Distributed Optimization for Control and Estimation

Shervin Parvini Ahmadi

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Cover illustration: A computational graph for solving an optimization problem in a distributed manner using message passing.

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To my parents

Abstract

Adopting centralized optimization approaches in order to solve optimization problem arising from analyzing large-scale systems, requires a powerful computational unit. Such units, however, do not always exist. In addition, it is not always possible to form the optimization problem in a centralized manner due to structural constraints or privacy requirements. A possible solution in these cases is to use distributed optimization approaches. Many large-scale systems have inherent structures which can be exploited to develop scalable optimization approaches. In this thesis, chordal graph properties are used in order to design tailored distributed optimization approaches for applications in control and estimation, and especially for model predictive control and localization problems. The first contribution concerns a distributed primal-dual interior-point algorithm for which it is investigated how parallelism can be exploited. In particular, it is shown how the computations of the algorithm can be distributed on different processors so that they can be run in parallel. As a result, the algorithm execution time is accelerated compared to the case where the algorithm is run on a single processor. Simulation studies on linear model predictive control and robust model predictive control confirm the efficiency of the framework. The second contribution is to devise a tailored distributed algorithm for nonlinear least squares with application to a sensor network location problem. It relies on the Levenberg-Marquardt algorithm, in which the computations are distributed using message passing over the computational graph of the problem, which is obtained from what is known as the clique tree of the problem. The results indicate that the algorithm provides not only a good localization accuracy, but also it requires fewer iterations and communications between computational agents in order to converge compared to known first-order methods. The third contribution is a study of extending the message passing idea in order to design tailored distributed algorithm for general non-convex problems. The framework relies on an augmented Lagrangian algorithm in which a primal-dual interior-point method is used for the inner iteration. Application of the framework for general model predictive control of systems with several interconnected sub-systems is extensively investigated. The performance of the framework is then compared with distributed methods based on the alternating direction method of multipliers, where the superiority of the framework is illustrated.

Populärvetenskaplig sammanfattning

Datorstödda beräkningar har öppnat upp för tillämpningar av optimering inom många olika ingenjörsvetenskapliga discipliner. Optimal styrning och reglering av elektriska och mekaniska system och kemiska processer kan lösas med hjälp av optimering. Huvudsakligen används centraliserad optimering för detta ändamål. Detta kräver att all information om det som ska styras och regleras liksom alla beräkningar kan samlas på ett ställe. Detta är dock en utmaning för komplexa och storskaliga system, som har blivit allt vanligare de senaste åren. Speciellt kan det finnas begränsningar i centraliserad beräkningskraft, i bandbredd för kommunikation och i konfidentialitet rörande data. För att överbrygga dessa problem har distribuerad optimering föreslagits som en lösning. Inom distribuerad optimering samarbetar ett antal beräkningsenheter med varandra för att lösa ett övergripande optimeringsproblem genom att varje beräkningsenhet löser enskilda delproblem och kommunicerar med vissa av de andra beräkningsenheterna. Målet med denna avhandling är att ta fram effektiva distribuerade optimeringsmetoder för komplexa och storskaliga problem. De tillämpningar som betraktas är bl.a. modellprediktiv reglering, som är en reglermetod speciellt lämpad för att styra system med begränsningar. En annan tillämpning som studeras är lokalisering av sensorer i ett nätverk baserat på avståndsmätningar. De distribuerade metoderna vi tar fram utnyttjar inneboende strukturer i problemen vi studerar för att ta fram effektiva och resurssnåla metoder.

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Linköping, January 2022
Shervin Parvini Ahmadi

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Part I

Background

1

Introduction

There has been a considerable interest in distributed optimization over the recent years due to the advances in technology. Distributed optimization is the procedure of solving an optimization problem using a network of computational agents. Particularly, the complete problem is decomposed into several local problems, which are assigned to the computational agents. The agents subsequently solve the complete optimization problem by collaborating and communicating with each other. The special case where only one computation agent is used, is called centralized optimization. This has been the primary way to solve optimization problems in the past. There are, however, a number of limitations associated with centralized optimization. The first one is the computational challenges, particularly memory and power insufficiency, (Boyd et al., 2011). These challenges arise when analyzing large-scale complex systems with a substantial number of decision variables. The second limitation is the communication challenges. Recent advances in wired and wireless technology have led to the appearance of large-scale networked systems. These systems comprise a significant number of interconnected sub-systems which cooperate with each other to achieve a desired objective. Therefore, using a centralized optimization approach will require a considerable number of communications due to the distributed nature of them which is not desired, (Yang et al., 2019). The third limitation is the incapability of the centralized optimization approaches in handling applications with structural constraints. For instance, if there is a requirement on preserving the privacy of data, then it is not always possible to form the optimization problem in a centralized manner, (Nedić and Liu, 2018; Pakazad, 2015; Zheng, 2019). In order to address these limitations, distributed optimization approaches have extensively been studied over the past years. Two pioneer early works are Bertsekas and Tsitsiklis (1997); Tsitsiklis (1984). The applications of distributed optimization in control and estimation are very broad, e.g., wireless sensor networks, (Biswas et al., 2006; Simonetto and Leus, 2014; Wang et al., 2008), power systems, (Kargarian et al., 2016; Molzahn et al., 2017), autonomous

vehicles, (Bian et al., 2019; Cao et al., 2012; Khayatian et al., 2020; Malikopoulos et al., 2018; Tajalli and Hajbabaie, 2018), machine learning, (Boyd et al., 2011; Nedic, 2020), robotic networks, (Bullo et al., 2008), communication networks, (Johansson, 2008; Nedić et al., 2018; Sayed, 2014), and multi-agent systems, (Fioretto et al., 2018; Ren and Cao, 2011; Zhu and Martínez, 2015), to name a few. For a thorough and recent review of distributed optimization algorithms, refer to Boyd et al. (2011); Molzahn et al. (2017); Nedić and Liu (2018); Nedich et al. (2015); Yang and Johansson (2010); Yang et al. (2019).

The classical distributed optimization approaches are based on applying first-order methods to the problem or some formulation of the problem. First-order methods are methods which make use of gradients of the objective and constraint functions. These approaches rely on exact gradient computations if the problem is smooth, otherwise they rely on subgradients or proximal operators, (Bertsekas and Tsitsiklis, 1997; Eckstein, 1989; Nedic and Ozdaglar, 2009). In this class of distributed approaches, some popular algorithms are based on the Alternating Direction Method of Multipliers (ADMM), which is a special case of the proximal point method, (Boyd et al., 2011). Although these approaches are effective, a major shortcoming is their slow rates of convergence. In order to alleviate this issue, there has recently been an interest in devising distributed approaches based on second-order or Newton type methods, such as interior-point methods, due to their superior convergence rate. Second-order methods are methods which make use of both gradients and Hessians of the objective and constraint functions. See Dinh et al. (2013); Houska et al. (2016); Necoara and Suykens (2009), for examples of distributed second-order approaches and the references therein. In Necoara and Suykens (2009), a distributed algorithm based on a Lagrangian dual decomposition technique is presented for convex optimization problems. The dual variables are updated in an interior-point framework using the second-order information of the dual function. The algorithm is limited to problems with affine constraints, and also it requires that each agent communicates its local primal variables with all the other agents at each iteration, in order to attain the necessary information for updating the dual variables. In Dinh et al. (2013), a dual decomposition based distributed method is presented for convex problems with affine constraints, which employs an interior-point framework with a smoothed dual function. A limitation of the algorithm is that it often requires a large number of iterations in order to converge to a solution with medium accuracy, (Houska et al., 2016). In Houska et al. (2016), a distributed algorithm for non-convex smooth optimization problems with coupled affine constraints is presented. The algorithm is a further development of ADMM, which contains ideas from augmented Lagrangian and sequential quadratic programming. The algorithm has a faster convergence rate compared to ADMM based algorithms. This however comes at the price that it requires more communications, and also the centralized update cannot be distributed, (Meyer-Huebner et al., 2019). Common for the distributed approaches based on second order information is that they do not address general nonlinear constraints.

In this thesis, we continue the lines of research initialized in Khoshfetrat Pakazad et al. (2017), for distributed optimization of loosely coupled convex problems. The algorithm is a the primal-dual interior-point method and enjoys superior conver-

gence properties compared to other existing distributed algorithms. We extend the applicability to non-convex smooth optimization problems with general nonlinear constraints. The algorithm is based on an augmented Lagrangian formulation. As we will see later, all the computations of the algorithm can be distributed, and also the required communications for convergence is much lower than for distributed ADMM based methods.

Most optimization problems arising in the aforementioned applications have inherent structures and are sparse in their topological connections. The main motivation of this thesis is to exploit the sparsity in these optimization problems. A common structure which is exploited in devising a majority of the distributed algorithms is what is known as partial separability. A partial separable function can be defined as a function which can be defined as summation of elementary functions, where each elementary function depends on a small subset of the variables, (Vandenberghe and Andersen, 2015). A more precise definition will be provided later. Examples of where partial separability is exploited within optimization algorithms include the distributed approaches based on the Douglas-Rachford method in Parikh and Boyd (2014); Spingarn (1985), the ADMM in Boyd et al. (2011), the quasi-Newton method in Griewank and Toint (1982a,b); (Nocedal and Wright, 2006, Ch. 7), the trust-region method in Colson and Toint (2005); Conn et al. (2000), the proximal method in Chen and Teboulle (1994); Tseng (1991), the dual block coordinate ascent method in Tseng (1993), and the interior-point method in Gondzio and Sarkissian (2003); Kojima et al. (1994). Also, the nonlinear optimization software, LANCELOT, (Conn et al., 1992), exploits partial separability. Several of the algorithms that exploit partial separability do this by exploiting so-called chordal properties; see Vandenberghe and Andersen (2015) and the references therein. Chordal graphs are well-studied in graph theory and have several applications in different fields. One well-known application is to solve sparse linear systems via Cholesky factorization, (Zheng, 2019). Also, there are fundamental properties that are shared between chordal graphs and sparse positive semidefinite matrices. These properties are extensively used in order to solve large semidefinite optimization problems, (Vandenberghe and Andersen, 2015). One other important application is in probabilistic inference and machine learning where chordal graphs are exploited in solving maximum likelihood estimation problems, (Koller and Friedman, 2009; Wainwright and Jordan, 2008). In this thesis, chordal graphs are used in order to design distributed optimization algorithms, tailored for two different applications, namely the Model Predictive Control (MPC) problem and the problem of sensor network localization. For the MPC application, different type of problems are considered. In particular, linear MPC and robust MPC problems are investigated, and a distributed primal-dual interior-point method is presented in which one can exploit parallelism. It should be noted that a main application of distributed optimization is in parallel implementation using multiple processors. This, in turn, accelerates the execution time of the algorithm compared to the case where the algorithm is run in a single processor. Nonlinear MPC problem is also explored for a network of inter-connected sub-systems and a distributed algorithm is devised based on the augmented Lagrangian and a primal-dual interior-point method. As for the sensor network localization problem, a distributed algorithm

is proposed which relies on the Levenberg–Marquardt method. In order to distribute the computations within each of the above-mentioned algorithms, the idea of message passing over a computational graph, known as a clique tree, is adopted from Khoshfetrat Pakazad et al. (2017). This can also be interpreted as doing dynamic programming over a tree, (Bertele and Brioschi, 1973).

1.1 Contributions

The main contributions of the thesis can be summarized as: (I) Parallel exploitation for a distributed primal-dual interior-point algorithm. (II) Development of a distributed Levenberg-Marquardt based algorithm for nonlinear least squares problems. (III) Development of a distributed augmented Lagrangian based algorithm for general non-convex problems. The detailed contributions are as follows:

- Implementation of a distributed primal-dual interior-point algorithm for loosely coupled Quadratic Programming (QP) problems with affine equality and convex inequality constraints in Julia. Algorithm execution time acceleration via parallel exploitation. Application of the algorithm to linear MPC. Application of the algorithm to robust MPC via reformulating it as a Robust Quadratic Programming (RQP) problem. (Papers A and B)
- Extension of the message passing idea to a nonlinear least squares problem. Development and implementation of a distributed algorithm based on the Levenberg-Marquardt algorithm. Application of the algorithm to the sensor network localization problem. Extensive simulations in Julia and performance analysis with regard to competitive methods in the literature. (Paper C)
- Extension of the message passing idea to non-convex problems with equality and inequality constraints. Development and implementation of a distributed algorithm based on a second-order augmented Lagrangian and primal-dual interior-point methods. Application of the algorithm to distributed general MPC. Performance analysis with regard to a competitive distributed method. Application of the method to a vehicle platooning problem. (Papers D and E)

1.2 Thesis Outline and Publications

The thesis is divided into two parts, where the first part provides the background material and the second part is a collection of the papers listed below. Here, a summary of each paper together with the contributions of the thesis author are also provided.

Paper A: Parallel Exploitation for Tree-Structured Coupled Quadratic Programming in Julia

Paper A is an edited version of

Shervin Parvini Ahmadi and Anders Hansson. Parallel exploitation for tree-structured coupled quadratic programming in Julia. In *2018 22nd International Conference on System Theory, Control and Computing (ICSTCC)*, pages 597–602. IEEE, 2018.

Summary: In this paper we implement a distributed primal-dual interior-point algorithm in Julia for loosely coupled QP problems. We consider affine equality and affine inequality constraints. The algorithm is based on message passing in which at each iteration we compute search direction and step size, distributedly. We exploit parallelism by running the algorithm on multiple processors. This, in turn, accelerate the execution time of the algorithm. We evaluate the performance of the algorithm on linear MPC problems.

Comment: The author of this thesis contributed with the majority of the work including implementations, numerical calculations, analysis and preparation of the manuscript.

Paper B: Efficient Robust Model Predictive Control Using Chordality

Paper B is an edited version of

Shervin Parvini Ahmadi, Anders Hansson, and Sina Khoshfetrat Pakazad. Efficient robust model predictive control using chordality. In *2019 18th European Control Conference (ECC)*, pages 4270–4275. IEEE, 2019.

Summary: In this paper we implement a distributed primal-dual interior-point algorithm in Julia for loosely coupled QP problems with affine equality and convex inequality constraints. We apply it on a robust MPC problem. To this end, we first reformulate the problem as a RQP problem. We then show how the chordal structure can be used to distribute the computations in the algorithm which allows us to exploit parallelism.

Comment: The author of this thesis contributed with the majority of the work including implementations, numerical calculations, analysis and preparation of the manuscript.

Paper C: Distributed Localization Using Levenberg-Marquardt Algorithm

Paper C is an edited version of

Shervin Parvini Ahmadi, Anders Hansson, and Sina Khoshfetrat Pakazad. Distributed localization using Levenberg-Marquardt algorithm. *EURASIP Journal on Advances in Signal Processing*, 2021(1):1–26, 2021.

Summary: In this paper we present a distributed algorithm for the sensor network localization problem which is formulated using maximum likelihood estimation.

The framework is based on the Levenberg-Marquardt algorithm in which the computations are distributed using message passing over the clique tree of the problem. We show that the framework provides a good localization accuracy and requires fewer iterations and communications between computational agents in comparison to first-order methods.

Comment: The author of this thesis contributed with the majority of the work including algorithm development, implementations, numerical calculations, analysis and preparation of the manuscript.

Paper D: A Distributed Second-Order Augmented Lagrangian Method for Distributed Model Predictive Control

Paper D is an edited version of

Shervin Parvini Ahmadi and Anders Hansson. A distributed second-order augmented Lagrangian method for distributed model predictive control. *IFAC-PapersOnLine*, 54(6):192–199, 2021a.

Summary: In this paper we present a distributed algorithm for nonlinear MPC of systems consisting of several interconnected sub-systems. The algorithm is based on a second-order augmented Lagrangian method in which the computations are distributed using message passing over the clique tree of the problem. We show the efficiency of the algorithm in terms of iterations and communications required for convergence.

Comment: The author of this thesis contributed with the majority of the work including algorithm development, implementations, numerical calculations, analysis and preparation of the manuscript.

Paper E: Distributed Model Predictive Control Using an Augmented Lagrangian Method

Paper E is an edited version of

Shervin Parvini Ahmadi and Anders Hansson. Distributed model predictive control using an augmented Lagrangian method. *Submitted for possible publication*, 2021b.

Summary: In this paper we present a distributed algorithm for nonlinear MPC of systems consisting of several interconnected sub-systems. Compared to the work in Paper D, in this paper we take inequality constraints into account as well. The algorithm relies on an augmented Lagrangian formalism in which a primal-dual interior-point method is used in the inner iteration. The computations of the algorithm are distributed using message passing over the clique tree of the problem. We compare performance of the algorithm to a competitive distributed ADMM based method in terms of iterations and communications required for convergence.

Additionally, we apply the proposed algorithm to a vehicle platooning problem and illustrate its efficiency.

Comment: The author of this thesis contributed with the majority of the work including algorithm development, implementations, numerical calculations, analysis and preparation of the manuscript.

1.3 Background

In this section we discuss how we can find the computational graphs used in devising the distributed algorithms in the appended papers. Throughout this thesis, we consider coupled optimization problems and we are particularly interested in the ones which are loosely coupled. Next, we present these type of problems.

1.3.1 Coupled Optimization Problems

A coupled optimization problem can be formulated as optimizing a sum of N objective functions, i.e.

$$\min_x F_1(x) + \cdots + F_N(x), \quad (1.1)$$

where $F_i : \mathbb{R}^n \rightarrow \mathbb{R}$ for all $i = 1, \dots, N$, and $x \in \mathbb{R}^n$ is the optimization variable. Notice that each F_i can describe equality or inequality constraints using indicator functions. We assume that the problem is loosely coupled meaning that each F_i depends only on some of the components of x . This type of problem is called *partially separable*. Let us denote the ordered set of indexes of x on which F_i depends, by $J_i \in \mathbb{N}_n$. The problem in (1.1) can then be rewritten as

$$\min_x \bar{F}_1(E_{J_1}x) + \cdots + \bar{F}_N(E_{J_N}x), \quad (1.2)$$

where E_{J_i} is a zero-one matrix obtained from an identity matrix of order n by deleting the rows indexed by $\mathbb{N}_n \setminus J_i$. The function $\bar{F}_i$ is lower dimensional description of F_i such that $F_i(x) = \bar{F}_i(E_{J_i}x)$ for all $x \in \mathbb{R}^n$ and $i = 1, \dots, N$. As an example, let us consider the following optimization problem:

$$\min_x F_1(x) + F_2(x) + F_3(x) + F_4(x) + F_5(x) + F_6(x) + F_7(x), \quad (1.3)$$

where $x \in \mathbb{R}^{15}$ and let us assume that $J_1 = \{1, 2, 5, 6\}$, $J_2 = \{2, 3, 11\}$, $J_3 = \{4, 5, 7, 8, 12\}$, $J_4 = \{3, 4, 13\}$, $J_5 = \{9, 10\}$, $J_6 = \{6, 7, 9, 14\}$ and $J_7 = \{8, 15\}$. Writing the problem in the same format as in (1.2) results in

$$\begin{aligned} \min_x \quad & \bar{F}_1(x_1, x_2, x_5, x_6) + \bar{F}_2(x_2, x_3, x_{11}) + \bar{F}_3(x_4, x_5, x_7, x_8, x_{12}) \\ & + \bar{F}_4(x_3, x_4, x_{13}) + \bar{F}_5(x_9, x_{10}) + \bar{F}_6(x_6, x_7, x_9, x_{14}) + \bar{F}_7(x_8, x_{15}). \end{aligned} \quad (1.4)$$

We can see the coupling structure of the problem more clearly in this formulation. The coupling structure can also be described via undirected graphs which is presented next.

1.3.2 Coupling and Sparsity Graphs

A graph G is denoted by $G(V, \mathcal{E})$, where $V = \{1, \dots, m\}$ is the set of vertices or nodes and $\mathcal{E} \subseteq V \times V$ denotes the set of edges. The coupling structure for the problem in (1.1), can be expressed using two undirected graphs, namely a *coupling*

(a) Coupling graph

(b) Sparsity graph

Figure 1.1: Coupling and sparsity graphs for the problem in (1.4).

graph and a *sparsity graph*. The coupling graph G_c for the problem in (1.1), is a graph with vertex set $V_c = \{1, \dots, N\}$ and edge set $\mathcal{E}_c$ with $(i, j) \in \mathcal{E}_c$ if and only if $J_i \cap J_j \neq \emptyset$. The sparsity graph G_s for the problem in (1.1), is a graph with vertex set $V_s = \{1, \dots, n\}$ and edge set $\mathcal{E}_s$ with $(i, j) \in \mathcal{E}_s$ if and only if there exists an $\bar{F}_k$ in the equivalent formulation in (1.2) which depends on both x_i and x_j . The coupling and sparsity graphs for the problem in (1.4) are illustrated in Figure 1.1. These graphs play a central role in designing distributed optimization methods for coupled structured problems. To be specific, depending on the coupling structure of the problem, one of these graphs will be used in order to obtain a so-called computational graph for the distributed optimization method. We will get back to this in the next section.

1.3.3 Chordal Graphs and Clique Trees

In order to define chordal graphs and clique trees, we first put forth a few definitions on graphs. Let us consider a graph $G(V, \mathcal{E})$ with vertex set V and edge set $\mathcal{E}$. Vertices $i, j \in V$ are *adjacent* if $(i, j) \in \mathcal{E}$. The set of adjacent vertices or *neighbors* of i are denoted by $\text{Ne}(i) = \{j \in \mathcal{E} | (i, j) \in \mathcal{E}\}$. A graph is said to be *complete* if all its vertices are adjacent. An *induced graph* $Q_I(V', \mathcal{E}')$ on $Q(V, \mathcal{E})$, is a graph where $V' \subseteq V$ and $\mathcal{E}' = \mathcal{E} \cap V' \times V'$. A *clique* C_i of $Q(V, \mathcal{E})$ is a maximal subset of V that induces a complete subgraph on Q , i.e. no clique is properly contained in another clique, (Blair and Peyton, 1993). A sequence of vertices $[v_0, v_1, v_2, \dots, v_l, v_0]$ is called a *cycle* of length $l + 1$ if $v_{i-1}, v_i \in \mathcal{E}$ for $i = 1, \dots, l$ and $v_l, v_0 \in \mathcal{E}$. Assume that all cycles of length at least four of $Q(V, \mathcal{E})$ have a *chord*, where a *chord* is an edge between two non-consecutive vertices in a cycle. This graph is then called *chordal*, (Golumbic, 2004, Ch. 4). It is possible to make a graph chordal by adding edges to the graph. This process is called *triangulation* and the resulting graph is referred to as a *chordal embedding*. Let $C_Q = \{C_1, \dots, C_q\}$ denote the set of its cliques, where q is the number of cliques of the graph. Then there exists a tree

Figure 1.2: Chordal embedding of the coupling graph in Figure 1.1(a), and its corresponding clique tree

defined on C_Q such that for every $C_i, C_j \in C_Q$ where $i \neq j$, $C_i \cap C_j$ is contained in all the cliques in the path connecting the two cliques in the tree. This property is called the clique intersection property, (Blair and Peyton, 1993). Trees with this property are referred to as *clique trees*.

Now as an example, let us reconsider the coupling and sparsity graphs in Figure 1.1 for the problem in (1.4). Neither of the graphs are chordal because they have cycles with a length greater than 3. For the coupling graph in Figure 1.1(a), one can simply find the chordal embedding by adding an edge between nodes 1 and 4, shown with the dashed line in Figure 1.2(a). By doing so, the corresponding cliques for this graph are

$$\begin{aligned} C_1 &= \{1, 2, 4\}, \\ C_2 &= \{1, 3, 4\}, \\ C_3 &= \{1, 3, 6\}, \\ C_4 &= \{5, 6\}, \\ C_5 &= \{3, 7\}. \end{aligned}$$

Note that the entries of the cliques obtained from coupling graph correspond to the indexes of $\bar{F}_i$ in (1.4). For the sparsity graph in Figure 1.1(b), it is not trivial to find the chordal embedding by adding edges manually. There are various methods to find a chordal embedding and in particular, there are exact algorithms to find an optimal chordal embedding. These algorithms, however, are too expensive in practice. A common alternative is to use greedy search methods, which are based on different heuristic cost functions, such as min-neighbors, min-weight, min-fill, weighted-min-fill. One such method for computing the chordal embedding and its cliques is given in (Koller and Friedman, 2009, Ch. 9), which is based

on the node-elimination technique. See also L'Excellent (2012), for a comparison between various chordal embedding methods and the resulting clique trees. Using a greedy search method in the MATLAB library Xie (2020), two different chordal embedding graphs for the sparsity graph in Figure 1.1(b), are illustrated in Figure 1.3(a) and Figure 1.4(a). The added edges are shown with the dashed lines. The corresponding cliques for the chordal embedding in Figure 1.3(a) are

$$\begin{aligned}
C_1 &= \{8, 15\}, \\
C_2 &= \{6, 7, 9, 14\}, \\
C_3 &= \{3, 4, 13\}, \\
C_4 &= \{4, 5, 7, 8, 12\}, \\
C_5 &= \{2, 3, 11\}, \\
C_6 &= \{9, 10\}, \\
C_7 &= \{4, 5, 6, 7\}, \\
C_8 &= \{1, 2, 4, 5, 6\}, \\
C_9 &= \{1, 2, 3, 4\}.
\end{aligned}$$

and for the chordal embedding in Figure 1.4(a), the corresponding cliques are

$$\begin{aligned}
C_1 &= \{1, 2, 3, 5, 6, 11\}, \\
C_2 &= \{1, 3, 4, 5, 6, 11, 13\}, \\
C_3 &= \{4, 5, 6, 7, 9, 13, 14\}, \\
C_4 &= \{4, 5, 7, 8, 9, 12, 13\}, \\
C_5 &= \{4, 7, 8, 9, 10, 12, 13\}, \\
C_6 &= \{8, 15\}.
\end{aligned}$$

Note that the entries of the cliques obtained from the sparsity graph correspond to the indexes of the variable x in (1.4). Having computed the chordal embedding graphs and their cliques, we need to find their corresponding clique trees as well. To do so, an efficient approach is the maximum spanning tree algorithm which is presented in Koller and Friedman (2009). The corresponding clique trees for the chordal embedding graphs in Figure 1.2(a), Figure 1.3(a) and Figure 1.4(a) are illustrated in Figure 1.2(b), Figure 1.3(b) and Figure 1.4(b), respectively.

Clique trees are used as the computational graphs for designing distributed algorithms in this thesis, where each clique acts a computational agent that collaborate with its neighboring cliques. Therefore, they play an important role in the process of algorithm development. Once the clique tree is found, we choose one clique as the root of the tree. By doing so, we can define the so-called *parent* and *children* of a clique. The parent clique of clique i , denoted by $\text{par}(C_i)$, is the adjacent clique of clique i which is in the path connecting clique i to the root clique. The children cliques of clique i , denoted by $\text{chi}(C_i)$, are the adjacent cliques of clique i which are not in the path connecting clique i to the root clique. For

Figure 1.3: First chordal embedding of the sparsity graph in Figure 1.1(b), and its corresponding clique tree

example, for the clique tree in Figure 1.2(b), if we choose C_1 to be the root clique, then C_1 is the parent clique of C_2 , and $\{C_3, C_4\}$ are the children cliques of C_2 . Once the root clique is selected, we assign each $\bar{F}_i$ in (1.4) to a clique. We denote the ordered set of indexes of $\bar{F}_i$ assigned to clique j by ϕ_j . The assignment is not unique and can be done in different ways. Next, we describe how this assignment can be done depending on if the given graph is a sparsity graph or a coupling graph. In the case of a coupling graph, the assignment strategy is straightforward. In particular, $\bar{F}_i$ can only be assigned to clique j if $i \in C_j \setminus \text{par}(C_j)$, i.e. i is an element of C_j which is not present in $\text{par}(C_j)$. Therefore, for the clique tree in Figure 1.2(b), the sets $\{\bar{F}_1, \bar{F}_2, \bar{F}_4\}$, $\{\bar{F}_3\}$, $\{\bar{F}_6\}$, $\{\bar{F}_5\}$ and $\{\bar{F}_7\}$ are assigned to cliques C_1 , C_2 , C_3 , C_4 and C_5 , respectively, i.e. $\phi_1 = \{1, 2, 4\}$, $\phi_2 = \{3\}$, $\phi_3 = \{6\}$, $\phi_4 = \{5\}$ and $\phi_5 = \{7\}$. In the case of a sparsity graph, however, a valid assignment is that $\bar{F}_i$ can only be assigned to clique j if $J_i \subseteq C_j$. It should be noted that there always exists a C_j for which $J_i \subseteq C_j$, see Khoshfetrat Pakazad et al. (2017), for the proof. Therefore, for the clique tree in Figure 1.3(b), the sets $\{\bar{F}_7\}$, $\{\bar{F}_6\}$, $\{\bar{F}_4\}$, $\{\bar{F}_3\}$, $\{\bar{F}_2\}$, $\{\bar{F}_5\}$ and $\{\bar{F}_1\}$ are assigned to cliques C_1 , C_2 , C_3 , C_4 , C_5 , C_6 and C_8 , respectively, and no term is assigned to cliques C_7 and C_9 . This means that $\phi_1 = \{7\}$, $\phi_2 = \{6\}$, $\phi_3 = \{4\}$, $\phi_4 = \{3\}$, $\phi_5 = \{2\}$, $\phi_6 = \{5\}$, $\phi_7 = \emptyset$, $\phi_8 = \{1\}$ and $\phi_9 = \emptyset$. Also, for the clique tree in Figure 1.4(b), the sets $\{\bar{F}_1, \bar{F}_2\}$, $\{\bar{F}_4\}$, $\{\bar{F}_6\}$, $\{\bar{F}_3\}$, $\{\bar{F}_5\}$ and $\{\bar{F}_7\}$ are assigned to cliques C_1 , C_2 , C_3 , C_4 , C_5 and C_6 , respectively, i.e. $\phi_1 = \{1, 2\}$, $\phi_2 = \{4\}$, $\phi_3 = \{6\}$, $\phi_4 = \{3\}$, $\phi_5 = \{5\}$ and $\phi_6 = \{7\}$.

As we stated before, either sparsity or coupling graphs can be used in order to find a computational graph for the distributed optimization method. The most commonly used one in the literature is the sparsity graph. For instance, see Khoshfetrat Pakazad et al. (2017); Vandenberghe and Andersen (2015); Wainwright and Jordan (2008). However, there are also works which use coupling graphs, for instance, see Garstka et al. (2021); Zheng et al. (2017, 2018, 2019). It should be

Figure 1.4: Second chordal embedding of the sparsity graph in Figure 1.1(b), and its corresponding clique tree

pointed out that an issue with using a sparsity graphs occurs when different terms of the optimization problem are assigned to the cliques. To be specific, checking the condition $J_i \subseteq C_j$ can be computationally demanding when the size of the sets J_i and C_j are large. This is not an issue when using a coupling graph. Notice that assigning terms to the clique tree in Figure 1.4(b) is more challenging than assigning terms to the clique tree in Figure 1.2(b). Also, when using a sparsity graph, the number of cliques in the resulting clique tree can be larger than the number of terms to be assigned. Therefore, in order to have a clique tree where all the cliques are assigned with optimization problem terms, the unassigned cliques should be merged with their neighboring cliques, which adds another task to the process of finding the computational graph. This is not the case when using a coupling graph, as the maximum number of the cliques in the resulting clique tree is bounded by the total number of the terms to be assigned. It should also be mentioned that using sparsity graphs can result in trees with a larger number of cliques compared to when using coupling graphs. In other words, given that a clique can be thought of as a computational agent, using sparsity graphs can potentially lead to algorithms which are more distributed in terms of computational agents used, than using coupling graphs. In this thesis, we use sparsity graphs in papers A and B and coupling graphs in papers C, D and E.

1.3.4 Optimization over Clique Trees Using Message Passing

Let us consider the optimization problem in (1.1) and let $G(V, \mathcal{E})$ be the chordal embedding of the sparsity or coupling graph of this problem. Let $T(V_t, \mathcal{E}_t)$ be its corresponding clique tree with cliques $C_Q = \{C_1, \dots, C_q\}$. Let us also assume that the terms in the equivalent problem in (1.4) are assigned based on the strategies discussed earlier, i.e. ϕ_i for $i = 1, \dots, q$ are selected. It is now possible to devise a distributed algorithm to solve the problem which uses the clique tree as its

computational graph. The cliques in the clique tree act as computational agents which collectively solve the problem by exchanging messages through the edges of the clique tree. Next we express how the messages can be exchanged between cliques. Starting from the cliques in the leaf of the tree, each clique i minimizes its assigned function with respect to the variables that are not shared with its parent clique j and send the resulting function to its parent clique j , i.e.

$$m_{ij}(x_{A_{ij}}) = \min_{x_{B_{ij}}} \left\{ \sum_{k \in \phi_i} \bar{F}_k(x_{J_k}) \right\},$$

$$A_{ij} = \begin{cases} C_i \cap C_j, & \text{if } G(V, \mathcal{E}) \text{ is a sparsity graph,} \\ \bigcup_{l \in C_i \cap C_j} J_l, & \text{if } G(V, \mathcal{E}) \text{ is a coupling graph,} \end{cases}$$

$$B_{ij} = \begin{cases} C_i \setminus (C_i \cap C_j), & \text{if } G(V, \mathcal{E}) \text{ is a sparsity graph,} \\ \left\{ \bigcup_{l \in C_i} J_l \right\} \setminus \left\{ \bigcup_{l \in (C_i \cap C_j)} J_l \right\}, & \text{if } G(V, \mathcal{E}) \text{ is a coupling graph.} \end{cases}$$

Notice that A_{ij} and B_{ij} are calculated differently depending on if $G(V, \mathcal{E})$ is a sparsity graph or a coupling graph, as the vertices in a sparsity graph correspond to the indexes of the variable x , whereas the vertices in a coupling graph correspond to the indexes of $\bar{F}_i$. The vectors $x_{A_{ij}}$, $x_{B_{ij}}$ and x_{J_k} include components of x that correspond to the elements in sets A_{ij} , B_{ij} and J_k , respectively. We denote by $m_{ij}(x_{C_i \cap C_j})$, a message function. For example for the clique tree in Figure 1.2(b), which is obtained from the coupling graph of the problem, the leaf cliques are C_4 and C_5 and the messages these two cliques send to their parent cliques, are

$$m_{43}(x_6, x_7, x_9, x_{14}) = \min_{x_{10}} \left\{ \bar{F}_5(x_9, x_{10}) \right\},$$

$$m_{52}(x_4, x_5, x_7, x_8, x_{12}) = \min_{x_{15}} \left\{ \bar{F}_7(x_8, x_{15}) \right\}.$$

Once each clique j receives the messages from all of its children, they are added to its assigned function. After that, the resulting function is minimized with respect to the variables that are not shared with its parent clique p and the resulting message function is sent to its parent clique p :

$$m_{jp}(x_{A_{jp}}) = \min_{x_{B_{jp}}} \left\{ \sum_{k \in \phi_j} \bar{F}_k(x_{J_k}) + \sum_{k \in \text{chi}(C_j)} m_{kj}(x_{D_{kj}}) \right\},$$

$$D_{kj} = \begin{cases} C_k \cap C_j, & \text{if } G(V, \mathcal{E}) \text{ is a sparsity graph,} \\ \bigcup_{l \in C_k \cap C_j} J_l, & \text{if } G(V, \mathcal{E}) \text{ is a coupling graph.} \end{cases}$$

The vector $x_{D_{kj}}$ includes components of x that correspond to the elements in set D_{kj} . Hence, for the clique tree in Figure 1.2(b), the message sent from C_3 to C_2 is

$$m_{32}(x_1, x_2, x_4, x_5, x_6, x_7, x_8, x_{12}) = \min_{x_9, x_{14}} \left\{ \bar{F}_6(x_6, x_7, x_9, x_{14}) + m_{43}(x_6, x_7, x_9, x_{14}) \right\},$$

and from C_2 to C_1 is

$$m_{21}(x_1, x_2, x_3, x_4, x_5, x_6, x_{13}) = \min_{x_7, x_8, x_{12}} \left\{ \bar{F}_3(x_4, x_5, x_7, x_8, x_{12}) \right. \\ \left. + m_{52}(x_4, x_5, x_7, x_8, x_{12}) + m_{32}(x_1, x_2, x_4, x_5, x_6, x_7, x_8, x_{12}) \right\}.$$

Finally, when the root clique r receives the messages from all its children, its optimal solution can be computed by solving the following optimization problem

$$x_{A_{rr}}^* = \operatorname{argmin}_{x_{A_{rr}}} \left\{ \sum_{k \in \phi_r} \bar{F}_k(x_{J_k}) + \sum_{k \in \operatorname{chi}(C_r)} m_{kr}(x_{D_{kr}}) \right\},$$

$$A_{rr} = \begin{cases} C_r, & \text{if } G(V, \mathcal{E}) \text{ is a sparsity graph,} \\ \bigcup_{l \in C_r} J_l, & \text{if } G(V, \mathcal{E}) \text{ is a coupling graph.} \end{cases}$$

Therefore, for the clique tree in Figure 1.2(b):

$$(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*, x_{11}^*, x_{13}^*) = \operatorname{argmin}_{x_1, x_2, x_3, x_4, x_5, x_6, x_{11}, x_{13}} \left\{ \bar{F}_1(x_1, x_2, x_5, x_6) \right. \\ \left. + \bar{F}_2(x_2, x_3, x_{11}) + \bar{F}_4(x_3, x_4, x_{13}) + m_{21}(x_1, x_2, x_3, x_4, x_5, x_6, x_{13}) \right\}.$$

The root clique r then sends its optimal solution to its children cliques and therefore, the children cliques can also compute their optimal solution as

$$x_{B_{ir}}^* = \operatorname{argmin}_{x_{B_{ir}}} \left\{ \sum_{k \in \phi_i} \bar{F}_k(x_{J_k}) + \sum_{k \in \operatorname{chi}(C_i)} m_{ki}(x_{D_{ki}}) \right\} \Big|_{x_{A_{rr}}^*}.$$

Therefore, for the clique tree in Figure 1.2(b):

$$(x_7^*, x_8^*, x_{12}^*) = \operatorname{argmin}_{x_7, x_8, x_{12}} \left\{ \bar{F}_3(x_4^*, x_5^*, x_7, x_8, x_{12}) + m_{52}(x_4^*, x_5^*, x_7, x_8, x_{12}) \right. \\ \left. + m_{32}(x_1^*, x_2^*, x_4^*, x_5^*, x_6^*, x_7, x_8, x_{12}) \right\}.$$

Similarly the other cliques can compute their optimal solution by receiving the optimal solution from their parents. Therefore, the cliques C_3 , C_4 and C_5 in Figure 1.2(b) find their solutions as

$$(x_9^*, x_{14}^*) = \operatorname{argmin}_{x_9, x_{14}} \left\{ \bar{F}_6(x_6^*, x_7^*, x_9, x_{14}) + m_{43}(x_6^*, x_7^*, x_9, x_{14}) \right\},$$

$$x_{10}^* = \underset{x_{10}}{\operatorname{argmin}} \left\{ \bar{F}_5(x_9^*, x_{10}) \right\},$$

$$x_{15}^* = \underset{x_{15}}{\operatorname{argmin}} \left\{ \bar{F}_7(x_8^*, x_{15}) \right\}.$$

respectively. In summary, by the end of an upward-downward pass through the clique tree, all the cliques compute their optimal solution. It can be shown that the obtained solution is an optimal solution, see (Koller and Friedman, 2009, Ch. 10), for the proof. It should be pointed out that message passing over clique tree is closely related to non-serial dynamical programming. Non-serial dynamical programming is used for solving problems with general coupling structures, which was introduced in Bertele and Brioschi (1973). For related works see e.g., Koller and Friedman (2009); Moallemi (2007); Wainwright et al. (2005) and the references therein. Message passing can also be used within optimization algorithms to distribute the computations of search directions. This is closely connected to multi-frontal factorization techniques for factorization of matrices, see e.g., Khoshfetrat Pakazad et al. (2017). Moreover, all other computations in an optimization algorithm, including step size and the termination criterion, can often also be distributed over the clique tree. This property is extensively used in developing various distributed optimization algorithms in this thesis.

2

Concluding Remarks and Future Work

Rapid advances in technology in recent years have limited the use of centralized optimization approaches for large-scale complex systems due to the computational or communication challenges and/or structural constraints, which were discussed in the introduction. Therefore, the main objective of this thesis was to develop distributed optimization approaches, tailored for various applications in control and estimation by exploiting the inherent structures of the problems. To this end, we made use of chordal graphs, which enabled us to carry out the computations of the optimization algorithms distributedly using message passing over the computational graph of the problem.

In particular, we implemented a distributed primal-dual interior-point algorithm for loosely coupled QP problems and we discussed how parallelism can be exploited. We applied the algorithm on a linear MPC problem and we showed how one can benefit from parallel computations. Specifically, we showed how one can run the algorithm on multiple processors in parallel, in order to accelerate execution time of the algorithm compared to the case where the algorithm is run on a single processors. Also, we applied it on a robust MPC problem by converting the problem to an equivalent RQP formulation and we then showed how the chordal structure can be used to distribute the computations.

In addition, in this thesis, we presented a distributed optimization framework for a sensor network localization problem formulated with maximum likelihood estimation. The framework is based on the Levenberg-Marquardt algorithm in which the computations are carried out over the computational graph of the problem. We illustrated how the computational graph can be obtained distributedly. The resulting algorithm converged to an accurate solution in much fewer iterations than first-order methods, which led to fewer exchanged messages between computational agents. We also discussed the conditions in which the algorithm outperforms other algorithms in terms of root-mean-square error.

Moreover, we presented a distributed second-order augmented Lagrangian method

for distributed MPC. In order to handle the inequality constraints, we adopted a primal-dual interior-point method in the inner iteration augmented Lagrangian method. We then showed how the computations of a centralized augmented Lagrangian method can be distributed using message passing over the computational graph of the problem. We illustrated the superiority of the algorithm in terms of number of iterations and communications compared to a distributed ADMM based method, using a set of numerical examples including a vehicle platooning problem.

Next, we point out a few directions for future research. The algorithms in papers A, B, D and E are based on Newton's method, meaning that the exact Hessian of the problem is used. When the Hessian is unavailable or is too expensive to compute, one can use a quasi-Newton method in which the Hessian is approximated with different formulas, such as Broyden-Fletcher-Goldfarb-Shanno (BFGS) or Broyden. Hence, one can investigate the possibility of distributing the computations of the approximate Hessian and use it instead of the exact Hessian.

In paper B, the obtained clique tree for the robust MPC problem has as many parallel branches as the number of what is called as scenarios in the paper. One possible extension is to introduce further parallelism for computations on each branch of the tree with the approach described in paper A.

In paper C, one can investigate different approaches for finding clique trees with parallel branches in order to facilitate parallel computations. In addition, different ways of initializing the algorithm in a cheap way can be explored.

In paper E, it is required that at each iteration of the algorithm, the equality constraints are preprocessed by going an upward pass through the clique tree. This can be avoided by using a slightly different augmented Lagrangian function, defined as the generalized primal-dual augmented Lagrangian in Gill and Robinson (2012). The reason is that by using this function, the resulting Karush-Kuhn-Tucker (KKT) system is guaranteed to be non-singular.

Moreover, in connection with paper E, an augmented Lagrangian interior-point approach is presented in Cao et al. (2016), where the linear KKT system is solved using a preconditioned conjugate gradient method, which is implemented efficiently on a Graphics Processing Unit (GPU) in parallel. For sufficiently sparse problems, one possible future work is to distribute the computations of the preconditioned conjugate gradient used for solving the KKT system, over the clique tree of the problem such that parallelism can be exploited. The results can then be compared with the ones in Cao et al. (2016).

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Part II

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