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Red, green, blue, and white clusters for daylight reproduction

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Abstract. Daylight is an inherent attribute of a sustainable building. Artificial reproduction of natural illumination under varying needs and environmental conditions has been made possible after the appearance of the new wave of solid-state light sources. Our work is devoted to the development of white light-emitting diode (LED) clusters consisting of red, green, blue, and white (RGBW) LEDs for implementation in a smart lighting system that is able to reproduce light with correlated color temperature (CCT) similar to daylight, high values of color rendering index, and luminous efficacy. A method for determination of the optimal contribution of each of the four LEDs is demonstrated and discussed. We show that only three LEDs—green, blue, and white (no red) can be used in many cases for reproducing daylight. Both luminous efficacy of radiation and actual luminous efficacy of the considered RGBW clusters as functions of the CCT are analyzed. © 2020 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.59.5.055102](https://doi.org/10.1117/1.OE.59.5.055102)]

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1 Introduction

Light is an important part of human life. In addition to the effective consumption of energy, a lot of attention in development of new light sources is paid to the vital aspect of illumination—quality of light.¹ This term is usually associated with similarity to daylighting, which in turn is the most appropriate for us in the daily routing. The eyes of humans, as well as of all living organisms, have passed a very long evolution process with final adaptation to nature. Therefore, natural illumination gives not only high color rendering of surrounding objects but also constitutionally governs the psycho-emotional state and has a positive impact on human's health in general. The issues of light quality, improvement of the energetic efficiency and luminous efficacy, and health and comfort of workers and students are being actively investigated.^{2–5} In particular, the studies on light influence on the human state of health show that hormone secretion,^{6–8} heart rate, and concentration of vitamin D⁹ depend on spectrum of light and its brightness.¹⁰ Also important is the change of light parameters during a day. The cyclical changes cause the circadian rhythms^{11–13} that are coupled with the biological cycles. The Nobel Prize in Physiology or Medicine 2017 for the work related to this field can be viewed as yet another piece of evidence of the prime importance of circadian rhythms and, as a result, a proper change of the lighting parameters for synchronization with our inner clock. Hence, when it comes to an illumination comfort and optimal lighting conditions, it is necessary to strive to reproduce all characteristics of daylight. Its broad spectrum is interpreted by brain as white color that can be

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characterized with the correlated color temperature (CCT) corresponding to the temperature of the black body.^{14,15} According to the principles studied in colorimetry,¹⁶ any color, including white, can be achieved by mixing two or more specific monochromatic light fluxes. This enables to create an artificial light source that will be perceived by the eye as daylight. Although this method works well in display technology,¹⁷ it is not applicable for comfortable lighting. The matter is that illumination by the light source with a discrete spectrum may give colors of the surrounding objects that differ from those objects observed with sunlight. To judge about the validity of a light source to reveal the true colors of various objects, the color rendering index (CRI) was introduced.¹⁸ Although new metrics such as the Illuminating Engineering Society color fidelity index, R_f , and gamut index, R_g , and color quality scale were developed, CRI is a widely accepted standardized metric.¹⁹ For reproducing natural illumination, a lighting system must have the same CCT as daylight and CRI as high as possible. In addition, it must be associated with low energy consumption and suitable illuminance for creating a comfortable lighting environment for work and leisure. In practice, this can be realized with a smart lighting system, the key element of which is a light-emitting diodes (LEDs) cluster.

Invention of blue LED (B LED) has brought in a new era in lighting with additional levels of functionality, efficiency, and performance.²⁰ Low driving voltage and fast response are other attractive features for the dimmable lighting. A programmable control of several types of LEDs irradiating in different spectral ranges enables one to obtain an illumination system possessing not only the functions of an on–off switch and brightness tunability but also spectral (or color) manipulation that increases the quality of light. Some manufacturers and research groups direct their efforts toward creation of white LED (W LED) clusters consisting of two or more LEDs of different types with the possibility to control the CCT. The simplest implementation is warm white/cool white clusters.^{21,22} However, such clusters cannot get CCTs along the daylight locus on the International Commission on Illumination (CIE) chromaticity diagram. Also, three component clusters, such as red, green, and blue (RGB) and red, yellow, and cyan (RYC) clusters, have been implemented for obtaining the desired light parameters in a wider range of CCT^{23–25} but they usually have low luminous efficacy.

The most optimal are red, green, blue, and white (RGBW) and red, green, blue, and amber (RGBA) clusters.^{22–24,26–30} When two or three LEDs are used, the LEDs contribution to the resulting white light is determined unequivocally. However, the situation is reversed in case of four or more LEDs where determination of the LEDs contributions is not unique and requires a study for finding optimal solutions. Researchers^{26,29} demonstrated an infinite number of spectral solutions with a specific target chromaticity coordinates due to changes of the LEDs weighting coefficients in the four-component systems and analyzed their visual and nonvisual parameters. The LED clusters proposed in other works^{22–24} have their advantages and disadvantages; however, most of the reported solutions cannot be considered as optimal in terms of high CRI and efficacy in a wide range of CCTs. To solve this problem, research activities are focused on development of new types of phosphors, study of the effect of quantum dots usage,³¹ creation of sophisticated algorithms for determining the LEDs contributions based on the fidelity-luminous efficacy of radiation (LER) Pareto boundaries,^{26,32} and other improvements such as applications that utilized more than 10-component systems.^{27,33} However, despite many advances in lighting technology, a simple algorithm is needed to determine the components contributions ratio in lighting systems using a typical set of LEDs with the primary colors, to escape complex computational algorithms and implementation of special non-typical components. We explored this issue. The purpose of this work is to develop algorithm for selection of LED's weighting coefficients in RGBW clusters for obtaining high luminous efficacy and CRI in a wide range of CCTs while three of four LEDs work simultaneously for additional system simplification. A second purpose is to study the influence of W LED's parameters on the resultant light. As a starting point, there are four chosen LED RGBW clusters with different W LEDs [warm (candle), warm, neutral, and cold]. A mathematical approach for obtaining high luminous efficacy and CRI values in a wide range of CCT is demonstrated.

This paper consists of the following parts: description of the RGBW cluster principle, solutions for optimal parameters of LEDs in RGBW cluster, and discussion of the results.

2 RGBW Clusters: Principle and Simulation

2.1 Condition on Relationship for RGB and White-Light Components

CCT of a light source is determined through the u, v diagram or the color coordinates, e.g., x, y , and z for CIE 1931.^{34,35} If an illuminating system consists of three LEDs emitting light of the basic colors (RGB), the resulting tristimulus values can be expressed as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \quad (1)$$

where the elements of the matrix are the tristimulus values of the LEDs,³⁶ and R, G , and B are the corresponding brightness of these LEDs. Such a procedure enables one to find an area in the CIE x, y chromaticity diagram ($z = 1 - x - y$ because of normalization), within which the resulting color is varied due to the change of R, G , and B . In the case of three primary colors (RGB), the area forms a triangle known as the Maxwell triangle.³⁷

From Eq. (1), the brightness of the LEDs is expressed uniquely through the resulting tristimulus values and the inverse matrix of the tristimulus values of the RGB LEDs in the following manner:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix}^{-1} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}. \quad (2)$$

As mentioned in the previous section, the use of the three primary colors works well in displays but it fails in lighting because of low CRI.³⁸ To increase CRI, it is possible to add a W LED characterized with a broad spectral range. This enables to reshape the resulting spectrum of the lighting or, in other words, to manipulate CRI at a fixed CCT. The Maxwell triangle formed by the RGB LEDs therewith is not changed.

The color of an RGBW light source can be implemented in a variety of ways due to four variables for three equations as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B & X_W \\ Y_R & Y_G & Y_B & Y_W \\ Z_R & Z_G & Z_B & Z_W \end{bmatrix} \cdot \begin{bmatrix} R \\ G \\ B \\ W \end{bmatrix}, \quad (3)$$

where W is the brightness of the W LED.

To get an appropriate solution of Eq. (3), a supplementary condition has to be introduced.²² Luminous efficacies of red, blue and, in most cases, green monochrome LEDs are significantly lower than that of W LEDs. Therefore, it can be assumed that minimization of the RGB LEDs contribution in color generation allows for receiving an optimal luminous efficacy of the system in general at its additional simplification. Moreover, when maximizing contribution of the W LED with the broad spectrum and minimizing contribution of the RGB LEDs with narrow spectra, the basic W LED's spectrum will be supplemented without sharp peaks by the contribution of the RGB components. This will result in an increase in CRI values of the system in comparison with the CRI of the W LED. Moreover, this condition will make it possible to determine unambiguously the components contributions to the resulting light.

Let us consider the color mixing in more details. There are three points of the color coordinates in the CIE x, y chromaticity diagram: (1) the resulting color that is needed to be obtained by the lighting system, C_{LS} , where C_{LS} is presented by the pair of the chromaticity coordinates x_{LS} and y_{LS} , (2) the color of the W LED C_W (x_W, y_W), and (3) the color generated by the RGB LEDs C_{RGB} (x_{RGB}, y_{RGB}). Since the first point is the sum of the two others, all three points lie on a straight line and C_{LS} is located between C_W and C_{RGB} . In the system under consideration, point

C_{LS} is selected on the Planckian locus, but it also can be placed close to the Planckian locus. The contribution of an LED in the color mixing is inversely proportional to the distance between the colors of this LED and C_{LS} . This means that C_{RGB} has to be as far as possible from C_{LS} . If so, C_{RGB} must lie on a side of the Maxwell triangle as shown in the examples of Fig. 1(a) where the generated colors C_{LS1} and C_{LS2} are on the Planckian locus and another pair C_{RGB1} and C_{RGB2} are the coordinates generated by the RGB LEDs.

It is pertinent to note that only two of the three LEDs are needed for generating a color on a side of the Maxwell triangle. In the demonstrated examples that are more typical for a real situation, they are B LED and green LED (G LED) whereas the red LED (R LED) is switched off. Therefore, the requirement of minimization of the RGB LEDs' irradiance in the resulting white light leads to the use of three LEDs—the W LED and two color LEDs for getting a color on the Planckian locus (C_{LS}). The choice of the two color LEDs depends on the location of C_{LS} on the Planckian locus and the chromaticity coordinates of the W LED (C_W). In particular, G, B, and W LEDs are used for C_W having lower CCT than for C_{LS} , i.e., C_{LS} lies within the BGC_W triangle, whereas G, R, and W LEDs must be switched on for C_W associated with high color temperature, when C_{LS} lies within the GRC_W triangle.

2.2 Algorithm for Calculation of Weighting Coefficients of the R, G, B, W Components

To generate a needed color C_{LS} by the RGBW LEDs, it is necessary to know how much each LED contributes. The contributions can be described in terms of weighting coefficients g_R , g_G , g_B , g_W , and g_{RGB} , which are possible to express through the color coordinates of the LEDs. Using the fact that the chromaticity coordinates can be treated in the same manner as geometrical coordinates in a two-dimensional space (Fig. 1), it is possible to work with the chromaticity coordinates as with components of vectors in the CIE x , y chromaticity diagram. In particular, $\vec{C}_{LS}$ can be considered as a vector that starts at the origin (0,0) and ends at C_{LS} (x_{LS} , y_{LS}) as shown in Fig. 1(b). Similarly, there are vectors $\vec{C}_W$, $\vec{C}_{RGB}$, $\vec{C}_R$, $\vec{C}_G$, and $\vec{C}_B$. The vector $\vec{C}_{RGB}$ can be expressed through $\vec{C}_W$ and $\vec{C}_{LS}$ as follows:

$$\vec{C}_{RGB} = \lambda_1(\vec{C}_{LS} - \vec{C}_W), \quad (4)$$

where λ_1 is a positive coefficient. On the other hand, the point C_{RGB} has to be on a side of the Maxwell triangle. This condition can be expressed as $\vec{C}_{RGB} = \vec{C}_G + \lambda_2(\vec{C}_B - \vec{C}_G)$, if C_{RGB} lies in the side BG, or $\vec{C}_{RGB} = \vec{C}_G + \lambda_2(\vec{C}_R - \vec{C}_G)$, if C_{RGB} lies in the side GR, or

Fig. 1 (a) CIE x , y chromaticity diagram with examples of the chromaticity coordinates C_{LS1} and C_{LS2} , which may be generated by the lighting system, chromaticity coordinates of the white LED (C_W), and the resulting chromaticity coordinates of RGB LEDs (C_{RGB1} and C_{RGB2}) and (b) vector representation of color mixing.

$\vec{C}_{RGB} = \vec{C}_R + \lambda_2(\vec{C}_B - \vec{C}_R)$, if C_{RGB} lies in the side RB, where λ_2 is another positive coefficient. These four expressions are transformed into three equations as follows:

$$\lambda_1(\vec{C}_{LS} - \vec{C}_W) = \vec{C}_G + \lambda_2(\vec{C}_B - \vec{C}_G), \quad (5a)$$

$$\lambda_1(\vec{C}_{LS} - \vec{C}_W) = \vec{C}_G + \lambda_2(\vec{C}_R - \vec{C}_G), \quad (5b)$$

$$\lambda_1(\vec{C}_{LS} - \vec{C}_W) = \vec{C}_R + \lambda_2(\vec{C}_B - \vec{C}_R). \quad (5c)$$

Each of Eq. (5) splits by two equations, if we substitute the chromaticity coordinates. This makes possible to find λ_1 and λ_2 . The solution that corresponds to the physical meaning, i.e., when C_{RGB} lies on one of the sides of the Maxwell triangle, is only for one of the equations, for which both λ_1 and λ_2 are positive (it is assumed that C_{LS} and C_W do not coincide). After substitution of the chromaticity coordinates in Eq. (5a), we have

$$\lambda_1 = \frac{x_G(y_B - y_G) - y_G(x_B - x_G)}{(x_{LS} - x_W)(y_B - y_G) - (y_{LS} - y_W)(x_B - x_G)}, \quad (6)$$

$$\lambda_2 = \frac{x_G(y_{LS} - y_W) - y_G(x_{LS} - x_W)}{(x_{LS} - x_W)(y_B - y_G) - (y_{LS} - y_W)(x_B - x_G)}; \quad (7)$$

for Eq. (5b):

$$\lambda_1 = \frac{x_G(y_R - y_G) - y_G(x_R - x_G)}{(x_{LS} - x_W)(y_R - y_G) - (y_{LS} - y_W)(x_R - x_G)}, \quad (8)$$

$$\lambda_2 = \frac{x_G(y_{LS} - y_W) - y_G(x_{LS} - x_W)}{(x_{LS} - x_W)(y_R - y_G) - (y_{LS} - y_W)(x_R - x_G)}; \quad (9)$$

and for Eq. (5c):

$$\lambda_1 = \frac{x_R(y_B - y_R) - y_R(x_B - x_R)}{(x_{LS} - x_W)(y_B - y_R) - (y_{LS} - y_W)(x_B - x_R)}, \quad (10)$$

$$\lambda_2 = \frac{x_R(y_{LS} - y_W) - y_R(x_{LS} - x_W)}{(x_{LS} - x_W)(y_B - y_R) - (y_{LS} - y_W)(x_B - x_R)}. \quad (11)$$

By knowing λ_1 and λ_2 , it is possible to find C_{RGB} . If, for example, C_{RGB} lies in the side BG Eq. (5a), the chromaticity coordinates C_{RGB} are found according to Eq. (4) through the coordinates x_{LS} , y_{LS} , x_B , y_B , x_G , y_G , x_W , and y_W .

$$x_{RGB} = \frac{x_G(y_B - y_G) - y_G(x_B - x_G)}{(x_{LS} - x_W)(y_B - y_G) - (y_{LS} - y_W)(x_B - x_G)}(x_{LS} - x_W), \quad (12)$$

$$y_{RGB} = \frac{x_G(y_B - y_G) - y_G(x_B - x_G)}{(x_{LS} - x_W)(y_B - y_G) - (y_{LS} - y_W)(x_B - x_G)}(y_{LS} - y_W). \quad (13)$$

There are similar operations for the other sides (GR and RB).

Using analogy with finding the center of mass of two points in physics, it is possible to write $|\vec{C}_{LS} - \vec{C}_W|g_W = |\vec{C}_{RGB} - \vec{C}_{LS}|g_{RGB}$ [Fig. 1(b)], where g_{RGB} is the weighting coefficient corresponding to $\vec{C}_{RGB}$. According to Eq. (4), we have

$$\frac{g_W}{g_{RGB}} = \lambda_1 - 1. \quad (14)$$

In this case, C_{RGB} is formed by two of three RGB LEDs. For example, let λ_1 and λ_2 be positive for Eq. (5a), i.e., the R LED is off ($g_R = 0$), the ratio between the weighting coefficients for G and B LEDs that are g_G and g_B , respectively, is given as

$$\frac{g_B}{g_G} = \lambda_2 - 1. \quad (15)$$

At the same time, it is necessary to take into account

$$g_B + g_G = g_{RGB}. \quad (16)$$

Another condition is that the brightness of each of the W, R, G, and B LEDs in a practical device should not exceed their maximum real values.

Thus, the coordinates x_{LS} , and y_{LS} of the resulting light can be written as

$$\begin{pmatrix} x_{LS} \\ y_{LS} \end{pmatrix} = g_R \begin{pmatrix} x_R \\ y_R \end{pmatrix} + g_G \begin{pmatrix} x_G \\ y_G \end{pmatrix} + g_B \begin{pmatrix} x_B \\ y_B \end{pmatrix} + g_W \begin{pmatrix} x_W \\ y_W \end{pmatrix}. \quad (17)$$

On the other hand, the tristimulus values of the resulting light can be defined from Eq. (3) as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \begin{pmatrix} X_R \\ Y_R \\ Z_R \end{pmatrix} + G \begin{pmatrix} X_G \\ Y_G \\ Z_G \end{pmatrix} + B \begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix} + W \begin{pmatrix} X_W \\ Y_W \\ Z_W \end{pmatrix}. \quad (18)$$

In this case, the coordinates $\begin{pmatrix} x_{LS} \\ y_{LS} \end{pmatrix}$ expressed through $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ can be defined as³⁶

$$\begin{pmatrix} x_{LS} \\ y_{LS} \end{pmatrix} = \frac{RC_R \begin{pmatrix} x_R \\ y_R \end{pmatrix} + GC_G \begin{pmatrix} x_G \\ y_G \end{pmatrix} + BC_B \begin{pmatrix} x_B \\ y_B \end{pmatrix} + WC_W \begin{pmatrix} x_W \\ y_W \end{pmatrix}}{C_{LS}}, \quad (19)$$

where $C_i = X_i + Y_i + Z_i$, the subindex i means R,G,B,W, and LS.

Since all coordinates can be normalized to C_{LS} or, equivalently, to $X_{LS} + Y_{LS} + Z_{LS} = 1$, we get

$$\begin{pmatrix} x_{LS} \\ y_{LS} \end{pmatrix} = RC_R \begin{pmatrix} x_R \\ y_R \end{pmatrix} + GC_G \begin{pmatrix} x_G \\ y_G \end{pmatrix} + BC_B \begin{pmatrix} x_B \\ y_B \end{pmatrix} + WC_W \begin{pmatrix} x_W \\ y_W \end{pmatrix}. \quad (20)$$

From Eqs. (17) and (20), $R = \frac{g_R}{C_R}$, $G = \frac{g_G}{C_G}$, $B = \frac{g_B}{C_B}$, $W = \frac{g_W}{C_W}$. Therefore, the radiant flux of the resulting light, $\Phi_e(\lambda)$, can be determined through the weighting coefficients, tristimulus values, and radiant fluxes of RGBW LEDs [$\Phi_{eR}(\lambda)$, $\Phi_{eG}(\lambda)$, $\Phi_{eB}(\lambda)$, and $\Phi_{eW}(\lambda)$, respectively] as

$$\begin{aligned} \Phi_e(\lambda) &= \Phi_{eR}(\lambda)R + \Phi_{eG}(\lambda)G + \Phi_{eB}(\lambda)B + \Phi_{eW}(\lambda)W = \\ &= \frac{g_R \Phi_{eR}(\lambda)}{X_R + Y_R + Z_R} + \frac{g_G \Phi_{eG}(\lambda)}{X_G + Y_G + Z_G} + \frac{g_B \Phi_{eB}(\lambda)}{X_B + Y_B + Z_B} + \frac{g_W \Phi_{eW}(\lambda)}{X_W + Y_W + Z_W}. \end{aligned} \quad (21)$$

By proportional increase or decrease of the LED's brightness, it is possible to ensure the correspondence of the resulting lighting.

2.3 Choice of the White LED's Optimal Parameters for the LED Cluster

The choice of a proper W LED from a great number of analogs, CCT and CRI of which differ in a wide range, is an important task. One of the key criteria for a smart lighting system is the high CRI in the full range of the CCT to be varied for generating the light. To develop a strategy for deciding on a proper W LED, we analyzed four RGBW clusters that differ from each other only

Table 1 Values of the CCT (CCT_W , K) and the CRI (CRI_W) of the W LEDs used in the studied RGBW clusters.

	CCT_W (K)	CRI_W
First W LED	2336	81
Second W LED	2985	82
Third W LED	4026	81
Fourth W LED	5897	79

Table 2 Values of the peak of the wavelength (λ_P , nm) and the half wavelength (λ_{hp} , nm) of RGB LEDs used in the studied RGBW clusters.

	λ_P (nm)	λ_{hp} (nm)
R LED	633.4	16.6
G LED	524.8	35.0
B LED	461.1	23.6

by W LEDs. All these W LEDs had close CRI but dissimilar CCT as shown in Table 1. Parameters of the RGB LEDs used in the clusters are listed in Table 2.

The spectra of the W LEDs in Table 1 are plotted in Fig. 2(a) and the spectra of the RGB LEDs in Table 2 are plotted in Fig. 2(b).

According to the above-described method, we developed a web program³⁹ to calculate the parameters (CCT, color rendering, and efficacy) of the resulting white light. Our program utilizes the open-source Python package Color⁴⁰ that provides a comprehensive number of algorithms and datasets for color science. An additional check was made using the well-proven Software LED ColorCalculator.⁴¹ Manipulation of the RGBW LEDs brightness enabled us to get the white light with the chromaticity coordinates following the Planckian locus within 2000 to 7000 K range. The simulation results for CRI versus CCT of each studied cluster are shown in Fig. 3. For additional information, all graphs are shown and analyzed in the CCT range of 2000 to 7000 K. Since CCT of daylight is usually higher than 2800 to 3000 K, the obtained values are analyzed in the CCT range of 3000 to 7000 K.

As shown in Fig. 3, all four CRI curves have similar behavior. Starting from the low values of CCT, CRI increases with increasing CCT then after reaching a peak falls down by about 10% from the maximum and begins to grow again. At the point where CCT of the resulting white light is equal to CCT_W , the CRI is close to CRI_W . This happens due to the chosen minimization of the

Fig. 2 Spectra of the (a) four white LEDs and (b) RGB LEDs used in the clusters.

Fig. 3 Calculated CRI versus CCT for the four studied RGBW clusters.

contribution of the RGB LEDs. When the required CCT differs from CCT_w , the contribution of the RGB LEDs increases with increasing the distance between CCT and CCT_w . In the case when $CCT > CCT_w$, the gradual increase in CRI is ~ 1 to 4 units per 1000 K for systems with neutral and cold W LEDs. At the same time for systems with warm W LEDs, this increase is ~ 7 to 8 units per the first 1000 K, then a weaker increase and subsequent decrease by 1 to 2 units for the last 1000 to 3000 K. When $CCT < CCT_w$, the behavior of the curves is more complex. At first, they have a sharp increase by more than 10 units with decreasing CCT when the contribution of the R and G LEDs (the B LED is switched off in this range of CCTs) to the resulting white light is of the order of 20%. The further increase of the contribution of the R and G LEDs in the resulting light having lower CCT causes the decrease of the CRI. The CRI goes down to 50 when the contribution of the R and G LEDs is of the order of 75%. An additional calculation of fidelity index Rf for the considered RGBW clusters showed dependence on CCT similar to the CRI values but with different numerical values.

In addition to high CRI values, it is necessary to obtain high values of luminous efficacy in the full range of the CCTs, and we investigated the dependence of the LER and K , as a function of CCT. K is defined as the ratio between luminous flux and radiant flux and is described as follows:^{15,24,36}

$$K = \frac{\Phi_v}{\Phi_e} = \frac{683 \int_{380}^{780} V(\lambda) \Phi_e(\lambda) d\lambda}{\int_{380}^{780} \Phi_e(\lambda) d\lambda}, \quad (22)$$

where $V(\lambda)$ is the spectral eye sensitivity, Φ_v is the luminous flux, and $\Phi_e(\lambda)$ is the radiant flux of the resulting light being determined by Eq. (21).

LER corresponding to the ideal case, when the LEDs in a cluster possess energetic efficiency near 100%, is often used for theoretical estimations of the upper limits of lighting systems.^{24,36} In our calculations, the studied LEDs had the following values: $K_R = 191$ lm/W, $K_G = 508$ lm/W, $K_B = 57$ lm/W, K_w (2336 K) = 304 lm/W, K_w (2985 K) = 334 lm/W, K_w (4026 K) = 336 lm/W, and K_w (5897 K) = 312 lm/W. The LERs obtained according to Eq. (17) as functions of CCT for the studied clusters are shown in Fig. 4. The ranges of CRI, Rf, Rg, and LER in the CCT range of 3000 to 7000 K for these four clusters are listed in Table 3.

There exists a similarity in behaviors of the curves as shown in Figs. 3 and 4. Since, at the point CCT_w , the contribution of RGB LEDs tends to zero, the value of LER at this point is equal to K_w . The increase of the contribution of RGB LEDs leads to decrease the luminous efficacy of the radiation. According to Fig. 4, the change of K constitutes 11% to 17% in the whole studied range of CCT (2000 to 7000 K). In other words, using the presented method for mixing the RGBW LEDs, relatively high CRI is achieved in the full range of CCTs without significant loss in LER.

Fig. 4 Calculated LER versus CCT for the four considered white LEDs.

Table 3 Ranges of CRI (CRI_{min} to CRI_{max}), fidelity index (Rf_{min} to Rf_{max}), gamut index (Rg_{min} to Rg_{max}), and LER (K_{min} to K_{max} , lm/W) in the CCT range of 3000 to 7000 K for four clusters.

	CRI_{min} to CRI_{max}	Rf_{min} to Rf_{max}	Rg_{min} to Rg_{max}	K_{min} to K_{max} (lm/W)
CCT_w (2336 K)	91 to 96	86 to 88	97 to 100	271 to 306
CCT_w (2985 K)	82 to 94	83 to 86	95 to 97	285 to 332
CCT_w (4026 K)	74 to 96	82 to 85	95 to 110	295 to 333
CCT_w (5897 K)	50 to 90	72 to 85	96 to 118	297 to 311

2.4 Comparison of Luminous Efficacy of the Clusters with Different Combinations of RGB and White LEDs

An actual luminous efficacy of LEDs (η , lm/W) differs (sometime even significantly) from the theoretical case—LER. As a result, the ratios between the efficacies of the R, G, B, and W LEDs in a practical device are different from the ideal situation that is only considered in some works.²⁶ To investigate the difference in this regard, the actual luminous efficacies of all four clusters were calculated using as examples of R, G, B two types of Osram LEDs: “type 1”—the LRTB GFTG series⁴² and “type 2”—the LRTB GRTG series,⁴³ and their parameters are listed in Table 4.

The W LEDs were the same for both types and had $\eta_w = 150$ lm/W. The obtained dependences are shown in Fig. 5. As expected, the behaviors of η for the system composed of the LEDs with real parameters slightly differ from the ideal case (Fig. 4). Starting from 2000 K, the luminous efficacy of all clusters increases and, after reaching the peak at CCT_w , gradually decreases and this fall is three to four times more than for LER. This is similar to the dependence of the contribution of the W LED to the resulting light from the CCT that is understandable because of the chosen condition for maximization of W and minimization of RGB LEDs. Therefore, the maximum light efficacy is observed at CCT_w , where it is equal to the light efficacy of the W LED. When the type 1 RGB LEDs⁴² are added to the lighting system, the luminous efficacy decreases, and its fall in the CCT range of 3000 to 7000 K is 42% to 58%. At the same time, in the case of “type 2” RGB LEDs,⁴³ the fall of the luminous efficacy is 39% to 54%. These

Table 4 Values of luminous efficacy (η , lm/W) of two types of Osram LEDs.

	Type 1 η (lm/W)	Type 2 η (lm/W)
R LED	45	42
G LED	50	66
B LED	11	12

Fig. 5 Calculated luminous efficacy versus CCT for the considered white LEDs and two types of RGB LEDs.

parameters as well as the ranges of luminous efficacy and the contributions of W LED to the resultant light in the CCT range of 3000 to 7000 K for all eight clusters are listed in Table 5.

An increase of the luminous efficacy of the G LED by 30%, which corresponds to 16 lm/W, does not cause a change of the behaviors and basic values of the dependences of the luminous efficacy of the lighting system on the CCT. At the same time, the lower limit of the light system efficacy increased by 6% to 15%.

Thus, the maximum value of the luminous efficacy of the system is determined by the luminous efficacy of the W LED and corresponds to the $CCT = CCT_W$. Its fall in the CCT range 3000 to 7000 K is 39% to 58% and depends on the light efficacies of the selected RGB LEDs.

Although the choice of specific LEDs among the variety presented on the market was not the goal of this work, it is important to note that the described method can be implemented for any selected LEDs and, as a result, to find LEDs that give maximum CRI with highest luminous efficacy in the desired range of CCT.

The obtained results suggest that the reproduced light with the warm W LEDs having $CCT_W = 2336$ K and $CCT_W = 2985$ K are more suitable. For the first, the CRI varies in the range 91 to 96 and its fall of η is 42%, and for the last the CRI is 82 to 94 and the fall of η is 53%. By comparison for the LEDs with $CCT_W = 4026$ K, there is 74 to 96 and 47%, respectively, and

Table 5 Values of the fall of luminous efficacy ($\Delta\eta$, %), the range of luminous efficacy ($\eta_{\min}$ to $\eta_{\max}$, lm/W), and the range of W LED luminous flux contribution to the resultant light ($W_{\max}$ to $W_{\min}$, %) in the CCT range of 3000 to 7000 K.

	Fall of luminous efficacy $\Delta\eta$ (%)	Range of luminous efficacy $\eta_{\min}$ to $\eta_{\max}$ (lm/W)	W LED contribution $W_{\max}$ to $W_{\min}$ (%)
Type 1 RGB			
CCT_W (2336 K)	42	62 to 107	86 to 59
CCT_W (2985 K)	53	70 to 147	99 to 69
CCT_W (4026 K)	47	78 to 146	99 to 56
CCT_W (5897 K)	58	62 to 147	99 to 34
Type 2 RGB			
CCT_W (2336 K)	39	71 to 116	86 to 59
CCT_W (2985 K)	47	78 to 148	99 to 69
CCT_W (4026 K)	44	83 to 146	99 to 56
CCT_W (5897 K)	54	67 to 147	99 to 34

Fig. 6 Examples of the spectra of the RGBW cluster with warm (candle) white basic LED ($CCT_W = 2336$ K) at different color temperatures: (a) 2500, 3000, and 4000 K and (b) 5000, 6000, and 7000 K.

for the LEDs with $CCT_W = 5897$ K there is 50 to 90 and 58%. It is pertinent to note that when the LEDs with $CCT_W = 2336$ K and $CCT_W = 2985$ K are used for the white-light generation, there are only three LEDs—green, blue, and white, needed, while the R LED has to be switched off in the range of the CCT 2500 to 7000 K and 3000 to 7000 K, respectively. This implies that it is appropriate to use W LEDs with low values of CCT, namely with values of CCT lying in the low part of the reproduction range and even less. Figure 6 shows few examples of the spectra of the RGBW cluster with warm (candle) white basic LED ($CCT_W = 2336$ K) that showed the highest CRI values and the lowest fall of η , and moreover when using it only three of the four LEDs are needed in the full range of CCT. For the resulting light with CCTs 2500 to 4000 K [Fig. 6(a)], the Rf and Rg values vary in the ranges 85 to 88 and 96 to 98, while for the light with CCTs 5000 to 7000 K [Fig. 6(b)] these values are 86 to 88 and 99 to 100, respectively.

From the results presented above, it can be assumed that the use of another W LED with CRI value higher than those considered by us, e.g., an LED with CRI above 90, will lead to an increase in the CRI of the resulting white light of the RGBW system in the full CCT range.

3 Conclusion Remarks

In this work, we demonstrate RGBW LED clusters suitable for a smart lighting system generating daylight in the full range of CCT (3000 to 7000 K). The main idea is based on the maximization of luminous efficacy and CRI for the desired CCT due to the minimization of the contributions of the color LEDs (RGB) possessing relatively narrow emission spectra and low luminous efficacies. Under this condition, only three of the four LEDs (white and two from RGB) are used at the same time. This enables one to simplify the lighting system driving electronics and to reduce energy consumption. This method is applicable to any four-component system consisting of three colored LEDs and one wide-band LED (white, amber, and lime) located inside the Maxwell triangle.

Analysis of the influence of the W LED CCT on the parameters of the lighting shows reasonability to use a W LED with CCT close to the lowest value from the CCT range required for illumination reproducing. In particular, a W LED with CCT about 2500 K and CRI about 81 enables to reach a final CRI above 90 in the range of CCT 3000 to 7000 K. The maximum luminous efficacy of the lighting system is equal to the luminous efficacy of the W LED, and its fall is about 39% to 53% in the range of 3000 to 7000 K when using warm white and neutral LEDs depending on CCT and parameters of the chosen LEDs.

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