# Are commuter train timetables consistent with passengers' valuations of waiting times and in-vehicle crowding? 

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#### Abstract

Social cost-benefit analysis is often used to analyse transport investments, and can also be used for transport operation planning and capacity allocation. If it is to be used for resolving capacity conflicts, however, it is important to know whether transit agencies' timetable requests are consistent with the cost-benefit framework, which is based on passenger preferences. We show how a public transport agency's implicit valuations of waiting time and crowding can be estimated by analysing timetables, apply the method to commuter train timetables in Stockholm, and compare the implicit valuations to the corresponding passenger valuations in the official Swedish cost-benefit analysis guidelines. The results suggest that the agency puts a slightly lower value on waiting time and crowding than the passenger valuations codified in the official guidelines. We discuss possible reasons for this and implications for using cost-benefit analysis for capacity allocation. We also find that optimal frequencies are more sensitive to the waiting time valuation than to that of crowding.


## 1. Introduction

Public transport is a central part of most urban transport systems, and the decisions of public transport agencies about which services to run are hence important. In this paper, we explore to what extent timetables determined by a local public transport agency (PTA) are consistent with passengers' valuations of waiting times and in-vehicle crowding, as codified in the guidelines for social cost-benefit analysis (CBA). We present a way to do this based on analysing the timetables' trade-offs between service frequency and operations costs, and apply the method on commuter trains in Stockholm.

Just as passengers' valuations can be inferred by observing their choices between travel options, analogous implicit valuations can be derived from the choices an agency makes when deciding timetables. These implied valuations can then be interpreted as the agency's implicit or "revealed preferences" (RP) regarding, e.g., waiting times and crowding. The question is then whether these coincide with passengers' valuations. Similar studies of the implicit preferences of public agencies have been conducted by, e.g., Mcfadden (1975), Mcfadden (1976), Nellthorp and Mackie (2000) and Eliasson and Lundberg (2012). These papers studied various administrations' decisions about infrastructure
investments, estimating implicit valuations of different kinds of benefits and costs. To our knowledge, this is the first similar study based on timetable decisions of a public transport agency.

There are several reasons why the question of consistency between an agency's timetable decisions and passenger preferences is important. First and most obvious, possible differences between agencies' and passengers' implicit valuations raise several interesting questions. Is the agency simply failing to construct cost-efficient timetables? In that case, the methods developed in this paper can indicate how timetables can be improved. Or is there something missing from the conventional CBA framework, that the agency correctly considers? In that case, such studies can identify improvements of the framework. Or are the agency's decisions affected by other considerations - perhaps (hidden) political pressure? Or are there some constraints on the agency's decisions which prevent it from making optimal decisions from the passengers' point of view? Exploring whether an agency's decisions are consistent with passenger valuations forms a starting point for interesting and deep discussions about an agency's objectives and efficiency, as well as the ability of the CBA framework to capture the relevant aspects of public transport service provision.

A second motivation for our interest in this issue is that it has been

[^0]suggested by Johnson and Nash (2008) and Ait-Ali et al. (2020) that CBA can be used to resolve capacity conflicts between commercial train services and publicly controlled commuter trains. The idea is to calculate the net loss of social benefits caused by adjustments of a commuter train timetable to make room for a commercial train service, and use this loss as a reservation price for the commercial train slot: capacity is allocated to the commercial train only if the commercial operator is willing to pay an access/congestion charge equal to this reservation price. However, this idea rests on the assumption that the CBA framework used to calculate the social loss from the timetable adjustment is consistent with the PTA's timetable preferences. If not, it is unlikely that the PTAs or commercial operators will accept the idea. At least, one would need to investigate the reasons for possible differences between the valuations in the CBA framework and the preferences of the PTA, and determine whether or how they can be reconciled.

A third reason for investigating the principles underlying agencies' timetable choices is that knowledge of these principles is necessary for evaluating infrastructure investments. As discussed in Eliasson and Börjesson (2014), the benefits of a railway capacity improvement are determined by the difference in timetables with and without the investment. To conduct a CBA of a railway investment, these timetables must hence be constructed, and the analyst needs a guiding principle to determine them. Eliasson and Börjesson (2014) suggest that, lacking better evidence, an analyst could assume that the PTA strives to maximize net social benefits - but empirical evidence of the implicit principles determining a PTA's timetable choices is obviously better.

Section 2 provides an overview of the relevant research literature. Section 3 describes the analytic model. Data for the numerical analysis is presented in section 4 and results in section 5 . We conclude the paper with section 6.

## 2. Literature review

There is a vast literature on passengers' valuations of trip characteristics, such as in-vehicle travel time, in-vehicle crowding, waiting time, walking time, and delays. Abrantes and Wardman (2011) provided an overview and meta-analysis for the valuation of in-vehicle travel time based on British evidence. Wardman and Whelan (2011) also performed a meta-analysis to evaluate the British value of crowding in rail trips. The two authors collected data on crowding valuations from the last 20 years from 15 different studies. The meta-analysis quantified the variations in the large set of time multipliers. The study aggregated these values into implied multipliers for seated and standing travellers for commuter and leisure trips. The valuations of in-vehicle time and waiting time used in this study are based on the Swedish value of time study, reported in Algers et al. (2010) and Börjesson and Eliasson (2012), and subsequently updated and included in the Swedish CBA guidelines (Trafikverket, 2016). The valuation of crowding is based on the study by Björklund and Swärdh (2017), who estimated crowding multipliers for different modes and areas from Swedish data, reaching similar results as the Wardman and Whelan (2011) meta-study in the UK.

Most valuation studies are based on stated preference (SP) experiments, but there are also studies based on revealed preferences (RP), i.e., observed behaviour. Kroes et al. (2014), Tirachini et al. (2016), Hörcher et al. (2017), estimate (relative) valuations of crowding based on passengers' route choices in public transport systems. So far, RP studies of crowding have tended to yield lower valuations than SP studies, in contrast to valuations of (in-vehicle) travel time where RP studies have often yielded higher valuations than SP studies (Wardman, 1988; Small et al., 2005; Isacsson, 2007). Hörcher et al. (2017) use RP data to estimate a waiting time multiplier (relative to in-vehicle time) of 1.76, which is in the lower part of the range of most SP studies.

Just as passengers' implicit valuations can be inferred by analysing their choices between options with different benefits and costs, agencies" "implicit preferences" can be inferred by analysing their
decisions. One of the earliest such studies is by Mcfadden (1975) who looked at the implicit valuations of benefits and costs of road investments implied by the decisions of a transport agency. The author inferred the implicit choice criteria and benefit valuations used by the agency when selecting infrastructure investments. The inference relies on ex-post evaluation of the consequences and outcomes of the selection decisions. Similar studies have been presented by Nellthorp and Mackie (2000) and Eliasson and Lundberg (2012). Compared to the many studies of travellers' valuations, there are only a few studies of the implicit preferences of agencies. Even fewer have compared the two sets of valuations, and as far as we know none in the context of public transport. There are, however, studies comparing optimal and actual public transport supply, such as Qin and Jia (2013), who studied a crowded rail transit line in China, Börjesson et al. (2017), who analysed optimal bus fares and frequencies in Stockholm, and Asplund and Pyddoke (2019) who did a similar study but in a medium-sized Swedish city. Seminal contributions to the analysis of optimal public transport supply and pricing were made by Mohring (1972) and Jansson (1980). A large literature on the topic has evolved; a good recent review is written by Hörcher and Tirachini (2021).

Basu (1980), in a book about the revealed preference of governments, formalized a model by Weisbrod and Chase (1966) which studied income redistribution weights in CBA studies. This formalization is based on a standard model of a social welfare function and the distinction between local and global welfare. Such a model allows to estimate the weights of a welfare function based on information about the projects chosen by the government. Another formalization by the same author used fuzzification for analysis of revealed binary preferences (Basu, 1984). Brent (1991) discussed the previous techniques for revealing government's distributional weights. The author contrasted stochastic methods, e.g., Mcfadden (1975), with deterministic ones, e.g. Basu (1980), indicating a preference for the former. The latter is applied and discussed in the case of the UK railway closure at that time. The stochastic approach is also used in the more recent work by Scarborough and Bennett (2012). They applied choice modelling techniques to estimate distributional weights in CBA models for environmental policy analysis.

A somewhat related literature considers consumer (i.e., personal and self-interested) versus citizen (i.e., social and moral) preferences. Im et al. (2014) looked at the extent to which citizen preferences are reflected in the resource allocations from the budget of the city of Seoul both at the city and district level. The authors found that there is no perfect reflection of such preferences, meaning that resource or budget allocation in the city seems to be non-participatory. The authors highlighted and discussed the potential of participatory budgeting which reflects the citizen preferences. Similar studies were also conducted in the US (Franklin and Carberry-George, 1999), the Netherlands (Michels and De Graaf, 2010) and Malaysia (Manaf et al., 2016). Most of these studies claim a positive impact of citizen involvement in decision making, and that participatory decision making is desirable in representative democracies. However, Bossert and Weymark (2004) found it difficult to include all the citizen groups and show that the social welfare-maximizing function can be dictatorial. Therefore and according to Arrovian social choice theory, certain individual preferences must be considered over others, see Arrow's impossibility theorem or paradox (Arrow, 1963). In a recent study, CBA is compared with Participatory Value Evaluation (PVE), where individuals select preferred projects given a budget limitation, in assessing the desirability of government projects (Mouter et al., 2021). The authors find that PVE and CBA produce different rankings, e.g., safety projects for cyclists/pedestrians rank higher in PVE whereas car traffic improvements are preferred when CBA is used.

Lewinsohn-Zamir (1998) criticized the distinction between consumer and citizen preferences in the context of the provision of public goods, e.g., public transport services. The author claimed that such a distinction is unrealistic, and no quantitative difference can be made,
arguing that both preferences are driven by other trade-offs that are less manifested in daily life. Moreover, since such preferences are successfully considered, in many cases, in the political arena, citizen preferences should be given more weight and be carefully used in tools such as cost-benefit analysis.

## 3. Benefits and costs of the commuter train timetable

### 3.1. The cost-benefit framework

The cost-benefit analysis framework used in this study includes passenger benefits and train operating costs. Passenger benefits include in-vehicle time, in-vehicle crowding and waiting time, while operating costs include fixed and variable vehicle costs, staff costs, maintenance costs and overhead costs. Since we are studying relatively minor changes in the timetables, we use a fixed origin-destination matrix, which means that there are no changes in external effects due to modal shifts from road transport, and no changes in fare revenues or tax revenues. Adding such effects is straightforward, provided that demand effects can be forecasted.

In the present study, we also ignore unexpected delays. This is an important issue for future research, since robustness towards incidents, minimizing knock-on delays, may be an important consideration when constructing optimal timetables.

Passenger benefits (consumer surplus) are calculated as follows. Let $D_{i j}(\tau)$ be the density of passengers going from station $i$ to station $j$ with preferred departure time (PDT) $\tau$. We distinguish train directions by letting stations have different indices depending on which direction passengers are travelling in, so each physical station will have at least two indices, one for each direction of each line serving the station.

Let the generalized travel cost be $c_{i j}(\tau)=\alpha(F) t_{i j}+\beta\left(w_{i j}(\tau)\right)+p_{i j}$, where $t_{i j}$ is the in-vehicle travel time from $i$ to $j$ (in our case study, this is independent of the time of day $\tau), \alpha(F)$ is the value of in-vehicle travel time (which depends on the number of passengers onboard the train $F$, as described below), $w_{i j}(\tau)$ is the waiting time for a passenger with PDT $\tau$, $\beta(w)$ is a non-linear function reflecting the disutility of waiting time (a piecewise linear function in the case study), and $p_{i j}$ is the fare. Since we assume constant demand, passenger benefits are the negative of passengers' total generalized travel costs $P C$, where $P C=\sum_{i j} \int D_{i j}(\tau) c_{i j}(\tau) d \tau$.

A timetable consists of a number of train services indexed $k$, connecting origin and destination stations at specified departure and arrival times. Passengers choose the train services which minimize their generalized travel cost, conditional on their PDT. Let $\delta_{i j}^{k}(\tau)$ be an indicator function which is 1 if a passenger going from $i$ to $j$ with PDT $\tau$ uses service $k$. This lets us define the number of boarding and alighting passengers on station $i$ and service $k$ as $B_{i}^{k}=\sum_{j} \int D_{i j}(\tau) \delta_{i j}^{k}(\tau) d \tau$ and $A_{i}^{k}=$ $\sum_{j} \int D_{j i}(\tau) \delta_{j i}^{k}(\tau) d \tau$, respectively. Given this, we can calculate passenger loads for each service and link as $F_{i}^{k}=\sum_{j(k) \leq i}\left(B_{j}^{k}-A_{j}^{k}\right)$, where the expression $j(k) \leq i$ denotes the set of all stations $j$ on service $k$ preceding station $i . F_{i}^{k}$ is hence the number of passengers onboard train service $k$ at the link following station $i$.

With these definitions, we can rewrite passengers' total generalized travel cost as
$P C=\sum_{i j} \int D_{i j}(\tau) c_{i j}(\tau) d \tau=\sum_{k} \sum_{i \in k} \alpha\left(F_{i}^{k}\right) F_{i}^{k} t_{i}+\beta\left(w_{i}^{k}\right) B_{i}^{k}+R$,
where $t_{i}$ is the travel time of the link segment following station $i, w_{i}^{k}$ is the average ${ }^{1}$ waiting time for service $k$ at station $i$, the expression $i \in k$ denotes the set of stations served by service $k$, and $R$ is total fare revenues. Total passenger travel costs hence consist of three components: total in-vehicle travel time weighted by the crowding-dependent factor $\alpha\left(F_{i}^{k}\right)$, total waiting time disutility, and total fare revenues.

The valuation (monetary disutility) of in-vehicle travel time $\alpha$ increases with the crowding in the vehicle since travelling in crowded conditions incurs a higher disutility per minute on travellers. Let the valuation of in-vehicle time be $\alpha=\alpha_{0}\left(1+\gamma\left(\frac{F}{S}\right)^{\theta}\right)$, where $\alpha_{0}$ is the baseline value of in-vehicle travel time,$S$ is the number of seats in the train, $F$ is the number of passengers onboard the train, and $\gamma$ and $\theta$ are parameters. This function is fitted to the Swedish CBA guidelines by estimating the coefficients $\gamma$ and $\theta$, taken from the study by Björklund and Swärdh (2017). The function also fits the study by Wardman and Whelan (2011) well.

Operating costs include staff, maintenance, wear and tear of vehicles and other operations-related costs. They consist of three components: one proportional to total vehicle operating hours, one proportional to total vehicle kilometres and one proportional to the number of vehicles in the fleet. The last component is determined by how many vehicles that are necessary to run peak-hour services, which means that marginal operating costs are higher in peak than in off-peak, since increasing peak-hour services makes it necessary to have more vehicles. Operating costs are taken from the Stockholm Public Transport Agency guidelines (SLL, 2017), presented in Table 1. Net operating costs NC is operating costs minus total fare revenues, multiplied by the marginal cost of public funds (MCPF) to reflect the deadweight loss of taxes (operating costs not covered by fare revenues are covered by taxes). This means that net operating costs will be proportional to the number of services for a given line and time period, so we can write $N C=K N-R$, where the factor $K$ depends on the distance and time of the line, $N$ is the number of services on the line and time period, and $R$ is the corresponding fare revenues. Note that $K$ will be different for peak and off-peak traffic, since the

Table 1
Parameter values in the CBA framework ( 10 SEK $\approx 1$ EUR).

| Parameters | Values | References |
| :---: | :---: | :---: |
| Travel time | $\alpha_{0}=73 \mathrm{SEK} / \mathrm{h}$ | Baseline value of travel time valuation; weighted average of leisure trips ( $57 \mathrm{SEK} / \mathrm{h}, 50 \%$ of trips), commuting ( 74 SEK/h, 46\%) and business ( 265 SEK/ $\mathrm{h}, 4 \%$ ) (Eliasson and Börjesson, 2014). |
| Waiting time | $\beta_{0}^{\text {peak }}=168 \mathrm{SEK} / \mathrm{h} \beta_{0}^{\text {off-peak }}=137$ <br> SEK/h | Valuation of average waiting time for headways during peak hours (less than 10 min ) and off-peak (11-30 min) (Algers et al., 2010). |
| Crowding | $\gamma_{0}=0.14 \theta_{0}=3$ | The function $1+\gamma\left(\frac{F}{S}\right)^{\theta}$ was fitted to the results by Björklund and Swärdh (2017). |
| Operation | $\begin{aligned} & K_{\text {distance }}=30 \mathrm{SEK} / \text { vehicle-km } \\ & K_{\text {time }}^{\text {peak }}=5205 \mathrm{SEK} / \text { vehicle-h } \\ & K_{\text {time }}^{\text {off-peak }}=2000 \mathrm{SEK} / \text { vehicle-h } \\ & K_{\text {overhead }}=9 \% \end{aligned}$ | SLL, (2017). |

[^1]necessary number of vehicles in the fleet is determined by peak services.
The net benefit of the public transport services is the negative of its total social cost, which is the sum of passenger costs PC and net operating costs $N C$. Maximizing net social benefits is hence equivalent to minimizing total social costs, since we assume constant demand. The total social cost for a given line and time period is hence:
$\mathrm{TC}=K N+\sum_{i k} \beta\left(w_{i}^{k}\right) B_{i}^{k}+\alpha_{0}\left(1+\gamma\left(\frac{F_{i}^{k}}{S}\right)^{\theta}\right) t_{i} F_{i}^{k}$
The valuations of passenger costs in a CBA framework are meant to reflect passenger preferences. Table 1 presents the CBA parameters taken from the official Swedish guidelines, which in turn build on various research studies. There are two relevant official CBA guidelines: the ASEK national guidelines (Trafikverket, 2016), which specify valuations of travel times, waiting times, crowding and various other benefits, and the guidelines specific to Stockholm public transport (SLL, 2017), which complement the ASEK guidelines by adding operating costs for commuter trains, travel purpose shares and various other benefits and costs.

In Table 1, references are given to the original research studies which Trafikverket (2016) and SLL (2017) have taken their valuations from. Valuations estimated in different years are scaled to a common price level. The passenger cost parameters $(\alpha, \beta, \gamma, \theta)$ are all indexed by 0 to distinguish them from the parameter estimates obtained from the timetable analysis presented below.
$K_{\text {distance }}$ is the marginal operating cost per vehicle kilometre, reflecting wear and tear, maintenance and similar costs proportional to total vehicle distance. $K_{\text {time }}^{\text {off-peak }}$ is the marginal operating cost per vehicle hour, reflecting staff costs and other costs that are proportional to total vehicle hours. $K_{\text {time }}^{\text {peak }}$ also includes the fixed capital cost for the vehicles, since this is determined by the number of vehicles necessary for train traffic during peak hours. Vehicle costs are at 5 million SEK per wagonyear, divided by the total number of peak hours per year ( 6 h per day and 260 days per year), which gives a capital cost of 3205 SEK/vehicle-hour in peak hours (SLL, 2017). K overhead reflects overhead cost such as administration, information and other costs proportional to traffic supply.

The seating capacity of a commuter train (with two coupled trainsets) is $S=748$ seats in total (ALSTOM, 2004). Most of the trains during the studied time periods are operated with two wagons even during midday (off-peak with lower passenger loads) due to other considerations such as infrastructure restrictions and punctuality. For comparison, we also analyse optimal off-peak frequency assuming operations with short trains (single trainset, $S=374$ ).

### 3.2. The optimal service frequency

The socially optimal timetable is the one that minimizes the net social cost TC. In general, finding the optimal timetable is hard optimization problem. However, with certain simplifying assumptions, analytic solutions can be obtained, which illuminates how the optimal timetable depends on the underlying demand and the parameters in the cost-benefit framework.

In most urban public transport systems, timetables are regular over certain time periods, so they can be expressed as service frequencies. Let $N$ be the number of trains per hour for a specific line and time period. If we further assume that the waiting time disutility function is linear and that passengers arrive uniformly to their origin station during the time period, it is possible to obtain analytic results for the optimal frequency, because passengers will be spread evenly across services: the number of passengers per train on link $i$ will be $\frac{F_{i}}{N}$, where $F_{i}$ is the total passenger flow on the link per hour, and the average boarding passengers per train will be $\frac{B}{N}$, where $B=\Sigma_{i} B_{i}$ is the total number of boarding passengers on the line during per hour. Since the waiting penalty is the waiting time
multiplied by a factor $\beta$, the total social costs $T C$ for this line and time period can then be rewritten as
$\mathrm{TC}(N)=K N+\alpha_{0} \sum_{i} t_{i} F_{i}+\frac{\alpha \gamma}{S^{\theta} N^{\theta}} \sum_{i} t_{i} F_{i}^{\theta+1}+\frac{\beta B}{2 N}$
The optimal service frequency $N^{*}$ is the one that minimizes TC $(N)$. If crowding is ignored $(\theta=0)$, the optimal frequency $N^{*}$ is the well-known square root rule from Mohring (1972):
$\frac{d T C}{d N}=K-\frac{\beta \mathscr{B}}{N^{2}}=0 \Rightarrow N^{*}=\sqrt{\frac{\beta \mathscr{B}}{K}}$
If $\theta>0$, the optimal frequency is given by
$\frac{d T C}{d N}=K-\frac{\beta B}{N^{2}}-\frac{\alpha_{0} \gamma \theta F_{\theta}}{S^{\theta} N^{\theta+1}}=0$
where $F_{\theta} \equiv \sum_{i} F_{i}^{\theta+1} t_{i}$. This equation has simple analytic solutions for $\theta=$ 1 and $\theta=3$. If the crowding penalty is linear in seating occupancy $(\theta=$ 1 ), the optimal frequency becomes
$N^{*}=\sqrt{\frac{\beta \mathscr{B}+\frac{\alpha_{0} \gamma}{S} F_{1}}{K}}$
For a cubic crowding penalty ( $\theta=3$ ), which is used in our case study, the optimal frequency becomes
$N=\sqrt{\frac{\beta B}{2 K}+\sqrt{\left(\frac{\beta B}{2 K}\right)^{2}+\frac{3 \alpha_{0} \gamma F_{3}}{K S^{3}}}}$
For general values of $\theta$, closed-form solutions are either complicated or non-existent (Abel, 1824).

In our case study, there are no closed-form solutions since passenger demand is not uniform across the time period, and hence the passenger load on each train is a more complicated function of the service frequency.

### 3.3. Estimating the implicit valuations of the public transport agency

The commuter train timetable in our case study is regular over certain time periods, so they can be expressed as service frequencies $N_{r}$ for each line and time period combination $r$. Passenger demand is not uniformly distributed across the time periods, however, so average waiting times and the number of passengers on each service will not simply be inversely proportional to the frequency as in the preceding section.

In order to estimate the agency's implicit valuation, we assume that it strives to minimize total social costs, i.e., service frequencies $N_{r}$ for each line and time period combination $r$ should fulfil $\frac{d T C}{d N_{r}}=0$. Based on this assumption, we can estimate the agency's implicit valuations such that the observed service frequencies $N_{r}$ are indeed the optimal choices, or as close to optimal as possible. In our case study, we have frequencies for two lines and three periods of time (we assume that frequencies have to be the same in each direction of a line), but only three valuation parameters $\gamma, \theta$ and $\beta$ (the baseline value of travel time $\alpha_{0}$ cannot be identified separately, since it is confounded with $\gamma$ ). We hence estimate the valuation parameters by minimizing the squared deviations from the optimality conditions $\frac{d T C}{d N_{r}}=0$, summed over line and time period combinations $r$ :

$$
\begin{align*}
\{\gamma, \theta, \beta\} & =\underset{\gamma, \theta, \beta}{\operatorname{argmin}} \sum_{r}\left(\frac{\partial \mathrm{TC}(\gamma, \theta, \beta)}{\partial N_{r}}\right)^{2}=\underset{\gamma, \theta, \beta}{\operatorname{argmin}} \sum_{r}\left(K_{r}+\frac{\alpha \gamma}{S^{\theta}} \frac{d \mathscr{F}_{r}}{d N_{r}}\left(\theta ; N_{r}\right)\right. \\
& \left.-\frac{d \mathscr{B}_{r}}{d N_{r}}\left(\beta ; N_{r}\right)\right)^{2} \tag{8}
\end{align*}
$$

Here, $K_{r}$ is the marginal operating cost (different for peak and off-peak
services as described above), and $\mathscr{F}_{r}=\sum_{k \in r} \Sigma_{i \in k} t_{i}\left(F_{i}^{k}\right)^{\theta+1}$ denotes crowding-weighted travel times, and $\mathscr{B}_{r}=\sum_{k \in r} \Sigma_{i \in k} \beta\left(w_{i}^{k}\right) B_{i}^{k}$ is total waiting time disutility. Since passenger demand is not uniform across time, these derivatives cannot be expressed analytically. Instead, we use numerical differentiation to calculate them.

### 3.4. Heterogenous demand and crowding variation

The crowding penalty function takes as its argument the ratio between passengers and seats on the train. That effectively assumes that passengers are spread evenly in the train, which is not true in reality; usually, there are more passengers near the ends of the train. Similarly, demand varies randomly between days, even after taking predictable demand variation into account.

Since the crowding penalty function is non-linear, this heterogeneity will not cancel out, but contribute to a higher average crowding penalty than if passengers were spread evenly across the train, and if demand did not vary between days. In this section, we explore the effect of this on the crowding penalty. We consider crowding variation as an example: day-to-day demand variation is analogous.

Let $F$ be the number of passengers on the train. Normalize the train length to 1 , so there are on average $F$ passengers per distance unit in the train. Passengers are spread out unevenly: there are more passengers at the ends of the train than in the middle. Assume that at the ends of the train there are $(1+\delta) F$ passengers per distance unit, and $(1-\delta) F$ in the middle. Hence, the average passenger-to-seat ratio is $\frac{F}{S}$, see Fig. 1 for an illustration. In other words, the passenger density $f$ at a distance from the train's end $x$ in the interval $x \in[0,0.5]$ is $f(x)=F(1+\delta-4 \delta x)$.

Given this passenger density $f$, the average in-vehicle value of time is obtained by integrating the value of time $\alpha_{0}\left(1+\gamma\left(\frac{f}{s}\right)^{\theta}\right)$ from one train end to the other, weighted by the passenger density:
$\int_{0}^{1} \alpha_{0}\left(1+\gamma\left(\frac{f(x)}{S}\right)^{\theta}\right) f(x) d x=\alpha_{0}\left(1+\gamma\left(\frac{F}{S}\right)^{\theta} \frac{(1+\delta)^{\theta+2}-(1-\delta)^{\theta+2}}{2 \delta(\theta+2)}\right) F$

When $\delta$ tends to zero, the cost in (9) will tend to the crowding penalty function used in equation (1). If $\delta=0.5$, which is a realistic value for crowding variation along a train, and with $\theta=3$, the crowding penalty factor $\gamma\left(\frac{F}{S}\right)^{\theta}$ is multiplied by a factor 1.5. As an illustration, this means that if the passenger-to-seat ratio is $\bar{F}=1.2$ and $\gamma=0.14$, the value of invehicle time increases by $49 \%$, compared to the $24 \%$ it would have increased if passengers had been evenly spread across the train. Further sensitivity analyses are presented below.


Fig. 1. Model for uneven in-vehicle crowding and spreading of passengers in the train.

### 3.5. Boarding and alighting time externalities

The cost-benefit framework presented above does not take into account that more boarding/alighting passengers on each train increases the total time passengers in the trains have to wait for other passengers to board/alight. Boarding/alighting passengers creates an externality for the other passengers that are already on the train, since the train has to wait. This effect is more pronounced for bus services than for train services, since buses usually only have one door through which to board or alight, while train boarding and alighting uses many doors, and an increase in the number of boarding and alighting passengers thus delays the train less than proportionally. Still, this effect is worth considering for large passenger volumes. In our case study, this effect is negligible, but in other cases it may be significant.

To clarify how this consideration affects the optimal frequency, we focus on boarding passengers and consider a time period with uniform passenger demand and a regular timetable with $N$ number of trains during the time period. Let $B_{i}$ be the number of boarding passengers on station $i$, and let $F_{i}$ be the total passenger flow on link $i$. Since passenger demand is uniform and the timetable is regular, the number of boarding passengers per train on station $i$ is $\frac{B_{i}}{N}$ and the number of passengers per train on link $i$ is $\frac{F_{i}}{N}$.

Consider the case where the train must wait on the station for passengers to board is proportional to the number of boarding passengers. This is realistic for buses where there is just one door to board through, but less so for trains with simultaneous boarding in many doors. Still, it is a reasonable approximation for large boarding volumes. Let $b$ be the boarding time per passenger, so total boarding time per train on station $i$ is $\frac{B_{i}}{N} b$. Ignoring crowding for simplicity, total social costs become the sum of operations costs, in-vehicle passenger time on the links, waiting times, and in-vehicle passenger time when the train is waiting for passengers to board:

$$
\begin{align*}
T C & =K N+\sum_{i} \beta \frac{B_{i}}{2 N}+\alpha_{0}\left(t_{i}+\frac{B_{i}}{N} b\right) F_{i} \\
& =K N+\sum_{i}\left(\beta+2 \alpha_{0} b F_{i}\right) \frac{B_{i}}{2 N}+\alpha_{0} t_{i} F_{i} \tag{10}
\end{align*}
$$

The factor $2 \alpha_{0} b F_{i}$ multiplying $B_{i}$ is new compared to the previous cost-benefit framework. It appears because boarding prolongs in-vehicle times for all in-vehicle passengers in proportion to the number of boarding passengers on each service, which is inversely proportional to service frequency, just as waiting times are. The expressions for optimal frequencies above are changed by this: $\beta$ in eqs. (4)-(7) is replaced by $\beta+$ $2 \alpha_{0} b F_{i}$, so the optimal frequency is increased by the boarding time externality. For low values of $b$ (such as trains with moderate crowding), this change is negligible, while for high values of $b$ (such as crowded bus services) it can be consequential.

## 4. Data

In this section, we present the input data for the numerical analysis performed on a commuter train line in Stockholm. Fig. 2 presents the commuter train network (as of 2015). We will first concentrate on one line and direction, i.e., the J35 line in Fig. 2 filled in black from Kungsängen (Kän) to Västerhaninge (Vhe). We then present summary results for the other main lines and directions (i.e., between Upplands Väsby and Tumba). The J35 line includes 17 stations (from a network total of around 50) with Stockholm central station as the largest passenger station. Part of the studied line (i.e., between Karlberg and Älvsjö) are shared with other lines.

For each pair of stations, we know the number of trips for every 15 min over a normal day in September 2015, i.e., the time-dependent OD matrix. This matrix is estimated from smart card data using entropy maximization, also used by Ait-Ali and Eliasson (2021). It also includes


Fig. 2. Studied (filled in black) line of the commuter train network in Stockholm, adapted from (SLL, 2015).
passengers transferring to and from other lines. Some services start or continue outside the studied line, but we assume that those passenger flows start (terminate) at the first (last) station. Travel times between stations $t_{i j}$ are known and constant for all trains. We study three main time periods: morning peak (6:00-9:00), afternoon peak (15:00-18:00) and midday off-peak (10:00-13:00). Fig. 3 illustrates the number of passengers entering each station per 15 min over the day.

SL, the public transport agency in Stockholm (also called SLL), adopted in the 2015 commuter train timetable summarized in Table 2. The table shows service frequencies from Kän to Vhe, including extra departures on parts of the line during peak hours. There is a regular service frequency during all the studied time periods, see column 2. During peak hours there are additional train departures which are not all regular, see column 3. Those extra departures do not operate the whole line but for the sake of simplicity, we assume they do which leads to the total frequency presented in column 4 . Thus, there are 7 departures per hour in total during the morning peak (i.e., a train every 8.6 min ), 6 departures per hour in the afternoon (i.e., a train every 10 min ) and 4 departures per hour during midday (i.e., train every 15 min ).

Given the OD-matrix and the train timetable, passenger link flows and the number of passengers boarding passengers are calculated for each train service, link and station. Fig. 4 shows passenger link flows per train service, one coloured line per service. The horizontal dashed line indicates the total number of available seats per train, i.e., $S=748$ (ALSTOM, 2004).

## 5. Results

Fig. 5 shows total social costs (in SEK) as a function of service frequency on the Kän-Vhe line for the three time periods. For the midday time period, costs are presented for long and short trains, respectively. Note that peak and off-peak have different scales on the y-axis, i.e., the right axis is for off-peak hours.

The optimal frequency is where the total social cost is minimal. If the frequency is higher than optimum, the increase in operations costs outweigh the decreased passenger costs, and vice versa for frequencies lower than optimum. Optimal frequencies for the different time periods are noted in column 2 of Table 3, and are compared to SL's (the Stockholm public transport agency) actual frequencies from the

Table 2
SL's service frequency from Kän to Vhe for different time periods during a working day (winter 2015).

| Time period | Regular <br> departures <br> (\#trains per hour) | Extra departures <br> (\#trains per hour) | Total frequency <br> (\#trains per <br> hour) |
| :---: | :--- | :--- | :--- |
| Morning peak <br> $(6: 00-9: 00)$ | $4^{1}$ | $3^{2}$ | 7.0 |
| Midday off-peak <br> $(10: 00-13: 00)$ | 4 | 0 | 4.0 |
| Afternoon peak <br> $(15: 00-18: 00)$ | 4 | 2 | 6.0 |

${ }^{1}$ Certain trains are running parts or beyond the studied line, e.g., to Älvsjö or Nynäshamn, from Jakobsberg.
${ }^{2}$ The provided frequency for extra departures is an average since not all are regularly running every X minute.

## timetable.

For all lines, directions and time periods, actual frequencies are lower than optimal ones according to the CBA framework. The difference is smallest for the Kän-Vhe line in morning, and largest for Upv-Tu line in the afternoon. Optimal frequencies are generally higher for the Upv-Tu line, due to the higher ridership on that line, but this is not reflected in the actual timetables.

Running short trains during off-peak hours leads to higher optimal frequencies, since operations costs are lower for shorter trains.

From an operational point of view (e.g., efficient rolling stock circulation), the frequencies for the two directions on the same line need to be similar. This constraint can be satisfied in different possible ways. For instance, the line frequency could be set as the average or maximum of the two optima. Another alternative is to modify the model in order to include all the line (i.e., both directions) with a single variable (line frequency).

That SL's frequencies are lower than what is optimal according to the CBA framework implies that the agency effectively puts a lower implicit valuation on either waiting time or crowding (or both) than the recommended CBA valuations in Table 1. We thus turn to the question of identifying the agency's implicit valuations, as implied by the chosen frequencies in the timetable. We concentrate first on the Kän-Vhe line in


Fig. 3. Variation of the number of passengers entering each station per 15 min over the day.


Fig. 4. Passenger link flows per train service from Kän to Vhe. Each line is one train service. The dashed horizontal line shows the total seating capacity of the train.


Fig. 5. Total social costs as a function of the frequency for different time periods.
some detail, and then present the most important results for the line and directions.

### 5.1. Valuation of waiting time

The higher the waiting time valuation $\beta$ is, the higher the optimal
service frequency will be. The fact that the actual frequency is lower than the optimal using the recommended parameter $\beta_{0}$ (from Table 1) suggests that SL's implicit valuation is lower than $\beta_{0}$. Fig. 6 shows how the optimal frequency depends on the waiting time valuation $\beta$. The horizontal dashed lines show SL's frequencies for the morning, mid-day and afternoon whereas the vertical dashed line is the baseline valuation $\beta_{0}$ (which is different for peak and off-peak since the waiting time disutility is a piecewise linear function).

SL's implicit valuation of waiting time (keeping other parameters fixed) can be seen from Fig. 6, where the optimal frequency curve intersects the actual frequency dashed line. The implicit valuations are different for different time periods, as summarized in Table 4.

For all time periods, the agency's implicit valuations (as presented in Table 4) are lower than the recommended valuation $\beta_{0}$ in the CBA guideline (from Table 1). The deviation is the lowest ( $15 \%$ ) during morning peak hours, and higher (37-39\%) during the other time periods. Note, though, that these are the implicit valuations obtained if only the waiting time valuation is changed, while the crowding valuation is kept constant. Next, we study the crowding valuation parameters.

### 5.2. Valuation of in-vehicle crowding

The crowding valuation depends on two variables, the factor $\gamma$ and the exponent $\theta$, and is more relevant for peak hours where crowding is present. Fig. 7 focuses on peak hours, and presents optimal frequencies as functions of $\gamma$ (left) and $\theta$ (right), keeping other valuations fixed. The

Table 3
Optimal service frequencies per line and time period according to the CBA framework, compared to SL's actual frequencies. The relative deviation from the actual frequency is given (in percent) between parentheses.

| Time period | SL | Kän $\rightarrow$ Vhe (Southwards) | Vhe $\rightarrow$ Kän (Northwards) | Upv $\rightarrow$ Tu (Southwards) | Tu $\rightarrow$ Upv (Northwards) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 7.0 | $7.3(+4 \%)$ | $7.3(+4 \%)$ | $8.3(+19 \%)$ | $8.6(+23 \%)$ |
| Midday (long) | 4.0 | $5.0(+25 \%)$ | $5.3(+33 \%)$ | $5.3(+33 \%)$ | $8.3(+33 \%)$ |
| Midday (short) | - | $7.0(+75 \%)$ | $7.6(+90 \%)$ | $8.3(+108 \%)$ | $8.3(+38 \%)$ |
| Afternoon | 6.0 | $7.0(+17 \%)$ | $7.0(+17 \%)$ | $8.0(+33 \%)$ |  |



Fig. 6. The optimal frequency as a function of the waiting time valuation for different time periods.

Table 4
Implicit waiting time valuations (in SEK/h) for train services from Kän to Vhe, other valuations fixed.

| Time period | Implicit waiting time valuation (in SEK/h) | Deviation from $\boldsymbol{\beta}_{0}^{\left({ }_{0}\right)}$ |
| :--- | :--- | :--- |
| Morning | 142 | $-15 \%$ |
| Midday (long) | 84 | $-39 \%$ |
| Afternoon | 106 | $-37 \%$ |

(*) $\rho_{0}^{\text {peak }}=168 \mathrm{SEK} / \mathrm{h}$ and $\beta_{0}^{\text {off-peak }}=137 \mathrm{SEK} / \mathrm{h}$.
meaning of the vertical and horizontal dashed lines is similar to that in Fig. 6.

As expected, higher crowding valuations generally yield higher optimal frequencies. After a certain value, increasing the valuations has a negligible effect on the optimal frequency. For afternoon peak hours, the numerical results indicate that optimal frequencies are still higher than actual frequencies even if the valuation of crowding is set to zero.

The crowding penalty function is based on average seating occupancy in the entire train, effectively assuming that passengers are spread evenly across the train. In fact, there are usually more passengers towards the ends of the train due to the layout of the stations (Peftitsi et al., 2020). Since the crowding valuation is a nonlinear function of the
seating occupancy, heterogenous occupancy along the train will increase the total crowding penalty, even more so if one also considers that more passengers will, by definition, experience high crowding than will experience low crowding (since there are, by definition, more passengers where there is higher crowding). The losses for high-crowded parts of the train will hence outweigh the benefits of the low-crowded parts. Below, we perform a sensitivity analysis to study the effect of taking heterogeneous occupancy into account.

Moreover, demand varies across days, and since the crowding valuation is nonlinear, the higher crowding penalties during highdemand days will outweigh the corresponding lower crowding penalties during low-demand days. Another sensitivity analysis is also presented later, looking at the effect of varying demand on the optimal frequency.

### 5.3. Joint changes in valuations

Fig. 8 illustrates how optimal frequencies vary when crowding and waiting time valuations are varied jointly. The contour lines show combinations of valuations (crowding $\gamma$ and waiting time $\beta$ ) with the same optimal frequency. The vertical and horizontal dashed lines refer, respectively to the baseline valuations $\beta_{0}$ and $\gamma_{0}$. Note that the contours are not continuous since costs are only calculated for certain discrete/


Fig. 7. Optimal frequency as a function of the crowding parameter during peak hours (left - factor, right - exponent).


Fig. 8. Contour lines showing the optimal frequency when varying both crowding and waiting time valuations during morning and afternoon time periods.
integer values of frequency during the studied time periods
Fig. 8 shows that the sensitivity of the optimal frequency to the waiting time valuation is much higher than to the crowding factor, i.e., level curves or contours are almost vertical, especially for afternoon peak hours. This is explained once again by the fairly low crowding in the trains on the studied line and direction.

### 5.4. The agency's average implicit valuations

In order to estimate SL's implicit valuation averaged over all the lines and time periods, we use equation (8) that was previously presented. The equation includes derivatives with respect to $N$ that can only be calculated numerically (due to demand not being uniform over time); we use the central difference method. Table 5 compares the implicit and baseline valuations of waiting time while fixing the crowding valuations, since crowding is so low that these parameters cannot be estimated. The table distinguishes between peak and off-peak hours, thus $r$ is over 4 combinations for peak hours and 2 for off-peak.

Table 5 indicates that SL's average implicit valuations of waiting time (over all lines and time periods) are generally higher than the valuations per time period for the Kän-Vhe line (as presented in Table 4). However, these average valuations are still lower than the recommended passenger valuations (from the CBA guidelines). The deviation is higher during peak-hours which is mainly due to the higher optimal frequencies on other lines and directions, see Table 3.

### 5.5. Heterogeneous crowding and demand

The presented results do not take into account that demand varies randomly between days, train services and across the train wagons. Since the crowding function is non-linear, this means that the passengerweighted average crowding will be higher than if passengers were spread evenly. To study the effect of these variations, we perform two sensitivity analyses, namely on the passenger demand and the train seat supply.

First, based on the already used OD matrix, we analyse the optimal

Table 5
Comparison between SL's estimated average implicit valuations of waiting time and baseline CBA guidelines (crowding valuation fixed).

|  |  | CBA guidelines | Average implicit <br> valuation | Deviation |
| :---: | :--- | :--- | :--- | :--- |
| Waiting <br> time | Peak | $\beta_{0}^{\text {peak }}=168 \mathrm{SEK} /$ | $\beta_{\mathrm{SL}}^{\text {peak }}=106 \mathrm{SEK} /$ | $-37 \%$ |
|  |  | h | h |  |
|  | Off-peak | $\beta_{0}^{\text {off-peak }}=137$ | $\beta_{\mathrm{SL}}^{\text {off-peak }}=$ | $-25 \%$ |
| Crowding | Factor | $\mathrm{SEK}_{0}=0.14$ | $103 \mathrm{SEK} / \mathrm{h}$ |  |
|  | Exponent | $\theta_{0}=3$ | $\gamma_{\mathrm{SL}}=\gamma_{0}$ | Fixed |
|  |  | $\theta_{\mathrm{SL}}=\theta_{0}$ | Fixed |  |

frequencies using varying passenger demand, i.e., 11 variants with OD matrix variation up to $-/+50 \%$ of the previously used demand for boarding/alighting passengers. The results are presented using the bar chart in Fig. 9 for the Kän-Vhe line. For each time period, the bars show the optimal frequency for the different demand (and supply) scenarios. The optimal frequencies are based on a total social cost function which is defined as the average over all OD matrix (demand) variants. The figure also shows the previously reported frequencies (bars to the left), i.e., baseline (SL's actual) and optimum.

When passenger demand is variable, the optimal frequency increases for peak hours and remains unchanged during mid-day time period. This insensitivity during off-peak hours can be due to the already low crowding levels, meaning that waiting times dominate the passenger costs.

Previously, we also considered that passengers are assumed to enter train wagons uniformly. However, passengers tend to choose wagons that are near the entrance of the train station (for boarding) or exit (for alighting) which leads to increased crowding in these wagons and decreased in others (Fang et al., 2019). Since the crowding penalty function is nonlinear, these variations do not necessarily cancel each other out. One way to study this difference in crowding levels between wagons is to vary the available seating capacity compared to the total number of seats in the trains. For instance, decreasing seating capacity in the analysis could reflect the unbalanced loads between train wagons.

Thus, we also perform a second sensitivity analysis on the available seating capacity (i.e., the total number of seats in the trains) in a way that is similar to the previous analysis on passenger demand, i.e., 11 variants with variation in vehicle capacity up to $-/+50 \%$ of a standard long train with two coupled trainsets. The results are presented using the bar (referring to variable supply) in Fig. 9.

The effects of variable supply on the optimal frequency is similar to that of variable demand. Moreover, varying both (passenger demand and seat supply) further increases the optimal frequency but only during peak hours, see the bars to the right in Fig. 9. Thus, the sensitivity analyses reveal that variations in day-to-day ridership (demand), disparities in crowding between wagons (seating capacity) or both can lead to an increase in the optimal frequency only during peak hours. These variations have no effect during off-peak.

## 6. Conclusions and future works

This study presents a method to estimate implicit valuations of crowding and waiting time implied by a public transport agency's (PTA) choice of timetable. The method is applied on a case study, the Stockholm commuter train system. In the case study, the PTA's implicit valuations turn out to be around $15-39 \%$ lower than the recommended valuations in the official CBA guidelines, which are based on passenger valuations as measured in stated preferences studies. Equivalently, the PTA's service frequencies are lower than the optimal ones according to


Fig. 9. Optimal frequency from Kän to Vhe when varying passenger demand and/or seat supply for different time periods, compared to SL's chosen frequency and the previously presented optimum for given passenger demand and seat capacity
recommended CBA valuations; optimal service frequencies are around $4-38 \%$ higher than actual frequencies, varying considerably between lines and time periods.

Optimal frequencies increase further when taking into account demand variations between days and varying crowding levels along the train. Since the crowding disutility is convex, these variations do not cancel out, but increase the optimal service frequency during peak hours. Our sensitivity analyses show that these effects are relatively small.

Further extensions of the model are possible, such as including valuations of punctuality and interchanges, and accounting for elastic demand and road traffic externalities.

One of the motivations of this study is that it has been suggested that CBA could be used for resolving capacity conflicts between commercial trains and (publicly controlled and subsidized) commuter trains, by setting a reservation price for a commercial train path equal to the social loss of the necessary adjustment of the commuter train timetable. For this to be practically applicable, CBA valuations cannot be too different from the PTA's implicit valuations, since this can lead to various inconsistencies. Our results suggest that the currently recommended CBA valuations would result in slightly higher reservation prices than what the PTA would in fact be willing to accept as compensation for an adjusted timetable. The differences are not very large, however, and there might be reasonable explanations for the differences in valuations, discussed below. Considering that CBA is rarely used for applied timetable construction in Sweden, the differences between implicit and recommended valuations are actually smaller than one might have expected. Overall, we would conclude that the PTA's implicit valuations are sufficiently close enough to the CBA valuations that it is reasonable to use CBA for capacity allocation, possibly after improving the CBA framework in the light of further explorations.

Another motivation for this study is that investments' CBAs need to assume timetables both with and without the investment - it is the difference between these timetables that determine the benefits of the investment. Our results suggest that PTA in the case study actually run slightly fewer services than implied by the CBA framework. This means that benefits of investments in increased capacity are likely to be overestimated if the CBA framework are used to calculate timetables.

Our results differ from the studies by Börjesson et al. (2017) and Asplund and Pyddoke (2019), who studied two different bus services in Sweden, finding frequencies to be higher than optimal, on average. There are indeed several reasons to expect PTA frequencies to be higher than optimal, for example local political lobbying and that marginal passenger benefits tend to be more salient than marginal taxpayers' expenses.

On the other hand, there are also several reasons why PTA frequencies may be lower than optimal, resulting in implicit valuations being lower than passengers valuations. Lack of capacity may be a factor; during peak hours, it may be impossible to increase frequencies further. However, our results show that PTA frequencies are lower in off-
peak hours as well - in fact, the difference is even larger then - so this is hardly the explanation.

A more plausible reason is lack of funds. Optimal frequencies are inversely proportional to the square root of the marginal operations cost, including the marginal cost of public funds. If the cost of public funds political or economical - is high, then this will decrease the optimal frequency. Finally, it is conceivable that the PTA actually (implicitly) puts a more correct value on crowding and waiting time than the CBA guidelines. The guidelines are based on stated preference (SP) studies, and the few studies that have compared SP with revealed preferences (RP) of waiting time and crowding suggest that SP valuations tend to be higher than RP valuations - contrary to similar comparisons of invehicle time valuations. The empirical evidence here is scarce, however. This is an area where more research is clearly needed; passengers' monetary valuations of waiting time and crowding are extremely important for public transport planning, since they directly affect the optimal trade-off between high service frequencies and low fares, which is central in all public transport planning.

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[^1]:    ${ }^{1}$ That we can use the average waiting time in the $\beta(w)$ function like this is because it is a piecewise linear function of the waiting time with different slopes, e.g., $\beta_{0}^{\text {peak }}$ or $\beta_{0}^{\text {off-peak }}$, depending on the headway, see Table 1 and below.

