Motion planning and feedback control techniques with applications to long tractor-trailer vehicles

Oskar Ljungqvist

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Cover illustration: A library of optimized maneuvers for a tractor-trailer vehicle.

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To my family!

Abstract

During the last decades, improved sensor and hardware technologies as well as new methods and algorithms have made self-driving vehicles a realistic possibility in the near future. At the same time, there has been a growing demand within the transportation sector to increase efficiency and to reduce the environmental impact related to transportation of people and goods. Therefore, many leading automotive and technology companies have turned their attention towards developing advanced driver assistance systems and self-driving vehicles.

Autonomous vehicles are expected to have their first big impact in closed environments, such as mines, harbors, loading and offloading sites. In such areas, the legal requirements are less restrictive and the surrounding environment is more controlled and predictable compared to urban areas. Expected positive outcomes include increased productivity and safety, reduced emissions and the possibility to relieve the human from performing complex or dangerous tasks. Within these sites, tractor-trailer vehicles are frequently used for transportation. These vehicles are composed of several interconnected vehicle segments, and are therefore large, complex and unstable while reversing. This thesis addresses the problem of designing efficient motion planning and feedback control techniques for such systems.

The contributions of this thesis are within the area of motion planning and feedback control for long tractor-trailer combinations operating at low-speeds in closed and unstructured environments. It includes development of motion planning and feedback control frameworks, structured design tools for guaranteeing closed-loop stability and experimental validation of the proposed solutions through simulations, lab and field experiments. Even though the primary application in this work is tractor-trailer vehicles, many of the proposed approaches can with some adjustments also be used for other systems, such as drones and ships.

The developed sampling-based motion planning algorithms are based upon the probabilistic closed-loop rapidly exploring random tree (CL-RRT) algorithm and the deterministic lattice-based motion planning algorithm. It is also proposed to use numerical optimal control offline for precomputing libraries of optimized maneuvers as well as during online planning in the form of a warm-started optimization step.

To follow the motion plan, several predictive path-following control approaches are proposed with different computational complexity and performance. Common for these approaches are that they use a path-following error model of the vehicle for future predictions and are tailored to operate in series with a motion planner that computes feasible paths. The design strategies for the path-following approaches include linear quadratic (LQ) control and several advanced model predictive control (MPC) techniques to account for physical and sensing limitations. To strengthen the practical value of the developed techniques, several of the proposed approaches have been implemented and successfully demonstrated in field experiments on a full-scale test platform. To estimate the vehicle states needed for control, a novel nonlinear observer is evaluated on the full-scale test vehicle. It is designed to only utilize information from sensors that are mounted on the tractor, making the system independent of any sensor mounted on the trailer.

Populärvetenskaplig sammanfattning

Under de senaste årtiondena har utvecklingen av sensor- och hårdvaruteknik gått i en snabb takt, samtidigt som nya metoder och algoritmer har introducerats. Samtidigt ställs det stora krav på transportsektorn att öka effektiviteten och minska miljöpåverkan vid transporter av både människor och varor. Som en följd av detta har många ledande fordonstillverkare och teknikföretag börjat satsa på att utveckla avancerade förarstödsystem och självkörande fordon. Även forskningen inom autonoma fordon har under de senaste årtiondena kraftigt ökat då en rad tekniska problem återstår att lösas.

Förarlösa fordon förväntas få sitt första stora genombrott i slutna miljöer, såsom gruvor, hamnar, lastnings- och lossningsplatser. I sådana områden är lagstiftningen mindre hård jämfört med stadsområden och omgivningen är mer kontrollerad och förutsägbar. Några av de förväntade positiva effekterna är ökad produktivitet och säkerhet, minskade utsläpp och möjligheten att avlasta människor från att utföra svåra eller farliga uppgifter. Inom dessa platser används ofta lastbilar med olika släpvagnskombinationer för att transportera material. En sådan fordonskombination är uppbyggd av flera ihopkopplade moduler och är således utmanande att backa då systemet är instabilt. Detta gör det svårt att utforma ramverk för att styra sådana system vid exempelvis autonom backning.

Självkörande fordon är mycket komplexa system som består av en rad olika komponenter vilka är designade för att lösa separata delproblem. Två viktiga komponenter i ett självkörande fordon är dels rörelseplaneraren som har i uppgift att planera hur fordonet ska röra sig för att på ett säkert sätt nå ett överordnat mål, och dels den banföljande regulatorn vars uppgift är att se till att den planerade manövern faktiskt utförs i praktiken trots störningar och modellfel. I denna avhandling presenteras flera olika algoritmer för att planera och utföra komplexa manövrar för lastbilar med olika typer av släpvagnskombinationer. De presenterade algoritmerna är avsedda att användas som avancerade förarstödsystem eller som komponenter i ett helt autonomt system. Även om den primära applikationen i denna avhandling är lastbilar med släp, kan många av de föreslagna algoritmerna även användas för en rad andra system, så som drönare och båtar.

Experimentell validering är viktigt för att motivera att en föreslagen algoritm är användbar i praktiken. I denna avhandling har flera av de föreslagna planerings- och reglerstrategierna implementerats på en småskalig testplattform och utvärderats i en kontrollerad labbmiljö. Utöver detta har även flera av de föreslagna ramverken implementerats och utvärderats i fältexperiment på en fullskalig testplattform som har utvecklats i samarbete med Scania CV. Här utvärderas även en ny metod för att skatta släpvagnens beteende genom att endast utnyttja information från sensorer monterade på lastbilen, vilket gör det föreslagna ramverket oberoende av sensorer monterade på släpvagnen.

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Linköping, April 2020
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Notation

ACRONYMS

Acronyms	Meaning
AABB	Axis-aligned bounding box
ADAS	Advanced driver assistance systems
ARA*	Anytime repairing A*
ARE	Algebraic Riccati equation
BVP	Boundary-value problem
CL-RRT	Closed-loop RRT
EKF	Extended Kalman filter
FFI	Fordonsstrategisk forskning och innovation
GNSS	Global navigation satellite system
GPS	Global positioning system
HLUT	Heuristic look-up table
IMU	Inertial measurement unit
LDI	Linear differential inclusion
LIDAR	Light detection and ranging
LMI	Linear matrix inequality
LPV	Linear parameter-varying
LQ	Linear quadratic
LTV	Linear time-varying
MIP	Mixed integer programming
MIQP	Mixed integer quadratic programming
MPC	Model predictive control
MSNT	Multi-steered N-trailer
NLP	Nonlinear programming
OBB	Oriented bounding box
OCF	Optimal control problem
PID	Proportional, integral, derivative
PMP	Pontryagin's minimum (maximum) principle
QP	Quadratic programming
RADAR	Radio detection and ranging
RRT	Rapidly-exploring random tree
SDP	Semidefinite programming
SQP	Sequential quadratic programming
SSNT	Single-steered N-trailer

NOTATION

Notation	Meaning
$\mathbb{R}$	Set of real numbers
$\mathbb{R}_+$	Set of positive real numbers
$\mathbb{Z}$	Set of integers
$\mathbb{Z}_+$	Set of positive integers
$\mathbb{Z}_{++}$	Set of strictly positive integers
$\mathbb{R}^n$	Set of real vectors with n components
$\mathbb{R}^{n \times m}$	Set of real matrices with n rows and m columns
$\mathbb{S}_+^n$	Set of positive semidefinite matrices with n columns
$\mathbb{S}_{++}^n$	Set of positive definite matrices with n columns
$A \succeq 0$	Matrix A is positive semidefinite
$A \succ 0$	Matrix A is positive definite
$\det(A)$	Determinant of a matrix A
$\text{rank}(A)$	Rank of a matrix A
$\sigma_{\max} \text{ (min)}(A)$	Maximum (minimum) singular value of matrix A
$\text{Cond}(A)$	Condition number of matrix A
$\mathbf{Co} P$	Convex hull of a set of points P
$ \mathcal{S} $	Cardinality of a set $\mathcal{S}$

Part I

Background

1

Introduction

This chapter introduces the research fields of self-driving vehicles and advanced driver assistance systems (ADAS). The chapter starts with a short background and motivation why research in this field is important. In Section 1.2, a simplified system architecture of a self-driving vehicle is presented and in Section 1.3, the main contributions of this thesis are summarized. Finally, Section 1.4 provides an overview of the contributions and an outline of the thesis.

1.1 Background and motivation

In the last decades, emerging sensor and hardware technologies have made the idea of developing self-driving vehicles a realistic possibility in the near future. Ever since the groundbreaking DARPA Grand Challenges (Buehler et al., 2007) and DARPA Urban Challenge (Buehler et al., 2009) were held, many leading automotive manufacturers and technology companies have turned their attention towards developing self-driving vehicles. Removing the human from the steering wheel and instead providing mobility as a service is predicted to have positive effects on road safety, reduced greenhouse gas emissions and enhanced utilization of the overall vehicle fleet (Burns, 2013). Car manufacturers see added value to their customers and the possibility to gain a competitive edge to their competitors by providing this technology.

At the same time, the transportation industry sees growing demands for delivering goods and transporting people. However, since the transportation industry is one of the largest emitter of greenhouse gases, the European Road Transport Research Advisory Council (2013) and the European Commission (2011) have set up strict goals for improvement of the European transportation system with an overall efficiency improvement by 50% in 2030 compared to 2010 while reducing emissions by 60%. To fulfill these requirements, the transportation industry sees large poten-

Figure 1.1: The test vehicle that is used as research platform. The truck is a modified version of a Scania R580 6x4, and neither the semitrailer or the dolly is equipped with any sensor. In Paper D and Paper E, this vehicle is used for evaluation of proposed motion planning and control solutions.

tial in automation, electrification and intelligent transportation systems, and are therefore also shifting their attention towards developing autonomous solutions.

Apart from environmental and efficiency aspects, safety is another concern. Annually, over 40 million people are injured in road traffic related accidents (World Health Organization, 2015), where most accidents occur due to human errors (European Commission, 2011). These statistics show that a car is by far the most dangerous transportation alternative (Ernst and Young, 2015). Systems to increase safety have been considered for many years in the automotive industry, where anti-lock braking and electronic stability program are examples of ADAS that have been developed for this purpose. Thanks to recent development in sensor technology, e.g., RADAR, LIDAR and camera sensors, more advanced ADAS have been developed and are now standard in many of today's (modern) cars. These systems can detect and handle critical situations much faster than an average driver. Examples of such systems are lane keeping assist, trailer assist, adaptive cruise control and queue assist. These systems are taking more and more control over the vehicle to aid the driver in critical or mentally exhausting situations.

With even more advanced systems, such as parallel parking assist and the Tesla autopilot, fully autonomous vehicles are getting closer and have the potential to revolutionize the transportation sector. Autonomous vehicles are not only predicted to reduce the traffic related accidents, but also to transform today's transportation towards a more service-based system. Sharing autonomous vehicles is predicted to result in a better overall utilization of the available vehicle fleet and contribute to a more sustainable future (Burns, 2013; Thrun, 2010).

Today, autonomous driving in urban areas with pedestrians, cyclists and other moving vehicles is still a hard challenge with many unsolved problems. This together with the legislation changes needed to drive autonomously on public roads make closed areas, such as mines, harbors and loading/offloading sites, perfect areas for initial deployment of self-driving vehicles. Here, expected positive outcomes are increased productivity and safety, reduced emissions, lower wear on the equipment and the possibility to relieve the human from performing complex or

dangerous tasks. Within these sites, different tractor-trailer vehicles can be used to efficiently transport goods and other material. One such example is the truck with a dolly-steered semitrailer that is depicted in Figure 1.1. This vehicle combination consists of a truck with front-wheel steering, a dolly and a semitrailer. This system will also be referred to as a general 2-trailer with a car-like tractor, where the word general refers to the non-zero off-axle hitch connection between the truck and the dolly Altafini et al. (2001).

In comparison to cars and trucks without any trailer, tractor-trailer vehicles are larger and have unstable dynamics while reversing. In particular, the dolly-steered semitrailer case is extra challenging. These properties increase the difficulty of designing efficient *motion planning* and *feedback control* frameworks for such systems, e.g., for autonomous reversing.

Despite the large amount of research efforts focusing on cars (Paden et al., 2016), existing work that explicitly targets motion planning and feedback control problems for general tractor-trailer vehicles is rather limited (Altafini et al., 2001; Kati et al., 2019b; Michalek, 2014; Pradalier and Usher, 2008; Rimmer and Cebon, 2017). Especially, prior to this work, no complete motion planning and control framework has been presented for these types of systems. Furthermore, before this work, neither has any complete such framework been successfully demonstrated in real-world experiments on a full-scale general 2-trailer with a car-like tractor.

Before presenting the contributions of this thesis, a brief overview of a simplified architecture for a self-driving vehicle is first presented.

1.2 System architecture

A self-driving vehicle is a complex system that is composed of multiple subsystems, where each subsystem is designed to solve separate tasks. These subsystems communicate by transmitting and retrieving information between each other using a well-defined interface. An overview of a simplified system architecture for a self-driving vehicle is depicted in Figure 1.2. This work is focusing on the subsystems that are colored in blue, i.e., the motion planner and the feedback controller.

First, a self-driving vehicle needs a mission or a task to perform. In industrial applications, a mission is in many cases defined in a complex way, where multiple vehicles need to cooperate to solve the task. A *mission planner* or *task planner* divides this problem into smaller subproblems and delegates them to suitable vehicles based on their capabilities. An example of a mission can be to load a tractor-trailer vehicle with rocks. A solution would be to instruct the tractor-trailer vehicle to go to a suitable loading area and the excavator to pick up rocks and load the trailer until it is fully loaded. These subproblems can be further divided into even smaller subproblems. In this thesis, the focus is on solving one subproblem which is of great importance: how to control a tractor-trailer vehicle such that it can automatically move to a desired location without a clearly defined driving path. In such scenarios, the vehicle is said to operate in an *unstructured environment* and a motion planner is here required to compute how the vehicle should move to the desired location, e.g., a loading bay or a parking spot.

Figure 1.2: An overview of a simplified system architecture for a self-driving vehicle where the blue subsystems are considered in this thesis. Inspired and adapted from (Evestedt, 2016; Lima, 2018).

Before a maneuver can be planned and executed, the system needs to obtain a representation of the surrounding environment and understand where the vehicle is located in the world. These tasks are taken care of by the *perception* and *localization* layers, which use measurements from the vehicle’s onboard sensors to construct this valuable information. A typical sensor platform on a self-driving vehicle is composed of multiple different sensors, such as global navigation satellite system (GNSS) receivers, e.g. global positioning system (GPS) receivers, inertial measurement units (IMUs), RADAR sensors, LIDAR sensors and cameras. The localization layer combines the sensor information with offline data, such as stored maps, to estimate the vehicle’s position, orientation and other important vehicle states (Skog and Händel, 2009). The perception layer makes use of the same sensor information to detect, classify and track different objects, such as people and other vehicles (Granström et al., 2019). Together, these layers provide a compressed representation of the surrounding environment in which motion planning and feedback control is performed.

The motion planner’s objective is to compute a motion plan from the vehicle’s current state to the desired goal state specified by the mission planner or a human operator. The motion plan is a nominal trajectory or path that, most often, satisfies a mathematical model of the vehicle and is computed such that the vehicle is predicted not to collide with any surrounding obstacle.

To guarantee safe execution of the motion plan, the feedback controller is designed to stabilize the vehicle around the nominal trajectory or path despite that disturbances are acting on the vehicle.

1.3 Main contributions

The main contributions of this thesis are within the area of motion planning and feedback control for tractor-trailer vehicles. The contributions include development of motion planning and feedback control frameworks, some design approaches for guaranteeing closed-loop stability, and experimental validation of the proposed frameworks through simulations, lab and field experiments. State estimation of a full-scale general 2-trailer with a car-like tractor is also considered for the case when neither the semitrailer nor the dolly is equipped with any sensor. Many of the developed solutions are also experimentally validated on either a physical small-scale or a full-scale test vehicle. In summary, the contributions of this thesis are:

- Development of a cascade controller for a reversing general 2-trailer system and the use of this controller within a closed-loop rapidly-exploring random tree (CL-RRT) framework (Paper A). To the authors' knowledge, this was the first sampling-based motion planner for this type of system.
- Derivation of a path-following error model for a general 2-trailer system for which a linear quadratic (LQ) path-following controller is proposed as well as a method for analyzing stability of the closed-loop system, where the resulting constraints on the motion plan can be enforced on a motion planner to guarantee safe path execution (Paper B).
- Modeling of the closed-loop system (a lattice-based planner, a state-feedback controller and a controlled vehicle) as a hybrid system and the development of a tailored controller synthesis and stability analysis tool to a priori guarantee stability (Paper C).
- Development of a path planning and path-following control framework for a general 2-trailer with a car-like tractor including a lattice-based planner, a path-following controller and an observer that only utilizes information from sensors that are mounted on the tractor (Paper D). To the authors' knowledge, this was the first planning and control framework for a full-scale tractor-trailer vehicle that has been successfully demonstrated in practice.
- Development, implementation and experimental evaluation of a model predictive path-following control approach for a general 2-trailer with a car-like tractor in which an advanced sensor's sensing limitations are modeled and incorporated as explicit constraints in the controller (Paper E).
- Generalization of the lattice-based planner in Paper D to also include multi-steered articulated vehicles and to propose a post-optimization step that enables the framework to compute locally optimal trajectories that start exactly at the vehicle's initial state and reaches the goal state exactly (Paper F).
- Generalization of the path-following error model in Paper B and parts of the model predictive path-following control approach in Paper E to also include multi-steered articulated vehicles (Paper G).

1.4 Thesis outline and contributions

This thesis is divided into two parts. The first part contains some background material regarding nonholonomic systems as well as popular motion planning and feedback control techniques for such systems. The second part of this thesis is a collection of scientific contributions including six accepted papers and one submitted paper which is an extension of an accepted conference paper. A brief summary of each paper is given below.

Paper A: Motion planning for a reversing general 2-trailer configuration using Closed-Loop RRT

N. Evestedt, O. Ljungqvist, and D. Axehill. Motion planning for a reversing general 2-trailer configuration using Closed-Loop RRT. In *Proceedings of the 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3690–3697, 2016b.

Summary: This paper presents a probabilistic motion planning framework based on closed-loop rapidly-exploring random tree (CL-RRT) for a general 2-trailer with a car-like tractor. The path-following controller developed in Evestedt et al. (2016a); Ljungqvist (2015) is used to enable efficient closed-loop simulations of the system within the CL-RRT framework presented in Evestedt et al. (2015). The framework is evaluated in a set of simulations in different kinds of environments and in a lab environment with a small-scale test platform. The results show that the planner has a high success rate in finding motion plans in complex and constrained environments.

Background and contribution: The idea of performing closed-loop simulations of a general 2-trailer with a car-like tractor within a CL-RRT framework was initiated from discussions between the Niclas Evestedt and Daniel Axehill. The concept was then developed by the author of this thesis during his Master’s thesis project (Ljungqvist, 2015) where Niclas acted as supervisor, and Daniel as examiner and supervisor. In an early stage, the author of this thesis and Niclas initiated a tight collaboration where the underlying CL-RRT platform was developed by Niclas (Evestedt et al., 2015) and the development as well as the integration of the stabilizing controller was performed by the author of this thesis. The experimental platform development and data collections were accomplished jointly between the author of this thesis and Niclas. The majority of the writing was done by Niclas and Daniel acted as supervisor and reviewed the manuscript.

Paper B: Path-following control for a reversing general 2-trailer system

O. Ljungqvist, D. Axehill, and A. Helmersson. Path following control for a reversing general 2-trailer system. In *Proceedings of the 55th IEEE Conference on Decision and Control*, pages 2455–2461, 2016.

Summary: A path-following controller for a reversing general 2-trailer with a car-like tractor is proposed for the case when the motion plan is kinematically

feasible to follow exactly. It is done by first deriving a path-following error model in which the vehicle's path-following error states are described in terms of deviation from the nominal path. A stabilizing LQ controller with feedforward action is then designed. Given that the motion planner computes motion plans for a specified set of possible motions, the origin of the closed-loop system is shown to be an exponentially stable equilibrium point. The theoretical results are verified through simulations of the closed-loop system around an eight-shaped path.

Background and contribution: The idea to this work was initiated through discussion between the author of this thesis, Daniel Axehill and Anders Helmersson. The modeling of the vehicle in the Frenet-Serret frame was performed by the author of this thesis as well as the theoretical derivations, implementations, numerical calculations and the written manuscript. Daniel and Anders acted as supervisors and reviewed the manuscript.

Paper C: On stability for state-lattice trajectory tracking control

O. Ljungqvist, D. Axehill, and J. Löfberg. On stability for state-lattice trajectory tracking control. In *Proceedings of the 2018 American Control Conference*, 2018.

Summary: This paper presents a systematic framework for analyzing stability of the closed-loop system consisting of a controlled vehicle and a feedback controller executing a motion plan computed by a lattice-based planner. When this motion planner is considered, it is shown that the closed-loop system can be modeled as a nonlinear hybrid system. Based on this, we propose a novel method for analyzing the behavior of the tracking error, how to design the low-level controller and how to potentially impose constraints on the motion planner, in order to guarantee that the tracking error is bounded and decays towards zero. The proposed method is applied on a tractor-trailer system and the results are verified in simulations.

Background and contribution: The idea to this work evolved after discussion between the author of this thesis and Daniel Axehill. Johan Löfberg was involved in some of the discussions and in particular during the development of Proposition 1. The author of this thesis contributed with the majority of the work including theoretical derivations and the written manuscript. Daniel contributed throughout the process and acted as supervisor and reviewed the manuscript.

Paper D: A path planning and path-following control framework for a general 2-trailer with a car-like tractor

O. Ljungqvist, N. Evestedt, D. Axehill, M. Cirillo, and H. Pettersson. A path planning and path-following control framework for a general 2-trailer with a car-like tractor. *Journal of Field Robotics*, 36(8):1345–1377, 2019.

Summary: This paper presents a complete motion planning and control framework for a general 2-trailer with a car-like tractor that can be used to automatically plan and execute difficult parking and obstacle avoidance maneuvers by combining forward and backward motion. A lattice-based motion planner is utilized to efficiently compute kinematically feasible and collision-free motion plans. To execute the motion plan, a path-following controller is developed that stabilizes the lateral and angular path-following error states of the vehicle. Moreover, a nonlinear observer for state estimation is developed which only utilizes information from sensors that are mounted on the tractor, making the system independent of any additional sensor mounted on the semitrailer. The proposed planning and control framework is implemented on a full-scale test vehicle and a series of field experiments are presented.

Background and contribution: This work summarizes a long project where the path planner from Ljungqvist et al. (2017) and the path-following controller from Paper C were implemented on a full-scale test platform. Many extensions from those works have been made to cope with the full-scale test platform and have mainly been performed by the author of this thesis together with Niclas Evestedt. Niclas started the implementation of the nonlinear observer before leaving academia. The author of this thesis took over and made many improvements to make the nonlinear observer work robustly for all relevant scenarios. The field experiments and the tuning of all modules were made by the author of this thesis together with Henrik Pettersson. The author of this thesis contributed with the majority of the work including theoretical derivations, numerical calculations, field experiments and the written manuscript. Niclas, Daniel Axehill and Marcello Cirillo contributed throughout the process and reviewed the manuscript.

Paper E: Estimation-aware model predictive path-following control for a general 2-trailer with a car-like tractor

O. Ljungqvist, D. Axehill, H. Pettersson, and J. Löfberg. Estimation-aware model predictive path-following control for a general 2-trailer with a car-like tractor. *Submitted to IEEE Transactions on Robotics*. Preprint: <https://arxiv.org/abs/2002.10291>, 2020b.

Summary: This paper presents a model predictive path-following control approach for a general 2-trailer with a car-like tractor. It is inspired by the work in Paper D where an advanced sensor with a limited field of view is placed in the rear of the tractor to solve the joint-angle estimation problem. This implies that the proposed estimation solution introduces restrictions on the joint-angle configurations that can be estimated with high accuracy. To model and explicitly consider these constraints in the controller, a model predictive path-following control approach is proposed. Two approaches with different computation complexity and performance are presented. In simulations and field experiments, the performance of the proposed path-following control approach is compared with the proposed control strategy in Paper D where the joint-angle constraints are neglected.

Background and contribution: The idea to this work initiated after discussions between the author of this thesis and Daniel Axehill. A preliminary version of the paper was presented in Ljungqvist et al. (2020a). Johan Löfberg assisted in the development of the mixed-integer programming formulation as well as with some implementation details in YALMIP (Löfberg, 2004). The implementation and field experiments were carried out by the author of this thesis together with Henrik Pettersson. The author of this thesis contributed with the majority of the work including theoretical derivations, numerical calculations, field experiments and the written manuscript. Daniel and Johan reviewed the manuscript.

Paper F: Optimization-based motion planning for multi-steered articulated vehicles

O. Ljungqvist, K. Bergman, and D. Axehill. Optimization-based motion planning for multi-steered articulated vehicles. *Accepted for publication in Proceedings of the 21th IFAC World Congress*. Preprint: <https://arxiv.org/abs/1912.06264>, 2020c.

Summary: This paper presents an optimization-based trajectory planner targeting low-speed maneuvers in unstructured environments for multi-steered N-trailer (MSNT) vehicles, which are composed of a car-like tractor and an arbitrary number of N interconnected trailers with fixed or steerable wheels. The proposed trajectory planner uses a lattice-based planner in a first step to compute a resolution optimal solution to a discretized version of the trajectory planning problem. The output from the lattice-based planner is then used in a second step to warm-start an optimal control problem solver, which enables the framework to compute locally optimal trajectories that start at the vehicle's initial state and reaches the goal state exactly. The performance of the trajectory planner is evaluated in a set of parking scenarios for an MS3T vehicle with a car-like tractor where the last trailer is steerable.

Background and contribution: The idea for this work was initiated after discussion between the author of this thesis, Kristoffer Bergman and Daniel Axehill. Throughout the process, the author of this thesis and Kristoffer maintained a tight collaboration. The proposed trajectory planner is based on the work in Bergman et al. (2019a,b) and is here tailored for MSNT vehicles. The implementation was accomplished jointly between the author of this thesis and Kristoffer. The author of this thesis contributed with all derivations, calculations and the written manuscript. Kristoffer and Daniel reviewed the manuscript.

Paper G: A predictive path-following controller for multi-steered articulated vehicles

O. Ljungqvist and D. Axehill. A predictive path-following controller for multi-steered articulated vehicles. *Accepted for publication in Proceedings of the 21th IFAC World Congress*. Preprint: <https://arxiv.org/abs/1912.06259>, 2020.

Summary: This paper presents a model predictive path-following control approach for automatic low-speed maneuvering of MSNT vehicles. The proposed path-following controller is tailored to follow nominal paths that contain full state and control-input information, and is designed to satisfy various constraints on the vehicle states as well as saturations and rate limitations on the tractor’s curvature and the trailer steering angles. The performance of the proposed model predictive path-following controller is evaluated in a set of simulations for an MS2T vehicle with a car-like tractor where the last trailer is steerable.

Background and contribution: The ideas for this work were mainly initiated by the author of this thesis. The work generalizes and combines parts of the work in Paper B, Paper D and Paper E to include MSNT vehicles. The author of this thesis contributed with the majority of the work including theoretical derivations, implementation, numerical calculations and the written manuscript. Daniel Axehill acted as supervisor and reviewed the manuscript.

1.5 Other publications

The following additional publications have been authored or co-authored by the author of this thesis:

N. Evestedt, O. Ljungqvist, and D. Axehill. Path tracking and stabilization for a reversing general 2-trailer configuration using a cascaded control approach. In *Proceedings of the 2016 IEEE Intelligent Vehicles Symposium*, pages 1156–1161, 2016a.

O. Ljungqvist, N. Evestedt, M. Cirillo, D. Axehill, and O. Holmer. Lattice-based motion planning for a general 2-trailer system. In *Proceedings of the 2017 IEEE Intelligent Vehicles Symposium*, pages 2455–2461, 2017.

G. Ling, K. Lindsten, O. Ljungqvist, J. Löfberg, C. Norén, and C. A. Larsson. Fuel-efficient model predictive control for heavy duty vehicle platooning using neural networks. In *Proceedings of the 2018 Annual American Control Conference*, pages 3994–4001, 2018.

O. Andersson, O. Ljungqvist, M. Tiger, D. Axehill, and F. Heintz. Receding-horizon lattice-based motion planning with dynamic obstacle avoidance. In *Proceedings of the 57th IEEE Conference on Decision and Control*, pages 4467–4474, 2018b.

O. Ljungqvist. *On motion planning and control for truck and trailer systems*. Licentiate’s thesis, Linköping University, 2019.

K. Bergman, O. Ljungqvist, and D. Axehill. Improved optimization of motion primitives for motion planning in state lattices. In *Proceedings of the 2019 IEEE Intelligent Vehicles Symposium*, 2019b.

K. Bergman, O. Ljungqvist, and D. Axehill. Improved path planning by tightly combining lattice-based path planning and optimal control. *Accepted for publication in IEEE Transactions on Intelligent Vehicles*. Preprint: <https://arxiv.org/abs/1903.07900>, 2019a.

R. Oliveira, O. Ljungqvist, P. F. Lima, and B. Wahlberg. Optimization-based on-road path planning for articulated vehicles. *Accepted for publication in Proceedings of the 21th IFAC World Congress*. Preprint: <https://arxiv.org/abs/2001.06827>, 2020a.

O. Ljungqvist, D. Axehill, and H. Pettersson. On sensing-aware model predictive path-following control for a reversing general 2-trailer with a car-like tractor. *Accepted for publication in Proceedings of the 2020 IEEE International Conference on Robotics and Automation*. Preprint: <https://arxiv.org/abs/2002.06874>, 2020a.

K. Bergman, O. Ljungqvist, T. Glad, and D. Axehill. An optimization-based receding horizon trajectory planning algorithm. *Accepted for publication in Proceedings of the 21th IFAC World Congress*. Preprint: <https://arxiv.org/abs/1912.05259>, 2020a.

R. Oliveira, O. Ljungqvist, P. F. Lima, and B. Wahlberg. A geometric approach to on-road motion planning for long and multi-body heavy-duty vehicles. *Accepted for publication in Proceedings of the 31st IEEE Intelligent Vehicles Symposium*, 2020b.

K. Bergman, O. Ljungqvist, J. Linder, and D. Axehill. An optimization-based motion planner for autonomous maneuvering of marine vessels in complex environments. *Submitted to the 59th IEEE Conference on Decision and Control*, 2020b.

2

Modeling of wheeled vehicles

This chapter is intended to present some models that are commonly used for motion planning and feedback control of wheeled vehicles during low-speed maneuvers. This chapter starts with a brief introduction to different modeling techniques. In Section 2.2, a short introduction to nonholonomic systems is presented. In Section 2.3 and Section 2.4, the kinematic bicycle model and the general N-trailer are presented, respectively.

2.1 Introduction

Modeling of wheeled vehicles has a long history and depending on the application, different modeling techniques are used (LaValle, 2006; Rajamani, 2011). Roughly speaking, a vehicle model is said to be *dynamic* if it is derived based on force balances, or *kinematic* if it is derived based on velocity constraints. These velocity constraints will be further referred to as *nonholonomic constraints*. For wheeled vehicles, these constraints naturally arise if the wheels of the vehicle are assumed to be rolling without slipping. During low-speed maneuvers on dry road-surface conditions, a kinematic model is often sufficient to describe the behavior of a wheeled vehicle (LaValle, 2006; Paden et al., 2016; Spong et al., 2006). On the contrary, during aggressive high-speed maneuvers, the dynamical effects become more prominent and a dynamic model is required to capture these effects (Anisratov et al., 2018; Berntorp et al., 2014; Falcone et al., 2007; Fors et al., 2019; Kati et al., 2019a,b). In this chapter, the basic tools for modeling the kinematic properties of a wheeled vehicle are presented. Here, only a brief introduction is provided and the reader is referred to, e.g., LaValle (2006); Spong et al. (2006) for a more detailed coverage. A major reason why more advanced vehicle models with higher fidelity are usually not used for planning is that advanced models imply a high-dimensional state-space. Since most practical applications require

efficient motion planning modules, kinematic models with a lower state dimension are commonly used (Paden et al., 2016). However, a more advanced vehicle model could preferably be used for simulation purposes (Lima, 2018; Rajamani, 2011) or in the design of safety modules to handle evasive maneuvering (Fors et al., 2019; Svensson et al., 2019).

2.2 Nonholonomic systems

A kinematic model of a wheeled vehicle is derived based on the assumption that the wheels of the vehicle are rolling without slipping. This implies that there exist velocity directions in which the vehicle cannot move. Systems subject to such velocity constraints are referred to as nonholonomic systems.

Let $q \in \mathbb{R}^n$ denote a configuration of the vehicle, i.e., an n -dimensional vector of generalized coordinates. The configuration space $\mathcal{C} \subseteq \mathbb{R}^n$ is defined as the manifold of all possible vehicle configurations and is assumed to be a smooth manifold (LaValle, 2006). For modeling of wheeled vehicles, we are particularly interested of constraints in the following form

$$g(q, \dot{q}) = 0, \quad (2.1)$$

where $\dot{q} = dq/dt$. These are constraints on the velocities of the system, e.g., a car cannot move sideways. In some cases, velocity constraints can be explicitly integrable, giving rise to constraints in the form

$$h(q) = 0, \quad (2.2)$$

which are algebraic constraints in the configuration of the vehicle. Such constraints are said to be *holonomic* and the motion of the vehicle is thus restricted to a level surface of h . If a velocity constraint is not explicitly integrable the constraint is said to be *nonholonomic*. There exist different holonomic and nonholonomic constraints that naturally arise due to physical limitations on the system. Two common examples are:

$$\begin{aligned} h(q) &\leq 0: && \text{configuration inequality constraint,} \\ h(\dot{q}) &\leq 0: && \text{velocity inequality constraint.} \end{aligned}$$

These are inequality constraints on the configuration q and the velocity $\dot{q}$ of the system and two examples of such constraints are limited steering angle and steering angle rate for a car. In the remainder of this chapter we focus on the special class of nonholonomic systems that have linear velocity constraints

$$\omega_i(q)\dot{q} = 0, \quad i = 1, \dots, k < n, \quad (2.3)$$

where $\omega_i(q) \in \mathbb{R}^{1 \times n}$. These constraints are called *Pfaffian constraints* and it is assumed that ω_i , $i = 1, \dots, k$ are smooth functions and linearly independent for all $q \in \mathcal{C}$. An interpretation of the constraints in (2.3) is that the vector fields of the system have to be orthogonal to each $\omega_i(q)$. The linear velocity constraints in (2.3) can be represented on matrix form as $\omega(q)\dot{q} = 0$, where $\omega(q) \in \mathbb{R}^{k \times n}$.

Instead of expressing which velocity directions the vehicle cannot move, it is more convenient to express which directions it can, i.e., transform the constraints in (2.3) to explicit form $\dot{q} = f(q, u)$ (LaValle, 2006). Let $m = n - k > 0$ and choose $g_1, \dots, g_m$ as a basis of right null-space of $\omega(q)$. By assigning each $g_j \in \mathbb{R}^n$ with a control signal u_j , a *control-affine driftless system* is obtained

$$\dot{q} = \sum_{j=1}^m g_j(q) u_j, \quad (2.4)$$

where each u_j , $j = 1, \dots, m$, typically corresponds to a physical actuator. A common property for nonholonomic systems is that they are underactuated, which means that the number of control signals m is less than the dimension of the configuration-space n . Generally, a nonholonomic system can be modeled as a *nonlinear system*

$$\dot{x} = f(x, u), \quad (2.5)$$

where the state vector x is typically chosen to coincide with the configuration $x = q$ or additional properties are also modeled, e.g., $x = [q^\top \dot{q}^\top]^\top$ when dynamics are also considered (LaValle, 2006). However, the degree of freedom m for the system will not change. In the remainder of this chapter, models of nonholonomic systems that are of particular importance for this thesis are derived.

2.3 The kinematic bicycle model

A vehicle model that is widely used in the motion planning and control literature is the kinematic bicycle model (LaValle, 2006; Paden et al., 2016). During low-speed maneuvers, cars and trucks (Lima, 2018; Paden et al., 2016; Werling et al., 2010) have been shown to be sufficiently well described by this relatively simple model. The model is derived based on the assumption that the wheels of the vehicle are rolling without slipping. The vehicle is also assumed to operate on a flat surface and assumed to be front-wheel steered with perfect Ackerman steering (see Figure 2.1). Briefly, the Ackerman steering geometry makes the absolute value of the turning radius for inner front wheel smaller than the absolute value of the outer front wheel's turning radius. As illustrated in Figure 2.1, this implies that there exists a resulting front-wheel steering angle α that can replace the inner and outer steering angles.

The wheelbase of the vehicle is denoted by L_1 and the longitudinal velocity of its rear axle is denoted by v_1 . The configuration of the vehicle is defined as $q = [x_1 \ y_1 \ \theta_1 \ \alpha]^\top$, where (x_1, y_1) is the position of the center of the vehicle's rear axle and θ_1 is the vehicle's orientation. Under no-slip assumptions, the component of the velocity vector that is orthogonal to the rear wheels is zero which leads to the following linear velocity constraint:

$$\underbrace{\begin{bmatrix} -\sin \theta_1 & \cos \theta_1 & 0 & 0 \end{bmatrix}}_{\triangleq \omega_1(q)} \dot{q} = 0. \quad (2.6)$$

Figure 2.1: A schematic illustration of the kinematic bicycle model.

Repeating this argument for the front wheel implies that the following nonholonomic constraint also has to be satisfied

$$-\dot{x}_{1f}\sin(\theta_1 + \alpha) + \dot{y}_{1f}\cos(\theta_1 + \alpha) = 0. \quad (2.7)$$

The position of the center of the front axle (x_{1f}, y_{1f}) can be expressed as a function of q as

$$\begin{aligned} x_{1f} &= x_1 + L_1 \cos \theta_1, \\ y_{1f} &= y_1 + L_1 \sin \theta_1. \end{aligned} \quad (2.8)$$

By differentiating (2.8) with respect to time and inserting the result in (2.7), the second nonholonomic constraint that has to hold is

$$\underbrace{\begin{bmatrix} -\sin(\theta_1 + \alpha) & \cos(\theta_1 + \alpha) & L_1 \cos \alpha & 0 \end{bmatrix}}_{\triangleq \omega_2(q)} \dot{q} = 0. \quad (2.9)$$

Provided that the steering angle satisfies $|\alpha| < \pi/2$, the row vectors $\omega_1(q)$ and $\omega_2(q)$ are linearly independent and a basis of the right null space to $\omega(q) = [\omega_1^T \ \omega_2^T]^T$ is given by

$$g_1 = \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \\ \frac{\tan \alpha}{L_1} \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (2.10)$$

That is, the kinematic bicycle model can be written as a control-affine driftless system (2.4) with $m = 2$, and one possible combination of control signals is the longitudinal velocity $u_1 = v_1$ and the steering angle rate $u_2 = \dot{\alpha}$.

In trajectory-tracking and path-following control applications during low-speed maneuvers, the dynamics in the steering angle is sometimes neglected and α is

assumed to be directly controlled (Paden et al., 2016). For this case, the state vector is defined as $x = [x_1 \ y_1 \ \theta_1]^\top$ and the control-input vector as $u = [v_1 \ \alpha]^\top$. The resulting simplified version of the kinematic bicycle model is

$$\dot{x} = v_1 \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \\ \frac{\tan \alpha}{L_1} \end{bmatrix}. \quad (2.11)$$

Here, the steering angle α can be substituted with the car-like vehicle's curvature κ which is defined as

$$\kappa = \frac{1}{R_1} = \frac{d\theta_1}{ds_x} = \frac{\tan \alpha}{L_1}, \quad (2.12)$$

where R_1 is the turning radius of the vehicle and $s_x(t) = \int_0^t |v_1(\tau)| d\tau$ is the distance traveled by the rear axle of the vehicle.

Dubins' version of the bicycle model (2.11) is obtained by restricting the control inputs as $v_1 \in \{0, 1\}$ and $\alpha \in \{-\alpha_{\max}, 0, \alpha_{\max}\}$, which means that the vehicle is only allowed to stand still or move straight, maximum left or maximum right at constant forward speed $v_1 = 1$ (Dubins, 1957). Reeds-Shepp's version is obtained by allowing the vehicle to also travel in backward motion $v_1 \in \{-1, 0, 1\}$ (Reeds and Shepp, 1990).

Since v_1 enters bilinearly into the model in (2.11), a method known as time-scaling (Houska et al., 2011b) can be applied to eliminate the longitudinal speed $|v_1|$ from the model. Since $\dot{s}_x = |v_1|$ holds, by applying the chain-rule $\frac{dx}{dt} = \frac{dx}{ds_x} |v_1|$ the model in (2.11) can be written on spatial form as

$$\frac{dx}{ds_x} = \bar{v}_1 \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \\ \kappa \end{bmatrix}, \quad (2.13)$$

where $\bar{v}_1 = \text{sign}(v_1) = \{-1, 1\}$, i.e., $\bar{v}_1 = 1$ denotes forward motion and $\bar{v}_1 = -1$ denotes backward motion. This result is frequently used as motivation for decoupling the motion planning problem into path planning and velocity planning when low-speed maneuvers are considered. It is also a strong motivation for decoupling the feedback control problem into lateral and longitudinal control.

2.4 The general N-trailer with a car-like tractor

A family of systems that is of special interest for this thesis is the so-called N-trailer vehicles (Altafini, 2001; Michałek, 2013b; Sjørdalen, 1993). These systems consist of $N + 1$ vehicle segments including a leading car-like tractor that is connected to N passive trailers with on-axle or signed off-axle hitch connections. Each vehicle segment is characterized by a segment length $L_i > 0$ and a signed hitching offset M_i that is positive if the connection is behind the preceding vehicle segment's axle. If the system has mixed hitching types it is called a general N-trailer, and in case of pure on-axle or off-axle hitching, it is referred to as a standard N-trailer or a non-standard N-trailer, respectively (Michałek, 2013b).

Figure 2.2: A schematic description of the general N -trailer with a car-like tractor. The system consists of a leading car-like tractor that is connected to N passive trailers with a mixture of on-axle and off-axle hitch connections.

The control inputs to an N -trailer vehicle with a car-like tractor are the tractor's longitudinal velocity of its rear axle v_1 and its front-wheel steering angle α (or the tractor's curvature $\kappa = \tan \alpha / L_1$). Similar to the kinematic bicycle model, the kinematic model for an N -trailer vehicle is also derived based on holonomic and nonholonomic constraints and a recursive formula is presented in Altafini (2001). The model is derived based on the assumptions that the wheels are rolling without slipping and that the vehicle is operating on a flat surface. A schematic description of the system is provided in Figure 2.2 and the state vector consists of $3 + N$ variables:

- the global position (x_l, y_l) and orientation θ_l of the l -th vehicle segment in a fixed coordinate frame

$$q_l = [x_l \quad y_l \quad \theta_l]^T \in \mathbb{R}^2 \times \mathbb{S}, \quad \mathbb{S} = (-\pi, \pi], \quad (2.14)$$

- for $i = 2, \dots, N+1$, a number of N joint angles

$$\beta_i = \theta_{i-1} - \theta_i \in \mathcal{B}_i = [-\bar{\beta}_i, \bar{\beta}_i], \quad \bar{\beta}_i \in (0, \pi). \quad (2.15)$$

The state vector for the general N -trailer with a car-like tractor is defined as

$$x = [q_l^T \quad \beta_2 \quad \dots \quad \beta_{N+1}]^T \in \mathcal{X}, \quad (2.16)$$

where $\mathcal{X} = \mathbb{R}^2 \times \mathbb{S} \times \mathcal{B}_2 \dots \times \mathcal{B}_{N+1}$.

The leading car-like tractor is described by a kinematic bicycle model (2.11) and thus, its yaw-rate is

$$\dot{\theta}_1 = v_1 \kappa, \quad (2.17)$$

where $\kappa = \tan \alpha / L_1$. The recursive formula for the transformation of the angular velocity $\dot{\theta}_i$ and the longitudinal velocity v_i between any two neighboring vehicle segments are given by (Altafini, 2001; Michałek, 2013b):

$$\begin{bmatrix} \dot{\theta}_i \\ v_i \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{M_{i-1} \cos \beta_i}{L_i} & \frac{\sin \beta_i}{L_i} \\ M_{i-1} \sin \beta_i & \cos \beta_i \end{bmatrix}}_{\triangleq J_i(\beta_i)} \begin{bmatrix} \dot{\theta}_{i-1} \\ v_{i-1} \end{bmatrix}, \quad i = 2, \dots, N+1. \quad (2.18)$$

For details regarding the derivation, the reader is referred to, e.g., Altafini (2001); Ljungqvist (2015); Michałek (2013b). The position of the l -th vehicle segment evolves according to standard unicycle kinematics

$$\begin{aligned} \dot{x}_l &= v_l \cos \theta_l, \\ \dot{y}_l &= v_l \sin \theta_l, \end{aligned} \quad (2.19)$$

where its angular rate $\dot{\theta}_l$ and longitudinal velocity v_l are given by

$$\begin{bmatrix} \dot{\theta}_l \\ v_l \end{bmatrix} = \prod_{i=N-l}^{N-2} J_{N-i}(\beta_{N-i}) \begin{bmatrix} v_1 \kappa \\ v_1 \end{bmatrix} = v_1 \prod_{i=N-l}^{N-2} J_{N-i}(\beta_{N-i}) \begin{bmatrix} \kappa \\ 1 \end{bmatrix}, \quad (2.20)$$

where $\prod_{i=N-l}^{N-2} J_{N-i}(\beta_{N-i}) = J_l(\beta_l) J_{l-1}(\beta_{l-1}) \cdots J_2(\beta_2)$. Note that (2.20) is derived by recursive usage of (2.18) for $l-1$ times together with (2.17). Now, by introducing the vectors $c = [1 \ 0]^\top$ and $d = [0 \ 1]^\top$, the longitudinal velocity v_l and the angular velocity $\dot{\theta}_l$ of the l -th vehicle segment can be written as

$$v_l = v_1 d^\top \prod_{i=N-l}^{N-2} J_{N-i}(\beta_{N-i}) \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \triangleq v_1 f_{v_l}(x, \kappa), \quad (2.21a)$$

$$\dot{\theta}_l = v_1 c^\top \prod_{i=N-l}^{N-2} J_{N-i}(\beta_{N-i}) \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \triangleq v_1 f_{\theta_l}(x, \kappa), \quad (2.21b)$$

and the model for the position of the l -th vehicle segment (2.19) can be written as

$$\begin{aligned} \dot{x}_l &= v_1 f_{v_l}(x, \kappa) \cos \theta_l, \\ \dot{y}_l &= v_1 f_{v_l}(x, \kappa) \sin \theta_l. \end{aligned} \quad (2.22)$$

Finally, by combining (2.17)–(2.18), the time derivative of (2.15) yields the kinematics for the joint angles $\dot{\beta}_i = \dot{\theta}_{i-1} - \dot{\theta}_i$, where

$$\dot{\beta}_i = v_1 c^\top \left(\prod_{j=N-i+1}^{N-2} J_{N-j}(\beta_{N-j}) - \prod_{j=N-i}^{N-2} J_{N-j}(\beta_{N-j}) \right) \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \triangleq v_1 f_{\beta_i}(x, \kappa), \quad (2.23)$$

for $i = 2, \dots, N+1$. To conclude, the model for the general N-trailer with a car-like tractor is given in (2.21b)–(2.23) and can compactly be written as

$$\dot{x} = v_1 f(x, \kappa). \quad (2.24)$$

The direction of motion is essential for the stability of the system (2.24), where the joint-angle kinematics (2.23) are structurally unstable in backward motion ($v_1 < 0$), where it risks to fold and enter what is called a jack-knife state (Altafini, 2001). In forward motion ($v_1 > 0$), these modes are stable. Moreover, as for the kinematic bicycle model (2.13), since the longitudinal velocity v_1 enters bilinearly into the model in (2.24), time-scaling (Houska et al., 2011b) can be applied to eliminate the dependence on the longitudinal speed $|v_1|$.

For the special case with only on-axle hitching ($M_i = 0$, $i = 1, \dots, N$) the model of the general N-trailer coincides with the standard N-trailer (Sørdalen, 1993). The standard N-trailer is *differentially flat* (Rouchon et al., 1993) and can be converted into chained form where the position of the N -th trailer (x_{N+1}, y_{N+1}) is the so-called flat output (Sørdalen, 1993). This result has lead to that the pose for the axle of the N -th trailer is commonly used to represent the global pose of the vehicle. However, if mainly forward motion is considered, by selecting the car-like tractor as guidance segment may drastically simplify, e.g., controller design, as the joint-angle kinematics is structurally stable in forward motion and can to some extent be neglected. The general 1-trailer with a car-like tractor is also differentially flat using a certain choice of flat output. However, when $N \geq 2$, the flatness property does not hold for the general N-trailer case (Rouchon et al., 1993).

The model of the general N-trailer is small-time locally controllable (Khalil and Grizzle, 2002), except from some singularities (Altafini, 2001). The practical interpretation of these singularities is that the vehicle segments are arranged such that the car-like tractor can move while some of the trailers remain static.

A general 2-trailer with a car-like tractor

The chapter is concluded with the derivation of the kinematic model for a general 2-trailer with a car-like tractor that will be extensively used in Part II of this thesis. The vehicle is illustrated in Figure 2.3 and is composed of a leading car-like tractor, an off-axle hitched dolly ($M_1 \neq 0$) and an on-axle hitched semitrailer ($M_2 = 0$). Since the vehicle consists of three segments, two joint angles β_2 and β_3 are needed to describe a configuration of the vehicle. Moreover, as both backward and forward motions are considered in this work, the axle of the semitrailer is used to represent the global pose $q_3 = [x_3 \ y_3 \ \theta_3]^\top$ of the vehicle. That is, the state vector (2.16) is defined as $x = [q_3^\top \ \beta_2 \ \beta_3]^\top$ and the control inputs are v_1 and κ .

Following the derivations in Section 2.4, the matrices $J_2(\beta_2)$ and $J_3(\beta_3)$ describing the longitudinal and angular velocity transformations between neighboring vehicle segments (2.18) are

$$J_2(\beta_2) = \begin{bmatrix} -\frac{M_1 \cos \beta_2}{L_2} & \frac{\sin \beta_2}{L_2} \\ M_1 \sin \beta_2 & \cos \beta_2 \end{bmatrix}, \quad J_3(\beta_3) = \begin{bmatrix} 0 & \frac{\sin \beta_3}{L_3} \\ 0 & \cos \beta_3 \end{bmatrix}, \quad (2.25)$$

Figure 2.3: An illustration of a general 2-trailer with a car-like tractor.

since $M_1 \neq 0$ and $M_2 = 0$. Using (2.25) the functions describing the longitudinal and angular velocity transformations from the car-like tractor to the semitrailer (2.21) are

$$\begin{aligned} f_{v_3}(x, \kappa) &= d^T J_3(\beta_3) J_2(\beta_2) \begin{bmatrix} \kappa \\ 1 \end{bmatrix} = \cos \beta_3 (\cos \beta_2 + M_1 \sin \beta_2 \kappa), \\ f_{\theta_3}(x, \kappa) &= c^T J_3(\beta_3) J_2(\beta_2) \begin{bmatrix} \kappa \\ 1 \end{bmatrix} = \frac{\sin \beta_3}{L_3} (\cos \beta_2 + M_1 \sin \beta_2 \kappa). \end{aligned} \quad (2.26)$$

Thus, the model for the pose q_3 of the semitrailer (2.21b)–(2.22) is

$$\begin{aligned} \dot{x}_3 &= v_1 f_{v_3}(x, \kappa) \cos \theta_3 = v_1 \cos \beta_3 (\cos \beta_2 + M_1 \sin \beta_2 \kappa) \cos \theta_3, \\ \dot{y}_3 &= v_1 f_{v_3}(x, \kappa) \sin \theta_3 = v_1 \cos \beta_3 (\cos \beta_2 + M_1 \sin \beta_2 \kappa) \sin \theta_3, \\ \dot{\theta}_3 &= v_1 f_{\theta_3}(x, \kappa) = v_1 \frac{\sin \beta_3}{L_3} (\cos \beta_2 + M_1 \sin \beta_2 \kappa). \end{aligned} \quad (2.27)$$

Finally, using (2.25) in (2.23) the models for the joint-angle kinematics are

$$\begin{aligned} \dot{\beta}_2 &= v_1 c^T [I - J_2(\beta_2)] \begin{bmatrix} \kappa \\ 1 \end{bmatrix} = v_1 \left(\kappa - \frac{\sin \beta_2}{L_2} + \frac{M_1}{L_2} \cos \beta_2 \kappa \right), \\ \dot{\beta}_3 &= v_1 c^T [J_2(\beta_2) - J_3(\beta_3) J_2(\beta_2)] \begin{bmatrix} \kappa \\ 1 \end{bmatrix} \\ &= v_1 \left(\frac{1}{L_2} (\sin \beta_2 - M_1 \kappa \cos \beta_2) - \frac{\sin \beta_3}{L_3} (\cos \beta_2 + M_1 \sin \beta_2 \kappa) \right). \end{aligned} \quad (2.28)$$

The kinematic model of the general 2-trailer with a car-like tractor is given in (2.27)–(2.28) and can compactly be written as $\dot{x} = v_1 f(x, \kappa)$.

3

Motion planning for self-driving vehicles

This chapter presents motion planning techniques for self-driving vehicles. The chapter starts with a short introduction to the motion planning concept. In Section 3.2, the motion planning problem is presented as an optimal control problem (OCP) followed by a discussion of why this OCP is hard to solve by directly applying numerical optimal control. In Section 3.3, the common steps for solving a motion planning problem are explained and in Section 3.4, the basic tools for nonholonomic motion planning are presented. Finally, Section 3.5 contains a definition of a general sampling-based motion planner together with descriptions of some of the popular deterministic and probabilistic versions.

3.1 Introduction

The concept of motion planning is and has been an active research topic in different research fields for many decades. However, depending on the field, the meaning of planning differs. In the field of artificial intelligence, planning historically refers to finding a sequence of logical decisions or actions that transforms an initial world state to a desired goal state where system dynamics is often neglected or simplified (Russell and Norvig, 2016). In this thesis, this planning domain will be referred to as *mission planning* or *task planning*. For a self-driving vehicle, this type of planner acts as a high-level planner that, e.g., specifies a mission to be executed by the low-level motion planning and control layer.

In this thesis, the motion planning problem is referred to as the problem of computing a feasible trajectory or path that moves the vehicle from its initial state to a desired goal state. The motion plan must satisfy the vehicle's physically imposed constraints as well as constraints imposed by surrounding obstacles while at the same time minimize a specified performance measure. From a control perspective, motion planning is an optimal control problem that in general is hard

solve due to its non-convex nature. In self-driving vehicle applications, it is also required that a motion plan is computed, and possibly updated, within seconds or even milliseconds. To fulfill these requirements, numerous of different motion planning algorithms have been proposed (Kuwata et al., 2009; LaValle, 1998; Paden et al., 2016; Pivtoraiko et al., 2009). Many of these algorithms originate from the robotics and computer science communities which has been adapted to also take system dynamics into account (LaValle, 2006; Lynch and Park, 2017).

This chapter focuses on motion planning for self-driving vehicles in *unstructured environments*, e.g., parking lots or other open areas where no clearly defined driver path is available and the vehicle's speed is low. This problem is fundamentally different from motion planning in *structured environments*, e.g., onroad driving, where information of the road center line is commonly used to enforce structure into the problem. This leads to a problem formulation that has strong similarities to *receding horizon control* (Batkovic et al., 2019; Lima et al., 2017b; Oliveira et al., 2020a; Paden et al., 2016; Werling et al., 2012; Ziegler and Stiller, 2009).

3.2 Problem formulation

The vehicle under consideration is assumed to be modeled as a time-invariant nonlinear system

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t_I) = x_I, \quad (3.1)$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $u(t) \in \mathbb{R}^m$ the denotes the control inputs and x_I is the vehicle's initial state. The vehicle is assumed to have physical limitations on its states $x(t) \in \mathbb{X}$ and control signals $u(t) \in \mathbb{U}$, where $\mathbb{X} \subseteq \mathbb{R}^n$ and $\mathbb{U} \subseteq \mathbb{R}^m$ are typically convex sets. The vehicle is operating in an environment where static and possibly also moving obstacles exist. The regions which are occupied with obstacles $\mathbb{X}_{\text{obs}}(t)$ are usually divided into static obstacles $\mathbb{X}_{\text{obs}}^s$ and dynamic obstacles $\mathbb{X}_{\text{obs}}^d(t)$. The free-space where the vehicle is not in collision with any obstacle at time t is defined as $\mathbb{X}_{\text{free}}(t) = \mathbb{X} \setminus \mathbb{X}_{\text{obs}}(t)$. For now, we make the assumption that the set $\mathbb{X}_{\text{free}}(t)$ can be described analytically. Since the free-space $\mathbb{X}_{\text{free}}(t)$ is defined as the complement set of $\mathbb{X}_{\text{obs}}(t)$, it is in general a non-convex set. The objective functional J to be minimized is often composed of different costs, e.g., time duration and/or energy.

The motion planning problem is defined as follows: compute a feasible and collision-free motion plan, *i.e.*, a trajectory $(x(t), u(t))$, $t \in [t_I, t_G]$ that moves the vehicle from its initial state x_I to a desired goal state x_G , while minimizing the objective functional J . This problem can be posed as an OCP in the form

$$\begin{aligned} \underset{u(\cdot), t_G}{\text{minimize}} \quad & J = \int_{t_I}^{t_G} L(t, x(t), u(t)) dt \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)), \\ & x(t_I) = x_I, \quad x(t_G) = x_G, \\ & x(t) \in \mathbb{X}_{\text{free}}(t), \quad u(t) \in \mathbb{U}, \end{aligned} \quad (3.2)$$

where $L > 0$ is the so-called *cost function*. It is assumed that $x_I \in \mathbb{X}_{\text{free}}(t)$ and $x_G \in \mathbb{X}_{\text{free}}(t)$, for $t \geq t_I$ to make the OCP in (3.2) well defined and it is also assumed that there exists a terminal control signal u_T such that the vehicle remains at the goal for $t \geq t_G$ (LaValle, 2006). The latter assumption is equivalent to that the vehicle ends up in an equilibrium point at the goal, i.e., $f(x(t), u_T) = 0$, for $t \geq t_G$. This assumption implies that the motion plan can be extended for an arbitrary time duration. The terminal control signal is easy to construct for wheeled vehicles by simply choosing the longitudinal velocity $v_{1,T} = 0$ which implies $f(x(t), u_T) = 0$. In some applications, the terminal constraint $x(t_G) = x_G$ is replaced with a goal region $x(t_G) \in \mathbb{X}_G$ (LaValle, 2006). The OCP in (3.2) is in general hard to solve by directly invoking a state-of-the-art numerical optimal control solver (Andersson et al., 2018a; Houska et al., 2011b). Three fundamental reasons for this are:

1. Even if $\mathbb{X}_{\text{obs}}(t) = \emptyset$, the OCP in (3.2) is a non-convex optimization problem because of the nonlinear vehicle model. Thus, for complex systems, a proper warm-start is needed to even find a feasible solution.
2. $\mathbb{X}_{\text{free}}(t)$ is often constructed by fusing information from precomputed maps and onboard sensors on the vehicle and an analytical expression for the obstacle region $\mathbb{X}_{\text{obs}}(t)$ might be difficult to construct in practice.
3. $\mathbb{X}_{\text{free}}(t)$ is often non-convex and the OCP in (3.2) therefore includes a combinatorial problem with possibly different classes of solution trajectories. A proper initial guess is therefore required to find a good locally optimal (or even a feasible) solution to the motion planning problem.

Besides the above mentioned problems, computation efficiency and the absence of real-time guarantees for nonlinear solvers are also strong reasons why standalone numerical optimal control is rarely used in practice. However, numerical optimal control still plays an important role in many motion planning algorithms.

For many nonholonomic systems, the trajectory planning problem in (3.2) can be converted into a path planning problem when dynamic obstacles are neglected. The relationship between a path and a trajectory is defined as follows.

Definition 3.1. A path is represented as $(x(s), u(s))$, $s \in [0, s_G]$ and a trajectory is a time-parametrized path $(x(s(t)), u(s(t)))$, $t \in [t_I, t_G]$. _____

Based on Definition 3.1, a trajectory can be constructed from a path by specifying velocity $\dot{s}(t) = v(t)$ of which the path should be executed. For path planning of ground vehicles, the direction of motion is already determined by the path planner and what remains for the *velocity planner* is to compute the desired longitudinal speed profile $|v(t)|$. For systems that are differentially flat, a motion plan can also be defined as a curve in the *flat outputs* (Paden et al., 2016). In that case, smoothness constraints on the curve have to be enforced such that the nominal state and control input trajectories can be recovered (Luca et al., 1998; Paden et al., 2016; Werling et al., 2012).

Figure 3.1: An example of typical steps that are performed to compute a motion plan.

3.3 A common solution concept

To solve, or rather to compute a suboptimal solution to the OCP in (3.2), motion planning algorithms are commonly used. A planning cycle for an autonomous vehicle is often divided into a couple of different phases, as illustrated in Figure 3.1 (LaValle, 2006; Lynch and Park, 2017). The perception and localization layer provides the motion planner with a compressed representation of the surrounding environment and information of the vehicle’s current state. The high-level task planner provides the low-level motion planning and control layer with a desired mission to be executed. Based on these inputs, a motion plan is computed through a series of steps that are illustrated in Figure 3.1. First, a collision-free path from the vehicle’s current state to a desired goal state is computed. In this planning phase, the system dynamics is sometimes neglected and the computed path is then not guaranteed to be feasible. The path can for example be composed of piecewise linear paths if a geometric planner is used during this step (LaValle, 1998, 2006; Likhachev et al., 2004). In the second step, the path is optimized, or smoothed, to generate a locally optimal motion plan that satisfies the system dynamics and its physically imposed constraints. Finally, a velocity profile is computed to obtain a trajectory. The velocity profile is computed such that the vehicle does not collide with any dynamic obstacle and such that the vehicle’s maximum allowed velocities and accelerations, and other comfort and safety constraints are satisfied.

The nominal trajectory $(x(t), u(t))$, $t \in [t_I, t_G]$ is then sent to a feedback controller for plan execution. The planning cycle is then repeated in case a new mission is received or if the current motion plan has become infeasible due to, e.g., inaccurate prediction of dynamic obstacles or if a previously unseen obstacle is detected by the perception layer.

Ideally, a unified motion planning algorithm that performs all three above mentioned steps simultaneously is preferred since separating the problem often leads to a suboptimal solution. This is however often intractable for most problem classes because of limited computation time. However, there exist efficient motion planning techniques where the vehicle's nonholonomic and physically imposed constraints are taken into account already while computing the path and some popular techniques will be presented in this chapter.

3.4 Nonholonomic motion planning

Motion planning for nonholonomic systems is conceptually not different compared to motion planning for a holonomic system, e.g., a quadcopter platform during low-speeds (Andersson et al., 2018b). The motion planning techniques that are presented here are therefore also applicable for holonomic systems, as long as the dimension of the state-space is sufficiently small and the algorithmic specific assumptions for the system dynamics are satisfied. However, the velocity constraints that are inherited by a nonholonomic system make different phases of the algorithms more demanding compared to the holonomic case.

For unstructured environments, e.g., parking lots and loading sites, a vast amount of different motion planning algorithms for ground vehicles exist and an overview can for example be found in LaValle (2006); Paden et al. (2016). Two popular branches for nonholonomic motion planning include *probabilistic methods* (Karaman and Frazzoli, 2011; Kuwata et al., 2009; LaValle and Kuffner, 2001) and *deterministic methods* (Cirillo et al., 2014; Pivtoraiko et al., 2009; Urmson et al., 2008) with different guarantees on the quality of the produced solution. Before presenting the planning algorithms, some important building blocks for these algorithms are presented.

3.4.1 Steering functions

Many motion planning algorithms require a method to generate a feasible trajectory between two nearby vehicle states (Karaman and Frazzoli, 2011; Pivtoraiko et al., 2009). Here, the surrounding obstacles are neglected and only the system dynamics $\dot{x} = f(x, u)$ and its physically imposed constraints $x \in \mathbb{X}$ and $u \in \mathbb{U}$ are considered. For some algorithms, this local trajectory generation problem needs to be solved online (Karaman and Frazzoli, 2011) and for others it is a procedure that is performed offline (Pivtoraiko et al., 2009). In the latter case, *position invariance* of the system is often exploited such that a precomputed trajectory can be translated and reused from all possible vehicle positions.

For unconstrained linear systems, *Pontryagin's minimum principle* (PMP) (Pontryagin, 2018) can be used to derive analytical solutions (Werling et al., 2012). PMP can also be used to derive analytical solutions for some nonlinear systems, including the kinematic bicycle model (2.11). Two examples are the Dubins (Dubins, 1957) and the Reeds-Shepp (Reeds and Shepp, 1990) minimal path lengths which have been extended to continuous and continuously differentiable vehicle curvature in (Fraichard and Scheuer, 2004) and (Oliveira et al., 2018), respectively.

For systems that are differentially flat, efficient techniques for local trajectory generation can be used (Laumond et al., 1994; Murray and Sastry, 1991; Švestka and Vleugels, 1995; Tilbury et al., 1995). However, in all above examples, a general objective functional J_{BVP} is hard to minimize and feasibility with respect to physically imposed constraints on the vehicle need to be checked a posteriori (Frazzoli et al., 2005). In general, the problem of connecting two nearby vehicle states $x_i \in \mathbb{X}$ and $x_f \in \mathbb{X}$ without considering obstacles can be posed as an OCP in the following form

$$\begin{aligned} \underset{u(\cdot), t_f}{\text{minimize}} \quad & J_{\text{BVP}} = \int_0^{t_f} L_{\text{BVP}}(x(t), u(t)) dt \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)), \\ & x(0) = x_i, \quad x(t_f) = x_f, \\ & x(t) \in \mathbb{X}, \quad u(t) \in \mathbb{U}. \end{aligned} \quad (3.3)$$

This OCP is sometimes referred to as a *boundary value problem* (BVP) and note its similarities to the formulation of the motion planning problem in (3.2). Here, the set of allowed vehicle states $\mathbb{X}$ is typically represented as a convex set, in contrast to (3.2) where $\mathbb{X}_{\text{free}}(t)$ is often non-convex. Moreover, as (3.3) is a subproblem of (3.2), the optimal final time t_f is also typically shorter. Together, these properties imply that (3.3) is easier to compute a locally optimal solution to. However, the OCP in (3.3) is still a non-convex problem due to the nonlinear system dynamics.

In many applications, it is important that $u(t)$ is sufficiently smooth which can be handled using an augmented state vector $\bar{x}(t) = (x(t), u(t), \dot{u}(t), \dots, u^{(r-1)}(t))$ and view the r -th time derivative $u^{(r)}(t)$ as an artificial control signal. It is now possible to penalize the derivatives of the nominal control signal and enforce limitations on them in a structured way. To guarantee that $u(t)$ is an $r-1$ times continuously differentiable function in time when multiple precomputed trajectories are combined during online planning, the boundary values for $\dot{u}(t)$, $\ddot{u}(t)$, $\dots$, $u^{(r-1)}(t)$ are typically constrained to zero (Andersson et al., 2018b; Ljungqvist et al., 2019; Werling et al., 2012).

The solution to (3.3) is a trajectory $(x(t), u(t))$, $t \in [0, t_f]$ that will be referred to as a *motion primitive* m_p and the solution satisfies

$$x(t_f) = x(0) + \int_0^{t_f} f(x(t), u(t)) dt. \quad (3.4)$$

Let $x_k = x(0)$ and $x_{k+1} = x(t_f)$, then the *state transition equation* in (3.4) is compactly defined as

$$x_{k+1} = f_p(x_k, m_p), \quad (3.5)$$

where the final vehicle state x_{k+1} when applying m_p from x_k is called a *successor state* of x_k . The cost function L_{BVP} in (3.3) may be the same as the cost function L

Figure 3.2: Precomputed motions for a general 2-trailer with a car-like tractor (a) and for a quadcopter platform (b). Only the trajectories for the position of the rear axle of the semitrailer and the center of gravity for the quadcopter are plotted, respectively.

in the original motion planning problem in (3.2), or some of the terms are neglected in L_{BVP} , e.g., only smoothness and comfort criteria are considered (Ljungqvist et al., 2017; Werling et al., 2012). In both cases, the *stage cost* $J(m_p)$ is computed by integrating the resulting trajectory using the cost function defined in the motion planning problem (3.2). Note however that from an optimality point of view it is preferred to use the same cost function in both steps (Bergman et al., 2019a).

One approach to find a locally optimal solution to the OCP in (3.3) is to use state-of-the-art numerical optimal control software, e.g., ACADO (Houska et al., 2011b) or CasADi (Andersson et al., 2018a). In these methods, the continuous-time optimal control problem is reformulated as a nonlinear programming (NLP) problem using for example direct multiple shooting combined with numerical integration. To solve the resulting NLP, a sequential quadratic programming (SQP) solver (Gill et al., 2005) or a nonlinear interior-point (IP) solver (Wächter and Biegler, 2006) can be used. Examples of different solutions for a general 2-trailer with a car-like tractor (see Section 2.4) and for a quadcopter platform (Andersson et al., 2018b) are provided in Figure 3.2. For systems with complex dynamics, it can be problematic to even find a feasible solution to the OCP in (3.3) and a proper initial guess is crucial. Moreover, systems with unstable dynamics can cause numerical problems while initializing the OCP solver. One such example is the general 2-trailer with a car-like tractor which is unstable in backward motion. In Paper D, we present a method to avoid this issue for tractor-trailer vehicles by establishing and exploiting time-symmetry properties of the system (Lamb and Roberts, 1998).

Figure 3.3: Illustration of two cases when the Euclidean distance (green dashed line) severely underestimates the true cost-to-go (black dashed line).

3.4.2 Heuristics

All sampling-based motion planning algorithms incrementally explore a sampled set $\mathbb{X}_s$ of the vehicle's state-space $\mathbb{X}$. This exploration is either performed in a deterministic or in a probabilistic fashion with the common goal of computing a motion plan to the desired goal. During this search process, an informative *heuristic function* h is needed to guide the exploration toward promising states (LaValle and Kuffner, 2001; Pivtoraiko et al., 2009). The heuristic function $h(x_k, x_G)$ is used to estimate the cost-to-go from a vehicle state $x_k \in \mathbb{X}_s$ to the goal state $x_G \in \mathbb{X}_s$. Let $h^*(x_k, x_G)$ denote the optimal cost-to-go for traveling from x_k to x_G and let $x_{k+1} \in \mathbb{X}_s$ represent a successor state of x_k achieved using motion primitive m_p with stage cost $J(m_p)$. A heuristic function $h(x_k, x_G)$ is said to be

Admissible if for all $x_k \in \mathbb{X}_s$, the heuristic function satisfies $h(x_k, x_G) \leq h^*(x_k, x_G)$

Consistent if for all $x_k \in \mathbb{X}_s$, and all its successor states x_{k+1} , the heuristic function satisfies $h(x_k, x_G) \leq J(m_p) + h(x_{k+1}, x_G)$

The admissibility criterion implies that the heuristic function always underestimates the optimal cost-to-go from x_k to x_G and the consistency criterion means that the heuristic cost from x_k to x_G cannot be larger than the stage cost $J(m_p)$ plus the heuristic cost from x_{k+1} to x_G , i.e., the triangle inequality must hold. These criteria are necessary to satisfy in order to preserve optimality guarantees for many motion planning algorithms (LaValle, 2006).

When path planning is considered, the simplest heuristic function is the Euclidean distance. However, due to the nonholonomic constraints that are inherited by a nonholonomic system and the surrounding obstacles, the Euclidean distance may in many scenarios severely underestimate the true cost-to-go. Two illustrative examples for a car-like vehicle are presented in Figure 3.3. In Figure 3.3a, the surrounding obstacles are not taken into account for and in Figure 3.3b, the nonholonomic constraints make the Euclidean distance a poor estimator of the actual cost-to-go. Using a heuristic function that generates poor cost-to-go estimates

causes the motion planner to spend unnecessary computation time on exploring vehicle states that seem promising according to the heuristics, but are in reality bad candidates since the true cost-to-go from these states may be very expensive.

A common approach when designing heuristic functions is to relax the motion planning problem by removing certain constraints to make the problem simpler to solve. For car-like vehicles, the problem in Figure 3.3b can be handled by neglecting obstacles and compute the analytical solution for the Reeds-Shepp car (Reeds and Shepp, 1990) and use its optimal path-length as heuristic function. For the case in Figure 3.3a, the system dynamics can be removed and a geometric planning problem could be solved to compute an underestimate of the cost-to-go. These two strategies are combined in Dolgov et al. (2010) where first a local trajectory generation problem without obstacles is solved, then the system dynamics is neglected and a geometric planning problem with obstacles is solved and the maximum of these two estimates is used as heuristic cost. At a first glance, this proposal may seem computationally expensive for nonlinear systems since a local trajectory generation problem without obstacles involves solving an OCP (3.3) at each state expansion. Fortunately, this problem can in many applications be performed offline on a finite grid of vehicle states and stored in a *heuristic look-up table* (HLUT) (Cirillo et al., 2014; Knepper and Kelly, 2006). The HLUT can then be used during online planning to efficiently retrieve the heuristic function values for many problems in the vicinity of the vehicle.

Depending on the complexity of the surrounding environment, the simplified geometric planning problem may yield a good or a poor estimate of the cost-to-go. For the problem in Figure 3.3b, the Euclidean distance and the solution to the geometric planning problem will yield the same poor estimate of the cost-to-go. For maze-like environments as in Figure 3.3a, the solution to a geometric planning problem will result in a good estimate of the cost-to-go and will guide the motion planning algorithm to not waste computation time on expanding states that lead into dead ends. To conclude, the development of a good and computationally efficient heuristic function is application dependent and depends on the surrounding environment the vehicle is intended to operate in.

Figure 3.4: Three different kinds of bounding regions: (a) circles, (b) axis-aligned bounding box, and (c) oriented bounding box.

Figure 3.5: Illustration of how obstacles with different bounding boxes are transformed to a binary occupancy grid map where blue means occupied and green means unoccupied.

3.4.3 Collision detection

All motion planning algorithms rely on efficient collision detection methods. The choice of method depends on how the surrounding obstacles $\mathbb{X}_{\text{obs}}(t)$ are represented and is in general invariant to the choice of motion planning algorithm. Typically, static obstacles $\mathbb{X}_{\text{obs}}^s$ and dynamic obstacles $\mathbb{X}_{\text{obs}}^d(t)$ are treated separately. Since the trajectory for a dynamic obstacle is unknown (unless the vehicles are communicating), its motion has to be predicted. The predictions are usually performed by a tracking and prediction module in the perception layer where probabilistic state-space models are used for trajectory prediction of the dynamic obstacles. The details of these algorithms are out of the scope of this thesis and the reader is referred to, e.g., Deo and Trivedi (2018); Deo et al. (2018); Houenou et al. (2013); Kim et al. (2017) and references therein for details.

Collision detection for static obstacles are typically performed using hierarchical methods where different bounding regions are used in each step. Figure 3.4 illustrates four different kinds of *bounding regions*: a bounding circle, multiple bounding circles, an axis-aligned bounding box (AABB) and an oriented bounding box (OBB). The complexity of performing collision checking for each type of bounding region is in increasing order where a circle is cheap and an oriented bounding box is costly. If the controlled vehicle is also represented as a bounding region (or a hierarchy of bounding regions), the criterion for collision is that the regions are intersecting. If the bounding circles are overlapping, AABB may be checked and finally OBB. As soon as one step reports no collision, the vehicle state is declared collision free. For multi-body vehicle's, separate bounding regions for each body are typically used (LaValle, 2006).

The collision detection procedure has to be performed thousands of times in a single planning cycle and thus precomputing certain steps significantly speeds up the planning process. A common way to perform 2D collision detection is to compute an occupancy grid map (Elfes, 1989) or an OctoMap for the 3D case (Hornung et al., 2013). The grid map is divided into cells where the obstacles' bounding

Figure 3.6: Collision detection using a binary occupancy grid map where a red cell means that the bounding circle of the controlled vehicle is intersecting an obstacle. In (a) the controlled vehicle is not in collision with any obstacle and in (b) the controlled vehicle is in collision with both obstacles.

regions are used to classify a cell as free or occupied¹. Figure 3.5 illustrates how an occupancy gridmap is computed where a green cell means that the cell is free and a blue cell means that the cell is occupied by an obstacle. The occupancy grid map can then be used to efficiently perform collision detection with the bounding region of the controlled vehicle, as illustrated in Figure 3.6. In Figure 3.6a, the vehicle is in free space $x \in \mathbb{X}_{\text{free}}$, whereas in Figure 3.6b the vehicle is in collision with an obstacle $x \in \mathbb{X}_{\text{obs}}^s$ and therefore not in free space $x \notin \mathbb{X}_{\text{free}}$. Note that the collision detection procedure may be rather conservative if a single circle is used as bounding region.

Collision detection of a motion primitive m_p is treated in analogy to a single state except that all states along the trajectory have to be collision free. In practice, only a finite number of states along the trajectory are used for collision detection and a motion primitive m_p applied from initial state x_k is declared collision free if $c(x_k, m_p) \in \mathbb{X}_{\text{free}}$, otherwise it is declared as in collision. If the motion primitives are precomputed, it is possible to speed up collision detection by precomputing and storing the vehicle's swept area as a grid map (called swath) for each maneuver (Pivtoraiko et al., 2009).

3.5 Sampling-based motion planning

We are now ready to present a general sampling-based motion planning framework for nonholonomic systems. For simplicity, only static obstacles are considered in the remainder of this chapter, i.e., $\mathbb{X}_{\text{obs}}^d(t) = \emptyset, \forall t$. A sampling-based motion planner performs its search process in a directed² search graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$ where a vertex $x_k \in \mathcal{V}$ represents a vehicle state and an edge $e_k \in \mathcal{E}$ encodes a motion

¹Probabilistic information and distances to obstacles can be incorporated in the cells instead of a binary representation (Urmson et al., 2008).

²It is also possible to use a bidirectional or an undirected search graph where the planner is searching from x_I to x_G , from x_G to x_I and possibly from other states as well at the same time.

primitive m_p , i.e., a feasible trajectory or path, that connects two vehicle states. The search graph can be incrementally constructed online or parts of the graph are precomputed offline. The general search process to compute a motion plan from the vehicle's initial state $x_I \in \mathbb{X}_{\text{free}}$ to its desired goal state $x_G \in \mathbb{X}_{\text{free}}$ involves the following main steps (LaValle, 2006):

1. **Initialization:** Let $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$ represent a directed search graph, for which only $x_I \in \mathbb{X}_{\text{free}}$ is contained in the vertex set $\mathcal{V}$ and the edge set $\mathcal{E}$ is empty. The cost-to-come is $J(x_I, x_I) = 0$ and the estimated cost-to-go is $h(x_I, x_G)$.
2. **Select expansion candidate:** Select a promising vehicle state $x_k \in \mathcal{V}$ by sorting $\mathcal{V}$ based on, e.g., estimated total cost $J(x_I, x_k) + h(x_k, x_G)$, and a desired vehicle state $x_{k+1} \in \mathbb{X}_{\text{free}}$ to connect.
3. **Steering function:** Try to generate a motion primitive m_p from x_k to x_{k+1} by solving an OCP as in (3.3) such that the state transition $x_{k+1} = f_p(x_k, m_p)$ is achieved and the resulting trajectory is collision free $c(x_k, m_p) \in \mathbb{X}_{\text{free}}$. If this stage fails, then go back to Step 2.
4. **Insert in graph:** If $x_{k+1} \notin \mathcal{V}$, it is added with cost-to-come $J(x_I, x_{k+1}) = J(x_I, x_k) + J(m_p)$. If x_{k+1} already exists in $\mathcal{V}$ and $J(x_I, x_{k+1})$ is more expensive, x_{k+1} is not inserted. On the contrary, if $J(x_I, x_{k+1})$ is cheaper, a rewiring step may be performed where the current edge to x_{k+1} is replaced with the new edge.
5. **Check for a solution:** If $x_{k+1} = x_G$ (or $x_{k+1} \in \mathbb{X}_G$), backtrack from x_{k+1} to x_I and mark its trajectory as a solution candidate with cost $J(x_I, x_{k+1})$.
6. **Check for termination:** Go back to Step 2 unless a termination condition is satisfied. Otherwise, if multiple solution candidates are found, return the solution with minimum total cost J or return failure if no solution has been found.

With minor modifications, most sampling-based motion planning algorithms follow the above mentioned steps. Two motion planning algorithms will now be presented, but first, some properties for motion planning algorithms are defined. A motion planning algorithm is said to be (LaValle, 2006):

Complete if the algorithm always finds a solution in finite time if one exists or terminates and reports failure if one does not exist.

Resolution complete if the algorithm always finds a solution within a specified resolution in finite time if one exists or terminates and reports failure if one does not exist.

Probabilistically complete if a solution exists, the probability of finding a solution approaches one as the number of iterations tends to infinity.

Completeness is an attractive property for any motion planning algorithm. However, when motion planning for nonholonomic systems is considered, the completeness property is hard to establish for an algorithm and weaker properties, such as resolution or probabilistic completeness, are often considered sufficient.

3.5.1 Motion planning in state lattices

The main idea with lattice-based motion planning is to precompute the building blocks for a directed search graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$ offline. These building blocks consist of vertices and edges that, when repeated, generate a search graph $\mathcal{G}$ that forms a regular and repeated pattern. This search graph often called a *state lattice* or a *lattice graph*, and thanks to its structure enables efficient graph search algorithms to be used during online planning (Cirillo et al., 2014; Pivtoraiko et al., 2009).

The state lattice is constructed offline by discretizing the vehicle's state space $x_k \in \mathcal{X}_d$ and computing a set of motion primitives $\mathcal{P}$ by solving a large number of OCPs in the form of (3.3). Each vertex $v_k \in \mathcal{V}$ is a vehicle state $x_k \in \mathcal{X}_d$ and each edge $e_k \in \mathcal{E}$ represents a motion primitive $m_p \in \mathcal{P}$. Each motion primitive $m_p \in \mathcal{P}$ is a feasible trajectory (3.4) that moves the vehicle from one discrete vehicle state $x_k \in \mathcal{X}_d$ to another $x_{k+1} \in \mathcal{X}_d$ in a bounded neighborhood in free-space. In other words, the initial and final states of a motion primitive must comply with the specified state-space discretization. The framework assumes that the vehicle is position invariant which implies that the set of motion primitives $\mathcal{P}$ can be computed from the position of the vehicle at the origin. A motion primitive can then be translated and reused from all other discrete positions during online planning. In general, all motion primitives are not applicable from each vehicle state $x_k \in \mathcal{X}_d$ and the motion primitives that can be used from x_k is denoted $\mathcal{P}(x_k) \subseteq \mathcal{P}$. The average size of $\mathcal{P}(x_k)$ represents the *branching factor* of the search problem and with increasing number of edges from each $x_k \in \mathcal{X}_d$, a good heuristic function becomes increasingly important during online planning.

By specifying the discretized state space $\mathcal{X}_d$ and precomputing the set of motion primitives $\mathcal{P}$, the continuous-time motion planning problem in (3.2) is approximated by a discrete-time OCP in the form

$$\underset{\{m_p^k\}_{k=0}^{N-1}, N}{\text{minimize}} \quad J_{\text{DP}} = \sum_{k=0}^{N-1} J(m_p^k) \quad (3.6a)$$

$$\text{subject to} \quad x_0 = x_I, \quad x_N = x_G, \quad (3.6b)$$

$$x_{k+1} = f_p(x_k, m_p^k), \quad (3.6c)$$

$$m_p^k \in \mathcal{P}(x_k), \quad (3.6d)$$

$$c(m_p^k, x_k) \in \mathbb{X}_{\text{free}}. \quad (3.6e)$$

The decision variables to the OCP in (3.6) are the motion primitive sequence $\{m_p^k\}_{k=0}^{N-1}$ and its length N . The set of applicable motion primitives from a certain vehicle state x_k is encoded in (3.6d) and a state transition $x_{k+1} = f_p(x_k, m_p^k)$ is feasible if the resulting trajectory remains in free space (3.6e). In (3.6), it is assumed that $x_I \in \mathcal{X}_d$ and $x_G \in \mathcal{X}_d$ to make the problem well defined. If $x_I \notin \mathcal{X}_d$ or $x_G \notin \mathcal{X}_d$, they have to be projected to their closest neighboring state in $\mathcal{X}_d$ using some distance metric. The projection of x_I to $\mathcal{X}_d$ should take into account the region of attraction for the closed-loop system, consisting of the feedback controller and the controlled vehicle. Otherwise, successful execution of the motion plan cannot be guaranteed. One approach to alleviate this problem is to include a post-

Figure 3.7: An illustration of the three steps that are performed to generate the state lattice. (1) Discretize the state-space, (2) select which states to connect, (3) solve the BPVs.

optimization step (Bergman et al., 2019a), where an OCP solver is warm-started with the solution computed by the lattice planner. In this framework, the lattice planner is used to solve the combinatorial aspects of the problem, e.g., to pass an obstacle to the left or to the right, and the OCP solver is then used to compute a locally optimal solution to the motion planning problem that starts at the vehicle’s initial state and reaches the goal state exactly. In Paper F, this technique is used to develop a trajectory planner for general N-trailer vehicles that can have a mixture of passively and actively steered trailers.

The construction of the state lattice is now explained more thoroughly.

State lattice construction

The offline construction of the state lattice is illustrated in Figure 3.7 and can be divided into three separate steps:

1. Specify the resolution of the search space $\mathcal{V}$ by discretizing the vehicle’s state-space.
2. Select which pairs of discrete vehicle states to connect based on the reachability of the system.
3. Generate the set of motion primitives $\mathcal{P}$ by solving the OCPs defined in the previous step.

In the first step, the vehicle’s state-space is discretized based on the maneuverability of the system and the environment in which the vehicle is intended to operate.

Figure 3.8: The first three levels of the reachability graph $\mathcal{G}_r(x_k, \mathcal{P})$ from $x_k = [0 \ 0 \ \pi/2]^\top$ where the vehicle is restricted to move according to the motion primitives in Example 3.1.

For path planning of a car-like vehicle, the first step is to specify the fidelity of the vehicle’s position, orientation and steering angle (Pivtoraiko et al., 2009).

The second step is to select which pairs of discrete vehicle states to connect. At this step, it is important to be aware of that the complexity of the discrete-time OCP in (3.6) scales exponentially in the number of applicable motion primitives $\mathcal{P}(x_k)$ from each discrete vehicle state x_k . Therefore, a careful selection of vehicle states to connect is most often performed. The selection should be performed such that the resulting *reachability graph* $\mathcal{G}_r(x_k, \mathcal{P})$ (the set of all possible trajectories from x_k given the motion primitive set $\mathcal{P}$) sufficiently resembles the vehicle’s *forward reachable set* from x_k .

The third step is to solve the specified OCPs, e.g., using some of the optimization methods presented in Section 3.4.1 or to employ the maneuver-based motion primitive generation framework presented in Bergman et al. (2019b). Examples of motion primitives $\mathcal{P}(x_k)$ from a selection of vehicle states x_k for a general 2-trailer with a car-like tractor and for a quadcopter platform are depicted in Figure 3.2 where CasADi (Andersson et al., 2018a) together with IPOPT (Wächter and Biegler, 2006) are used to solve the resulting OCPs. To clarify the above mentioned steps, Example 3.1 shows how to construct a minimal state lattice for a kinematic bicycle model.

Example 3.1: The Kinematic bicycle model

The vehicle states for the Kinematic bicycle model (2.11) are the position $(x_1, y_1) \in \mathbb{R}^2$ and orientation $\theta_1 \in (-\pi, \pi]$. The control signals are the steering angle α and longitudinal velocity $v = 1$, i.e., the vehicle is only allowed to move forward with constant longitudinal velocity. The position of the vehicle is discretized to a uniform grid with resolution r in both x and y . The orientation is discretized

into four different orientations $\Theta = \{-\pi/2, 0, \pi/2, \pi\}$. The steering angle α is constrained to zero at each vertex to make sure that it is continuous when multiple edges are combined during online planning.

The pair of discrete vehicle states to connect are illustrated in Figure 3.7 where a green box with a green arrow encodes that a discretized vehicle state is included. The resulting motion primitives are the blue paths where only $\pm 90^\circ$ -turns and straight forward motions are generated from each initial orientation $\theta_i \in \Theta$ from the position of the vehicle at the origin. The first three levels of the resulting reachability graph $\mathcal{G}_r(x_k, \mathcal{P})$ from $x_k = [0 \ 0 \ \pi/2]^\top$ are illustrated in Figure 3.8.

To improve the online planning time, the motion primitive set $\mathcal{P}$ can be reduced using the reduction technique presented in Cirillo et al. (2014); Pivtoraiko and Kelly (2011). A motion primitive $m_p \in \mathcal{P}$ with stage cost $J(m_p)$ is removed if its state transition can be achieved by a combination of the other motion primitives in $\mathcal{P}$ with a combined total stage cost J_{comb} that satisfies $J_{\text{comb}} \leq \eta J(m_p)$, where $\eta \geq 1$ is a user specified constant. This procedure can also be used to verify that redundant motion primitives do not exist by selecting $\eta = 1$. Moreover, as discussed in Section 3.4.2, an informative heuristic function h is needed to guide the search algorithm during online planning where, e.g., a precomputed HLUT can be used to quickly access the heuristic cost at runtime (Cirillo et al., 2014; Pivtoraiko et al., 2009).

Online planning

During online planning, the discrete-time OCP in (3.6) is solved using a graph-search algorithm, such as, A* (Hart et al., 1968) or anytime repairing A* (ARA*) (Likhachev et al., 2004). Since these methods perform *deterministic searches* in a directed search graph, the lattice planner automatically inherits the properties and guarantees of the chosen graph-search algorithm. The solution to the discrete-time OCP in (3.6) is an ordered sequence of N motion primitives $\{m_p^k\}_{k=0}^{N-1}$, i.e., a feasible and collision-free trajectory $(x(t), u(t))$, $t \in [t_I, t_G]$ or path $(x(s), u(s))$, $s \in [0, s_G]$ that minimizes the objective function J_{DP} . If A* is used as search method and enough planning time is given, the algorithm is guaranteed to return a solution that is *resolution optimal* (Pivtoraiko et al., 2009).

The A* graph-search algorithm is presented in Algorithm 1. With some minor adjustments the algorithm follows the steps presented in Section 3.5. Firstly, since the motion primitive set is precomputed, no OCP needs to be solved online. Secondly, not only one but all possible edges $\mathcal{P}(x_k)$ from the best candidate vertex x_k are explored. In the algorithm, the *closedSet* stores the discrete vehicle states that have been expanded from and the *openSet* stores the vehicle states that have been explored but remains to be expanded. The *openSet* is commonly referred to as the *frontier* and is typically implemented as a priority queue. If the heuristic function h is admissible, the termination condition is trivial (line 10). That is, if the best candidate vertex x_k from the *openSet* is $x_k = x_G$, the optimal motion primitive sequence $\{m_p^k\}_{k=0}^{N^*-1}$ is found with cost $J(x_k, x_I)$ and the motion primitive sequence can then be obtained by *backtracing* from the found goal node x_k

Algorithm 1 Online lattice-based motion planning using A*

```

1: Inputs:  $x_I$ ,  $x_G$  and  $\mathbb{X}_{\text{free}}$ 
2:  $\text{closedSet} = \emptyset$ 
3:  $\text{openSet} = \emptyset$ 
4:  $x_I.\text{arrivalEdge} = 0$ 
5:  $x_I.\text{costToCome} = 0$ 
6:  $x_I.\text{costToGo} = h(x_I, x_G)$ 
7:  $\text{openSet.add}(x_I)$ 
8: while  $\text{openSet} \neq \emptyset$  do
9:    $x_k \leftarrow \text{openSet.popBestCandidate}()$ 
10:  if  $x_k = x_G$  then
11:    return  $\text{backtrace}(x_k)$ 
12:   $\text{closedSet.add}(x_k)$ 
13:  for each  $m_p \in \mathcal{P}(x_k)$  do
14:     $x_{k+1} = f_p(x_k, m_p)$ 
15:    if  $x_{k+1} \in \text{closedSet}$  or  $c(m_p, x_k) \notin \mathbb{X}_{\text{free}}$  then
16:      continue
17:     $\text{tentativeCost} = J(x_k, x_I) + J(m_p)$ 
18:    if  $x_{k+1} \notin \text{openSet}$  then
19:       $\text{openSet.add}(x_{k+1})$ 
20:    else if  $\text{tentativeCost} \geq \text{openSet}(x_{k+1}).\text{costToCome}$  then
21:      continue
22:    else
23:       $\text{openSet.swap}(x_{k+1})$ 
24:       $x_{k+1}.\text{parent} = x_k$ 
25:       $x_{k+1}.\text{arrivalEdge} = m_p$ 
26:       $x_{k+1}.\text{costToCome} = \text{tentativeCost}$ 
27:       $x_{k+1}.\text{costToGo} = h(x_{k+1}, x_G)$ 
28:  end for
29: end while
30: return failure

```

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to the root node x_I (line 11). Moreover, *the principle of optimality* ensures that every subtrajectory is optimal for its corresponding subproblem.

If $x_G \notin \mathcal{X}_d$ and an accurate goal state x_G is important, an OCP in the form of (3.3) can be solved online to optimize the final part of the motion plan $(x(t), u(t))$, $t \in [t_I, t_G]$ such that the goal state is reached exactly. The same approach can be used for the initial state x_I but is often omitted and the initial transient is left for the feedback controller to suppress.

Due to the algorithm's efficiency, lattice-based motion planning has been successfully deployed on various robotic platforms (Andersson et al., 2018b; Pivtoraiko et al., 2009; Urmson et al., 2008). To illustrate the capability of lattice-based motion planners, two planning examples are illustrated in Figure 3.9. In Figure 3.9a, the quadcopter problem considered in Andersson et al. (2018b) is shown and in Figure 3.9b, the general 2-trailer vehicle from Paper D is shown.

(a)

(b)

Figure 3.9: Two online planning examples using a lattice-based motion planner. In (a), trajectory planning for a quadcopter platform is shown and in (b), path planning for a general 2-trailer with a car-like tractor is shown. The blue and the white paths are the computed motion plans for the position of the quadcopter and the position for the axle of the semitrailer, respectively.

Other deterministic approaches

There exist other deterministic motion planning algorithms that rely on *control-input sampling* $u \in \mathcal{U}_d$ instead of state-space discretization. In this case, a model of the vehicle is used online to simulate the system for Δt seconds, or Δs meters, using a constant or parametrized control signal $u(t)$, $t \in [0, \Delta t]$ from $\mathcal{U}_d$. In general, the resulting trajectory segment does not end up at a specified final state. This implies that the search graph becomes irregular which results in a frontier that grows exponentially during online graph search (Paden et al., 2016; Pivtoraiko et al., 2009). To resolve this, the vehicle's state-space is often divided into cells where a cell is only allowed to contain one state. One motion planning algorithm that

uses control-input sampling is the hybrid A* (Dolgov et al., 2010). Common for these approaches is that they lack all completeness and optimality guarantees and control-input sampling is not practical for unstable systems, unless the simulations are performed in closed-loop with a stabilizing feedback controller.

On the contrary, control-input sampling may be more suitable for systems that are not position invariant or if some vehicle states (except for the position of the vehicle) have to be discretized with high resolution. The latter case arises in dynamic on-road driving scenarios where trajectory planning is needed. A state-lattice approach is here impractical to use since the discretization of a sufficient number of possible vehicle speeds will result in a huge search graph during online planning (Ziegler and Stiller, 2009). An efficient algorithm that is tailored for on-road driving is the so-called spatiotemporal lattice formulation (McNaughton et al., 2011; Ziegler and Stiller, 2009) where parts of the vehicle's state-space is regularly discretized offline and other parts are implicitly discretized during online planning by sampling the vehicle's control inputs.

3.5.2 Motion planning using RRT

Another popular family of motion planning algorithms includes the probabilistic *rapidly-exploring random tree* (RRT) algorithm (LaValle, 1998). Only a brief overview will be presented and the reader is referred to, e.g., Karaman and Frazzoli (2010); LaValle (2006); Paden et al. (2016) for more details. In contrast to the deterministic approaches presented in the previous section, the RRT algorithm is based on randomized exploration of the search space. The original RRT algorithm was developed for holonomic systems and was later extended to nonholonomic systems in LaValle and Kuffner (2001) where the algorithms follow the main steps presented in Section 3.5.

The RRT algorithm incrementally constructs a directed search tree³ during online planning. The search tree is expanded by iteratively drawing random samples $x_{\text{rand}} \in \mathbb{X}$. A steering function is then used to connect from its nearest neighboring state x_{near} in the tree towards x_{rand} . If the resulting edge is collision-free, a new vertex is added to the tree where x_{near} is determined by sorting the vertices in the search tree using a heuristic function, e.g., a look-up table for a nonholonomic system (LaValle and Kuffner, 2001; Pivtoraiko et al., 2009). If an exact steering function exists, the RRT algorithm can connect to a specified goal state x_G by performing goal sampling. Otherwise, the desired goal state has to be replaced with a goal region $\mathbb{X}_G$, e.g., a ball centered around the goal state: $X_G = \{x \in \mathbb{X} \mid \|x - x_G\|_Q \leq \epsilon\}$ where $Q \succ 0$ and $\epsilon > 0$.

For holonomic systems, the RRT algorithm is *probabilistically complete*, but is also guaranteed with probability one to converge to a suboptimal solution (Karaman and Frazzoli, 2010). For the nonholonomic case, the RRT algorithm lacks all completeness guarantees (Karaman and Frazzoli, 2010; Kunz and Stilman, 2015). An extension of the basic RRT is the so-called RRT* (Karaman and Frazzoli, 2011) where a rewiring procedure is performed in each tree expansion. In this step, the edges for the neighboring vertices in a ball around the new sample is

³A search tree is a special type of a search graph where each vertex only has one parent.

rewired if a lower cost-to-come can be achieved. With this extension, RRT* is shown to be *probabilistically optimal* meaning that the algorithm will converge to an optimal solution when the number of samples approaches infinity. However, the rewiring procedure involves solving several OCPs online. Therefore, its practical applicability is limited to systems where analytical steering functions exist.

Many modifications to the original RRT algorithm have been proposed to speed up the search using different pruning strategies and heuristic functions that bias the sampling towards interesting regions. These methods are out of the scope of this thesis and some extensions can be found in, e.g., Gammell (2017); Kalisiak and van de Panne (2006); Nasir et al. (2013); Urmson and Simmons (2003).

Anytime implementations of RRTs have been successfully deployed in dynamical environments by Kuwata et al. (2009) using biased sampling of reference signals to a closed-loop system. The closed-loop RRT (CL-RRT) algorithm was developed to enable a longer tree expansion without solving expensive OCPs online, i.e., the stabilizing controller is acting as an inexact solver. Moreover, the CL-RRT framework is suitable for unstable systems which is exploited in Paper A where a motion planner for a reversing general 2-trailer with a car-like tractor is presented.

4

Feedback control for self-driving vehicles

This chapter presents different *feedback-control* techniques for self-driving vehicles. In this context, the objective of the feedback controller is to ensure that the controlled vehicle executes a computed motion plan with a small control error even if disturbances and initial control errors may be present. Depending on the reference obtained from the motion planner, the feedback-control problem will either be referred to as a *path-following control problem* or a *trajectory-tracking control problem*. We recall that the vehicle is assumed to be modeled as a time-invariant nonlinear system $\dot{x}(t) = f(x(t), u(t))$. During the control design, it is assumed that the vehicle state $x(t)$ is known¹ and that the system is controllable, at least locally around the nominal trajectory or path (Khalil and Grizzle, 2002). Moreover, the motion plan is assumed to be feasible and collision-free, i.e., it satisfies the vehicle model and does not collide with any surrounding obstacles. Here, the theory will be presented in continuous-time but most of the material has a discrete-time counterpart (Anderson and Moore, 2007; Garcia et al., 1989; Rawlings, 2000).

The remainder of this chapter is structured as follows. In Section 4.1, the trajectory-tracking and path-following control concepts are explained. In Section 4.2, the trajectory-tracking problem is formally derived and some controller synthesis techniques are presented. In Section 4.3, path following for wheeled vehicles is presented together with some path-following techniques tailored for self-driving vehicles.

4.1 Problem formulations

The objective of the feedback controller is to stabilize the vehicle around a nominal path or trajectory computed by a motion planner. Even though the control tasks

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¹In practice, the vehicle states are estimated using information from sensors on the vehicle. The sensor information is fused using nonlinear observers to obtain a state estimate $\hat{x}(t) \approx x(t)$.

differ, the solution is in the form of a *state-feedback controller*. The control tasks are defined as follows:

Trajectory tracking: The closed-loop system consisting of the controlled vehicle and the trajectory-tracking controller should execute a nominal trajectory $(x_r(t), u_r(t))$, $t \in [0, \infty]$ with a small tracking error $\bar{x}(t) = x(t) - x_r(t)$.

Path following: The closed-loop system consisting of the controlled vehicle and the path-following controller should execute a nominal path $(x_r(s), u_r(s))$, $s \in [0, \infty]$ with a small path-following error $\tilde{x}(t) = x(t) - x_r(s(t))$. Here, the path parameter $s(t)$, satisfying $\dot{s} > 0$, is defined as the progression of the vehicle along the nominal path.

In *trajectory tracking*, the reference trajectory consists of nominal vehicle states $x_r(t)$ and control inputs $u_r(t)$ that are parametrized in time. In other words, the objective of the trajectory-tracking controller is to stabilize the vehicle around a nominal state trajectory. Here, the nominal trajectory $(x_r(\cdot), u_r(\cdot))$ at each time instant is predetermined and is independent of the vehicle's current state $x(t)$. The trajectory-tracking problem is illustrated in Figure 4.1a. Note that *point-stabilization* is a special case of trajectory tracking where the nominal trajectory collapses to a single point (x_r, u_r) .

In *path following*, the obtained reference from the motion planner consists of nominal vehicle states $x_r(s)$ and control inputs $u_r(s)$ that are parametrized in space rather than in time. In other words, the path-following controller acts upon a space-parametrized path-following error that is implicitly parametrized in time. Given the vehicle's current state $x(t)$, the nominal vehicle state $x_r(s(t))$ and control input $u_r(s(t))$ are calculated by projecting the position $p(t)$ of the vehicle onto the nominal path in $p_r(s)$, $s \in [0, \infty]$, as illustrated in Figure 4.1b.

For wheeled vehicles, the control signals typically consist of the steering angle $\alpha(t)$ and the longitudinal velocity $v_1(t)$ of the vehicle. The path-following controller uses the steering angle to stabilize the vehicle around the nominal path and

Figure 4.1: Illustration of the difference between trajectory tracking (a) and path following (b). The black vehicle is the controlled vehicle, the blue vehicle represents the reference vehicle and the red line is the positional control error. The green line represents a typical convergence behavior of the closed-loop system when trajectory-tracking control (a) and path-following control (b) is used, respectively.

the longitudinal velocity is controlled such that the nominal path is executed at a desired set speed $v_r(t)$. In this setup, the control task is said to be separated into *lateral* and *longitudinal* control. Figure 4.1b illustrates how a path-following controller operates to stabilize the vehicle around the nominal path. For small control errors, a trajectory-tracking controller and a path-following controller performs similarly. However, if a large longitudinal tracking error is present in a sharp corner, a trajectory-tracking controller can behave undesirably and take shortcuts since longitudinal and lateral control are coupled.

4.2 Trajectory-tracking control

In trajectory-tracking control applications, the motion planner is assumed to provide the feedback controller with a collision-free nominal trajectory² $(x_r(t), u_r(t))$, $t \in [0, \infty)$. Since the nominal trajectory is feasible, it satisfies the vehicle model

$$\dot{x}_r(t) = f(x_r(t), u_r(t)), \quad x_r(0) \text{ given}, \quad (4.1)$$

and the vehicle's physically imposed constraints $x_r(t) \in \mathbb{X}$ and $u_r(t) \in \mathbb{U}$, $\forall t \geq 0$. The objective of the trajectory-tracking controller is to control the vehicle's state trajectory $x(\cdot)$ such that the nominal trajectory (4.1) is executed with a small tracking error $\bar{x}(t) = x(t) - x_r(t)$. Denote the control-signal deviation as $\bar{u}(t) = u(t) - u_r(t)$. Then the *tracking-error system* becomes

$$\dot{\bar{x}}(t) = f(\bar{x}(t) + x_r(t), \bar{u}(t) + u_r(t)) - f(x_r(t), u_r(t)) \triangleq \bar{f}(t, \bar{x}(t), \bar{u}(t)), \quad (4.2)$$

where $(\bar{x}(t), \bar{u}(t)) = (0, 0)$ is an equilibrium point since $\bar{f}(t, 0, 0) = 0$, $\forall t \geq 0$ and the initial tracking error $\bar{x}(0) = x(0) - x_r(0) = \bar{x}_0$ is given. Note that the nominal trajectory $(x_r(\cdot), u_r(\cdot))$ makes the tracking-error system in (4.2) become time-varying. Define the feedback controller as $\bar{u}(t) = g(t, \bar{x}(t))$ where $g(t, 0) = 0$. With this feedback control-law, the *closed-loop tracking-error system* can compactly be written as

$$\dot{\bar{x}}(t) = \bar{f}(t, \bar{x}, g(t, \bar{x}(t))) \triangleq \bar{f}_{cl}(t, \bar{x}(t)), \quad \bar{x}(0) = \bar{x}_0, \quad (4.3)$$

where the origin $\bar{x}(t) = 0$ is an equilibrium point since $\bar{f}_{cl}(t, 0) = \bar{f}(t, 0, 0) = 0$, $\forall t \geq 0$. The closed-loop tracking-error system in (4.3) is a nonlinear time-varying system and the objective is to design the trajectory-tracking controller $\bar{u}(t) = g(t, \bar{x}(t))$ such that the origin of (4.3) obtains some of the following properties.

Definition 4.1 (Khalil and Grizzle (2002) Definition 4.4). The equilibrium point $\bar{x} = 0$ of (4.3) is

- *Stable*, if, for each $\epsilon > 0$, there is a $\delta = \delta(\epsilon, t_0) > 0$ such that

$$\|\bar{x}(t_0)\| < \delta \Rightarrow \|\bar{x}(t)\| < \epsilon, \quad \forall t \geq t_0 \geq 0. \quad (4.4)$$

²In the previous chapter, it was assumed that the motion plan is constructed such that the goal is an equilibrium point, i.e., $f(x_r(t), u_r(t)) = 0$, $t \geq t_G$. This assumption implies that the motion plan can be extended to infinity where the vehicle is specified to remain at the goal state $\forall t \geq t_G$.

- *Uniformly stable* if, for each $\epsilon > 0$, there is a $\delta = \delta(\epsilon) > 0$, independent of t_0 , such that (4.4) is satisfied.
- *Asymptotically stable* if it is stable and there is a positive constant $c = c(t_0)$ such that $\bar{x}(t) \rightarrow 0$ as $t \rightarrow \infty$, for all $\|\bar{x}(t_0)\| < c$.

Other desirable stability properties of the origin includes *exponential stability* where the rate of convergence is upper-bounded by an exponential decay and *uniform asymptotic stability* which means that asymptotic stability around the origin is independent of the initial time (Khalil and Grizzle, 2002). In many applications, global stability properties of the origin for the closed-loop tracking-error system (4.3) are hard to establish and only local stability properties can be verified (Paden et al., 2016).

Generally, the problem of determining uniform stability (uniform asymptotic stability) of the closed-loop system in (4.3) is to find a *Lyapunov function candidate* $V(t, \bar{x}) \geq 0$, where its time derivative

$$\dot{V}(t, \bar{x}) = \frac{\partial V(t, \bar{x})}{\partial \bar{x}} \bar{f}_{cl}(t, \bar{x}(t)) + \frac{\partial V(t, \bar{x})}{\partial t}, \quad (4.5)$$

is non-positive (negative) for all $t \geq 0$ in a neighborhood around the origin (Khalil and Grizzle, 2002). For more details regarding stability of nonlinear systems, the reader is referred to (Khalil and Grizzle, 2002). Some techniques are now presented for designing the trajectory-tracking controller $\bar{u}(t) = g(t, \bar{x}(t))$.

4.2.1 Linear trajectory-tracking techniques

Linear control techniques is a simple but yet efficient design tool for trajectory tracking of nonlinear systems where the nonlinear tracking-error system (4.2) is first linearized around the nominal trajectory. Since the nominal trajectory is feasible, it satisfies the vehicle model and the vehicle's physically imposed constraints $x_r(t) \in \mathbb{X}$ and $u_r(t) \in \mathbb{U}$, $\forall t \geq 0$. Typically, the constraints are assumed to be satisfied with a certain margin, making it possible for the trajectory-tracking controller to suppress disturbances during the execution of the nominal trajectory. Then, depending on the application, it can sometimes be reasonable to neglect the constraints during the control design if the initial tracking error $\bar{x}(0)$ is small and the designed controller is able to locally stabilize the vehicle around the nominal trajectory. A first-order Taylor series expansion of the tracking-error system (4.2) around the nominal trajectory yields

$$\dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)\bar{u}(t), \quad \bar{x}(0) = \bar{x}_0, \quad (4.6)$$

where

$$A(t) = A(x_r(t), u_r(t)) = \left. \frac{\partial f(x(t), u(t))}{\partial x} \right|_{(x_r(t), u_r(t))}, \quad (4.7a)$$

$$B(t) = B(x_r(t), u_r(t)) = \left. \frac{\partial f(x(t), u(t))}{\partial u} \right|_{(x_r(t), u_r(t))}. \quad (4.7b)$$

The system in (4.6) is a *linear time-varying* (LTV) system. Here, we are interested in stabilizing this system around the origin using a *linear state-feedback controller* with *feedforward action*

$$u(t) = u_r(t) + K(t)\bar{x}(t), \quad (4.8)$$

where $K(t) \in \mathbb{R}^{m \times n}$ is the *state-feedback gain*. Note that the feedforward $u_r(t)$ is provided by the motion planner and therefore included in (4.8). With this state-feedback controller inserted in the nonlinear tracking-error system (4.2), the closed-loop system is on the form in (4.3). The objective is now to design the feedback gain $K(t)$ such that the origin to this closed-loop system obtains desired stability properties. By inserting the control-law in (4.8) into the linearized tracking-error system (4.6), the closed-loop system becomes

$$\dot{\bar{x}}(t) = (A(t) + B(t)K(t))\bar{x}(t) \triangleq A_{cl}(t)\bar{x}(t), \quad \bar{x}(0) = \bar{x}_0. \quad (4.9)$$

Global exponential stability of the origin for this system follows from Theorem 4.1.

Theorem 4.1 (Khalil and Grizzle (2002)). *Let $A_{cl}(t)$ be continuous $\forall t \geq 0$. Suppose there exists a continuously differentiable $P(t) \succ 0$ that satisfies*

$$0 \prec c_1 I \preceq P(t) \preceq c_2 I, \quad \forall t \geq 0,$$

for positive constants c_1 and c_2 , where $c_1 < c_2$. If $P(t)$ satisfies the Lyapunov inequality

$$\dot{P}(t) + P(t)A_{cl}(t) + A_{cl}^T(t)P(t) \preceq -Q, \quad \forall t \geq 0, \quad (4.10)$$

with $Q \succ 0$, then, the origin to (4.9) is a globally exponentially stable equilibrium point.

Proof: See proof of Theorem 4.10 in Khalil and Grizzle (2002). $\square$

The condition in (4.10) is equivalent to the existence of a Lyapunov function $V(t, \bar{x}) = \bar{x}^T P(t) \bar{x}$ that satisfies $\dot{V}(t, \bar{x}) \leq -\bar{x}^T Q \bar{x}$, $\forall t \geq 0$, for the closed-loop LTV system in (4.9). In fact, exponential stability of the origin of the closed-loop LTV system in (4.9) is a sufficient condition for exponential stability of the origin of the nonlinear closed-loop system in (4.3). The result is formalized in Theorem 4.2.

Theorem 4.2 (Khalil and Grizzle (2002)). *Let $\bar{x}(t) = 0$ be an equilibrium point for the nonlinear system in (4.3) where $f_{cl} : [0, \infty) \times D \rightarrow \mathbb{R}^n$ is continuously differentiable, $D = \{\bar{x} \in \mathbb{R}^n \mid \|\bar{x}\|_2 < r\}$, and the Jacobian matrix $A_{cl}(t)$ is bounded and Lipschitz on D , uniformly in $t \geq 0$. Then, the origin is an exponentially stable equilibrium point for the nonlinear system in (4.3) if it is an exponentially stable equilibrium point for the LTV system in (4.9).*

Proof: See proof of Theorem 4.15 in Khalil and Grizzle (2002). $\square$

Remark 4.1. Both Theorem 4.1 and Theorem 4.2 have a finite-time counter part. In that case, exponential stability is replaced with the norm of the tracking error $\|\bar{x}(t)\|$ being upper bounded by a positive and exponentially decreasing function in time. $\square$

To conclude, if $K(t)$ is designed such that the closed-loop LTV system in (4.9) satisfies Theorem 4.1. Then, Theorem 4.2 ensures exponential stability of the origin for the nonlinear closed-loop system in (4.3). Now, some methods are presented for designing the feedback gain $K(t)$ such that Theorem 4.1 is satisfied.

4.2.2 Robust control design using linear matrix inequalities

In some applications, a static state-feedback gain K is enough to stabilize the LTV system in (4.9). One way to make use of Theorem 4.1 is to assume that the pairs $[A(t), B(t)]$ lie in the *convex polytope* $\mathbb{P}$, $\forall t \geq 0$, where $\mathbb{P}$ is represented by L vertices

$$[A(t), B(t)] \in \mathbb{P} = \mathbf{Co}\{[A_1, B_1], \dots, [A_L, B_L]\}, \quad (4.11)$$

where $\mathbf{Co}$ denotes the convex hull. Denote the *polytopic linear differential inclusion* (LDI) (Boyd et al., 1994) as

$$\dot{\bar{x}}(t) \in \mathbb{P} \begin{bmatrix} \bar{x}(t) \\ \bar{u}(t) \end{bmatrix}, \quad \bar{x}(0) = \bar{x}_0. \quad (4.12)$$

We can interpret the polytopic LDI in (4.12) as describing a family of LTV systems including (4.6). This implies that a solution to (4.6) is also a trajectory of the LDI (4.12). Thus, with $\bar{u}(t) = K\bar{x}(t)$ in (4.12), global exponential stability of the closed-loop version of (4.12) implies global exponential stability of the closed-loop LTV system in (4.9). With a constant Lyapunov matrix $P(t) = P \succ 0$ in Theorem 4.1, the condition (4.10) for global exponential stability can be reformulated as a controller synthesis problem (Boyd et al., 1994) in the form of a *matrix inequality* feasibility problem in the variables $P \succ 0$ and $K \in \mathbb{R}^{m \times n}$

$$(A_i + B_i K)^\top P + P(A_i + B_i K) \preceq -2\epsilon P, \quad i = 1, \dots, L, \quad (4.13)$$

where $\epsilon > 0$ is a constant and $Q \succ 0$ in (4.10) has been replaced with $2\epsilon P \succ 0$. Here, $\epsilon > 0$ denotes the *decay-rate* of the Lyapunov function (Boyd et al., 1994). This matrix inequality is not jointly convex in P and K . However, if $\epsilon > 0$ is fixed, with the bijective transformation $S = P^{-1} \succ 0$ and $Y = KP^{-1} \in \mathbb{R}^{m \times n}$, the matrix inequality in (4.13) can be rewritten as a *linear matrix inequality* (LMI) in S and Y (Wolkowicz et al., 2012):

$$SA_i^\top + Y^\top B_i^\top + A_i S + B_i Y + 2\epsilon S \preceq 0, \quad i = 1, \dots, L. \quad (4.14)$$

In other words, it is an LMI feasibility problem to find a linear state-feedback controller $\bar{u}(t) = K\bar{x}(t)$ that satisfies condition (4.10) in Theorem 4.1. If S and Y are feasible solutions to (4.14), the *quadratic Lyapunov function* is $V(\bar{x}(t)) = \bar{x}(t)^\top S^{-1} \bar{x}(t)$ and the linear state-feedback controller is $\bar{u}(t) = YS^{-1}\bar{x}(t)$. Moreover, by Theorem 4.2, the origin of the nonlinear closed-loop system in (4.3) is an exponentially stable equilibrium point. The constraint in (4.14) can be used in a semi-definite programming (SDP) problem by, e.g., minimizing $\|K - K_{\text{des}}\|$, where K_{des} is a nominal feedback gain. Another possibility is to minimize the condition number of S^{-1} (Boyd et al., 1994).

If a single static state-feedback controller is not enough to stabilize the vehicle around the nominal trajectory, *gain-scheduling* can be used (Rugh and Shamma, 2000). Here, a set of working points is selected and a state-feedback gain $K(x_r, u_r)$ is designed for each working point (x_r, u_r) (Rugh and Shamma, 2000). The resulting gain-scheduled linear feedback controller with feedforward action is

$$u(t) = u_r(t) + K(x_r(t), u_r(t))\bar{x}(t), \quad (4.15)$$

where the state-feedback gain is typically obtained using a convex-combination of neighboring working points to $(x_r(t), u_r(t))$. Unless the matrices $A(x_r(t), u_r(t))$ and $B(x_r(t), u_r(t))$ are *slowly-varying*, stability for the resulting closed-loop LTV-system cannot be formally guaranteed (Rugh and Shamma, 2000). Thus, extensive simulations and experiments need to be conducted to validate a gain-scheduled controller. This is however the case for any feedback-control strategy in practice.

4.2.3 Linear quadratic control

In this section, we turn to the problem of designing the linear state-feedback controller $\bar{u}(t) = K(t)\bar{x}(t)$ in an optimal way. One possibility is to use *linear quadratic* (LQ) control techniques. The time-varying finite-horizon LQ control problem is (Anderson and Moore, 2007):

$$\begin{aligned} & \underset{\bar{u}(\cdot)}{\text{minimize}} \quad \|\bar{x}(T)\|_{P_N}^2 + \int_0^T \left(\|\bar{x}(t)\|_{Q(t)}^2 + \|\bar{u}(t)\|_{R(t)}^2 \right) dt \\ & \text{subject to} \quad \dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)\bar{u}(t), \\ & \quad \bar{x}(0) = \bar{x}_0 \text{ given,} \end{aligned} \quad (4.16)$$

where $T > 0$ is the prediction horizon, $P_N \succeq 0$ is the terminal cost, $R(t) \succ 0$ and $Q(t) \succeq 0$ are design matrices that are used to trade-off between large tracking errors and control-signal deviations. Here, $A(t)$ and $B(t)$ describe the time-varying linearization (4.6) of the nonlinear tracking-error system in (4.2) around the nominal trajectory $(x_r(\cdot), u_r(\cdot))$. Since $R(t) \succ 0$, the Hamilton-Jacobi-Bellman (HJB) equation (Anderson and Moore, 2007) is strictly convex in $\bar{u}(t)$ and the optimal solution to the finite-horizon LQ control problem is a linear state-feedback control law $\bar{u}(t) = K(t)\bar{x}(t)$ where the time-varying feedback gain is

$$K(t) = -R^{-1}(t)B^\top(t)P(t), \quad (4.17)$$

and $P(t) = P^\top(t)$ is obtained by solving a *matrix Riccati partial differential equation*

$$-\dot{P}(t) = A^\top(t)P(t) + P(t)A(t) - P(t)B(t)R^{-1}B^\top(t)P(t) + Q(t), \quad (4.18)$$

with boundary condition $P(T) = P_N$ (Kalman et al., 1960). Furthermore, the optimal cost for the finite-horizon LQ control problem is $V(t, \bar{x}) = \bar{x}^\top(t)P(t)\bar{x}(t)$. In general, an explicit solution to (4.18) does not exist but it can be computed

numerically (Anderson and Moore, 2007; Kalman et al., 1960; Zhou et al., 1996). The discrete-time version of (4.16) instead yields a solution $P(t_k)$ that satisfies a *matrix Riccati recursion* (Anderson and Moore, 2007; Axehill, 2005; Nielsen, 2017).

The feedback controller defined in (4.17) and (4.18) can be deployed in practice using a *receding horizon control* strategy where the problem is solved over a moving time window. Here, a matrix Riccati partial differential equation (4.18), or a matrix Riccati recursion for the discrete-time case (Anderson and Moore, 2007; Axehill, 2005), has to be computed at each sampling instance.

If the matrices $A(x_r(t), u_r(t))$ and $B(x_r(t), u_r(t))$ are almost constant matrices, one single operating point (x_r, u_r) can be used to design the feedback gain. This motivates treating the LTV system as a *linear time-invariant* (LTI) system given by the constant system matrices $A = A(x_r, u_r)$ and $B = B(x_r, u_r)$. The time-invariant infinite-horizon LQ control problem is

$$\begin{aligned} & \underset{\bar{u}(\cdot)}{\text{minimize}} \quad \int_0^\infty (\|\bar{x}(t)\|_Q^2 + \|\bar{u}(t)\|_R^2) dt \\ & \text{subject to} \quad \dot{\bar{x}}(t) = A\bar{x}(t) + B\bar{u}(t), \\ & \quad \bar{x}(0) = \bar{x}_0 \text{ given.} \end{aligned} \tag{4.19}$$

Given that the pair (A, B) is controllable and $(A, Q^{1/2})$ is observable, this problem has an explicit solution and the optimal state-feedback controller is $\bar{u}(t) = K\bar{x}(t)$ where the feedback gain is

$$K = -R^{-1}B^\top P, \tag{4.20}$$

and $P \succ 0$ is the unique solution the *algebraic Riccati equation* (ARE)

$$A^\top P + PA - PBR^{-1}B^\top P + Q = 0. \tag{4.21}$$

The optimal cost for (4.19) is $V(\bar{x}(t)) = \bar{x}^\top(t)P\bar{x}(t)$ and the resulting closed-loop system $\dot{\bar{x}} = (A + BK)\bar{x}$ is exponentially stable (Anderson and Moore, 2007) where $V(\bar{x}(t))$ is a valid Lyapunov function candidate. Note that the solution P in (4.21) is often used as terminal cost P_N for the finite-horizon LQ control problem in (4.16) where a working point (x_r, u_r) is selected for its derivation (Anderson and Moore, 2007). The location of the poles to the closed-loop system are the negative eigenvalues to the *Hamiltonian matrix* (Anderson and Moore, 2007):

$$H = \begin{bmatrix} A & -BR^{-1}B^\top \\ -Q & -A^\top \end{bmatrix}. \tag{4.22}$$

The location of these poles provides information about the system's time constants and thus the required length of the prediction horizon T to obtain a good trade-off between tracking performance and computational complexity. Infinite-horizon LQ control is a popular control strategy due to its simplicity, robustness margins and stability guarantees (Anderson and Moore, 2007). However, if an *observer* is used to estimate the vehicle states needed for control, all guarantees are lost (Doyle,

1978). The solution to the LQ problem is optimal in the sense that it minimizes the specified cost function given that the model is correct. However, in practice model uncertainties and other disturbances are always present. H_∞ control (Helmersson, 1995; Ravi et al., 1991; Zhou et al., 1996) is a structured design approach for taking uncertainties and disturbances into account while synthesizing the controller. H_∞ control has been proposed for robust lateral control of different tractor-trailer combinations in, e.g., Kati et al. (2016, 2019a,b); Tai et al. (1999); Varga et al. (2018) to account for, e.g., uncertain moments of inertia and varying road-surface conditions during onroad driving.

4.2.4 Model predictive control

One of the most powerful tools for designing the feedback controller is to deploy online optimization techniques where an OCP is solved online at a specified sampling rate. For the previously presented control strategies, the physically imposed constraints on the vehicle states $x(t) \in \mathbb{X}$ and control input $u(t) \in \mathbb{U}$ are hard to take into account for during the control design. This can be motivated during moderate driving conditions where the nominal trajectory is feasible. However, if the motion planner uses a too simplistic model of the vehicle or if large disturbances are present, it is not certain that the actual motion plan is feasible to follow. This can for example be the case during slippery road surface conditions or during emergency maneuvers where vehicle models with higher fidelity have to be used (Rajamani, 2011). To handle such situations, more responsibility is commonly laid on the feedback controller to take care of both local replanning and feedback control (Paden et al., 2016; Svensson et al., 2019).

Model predictive control (MPC) (Garcia et al., 1989) is a powerful control strategy which can be very efficient for such scenarios. Similar to the finite-horizon LQ controller (4.16), an MPC controller uses a model of the vehicle to predict and optimize the vehicle's future state and control-input trajectories. Here, a nonlinear model of the vehicle could be used and the vehicle's physical constraints can be taken into account during the control design.

To obtain feedback properties, the idea with MPC is to only use the first portion of the control input and instead repetitively solve a receding horizon OCP at every time step Δt . Given the current vehicle state $x(t)$, the MPC control law $u(t)$ is computed by solving the *continuous-time nonlinear MPC problem*

$$\begin{aligned} & \underset{u(\cdot)}{\text{minimize}} \quad \Phi_T(\bar{x}(t+T)) + \int_t^{t+T} (||\bar{x}(\tau)||_Q^2 + ||\bar{u}(\tau)||_R^2) d\tau \quad (4.23) \\ & \text{subject to} \quad \frac{dx}{d\tau} = f(x(\tau), u(\tau)), \\ & \quad u(\tau) \in \mathbb{U}, \quad x(\tau) \in \mathbb{X}, \\ & \quad x(t) \text{ given.} \end{aligned}$$

Here, T denotes the prediction horizon, $\Phi_T(\bar{x}(t+T)) \geq 0$ is the terminal cost, and $Q \succeq 0$ and $R \succ 0$ are design matrices. The terminal cost is often chosen

as $\Phi_T(\bar{x}(t+T)) = \|\bar{x}(t+T)\|_P^2$, where $P \succeq 0$ is a design matrix. The matrix P is often selected as the solution to a continuous-time algebraic Riccati equation (4.21) where the nonlinear system is linearized around a working point (x_r, u_r) , e.g., a straight reference trajectory for a wheeled vehicle. The idea is that $\|\bar{x}(t+T)\|_P^2$ is used to approximate the remaining cost-to-go after the prediction horizon and thus emulate the infinite-horizon cost. An appropriate design of the terminal cost is especially important when the selection of T is small compared to the vehicle's time constants.

In contrast to the motion planning problem in (3.2), the obstacle imposed constraints are often neglected at the trajectory-tracking level and only the vehicle's physical limitations are considered. Instead, a quadratic penalty on the deviation from the nominal trajectory $(x_r(\cdot), u_r(\cdot))$ is used as performance measure.

To solve the continuous-time OCP in (4.23), it is first discretized to obtain an NLP. Since the vehicle model in general is nonlinear, the resulting NLP problem is non-convex and thus hard to solve to optimality. However, efficient method exists for real-time deployment of approximate solution to the resulting NLP problem. One example is linear MPC, where the NLP is simplified to a quadratic programming (QP) problem, e.g., by linearizing the nonlinear vehicle model around the nominal trajectory or path. Another example is real-time iterations (RTI) (Gros et al., 2020; Houska et al., 2011a) where instead of solving the discretized version of (4.23) to local optimality, only a few (commonly one) SQP iterations are performed at each sampling instance. The idea is that if several sampling instances are considered, an RTI scheme is basically resembling an SQP method. However, in RTI the current vehicle state can be updated as new information arrives and the preliminary solution can be used as control signal before the solver has completely converged. One example of open source software for RTI is the ACADO code generation toolkit (Houska et al., 2011a) which can be used to automatically generate C-code for efficient approximate nonlinear MPC. The generated code solves a sequence of quadratic programs using qpOASES (Ferreau et al., 2014) which is a parametric active-set solver with warm starts.

Similar to H_∞ control (Zhou et al., 1996), robust MPC (Löfberg, 2003; Paden et al., 2016; Pereira et al., 2018) is a robust control strategy to take model uncertainties and disturbances into account while designing the MPC controller.

Trajectory-tracking control using MPC was first utilized in the process industry to control slowly-varying systems (Rawlings, 2000). Advances in computing power as well as mathematical programming algorithms have made MPC feasible for real-time deployment in self-driving vehicles. Surveys of existing work for car-like vehicles can be found in, e.g., Lima (2018); Paden et al. (2016).

4.2.5 Trajectory-tracking control using LQ control: An example

A trajectory-tracking example for a kinematic bicycle model (2.11) is now presented. The state vector is $x(t) = [x_1(t) \ y_1(t) \ \theta_1(t)]^\top$, where (x_1, y_1) is the position for the center of the vehicle's rear axle and θ_1 is its orientation, see Figure 4.2. The control-input vector is $u(t) = [v_1(t) \ \kappa(t)]^\top$ where v_1 is the longitudinal velocity for the vehicle's rear axle and κ is its curvature. Assume a nominal trajectory

Figure 4.2: The kinematic bicycle model at state $x(t)$ and the nominal vehicle at state $x_r(t)$.

$\dot{x}_r(t) = f(x_r(t), u_r(t))$ is provided by a motion planner. Denote the tracking error as $\bar{x}(t) = x(t) - x_r(t)$ and the control-input deviation as $\bar{u}(t) = u(t) - u_r(t)$. Then, the tracking-error model can compactly be written as $\dot{\bar{x}}(t) = f_{cl}(t, \bar{x}(t), \bar{u}(t))$. Since the model is defined in a global coordinate system, it is not possible to design a static linear state-feedback controller $u(t) = u_r(t) + K\bar{x}(t)$ which stabilizes the tracking-error system unless a coordinate transformation, similar to Kanayama et al. (1990), is first performed. By the following change of basis

$$\begin{bmatrix} x_{1,e}(t) \\ y_{1,e}(t) \\ \theta_{1,e}(t) \end{bmatrix} = T(t, \bar{x}(t)) \bar{x}(t) = \begin{bmatrix} \cos \theta_1(t) & \sin \theta_1(t) & 0 \\ -\sin \theta_1(t) & \cos \theta_1(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_1(t) \\ \bar{y}_1(t) \\ \bar{\theta}_1(t) \end{bmatrix}, \quad (4.24)$$

the tracking-error system in the new coordinate frame is (see Appendix A)

$$\dot{x}_{1,e}(t) = y_{1,e}(t)v_1(t)\kappa(t) + v_1(t) - v_{1,r}(t)\cos\theta_{1,e}(t), \quad (4.25a)$$

$$\dot{y}_{1,e}(t) = -x_{1,e}(t)v_1(t)\kappa(t) + v_{1,r}(t)\sin\theta_{1,e}(t), \quad (4.25b)$$

$$\dot{\theta}_{1,e}(t) = v_1(t)\kappa(t) - v_{1,r}(t)\kappa_r(t). \quad (4.25c)$$

Define $x_e = [x_{1,e} \ y_{1,e} \ \theta_{1,e}]^\top$ and represent the tracking-error model in (4.25) as $\dot{x}_e = f_e(t, x_e, \bar{u})$. Clearly, $x_e(t) = 0$ if and only if $\bar{x}(t) = 0$ because $T(t, \bar{x})$ is nonsingular ($\det T(t, \bar{x}) = 1$). Since the origin $(x_e, \bar{u}) = (0, 0)$ to (4.25) is an equilibrium point, a first-order Taylor-series expansion around the origin yields

$$\dot{x}_e = \underbrace{\begin{bmatrix} 0 & v_{1,r}(t)\kappa_r(t) & 0 \\ -v_{1,r}(t)\kappa_r(t) & 0 & v_{1,r}(t) \\ 0 & 0 & 0 \end{bmatrix}}_{\triangleq A(t)} x_e(t) + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \kappa_r(t) & v_{1,r}(t) \end{bmatrix}}_{\triangleq B(t)} \bar{u}(t). \quad (4.26)$$

This system is an LTV system with time-varying matrices $A(t)$ and $B(t)$ that depend on the nominal velocity $v_{1,r}(t)$ and nominal curvature $\kappa_r(t)$ trajectories. This system can also be viewed as a linear parameter-varying (LPV) system where

Figure 4.3: Trajectory tracking for a kinematic bicycle model with initial tracking error $x_e(0) = [-1 \text{ m } 1 \text{ m } 0.2 \text{ rad}]^\top$. The controlled vehicle is plotted in blue and the nominal vehicle in red. The vehicles are plotted at $t = 0, 7$ and 14 s . The red dotted path displays the trajectory for the controlled vehicle's position and the blue path is its nominal trajectory.

$v_{1,r}(t)$ and $\kappa_r(t)$ are the time-varying parameters. As long as $v_{1,r} \neq 0$, the system is controllable but it can be difficult to stabilize the vehicle over a wide range of nominal curvature and velocity profiles. Some approaches to handle this include finite-horizon LQ control (4.16) or gain-scheduling techniques (4.15). As a simple example, a single infinite-horizon LQ controller (4.19) is designed around the working point $(v_{1,r}, \kappa_r) = (1, 0)$. With the weight matrices $Q = \text{diag}([1, 1, 10])$ and $R = \text{diag}([10, 40])$, the LQ controller is given by

$$u(t) = u_r(t) + \underbrace{\begin{bmatrix} -0.3162 & 0 & 0 \\ 0 & -0.1581 & -0.7525 \end{bmatrix}}_{\triangleq K} x_e(t). \quad (4.27)$$

From the structure of K it is clear that the longitudinal error $x_{1,e}$ is controlled by $v_1(t)$ and the lateral $y_{1,e}$ and angular $\theta_{1,e}$ errors are controlled by $\kappa(t)$. This is, however, not the case if the system is linearized around a working point $\kappa_r \neq 0$. To evaluate the controller's performance, a nominal trajectory is generated by simulating the vehicle using a sinusoidal velocity and curvature profile

$$v_{1,r}(t) = 1 + A_v \sin(w_v t), \quad (4.28)$$

$$\kappa_r(t) = A_\kappa \sin(w_\kappa t), \quad (4.29)$$

with $w_v = w_\kappa = 0.1 \text{ rad/s}$, $A_v = 0.5 \text{ m/s}$ and $A_\kappa = 0.1 \text{ m}^{-1}$. To perform more realistic simulations, additional first-order systems for the curvature and the longitudinal velocity with time constants $T_\kappa = 0.2 \text{ s}$ and $T_v = 0.5 \text{ s}$, respectively, are included. The closed-loop simulation is depicted in Figure 4.3, where an initial tracking error $x_e(0) = [-1 \text{ m } 1 \text{ m } 0.2 \text{ rad}]^\top$ is used to analyze how the feedback controller handles disturbance rejection. As can be seen from Figure 4.3, the tracking errors converge towards zero. In Figure 4.4, the control signals are plotted together with their nominal trajectories. The results illustrate that the feedback part handles disturbance rejection and the feedforward part is responsible for tracking.

Figure 4.4: The control signals (blue line) and their feedforwards (dashed red) during the trajectory-tracking example.

4.3 Path-following control

In self-driving vehicle applications, it is often convenient to separate the control problem into lateral and longitudinal control (Paden et al., 2016). The lateral controller is called a path-following controller and its objective is to make sure that a nominal path $(x_r(s), u_r(s))$, $s \in [0, s_G]$ obtained from the motion planner is followed with a small and bounded path-following error. After initial transients have been eliminated, the longitudinal controller is designed such that the motion plan is executed at a desired velocity, i.e., $\dot{s}(t) = v_{1,r}(t)$. In the sequel, it is assumed that the system is a wheeled vehicle³ represented by a state vector $x(t)$ that can be partitioned as $x = [p^\top \ \theta_1^\top \ \eta^\top]^\top$, where $p(t) = (x_1(t), y_1(t))$ denotes the position of the vehicle, $\theta_1(t)$ its orientation and $\eta(t)$ represents additional vehicle states, e.g., N joint angles for the general N-trailer vehicle (2.24). Moreover, the position $p(t)$ of the vehicle is assumed to have a velocity vector $v_1(t)$ that is aligned with its orientation $\theta_1(t)$, i.e., the position of the vehicle evolves as

$$\dot{p}(t) = v_1(t) \begin{bmatrix} \cos \theta_1(t) \\ \sin \theta_1(t) \end{bmatrix}. \quad (4.30)$$

Examples of systems that satisfy these assumptions are the kinematic bicycle model (2.11) and the general N-trailer vehicle (2.24).

The separation between lateral and longitudinal control is naturally performed by representing the vehicle in a path-coordinate system called the *Frenet-Serret frame* (Bishop, 1975). The Frenet-Serret frame is a coordinate system that moves along with the controlled vehicle. It is used to represent the position of the vehicle $p(t)$ in terms of its progression $s(t)$ along the path in $p_r(s)$ and its signed lateral distance $\tilde{z}(t)$ with respect to $p_r(s(t))$, see Figure 4.5. Here, $s(t)$ is defined as the projection of the vehicle's position $p(t)$ with respect to its nominal path $p_r(\cdot)$ at time t :

$$s(t) = \arg \min_{\tau \in [0, s_G]} \|p(t) - p_r(\tau)\|_2. \quad (4.31)$$

³Note that path following can be generalized to the 3D-case as well (Kaminer et al., 2006; Sujit et al., 2014).

Figure 4.5: A wheel vehicle in the Frenet-Serret frame where the red-dashed line represents the nominal path in $(x_{1,r}(\cdot), y_{1,r}(\cdot))$. The black vehicle is the controlled vehicle and the gray vehicle is the nominal vehicle at $s(t)$.

Note that at $s(t)$, there exists a nominal vehicle state $x_r(s(t))$ and a nominal control input $u_r(s(t))$. Define the curvature $\kappa_r(s)$ of the nominal path in $p_r(s)$ as

$$\kappa_r(s) = \frac{d\theta_{1,r}(s)}{ds}, \quad (4.32)$$

which is assumed bounded for all $s \in [0, s_G]$. In the Frenet-Serret frame, the position of the vehicle $(x_1(t), y_1(t))$ is transformed to $(s(t), z(t))$ and its model is given by (Luca et al., 1998):

$$\dot{s}(t) = v_1 \frac{\cos \tilde{\theta}_1}{1 - \kappa_r(s) \tilde{z}}, \quad (4.33a)$$

$$\dot{\tilde{z}}(t) = v_1 \sin \tilde{\theta}_1, \quad (4.33b)$$

where $\tilde{\theta}_1(t) = \theta_1(t) - \theta_{1,r}(s(t))$ denotes the orientation error of the vehicle. The transformation to the Frenet-Serret frame is defined in a tube around the nominal path in $p_r(s)$ for which $1 - \kappa_r(s) \tilde{z} > 0$. The width of this tube depends on the nominal curvature $\kappa_r(s)$ and when the curvature tends to zero (a straight line) $\tilde{z}$ can vary arbitrarily. Essentially, this coordinate transformation is well-defined if $|\tilde{z}| < |\kappa_r^{-1}|$ when $\tilde{z}$ and κ_r have the same sign. Moreover, to guarantee $\dot{s}$ to be a monotonic function in time, the orientation error $\tilde{\theta}_1$ is assumed to satisfy $|\tilde{\theta}_1| < \pi/2$. With this assumption, $\dot{s} > 0$ along forward motion segments and $\dot{s} < 0$ along backward motion segments. Differentiating the orientation error $\tilde{\theta}_1(t) = \theta_1(t) - \theta_{1,r}(s(t))$ with respect to time yields

$$\dot{\tilde{\theta}}_1(t) = \dot{\theta}_1(t) + \dot{s}(t) \kappa_r(s(t)), \quad (4.34)$$

where the chain-rule has been applied and $\kappa_r(s(t))$ is defined in (4.32). The additional error states are defined as $\tilde{\eta}(t) = \eta(t) - \eta_r(s(t))$ and their corresponding

differential equations $\dot{\eta}(t)$ can be obtained in analogy to (4.34). Define the path-following error state vector as $\tilde{x} = [\tilde{z} \ \tilde{\theta}_1 \ \tilde{\eta}^\top]^\top$ and the control-input deviation as $\tilde{u}(t) = u(t) - u_r(s(t))$, then the wheeled vehicle can compactly be described in the path-coordinate system as

$$\dot{s}(t) = f_s(s(t), \tilde{x}(t), \tilde{u}(t)), \quad (4.35a)$$

$$\dot{\tilde{x}}(t) = f_e(s(t), \tilde{x}(t), \tilde{u}(t)). \quad (4.35b)$$

The origin $(\tilde{x}(t), \tilde{u}(t)) = (0, 0)$ to the path-following error model in (4.35b) is an equilibrium point since $f_e(s(t), 0, 0) = 0$ for all $s(t) \in [0, s_G]$. The model in (4.35a) describes the progression of the vehicle along the nominal path, and at the origin, $\dot{s} = v_1(t)$.

The path-following control problem can now be formulated as follows (Paden et al., 2016): Design the feedback controller $\tilde{u}(t) = g(s(t), \tilde{x}(t))$, with $g(s(t), 0) = 0$, such that the origin to the closed-loop path-following error system

$$\dot{\tilde{x}}(t) = f_e(s(t), \tilde{x}(t), g(s(t), \tilde{x}(t))) = \tilde{f}_{cl}(s(t), \tilde{x}(t)), \quad (4.36)$$

is, *e.g.*, exponentially stable and such that the nominal path is executed at a desired set-speed $v_{1,r}(t)$. With minor adjustments, the model in (4.36) can now be stabilized using the feedback-control techniques presented in Section 4.2.

A path-following controller can easily be converted into a trajectory-tracking controller by letting the longitudinal velocity of the vehicle control the progression $s(t)$ such that a nominal trajectory in $s_r(t)$ is tracked. As an example, define the progression error as $\tilde{s}(t) = s(t) - s_r(t)$. Since $\dot{s}_r(t) = v_{1,r}(t)$, the longitudinal error model becomes

$$\dot{\tilde{s}}(t) = \dot{s}(t) - \dot{s}_r(t) = v_1 \frac{\cos \tilde{\theta}_1}{1 - \kappa_r(s) \tilde{z}} - v_{1,r}. \quad (4.37)$$

If $v_1(t)$ is a control signal, the input substitution $\mu(t) = v_1 \frac{\cos \tilde{\theta}_1}{1 - \kappa_r(s) \tilde{z}} - v_{1,r}$ transforms the longitudinal error model into a single integrator system $\dot{\tilde{s}}(t) = \mu(t)$ where, *e.g.*, a proportional-integral-derivative (PID) controller can be designed to stabilize the longitudinal error model around $\tilde{s}(t) = 0$.

4.3.1 Pure-pursuit controller

An early proposed solution to the path-following control problem is the so-called *pure-pursuit controller* (Coulter, 1992; Wallace et al., 1985). This controller uses a *lookahead distance* R to find its *lookahead point* $P(t) = (x_p(t), y_p(t))$ on the reference path in $p_r(s)$. This point can be computed by finding the intersection between the reference path and a semi-circle with radius R that is centered at the vehicle's current position $(x_1(t), y_1(t))$, as illustrated in Figure 4.6. The pure-pursuit controller computes a desired curvature $\kappa(t)$ of the vehicle such that the lookahead point $P(t)$ is reached from the vehicle's current position $(x_1(t), y_1(t))$. However, since the lookahead point moves along with the vehicle, a smooth motion towards the path is obtained. The pure-pursuit controller is schematically illustrated in

Figure 4.6: Geometric illustration of the pure-pursuit controller. The controller uses a look-ahead distance R to compute the lookahead point P on the path where a feedback law is derived based on the look-ahead distance and the orientation error θ_e .

Figure 4.6 and the derivation of the feedback-law can be found in, e.g., Coulter (1992); Kuwata et al. (2009); Ljungqvist (2015). Define $\theta_e(t)$ as the orientation error of the vehicle with respect to the lookahead point $P(t)$. In forward motion, $\theta_{e,\text{fwd}}(t)$ can be computed as

$$\theta_{e,\text{fwd}}(t) = \theta_1(t) - \text{atan2}(y_p(t) - y_1(t), x_p(t) - x_1(t)), \quad (4.38)$$

where $\text{atan2}(\Delta y, \Delta x)$ denotes the generalization of $\arctan(\Delta y / \Delta x)$ over all four quadrants. Using basic geometry, the pure-pursuit feedback law in forward motion is (Kuwata et al., 2009):

$$\kappa(t) = -\frac{2\sin\theta_{e,\text{fwd}}(t)}{R}, \quad (4.39)$$

which for the kinematic bicycle model (2.11) relates to the steering angle as $\alpha(t) = \arctan(L_1\kappa(t))$. In backward motion, the semi-circle is flipped and if the orientation error $\theta_{e,\text{rev}}(t)$ is defined as follows

$$\theta_{e,\text{rev}}(t) = \text{atan2}(y_1(t) - y_p(t), x_1(t) - x_p(t)) - \theta_1(t), \quad (4.40)$$

the same control-law as in (4.39) can be used in backward motion with $\theta_{e,\text{fwd}}(t)$ replaced by $\theta_{e,\text{rev}}(t)$. The design parameter for the pure-pursuit controller is the lookahead distance R and the feedback law becomes less aggressive with increasing value of R (Kuwata et al., 2009). However, its tracking performance degrades with increasing value of R . Commonly, the lookahead distance is parametrized

with respect to the vehicle's current speed to trade-off between smoothness during high speeds (large R) and improved tracking performance during low speeds (small R) (Kuwata et al., 2009).

The pure-pursuit controller has been widely used in practice due to its simplicity and relatively satisfactory performance. For instance, two vehicles in the DARPA Grand Challenge (Buehler et al., 2007) and three vehicles in the DARPA Urban Challenge (Buehler et al., 2009) reported using a pure-pursuit controller for path-following control. One of its strengths is that the reference path does not necessarily need to be feasible to follow. However, the controller will never follow the reference path with zero path-following error unless the reference path is a straight line.

4.3.2 Nonlinear path-following techniques

Path-following control for nonholonomic wheeled vehicles has received a lot of attention from the automatic control and robotic communities over the last decades. A vast amount of advanced nonlinear control techniques have been proposed where the natural approach of modeling the vehicle in the Frenet-Serret frame (Bishop, 1975) is usually explored (Altafini, 2003; David and Manivannan, 2014; Fliess et al., 1995; Lima, 2018; Paden et al., 2016). In recent years, path following using MPC has started to be widely used in practice for cars (Cairano et al., 2016; Falcone et al., 2007; Plessen et al., 2018) and trucks (Lima et al., 2017a) and a survey of path-following techniques for such systems can be found in Paden et al. (2016). In Paper E and Paper G, path-following control approaches using MPC for different tractor-trailer vehicles are proposed.

For tractor-trailer vehicles with pure on-axle hitched trailers (standard N-trailer), its flatness property can be explored to design path-following controllers using feedback linearization (Fliess et al., 1995; Rouchon et al., 1993). However, for tractor-trailer systems with a mixture of off-axle and on-axle hitching (general N-trailer), feedback linearization is not applicable (Astolfi et al., 2004; Rouchon et al., 1993). To circumvent this problem, approximate solutions have been proposed (Bolzern et al., 1998), where a simplified reference vehicle with similar stationary but different transient behavior is used for control design.

Classical path-following techniques for the general 1-trailer with a car-like tractor involve input-output linearization (Werling et al., 2014) and Lyapunov-based control design (Astolfi et al., 2004). The proposed path-following controller in Werling et al. (2014) is evaluated in field experiments for reversing a car with a trailer. Path following in forward motion for the general N-trailer using input-output linearization is proposed in Altafini (2003). The proposed solution is however limited to forward motion since the introduced zero-dynamics is unstable in backward motion.

Path-following approaches in backward and forward motion for N-trailer vehicles with pure off-axle hitched trailers (nonstandard N-trailer) using nonlinear cascade-like control methods are proposed in Michałek (2013a, 2014, 2017). In those works, the path is assumed to be represented as an analytical equation in the nominal position of the last trailer. Compared to many other path-following

control approaches, these methods do not need to find the closest distance to the path and the complexity of the feedback controllers scales well with increasing number of trailers. By introducing artificial off-axle hitches, the proposed approaches can also be used for the GNT-case (Michalek, 2013a). However, the control approaches become sensitive to measurement noise when the distance of any signed off-axle hitch connection is close to zero.

Due to the complex kinematics of the general N-trailer, many path-following solutions rely on Jacobian linearization and linear control techniques (Altafini et al., 2001; Divelbiss and Wen, 1997; Hoel and Falcone, 2013). A similar approach for path following is proposed in Rimmer and Cebon (2017) for reversing a general 2-trailer with a car-like tractor, which has been successfully demonstrated in practice. A historic survey of path-following techniques for tractor-trailer vehicles can be found in David and Manivannan (2014).

4.3.3 Path-following control using LQ control: An example

To conclude this chapter, a path-following example for a kinematic bicycle model is presented. The model of the vehicle is given in (2.11) where the state vector is $x(t) = [x_1(t) \ y_1(t) \ \theta_1(t)]^\top$, and the control input is $u(t) = [v_1(t) \ \kappa(t)]^\top$. Here, $v_1(t)$ is the longitudinal velocity of the vehicle's rear axle and $\kappa(t) = \tan \alpha(t)/L_1$ is its curvature. The wheelbase of the vehicle is L_1 and $\alpha(t)$ is its steering angle. The kinematic bicycle model perfectly fits into the path-following framework presented in Section 4.3. The progression $s(t)$ is defined as the location of the vehicle's rear axle $(x_1(t), y_1(t))$ onto its projection to its nominal path in $(x_{1,r}(\cdot), y_{1,r}(\cdot))$ at time t , see Figure 4.7. The orientation of the vehicle evolves as $\dot{\theta}_1(t) = v_1(t)\kappa(t)$ and by inserting this in (4.34), the complete model of the vehicle in the Frenet-Serret frame (4.35) is

$$\dot{s} = v_1 \frac{\cos \tilde{\theta}_1}{1 - \kappa_r(s)\tilde{z}}, \quad (4.41a)$$

$$\dot{\tilde{z}} = v_1 \sin \tilde{\theta}_1, \quad (4.41b)$$

$$\dot{\tilde{\theta}}_1 = v_1 \left(\kappa - \frac{\kappa_r(s) \cos \tilde{\theta}_1}{1 - \kappa_r(s)\tilde{z}} \right). \quad (4.41c)$$

The longitudinal model is represented by (4.41a) and the path-following error model by (4.41b)–(4.41c). For simplicity, we only consider forward motion and assume that $v_{1,r}(t) > 0$ and $v_1(t) > 0$. In this setup, the path-following error is $\tilde{x} = [\tilde{z} \ \tilde{\theta}_1]^\top$. Denote the controlled curvature deviation as $\tilde{\kappa} = \kappa - \kappa_r$ and the path-following error model in (4.41b)–(4.41c) can in a compact form be written as

$$\dot{\tilde{x}} = v_1 f_e(s(t), \tilde{x}, \tilde{\kappa}), \quad (4.42)$$

where the origin $(\tilde{x}, \tilde{\kappa}) = (0, 0)$ is an equilibrium point. Since $v_1(t)$ enters bilinearly in (4.42), in analogy with Section 2.3, time-scaling (Sampei and Furuta, 1986) can be applied to eliminate the speed dependence $|v_1(t)|$ from the model. Thus, during the design of the path-following controller it is, without loss of generality,

Figure 4.7: The kinematic bicycle model in the Frenet-Serret frame where the red-dashed line represents the nominal path in $(x_{1,r}(s), y_{1,r}(s))$. The black vehicle illustrates the controlled system and the gray vehicle is the reference vehicle at $s(t)$.

assumed⁴ that the longitudinal velocity $v_1(t)$ of the vehicle only takes on the value $v_1(t) = 1$ m/s.

The design of the path-following controller is now equivalent to designing a stabilizing feedback controller $\kappa(t) = g(s(t), \tilde{x}(t))$ for the nonlinear system in (4.42) and any method presented in Section 4.2 can now be used with minor adjustments. As an example, we design the path-following controller using robust linear control techniques. Since the origin to (4.42) is an equilibrium point, the Jacobian linearization is

$$\dot{\tilde{x}}(t) = A(s(t))\tilde{x} + B\tilde{\kappa}, \quad (4.43)$$

where

$$A(s(t)) = \left. \frac{\partial f_e}{\partial \tilde{x}} \right|_{(0,0)} = \begin{bmatrix} 0 & 1 \\ -\kappa_r^2(s(t)) & 0 \end{bmatrix}, \quad B = \left. \frac{\partial f_e}{\partial \tilde{\kappa}} \right|_{(0,0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (4.44)$$

The objective is to design a static state-feedback controller with feedforward action $\kappa(t) = \kappa_r(s(t)) + K\tilde{x}(t)$ such that the closed-loop system

$$\dot{\tilde{x}}(t) = (A(s(t)) + BK)\tilde{x}(t), \quad (4.45)$$

is exponentially stable. Assume the nominal path is constructed such that the nominal curvature is bounded as $|\kappa_r(s(t))| \leq \kappa_{\max}$. Then, similar to (4.12), we can describe the linearized path-following error model in (4.43) as a polytopic LDI. The matrix B is a constant and the matrix $A(s(t))$ can be contained in a polytope (4.11) with 2 vertices

$$A(s(t)) \in \mathbb{P} = \text{Co}\{A_1, A_2\}, \quad (4.46)$$

⁴Note that in practice, this assumption is only feasible for low-speed maneuvers since the model is derived based on no-slip conditions and rate-limitations on the steering angle are neglected.

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -\kappa_{max}^2 & 0 \end{bmatrix}. \quad (4.47)$$

Similarly to (4.14), with a constant decay rate $\epsilon > 0$, the LMI feasibility problem to find a stabilizing feedback gain $K = YS^{-1}$ and a quadratic Lyapunov function $V(\tilde{x}) = \tilde{x}^\top S^{-1} \tilde{x}$ with $S^{-1} \succ 0$ is

$$SA_i^\top + Y^\top B^\top + A_i S + BY + 2\epsilon S \preceq 0, \quad i = 1, 2. \quad (4.48)$$

It is desired that the feedback gain K inherits properties of an infinite-horizon LQ controller. A nominal LQ controller is thus designed around a straight reference path ($\kappa_r(s) = 0$) in forward motion using the weight matrices $R = 1$ and $Q = \frac{1}{20} \text{diag}([1, 1])$. The resulting feedback gain is $K_{LQ} = -[0.22 \quad 0.70]$. This controller is not guaranteed to stabilize the LPV system in (4.43) for all nominal curvatures. To establish stability guarantees, we can minimize the difference between K and K_{LQ} subject to the LMI in (4.48). Since $S \succ 0$, this can be posed as an SDP problem⁵ (Ljungqvist et al., 2018)

$$\begin{aligned} & \underset{Y, S}{\text{minimize}} \quad \|Y - K_{LQ}S\| \\ & \text{subject to} \quad (4.48) \text{ and } S \succ I. \end{aligned} \quad (4.49)$$

The SDP problem in (4.49) is solved using YALMIP (Löfberg, 2004) together with the SDP solver MOSEK (MOSEK, 2019). By selecting $\epsilon = 0.2$ and $\kappa_{max} = 0.3 \text{ m}^{-1}$, the optimal objective function value to (4.49) is zero. This implies that $K = K_{LQ}$ and one quadratic Lyapunov matrix $P = S^{-1} \succ 0$ is

$$P = \begin{bmatrix} 0.16 & 0.17 \\ 0.17 & 0.63 \end{bmatrix}, \quad (4.50)$$

that guarantees that the closed-loop LTV system $\tilde{x}(t) = (A(s(t)) + BK)\tilde{x}(t)$ is exponentially stable as long as the nominal curvature is bounded as $|\kappa_r(s(t))| \leq \kappa_{max}$. Moreover, with this feedback-law $\tilde{\kappa}(t) = K\tilde{x}(t)$, the origin of the closed-loop version of the path-following error system in (4.42) is an exponentially stable equilibrium point when $v_1(t) > 0$.

To illustrate how the path-following controller handles disturbance rejection, a simulation of the nonlinear closed-loop system is performed. The nominal path is generated using a constant longitudinal velocity profile $v_{1,r}(s) = 1$ and a sinusoidal curvature profile $\kappa_r(s) = 0.2 \sin(0.1s)$. Figure 4.8 provides simulation results with initial path-following error $\tilde{x}(0) = [\tilde{z}(0) \quad \tilde{\theta}_1(0)]^\top = [1 \text{ m} \quad 0.2 \text{ rad}]^\top$. As can be seen, the initial lateral and angular path-following errors are eliminated after roughly 15 s. The value of the quadratic Lyapunov function $V(\tilde{x}(t)) = \tilde{x}^\top(t)P\tilde{x}(t)$ for the simulation is provided in Figure 4.9. The result shows that the closed-loop system is exponentially stable. Note that a similar control design can be performed for backward motion by simply changing the sign of the matrices $A(s(t))$ and B in (4.43). However, it is not possible to design the linear state-feedback controllers such that a common quadratic Lyapunov function exists in forward and backward motion. This will be shown in Paper C.

⁵If $\|K - K_{LQ}\| = \|YS^{-1} - K_{LQ}\| = \|(Y - K_{LQ}S)S^{-1}\| = 0$, then $K = K_{LQ}$ since $S \succ 0$.

(a) The nominal path (blue line) and the motion of the controlled vehicle (red-dashed line).

(b) Feedback-control signal $\tilde{\kappa}(t) = K\tilde{x}(t)$.

(c) The orientation error $\tilde{\theta}_1(t)$.

(d) The lateral error $\tilde{z}(t)$.

Figure 4.8: Simulation results for the closed-loop system using a path-following controller designed using LQ control.

Figure 4.9: The quadratic Lyapunov function $V(\tilde{x}(t)) = \tilde{x}^\top(t)P\tilde{x}(t)$ during the simulation.

5

Concluding remarks

In this chapter, a summary of the contributions of this thesis is given in more technical terms and some directions for future work are discussed. The first part of this thesis gave an introduction to nonholonomic systems and some common motion planning and feedback control techniques for such systems. These techniques have been combined and modified for several practical problems and the results are presented in the papers that constitute the second part of this thesis.

5.1 Summary of contributions

The contributions of this thesis are within the area of motion planning and feedback control for different kinds of tractor-trailer combinations during low-speed maneuvers in unstructured environments. The proposed motion planning techniques include probabilistic and deterministic sampling-based approaches. Furthermore, numerical optimal control techniques are used offline to precompute libraries of optimized maneuvers as well as during online planning as a warm-started optimization step. To follow the computed motion plans, different path-following control approaches are proposed that use a path-following error model of the vehicle. Several of the proposed approaches have been successfully demonstrated on a small-scale or on a full-scale general 2-trailer with a car-like tractor. Furthermore, the first complete motion planning and control framework for this type of system is presented which is also accompanied with the first successfully results from real-world experiments. In this framework, a novel nonlinear observer for state estimation is evaluated which only utilizes information from sensors mounted on the tractor, making the framework independent of any sensor mounted on the trailer.

In Paper A, a probabilistic motion planner for a general 2-trailer with a car-like tractor is proposed. The cascade-like path-following controller presented in Evestedt et al. (2016a); Ljungqvist (2015) is used to enable efficient closed-loop simu-

lations of the vehicle within a CL-RRT framework. The framework is evaluated in a series of Monte-Carlo runs and is shown to have a high success rate in finding motion plans in complex and constrained environments. Experiments on a small-scale test platform is also presented.

In Paper B, a path-following controller for a reversing general 2-trailer with a car-like tractor is proposed for the case when the nominal path contains full state and control-input information. A path-following error model of the vehicle is derived and a stabilizing LQ controller with feedforward action is proposed. Given that the nominal path satisfies certain properties, exponential stability of the closed-loop system is established by combining global optimization, theory from LDIs, and LMI techniques. The theoretical results are verified through simulations of the closed-loop system around a figure-eight shaped reference path.

In Paper C, a systematic framework is proposed for analyzing stability of the closed-loop system consisting of a controlled vehicle and a feedback controller, executing a motion plan computed by a lattice-based planner. When this motion planner is considered, it is shown that the closed-loop system can be modeled as a time-switched nonlinear hybrid system. Based on this, a method is proposed for analyzing the behavior of the tracking error, how to design the low-level controller and how to potentially impose restrictions on the motion planner to guarantee that the tracking error is bounded and decays towards zero. The synthesis and analysis tools are based on LMI techniques. The proposed method is applied on a general 2-trailer with a car-like tractor and the results are illustrated in simulations.

In Paper D, a path-planning and path-following control framework for a general 2-trailer with a car-like tractor is presented. A lattice-based path planner is developed that uses a library of optimized maneuvers together with graph search to compute a path to the goal. To follow the planned path, the LQ controller in Paper B is extended and used in practice. To estimate the vehicle states needed for control, a nonlinear observer is developed which only utilizes information from a rear-view LIDAR sensor mounted on the tractor, making the system independent of any sensor mounted on the semitrailer or the dolly. The proposed framework is implemented on a full-scale test vehicle and results from simulations and field experiments are presented.

In Paper E, an estimation-aware MPC controller for a general 2-trailer with a car-like tractor is proposed. The approach is driven and inspired by the work in Paper D where an advanced sensor with a limited field of view is placed in the rear of the tractor to solve the joint-angle estimation problem. To model and explicitly consider the sensor's sensing limitations in the controller, a model predictive path-following control approach is proposed. Two approaches with different computation complexity and performance are presented. In the first approach, the constraint on the joint angles is modeled as a union of convex polytopes, making it necessary to incorporate binary decision variables in the MPC controller. The second approach, which is a special case of the former, avoids binary variables at the expense of a more restrictive approximation of the joint-angle constraints. The performance of the proposed path-following control approach is evaluated in simulations as well as in field experiments on a full-scale test vehicle.

In Paper F, a two-step optimization-based trajectory planner is proposed for

MSNT vehicles operating at low-speeds. The proposed trajectory planner uses a lattice-based trajectory planner in a first step to compute a resolution optimal solution to the trajectory planning problem. The output from the lattice-based planner is then used to warm-start a second numerical optimal control step, which enables the framework to compute locally optimal trajectories that start at the vehicle's initial state and reaches the goal state exactly. The performance of the proposed optimization-based trajectory planner is evaluated in a set of parking scenarios for a MS3T vehicle with a car-like tractor where the last trailer is steerable.

Finally, in Paper G, a model predictive path-following controller is proposed for automatic low-speed control of MSNT vehicles. The proposed MPC controller uses a linear path-following error model of the vehicle for predictions and is designed to satisfy various constraints on the vehicle states as well as saturations and rate limitations on the tractor's curvature and the trailer steering angles. The performance of the proposed model predictive path-following controller is evaluated in a set of simulations for a MS2T vehicle with a car-like tractor where the last trailer is steerable.

5.2 Future work

In this section, some ideas for future work and possible extensions of the contributions that are presented in this thesis are summarized.

Motion planning and feedback control

The CL-RRT algorithm developed for a general 2-trailer with a car-like tractor in Paper A is possible to generalize to the general N-trailer and the MSNT cases. It would also be interesting to benchmark its performance, as well as the lattice-based approaches, with previously presented motion planning strategies for standard N-trailer vehicles, e.g., Laumond et al. (1994); Sekhavat et al. (1998).

To increase the computational efficiency of the two-step optimization-based motion planner in Paper F, it is possible to run the optimization step in receding horizon as proposed in Bergman et al. (2020a). Using this technique, it would be interesting to let the optimization step perform local adjustments of the motion plan to become reactive against, e.g., inaccurate predictions of moving obstacles or if previously unseen obstacles are detected by the perception layer (Batkovic et al., 2019, 2020). In such a reactive setup, it will be important to understand and analyze which layer, i.e., sampling-based planner or optimization step, that is responsible for modifying the motion plan to obtain guarantees in terms of feasibility and convergence to the goal. It would also be interesting to investigate if the optimization-based motion planning concept can be applied on other systems, such as drones (Andersson et al., 2018b) and ships (Bergman et al., 2020b).

The estimation-aware control concept presented in Paper E would be interesting to extend to other systems as well as other scenarios where onboard or external sensors with similar sensing limitations are used to enhance the performance of the vehicle's estimation and perception layers. As an example, one practical scenario is reversing a tractor-trailer vehicle to a loading bay where, e.g., external sensors

placed on the loading bay can be used to improve the performance of the estimation and control modules. These sensing limitations can also be incorporated in the motion planner either as constraints or using techniques from informative path planning and sensor management (Boström-Rost, 2019) to, e.g., optimize the system's future perception of the surrounding environment.

Onroad motion planning for tractor-trailer vehicles is also an interesting and natural research direction to pursue. In forward motion, the joint-angle kinematics are stable and do not necessarily need to be stabilized. However, during evasive maneuvers, a dynamic vehicle model needs to be used in order to capture dynamic effects and to compute motion plans that are guaranteed to be feasible to follow. As long as the system can be simulated, it would be possible to develop RRT frameworks or lattice-based frameworks using input-sampling for this task. Another possibility is to leave more responsibility to the feedback controller to perform both local planning and control. Other interesting aspects related to on-road motion planning for tractor-trailer vehicles are interaction-aware trajectory planning (Evestedt, 2016; Ward et al., 2017), fall-back solutions using safe-stop trajectory planning (Svensson et al., 2018), as well as methods to incorporate human-like driving behaviors (Rosbach et al., 2019) and traffic rules (Tumova et al., 2013).

Robust and safe trajectory planning (Althoff and Dolan, 2014; Majumdar and Tedrake, 2017; Singh et al., 2018), lateral control of tractor-trailer vehicles at high speeds and during at the limit-handling (Anistratov et al., 2018; Berntorp et al., 2014; Fors et al., 2019; Pereira et al., 2018; Svensson et al., 2019) are also interesting future research directions. Example of disturbances that the system need to be robust against include parameter uncertainties, strong winds and slippery road surface conditions. In a lattice-based motion planning framework, it would be interesting to compute several libraries of maneuvers that are robust against different values of the disturbances. During online planning, the current estimate of the disturbances can then be used to select the most appropriate library. Compared to computing only one library for any type of disturbance as proposed in Majumdar and Tedrake (2017), the uncertainties of slowly-varying disturbances can be reduced. As a result, the motion planner will become less conservative without increasing the risk of inducing instability or collision with surrounding obstacles.

Learning and system identification

The incorporation of learning in a motion planning and control architecture is also an interesting direction of future research. If similar motion plans are used multiple times, it would be possible to evaluate and adapt the controller and/or the motion plans in order to enhance the performance of the closed-loop system (Rosolia and Borrelli, 2017). These ideas have strong similarities with iterative learning control (ILC) (Bristow et al., 2006; Gunnarsson and Norrlöf, 2001) and learning MPC (Rosolia and Borrelli, 2017; Rosolia et al., 2017) tailored for repetitive tasks.

It would also be interesting to perform online estimation of important vehicle parameters, such as road friction coefficients and side-slip angles (Morrison and Cebon, 2016; Svensson et al., 2019), moment of inertia (Cheng and Cebon,

2011) and vehicle lengths (Arnström, 2018). Reinforcement learning approaches are also interesting directions for future work where, e.g., cost functions, motion primitives and heuristics used in a motion planner could be learned from an expert driver (Rosbach et al., 2019).

Appendix

Derivation of the tracking-error system

In this section, the derivation of the tracking-error system in (4.25) is presented. The kinematic bicycle model in a global coordinate frame is given by

$$\dot{x}(t) = v_1(t) \begin{bmatrix} \cos \theta_1(t) \\ \sin \theta_1(t) \\ \kappa(t) \end{bmatrix}, \quad (\text{A.1})$$

where $x(t) = [x_1(t) \ y_1(t) \ \theta_1(t)]^\top$ is the state vector and $u(t) = [v_1(t) \ \kappa(t)]^\top$ is the control-input vector. Represent the model in (A.1) as $\dot{x}(t) = f(x(t), u(t))$. The nominal trajectory is feasible and thus satisfies $\dot{x}_r(t) = f(x_r(t), u_r(t))$. Denote the tracking error as $\bar{x}(t) = x(t) - x_r(t)$. By taking the time derivative of the proposed state transformation

$$\begin{bmatrix} x_{1,e}(t) \\ y_{1,e}(t) \\ \theta_{1,e}(t) \end{bmatrix} = T(t, \bar{x}(t)) \bar{x}(t) = \begin{bmatrix} \cos \theta_1(t) & \sin \theta_1(t) & 0 \\ -\sin \theta_1(t) & \cos \theta_1(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_1(t) \\ \bar{y}_1(t) \\ \bar{\theta}_1(t) \end{bmatrix}, \quad (\text{A.2})$$

and using the relations $\dot{x}_1 \sin \theta_1 = \dot{y}_1 \cos \theta_1$ and $v_1 = \dot{x}_1 \cos \theta_1 + \dot{y}_1 \sin \theta_1$, the tracking-error system in (4.25) is

$$\begin{aligned} \dot{x}_{1,e} &= (\dot{x}_1 - \dot{x}_{1,r}) \cos \theta_1 + (\dot{y}_1 - \dot{y}_{1,r}) \sin \theta_1 - (x_1 - x_{1,r}) \dot{\theta}_1 \sin \theta_1 + (y_1 - y_{1,r}) \dot{\theta}_1 \cos \theta_1 \\ &= v_1 + y_{1,e} v_1 \kappa - \dot{x}_{1,r} \cos \theta_1 - \dot{y}_{1,r} \sin \theta_1 \\ &= v_1 + y_{1,e} v_1 \kappa - \dot{x}_{1,r} \cos(\theta_{1,r} + \theta_{1,e}) - \dot{y}_{1,r} \sin(\theta_{1,r} + \theta_{1,e}) \\ &= v_1 + y_{1,e} v_1 \kappa - \dot{x}_{1,r} (\cos \theta_{1,r} \cos \theta_{1,e} - \sin \theta_{1,r} \sin \theta_{1,e}) - \dot{y}_{1,r} (\sin \theta_{1,r} \cos \theta_{1,e} + \cos \theta_{1,r} \sin \theta_{1,e}) \\ &= v_1 + y_{1,e} v_1 \kappa - (\dot{x}_{1,r} \cos \theta_{1,r} + \dot{y}_{1,r} \sin \theta_{1,r}) \cos \theta_{1,e} + (\dot{x}_{1,r} \sin \theta_{1,r} - \dot{y}_{1,r} \cos \theta_{1,r}) \sin \theta_{1,e} \\ &= v_1 + y_{1,e} v_1 \kappa - v_{1,r} \cos \theta_{1,e}, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned}
\dot{y}_{1,e} &= -(\dot{x}_1 - \dot{x}_{1,r}) \sin \theta_1 + (\dot{y}_1 - \dot{y}_{1,r}) \cos \theta_1 - (x_1 - x_{1,r}) \dot{\theta}_1 \cos \theta_1 - (y_1 - y_{1,r}) \dot{\theta}_1 \sin \theta_1 \\
&= -x_{1,e} v_1 \kappa + \dot{x}_{1,r} \sin(\theta_{1,r} + \theta_{1,e}) - \dot{y}_{1,r} \cos(\theta_{1,r} + \theta_{1,e}) \\
&= -x_{1,e} v_1 \kappa + \dot{x}_{1,r} (\sin \theta_{1,r} \cos \theta_{1,e} + \sin \theta_{1,e} \cos \theta_{1,r}) - \dot{y}_{1,r} (\cos \theta_{1,r} \cos \theta_{1,e} - \sin \theta_{1,r} \sin \theta_{1,e}) \\
&= -x_{1,e} v_1 \kappa + (\dot{x}_{1,r} \sin \theta_{1,r} - \dot{y}_{1,r} \cos \theta_{1,r}) \cos \theta_{1,e} + (\dot{x}_{1,r} \cos \theta_{1,r} + \dot{y}_{1,r} \sin \theta_{1,r}) \sin \theta_{1,e} \\
&= -x_{1,e} v_1 \kappa + v_{1,r} \sin \theta_{1,e}, \tag{A.4}
\end{aligned}$$

and

$$\dot{\theta}_{1,e} = \dot{\bar{\theta}}_1 = \dot{\theta}_1 - \dot{\theta}_{1,r} = v_1 \kappa - v_{1,r} \kappa_r. \tag{A.5}$$

Together, the model in (A.3)–(A.5) represents the tracking-error system (4.25) in the local coordinate frame.

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Part II

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Papers

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