((Q_1 - p_1)m_1, (Q_2 - p_2)m_2) \\ ((Q_2 - p_2)m_2, (Q_1 - p_1)m_1) & \left(\frac{m_2}{\alpha_2}, \frac{m_2}{\alpha_2} \right) \end{bmatrix}$$

For small $Q_2 : Q_2 \leq p_2 + \frac{m_1}{m_2\alpha_1}$ the equilibrium is to compete for players in the first market, and for small $Q_1 : Q_1 \leq p_1 + \frac{m_2}{m_1\alpha_2}$ the equilibrium is to compete for players in the second market. Otherwise, it is profitable to use different markets.

6.2 Stable allocation of identical players in two markets

Here we consider a problem of stable allocation of n identical players between markets M_1 and M_2 . Suppose that the market M_1 is more attractable than M_2 , i.e.

$$\frac{m_1}{\alpha_1} \geq \frac{m_2}{\alpha_2}.$$

Consider the case when $n_1 \geq 2$ identical players compete on the market M_1 and $n_2 \geq 2$ players compete on the market M_2 , $n_1 + n_2 = n$.

This allocation will be stable if for any player it is not profitable to deviate from the market. Consider market M_1 . Here the payoff of a player V_1 is

$$u_1^1 = \frac{m_1}{\alpha_1(n_1 - 1)}.$$

If it moves to market M_2 its payoff will be

$$u_1^2 = \frac{m_2}{\alpha_2 n_2}.$$

In case

$$\frac{m_1}{\alpha_1(n_1 - 1)} \geq \frac{m_2}{\alpha_2 n_2}$$

a player from market M_1 is not interested in the market M_2 . Arguing the same way if

$$\frac{m_2}{\alpha_2(n_2 - 1)} \geq \frac{m_1}{\alpha_1 n_1}$$

the players from market M_2 are not interested in the market M_1 .

We conclude that a stable allocation takes place for

$$\frac{\frac{m_1}{\alpha_1}}{\frac{m_1}{\alpha_1} + \frac{m_2}{\alpha_2}}(n - 1) \leq n_1 \leq \frac{\frac{m_1}{\alpha_1}}{\frac{m_1}{\alpha_1} + \frac{m_2}{\alpha_2}}(n - 1) + 1.$$

That is the same allocation as in Section 2 where we suppose that in each market we have at least two players. But it is possible that it is profitable for players to allocate the such way where $n - 1$ players act on the market M_1 and one player acts on the market M_2 .

In Section 5 for two players we show that sometimes it is profitable to choose different markets.

Consider here the case when the stable allocation is such that $n - 1$ players compete in the first market and player M_n occupies the second market.

In case when the players $V_1, \dots, V_{n-1}$ compete in M_1 then the equilibrium prices are

$$q_i^* = p_1 + \frac{n - 1}{\alpha_1(n - 2)}, i = 1, \dots, n - 1,$$

and the optimal payoffs are

$$u_i^1 = \frac{m_1}{\alpha_1(n - 2)}, i = 1, \dots, n - 2.$$

The payoff of player n is

$$u_n^2 = (Q_2 - p_2)m_2.$$

If player V_n joins the first market its payoff will be

$$u_n^1 = \frac{m_1}{\alpha_1(n-1)}.$$

It is less than u_n^2 if

$$Q_2 \geq p_2 + \frac{m_1}{m_2 \alpha_1(n-1)}. \quad (9)$$

So, in case of (9) player V_n will stay in the market M_2 .

Assume that one of players $V_1, \dots, V_{n-1}$ decides to change the current market M_1 for market M_2 . In this case its payoff will be

$$u_1^2 = \frac{m_2}{\alpha_2}.$$

It is unprofitable if

$$\frac{m_2}{\alpha_2} \leq \frac{m_1}{\alpha_1(n-2)}. \quad (10)$$

We have a stable allocation if both conditions (9), (10) are satisfied. Collecting together we obtain the condition for the monopoly in one of markets

$$1 + \frac{m_1}{\alpha_1} \frac{1}{m_2(Q_2 - p_2)} \leq n \leq 2 + \frac{\frac{m_1}{\alpha_1}}{\frac{m_2}{\alpha_2}}.$$

We see that monopoly case takes place for a small number of players and for $n > 2 + \frac{m_1}{\alpha_1} / \frac{m_2}{\alpha_2}$ the stable allocation yields the competition in all markets.

6.3 Stable allocation of non-identical players

In above we assume that the parameters of players which show the affect of changing of the price on the number of consumers α_{ij} depend on the market but the players are identical. Here we demonstrate the stable allocation of non-identical MVNOs among MNOs. As now the market share M_j for service i depends on the service itself, let us assume $\alpha_{ij} = \alpha^i, i = 1, \dots, n$.

Suppose that at the market there are two types of MVNOs. For example, there are "strong" and "weak" mobile virtual operators. Formally, it corresponds to the parameters $\alpha_{ij} = \alpha^1$, or $\alpha_{ij} = \alpha^2$, for all $j = 1, \dots, m$, and $\alpha^1 \leq \alpha^2$.

Suppose that there are only two markets M_1 and M_2 and $n = n_1 + n_2$ mobile virtual operators where n_1, n_2 is the number of "strong" and "weak" MVNO. First of all, the players select a market. Then they play in "pricing game".

Assume that the players are distributed in the following manner. On the market M_1 the distribution of "strong" and "weak" players is (k_1, k_2) , and on the market M_2 the distribution is (l_1, l_2) , $k_1 + l_1 = n_1$ and $k_2 + l_2 = n_2$. Let us find the equilibrium prices in each market. For simplicity assume that all parameters $k_{ij} = 1, \forall i, j$.

Consider the market M_1 . the price of MNO here is p_1 . The profile of prices of MVNO is divided for two parts $q = (q_1^1, \dots, q_{k_1}^1; q_1^2, \dots, q_{k_2}^2)$, corresponding to "strong" and "weak players", and the payoffs are

$$u_i^1(q) = (q_i^1 - p_1) m_1 \frac{\exp\{-\alpha^1 q_i^1\}}{\sum_{l=1}^{k_1} \exp\{-\alpha^1 q_l^1\} + \sum_{l=1}^{k_2} \exp\{-\alpha^2 q_l^2\}} - c_1, i = 1, \dots, k_1,$$

for "strong" players and

$$u_i^2(q) = (q_i^2 - p_1)m_1 \frac{\exp\{-\alpha^2 q_i^2\}}{\sum_{l=1}^{k_1} \exp\{-\alpha^1 q_l^1\} + \sum_{l=1}^{k_2} \exp\{-\alpha^2 q_l^2\}} - c_2, i = 1, \dots, k_2,$$

for "weak" players.

The first order condition for the equilibrium $\partial u_i^j(q)/\partial q_i = 0, \forall i, j = 1, 2$, and symmetry of players inside the groups yields that the equilibrium prices for "strong" and "weak players" are equal to q_1^*, q_2^* , respectively and satisfy the system of equation

$$\begin{aligned} (q_1 - p_1)\alpha^1 ((k_1 - 1) \exp(-\alpha^1 q_1) + k_2 \exp(-\alpha^2 q_2)) &= k_1 \exp(-\alpha^1 q_1) + k_2 \exp(-\alpha^2 q_2) \\ (q_2 - p_1)\alpha^2 (k_1 \exp(-\alpha^1 q_1) + (k_2 - 1) \exp(-\alpha^2 q_2)) &= k_1 \exp(-\alpha^1 q_1) + k_2 \exp(-\alpha^2 q_2). \end{aligned} \quad (11)$$

Hence, the optimal payoff of "strong" player on the first market is

$$u^1(k_1, k_2, m_1) = (q_1^* - p_1) \frac{m_1}{\alpha^1} \cdot \frac{\exp\{-\alpha^1 q_1^*\}}{(k_1 - 1) \exp\{-\alpha^1 q_1^*\} + k_2 \exp\{-\alpha^2 q_2^*\}} - c_1, i = 1, \dots, k_1,$$

and

$$u^2(k_1, k_2, m_1) = (q_2^* - p_1) \frac{m_1}{\alpha^2} \cdot \frac{\exp\{-\alpha^2 q_2^*\}}{k_1 \exp\{-\alpha^1 q_1^*\} + (k_2 - 1) \exp\{-\alpha^2 q_2^*\}} - c_2, i = 1, \dots, k_2,$$

for "weak" players.

The same arguments are true for the equilibrium prices at the market M_2 . Now we can determine when the allocation of n MVNOs among two markets M_1 and M_2 will be stable. Allocation $[(k_1, k_2); (l_1, l_2)]$ is Nash-stable if for each player it is not-profitable to deviate from the current market. Formally, it means that the following inequalities must be satisfied

$$\begin{aligned} u^1(k_1, k_2, m_1) &\geq u^1(l_1 + 1, l_2, m_2), \quad u^2(k_1, k_2, m_1) \geq u^2(l_1, l_2 + 1, m_2), \\ u^1(k_1 + 1, k_2, m_1) &\leq u^1(l_1, l_2, m_2), \quad u^2(k_1, k_2 + 1, m_1) \leq u^2(l_1, l_2, m_2). \end{aligned} \quad (12)$$

6.4 Numerical example

Consider the mobile network market with two MNOs, see Figure 2. The first market is large $m_1 = 1000$, the second is twice smaller $m_2 = 500$. There are twenty two MVNOs competing for the consumers at these markets. Suppose that among these MVNOs there are $n_1 = 8$ "strong" players and $n_2 = 14$ "weak" players, and $\alpha^1 = 1, \alpha^2 = 2$. Let the prices for the resource in both markets be equal $p_1 = p_2 = 1$, and the costs are $c_1 = 5, c_2 = 2$. Let us show that the allocation

$(k_1 = 5, k_2 = 10), (l_1 = 3, l_2 = 4)$ is Nash-stable.

From the equations (11) we find the equilibrium prices in both markets. On the market M_1

$$q_1^* = 2.125, q_2^* = 1.523.$$

The payoffs of the both types players in the equilibrium on the market M_1 are

$$u^1(5, 10, 1000) = 120.299, u^2(5, 10, 1000) = 21.191.$$

On the market M_2 we find

$$q_1^* = 2.267, q_2^* = 1.550.$$

The payoffs of the both types players in the equilibrium on the market M_2 are

$$u^1(3, 4, 500) = 128.755, u^2(3, 4, 500) = 23.242.$$

We see that the market M_2 is more profitable for both types of players. Prove the conditions (12). Suppose, that a "strong" player from the market M_1 decides to move to the market M_2 . We find that its payoff here is $u^1(4, 4, 500) = 102.598$. It is less than its payoff on the market M_1 . So, it is not reasonable to move to another market. Now assume that the "weak player" moves from market M_1 to the market M_2 . Its payoff here is $u^2(3, 5, 500) = 20.700$. It is less than on the market M_1 . So, we see that the allocation $(k_1 = 5, k_2 = 10), (l_1 = 3, l_2 = 4)$ is Nash-stable.

7 Related Work

Today mobile virtual network operators (MVNO) play an important role in service diversification. However, alone without proper regulatory rules MVNO would not be able to operate, since mobile network operators (MNO) can easily push them out of market [3]. In this paper we do not consider the competition between MVNO and MNO as such, but rather assume that the latter one, in order to compete for the market share, needs to organize a separate child company which will compete for the market with MVNO. Thus, similarly to [6], we consider only the case when end-users select MVNOs based on preferences (such as price, Service Level Agreement (SLA), etc) and MVNOs compete with each other while trying to maximize own revenue and reduce the operational costs. The problem is therefore reduced to a classical setup for the game theory, in a similar way as it was done in [1].

Game theory has a long history and finds its roots in economy where market players compete with each for resources [5]. Since then, the game theory paved its way to other relevant fields in which competition between strategic players appears. Recently the game theoretic approach found its way in cloud computing [1, 6]. Here multiple brokers compete with each other for users who make decisions which provider to use based on price for computational resources. As indicated in [6], the price is not the only parameter in utility function of an end-user: SLA [1], energy consumption and added value services can be introduced in calculation of Nash equilibrium (NE), too.

To the best of our knowledge the scenario which we consider in this paper was not yet covered fully in the literature. In [6] the authors consider the

similar problem and introduce a game theoretical view on it, however they do not consider mobile network operators but rather cloud service providers as players of the game. On the other hand, in [4, 2, 9, 11] the authors consider the problem of competition between MNOs and MVNOs but they work toward solving the radio resource allocation problem in optimal way, which is different from the problem suggested in this paper.

8 Conclusion

This paper discussed a competition model on the three-level mobile operator market. The model is presented as a two-staged non-cooperative game. On the first stage that is an allocation game where the players (MVNO) select the MNO market. On the second stage that is a pricing game where they compete between themselves for the consumers. We show that the pricing game is a potential game. It simplifies finding the equilibrium. Moreover, the payoffs of the players are decreasing functions if the number of MVNOs on the same market is increasing. It yields that in some cases the allocation game on the first stage can be investigated by the methods of the congestion game theory. We analyze in detail the cases of identical MVNOs and non-identical MNOs. A useful characteristic of the MNO markets is developed, that is the ratio of the number of consumers covered by MNO and the influence of the price on attractability of this operator. All mobile network operators can be ranked on the market using this characteristic.

We make accent in the paper mainly on the analytic investigation of the problem. But the proposed game-theoretical model can be applied in any real mobile network market. The model can be extended to include to the game MNO as a player. They can also compete in the market determining the price for their resources. This three-stage game is an interesting topic for future study.

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