Case Study

The effect of flexural–torsional buckling on the stability of timber members: a case study

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Abstract
This study investigates the stability of timber members subjected to simultaneously acting axial compression and bending moment, with possible risk for torsional and flexural–torsional buckling. This situation can occur in laterally supported members where one side of the member is braced but the other side is unbraced. In this case, the free side will buckle out of plane while the braced side will be prevented from torsional and flexural–torsional buckling. This problem can be evident for long members in timber-frame structures, which are subjected to high axial compression combined with bending moments in which the member is not sufficiently braced at both sides. This study is based on the design requirement stated in Eurocode 5. Solution methods discussed in this paper can be of interest within the framework of structural and building Engineering practices and education in which the stability of structural elements is investigated.

Article Highlights

- This case study investigates some design situations where the timber member is not sufficiently braced. In this case, a stability problem associated with combined torsional buckling and flexural buckling can arise.
- The study shows that the torsional and/or flexural–torsional buckling of timber members can be important to control in order to fulfil the criteria of the stability of the member according to Eurocode 5 and help the structural engineer to achieve safer designs.
- The study investigates also a simplified solution to check the effect of flexural torsional buckling of laterally braced timber members.

Keywords Flexural–torsional buckling · Stability of timber structures · Eurocode 5 · Ultimate limit state

1 Introduction

In modern timber-frame structures, instability of structural members such as beams and columns is one of the most important problems that structural engineers must address. In general, column-beams that are centrally loaded will have three different buckling loads, at least one of which corresponds to torsional or flexural–torsional mode in rectangular sections or doubly symmetric sections. Since the flexural buckling load about the weak axis is in many cases the lowest, the torsional buckling load is disregarded in doubly symmetric sections. However, in non-symmetric sections, buckling will in many cases always be in flexural–torsional mode, irrespective of its shape and dimensions. In practice, however, non-symmetric sections are seldom used in timber constructions.

Flexural–torsional stresses in timber-framed structures are seldom serious enough to require structural

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design analysis. However, there are conditions in which flexural–torsional loads will produce stresses of sufficient magnitude to require torsional analysis of a structural member. Technically, this situation can occur in laterally-supported members when one side of the member is braced at one side but the other side is unbraced. In this case, the free side will buckle laterally (out of plane), while the braced side will be prevented (totally or partially) from flexural–torsional buckling. As a result, the member will twist and the final buckling deformation will be a combination of torsion and bending. In this case, such a deflection mode should be controlled for design purposes.

Eurocode 5 [1] proposes interaction formulae to account for both buckling and lateral-torsional buckling. In this formulation, the interaction of combined bending moment and compression is considered. For the ultimate limit state analysis, Eurocode 5 [1] recommends using a linear interaction model for combined axial compression and bending for members with buckling failure.

With the increasing trend of building residential structures in timber, laterally supported beams or columns are used, with long spans. This design results in high axial compression forces combined with bending moments, and the stability problem of the structural member should be adequately controlled. In many cases in practice, the case of flexural buckling and lateral-torsional buckling are the two decisive stability problems, and these must be checked to guarantee design safety. These two problems are well addressed in Eurocode 5 [1]. However, in some design situations where the structural member is not adequately braced, problems associated with combined torsional buckling and flexural buckling can arise, as indicated earlier. This leads to a reduction in the compressional capacity of the member. Consequently, the torsional and/or flexural–torsional buckling of the member should be checked, to ensure that the lateral supports and bracing system are sufficient to withstand the unwanted buckling of the unbraced part of the member. This study will investigate this problem.

A number of previous works have addressed in general this problem. Raven et al. [2] studied elastic compressive-flexural–torsional buckling in structural members and developed methods to treat this issue in structures. Wong and Driver [3] compared a number of methods to determine the equivalent moment factors used in evaluating the elastic critical moment of laterally unsupported beams for a wide variety of moment distributions using the Canadian design standard. Serna et al. [4], proposed eequivalent uniform moment factors for lateral-torsional buckling of steel members. Secer and Uzun [5] carried out inelastic ultimate load analysis of steel frames considering lateral torsional buckling under distributed loads. However, it is noted that these works are not specifically developed to laterally supported timber structures according to Eurocode 5. Steiger and Fontana [6] and Hassan [7] considered the stability of timber members with respect to bending moment and axial force without considering the effect of torsional or flexural torsional buckling. The stability of timber members with respect to lateral-torsional buckling considering different structures was investigated further by Challamel and Girhammar [8], Hofmann and Kuhlmann [9], Bedon and Fragiacomo [10] and Koris and Bódi, [11]. However, torsional and flexural–torsional buckling in laterally supported timber are not thoroughly addressed. In the same way, Sahraei et al. [12] derived expressions for elastic lateral torsional buckling of wooden beams. Bresser et al. [13] proposed a solution in the form of a general formulation to determine equivalent moment factors for both I-sections and rectangular slender sections. It is noticed here that both papers only investigated the effect of lateral torsional buckling of beams. However, the effects of torsional and flexural–torsional buckling are not investigated.

Consequently, it is attempted in this paper to examine the elastic torsional and flexural–torsional buckling (due to the flexural–torsional buckling mode) in laterally supported timber structures in terms of the Eurocode 5 standard. Specifically, the case study will consider the practical consequences of the load-bearing behaviour of laterally-supported timber members subjected to simultaneously acting axial compression and bending moment, with risk for torsional and/or flexural–torsional buckling. Further, the study is limited to typical cross-sections commonly used in timber structures, such as doubly symmetric rectangular sections considering the ultimate limit state as stated in Eurocode 5 [1].

The paper is organised as follows. Firstly, the interactions formula as described in the Eurocode 5 are presented. Secondly, a theoretical approach of the problem is introduced. Thirdly, a case study is developed to investigate the problem, followed by the discussion of the results.

2 Interaction formulae to Eurocode 5

Below is a review of the stability criteria as stated in the Eurocode 5 [1]. Eurocode 5 proposes interaction formulae in order to account for both flexural buckling and lateral-torsional buckling, as stated below.

2.1 Flexural buckling

For members subjected to combined bending and axial compression parallel to the grain with risk for flexural buckling, the following interaction formulae must be satisfied:

$$\frac{\sigma_{c,0,d}}{k_{c,y}f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m\frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1 \tag{1}$$

$$\frac{\sigma_{c,0,d}}{k_{c,z}f_{c,0,d}} + k_m\frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \le 1 \tag{2}$$

where $\sigma_{c,0,d}$ is the design compressive stress; $f_{c,0,d}$ is the design compressive strength along the grains; k_m is = 0.7 for rectangular sections; $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the design bending stress about the principal *y*- and *z*-axis respectively; $f_{m,y,d}$ and $f_{m,z,d}$ are the design bending strength about the principal *y*- and *z*-axis respectively. Equations (1) and (2) are valid only for the case where $\lambda_{rel,y}$ and/or $\lambda_{rel,z} > 0.3$, where

$$\lambda_{rel,y} = \frac{\lambda_y}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}} \tag{3}$$

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}} \tag{4}$$

where $E_{0,05}$ is the fifth percentile value of the modulus of elasticity parallel to the grain; $f_{c,0,k}$ is the characteristic compression strength along the grains. The design compressive strength along the grains reads:

$$f_{c,0,d} = \frac{k_{mod}f_{c,0,k}}{\gamma_M} \tag{5}$$

and the design bending strength:

$$f_{m,d} = \frac{k_{mod}f_{m,k}k_h}{\gamma_M} \tag{6}$$

where k_{mod} is a modification factor taking into account the effect of the duration of load and moisture content; $f_{m,k}$ is the characteristic value of bending moment capacity; γ_M is the partial factor for a material property; k_h is factor to take into account the volume effect and λ is the slenderness ratio given by:

$$\lambda_y = \frac{L_{ef,y}}{i_y};\lambda_z = \frac{L_{ef,z}}{i_z} \tag{7a,b}$$

where L_{ef} is the effective buckling length and i is the radius of gyration. The buckling reduction factors $k_{c,y}$ and $k_{c,z}$ can be obtained as:

$$k_{c,y} = \frac{1}{k_y + \sqrt{\left[k_y^2 - \lambda_{rel,y}^2\right)}} \tag{8}$$

$$k_{c,z} = \frac{1}{k_z + \sqrt{\left[k_z^2 - \lambda_{rel,z}^2\right)}} \tag{9}$$

in which

$$k_y = 0.5\left(1 + \beta_c\left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^2\right) \tag{10}$$

$$k_z = 0.5\left(1 + \beta_c\left(\lambda_{rel,z} - 0.3\right) + \lambda_{rel,z}^2\right) \tag{11}$$

where $\beta_c = 0.2$ for solid timber and = 0.1 for glue-laminated timber.

2.2 Flexural and lateral-torsional buckling

According to Eurocode [1], lateral-torsional stability may be verified both in the case where only a moment M_y exists about the strong axis *y* and where a combination of moment M_y and compressive force N_c exists. In the latter case, the stresses should satisfy the following expression:

$$\frac{\sigma_{c,0,d}}{k_{c,z}f_{c,0,d}} + \left(\frac{\sigma_{m,y,d}}{k_{crit}f_{m,y,d}}\right)^2 \le 1 \tag{12}$$

where $\sigma_{m,d}$ is the design bending stress; $\sigma_{c,d}$ is the design compressive stress; $f_{c,0,d}$ is the design compressive strength parallel to grain and $k_{c,z}$ is given by Eq. (9).

The relative slenderness for bending should be taken as:

$$\lambda_{rel,m} = \sqrt{\frac{f_{mk}}{\sigma_{m,crit}}} \tag{13}$$

where $\sigma_{m,crit}$ is the critical bending stress calculated according to the classical theory of stability, using 5-percentile stiffness values. The critical bending stress should be taken as:

$$\sigma_{m,crit} = \frac{M_{y,crit}}{W_y} = \frac{\pi\sqrt{E_{0.05}I_zG_{0.05}I_{tor}}}{L_{ef,z}W_y} \tag{14}$$

where $E_{0,05}$ is the fifth percentile value of modulus of elasticity parallel to grain; $G_{0,05}$ is the fifth percentile value of shear modulus parallel to grain; I_z is the second moment of area about the weak axis *z-z*; I_{tor} is the torsional moment of inertia; l_{ef} is the effective length of the beam, depending on the support conditions and the load; and W_y is the section modulus about the strong axis *y*. The quantity k_{crit} is a factor which takes into account the reduced bending strength due to lateral buckling. k_{crit} may be determined from the expression:

$$k_{crit} = \begin{cases} 1.0 & \text{for } \lambda_{rel,m} \le 0.75 \\ 1.56 - 0.75\lambda_{rel,m} & \text{for } 0.75 \le \lambda_{rel,m} \le 1.4 \\ \dfrac{1}{\lambda_{rel,m}^2} & \text{for } \lambda_{rel,m} > 1.4 \end{cases} \tag{15}$$

where $\sigma_{m,y,d} = M_{y,Ed}/W_y$, where $M_{y,Ed}$ is here the maximum initial moment according to first-order theory, which coincides with maximum deflection of the beam/column and $\sigma_{c,0,d} = N_{Ed}/A$, where N_{Ed} is the design normal force and A is the cross-sectional area of the structural member. Equation (14) is derived on the basis that the torsional rotation is prevented at its supports. The factor k_{crit} can be taken as 1.0 for a beam/column where lateral displacement of its compressive edge is prevented throughout its length.

3 Theoretical background

Below is a presentation of the theoretical background of the investigated problem. The purpose is to present the design requirements for members subjected to combined bending and axial compression parallel to the grain with risk for flexural–torsional buckling and torsional buckling.

3.1 Flexural–torsional buckling

For laterally supported members according to Fig. 1, the elastic flexural–torsional buckling force or critical load can be calculated according to the classical theory of stability. It may be expressed as [14]:

$$N_{cr,FT} = \frac{\left[\left(EI_y b_y^2 + EI_z b_z^2\right)\left(\pi/L_{cr}\right)^2 + GI_t\right]}{b_y^2 + b_z^2 + i_p^2} \tag{16}$$

where b_y and b_z are distances from load's application point to section shear centre, parallel to y- and z-axis, respectively; L_{cr} is the relevant buckling length for the torsional buckling mode; i_p is the polar radius of gyration which in this case reads:

$$i_p = \sqrt{\frac{I_y + I_z}{A}} \tag{17}$$

where I_z and I_y are the second moments of area about the weak axis z-z and strong axis y-y respectively; G is the shear modulus, which should here be taken as the fifth percentile value of shear modulus parallel to grain: $G = G_{0,05}$; E is the modulus of elasticity, taken as the fifth percentile value of the modulus of elasticity parallel to the grain: $E = E_{0,05}$; I_{tor} is the torsional moment of inertia; A is the cross-sectional area b is the width of the beam and h is the depth of the beam as shown in Fig. 1.

Accordingly, the flexural–torsional stress will be expressed as:

$$\sigma_{FT} = \frac{N_{FT}}{A} \tag{18}$$

The slenderness parameter λ_{FT} can now be calculated as:

$$\lambda_{FT} = \sqrt{\frac{f_{c,0,k}}{\sigma_{FT}}} \tag{19}$$

Equation (18) may further be simplified by setting $I_{tor} \approx hb^3/3$ and assuming the alignment of the lateral supports to be at the extreme edges of the member: $b_z = h/2$ and $b_y = b/2$:

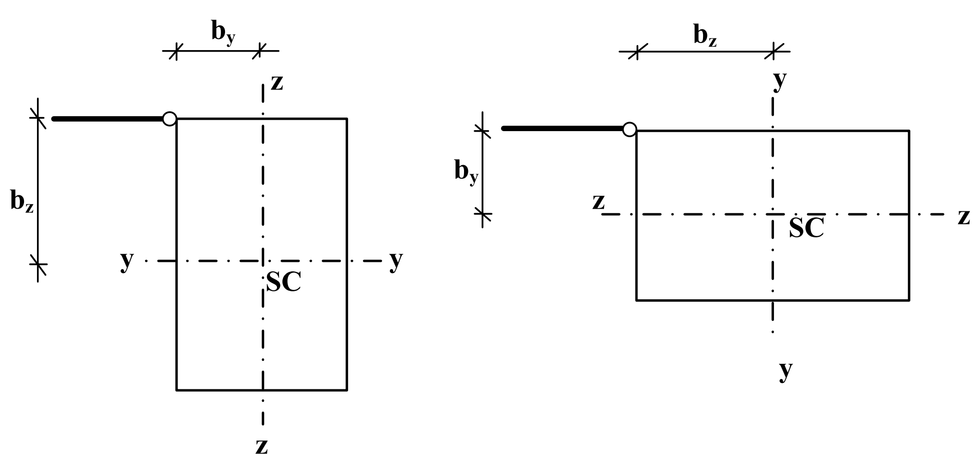


Fig. 1 Cross-section of beam supported by lateral restraints. SC is the shear centre. For doubly symmetric sections, SC coincides with the centroid

$$\sigma_{cr,FT} = \frac{b^2\left[\pi^2 h^2 E_{0.05} + 8L_c^2 G_{0.05}\right]}{8L_{cr}^2\left(b^2 + h^2\right)} \tag{20}$$

Further simplification can be obtained if $E_{0.05}/G_{0.05} \approx 16$, then Eq. (20) is reformulated as:

$$\sigma_{cr,FT} = \frac{b^2 E_{0.05}\left[2\pi^2 h^2 + L_c^2\right]}{8L_{cr}^2\left(b^2 + h^2\right)} \tag{21}$$

The buckling reduction factors k_{FT} and $k_{c,FT}$ can be obtained as:

$$k_{c,FT} = \frac{1}{k_{FT} + \sqrt{\left[k_{FT}^2 - \lambda_{FT}^2\right)}} \tag{22}$$

in which

$$k_{FT} = 0.5\left(1 + \beta_c\left(\lambda_{FT} - 0.3\right) + \lambda_{FT}^2\right) \tag{23}$$

where $\beta_c = 0.2$ for solid timber and $= 0.1$ for glue-laminated timber. The factor $k_{c,FT}$ is a factor which takes into account the reduced compressive strength due to flexural–torsional buckling.

For members subjected to the combined effect of flexural buckling and torsional buckling parallel to the grain, the stresses should in this case satisfy the following expression:

$$\frac{\sigma_{c,0,d}}{k_{c,FT} f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \tag{24}$$

Equation (24) is, typically, valid for combination of moment M_y about the strong axis y and compressive force N_c.

Equation (24) is formed as with Eq. (1) and Eq. (2) by considering a linear model, which is conservative for timber members that are characterized by having higher stiffness in the major direction than in the minor direction. Therefore, timber beam sections can safely be studied by the linear approach.

3.2 Validity of the approximate solution

In order to compare the approximate solutions as expressed by Eq. (20) and Eq. (21) with Eq. (18), different standard dimensions of timber beam of type glue-laminate timber with strength class GL32c (Table 1) are used to calculate the reduction factor, $k_{c,FT}$, Eq. (22). The beam is assumed to be free at the bottom, and the top is restrained partially from lateral displacement by single lateral supports; see Fig. 2.

The results are presented in Table 2, where one can see that the approximate solution of Eq. (20) agrees well with Eq. (18), although somewhat higher values were obtained.

Table 1 Material properties of glulam timber with strength class GL32c

Property	$f_{c,0,k}$	f_{mk}	$E_{0.05}$	$G_{0.05}$	k_{mod}
Value	24.5 MPa	32 MPa	11.2 GPa	540 MPa	0.8

Table 2 Comparison between the different solutions to calculate k_{cFT} for glulam timber with strength class GL32c

Beam dimension	k_{cFT} with Eq. (18)	k_{cFT} with Eq. (20)	k_{cFT} with Eq. (21)
1080 × 78	0.143	0.148	0.18
1215 × 90	0.161	0.166	0.199
360 × 165	0.958	0.974	0.983
1620 × 140	0.266	0.274	0.317
810 × 66	0.159	0.161	0.206
1440 × 140	0.301	0.312	0.365
1620 × 215	0.567	0.59	0.663
810 × 140	0.600	0.646	0.748
810 × 165	0.741	0.787	0.858

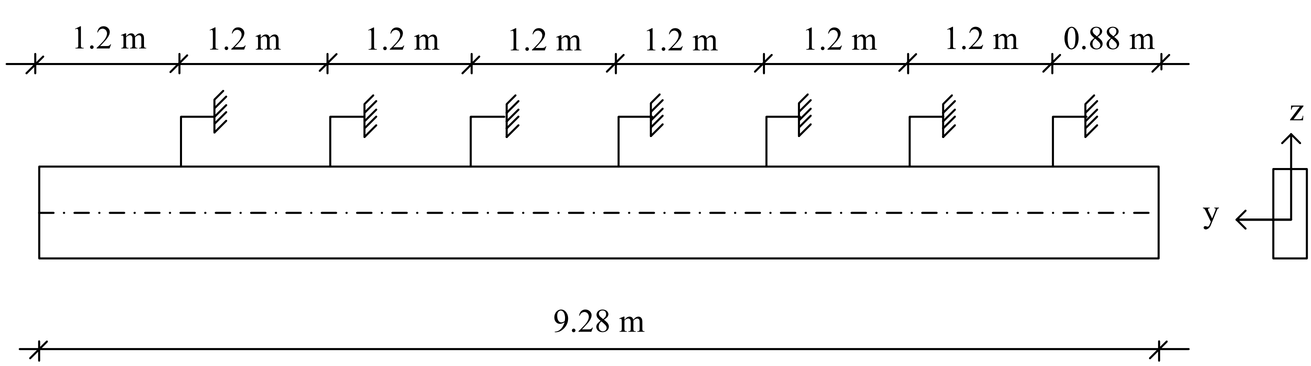


Fig. 2 Discrete bracing of the roof beam at the upper level of the beam (compressive edge) with lateral supports

The calculation difference between Eqs. (18) and (21) is relatively large. At any rate, Eq. (21) can be a useful tool if suitable $G_{0,05}$ values for the timber sections are not available.

3.3 Torsional buckling

In principle, the torsional buckling, $N_{cr,T}$ of beam-columns may also be examined to have a complete picture of the stability problem. The elastic critical load with regard to torsional buckling may be calculated according to [15]:

$$N_{cr,T} = \frac{1}{i_p^2}\left(GI_t + \frac{\pi^2 EI_w}{L_{cr}^2}\right) \tag{25}$$

where I_w is the warping constant and L_{cr} is buckling length for the torsional buckling mode. The slenderness parameter λ_T can now be calculated as:

$$\lambda_T = \sqrt{\frac{Af_{c,0,k}}{N_{cr,T}}} \tag{26}$$

The buckling reduction factors k_T and $k_{c,T}$ can be obtained as:

$$k_{c,T} = \frac{1}{k_T + \sqrt{\left[k_T^2 - \lambda_T^2\right)}} \tag{27}$$

in which

$$k_T = 0.5\left(1 + \beta_c\left(\lambda_T - 0.3\right) + \lambda_T^2\right) \tag{28}$$

In the case of combined effect of "flexural and torsional" buckling, the stresses should in this case satisfy the following expression:

$$\frac{\sigma_{c,0,d}}{k_{c,T} f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \tag{29}$$

Alternatively, the value of elastic critical load may be taken as the smallest of torsional buckling, $N_{cr,T}$ and flexural–torsional buckling $N_{cr,FT}$ and execute computations accordingly.

However, in many cases in practice, it is sufficient to control only the flexural–torsional buckling and flexural buckling. Consequently, Eq. (29) may be considered in this case as an extra stability check. It is worth mentioning that there is no need to control Eqs. (24) and (29), if lateral displacements of member's two sides are prevented throughout its length (continuous bracing) and torsional rotation is prevented at its supports.

4 Example application

An industrial timber building with a plan of 18 m × 30 m with the applied vertical loads is shown in Fig. 3. The frames are spaced at 6 m centre-to-centre. Timber purlins sit over the frames at 1.2 m c/c, supporting the roof construction. The purlins have a cross-section of 90 mm × 270 mm and the rafter beams' cross-section is 140 mm × 810 mm ($b \times h$). The purlins function as bracing restraints and are placed at the compressed edge, mainly to reduce the effects of the lateral-torsional buckling but also to minimize the flexural buckling of the beam around the weak direction, *z-z* axis. The other beam edge level is not is restrained so that torsional displacement could occur throughout its length, as shown in Fig. 2. The column height is 4 m with cross-section 140 mm × 315 mm. The columns are provided with external wall construction between the columns at 6 m c/c. The columns and beams are made of timber of class GL32c, see Table 1 for material properties. There are hinges between beams and column-beams. The design value of the effects of actions for the roof beams, q_{Ed} for ultimate limit state is determined using structural load combination as stated in Eurocode 5 [1] as follows. The distributed loads on the beams are calculated using a partial safety factor, $\gamma_d = 1.0$, characteristic permanent action on the beam, $g_k = 3.68$ kN/m, and characteristic value of snow action on the beams, $S = 12$ kN/m. Accordingly,

$$\begin{aligned} q_{Ed} &= \gamma_d.1.2g_k + \gamma_d.1.5S \\ &= 1.0 \times 1.2 \times 3.68 + 1.0 \times 1.5 \times 12 \\ &= 22.42\text{kN/m} \end{aligned}$$

The distributed loads on the columns are calculated using a partial safety factor, $\gamma_d = 1.0$, and characteristic value of wind action on the columns, $W = 2.88$ kN/m. Accordingly,

$$q_{Ed} = \gamma_d.1.5W = 1.0 \times 1.5 \times 2.88 = 4.32\,\text{kN/m}$$

The moment and normal force diagrams are shown in Figs. 4 and 5.

The task is to check the stability of roof beams with respect to buckling failure in accordance with Sects. 2 and 3. The design section for the interaction formulae is taken at $x = 4.64$ m, so the design forces can be read out of Figs. 4 and 5 as $N_{Ed} = 417.8$ kN and $M_{Ed} = 227$ kNm, where $M_{y,Ed}$ is here the maximum initial moment around the strong axis (*y*-axis) according to first-order theory, which coincides with maximum deflection, $\delta_{max} = 68.12$ mm of the beam. The flexural–torsional buckling load is calculated according to Eq. (16).

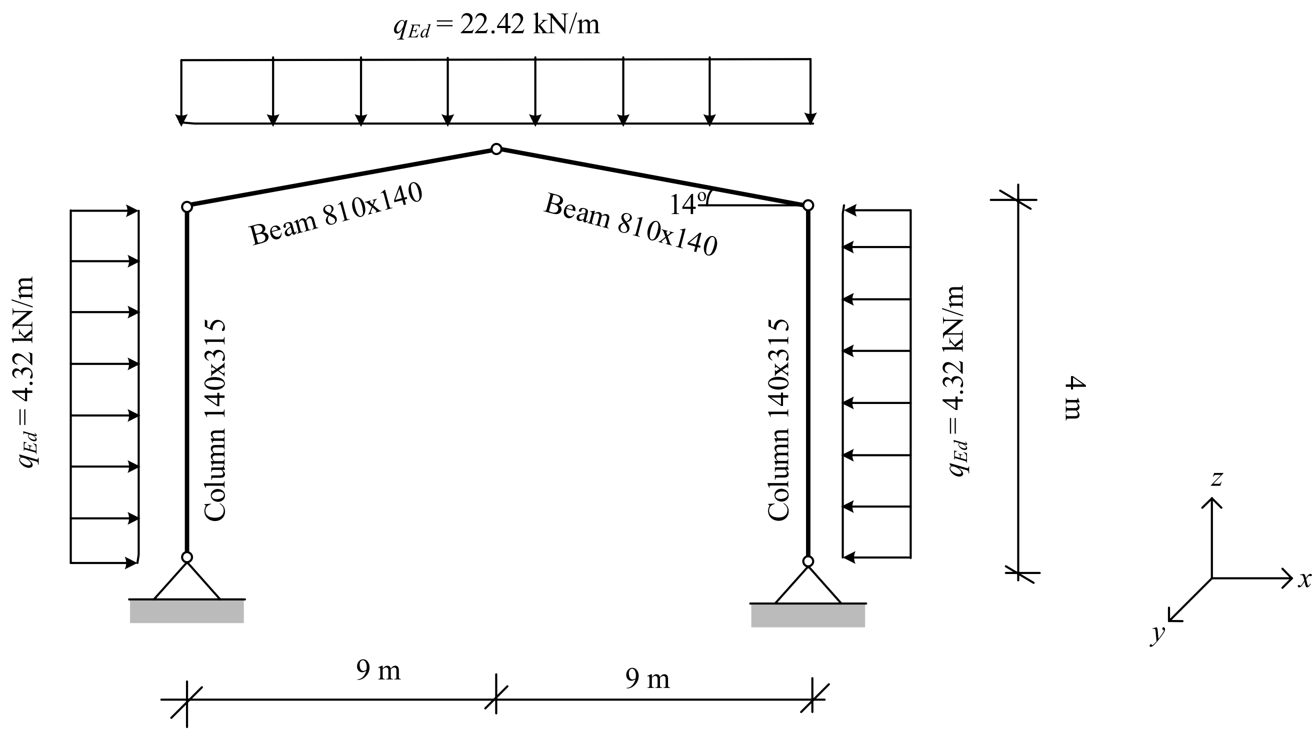


Fig. 3 Cross section through building structure with the applied variable characteristic actions

Fig. 4 Bending moment diagram of the frame structure. All units are kNm

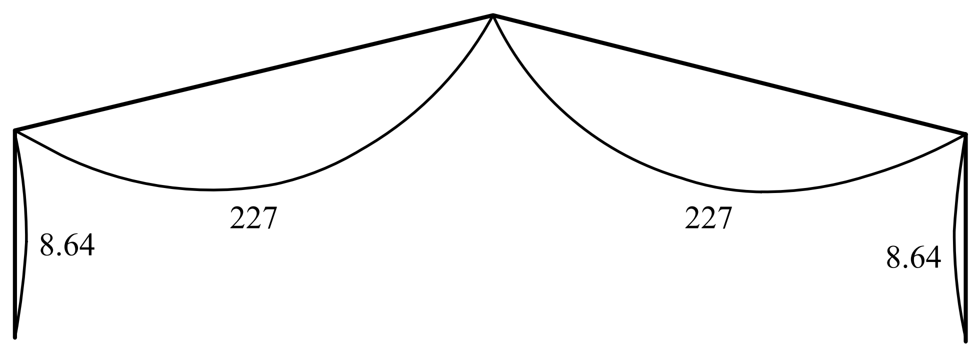
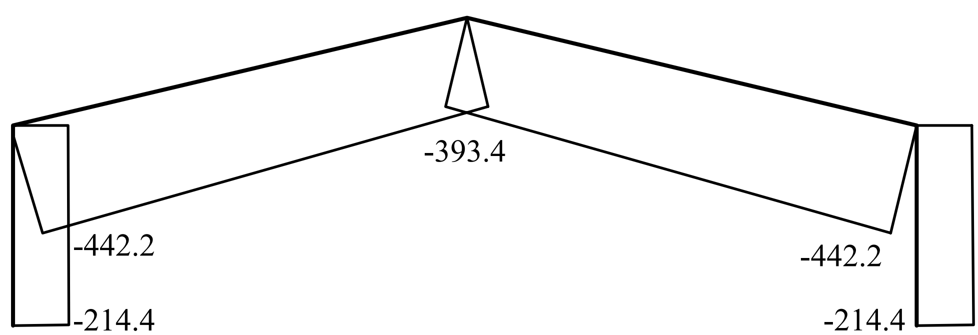


Fig. 5 Normal force diagram of the frame structure. All units are kN



The buckling lengths for flexural buckling modes around *y-y* and *z-z* can be taken as 9.28 m and 1.2 m, respectively. The buckling length for the torsional buckling mode varies here due to different unbraced lengths at both sides. It is, however, sufficient to check the worst case with L_{cr} = 9.28 m.

Table 3 Parameters to calculate the interaction formulae

Quantity	$k_{c,z}$	$k_{c,y}$	$k_{c,FT}$	$k_{c,T}$	k_{crit}
Result	0.98	0.96	0.60	0.94	1.0

Table 4 Results of the interaction formulae the roof beam

Interaction formula	Flexural buckling around y-y Eq. (1)	Flexural buckling around z-z Eq. (2)	Flexural and lateral-torsional buckling Eq. (12)	Flexural-torsional buckling Eq. (24)	Flexural and torsional buckling Eq. (29)
Result	0.97	0.75	0.76	1.12	0.97

Table 5 The effect of beam dimensions on beam stability

Beam dimensions (mm)	Flexural buckling around y-y Eq. (1)	Flexural buckling around z-z Eq. (2)	Flexural and lateral-torsional buckling Eq. (12)	Flexural-torsional buckling Eq. (24)	Flexural and torsional buckling Eq. (29)
855 × 140	0.88	0.68	0.65	1.05	0.89
810 × 165	0.82	0.63	0.58	0.88	0.82

5 Results and discussion

The calculated results are shown in Tables 3 and 4. As can be seen, the results of the interaction formulae indicate that although the stability design requirements for flexural buckling and "flexural and lateral-torsional" buckling are satisfied, the result of Eq. (24) indicates that there is a risk of beam instability due to the effect of flexural–torsional buckling mode, which is dominant in this case. While the stability of the beam is accepted in view of Eurocode formulation Eqs. (1), (2) and (12), it is not accepted in view of Eq. (24). Note that since the value of k_{crit} = 1.0, there will be no risk for lateral-torsional buckling; however, the calculations are carried out according to Eq. (12) merely for comparison.

To further investigate the influence of different parameters on the stability criteria, a parametric study is performed.

5.1 Parametric study

The effect of beam dimensions (*h* x *b*) on beam stability are presented in Table 5. Here the choice was made to investigate the case where *h* and *b* are increased to nearest standard dimension. The results indicate that the flexural–torsional buckling mode of the beam is most affected by beam width. By increasing *b* from 140 to 165 mm, all the stability criteria will be satisfied. Evidently, inspection of Eq. (20) can verify this conclusion. Moreover, beam width seems to influence the other stability criteria to a greater extent than beam depth. It is interesting to see that the value of "flexural and torsional" buckling, Eq. (29), is almost comparable to the value of flexural buckling around the strong axis *y-y*.

The effect of bracing of the roof beam is presented further in Table 6. In this study, the lateral supports are placed in the weak direction, considering four different cases:

Case 1. Discrete restrains are placed on both sides of the beam with the same distribution, at 1.2 m, see Fig. 6. In this case, L_{cr} = 1.2 m, $L_{ef,y}$ = 9.28 m, $L_{ef,z}$ = 1.2 m.

Case 2. Continuous bracing is used at the bottom side of the beam, while discrete restraints are placed at the

Table 6 The effect of lateral supports on beam stability. Beam dimension 810 × 140, strength class GL32c

Case	Flexural buckling around y-y Eq. (1)	Flexural buckling around z-z Eq. (2)	Flexural and lateral- torsional buckling Eq. (12)	Flexural- torsional buckling Eq. (24)	Flexural and torsional buckling Eq. (29)
Case 1	0.97	0.75	0.76	0.96	0.97
Case 2	0.97	Continuous side support (out of plane), no flexural buckling in this direction possible	0.76	0.96	0.97
Case 3	0.97	Continuous side support (out of plane), no flexural buckling in this direction is possible	Side support condition prevents lateral torsional buckling	Side support condition prevents flexural torsional buckling	Side support condition prevent torsional buckling
Case 4	0.97	Continuous side support (out of plane), no flexural buckling in this direction possible	Side support condition prevent lateral torsional buckling	0.96	0.97

upper side at distances of 1.2 m. In this case, $L_{cr} = 1.2$ m, $L_{ef,y} = 9.28$ m.

Case 3. Continuous bracing is used at both sides of the beam. In this case, $L_{ef,y} = 9.28$ m.

Case 4. Continuous bracing is used only at the top side of the beam (compressive edge) while discrete lateral supports are placed at the bottom side of the beam at distances of 1.2 m. In this case, $L_{cr} = 1.2$ m, $L_{ef,y} = 9.28$ m.

As can be seen, the continuous side support at both beam sides will prevent the modes activation of torsional and flexural–torsional buckling. From an instructive point of view, the buckling length of the flexural–torsional buckling mode is a decisive factor in this context. It depends on the unbraced length between lateral supports at both sides of the beam. Moreover, the continuous bracing of the compressive side of the beam will prevent flexural and lateral torsional buckling of the beam. From an instructive point of view, Eq. (12) is based on the fact that nonlinear addition of stresses is made so that no plastic behaviour should be allowed to occur under the effects of the axial load but is allowed under the effect of the moment about the major axis. The interaction between axial load and moment (at failure) is based on a model considering plastic behavior.

In the previous results presented in Tables 4, 5 and 6, it is shown that "flexural and torsional buckling", Eq. (29), is not a decisive factor for the stability criteria, as mentioned earlier. However, it is worth noting that the interaction formula for "flexural and torsional" buckling yielded sometimes relatively higher value than the interaction formula for flexural–torsional buckling. This is mainly due to the shortness of the buckling length of torsional buckling mode.

As a final observation, there is no need to control the flexural–torsional buckling if lateral displacements of member's two sides are prevented throughout its length (continuous bracing) and torsional rotation is prevented at its supports.

6 Concluding remarks

This study investigates the load-bearing behaviour of timber members, subjected to simultaneously acting axial compression and bending moment, with risk for torsional and/or flexural–torsional buckling. In certain design situation, the flexural–torsional buckling can reduce the compressional capacity of the member. Consequently, the flexural–torsional buckling mode of the member should be checked to ensure that the lateral supports and bracing system are sufficient to withstand the flexural–torsional buckling of the unbraced part of the member. This case can be evident for long members in frame structures, which are subjected to high axial compression combined with bending moments in which the member is not sufficiently braced at both sides. To control this situation, interaction formula of flexural–torsional buckling may be determined. Technically, flexural–torsional buckling will occur about the weak and strong directions (z and y-axis), depending on the applied moments at both directions. The smaller of the two will govern the design strength.

It has been shown in the examples provided that although the design requirement for flexural buckling is satisfied, a problem with flexural–torsional buckling can arise. The results indicate that the flexural–torsional buckling mode of the beam is mostly affected by the beam width and the bracing system. To minimize the risk of flexural–torsional buckling in braced members, the beam-columns can be provided with lateral supports at both sides of the beams, at suitable distances between the restraints. Continuous side supports at both beam-column sides is the best method to tackle the problem. This will not only prevent flexural–torsional buckling, but also "flexural and

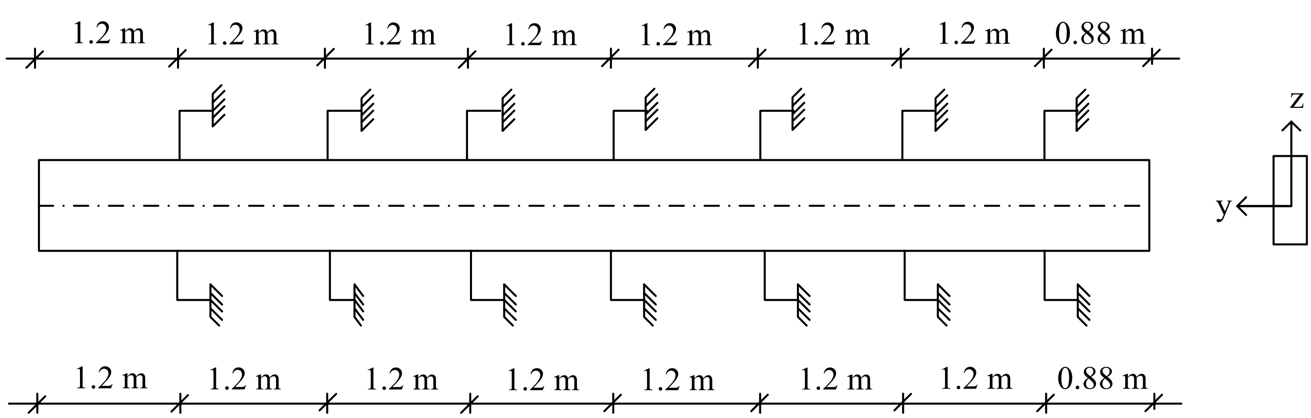


Fig. 6 Discrete bracing of the roof beam at the upper and bottom level of the beam with lateral supports

lateral-torsional buckling", and even "flexural and torsional buckling".

Continuous lateral bracing can be provided by some types of roofing and stabilizing systems. However, this situation depends on the roofing component's thickness and configuration in addition to attachments/connections and joints for the purlins. This can be acceptable if only lateral side bracing in the weak direction is required. Alternatively, if a bracing system is not provided at both sides of the member, increasing the member width will increase the stability of the member exposed to both axial forces and bending moment.

An approximate expression for the flexural–torsional stress has been derived for the case of rectangular timber sections, which can be a practical tool if suitable $G_{0,05}$ values for the timber sections are not obtainable.

From the perspective of architectural and structural engineering education, the analytical modelling discussed in this paper may be used by tutors to develop suitable case studies for architectural and building engineering students. It is demonstrated how problems, not easily found in the handbook literature, can be solved analytically without too much effort. Technically, by investigating the effect of flexural–torsional buckling of laterally braced timber, structural engineer can achieve safer designs for the stability of timber beam-columns.

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Declaration

Conflict of interest The author declares no potential conflict of interest in the manuscript.

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