

Linköping Studies in Science and Technology  
Dissertation No. 2162

# Autonomous Avoidance Maneuvers for Vehicles using Optimization

Pavel Anistratov





Linköping Studies in Science and Technology  
Dissertations No. 2162

# **Autonomous Avoidance Maneuvers for Vehicles using Optimization**

**Pavel Anistratov**



Division of Vehicular Systems  
Department of Electrical Engineering  
Linköping University  
SE-581 83 Linköping, Sweden

Linköping 2021

Linköping Studies in Science and Technology  
Dissertations No. 2162



This work is licensed under a Creative Commons Attribution 4.0  
International License.

<https://creativecommons.org/licenses/by/4.0>

Pavel Anistratov  
pavel.anistratov@liu.se  
www.vehicular.isy.liu.se  
Division of Vehicular Systems  
Department of Electrical Engineering  
Linköping University  
SE-581 83 Linköping, Sweden

Cover picture: Avoiding a suddenly appearing moose on the road is a common illustrative example in this thesis. The cover-page picture shows a moose on a foggy road and an avoidance path for an autonomous vehicle. The picture is an edited version of a photo provided through the courtesy of *Vermont Fish And Wildlife* by Benjamin Young.

Copyright © 2021 Pavel Anistratov, unless otherwise noted.  
Published articles have been reprinted with permission from the  
respective copyright holder.

Anistratov, Pavel  
Autonomous Avoidance Maneuvers for Vehicles using Optimization  
ISBN 978-91-7929-007-8  
ISSN 0345-7524  
URL <http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-176515>

Typeset using L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub>  
Printed by LiU-Tryck, Linköping, Sweden 2021

*To my family*



## Populärvetenskaplig sammanfattning

Moderna personbilar konstrueras mer och mer automatiserade, med syftet att bli både säkrare och bekvämare. Redan i dag finns det bilar på marknaden som automatiskt anpassar sin hastighet till omgivande bilar och andra hinder samt till rådande hastighetsbegränsningar, speciellt så länge bilen kör i samma fil. Det förekommer även bilar på marknaden som kan byta körfält i normal trafik nästan utan, eller till och med helt utan, förarens hjälp.

För att undvika en kollision med hinder krävs ibland hastiga undanmanövrar som innebär att bilen byter körfält en eller flera gånger mycket snabbare än vanligt. I dessa fall är det viktigt att ta hänsyn till olika begränsningar, till exempel hur snabbt det går att förflytta bilen i sidled utan att tappa kontroll över den. Utförande av den här typen av manövrar är nödvändiga för helt automatiserade bilar. För att kunna förutse bilens rörelse under inverkan av olika styrsignaler används matematiska modeller som är baserade på Newtons rörelselagar tillsammans med experimentella samband för att beskriva friktionen mellan däck och väg. Med hjälp av modellerna finns det olika sätt att automatiskt planera bilens rörelse. En ansats som är speciellt anpassad för att ta hänsyn till förekommande begränsningar är rörelseplanering baserad på dynamisk optimering. Denna metod hittar den rörelse för bilen som är bäst med avseende på ett kriterium, under bivillkoret att alla begränsningar är uppfyllda. Exempel på kriterier som används i den här typen av formuleringar är manövernens totala tid och hastighet.

För hastiga undanmanövrar som kräver ett snabbt filbyte till ett körfält för trafik i motsatt riktning, har den totala tiden inte avgörande betydelse, men tiden utanför den egna filen är oerhört viktig för att undvika kollision. Ett nytt kriterium som är baserat på hur mycket bilen avviker från den egna filen under manövern har därför utvecklats. Kriteriet är konstruerat för att leda till minskad tid utanför det egna körfältet och därmed minskad risk för en olycka. Vidare visas att den avslutande fasen av en undanmanöver tjänar på en särskild hantering för att bilen ska återgå till normal körning på önskat sätt. Genom att inkludera ytterligare termer i kriteriet som används i optimeringen förbättras den övergripande prestandan för bilens återgång till körning i den egna filen. En modellanalys som presenteras i avhandlingen ger en teoretisk grund för att de införda termerna förbättrar den avslutande delen av manövern oberoende av andra termer i kriteriet.

Den särskilda hanteringen av den avslutande fasen av en undanmanöver visar att en undanmanöver är uppbyggd av flera delar. Det är därför intressant att studera om, och i så fall hur, manövern kan delas upp i ytterligare delar och vilka egenskaper hos manövern som kan utnyttjas om detta görs. Exempelvis kan det vara tidskrävande att numeriskt lösa ett dynamiskt optimeringsproblem i en dator, och det är därför lockande att försöka dela upp problemet i delar där varje del är kopplad till en specifik fas av manövern. En matematisk algoritm har tagits fram för att lösa dessa faser parallellt, med det övergripande syftet att spara beräkningstid. Här är lösningen av ett överordnat koordinationsproblem centralt, vilket säkerställer att segmenten som utgör de olika faserna hänger ihop när beräkningarna är färdiga. Om det finns flera hinder, eller modellen för bilen som används i optimeringen är mer komplicerad, så har numeriska lösare för dynamiska optimeringsproblem ibland svårigheter att hitta en lösning. Därvid kan rörelsekandidater, genererade utifrån en modell av bilen, som tar hänsyn till hinder vara ett sätt att först beräkna en preliminär bana och detta är en metod som utvecklas i avhandlingen. Den preliminära banan används sedan som initialisering i en optimering som utnyttjar parallella beräkningar av manöversegmenten.

I vissa fall är det önskvärt med en manöver där bilen både behöver styra och bromsa hårt, utan att tappa kontroll. Om hindren är rörliga eller körförhållandena ändras över tiden behövs omplanering för att snabbt hitta en uppdaterad manöver som tar hänsyn till den nya informationen. Resultaten i avhandlingen visar att det är möjligt att använda en enkel modell för att beräkna en approximativ accelerationsreferens som visar både i vilken riktning bilen behöver styra och hur hårt den ska bromsa. För att följa denna referens i realtid används sedan en utvecklad regulator som baserat på denna referens beräknar hur bilen ska styras och bromsas.



## Abstract

To allow future autonomous passenger vehicles to be used in the same driving situations and conditions as ordinary vehicles are used by human drivers today, the control systems must be able to perform automated emergency maneuvers. In such maneuvers, vehicle dynamics, tire–road interaction, and limits on what the vehicle is capable of performing are key factors to consider. After detecting a static or moving obstacle, an avoidance maneuver or a sequence of lane changes are common ways to mitigate the critical situation. For that purpose, motion planning is important and is a primary task for autonomous-vehicle control subsystems. Optimization-based methods and algorithms for such control subsystems are the main focus of this thesis.

Vehicle-dynamics models and road obstacles are included as constraints to be fulfilled in an optimization problem when finding an optimal control input, while the available freedom in actuation is utilized by defining the optimization criterion. For the criterion design, a new proposal is to use a lane-deviation penalty, which is shown to result in well-behaved maneuvers and, in comparison to minimum-time and other lateral-penalty objective functions, decreases the time that the vehicle spends in the opposite lane.

It is observed that the final phase of a double lane-change maneuver, also called the recovery phase, benefits from a dedicated treatment. This is done in several steps with different criteria depending on the phase of the maneuver. A theoretical redundancy analysis of wheel-torque distribution, which is derived independently of the optimization criterion, complements and motivates the suggested approach.

With a view that a complete maneuver is a sequence of two or more sub-maneuvers, a decomposition approach resulting in maneuver segments is proposed. The maneuver segments are shown to be possible to determine with coordinated parallel computations with close to optimal results. Suitable initialization of segmented optimizations benefits the solution process, and different initialization approaches are investigated. One approach is built upon combining dynamically feasible motion candidates, where vehicle and tire forces are important to consider. Such candidates allow addressing more complicated situations and are computed under dynamic constraints in the presence of body and wheel slip.

To allow a quick reaction of the vehicle control system to moving obstacles and other sudden changes in the conditions, a feedback controller capable of replanning in a receding-horizon fashion is developed. It employs a coupling between motion planning using a friction-limited particle model and a novel low-level controller following the acceleration-vector reference of the computed plan. The controller is shown to have real-time performance.



## Acknowledgments

I would like to thank my main supervisor Lars Nielsen for taking me on this journey of post-graduate studies. Björn Olofsson and Jan Åslund were my co-supervisors. Lars and Björn guided me through research challenges and were my co-authors together with Victor Fors and the late Oleg Burdakov. Björn's extensive help with proofreading and guidance in scientific writing is very much appreciated.

I would like to thank the people at the Division of Vehicular Systems for providing a nice working environment and helping me with tips about life in Sweden. My colleagues are acknowledged for keeping me busy with teaching activities, helping me to get started with it, and for developing my teaching skills.

Thank you, the frequent visitors to the *fika* room (as well as the digital *fika* room), for interesting discussions during coffee breaks as well as for contributing to my language skills. Friday *fika* pastries are appreciated as well.

Thanks to Victor for showing me around when I was new at the division, for being a nice colleague to share office with, and for being a reliable and knowledgeable partner in course assignments and manuscript work.

The students from the first batch of the WASP Autonomous Systems graduate school are acknowledged for the pleasant time during our joint courses and study trips. Organization efforts for these activities from the WASP seniors are also appreciated.

Thanks to the people who contributed to my social life outside office hours with various activities.

I would also like to thank my family and friends for supporting me on my way. Thank you very much for that!

Linköping, September 2021

Pavel Anistratov

## Funding

This work was partially supported by the Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundation.



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Background . . . . .	2
1.2	Thesis Focus . . . . .	6
1.3	Contributions . . . . .	6
1.4	Outlook . . . . .	12
	<b>References</b>	<b>15</b>
	<b>Papers</b>	<b>21</b>
<b>I</b>	<b>Lane-Deviation Penalty Formulation and Analysis for Autonomous Vehicle Avoidance Maneuvers</b>	<b>23</b>
1	Introduction . . . . .	24
2	Modeling . . . . .	26
3	Optimal Control Problem . . . . .	35
4	Scenario Variations . . . . .	40
5	Comparison with Other Criteria . . . . .	46
6	Discussion . . . . .	51
7	Conclusions . . . . .	52
	References . . . . .	53
<b>II</b>	<b>Analysis and Design of Recovery Behaviour of Autonomous-Vehicle Avoidance Manoeuvres</b>	<b>59</b>
1	Introduction . . . . .	60
2	Vehicle Model . . . . .	61
3	Road and Obstacle . . . . .	66
4	Baseline Formulation of Optimal Control Problem . . . . .	68
5	Extension of the Optimisation Formulation for Recovery Behaviour . . . . .	70

6	Analysis of Actuation Distribution . . . . .	75
7	Demonstration of Performance for other Evasive Maneuvers . . . . .	83
8	Conclusions . . . . .	88
	References . . . . .	88
<b>III</b>	<b>Autonomous-Vehicle Maneuver Planning Using Segmentation and the Alternating Augmented Lagrangian Method</b>	<b>91</b>
1	Introduction . . . . .	92
2	Vehicle Model . . . . .	93
3	Separable Optimal Control Problem . . . . .	96
4	Parameters and Implementation . . . . .	100
5	Results . . . . .	103
6	Conclusion . . . . .	109
	References . . . . .	110
<b>IV</b>	<b>Dynamic Segment-Based Optimal Motion Planning of Multiple Lane Changes</b>	<b>113</b>
1	Introduction . . . . .	114
2	Modeling . . . . .	116
3	IFM Planner . . . . .	119
4	SOM Planner . . . . .	125
5	Parameters and Implementation . . . . .	132
6	Results . . . . .	134
7	Conclusions . . . . .	144
	References . . . . .	145
<b>V</b>	<b>Predictive Force-Centric Emergency Collision Avoidance</b>	<b>149</b>
1	Introduction . . . . .	150
2	Real-Time Motion Planning . . . . .	151
3	Acceleration References . . . . .	153
4	Acceleration Following . . . . .	158
5	Scenarios . . . . .	166
6	Evaluation Model . . . . .	167
7	Results . . . . .	169
8	Discussion . . . . .	180
9	Conclusions . . . . .	180
	References . . . . .	181

# Chapter 1

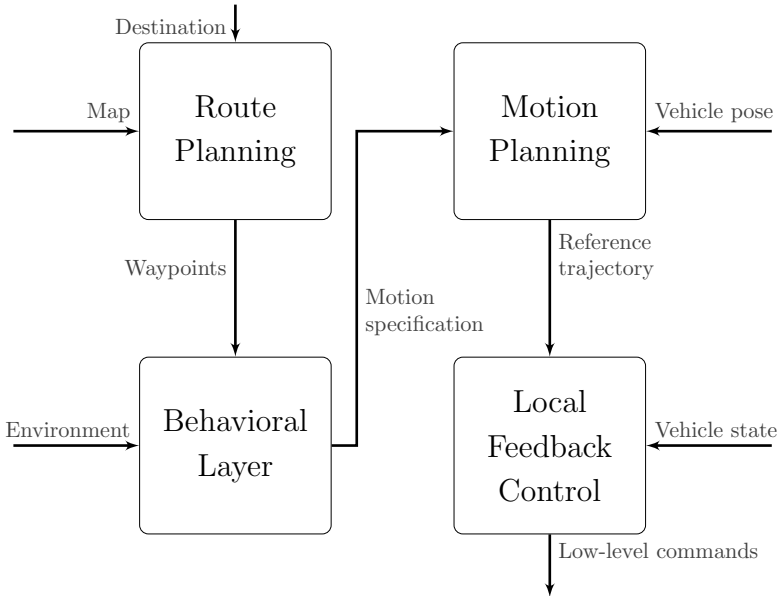
---

## Introduction

TO allow future autonomous vehicles to be used in similar driving situations and conditions as ordinary vehicles are used by human drivers today, many systems need to be further developed. A control system that can perform safety or emergency maneuvers is one example. For such a system and maneuvers, the vehicle dynamics, or limits on what the vehicle is capable of performing, is crucial to consider.

One driving force for the development of such systems is a desire to increase driving safety so as to have no fatal accidents on public roads (see, e.g., VisionZero [1] for Sweden). Autonomous vehicles may allow the driver: to be just a passenger and pursue other activities while being transported; and to save time looking for a parking spot or walking from a remote parking lot [2]. Autonomous vehicles could as well be made available at a requested location [2]. Additional motives include interest in higher coverage and extended opportunities of public transportation with new mobility concepts made possible by autonomous vehicles [3].

The topic of autonomous vehicles is very broad, and some involved key systems could be categorized using the hierarchy of a decision-making process for autonomous cars presented in [4], see Figure 1. In the presence of a sudden obstacle or appearance of any other situation requiring emergency action, these vehicle systems typically coordinate with each other to control the vehicle. Their functions are, e.g., to provide an estimate of the vehicle state and surrounding situation, to select a suitable action among candidates, to compute a dynamically feasible trajectory for the vehicle to follow, and to control the vehicle actuators such that the vehicle



**Figure 1:** Hierarchy of a decision-making process used in driverless cars (adopted from [4]).

follows the decided trajectory. In this thesis, motion planning for ground vehicles is a primary focus. Different approaches to motion planning exist, and a brief overview of the area is given in the following section.

## 1.1 Background

A survey of motion planning and control techniques in autonomous vehicles is found in the previously mentioned [4], where the hierarchy of a decision-making process used in driverless cars is presented as Route Planning, Behavioral Decision Making, Motion Planning, and Local Feedback Control. According to [4], the motion-planning layer is responsible for computing a safe, comfortable, and dynamically feasible trajectory from the current configuration of the vehicle to the goal configuration provided by the Behavioral Decision-Making level. Several approaches are outlined for path/trajectory planning, including approaches based on graph-search methods (e.g., motion primitives used to plan on a state lattice [5, 6]), the RRT method and its extensions [7, 8], and variational (optimization) methods [9, 10]. An introduction to interior-point methods for trajectory optimization is given in [11]. Interior-point methods have



found broad application nowadays, even though they were in limited use for a long time, see, e.g., [12] for a brief history of the methods.

Motion planning is also possible to approach by considering an artificial potential field to guide the vehicle [13]. This approach is experimentally validated in [14] for lane-keeping driver assistance. In [15], the idea has been extended by including an MPC applied after the path has been computed using the defined artificial potential field. By considering simplified models and assuming limited coupling between the longitudinal and lateral dynamics in the motion planning [16], it is possible to obtain lane-change maneuvers on highways with low computational resources using a quadratic-programming formulation of the optimization problem [17]. When multiple autonomous vehicles should be coordinated, a distributed cooperative MPC with a compatibility constraint is a way to both make the problem computationally feasible in real-time and to ensure collision avoidance [18]. Using an acceleration-following approach, computations for motion planning in certain scenarios could be done even explicitly without knowledge of the friction coefficient, while still taking advantage of all available friction for increased driving safety and robustness [19].

Improved driving safety could as well be reached by control including a suitable tire-model adaptation while driving, based on measurements of driving conditions [20]. Examples of additional ways to make autonomous vehicles safer include: motion planning with focus on safe stop trajectories [21]; a computationally efficient departure prediction algorithm [22]; and a model predictive controller formulation for trucks with included factors related to controller stability [23]. Especially for large vehicles, questions on how to model and formulate motion problems [24, 25] and how to keep trip time while decreasing fuel consumption in a computationally efficient way [26–29] are important as well.

### 1.1.1 Optimization for Safety-Critical Motion Planning

Results of offline optimization problems are fruitful in understanding and designing safety systems, see, e.g., the survey on optimal control in automotive applications [30]. Examples of the use of offline optimization methods include: a study of at-the-limit maneuvers and a comparison of vehicle models of different complexity used in optimizations [31]; an analysis of minimum distance when an avoidance maneuver is still possible [32]; or the utilization of an optimal-control based method for quantifying the maneuverability of autonomous vehicles during emergency highway-speed situations [33]. According to [34], optimization utilizing information in

crash databases allows estimating the potential of not yet existing autonomous systems to mitigate accidents to save lives.

Offline optimization allows generation of fuel-saving look-up tables for velocity and freewheeling under varying speed limit [29] and to improve controller robustness when parameters like tire cornering stiffness and body moment of inertia are known only as ranges [35]. In [36], optimal braking patterns using offline optimization have been formulated and interpreted to provide new insights for future safety systems with adaptation of the level of braking. Using the concept of attainable force volumes of optimal maneuvers, a set of control principles for lane-keeping control with close to optimal behavior was formulated [37].

Optimal control results for a simplified vehicle model (particle model) have been used as reference values by the control system, e.g., to perform an avoidance maneuver by applying the modified Hamiltonian algorithm for nonlinear optimal control allocation [38] or to prevent road departure on curved roads [39]. A control design for evasive maneuver assist for collision mitigation with oncoming vehicles has been presented in [40] based on optimal control results for a particle model.

### 1.1.2 Computational Aspects of Optimization

The use of optimization-based methods is often connected with computational challenges. For offline applications, these challenges are not as significant compared to online applications, and there are many useful applications only requiring offline computation. A number of different approaches improving online applicability of optimization-based methods have been proposed in the literature.

The load of the motion-planning problem could be divided between the offline and online parts. Instead of defining motion constraints directly as optimization constraints, it is possible to define the motion-planning problem as a search for a suitable combination of motion primitives, which are precomputed offline [41, 42], and in special cases such an approach also works for dynamic applications [43–45]. Several approaches have been studied to use a precomputed offline trajectory in an online model predictive controller: to calculate the required steering angle for vehicles on slippery roads when a trajectory is known [46]; to perform local trajectory re-planning for autonomous off-road ground vehicles [47]; or to control a racing-car with a high re-planning frequency and a long planning horizon [48].

A motion-planning problem could be decoupled into path planning

and subsequent trajectory planning [7], i.e., by first finding a path and then a suitable velocity profile. In [49], a racing application is considered where a sequential two-step algorithm alternating between path and velocity profile optimization is used to generate minimum-time trajectories. Optimization could be applied on initial trajectories obtained by other means, e.g., by first applying graph search of some predefined motion primitives and then by optimizing the resulting path [50, 51]. Even better performance could be reached if the predefined maneuvers are computed for the same criterion as used in the optimization step [52].

For many dynamic situations, where it is challenging to precompute and store sufficiently many maneuvers, sampling techniques, such as input-space sampling or state sampling [53], are popular since they allow computation of candidate trajectories online [54]. Combined with a real-time iteration scheme for optimal control [55], these trajectories could be improved online [56].

The original motion-planning problem could be decomposed into several connected subproblems, which potentially are easier to solve. One of these approaches is the concept of duality decomposition presented in [57]. Another decomposition method is the alternating direction method of multipliers, which is covered in [58]. The mentioned approaches are targeting the situation when the objective function of the resulting optimization is decomposable.

Other approaches to improve online applicability of optimization-based methods are based on making the computational process faster. This could be achieved by improving the computational performance of the underlying methods, e.g., obtaining a solution of the linear system of equations arising at each step of many iterative optimization solvers faster [59], or by not fully solving the optimization problem at each step when a sequence of slightly changing problems is computed. Solutions using sequentially quadratic methods could in some cases be of sufficient quality for feedback control after just one iteration [55]. For interior-point methods, computational improvements could be achieved by early termination based on a small number of maximum iterations, or not updating the barrier penalty parameter, which also allows taking advantage of warm starting the solver [60].

## 1.2 Thesis Focus

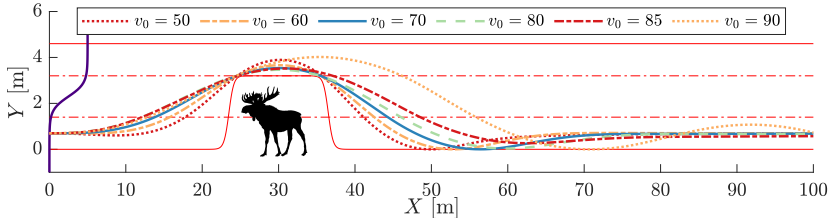
Using an avoidance maneuver as an example of a demanding driving situation, the focus in this thesis is on autonomous avoidance maneuvers for vehicles in critical situations. The key approach to motion planning pursued here is to use dynamic optimization. As an example of an avoidance maneuver, a double lane-change is a frequently used scenario with dimensions inspired by the ISO standard [61], where the vehicle is operating with high actuation usage at typical velocities possible for such a maneuver.

Primarily building on existing single-track or double-track vehicle models with Pacejka's tire models [62] and previously developed optimal control problem formulations, the focus in the thesis is on investigating criteria particularly suited for the considered avoidance maneuvers and on developing decomposition approaches to the corresponding optimal control problems. This is done with a view that a complete maneuver is a sequence of two or more sub-maneuvers, which is inspired by the use of motion primitives in low-speed motion-planning problems. However, since dynamic systems are considered, it is challenging to precompute and store sufficiently many primitives, and this view is used instead to tune a specific vehicle behavior or to perform parallel computations of sub-maneuvers. An additional perspective considered is how to use the results of optimal control problems for closed-loop control design for real-time obstacle avoidance, even in the case of moving obstacles.

## 1.3 Contributions

All included publications are about avoidance maneuvers in critical situations, such that the limits of friction and vehicle dynamics need to be considered. In Paper I, a new suggestion for the optimization criterion to use is investigated. A general treatment of the recovery behavior after passing an obstacle is studied in Paper II. Paper III investigates possibilities offered by segmenting the whole maneuver, which are extended in Paper IV to approach the case of multiple obstacles. Finally, Paper V shows how a real-time controller can be devised.

The publications included and the main contributions are summarized in the following.



**Figure 2:** Examples from Paper I of a double lane-change maneuver using the presented formulation for different initial velocities (km/h).

### **Paper I: Lane-Deviation Penalty Formulation and Analysis for Autonomous Vehicle Avoidance Maneuvers**

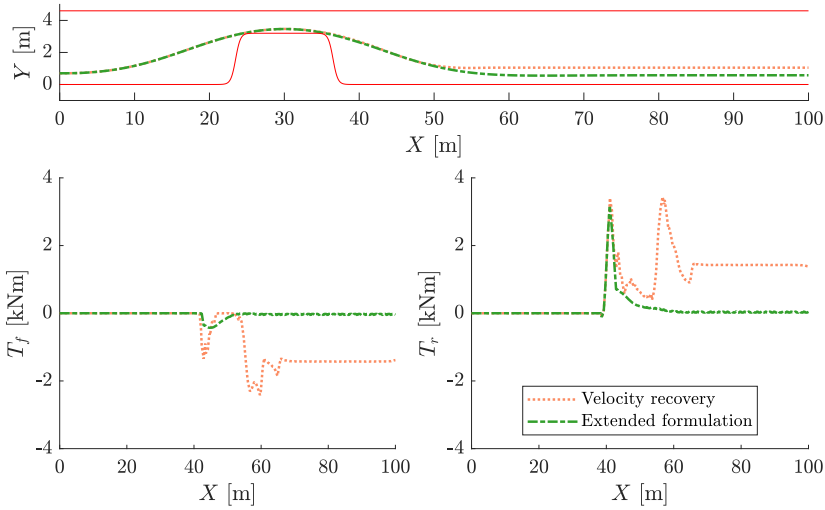
by Pavel Anistratov, Björn Olofsson, and Lars Nielsen. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 235(12):3036-3050, 2021.

The paper considers a double lane-change maneuver as a safety-critical maneuver requiring at-the-limit operation. The observation that it is dangerous for the vehicle to be in the opposing lane, but it is safe to drive in the original lane again after the obstacle, is captured using a lane-deviation penalty (LDP) objective function. For varying parameters of the emergency situation, such as the initial speed of the vehicle (see Figure 2 for examples) and the size and placement of the obstacle, the use of the objective function is observed to result in well-behaved maneuvers. A comparison with minimum-time and other lateral-penalty objective functions shows that the investigated objective function decreases the time that the vehicle spends in the opposing lane.

### **Paper II: Analysis and Design of Recovery Behaviour of Autonomous-Vehicle Avoidance Manoeuvres**

by Pavel Anistratov, Björn Olofsson, and Lars Nielsen. *Vehicle System Dynamics*, 2021. DOI: 10.1080/00423114.2021.1900577

An avoidance maneuver is typically composed of an evasive phase avoiding an obstacle followed by a recovery phase where the vehicle returns to normal driving. An analysis of the different aspects of the recovery phase is presented and a subsequent optimization formulation is developed in several steps based on theory and simulation of a double lane-change scenario. Each step leads to an extension of the optimization criterion. Key results are a theoretical redundancy analysis of wheel-



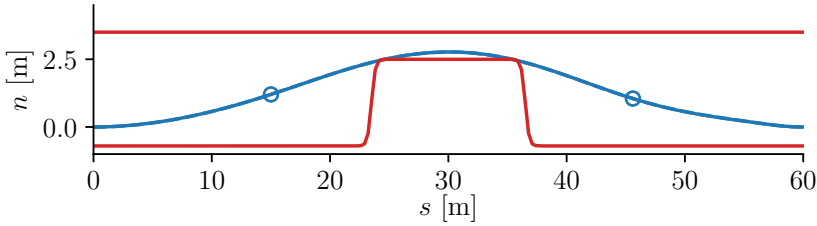
**Figure 3:** The combined torque for the front  $T_f$  and rear  $T_r$  axle illustrates the torque distribution for different formulations of the objective function from Paper II.

torque distribution and the subsequent handling of it (see Figure 3). The redundancy in the wheel-torque distribution appears for the situation of constant speed driving, where only the sum of the driving forces is defined and not individual quantities. The overall contribution is a general treatment of the recovery phase in an optimization framework, and the method is successfully demonstrated in combination with three different formulations for the evasive phase: lane-deviation penalty, minimum time, and squared lateral-error norm.

### **Paper III: Autonomous-Vehicle Maneuver Planning Using Segmentation and the Alternating Augmented Lagrangian Method**

by Pavel Anistratov, Björn Olofsson, Oleg Burdakov, and Lars Nielsen. *IFAC-PapersOnLine (21st IFAC World Congress Proceedings, Berlin, Germany)*, 53(2):15558–15565, 2020.

The paper considers computation of evasive maneuvers divided into maneuver segments (see Figure 4). The basic idea here is to use a vehicle-dynamics perspective, i.e., studying vehicle variables such as the vehicle orientation and yaw rate, to find favorable splitting points. A segmentation approach is investigated to decrease the complexity of motion-



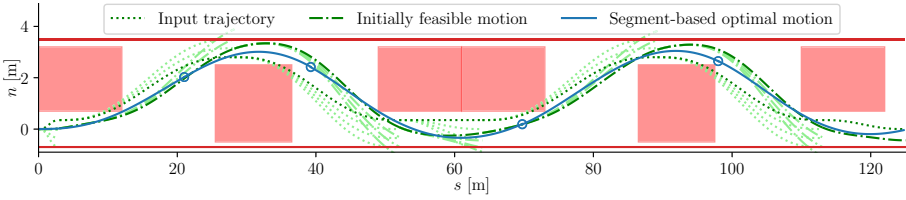
**Figure 4:** Illustration of a maneuver divided into three maneuver segments (subproblems) from Paper III.

planning optimizations. The original optimization problem is modified by adopting the alternating augmented Lagrangian method. This modification allows decomposition of the problem into subproblems, one for each segment, and to compute them in parallel. The coupling constraints introduced to connect the sub-maneuvers are moved into a separate coordination problem, which is possible to solve analytically. By using the solution of a low-complexity initialization problem and applying warm-start techniques in the optimization, a solution is possible to obtain after just a few alternating iterations using the developed approach. If necessary, sequentially improved solutions are obtained after more iterations.

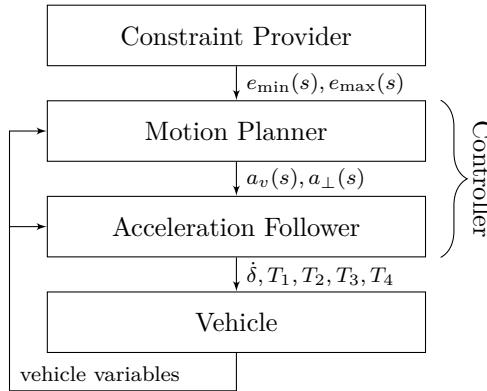
## **Paper IV: Dynamic Segment-Based Optimal Motion Planning of Multiple Lane Changes**

by Pavel Anistratov, Björn Olofsson, and Lars Nielsen. *Manuscript*.

Compared to Paper III, the approach is applied to larger problems including multiple lane changes. The approach to initialize the optimization of maneuver segments from Paper III is in this paper developed such that it is no longer dependent on solving an optimization problem. It is instead based on selecting and combining motion candidates obtained using simulation, while taking obstacles into account both for safety and efficient computations. Such an approach gives an initially feasible motion, which is improved by applying segment-based optimization, see Figure 5 for an illustration. The technique is also developed to work with a receding-horizon approach, meaning that maneuver segments not yet reached are iteratively updated before they are reached by the vehicle for improved overall quality of the solution.



**Figure 5:** The straight-road scenario from Paper IV. Several input trajectories and motion candidates are computed (shown in light green); the selected motion candidates are shown in dark green and compose the initially feasible motion, which is improved using segment-based optimization.



**Figure 6:** Control layout involving acceleration-reference following in Paper V, where  $a_v(s)$  and  $a_{\perp}(s)$  are components of the acceleration vector.

## Paper V: Predictive Force-Centric Emergency Collision Avoidance

by Victor Fors, Pavel Anistratov, Björn Olofsson, and Lars Nielsen.  
*ASME, Journal of Dynamic Systems, Measurement, and Control*, 143(8): 081005, 2021.

A force-centric or acceleration-centric perspective to motion planning and control is considered in the paper. The paper designs a controller, for critical vehicle maneuvering, consisting of a motion planner and a low-level controller called acceleration follower (see Figure 6). The low-level controller is provided with an acceleration reference expressed as a function of position instead of a path or a trajectory. The motion planner is formulated as an optimization problem using a low-complexity friction-limited particle model, and it is executed in a receding-horizon fashion to obtain acceleration-vector references. The low-level controller follows



these acceleration references and is transforming them into actuator commands. Real-time capabilities of the overall system are demonstrated; the planning step can run each 2 m of driven longitudinal distance and the acceleration follower can run at 1000 Hz. The method is evaluated in simulation for a number of challenging scenarios, including turns and moving obstacles, and it results in a well-behaved vehicle avoiding the obstacles and staying on the road while achieving heavy braking.

### Other Publications by the Author

Additional publications by the author include papers presented at conferences and a Licentiate Thesis. These publications cover some initial investigations of the ideas and methods presented in this thesis.

- Pavel Anistratov, Björn Olofsson, and Lars Nielsen. Lane-Deviation Penalty for Autonomous Avoidance Maneuvers. In: *14th International Symposium on Advanced Vehicle Control*, Beijing, China, 2018.

Preliminary first results, which later were developed into the first two journal publications (included as Papers I and II).

- Pavel Anistratov, Björn Olofsson, and Lars Nielsen. Segmentation and Merging of Autonomous At-the-Limit Maneuvers for Ground Vehicles. In: *14th International Symposium on Advanced Vehicle Control*, Beijing, China, 2018.

Initial investigations on maneuver segmentation.

- Pavel Anistratov, Björn Olofsson, and Lars Nielsen. Efficient Motion Planning for Autonomous Vehicle Maneuvers Using Duality-Based Decomposition. In: *IFAC-PapersOnLine (9th IFAC Symposium on Advances in Automotive Control Proceedings*, Orléans, France), 52(5):78–84, 2019.

Initial investigations on maneuver segmentation approaches, which led to the publication included as Paper III.

- Pavel Anistratov. *Computation of autonomous safety maneuvers using segmentation and optimization*. Linköping Studies in Science and Technology, Licentiate Thesis No. 1860, Department of Electrical Engineering, Linköping University, 2019.

Published in partial fulfillment of the Doctor's degree. It contains

earlier investigations made for a vehicle model with lower complexity compared to the vehicle model used in Papers I and II. It also includes the initially submitted version of Paper III.

## 1.4 Outlook

The investigated motion-planning approaches are tailored to a selection of traffic situations, namely avoidance and double lane-change maneuvers. Of course, for a complete system more situations need to be handled, e.g., normal driving situations which are preferably addressed by complementary approaches. These different algorithms need to be integrated in a common architecture with automatic hand-over procedures between them depending on the specific driving situation.

An interior-point optimization solver (IPOPT) has shown to be powerful to solve different nonlinear optimization problems in the papers. When iterative approaches in Papers III–V are used to compute slightly updated optimization problems, other solvers could be investigated to look for possibly more computationally efficient solvers. Models of different fidelity could also be considered and compared when the decomposition techniques in Papers III and IV are applied.

Two methods have been considered to initialize decomposed problems: using coarse optimization in Paper III and a motion-candidate approach in Paper IV. In the latter case, information about the state variables is available to start the solution process of the decomposed problems, but the Lagrange multipliers are set to zero. An interesting investigation is to see if other approaches to initialize Lagrange multipliers without prior optimization are possible that lead to improved computational performance.

This idea of sub-maneuvers has been fruitful for situations when the kinematic model assumption is valid. In Papers II–IV, the sub-maneuver view has been successfully used for dynamic systems to show that individual phases could be tuned and parallel computations of sub-maneuvers with close to optimal results are possible. An interesting question is if there are more extensions of the sub-maneuver view for dynamic systems. For example, it could be a selection of precomputed characteristic sub-maneuvers that, when combined into a maneuver after simple transformations, gives a good initialization trajectory.

The force-centric perspective in Paper V has shown a computationally effective way of capturing challenging braking situations. The approach

uses an objective function focusing on velocity reduction in the motion planner together with a dedicated acceleration follower assuming at-the-limit operation of the vehicle. Potentially, other objective functions, e.g., those developed in Papers I and II, are possible to fit within the considered framework, which would be an interesting investigation.

The developed methods are evaluated in example scenarios using computer simulations. It is natural to continue with more investigations in other scenarios and in other environments including more advanced simulation environments or hardware platforms. From personal experience outside the included papers, such investigations would be much easier to perform with access to an open system with standardized interfaces. Such a system should have necessary autonomous-vehicle components from environment sensing to vehicle control implemented, while allowing substitution of selected functions with own methods. For use in such a system, the developed methods should be extended to work with time-varying driving conditions, e.g., about actual vehicle load and actual tire–road interaction.



---

## References

- [1] Trafikverket. Vision Zero Academy. <https://www.trafikverket.se/en/startpage/operations/Operations-road/vision-zero-academy>, 2021. (Date accessed: 2021-08-12).
- [2] W. Wachenfeld, H. Winner, J. C. Gerdes, B. Lenz, M. Maurer, S. Beiker, E. Fraedrich, and T. Winkle. Use cases for autonomous driving. In *Autonomous Driving*, pages 9–37. Springer, Berlin Heidelberg, 2016.
- [3] B. Lenz and E. Fraedrich. New mobility concepts and autonomous driving: The potential for change. In *Autonomous Driving*, pages 173–191. Springer, Berlin Heidelberg, 2016.
- [4] B. Paden, M. Čáp, S. Z. Yong, D. Yershov, and E. Frazzoli. A survey of motion planning and control techniques for self-driving urban vehicles. *IEEE Transactions on Intelligent Vehicles*, 1(1):33–55, 2016.
- [5] M. Likhachev and D. Ferguson. Planning long dynamically feasible maneuvers for autonomous vehicles. *The International Journal of Robotics Research*, 28(8):933–945, 2009.
- [6] O. Ljungqvist. *Motion planning and feedback control techniques with applications to long tractor-trailer vehicles*. Linköping studies in science and technology, Dissertation No. 2070, Department of Electrical Engineering, Linköping University, 2020.
- [7] S. M. LaValle. *Planning Algorithms*. Cambridge University Press, New York, NY, USA, first edition, 2006.
- [8] S. Karaman and E. Frazzoli. Sampling-based algorithms for optimal motion planning. *The International Journal of Robotics Research*, 30(7):846–894, 2011.
- [9] D. Limebeer and A. Rao. Faster, higher, and greener: Vehicular optimal control. *Control Systems Magazine*, 35(2):36–56, 2015.

- [10] K. Bergman. *Exploiting Direct Optimal Control for Motion Planning in Unstructured Environments*. Linköping studies in science and technology, Dissertation No. 2133, Department of Electrical Engineering, Linköping University, 2021.
- [11] M. Kelly. An introduction to trajectory optimization: How to do your own direct collocation. *SIAM Review*, 59(4):849–904, 2017.
- [12] A. Forsgren, P. E. Gill, and M. H. Wright. Interior methods for nonlinear optimization. *SIAM Review*, 44(4):525–597, 2002.
- [13] J. C. Gerdes and E. J. Rossetter. A unified approach to driver assistance systems based on artificial potential fields. *Journal of Dynamic Systems, Measurement, and Control*, 123(3):431–438, 2001.
- [14] E. Rossetter, J. Switkes, and J. Gerdes. Experimental validation of the potential field lanekeeping system. *International Journal of Automotive Technology*, 5(2):95–108, 2004.
- [15] J. Ji, A. Khajepour, W. W. Melek, and Y. Huang. Path planning and tracking for vehicle collision avoidance based on model predictive control with multiconstraints. *IEEE Transactions on Vehicular Technology*, 66(2):952–964, 2017.
- [16] J. Nilsson, M. Brännström, E. Coelingh, and J. Fredriksson. Longitudinal and lateral control for automated lane change maneuvers. In *American Control Conference (ACC)*, pages 1399–1404, Chicago, IL, USA, 2015.
- [17] J. Nilsson, J. Silvlin, M. Brännström, E. Coelingh, and J. Fredriksson. If, when, and how to perform lane change maneuvers on highways. *IEEE Intelligent Transportation Systems Magazine*, 8(4):68–78, 2016.
- [18] F. Mohseni, E. Frisk, and L. Nielsen. Distributed cooperative MPC for autonomous driving in different traffic scenarios. *IEEE Transactions on Intelligent Vehicles*, 6(2):299–309, 2021.
- [19] V. Fors, B. Olofsson, and L. Nielsen. Autonomous wary collision avoidance. *IEEE Transactions on Intelligent Vehicles*, 6(2):353–365, 2021.
- [20] K. Berntorp, R. Quirynen, T. Uno, and S. D. Cairano. Trajectory tracking for autonomous vehicles on varying road surfaces by friction-adaptive nonlinear model predictive control. *Vehicle System Dynamics*, 58(5):705–725, 2019.
- [21] L. Svensson, L. Masson, N. Mohan, E. Ward, A. P. Brenden, L. Feng, and M. Törngren. Safe stop trajectory planning for highly automated vehicles: An optimal control problem formulation. In *IEEE Intelligent Vehicles Symposium (IV)*, Changshu, China, 2018.

- [22] J. Dahl, G. R. de Campos, and J. Fredriksson. Performance and efficiency analysis of a linear learning-based prediction model used for unintended lane-departure detection. *IEEE Transactions on Intelligent Transportation Systems*, 2021. doi: 10.1109/tits.2021.3090941.
- [23] P. F. Lima, G. C. Pereira, J. Mårtensson, and B. Wahlberg. Experimental validation of model predictive control stability for autonomous driving. *Control Engineering Practice*, 81:244–255, 2018.
- [24] R. Oliveira, P. F. Lima, G. Collares Pereira, J. Mårtensson, and B. Wahlberg. Path planning for autonomous bus driving in highly constrained environments. In *IEEE Intelligent Transportation Systems Conference (ITSC)*, Auckland, New Zealand, 2019.
- [25] R. Oliveira, O. Ljungqvist, P. F. Lima, and B. Wahlberg. Optimization-based on-road path planning for articulated vehicles. *IFAC-PapersOnLine*, 53(2):15572–15579, 2020.
- [26] E. Hellström, M. Ivarsson, J. Åslund, and L. Nielsen. Look-ahead control for heavy trucks to minimize trip time and fuel consumption. *Control Engineering Practice*, 17(2):245–254, 2009.
- [27] E. Hellström, J. Åslund, and L. Nielsen. Design of an efficient algorithm for fuel-optimal look-ahead control. *Control Engineering Practice*, 18(11): 1318–1327, 2010.
- [28] A. Hamednia, N. K. Sharma, N. Murgovski, and J. Fredriksson. Computationally efficient algorithm for eco-driving over long look-ahead horizons. *IEEE Transactions on Intelligent Transportation Systems*, 2021. doi: 10.1109/tits.2021.3058418.
- [29] M. Held, O. Flärdh, and J. Mårtensson. Experimental evaluation of a look-ahead controller for a heavy-duty vehicle with varying velocity demands. *Control Engineering Practice*, 108:104720, 2021.
- [30] R. S. Sharp and H. Peng. Vehicle dynamics applications of optimal control theory. *Vehicle System Dynamics*, 49(7):1073–1111, 2011.
- [31] K. Berntorp, B. Olofsson, K. Lundahl, and L. Nielsen. Models and methodology for optimal trajectory generation in safety-critical road-vehicle manoeuvres. *Vehicle System Dynamics*, 52(10):1304–1332, 2014.
- [32] Z. Shiller and S. Sundar. Emergency lane-change maneuvers of autonomous vehicles. *Journal of Dynamic Systems, Measurement, and Control*, 120(1): 37–44, 1998.
- [33] P. Dingle and L. Guzzella. Optimal emergency maneuvers on highways for passenger vehicles with two- and four-wheel active steering. In *American Control Conference*, pages 5374–5381, Baltimore, MD, USA, 2010.

- [34] B. Olofsson and L. Nielsen. Using crash databases to predict effectiveness of new autonomous vehicle maneuvers for lane-departure injury reduction. *IEEE Transactions on Intelligent Transportation Systems*, 22(6):3479–3490, 2021.
- [35] M. S. Kati, H. Köroğlu, and J. Fredriksson. Robust lateral control of long-combination vehicles under moments of inertia and tyre cornering stiffness uncertainties. *Vehicle System Dynamics*, 57(12):1847–1873, 2018.
- [36] V. Fors, B. Olofsson, and L. Nielsen. Formulation and interpretation of optimal braking and steering patterns towards autonomous safety-critical manoeuvres. *Vehicle System Dynamics*, 57(8):1206–1223, 2019.
- [37] V. Fors, B. Olofsson, and L. Nielsen. Attainable force volumes of optimal autonomous at-the-limit vehicle manoeuvres. *Vehicle System Dynamics*, 58(7):1101–1122, 2019.
- [38] Y. Gao, T. Gordon, and M. Lidberg. Optimal control of brakes and steering for autonomous collision avoidance using modified Hamiltonian algorithm. *Vehicle System Dynamics*, 57(8):1224–1240, 2019.
- [39] Y. Gao and T. Gordon. Optimal control of vehicle dynamics for the prevention of road departure on curved roads. *IEEE Transactions on Vehicular Technology*, 68(10):9370–9384, 2019.
- [40] A. Arikere, D. Yang, M. Klomp, and M. Lidberg. Integrated evasive manoeuvre assist for collision mitigation with oncoming vehicles. *Vehicle System Dynamics*, 56(10):1577–1603, 2018.
- [41] M. Pivtoraiko and A. Kelly. Kinodynamic motion planning with state lattice motion primitives. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 2172–2179, San Francisco, CA, USA, 2011.
- [42] K. Bergman, O. Ljungqvist, and D. Axehill. Improved optimization of motion primitives for motion planning in state lattices. In *IEEE Intelligent Vehicles Symposium (IV)*, Paris, France, 2019.
- [43] E. Frazzoli, M. A. Dahleh, and E. Feron. Real-time motion planning for agile autonomous vehicles. *Journal of Guidance, Control, and Dynamics*, 25(1):116–129, 2002.
- [44] E. Frazzoli, M. A. Dahleh, and E. Feron. Maneuver-based motion planning for nonlinear systems with symmetries. *IEEE Transactions on Robotics*, 21(6):1077–1091, 2005.
- [45] A. Gray, Y. Gao, T. Lin, J. K. Hedrick, H. E. Tseng, and F. Borrelli. Predictive control for agile semi-autonomous ground vehicles using motion primitives. In *American Control Conference (ACC)*, Montreal, QC, Canada, 2012.



- [46] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat. Predictive active steering control for autonomous vehicle systems. *IEEE Transactions on Control Systems Technology*, 15(3):566–580, 2007.
- [47] Y. Yoon, J. Shin, H. J. Kim, Y. Park, and S. Sastry. Model-predictive active steering and obstacle avoidance for autonomous ground vehicles. *Control Engineering Practice*, 17(7):741–750, 2009.
- [48] J. Subosits and J. C. Gerdes. From the racetrack to the road: Real-time trajectory replanning for autonomous driving. *IEEE Transactions on Intelligent Vehicles*, 4(2):309–320, 2019.
- [49] N. R. Kapania, J. Subosits, and J. C. Gerdes. A sequential two-step algorithm for fast generation of vehicle racing trajectories. *Journal of Dynamic Systems, Measurement, and Control*, 138(9): 091005, 2016.
- [50] W. Xu, J. Wei, J. M. Dolan, H. Zhao, and H. Zha. A real-time motion planner with trajectory optimization for autonomous vehicles. In *IEEE International Conference on Robotics and Automation*, pages 2061–2067, Saint Paul, MN, USA, 2012.
- [51] O. Ljungqvist, K. Bergman, and D. Axehill. Optimization-based motion planning for multi-steered articulated vehicles. *IFAC-PapersOnLine*, 53(2): 15580–15587, 2020.
- [52] K. Bergman, O. Ljungqvist, and D. Axehill. Improved path planning by tightly combining lattice-based path planning and optimal control. *IEEE Transactions on Intelligent Vehicles*, 6(1):57–66, 2021.
- [53] T. M. Howard, C. J. Green, A. Kelly, and D. Ferguson. State space sampling of feasible motions for high-performance mobile robot navigation in complex environments. *Journal of Field Robotics*, 25(6-7):325–345, 2008.
- [54] Y. Kuwata, S. Karaman, J. Teo, E. Frazzoli, J. How, and G. Fiore. Real-time motion planning with applications to autonomous urban driving. *IEEE Transactions on Control Systems Technology*, 17(5):1105–1118, 2009.
- [55] M. Diehl, H. G. Bock, and J. P. Schlöder. A real-time iteration scheme for nonlinear optimization in optimal feedback control. *SIAM Journal on Control and Optimization*, 43(5):1714–1736, 2005.
- [56] L. Svensson, M. Bujarbaruah, N. R. Kapania, and M. Törngren. Adaptive trajectory planning and optimization at limits of handling. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 3942–3948, Macau, China, 2019.
- [57] L. Lasdon. Duality and decomposition in mathematical programming. *IEEE Transactions on Systems Science and Cybernetics*, 4(2):86–100, 1968.

- [58] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3:1–122, 2011.
- [59] Y. Cao, A. Seth, and C. D. Laird. An augmented lagrangian interior-point approach for large-scale NLP problems on graphics processing units. *Computers & Chemical Engineering*, 85:76–83, 2016.
- [60] Y. Wang and S. Boyd. Fast model predictive control using online optimization. *IEEE Transactions on Control Systems Technology*, 18(2):267–278, 2010.
- [61] ISO 3888-2:2011. Passenger cars — Test track for a severe lane-change manoeuvre — Part 2: Obstacle avoidance. Technical report, International Organization for Standardization (ISO). Geneva, Switzerland, 2011.
- [62] H. Pacejka. *Tyre and vehicle dynamics*. Butterworth-Heinemann, Oxford, UK, second edition, 2006.

# Papers

# Papers

The papers associated with this thesis have been removed for copyright reasons. For more details about these see:

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-176515>

Dissertations  
Division of Vehicular Systems  
Department of Electrical Engineering  
Linköping University

- No. 1 Magnus Pettersson, *Driveline Modeling and Control*, 1997.
- No. 2 Lars Eriksson, *Spark Advance Modeling and Control*, 1999.
- No. 3 Mattias Nyberg, *Model Based Fault Diagnosis: Methods, Theory, and Automotive Engine Applications*, 1999.
- No. 4 Erik Frisk, *Residual Generation for Fault Diagnosis*, 2001.
- No. 5 Per Andersson, *Air Charge Estimation in Turbocharged Spark Ignition Engines*, 2005.
- No. 6 Mattias Krysander, *Design and Analysis of Diagnosis Systems Using Structural Methods*, 2006.
- No. 7 Jonas Biteus, *Fault Isolation in Distributed Embedded Systems*, 2007.
- No. 8 Ylva Nilsson, *Modelling for Fuel Optimal Control of a Variable Compression Engine*, 2007.
- No. 9 Markus Klein, *Single-Zone Cylinder Pressure Modeling and Estimation for Heat Release Analysis of SI Engines*, 2007.
- No. 10 Anders Fröberg, *Efficient Simulation and Optimal Control for Vehicle Propulsion*, 2008.
- No. 11 Per Öberg, *A DAE Formulation for Multi-Zone Thermodynamic Models and its Application to CVCP Engines*, 2009.
- No. 12 Johan Wahlström, *Control of EGR and VGT for Emission Control and Pumping Work Minimization in Diesel Engines*, 2009.
- No. 13 Anna Pernestål, *Probabilistic Fault Diagnosis with Automotive Applications*, 2009.
- No. 14 Erik Hellström, *Look-ahead Control of Heavy Vehicles*, 2010.

- No. 15** Erik Höckerdal, *Model Error Compensation in ODE and DAE Estimators with Automotive Engine Applications*, 2011.
- No. 16** Carl Svärd, *Methods for Automated Design of Fault Detection and Isolation Systems with Automotive Applications*, 2012.
- No. 17** Oskar Leufvén, *Modeling for Control of Centrifugal Compressors*, 2013.
- No. 18** Christofer Sundström, *Model Based Vehicle Level Diagnosis for Hybrid Electric Vehicles*, 2014.
- No. 19** Andreas Thomasson, *Modeling and control of actuators and co-surge in turbocharged engines*, 2014.
- No. 20** Emil Larsson, *Model Based Diagnosis and Supervision of Industrial Gas Turbines*, 2014.
- No. 21** Andreas Myklebust, *Dry Clutch Modeling, Estimation, and Control*, 2014.
- No. 22** Tomas Nilsson, *Optimal Engine Operation in a Multi-Mode CVT Wheel Loader*, 2015.
- No. 23** Daniel Jung, *Diagnosability Performance Analysis of Models and Fault Detectors*, 2015.
- No. 24** Martin Sivertsson, *Optimal Control of Electrified Powertrains*, 2015.
- No. 25** Peter Nyberg, *Evaluation, Generation, and Transformation of Driving Cycles*, 2015.
- No. 26** Kristoffer Lundahl, *Models and Critical Maneuvers for Road Vehicles*, 2016.
- No. 27** Vaheed Nezhadali, *Modeling and Optimal Control of Heavy-duty Powertrains*, 2016.
- No. 28** Xavier Llamas, *Modeling and Control of EGR on Marine Two-Stroke Diesel Engines*, 2018.
- No. 29** Sergii Voronov, *Machine Learning for Predictive Maintenance*, 2020.
- No. 30** Victor Fors, *Autonomous Vehicle Maneuvering at the Limit of friction*, 2020.

- No. 31** Fatemeh Mohseni, *Decentralized Optimal Control for Multiple Autonomous Vehicles in Traffic Scenarios*, 2021.
- No. 32** Mahdi Morsali, *Trajectory Planning for an Autonomous Vehicle in Multi-Vehicle Traffic Scenarios*, 2021.
- No. 33** Kristoffer Ekberg, *Modeling and Optimal Control for Dynamic Driving of Hybridized Vehicles with Turbocharged Diesel Engines*, 2021.





## **FACULTY OF SCIENCE AND ENGINEERING**

Linköping Studies in Science and Technology, Dissertation No. 2162, 2021  
Department of Electrical Engineering

Linköping University  
SE-581 83 Linköping, Sweden

[www.liu.se](http://www.liu.se)