Pressure and Level Control of a River Water Pumping Station

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Master of Science Thesis in Electrical Engineering

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Abstract

The river water pumping station at SSAB in Borlänge is a critical part of the factory since it supplies the whole factory with cooling water. The problem with the river water pumping station is that the pressure in the pipes is very dependent on the large water consumers in the factory. The large consumers causes large variations in pressure and water level when they suddenly turns on and off. The second problem in the river water pumping station is that it can not pump enough water to the consumers during the warmer periods of the year. The aim of this thesis is to improve the control system in the river water pumping station by first creating a model of the system which then can be used to test a new controller. The model is verified against measurements from the real process.

The results show that the developed model captures the general behavior of the system. Further analysis of the system show that a smaller pump could be the cause of the problem in the control system. In the development of the new controller the smaller pump was then removed and replaced with a larger pump. The new and old controller perform similarly when it comes to pressure and flow rate however the new controller is slightly better when taking the control signal into account because it does not make any large sudden changes.
Acknowledgments

This thesis marks the end of my education at Linköping University. I have gained new friends whom has been an invaluable support through the education and are the reason why these last five years has been the best years of my life.

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Borlänge, June 2021
Lilli Zickerman Bexell
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### Abbreviations

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<tr>
<td>RPM</td>
<td>Revolutions per minute</td>
</tr>
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<td>LI</td>
<td>Level indicator</td>
</tr>
<tr>
<td>LIC</td>
<td>Level indicator controller</td>
</tr>
<tr>
<td>TI</td>
<td>Temperature indicator</td>
</tr>
<tr>
<td>PI</td>
<td>Pressure indicator</td>
</tr>
<tr>
<td>PIC</td>
<td>Pressure indicator controller</td>
</tr>
<tr>
<td>ARMAX</td>
<td>Auto-regressive moving average model with exogenous inputs</td>
</tr>
<tr>
<td>ARX</td>
<td>Auto-regressive model with exogenous inputs</td>
</tr>
<tr>
<td>BJ</td>
<td>Box-Jenkins</td>
</tr>
<tr>
<td>OE</td>
<td>Output-Error</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional, integral, differential (controller)</td>
</tr>
<tr>
<td>SP</td>
<td>Set point</td>
</tr>
<tr>
<td>PV</td>
<td>Process value</td>
</tr>
<tr>
<td>MV</td>
<td>Manipulated value</td>
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The purpose of this chapter is to give an introduction to the master thesis and how the river water pumping station works.

1.1 Background

SSAB in Borlänge manufactures hot rolled and cold rolled high strength steel of slabs from mostly Luleå and also sometimes from Oxelösund. The river water pumping station at SSAB in Borlänge supplies the whole factory with cooling water which makes it an important and critical part of the factory. The problem with the river water pumping station is that the water pressure in the pipes is very dependent on the large water consumers in the factory. When a large consumer instantaneously turns on, the water pressure in the pipes leading to the consumers suddenly drops which causes the pumps to accelerate. Additionally, very large consumption of water can lead to the hatch between the reservoir and river not opening fast enough, there is then a risk that the water level in the reservoir sinks too low. The opposite happens when a large water consumer suddenly turns off. The pressure in the pipes leading to the consumers becomes too high, the pumps slow down which leads to the water level in the reservoir raising too fast and becomes too high. In other words the problem is that there are too large variations in pressure and the water level. There is also a problem that the river water pumping station can not pump enough water to the consumers during the warmer periods of the year.
1.2 Problem Formulation

The purpose of the thesis is to make a model of the water pumping station in order to develop a better way to control the system. The questions this thesis aims to answer are the following:

- What is it in the current control system that causes the control problems?
- How do the dynamics in the system affect the choice of control structure and is there a better way to control this system?
- How should the natural nonlinearities in the system be handled?

1.3 Delimitations

In this project it is necessary to limit what is seen as a large water consumer since there are a lot of consumers in reality and it would take too much time to model all of them. In this thesis all of the consumers that use less water than $300 \text{m}^3/\text{h}$ are considered small and will not be part of the modeled system even though they are part of the real system. Only regular operation modes are treated in the development of the model and the controller. In other words, all special cases that are important in reality in case there is a breakdown are not part of the model or considered in the development of the controller.

1.4 Related Work

Pressure control in water distribution networks is a widely researched area due to its importance in both industry and water supply to households [12]. Much of the work concerning pressure control in water distribution networks is focused on different control strategies to reduce water loss and leakage in water distribution networks [4, 10, 14]. Most of them use pressure control valves that control the pressure in a certain part of the network. There are several different types of controllers that can be used to control the pressure in water distribution networks and in [8] many of them are introduced and compared.

Another interesting aspect in pressure control systems is the efficiency concerning the control of the pumps because pumps are responsible for a large amount of the electricity consumption in the world [15]. In [5] it has been shown that the most cost effective and efficient way to control the pressure in a pipe is with a pump that has a variable speed drive. It is proved in [16] that the most efficient way to control parallel pump systems is with the same control signal to all pumps, if the pumps are of the same size and more than one is turned on. There have also been a few studies on how to optimize the work schedule for the pumps, i.e. when to turn them on and off. In [11] an optimal method based on the pumps individual efficiency is developed, in [17, 20] genetic algorithms are used and in [19] adaptive and gradient-based algorithms are used.
In this chapter, the river water pumping station is described.

2.1 General Description of the System

In Figure 2.1 a simplified overview of the river water pumping station is presented, which shows how the water flows through the river water pumping station.

*Figure 2.1: An overview of how the water flows through the river water pumping station.*

The water from the river is directed into the pump station through an inlet port...
from the weir. The water is cleaned from larger materials in a wreck grate before the inlet. The water level in the weir is measured after the wreck gate.

There is an inlet hatch, placed in the inlet port after the wreck gate that controls the water flow into the pump station. The inlet hatch is a hydraulic hatch and it is controlled by the water level in the pump well such that there is a continuous overflow to an outlet tunnel. The inlet hatch can maximally open 2.5 m but SSAB is only allowed to take a certain amount of water from the river, because of this the opening of the hatch is limited in the control system and can only open 1.465 m. There is also a limit in the system such that the inlet hatch cannot close more than 0.17 m to make sure that there always is a flow of water to the pumping station.

The hatch has 50 fixed positions and there are 50 mm between each position and these positions correspond to a change in flow of 450-750 $m^3/h$. The hatch opens and closes depending on the water level in the pump well and the overflow to the outlet tunnel. The hydraulic components that control the inlet hatch are a hydraulic pump, oil cooler, warmer, valves for control and corresponding guards. All of the automatic hydraulics are controlled by the central PLC.

After the inlet hatch, the water is directed into the pump station and through a second grate with a column width of 15 mm. The purpose of this grate is once again to clear the water from materials. If there are any larger problems, such that the water cannot flow through the grate the water can be led directly to the pump well.

The water is then led to four disc filters after the second grate. The disc filters have a filtering degree of 100 $\mu m$ and the smaller particles are separated from the water in these disc filters. The disc filters are frequency controlled and rotate continuously with an adjustable velocity and in normal operation all of the disc filters are used. The water level through the disc filters and the up stream distribution conduit, are controlled by the stationary overshoot over the disc filters, the height of the overshoot and the fall of pressure. The level before the disc filters is mostly controlled by the pressure drop through the filter and after that the overshoot. An overflow hatch will open and let water into the pump well if the water level before the disc filters is high and if the water level is really there will be an overflow over an edge.

The water is directed to the pump well after it has passed through the disc filters and the water level in there is controlled by the inlet hatch, as said before, by measuring the water level.

There are six pumps in the pump station, four of them are for a larger distribution system and two are for a smaller. All of these pumps take water from the pump well. The pressure in the large distribution system should be between 4 and 5 bar and the pressure in the smaller system should be around 6 bar. The pumps are pressure and variable-speed controlled to keep the pressure constant in the two systems.
There is also a diesel pump that starts if there is a power failure to provide the factory with emergency cooling water. The diesel pump starts when the pressure in the 5 bar system is less than 4 bar or if the pressure is less than 5 bar in the 6 bar system. The back-up system first prioritizes the 6 bar system and then the 5 bar system. The diesel pump has an autonomous control system with a separate control cabin and in addition, it also has its own battery powered starter motor, cooling system and pressure sensor.

2.2 Current Control System

The system is currently controlled by two separate controllers, one controller controls the pressure in the pipes and the second controller controls the water level in the pump well

2.2.1 Pressure Control in the Pipes

The pressure in the pipes is controlled with a PID-controller that controls four pumps, three large pumps and one smaller and all of these work in parallel with each other. The input to the PID-controller is the difference in pressure between the set point (SP) and the process value (PV) in the pipes and the manipulated value (MV) from the controller is a value between 0-100 %. The output from the PID-controller is rescaled to rpm which is used as the input to the pumps. One of the larger pumps is always on, if the pressure drops the smaller pump is turned on first then the other large pumps if necessary. If the pressure increases too much and all of the pumps are on, the small pump will turn off first, then the larger pumps where the one that was turned on last will turn off first and so on.

2.2.2 Level Control in the Pump Well

The water level in the pump well is controlled by opening and lowering the hatch in the inlet channel. The hatch opens one step (50 mm) if the water level is lower than 112.65 meters above sea level and closes one step (50 mm) if the water level is above 112.72 meters above sea level. Once the inlet hatch has changed position there will be a pause of 30 seconds before feedback and no new control will be applied within the next 8 minutes.

2.3 Model Structure

The flow through the river water pumping station during normal operation is through the grates, inlet hatch, filters, then down to the pump well and after that to the consumers through the pumps and pipes. As said before, only large consumers will be considered in the model and there are only two large consumers both of which are using the 4-5 bar system. Considering all of this, the river water pumping station can be modeled as in Figure 2.2 where the inlet hatch can be seen as a valve, the inlet channel can be seen as a tank, the filters as an orifice
followed by a tank and finally the pump well which can also be seen as a tank. The consumers can be seen as valves at the end of the pipes. The theory for all of the modeled parts is described later in Chapter 3.

![Diagram of the river water pumping station](image)

**Figure 2.2: Model of the river water pumping station.**

### 2.4 Dimensions and Measurements

In order to be able to model the system is it necessary to know the dimensions of the inlet channel, pump well, pipes etc. The inlet channel, pump well and filter tanks all have a cuboid form. It is also necessary to know what kind of sensors there are and where they are placed. Table 2.1 shows the dimensions of the tanks and inlet hatch. Figure 2.3 gives an overview of the location of the sensors, the type of sensor and if the sensor is connected to a controller, the figure also shows the dimensions of the pipes.

**Table 2.1: Dimensions of the different parts of the river water pumping station.**

<table>
<thead>
<tr>
<th>Dimensions</th>
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<tbody>
<tr>
<td>Inlet channel</td>
<td>Bottom area: 76.779(m^2)</td>
</tr>
<tr>
<td>Filter tank</td>
<td>Bottom area: 23.6925(m^2)</td>
</tr>
<tr>
<td>Pump well</td>
<td>Bottom area: 192.825(m^2)</td>
</tr>
<tr>
<td>Inlet hatch</td>
<td>Width: 2m</td>
</tr>
</tbody>
</table>
Figure 2.3: Dimensions of the pipes, the location of the sensors and type of sensor in the river water pumping station.
In this chapter, the theory that is used to model the system is introduced and the method to validate the model is described. The model was implemented in MATLAB Simulink.

### 3.1 Tank

Figure 3.1 shows a tank with a flow in, $q_{in}$, and out $q_{out}$ and the volume of fluid in the tank at time $t$ is $h(t)A$. The rate at which the volume of fluid in the tank changes depends on the flow rate in and out of the tank and this can be described by:

$$\frac{d}{dt}Ah(t) = q_{in}(t) - q_{out}(t) \quad (3.1)$$

### 3.2 Orifice

A model of an orifice can be derived from the Bernoulli Equation (3.2), where it is assumed that there is no difference in elevation.

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad (3.2)$$

In (3.2), $\rho$ is the density of the fluid, $p_1$ and $v_1$ is the pressure and velocity of the fluid before the orifice and $p_2$ and $v_2$ is the pressure and velocity of the fluid after
Figure 3.1: A model of a tank with a flow in and out.

the orifice. Figure 3.2 illustrates the flow of a liquid through an orifice and where the measuring points $p_1, v_1, p_2$ and $v_2$ are.

Figure 3.2: A pipe with an orifice in it.

The flow rate in and out of the orifice must be the same according to the continuity equation which means that $q = A_1 v_1 = A_2 v_2$. Replacing the velocities with flow rate divided by the area and solving for $q$ gives the following equation:

$$q = A_2 \sqrt{\frac{2}{1 - \left(\frac{A_1}{A_2}\right)^2}} \sqrt{\frac{p_1 - p_2}{\rho}}$$  (3.3)

The $A_2 \sqrt{\frac{2}{1 - \left(\frac{A_1}{A_2}\right)^2}}$ term is usually replaced with $k_v$. The orifice and valve are modeled the same way except that for a valve the area changes and $k_v$ then depends on how open the valve is. Equation 3.3 then becomes the following equation:

$$q = k_v \sqrt{\frac{p_1 - p_2}{\rho}}$$  (3.4)
3.3 Flow Over an Edge

A model for flow over an edge can also be derived from the Bernoulli equation written on the form as in (3.5), where $z_0$ is the highest level of the water above the edge, $z$ is any point of the water above the edge, $v$ is the velocity at point $z$ and $v_0$ is the velocity at point $z_0$, Figure 3.3 shows water flow over an edge. [6]

\[
\left( z_0 g + \frac{v_0^2}{2} \right) - \left( z g + \frac{v^2}{2} \right) = 0 \implies v = \sqrt{2g} \sqrt{z_0 g + \frac{v_0^2}{2g} - z} \tag{3.5}
\]

Figure 3.3: A figure of the flow over an edge.

This equation is based on the assumptions that $v_0$ is uniform and parallel, the streamlines are horizontal above the edge and that the flow is frictionless. If the width of the edge is $b$ and the height of a cross-sectional flow area is $dz$, the volumetric flow rate through this cross-sectional flow area can then be written as in the following equation: [6]

\[
dQ = bvdz \tag{3.6}
\]

Substituting $v$ in (3.6) with (3.5) gives:

\[
dQ = b\sqrt{2g} \sqrt{z_0 g + \frac{v_0^2}{2g} - z}dz \tag{3.7}
\]

The $b$ in (3.7) is constant which makes it possible to integrate the equation. This results in:
\[ Q = -\frac{2}{3} b \sqrt{2g} \left( z_0 g + \frac{v_0^2}{2g} - z \right)^{3/2} + c \]  

The constant \( c \) can be calculated when \( z = 0 \) which also leads to \( Q = 0 \). Given \( c \) and that the approach velocity \( v_0 \) can be neglected [6], the flow rate for when \( z = z_0 \) is then given by the following equation:

\[ Q = \frac{2}{3} b \sqrt{2g} (z_0)^{3/2} \]  

(3.9)

The flow rate \( Q \), in practice, is given by (3.10), where \( C_d \) is the dimensionless discharge coefficient [6].

\[ Q = \frac{2}{3} C_d b \sqrt{2g} (z_0)^{3/2} \]  

(3.10)

### 3.4 Fluid Mechanics

Fluid mechanics is the part of mechanics that describes the movements of fluids, where fluids are gases or liquids.

The equations that describe the movement of a fluid is given by two physical laws, the first is the law of the indestructibility of mass, which gives the continuity equation. The second law is Newton’s second law, \( \text{force} = \text{mass} \cdot \text{acceleration} \), which gives the equations of motion.

Many flow processes are of a steady state type of flow which means that hydrodynamic properties such as velocity and pressure drop are independent of time, except at perhaps process start-up or shut-down. Unsteady flow is when hydrodynamic properties changes with time. The flow in this case is unsteady because the flow in the pipe is constantly disturbed by the pumps and by valves suddenly opening and closing, this results in sudden changes in pressure and velocity. This type of flow is called transient flow. [13]

#### 3.4.1 Equation of Motion

To derive the equation of motion for transient flow in a pipe the small control volume in Figure 3.4 is considered [21]. The area of the cross section in the control volume is \( A \), the pressure is \( p \), the diameter is \( D \), the velocity is \( V \), the shear stress is \( \tau_0 \), the density is \( \rho \), the gravity is \( g \), the inclination of the pipe is \( \alpha \) and \( dx \) is the length of the control volume. The hydraulic grade line, \( H \), also called head is a measurement of pressure above the reference point. The shear stress acts along the pipe wall and is a friction that counter-acts the fluids motion. The pressure, velocity and head depends on distance, \( x \), and time, \( t \).
The forces acting on the control volume in the $x$-direction are the normal pressure forces on the transverse surfaces, the shear force, the gravitational force and the pressure along the pipe wall [21], see Figure 3.4, the equations are shown in the following equation:

\[
\begin{align*}
\text{Normal pressure force:} & \quad pA - \left( pA + \frac{\delta p}{\delta x} A dx \right) \\
\text{Pressure force along pipe wall:} & \quad \left( p + \frac{\delta p}{\delta x} \right) \frac{\delta A}{\delta x} dx \\
\text{Shear force:} & \quad - \tau_0 \pi D dx \\
\text{Gravity force:} & \quad - \rho g \left( A + \frac{\delta A}{\delta x} \right) \frac{dx}{2} dx \sin \alpha
\end{align*}
\]  

(3.11)

Newton’s second law and the equations in (3.11) result in:

\[
\rho A dx \frac{dV}{dt} = pA - \left( pA + \frac{\delta p}{\delta x} A dx \right) + \left( p + \frac{\delta p}{\delta x} \right) \frac{\delta A}{\delta x} dx - \tau_0 \pi D dx - \rho g \left( A + \frac{\delta A}{\delta x} \right) \frac{dx}{2} dx \sin \alpha
\]

(3.12)

The expression in (3.12) are simplified to (3.13), where all $dx^2$ can be neglected because $dx$ is very small, which means that $dx^2$ is close to zero.

\[
\frac{\delta p}{\delta x} Adx + \tau_0 \pi D dx + \rho g Adx \sin \alpha + \rho Adx \frac{dV}{dt} = 0
\]

(3.13)

The shear stress, $\tau_0$, is calculated from the Darcy-Weisbach equation, see (3.14),
where $L$ is the length of the pipe and $f$ is the dimensionless friction factor called Darcy friction factor. The Darcy-Weisbach equation is an empirical equation that relates the pressure loss that comes from friction along a pipe of a certain length where an incompressible fluid flows with an average velocity [21].

$$\Delta p = \frac{\rho f L V^2}{D}$$  \hspace{1cm} (3.14)

The force-balance in steady flow and the force from shear stress gives the following equation:

$$\Delta p \frac{\pi D^2}{4} = \tau_0 \pi DL$$  \hspace{1cm} (3.15)

The $\Delta p$ in (3.15) is exchanged with the Darcy-Weisbach equation in (3.14) and gives (3.16), where the absolute value of the velocity in the equation is there to ensure that the stress direction is always opposite to the velocity direction.

$$\tau_0 = \frac{\rho f |V|}{8}$$  \hspace{1cm} (3.16)

The $\tau_0$ in (3.13) is replaced with the expression on the left-hand-side in (3.16) which results in (3.17) and this equation is the general form of the equation of motion.

$$\frac{\delta p}{\delta x} Adx + \rho f |V| \frac{\pi D}{8} dx + \rho g Adx \sin \alpha + \rho Adx \frac{dV}{dt} = \left[ \frac{dV}{dt} = V \frac{\delta V}{\delta x} + \frac{\delta V}{\delta t} \right]$$

$$= \frac{1}{\rho} \frac{\delta p}{\delta x} + \frac{f |V|}{2D} + g \sin \alpha + V \frac{\delta V}{\delta x} + \frac{\delta V}{\delta t} = 0$$  \hspace{1cm} (3.17)

In most cases the term $V \frac{\delta V}{\delta x}$ in (3.17) is small compared to the other terms and the velocity $V$ can be replaced with the flow rate $Q = A \cdot V$, the equation of motion can then be expressed as in:

$$\frac{A}{\rho} \frac{\delta p}{\delta x} + \frac{f Q |Q|}{2AD} + \frac{\delta Q}{\delta t} + A \sin \alpha = 0$$  \hspace{1cm} (3.18)

### 3.4.2 Continuity Equation

To derive the continuity equation for transient flow in a pipe the small control volume in Figure 3.5 is considered. The $u$ in the control volume is the velocity of the pipe wall at point $x$, everything else $A$, $V$, $\rho$, $dx$, $\alpha$, $H$ is the same as before.
The control volume in Figure 3.5 is considered to be fixed relative to the pipe and only move and stretch if the surface on the inside of the pipe moves and stretches. This gives a total derivative with respect to the axial motion of the pipe which is \( \frac{D'}{Dt} = u \frac{\delta}{\delta x} + \frac{\delta}{\delta t} \) and the time rate that the length \( dx \) of the control volume changes is \( \frac{D'}{Dt} dx = \frac{\delta u}{\delta x} dx \).[21]

![Figure 3.5: Control volume for transient flow in pipe to help derive the continuity equation.](image)

The continuity equation comes, as said before, from the indestructibility of mass which means that the rate of mass in- and outflow is equal to the rate of collected mass within the control volume [21], which results in the following equation:

\[
\rho A (V - u) - \left( \rho A (V - u) + \frac{\delta \rho A (V - u)}{\delta x} dx \right) = \frac{D'}{Dt} (\rho A dx)
\]

\[
\Rightarrow \left| \frac{D'}{Dt} dx = \frac{\delta u}{\delta x} dx, \quad \frac{D'}{Dt} = u \frac{\delta}{\delta x} + \frac{\delta}{\delta t} \right| \implies (3.19)
\]

\[
= V \frac{\delta \rho A}{\delta x} + \rho A \frac{\delta V}{\delta x} + \frac{\delta \rho A}{\delta t} = 0
\]

The total derivative with respect to motion of a mass particle is given by \( \frac{D}{Dt} = V \frac{\delta}{\delta x} + \frac{\delta}{\delta t} \). Equation 3.19 can then be rewritten to (3.20) which holds for converging, diverging or flexible tubes as well as for cylindrical pipes.

\[
\frac{1}{\rho A} \frac{D}{Dt} (\rho A) + \frac{\delta V}{\delta x} = \frac{1}{A} \frac{DA}{Dt} + \frac{1}{\rho} \frac{D \rho}{Dt} + \frac{\delta V}{\delta x} = 0
\]

(3.20)
The bulk modulus of elasticity of fluids is a constant, $K$, that describes how resistant the fluid is of compression and the definition of the bulk modulus gives the following equation:

$$\frac{1}{\rho} \frac{D\rho}{Dt} = \frac{1}{K} \frac{Dp}{Dt}$$  \hspace{1cm} (3.21)

Substituting $\frac{1}{\rho} \frac{D\rho}{Dt}$ in (3.20) with the expression in (3.21) gives:

$$\frac{1}{A} \frac{DA}{Dt} + \frac{1}{K} \frac{Dp}{Dt} + \frac{\delta V}{\delta x} = \begin{cases} \text{if area change is} \quad \frac{\delta V}{\delta x} = \frac{Dp}{Dt} \left( \frac{K}{A} \frac{DA}{Dp} + 1 \right) + K \frac{\delta V}{\delta x} = 0 \end{cases} \quad (3.22)$$

The velocity, $a$, for the propagating pressure wave in transient flow in a pipe is given by (3.23) [21].

$$a^2 = \frac{K/\rho}{1 + (K/A)(DA/Dp)}$$  \hspace{1cm} (3.23)

The expression in (3.23) can be put into (3.22) which gives the following equation:

$$\frac{Dp}{Dt} + \rho a^2 \frac{\delta V}{\delta x} = 0$$  \hspace{1cm} (3.24)

The velocity can once again be replaced with the flow rate $Q = AV$ and $\frac{Dp}{Dt} = V \frac{\delta p}{\delta x} + \frac{\delta p}{\delta t}$, the term $V \frac{\delta p}{\delta t}$ is in most cases small compared to the other terms and this gives the continuity equation as in the following equation:

$$\frac{\delta p}{\delta t} + \rho a^2 \frac{\delta Q}{A} = 0$$  \hspace{1cm} (3.25)

### 3.4.3 Method of Characteristics

The method of characteristics is a way to covert the non-linear partial differential equations into ordinary differential equations. Equation 3.25 is multiplied with an unknown multiplier $\lambda$ and added with (3.18), which gives (3.26). [21]

$$\lambda \left( \frac{A}{\rho} \frac{\delta p}{\delta t} + a^2 \frac{\delta Q}{\delta x} \right) + \frac{A}{\rho} \frac{\delta p}{\delta x} + f \frac{|Q|}{2AD} + \frac{\delta Q}{\delta t} + Agsina =$$

$$\lambda \frac{A}{\rho} \left( \frac{\delta p}{\delta t} + \frac{1}{\rho} \frac{\delta p}{\delta x} \right) + \left( \frac{\delta Q}{\delta t} + \lambda a^2 \frac{\delta Q}{\delta x} \right) + f \frac{|Q|}{2AD} + Agsina = 0$$  \hspace{1cm} (3.26)
The total derivative of \( p \) and \( Q \) are given by the following equation:

\[
\begin{align*}
\frac{dQ}{dt} &= \frac{\delta Q}{\delta t} + \frac{\delta Q}{\delta x} \frac{dx}{dt} \\
\frac{dp}{dt} &= \frac{\delta p}{\delta t} + \frac{\delta p}{\delta x} \frac{dx}{dt}
\end{align*}
\] (3.27)

The multiplier \( \lambda \) can then be defined as in the following equation:

\[
\frac{1}{\lambda} = \frac{dx}{dt} = \lambda a^2 \iff \lambda = \pm \frac{1}{a}
\] (3.28)

Equation 3.26, (3.27) and (3.28) give two sets of ordinary differential equations, (3.29b), which is valid when \( \frac{dx}{dt} = a \) is satisfied and (3.30b), which is valid when \( \frac{dx}{dt} = -a \) is satisfied. In the x-t plane, \( \frac{dx}{dt} = \pm a \) are two straight lines with slopes \( \pm a \) and they are called characteristic lines.

\[
\begin{align*}
\frac{dQ}{dt} + \frac{A}{a} \frac{dp}{dt} + \frac{fQ|Q|}{2AD} + Agsina &= 0 \\
\frac{dx}{dt} &= a
\end{align*}
\] (3.29a)

\[
\begin{align*}
\frac{dQ}{dt} - \frac{A}{a} \frac{dp}{dt} + \frac{fQ|Q|}{2AD} + Agsina &= 0 \\
\frac{dx}{dt} &= -a
\end{align*}
\] (3.30a)

\[
\begin{align*}
\frac{dQ}{dt} + \frac{A}{a} \frac{dp}{dt} + \frac{fQ|Q|}{2AD} + Agsina &= 0 \\
\frac{dx}{dt} &= a
\end{align*}
\] (3.29b)

\[
\begin{align*}
\frac{dQ}{dt} - \frac{A}{a} \frac{dp}{dt} + \frac{fQ|Q|}{2AD} + Agsina &= 0 \\
\frac{dx}{dt} &= -a
\end{align*}
\] (3.30b)

### 3.4.4 Finite Difference

Finite differences method is used to be able to solve the the ordinary differential equations given by the method of characteristics. A pipe of length \( L \) is divided into \( N \) parts with length \( \Delta x = L/N \) as shown in Figure 3.6a), where \( \frac{dx}{dt} = a \) is satisfied along the \( C^+ \) diagonal and \( \frac{dx}{dt} = -a \) is satisfied along the \( C^- \) diagonal. The flow rate \( Q \) and pressure \( p \) is known at the points \( A \) and \( B \) in Figure 3.6a). This makes it possible to integrate between \( AP \) and \( PB \) which gives two equations and makes it possible to calculate the unknown flow rate and pressure at point \( P \). By multiplying (3.29b) with \( a dt = dx \) and (3.30b) with \( -a dt = dx \), it is possible to integrate along the \( C^+ \) and \( C^- \) characteristics as in (3.31).
Figure 3.6: Illustration on how flow rate and pressure is calculated at a new time instant.

The integration of $Q$ with respect to $x$ in (3.31) is unknown and is approximated with a first order approximation. The integration along the characteristic lines is then given by:

$$p_P - p_A + \frac{aP}{A} (Q_P - Q_A) + \frac{fP}{2DA^2} \int_{Q_A}^{Q_P} Q|Q| dx + \rho g \sin \alpha \int_{x_A}^{x_P} dx = 0$$

(3.31)

$$p_P - p_B - \frac{aP}{A} (Q_P - Q_B) - \frac{fP}{2DA^2} \int_{Q_B}^{Q_P} Q|Q| dx - \rho g \sin \alpha \int_{x_P}^{x_B} dx = 0$$

Solving (3.32) for $H_p$ gives (3.33), where $B = \frac{aP}{A}$ and $R = \frac{fP \Delta x}{2DA^2}$

$$C^+ : p_P = p_A - B(Q_P - Q_A) - RQ_A|Q_A| - \rho g \Delta x \sin \alpha$$

$$C^- : p_P = p_B + B(Q_P - Q_B) + RQ_B|Q_B| + \rho g \Delta x \sin \alpha$$

(3.33)

To solve the equations, initial conditions at $t = 0$ for $p$ and $Q$ are set which are usually steady state conditions. $Q$ and $p$ are then found for each grid point at time $t = \Delta t$ first then at time $t = 2\Delta t$ and so on until the desired time limit has been reached as is shown in Figure 3.6b. At all of the interior grid points are the equations in (3.33) solved simultaneously for the unknown $p_P$ and $Q_P$. The equations are written in a more simple form as in the following equation:
\[ C^+ : p_P = C_p - BQ_P \]
\[ C^- : p_P = C_m + BQ_P \]
\[ C_p = p_{i-1} + BQ_{i-1} - RQ_{i-1}\left|Q_{i-1}\right| - \rho g \Delta x \sin \alpha \]
\[ C_m = p_{i+1} - BQ_{i+1} + RQ_{i+1}\left|Q_{i+1}\right| + \rho g \Delta x \sin \alpha \]  \hspace{1cm} (3.34)

Equation 3.34 gives that \( p_P = \frac{C_p + C_m}{2} \) and \( C^- \) or \( C^+ \) can then be used to solve for \( Q_p \). The equations can be used to solve the interior points in the pipes but at the two end points some boundary conditions are needed.

### 3.4.5 Boundary Conditions

Only one of the characteristic lines holds for the boundaries, at the upstream end only \( C^- \) holds and at the downstream end only \( C^+ \) holds. An additional equation that specifies \( Q_p, p_p \) or some relation between them is needed in order to be able to solve for \( p_P \) and \( Q_P \) at the end points. The boundaries are solved independently of each other and the interior points. \[21\]

**Centrifugal Pump** If there is a centrifugal pump at the upstream end with a known pump characteristic curve for a certain rpm the pressure \( p_P \) can be modeled with (3.35) \[21\], where \( a_0, a_1 \) and \( a_2 \) are constants that describes the pump characteristic curve. Figure 3.7 shows how characteristic curves can look like for a pump where each line represents one characteristic curve for a certain rpm.

\[ p_P = a_0 + Q_P(a_1 + a_2 Q_P) \]  \hspace{1cm} (3.35)

The constants \( a_0, a_1 \) and \( a_2 \) can be calculated by choosing a few known points on the pump characteristic curve for a certain rotational velocity and then applying the least squares method to approximate the constants. In (3.36) \( Q_1 \) to \( Q_n \) and \( p_1 \) to \( p_n \) are known and \( a_0, a_1 \) and \( a_2 \) are unknown.

\[
A = \begin{bmatrix}
Q_1^2 & Q_1 & 1 \\
\vdots & \vdots & \vdots \\
Q_n^2 & Q_n & 1 \\
\end{bmatrix}, \quad X = \begin{bmatrix}
a_2 \\
a_1 \\
a_0 \\
\end{bmatrix}, \quad Y = \begin{bmatrix}
p_1 \\
\vdots \\
p_n \\
\end{bmatrix}, \quad AX = Y
\]  \hspace{1cm} (3.36)

The least squares method can then be used as in (3.37) which fits the constants \( a_0, a_1 \) and \( a_2 \) to the known points on the pump characteristic curve for a certain rotational speed.

\[ X = (A^T A)^{-1} A^T Y \]  \hspace{1cm} (3.37)
The pump is controlled in such a way that it changes rotational speed and jumps between characteristic curves which means that it is also necessary for the model to be able to jump between characteristic curves for different rotational speeds. The affinity laws can then be used to scale the constants $a_0$, $a_1$, and $a_2$, the affinity laws are defined in (3.38) [18] where $Q_1$, $p_1$, and $N_1$ are flow rate, pressure and rpm for one characteristic curve and $Q_2$, $p_2$, and $N_2$ are flow rate, pressure and rpm for a second characteristic curve.

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}, \quad \frac{p_1}{p_2} = \left(\frac{N_1}{N_2}\right)^2$$ \hspace{1cm} (3.38)

Equation 3.39 shows how the constants can be scaled and the scaling of the constant is derived in Appendix A.

$$a_2, \quad a_1 \frac{N_1}{N_2}, \quad a_0 \left(\frac{N_1}{N_2}\right)^2$$ \hspace{1cm} (3.39)

Once the constants are determined, (3.35), the scaling from (A.2) and the $C^-$ characteristic equation are used to give an equation for $Q_P$ which is (3.40), $p_P$ can then be calculated from the $C^-$ equation.
\[ Q_P = \frac{1}{2a_2} \left( B - a_1 \frac{N_1}{N_2} - \sqrt{\left( B - a_1 \frac{N_1}{N_2} \right)^2 + 4a_2 \left( C_m - a_0 \left( \frac{N_1}{N_2} \right)^2 \right)} \right) \] (3.40)

**Series Connection**  This boundary condition is used when the diameter, roughness, thickness or anything else changes in the pipe. The $C^-$ equation is available for the second pipe and the $C^+$ equation is available for the first equation. The condition of a common hydraulic grade line elevation and the continuity expression gives two equations in (3.41), where the subscripts 1,NS and 2,1 stands for first pipe downstream boundary and second pipe upstream boundary. [21]

\[ Q_{P_{1,NS}} = Q_{P_{2,1}}, \quad p_{P_{1,NS}} = p_{P_{2,1}} \] (3.41)

Equation 3.41 and the characteristic equations $C^+$ and $C^-$ gives (3.42) and the pressure $p_{P_{1,NS}} = p_{P_{2,1}}$ can then be determined with either the $C^-$ or $C^+$ equation.

\[ Q_{P_{1,NS}} = Q_{P_{2,1}} = \frac{C_{p_1} - C_{m_2}}{B_1 + B_2} \] (3.42)

**Valve**  It is assumed that the transient flow through a valve is given by a similar expression as in the steady state flow [21]. This is the same as (3.4) but with a $k_v$ that changes depending on how open the valve is. It is also assumed that the pressure on the other side of the valve is zero which results in the following equation.

\[ Q_P = k_v \sqrt{\frac{p_p}{\rho}} \] (3.43)

The $C^+$ equation and (3.43) is solved simultaneously and gives (3.44). The pressure $p_p$ can then be determined from the $C^+$ equation or (3.43).

\[ Q_P = -\frac{Bk_v^2}{2\rho} + \sqrt{\left( \frac{Bk_v^2}{2\rho} \right)^2 + \frac{C_p k_v^2}{\rho}} \] (3.44)

### 3.5 Model Validation

The model is validated by using measured data as input to the model and then comparing the simulated outputs with equivalent measured data. It is done to see that model behaves in a similar way to reality.
To be able to develop a controller for the system it is necessary to breakdown the model to be able to see how different variables affect each other. It is done separately for the pressure control and the level control.

### 4.1 Pressure Control

The interesting part of the model for the pressure control is the pumps, pipes and valves. The consumers in the system are seen as disturbances from the controllers point of view. Because of that all of the consumers, in this simplification of the model, are lumped together to one valve. In the first step only one pump and valve is considered without the dynamic in the pipes between them. Figure 4.1 shows a block diagram of a pump and a valve, where $u_r(t)$, is the control signal in rpm, $a_0$, $a_1$ and $a_2$ are the constants describing the pumps characteristic curve for rotational speed $N$ and $u_v(t)$ is the control signal to the valve.

*Figure 4.1: Block diagram of simplified model with one pump.*
In the second step more pumps are considered. Since the pumps, work in parallel the pressure at the pumps will be the same for all pumps, but the flow rate that each of the pumps provides will be summed up. Equation 3.35, (3.39) and the fact that the pressure is the same and the flow rate is summed up gives (4.1). In other words if several pumps are running at once each pump will contribute with the flow rate \( \frac{Q_{tot}}{N_{pumps}} \) where \( N_{pumps} \) in the number of pumps running. The small pump is replaced with a large pump because of the results that are presented later in Chapter 6.

\[
\begin{align*}
Q_i &= -\frac{a_1 u_p}{2a_2} + \sqrt{p-a_0 \left( \frac{u_p}{N} \right)^2} - \left( \frac{a_1 u_p}{2a_2} \right)^2 \\
Q_{tot} &= \sum_{i=0}^{N_{pumps}} Q_i \\
p &= a_0 \left( \frac{u_p}{N} \right)^2 + a_1 \frac{u_p}{N} \frac{Q_{tot}}{N_{pumps}} + a_2 \left( \frac{Q_{tot}}{N_{pumps}} \right)^2
\end{align*}
\]

(4.1)

The block diagram for several pumps running at the same time is then almost the same as the one for just one pump running, see Figure 4.2, the only difference is the \( \frac{1}{N_{pumps}} \) block.

![Figure 4.2: Block diagram of simplified model with several pumps.](image_url)

### 4.1.1 Pipe Dynamics

There is no simple transfer function between the pressure and flow rate, which makes it hard to understand the dynamics in the pipe. A black box model is used to create a simplified model of the relation between the pressure and flow rate. There are a few different types of time discrete black box models and the most common are, ARX, ARMAX, BJ and OE. The equations for the black box models are given in (4.2), where \( u(t) \) is the input, \( y(t) \) is the output, \( q \) is the delay operator and in (4.3) are \( a, b, c, d \) and \( f \) are parameters that are fit to data and \( na, nb, nc, nd, nf \) and \( nk \) are chosen parameters. [7]
4.1 Pressure Control

\[ ARX: A(q)y(t) = B(q)u(t) + e(t) \]
\[ ARMAX: A(q)y(t) = B(q)u(t) + C(q)e(t) \]
\[ BJ: y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \]
\[ OE: y(t) = \frac{B(q)}{F(q)}u(t) + e(t) \]

Where \( A(q), B(q), C(q), D(q) \) and \( F(q) \) are given by the following equations:

\[ A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \]
\[ D(q) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd} \]
\[ F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf} \]
\[ C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \]
\[ B(q) = b_1 q^{-nk} + b_2 q^{-nk-1} + \ldots + b_{nb} q^{-nk-nb+1} \]
When considering what type of controller to use in this system it is important to consider controller performance versus simplicity of the controller. The river water pumping station is a crucial part of the factory which makes it very important that the controller is easy to understand and maintain. The most common controller in the process industry is the PI controller and it is in most cases a sufficiently good controller [22]. Different types of common controllers for water distribution networks have been studied in [8], where one of the conclusions is that a classical type of controller such as PI gives sufficient performance for smaller water distribution networks and it is also more cost-effective and easier to implement than more advanced controllers. The PI controller should be sufficient in this case since it is a small distribution system and the PI controller is something most of the people that work with controllers understand.

Regarding the distribution of the control signal between the multiple actuators, the most efficient way to control pumps working in parallel, according to [16], is with the same control signal, when more than one pump is turned on.

Another thing to take into account when considering the controller is that resonances can occur in the system because of the transient flow [1]. To avoid introducing resonance in the pipe, a low pass filter can be used in the feedback [3] so that the controller will not react to the resonance frequencies. The poles for the low pass filter are placed in such a way that they become dominant in the feedback and dampen the resonance frequencies. Since the feedback is dominated by a low pass filter, the modulus optimum tuning method is a good tuning alternative, because the it gives a good frequency response for lower frequencies [2].
5.1 PID-Controller

The PID-controller, proportional-integral-derivative controller, is a feedback controller. Figure 5.1 shows a block diagram of a feedback controller. The output from the process, $y(t)$, is fed back to calculate the error, $e(t)$, between the actual output from the process and the wanted output value, $r(t)$, the error is then used to calculate a control signal, $u(t)$.

$$u(t) = K_c \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$  \hspace{1cm} (5.1)

The equation for the PID-controller is given by Equation 5.1, where $K$ is the proportional gain, $\frac{1}{T_i}$ the integral gain and $T_d$ the derivative gain.

The proportional part of the PID-controller is used to speed up the controller, but there is often a static error if just the proportional part is used, the integral part is used to remove such errors. The proportional and integral part of the controller can contribute to making the controller more unstable. For this reason it is sometimes necessary to add a derivative part that makes the controller more stable because it takes the future error into account, but it also reacts much more to disturbances and noise. The derivative gain, in this case, would not add anything to the controller.

5.2 Low Pass Filter

A low pass filter is a filter that lets lower frequencies pass while higher frequencies are dampened. The second order low pass filter is commonly used in control feedback loops because it has a steeper slope and introduces less phase lag in the system than first order low pass filters [3]. The equation for a second order low pass filter is given in (5.2) where $\omega_N$ is the natural frequency for the filter and $\zeta$ is the damping ratio.

$$G_{lp}(s) = \frac{\omega_N^2}{s^2 + 2\zeta\omega_N + \omega_N^2}$$  \hspace{1cm} (5.2)
5.3 Modulus Optimum Tuning

In modulus optimum tuning the controller parameters are chosen in such a way that the transfer function between SP and PV is as close to one as possible for low frequencies. The desired loop transfer function for modulus optimum method is given by (5.3), where $\omega_0$ is a parameter. [2]

$$G_{MO} = \frac{\omega_0}{s(s + \sqrt{2}\omega_0)} \quad (5.3)$$

When having a second order transfer function for the process as in (5.4), where $K_p$ is the process gain and $T_1$ and $T_2$ are time constants, a PI-controller can be used. The control parameters are set such that the loop transfer function becomes the desired loop transfer function in (5.3), which means that $T_i = T_1$ which cancels the dominant pole and $K = \frac{T_i}{2K_pT_2}$ which gives the desired gain.

$$G(s) = \frac{K_p}{(sT_1 + 1)(sT_2 + 1)}, \quad T_1 > T_2 \quad (5.4)$$

The process, in this case, is nonlinear which means that the process gain can vary depending on operating point. To help determine the controller gain an adaptive controller can be used, that continuously calculates the process gain.

5.3.1 Adaptive Controller

Adaptive controllers adjust their parameters continuously to the process dynamics or disturbances [22]. Figure 5.2 shows a block diagram of an adaptive controller where the parameter estimation block estimates the process, which is then used to tune the control parameters. The type of adaptive controller used in this case is similar to gain scheduling which changes control parameters depending on the operation point of the system.

![Figure 5.2: Block diagram of an adaptive controller](Image)
The simplification of the model, from Chapter 4, is used to estimate the process gain, however the transfer function from control signal to pressure is nonlinear therefore the transfer function is linearized in order to use it to estimate the gain. Equation 5.5 shows how the linearization of this system can be done, where \( p \) is pressure, \( u_r \) is the control signal, \( Q \) is the flow rate, the subscript 0 stands for the operating point and \( f \) is the nonlinear function that is linearized [9].

\[
\begin{aligned}
\Delta p &= p - p_0 \\
\Delta u_r &= u_r - u_{r0} \\
\Delta Q_{tot} &= Q_{tot} - Q_{tot0} \\
\Delta p &= \frac{\delta f}{\delta u_r} \Delta u_r + \frac{\delta f}{\delta Q_{tot}} \Delta Q_{tot}
\end{aligned}
\]  
(5.5)

Linearizing the pump and the valve model from Figure 4.2 this results in:

\[
\Delta p = \frac{2a_0 u_{r0}}{N^2} \Delta u_r + \frac{a_1 Q_{tot0}}{NN_{pumps}} \Delta u_r + \frac{a_1 u_{r0}}{NN_{pumps}} \Delta Q_{tot} + \frac{2a_2 Q_{tot0}}{N^2_{pumps}} \Delta Q_{tot}
\]

\[
\Delta Q = \frac{k_v}{2 \sqrt{p_0}} \Delta p = \left| k_v \right| = \frac{Q_{tot0}}{2p_0}
\]  
(5.6)

From the linearization of the system it is possible to get a linear function that describes the relationship between control signal to pressure as is shown in the following equation:

\[
\begin{aligned}
\frac{\Delta p}{\Delta u_p} &= \frac{\frac{2a_0 u_{r0}}{N^2} + \frac{a_1 Q_{tot0}}{N}}{1 - \left( \frac{a_1 u_{r0}}{N} + \frac{2a_2 Q_{tot0}}{N^2_{pumps}} \right) \frac{Q_{tot0}}{2p_0}}
\end{aligned}
\]  
(5.7)

The operating points are low pass filtered measured signals such that high frequency disturbance does not affect the calculation of the gain. In other words the estimate of the process gain is given by (5.7).

5.3.2 Scaling

Another part that has to be considered is the scaling between different units in the system. Figure 5.3 shows a block diagram of how the signals are scaled in the system.

Following the block diagram one can see that the total scaling ends up at \( \frac{rpm_{max}}{bar_{max}} \). This is what the estimated process gain has to be multiplied with in order for the scaling in the system to be correct.
Figure 5.3: Block diagram that shows how the signals are scaled.
In this chapter are the results presented and discussed.

6.1 Model Validation

In this section the results from the modeling of the system are presented. Measured data from the real process has been used in the simulations as inputs. Measured data has also been plotted in figures with the outputs from the simulated data to qualitatively evaluate the model.

6.1.1 Pumps

The pump characteristic curve is approximated by first taking a few points on the pump characteristic curve, for a certain rotational speed, given by the data sheet of the pump and then using the least squares method, (3.37), as in Section 3.4. The operating point for the large pump is 920 rpm and small pump is 1078 rpm. The result from the approximation of the pump characteristic curve for both the large and small pump is shown in Figure 6.1, where real curve refers to the points on the pump characteristic curve given by the data sheet, and approximated curve is the least squares approximation of the real curve.

As can be seen in Figure 6.1 the approximation of the pump characteristic curves with a second order polynomial is quite good and the approximated curves follow the real curves well.

The model of the pumps were tested by using measured pressure and control signals in rotational speed as inputs to the model. The results from the simulations are shown in Figure 6.2, where Figure 6.2(a) shows the measured total flow rate
and total simulated flow rate and Figure 6.2(b) shows the simulated flow rate for each of the pumps.

Figure 6.3 shows real data on how the controlled pressure reacts when the smaller pump is on and off.

The three large pumps give the same flow rate and the smaller pump has a much lower flow rate which is sometimes even a zero, see Figure 6.2(b). This happens because the pressure in the pipe is too high for the smaller pump and in those moments it does not have the capacity to push the water back. In other words the installment of the smaller pump was quite problematic and could be part of the problem why the current controller does not work very well. This theory is also supported by Figure 6.3 where it can be seen that when the small pump is turned on the pressure oscillates more than when it is off. The small pump has also never worked the way it was supposed to. The main reason it was installed was to run by itself, which it never has, when the flow rate was too low for the large pumps. The circumstances has also changed since the small pump was installed and the overall need for water has increased. Therefore when the new controller is developed the smaller pump is replaced with a larger pump. This could solve the problem with the disturbances and the shortage of water that occurs during the warmer periods.

To validate that the logic for when to start and stop the pumps work. The real control signal, from a period when three of the pumps are turned on and off, was used as input to the model of the logic. The result from this simulation is shown

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**Figure 6.1:** Real and approximated pump characteristic curve for both the large and the small pump.
6.1 Model Validation

(a) Measured and simulated total flow rate.

Figure 6.2: Simulated and measured flow rate for the small pump and the large pumps.

(b) Simulated flow rate for all of the pumps.

in Figure 6.4 where the simulated output from the logic, which is the rotational speed for each of the pumps, is plotted against the measured rotational speed for each of the pumps.

The logic for the pumps works well and the pumps are turned on and off when they are supposed to as seen in the figure.

6.1.2 Pipes

The pumps, the pipes leading to the consumers, valves at the large consumers and the pressure controller that control the pumps is simulated with the the flow rate to the consumers as the input. Figure 6.5 shows result from this simulation, where Figure 6.5(a) shows the simulated and measured pressure in the pipes right after the pumps and Figure 6.5(b) shows the measured and simulated flow rate in the pipes right after the pumps.

The result of the pipe simulation in Figure 6.5 is a bit off the measured values when it comes to the total flow rate because in the model there are only two consumers, but in reality there are several more of them. This resulted in the simulation deviating slightly in total flow rate compared to reality however the general behavior is similar.

6.1.3 Inside the Pumping Station

The model of everything, except for the pumps and pipes, is validated by running the model with measured values of the total flow rate out of the river water pumping station as input to the model. The result from this is shown in Figure 6.6 and Figure 6.7.
Results and Discussion

The result from this seems to be good in general and has similar behavior as the measured data, as seen in the figures. The parameters in the model were chosen by hand. An alternative to this is to instead fit them directly to the data. This would improve the model fit, however it is not very important in this case because it is the general dynamics in the system that are interesting. The controller for the inlet hatch is also not perfect, which is probably due to the delays in the system. Once the inlet hatch is done moving one position there is a pause of 30 seconds before feedback and no new control done within the next 8 minutes. The time for how long it takes to move the inlet hatch one position varies which makes it hard to capture the delays in the simulation.

6.2 Black Box

Estimation and validation data is created by running the pipe model with a pseudo-random signal as input, which is the flow rate out of the pipe. The output is then the pressure at the upstream end of the pipe. The different black box models described in 4.1.1 were estimated using the estimation data and their level of fit to the validation data is shown in Figure 6.8. Figure 6.9 shows the frequency response for the estimated black box models.

All of the different types of black box models fit the validation data quite well. However the black box models are made in order to get an estimation of the transfer function for the pipe, this makes the frequency response for the models interesting to look at because resonance in the pipe can then be investigated. In
6.3 Filter

Figure 6.4: Measured and simulated rpm for all pumps to see that the logic works.

Figure 6.9 it can be seen that all of the black box models have a resonance peak at around 0.1 Hz.

6.3 Filter

Since the filter is supposed to suppress the resonance in the system, the black box model with the largest resonance peak is chosen when tuning the low pass filter. The result from this is shown in Figure 6.10 with a Bode plot of the black box model and the filter.

When it comes to creating the filter the important part is to suppress the resonance peak. Figure 6.10(a) shows that the resonance peak is 24 dB high. In order to suppress that peak the low pass filter has to attenuate that frequency by at least 24 dB. Figure 6.10(b) shows a manually tuned filter which fulfills that demand.

6.4 Controller

The new controller is tested by simulating two different ways of closing the valves, as a step and as a ramp that takes 30 s, at the consumers. The pressure, flow rate and control signal from the new controller is compared to the corresponding signals from the old controller in the same simulations. Figure 6.11 and 6.12 show the result from the simulations when the valves close as step for the old and new controller respectively.

Figure 6.13 and 6.12 shows the result from closing the valves as a ramp that takes 30 s for the old and new controller respectively.
Results and Discussion

(a) Simulated and measured pressure in the pipe.

(b) Simulated and measured flow rate in the pipe.

Figure 6.5: Simulated and measured flow rate and pressure in the pipe

(a) Measured and simulated data of the water level in the inlet channel.

(b) Measured and simulated data of the water level in the pump well.

Figure 6.6: Simulated and measured water level for the inlet channel and pump well.

There does not seem to be that much of a difference in the controllers when considering the figures showing the pressure and flow rate, see Figure 6.11(a)-6.14(a). On the other hand when it comes to the control signal there is a difference, as seen in Figure 6.11(b)-6.14(b). The control signals from the old controller oscillates more when the valves close than what the new controller does. The new controller is therefore nicer to the pumps because it does not make any large sudden changes in the control signal.
Figure 6.7: Measured and simulated positions for the inlet hatch.

Figure 6.8: A few black box models fit to validation data.
Figure 6.9: Frequency response from the black box models.

Figure 6.10: Bode plot of one of the black box models with 24 dB high resonance peak at 0.737 Hz and the filter which is -24.6 dB at 0.73 Hz
(a) Simulated pressure and flow rate in the pipe with the old controller.  
(b) Simulated control signal to the pumps with the old controller.

**Figure 6.11:** Simulation of old controller with the two consumers valve closing as a step.

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(a) Simulated pressure and flow rate in the pipe with the new controller.  
(b) Simulated control signal to the pumps with the new controller.

**Figure 6.12:** Simulation of new controller with the two consumers valve closing as a step.
Results and Discussion

(a) Simulated pressure and flow rate in the pipe with the old controller.

(b) Simulated control signal to the pumps with the old controller.

Figure 6.13: Simulation of old controller with the two consumers valve closing as in reality.

(a) Simulated pressure and flow rate in the pipe with the new controller.

(b) Simulated control signal to the pumps with the new controller.

Figure 6.14: Simulation of new controller with the two consumers valve closing as in reality.
In conclusion, the problem with the river water pumping station is the control system and that it cannot pump enough water during the warmer periods of the year and the questions that this thesis aimed to answer were:

- What is it in the current control system that causes the control problems?
- How does the dynamics in the system affect the choice of control structure and is there a better way to control this system?
- How should the natural nonlinearities in the system be handled?

In this thesis a model has been created to capture the overall dynamics of the system. The model is then used to test the newly develop controller to evaluate its performance. The model captured the overall dynamics of the system. When modeling the system it was found that the smaller pump contributes a much smaller flow rate than the larger pumps and it also introduces oscillations to the system. Because of that the smaller pump is removed in the new controller and four large pumps are used instead. This could solve the problem with the disturbances and the shortage that occurs during the warm periods of the year. In other words this smaller pump could be the cause of the control problems in the system.

The nonlinearities in the system are handled by linearizing the model and with the help of the linearized model adapt the control gain to different operating points.

Resonance could occur in the pipes so when it came to the choice of controller this aspect had to be taken into account to not trigger the resonance frequency. The newly developed controller preforms similarly to the old controller when
just comparing pressure and flow rate. However when taking the control signal into account the new controller is a little bit better because it does no make any large sudden changes.
There are a few different things that would be interesting to look into in the future related to this thesis.

There is a problem with the control of the inlet hatch in the river water pumping station. The controller is not fast enough when large amounts of water are suddenly drawn or large amounts have been drawn and it suddenly stops. One possible solution to the problem could be to use the flow rate measurement and to also open the hatch when the flow rate increases a certain amount.

The controller could be improved by a feedforward controller from the consumers. That way the controller would know when the consumers will open their valves and thereby be able to compensate for the valves opening before it affects the pressure too much.

Further development that could be done with the model is to use it for fault diagnosis in the system. In other words the model could be used to create residuals and variables that can give an indication on the status of the pumps, sensors etc. If such variables are found it may be possible to also use them to adapt the parameters in the controller to compensate for the faults.
Appendix
The scaling of the constants that describes the pump characteristic curve can be derived the following way. The inverse of $A^T A$ in (3.37) can be calculated with $\frac{1}{|A^T A|} \text{adj}(A^T A)$ this gives (A.1), where $n$ are all of the points on the pump characteristic curve and $Q_i$ and $p_i$ are the flow rate and corresponding pressure of one point on the characteristic curve.

$$|A^T A| = \sum_n Q_i^4 \sum_n Q_i^2 \sum_n 1 + \sum_n Q_i^3 \sum_n Q_i^2 \sum_n Q_i + \sum_n Q_i^2 \sum_n Q_i^3 \sum_n Q_i -$$

$$\sum_n Q_i^2 \sum_n Q_i^2 \sum_n Q_i^2 - \sum_n Q_i^3 \sum_n Q_i^3 \sum_n Q_i - \sum_n Q_i^3 \sum_n Q_i^3 \sum_n 1$$

$$\text{adj}(A^T A) =$$

$$\begin{bmatrix}
\sum_n Q_i^2 \sum_n 1 - \sum_n Q_i \sum_n Q_i & -\sum_n Q_i^2 \sum_n 1 + \sum_n Q_i^3 \sum_n Q_i & \sum_n Q_i^4 \sum_n Q_i - \sum_n Q_i^5 \sum_n Q_i \\
-\sum_n Q_i^2 \sum_n 1 + \sum_n Q_i \sum_n Q_i & \sum_n Q_i^3 \sum_n 1 - \sum_n Q_i^2 \sum_n Q_i & -\sum_n Q_i^4 \sum_n Q_i + \sum_n Q_i^5 \sum_n Q_i \\
\sum_n Q_i^3 \sum_n Q_i - \sum_n Q_i^2 \sum_n Q_i & -\sum_n Q_i^4 \sum_n Q_i + \sum_n Q_i^5 \sum_n Q_i & \sum_n Q_i^5 \sum_n 1 - \sum_n Q_i^5 \sum_n Q_i
\end{bmatrix}$$

$$X = \frac{1}{|A^T A|} \text{adj}(A^T A) \begin{bmatrix}
\sum_n Q_i^2 p_i \\
\sum_n Q_i p_i \\
\sum_n p_i
\end{bmatrix}$$

(A.1)

It is assumed that $N_1$, $N_2$, $p_2$ and $Q_2$ are known when using the affinity laws in (3.38), which means that $p_1$ and $Q_1$ can be calculated with the affinity laws. All the points $Q_i$ and $p_i$, in Equation A.1, are known and they are on the characteristic curve with rpm $N_2$. The control signal to the pumps is rpm $N_1$, new points along the characteristic curve with rpm $N_1$ can then be calculated with the affinity laws. This results in (A.2) which is the calculation of the constants for the
characteristic curve for rpm $N_1$, the equation results in how the constants $a_0$, $a_1$ and $a_2$ can be scaled in order to jump from the characteristic curve with rpm $N_2$ to the characteristic curve with rpm $N_1$.

$$\text{adj}(A^T A) = \begin{bmatrix}
\sum_n Q_i^2 \sum_n Q_i - \sum_n Q_i \sum_n Q_i (\frac{N_1}{N_2})^2 \\
-\sum_n Q_i^2 \sum_n Q_i + \sum_n Q_i \sum_n Q_i (\frac{N_1}{N_2})^3 \\
(\sum_n Q_i^2 \sum_n Q_i - \sum_n Q_i \sum_n Q_i) (\frac{N_1}{N_2})^3
\end{bmatrix}
\begin{bmatrix}
\sum_n Q_i^2 p_i (\frac{N_1}{N_2})^4 \\
\sum_n Q_i p_i (\frac{N_1}{N_2})^3 \\
\sum_n p_i (\frac{N_1}{N_2})^2
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
a_2 \\
a_1 \\
a_0
\end{bmatrix}
= \frac{1}{|A^T A| (\frac{N_1}{N_2})^6} \text{adj}(A^T A) \begin{bmatrix}
\sum_n Q_i^2 p_i (\frac{N_1}{N_2})^4 \\
\sum_n Q_i p_i (\frac{N_1}{N_2})^3 \\
\sum_n p_i (\frac{N_1}{N_2})^2
\end{bmatrix}$$

(A.2)
Bibliography


