Incremental fatigue damage modeling of 7050-T7 aluminum alloy at stress-raisers

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A B S T R A C T

The Ottosen–Stenström–Ristinmaa (OSR) incremental fatigue damage model, based on an moving endurance surface centered around a backstress, is adapted for high-cycle fatigue at stress-raisers in AA7050-T7 specimens. Fatigue experiments are carried out for circumferentially-notched, axisymmetric specimens subjected to constant-amplitude (CA) load. The OSR model parameters are fitted to CA fatigue data, showing fair agreement for one set of model parameters across different stress ratios, stress concentration factors, uniaxial stress and biaxial stress. To demonstrate predictive capability, the fatigue life is integrated for an aircraft load spectrum (TWIST), and compared with experimental fatigue life data for holeplate specimens in the literature.

1. Introduction

To reduce the number of structural components and simplify assembly, air vehicle development is increasing the use of integrated structural parts. Such integrated structures, e.g. large machined frames, are geometrically complex and are often subjected to nonproportional, spectrum loads. This makes it difficult to apply traditional fatigue-assessment methods based on simplifying assumptions, such as uniaxial proportional stress, simple notch criteria and cycle-counting techniques. Integrated structures are also difficult to inspect, repair or replace in case of fatigue damage. Therefore, in order to ensure fatigue-reliable designs, the assessment method must be capable of handling the mentioned complexities. The fatigue damage model needs to account, not only for the biaxial, nonproportional stress history, but also for the stress gradient effects induced by the complex geometry.

Herein, we consider the high-cycle fatigue (HCF) behavior of 7050-T7 aluminum alloy, which is a common structural material in high-performance air vehicles. We present an adaptation of the incremental fatigue damage model of Ottosen and co-workers [1], and perform constant-amplitude (CA) tests to determine the model parameters. For that purpose, we use axisymmetric, circumferentially-notched specimens, that induce biaxial stress states, as well as unnotched specimens. This complements experiments with holeplate specimens in the literature [2]. Our purpose is to explore the predictive capabilities of the model when applied to a wide range of experimental conditions, where stress ratio, stress concentration factor (SCF) and degree of biaxiality are varied.

Incremental fatigue damage models (see e.g. [3]) treat damage as a time-continuous state variable, which is integrated without cycle-counting. One such model is the Ottosen–Stenström–Ristinmaa (OSR) model [1], also known as the continuum damage approach, which is based on an endurance surface that moves with a backstress. In the original model, this endurance surface has the shape of a Drucker–Prager yield surface, but more general endurance surfaces have been proposed [4,5]. The backstress evolves and damage develops during onloading, when the stress state reaches outside this endurance surface. The damage variable can be related to physical damage by employing, e.g., the Lemaitre–Chaboche damage rule [1,6]. For periodic, proportional stress a steady-state develops, where the backstress exhibits repeatable cycles [1,7]. Such repeatable cycles are also observed for many instances of periodic, nonproportional stress, and the damage per period in the steady-state may be used to calculate the fatigue life [7]. The OSR model has been further developed to predict HCF of transversely isotropic materials [8,9], and to capture stress-gradient effects [10]. Moreover, a Bresler–Pister type endurance surface, which has a nonlinear dependence of the mean stress, has been proposed and demonstrated to give a fair prediction for 7075-T6 aluminum alloy subjected to nonproportional stress [11].

Some of the benefits of the OSR model is that it has a small number of free parameters [11], that it is easy to implement using...
standard ordinary differential equation (ODE) solvers [7], and that it is computationally inexpensive allowing for large-scale simulations [12]. Moreover, it presents a unified framework for variable-amplitude [13], multiaxial, and nonproportional stress [1,11]. A drawback is the scalar representation of damage, which precludes a more detailed analysis of orientation-dependent damage.

In this work, we combine a stress-gradient correction similar to the one introduced for steel in the original OSR model [10] with the quadratic endurance surface [11]. We fit this modified OSR model to new fatigue life experiments for circumferentially-notched 7050-T7 specimens and fatigue life data from the literature [2] at the same time. Thus, we show for the first time that the OSR model [1], augmented with previously suggested modifications [10,11], interpolates data for different stress-raisers, SCFs, stress ratios and material batches. We also compare OSR model predictions to variable-amplitude fatigue data in the literature; the OSR model exhibits a fair to conservative fatigue life prediction for a transport aircraft load spectrum.

2. Theory

The OSR model with stress-gradient correction [10] and a quadratic endurance surface [11] is described. This model is based on linear-elastic finite element (FE) analysis with static material parameters.

2.1. Tensor operators and terminology

For all tensors, a superscripted "T" denotes the transpose, and tr(\(\cdot\)) denotes the trace of a tensor. Let \(I\) be the second-order identity tensor. The colon operator denotes the Frobenius product, \(\varepsilon(\varepsilon) = \varepsilon^T \varepsilon\), for second-order tensors \(\varepsilon\) and \(\varepsilon\). The Frobenius norm of a second-order tensor is \(\|\varepsilon\| = \sqrt{\varepsilon^T \varepsilon}\). The vector space of symmetric, second-order tensors is \(\mathcal{V}_{sym} = \{X \in \mathbb{R}^{3 \times 3} | X = X^T\}\) with the Frobenius norm as the inner product. The deviatoric subspace of \(\mathcal{V}_{sym}\) is \(\mathcal{V}_{dev} = \{X \in \mathcal{V}_{sym} | \varepsilon(X) = 0\}\).

2.2. Linear elastic model

Consider a body \(\Omega\), whose surface is partitioned into two disjoint boundaries \(\partial \Omega_v\), with prescribed displacement \(u_0\), and \(\partial \Omega_u\), with prescribed traction \(\tau(t)\), for times \(t \in [0,T]\) with \(T\) the duration. It is assumed that the body is in quasistatic equilibrium, and we seek the stress field \(\sigma \in \mathcal{V}_{sym}\) and strain field \(\varepsilon\) related by a linear elastic material model. Hence, based on the principle of virtual work,

\[
\int_\Omega \sigma : \varepsilon^T \psi \, dV = \int_{\partial \Omega_v} \tau^T \psi \, dA, \quad \forall \psi \in \mathcal{U}, \quad u = u_0, \quad \text{on} \ \partial \Omega_u, \tag{1a}
\]

where \(\mathcal{U}\) is a set of smooth test functions that vanish on \(\partial \Omega_u\). We restrict this investigation to isotropic materials,

\[
E \frac{\varepsilon}{1 + \nu} \sigma - \nu \frac{\varepsilon}{E} \varepsilon^{tr(\sigma)} I \tag{2}
\]

with \(E\) the Young modulus and \(\nu\) the Poisson ratio.

2.3. Incremental fatigue damage model

The incremental fatigue damage model used herein is that of Ref. [11], which is a modification of the OSR model [1]. Fatigue is primarily driven by an effective stress

\[
\sigma(\sigma, \alpha) = \sqrt{\frac{3}{2}} \|s - \alpha\|, \tag{3}
\]

where \(s = \sigma - \frac{1}{2} \varepsilon^{tr(\sigma)} I \in \mathcal{V}_{dev}\) and \(\alpha \in \mathcal{V}_{dev}\) is the backstress tensor. This effective stress together with the first stress invariant \(I_1 = \varepsilon^{tr(\sigma)} I\) defines an endurance surface \(\Pi(\alpha) = \{\sigma \in \mathcal{V}_{sym} | \alpha(\sigma, I_1) = 0\}\) by means of an endurance function [11]

\[
\alpha(y) = \frac{y^T A y + a^T y - 1}{y^T E I}, \tag{4}
\]

where \(A\) is a symmetric material parameter matrix, and \(a\) is a material parameter vector. Sample endurance surfaces and a stress path are illustrated in Fig. 1. A damage variable \(D\) is introduced with \(D = 0\) for the virgin material and \(D = 1\) for failure. The evolution of state is governed by initial value problems (IVPs) [1,11]

\[
a = (s - a) \cdot H(\beta(\alpha)), \quad a(0) = 0, \tag{5a}
\]

\[
D = K \int_0^1 H(\beta(\alpha)), \quad D(0) = 0, \tag{5b}
\]

where \((s) = \frac{1}{2}(s + |s|)\) denotes the unit ramp function, \(H(x) = \frac{1}{2} x, x \neq 0\) is the Heaviside step function, \(C > 0\) is a model parameter, and \(K > 0\) and \(L > 0\) are material constants. Following Ref. [11], we have that \(H(\beta(\alpha)) = H(\beta(\alpha))\) with

\[
\alpha(\sigma, \alpha, \alpha) = \frac{1}{1 + C \sigma \frac{\partial \sigma}{\partial \alpha}} \left[ \sqrt{\frac{3}{2}} \frac{\partial \beta}{\partial \alpha} (s - a) + \frac{\partial \beta}{\partial \alpha} I_1 \right] : \sigma, \tag{6}
\]

where

\[
\frac{\partial \beta}{\partial \alpha}, \frac{\partial \beta}{\partial \alpha} I_1 = \left(2y^T A + a^T\right), \tag{7}
\]

Thus, by substituting \(\langle \beta \rangle\) for \(\langle \alpha \rangle\) into Eqs. (5a) and (5b), ODEs are obtained that can be integrated using standard methods.

2.4. Stress-gradient effects

To capture the size-dependence of fatigue at stress-raisers, each point in the material needs to be associated with a length scale. As previously advocated [10], we use the relative stress gradient \(x_\alpha\) [m^{-1}], here defined as

\[
x_\alpha = \frac{\int_0^1 \|V \sigma_{\alpha M}\| \|\sigma\| \, dV}{\int_0^1 \sigma_{\alpha M} \|\sigma\| \, dV}, \quad \sigma_{\alpha M} = \sqrt{\frac{3}{2} \|\alpha\|}, \tag{8}
\]

to quantify this length scale, where \(V\) denotes the gradient operator, and \(\sigma_{\alpha M}\) is the von Mises equivalent stress. For proportional stress, Eq. (8) simplifies to \(x_\alpha = \|V \sigma_{\alpha M}\| / \sigma_{\alpha M}\) whenever \(\sigma_{\alpha M} \neq 0\), which was previously used for the definition of \(x_\alpha\) [10]. Definition (8) has the added advantage of being time-independent for nonproportional stress.
We assume that the stress gradient does not affect material properties, but rather modifies the apparent stress state. This can be captured by introducing a diagonal matrix \( \mu = \text{diag} [\mu_1(x), \mu_2(x)] \), which modifies \( \gamma \) in the endurance function,

\[
\beta(y) = (\mu_1 y)^\top A_0 (\mu_2 y) + (\mu_2^\top y - 1) = y^\top (\mu_1 y A_0 y + (\mu_2^\top y a_0 y - 1),
\]

where \( A_0 \) and \( a_0 \) denote the parameters of homogeneous stress. The modified model parameters are identified as \( \lambda = \mu_1 A_0 \mu_2 \) and \( \alpha = \mu_2^\top a_0 \mu_1 \) by comparing Eq. (9) with Eq. (4). Here, \( \mu_1(x) \) and \( \mu_2(x) \) are positive functions with \( \mu_1(0) = \mu_2(0) = 1 \).

In the work of Ottosen et al., with \( A_0 = 0 \) for steel, the modulating functions were effectively chosen as [10]

\[
\mu_1(x) = \mu_2(x) = \begin{cases} 1 & \gamma \leq \gamma_\infty, \\ \gamma_\infty/\gamma_\infty & \text{otherwise}. \end{cases}
\]

Here, \( \gamma_\infty \) is a material constant, and \( \gamma_\infty \) is a limiting stress gradient introduced to prevent the fatigue sensitivity from vanishing for steep gradients. It is demonstrated by Ottosen et al. [10], that this choice leads to a fatigue sensitivity similar to the one formulated by Siebel and Stieler [14,15].

As demonstrated in later Section 3.3, the notch sensitivity factor of 7050-T7 is well-described by the model of Neuber [16], Kuhn and Hardrath [17]. For a constant notch shape but varying length-scale, the stress reduction factor at the notch root of the Neuber–Kuhn–Hardrath model, see Eq. (16), can be written as

\[
\sigma_1(x) = \sigma_2(x) = \frac{1 + \mu_2 E_0 \sqrt{\gamma_\infty}}{1 + \mu_2 E_0 \gamma_\infty},
\]

where \( \eta \) is a material constant. This material-specific modulating function monotonically approaches the limit \( \mu_2 \in (0, 1) \) as \( \gamma \to \infty \).

In summary, the history-dependent linear elastic problem is solved for \( \sigma(t) \), and the local parameters of the endurance function are determined using Eqs. (8) and (11). Then, the evolution of the state variables, \( \alpha \) and \( D_0 \), is obtained by solving the IVPs (5a) and (5b). The tensorial formulation of the IVPs is specialized to proportional stress in Appendix A to further increase computational speed.

## 3. Experiments

In this work, we consider axisymmetric specimens with circumferential notches and holeplate specimens subjected to uniaxial loading. To characterize these notches, we introduce a coordinate system with basis vector \( e_1 \) in the load direction, \( e_2 \) in the trough direction and \( e_3 = e_1 \times e_2 \) in the outward surface normal at the notch root (Figs. 2a-b).

For an applied nominal stress \( \sigma_{\text{nom}}(t) \), the stress at the notch root is

\[
\sigma = \sigma_{\text{nom}}(t) F,
\]

where \( F \in V_{\text{sym}} \) is a normalized stress tensor with SCF \( K_i = F_{11} > 0, F_{12} > 0 \) and \( F_{13} = F_{21} = F_{23} = 0 \). The \( F_{11} \) and \( F_{22} \) components and the notch root stress gradient \( \gamma \) are accessible through FE analysis.

### 3.1. Test specimens

The material considered in our experiments is 7050-T7451 aluminum alloy (Constellium, France), delivered as a plate with thickness 75 mm. Rectangular bars with dimensions 22 x 22 x 190 mm³ are cut in the longitudinal direction of the plate. Axisymmetric specimens (following ASTM E466 [18]) are manufactured using a lathe machine with a cutting rate of 80 m/min in the region of the notch root, and a stereoscopic microscope is used to assert that the surface roughness does not exceed 0.8 µm in that region. Smooth axisymmetric specimens are denoted by A1.0, while two types of circumferentially-notched, axisymmetric specimens, one with notch radius \( R = 1.745 \) mm and one with notch radius \( R = 0.687 \) mm, are denoted by A1.8 and A2.5, respectively, where the numbers in the designation is the SCF (Fig. 3).

In the work of Hillbrecht and Hoffer [2], we find HCF data for an hourglass-shaped plate and two types of holeplate specimens of 7050-T73651, all with plate thickness 4 mm. The hourglass-shaped plate, denoted by P1.05, is essentially smooth with \( K_i = 1.053 \), see Sect. 2 of the supplemental information (SI). The first holeplate type, denoted by P2.5, has a centered hole with radius \( R = 4 \) mm. In the second specimen type, denoted by P3.4, a central hole is flanked by two smaller holes with radii \( R = 2 \) mm forming a keyhole shape (Fig. 3).

An FE characterization using the second-order, continuous Galerkin element with the DOLFIN FE library [19] of FEniCS [20] (SI Sect. 1) is conducted to find the normalized stress state \( F \) and the stress gradient \( \gamma \) at the notch root of each specimen (SI Sect. 2), with the results compiled in Table 1.

### 3.2. Stress–strain relation

The stress–strain relation is determined using the same experimental setup as for the CA fatigue test with the additional use of an axial extensometer with a gauge length of 12.5 mm. A tensile strain ramp with strain \( \epsilon_\tau \) is applied for the 7050-T7451 A1.0 specimen with a strain rate of 0.001 s⁻¹, while measuring the tensile stress \( \sigma_\tau \). A Ramberg–Osgood model [21,22]

\[
\epsilon_\tau = \frac{\sigma_\tau}{E} + \left( \frac{\sigma_\tau}{K_\tau} \right)^{1/n_{\text{OH}}}
\]

is fitted (nlparci [23], Matlab) to two such stress–strain measurements (Fig. 4a), giving the Young modulus \( E = 70.3 \pm 0.4 \) GPa and the Hollomon parameters \( K_\tau = 594 \pm 12 \) MPa and \( n_{\text{OH}} = 0.0373 \pm 0.0034 \), with 95% confidence intervals (nlparci, Matlab). We take the Poisson ratio \( \nu = 0.33 \) from the literature [24]. In the following, we use the \( K_\tau = 471 \) MPa proof stress, as obtained from the Ramberg–Osgood relation, as a characteristic stress level for yielding.

### 3.3. Constant-amplitude HCF

Fatigue testing is performed according the ASTM E466 [18] standard using a servo-hydraulic fatigue test rig with an Instron 8800
controller and an Instron ±50 kN load cell. A CA nominal stress is applied in the experiments considered herein,
\[
\sigma_{\text{nom}}(t) = \sigma_m + \sigma_a \sin(2\pi ft),
\]
where \(\sigma_m\) is the mean nominal stress, \(\sigma_a\) is the nominal stress amplitude, and \(f = 10\) Hz is the frequency. The experiments are conducted in stress-control until failure, or until runout at \(5 \cdot 10^6\) cycles. Wöhler curves are obtained at approximately constant stress ratios \(R \in \{-1, 0.1\}\), where \(\sigma_m = \sigma_a (1+R)/(1-R)\). Our measurements are presented in tabulated form for unnotched (SI, Table 2) and notched (SI, Table 3) specimens.

The fatigue limit \(\sigma_{\text{lim}}(R)\) of a notched specimen is reduced by a fatigue notch factor \(K_f\) as compared to an unnotched specimen. According to Neuber [16], Kuhn and Hardrath [17], \(K_f\) can be expressed as
\[
K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\lambda_N/\rho}},
\]
where \(\rho\) is the notch root radius (Fig. 2), and \(\lambda_n\) is a material constant.

To show all experimental HCF data from our work and Ref. [2] (digitized using Engauge [25]) in one diagram, we plot \(K_f\sigma_m\) against fatigue life (Fig. 4b), where \(\lambda_N = 1.5\) mm was obtained by manual fitting, with the objective to collapse fatigue data with the same stress ratio \(R = -1\). This underscores the importance of the size effect of notches in 7050-T7, and how it explains many notch effects. By rearranging Eq. (15), we find that
\[
\frac{K_f}{K_t} = 1 + \frac{1}{\sqrt{\lambda_N/\rho}}.
\]

This stress reduction factor \(K_f/K_t\) for notched specimens serves as a motivation for our choice of stress reduction matrix \(\mu\), Eq. (11), which has a similar form, but the measure of size is taken as \(\chi^2\) instead of \(\rho\).

Attempts were also made to collapse the 7050-T7 HCF data using many different fatigue notch factor expressions, as listed in Ref. [15]. Of these, Eq. (15) is the most successful (not shown).

4. Results and discussion

4.1. Fatigue life simulations

As previously demonstrated [11], Wöhler curves can be computed efficiently for proportional stress (see Appendix A). The IVPs (A.3a) and (A.3b) of the OSR model are integrated numerically (ode45 [26], Matlab). As previously reported for CA load [1,7], we observe convergence of the stress and backstress fluctuations into steady-state with a damage per cycle \(\Delta D = \lim_{t \to \infty} D(t+1/f) - D(t)\). In this work, we take \(\Delta D = D(t_m + 1/f) - D(t_m)\) for a \(t_m\) that fulfills the convergence criterion
\[
\left| \frac{D(t_m + 1/f) - D(t_m)}{D(t_m/2 + 1/f) - D(t_m/2)} - 1 \right| < 10^{-3},
\]
and the fatigue life is taken as \(N_f = 1/\Delta D\), thus neglecting transient behavior.

4.2. OSR model parameters

In the general case, the modified OSR model used in this work comprises a large number of parameters. However, a previous investigation revealed that a preset constant \(C = 10\) is appropriate to eliminate an interdependence between the \(K\), \(L\) and \(C\) parameters, and that
Table 2
Material and OSR model parameters for 7050-T7. The OSR model parameters are fitted with standard deviation $s = 0.043$ w.r.t. the 10 logarithm of stress amplitude. A 95% confidence interval is indicated for each parameter. *From Ref. [24]. †Numerical parameter not fitted.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Values</th>
</tr>
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<tr>
<td>Elasticity</td>
<td>$E = 70.3 \pm 0.4$ GPa, Poisson = 0.33*</td>
<td></td>
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<tr>
<td>Hardening</td>
<td>$K_u = 594 \pm 12$ MPa, $n_u = 0.0373 \pm 0.0033$</td>
<td></td>
</tr>
<tr>
<td>OSR model</td>
<td>$A_{0i}^0 = -12700 \pm 1400$, $a_i^0 = 501 \pm 22$, $a_i^2 = 180 \pm 10$, $K = 4.09 \cdot 10^{-5}$, $L = 12.9 \pm 1.2$, $C = 10^7$, $\delta = 9.7 \pm 1.9 \text{ mm}$, $\mu_{wa} = 0.626 \pm 0.027$</td>
<td></td>
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Fig. 5. Comparison between experimental data for unnotched specimens and fitted OSR model.

AA7050-T7, unnotched

4.3. Haigh diagrams

The fatigue limit, $\sigma_{\text{fl}}$, is analytically available for the BD endurance function used herein $A$. Moreover, the damage per cycle, $\Delta D = \Delta D(\sigma_m, R)$ or $\Delta D = \Delta D(\sigma_m, R)$, is implicitly defined for CA load, and is computed numerically (Section 4.1). From this information, we construct Haigh diagrams associated with the OSR model, as depicted in Fig. 7 for A1.8 and A2.5.

Since the OSR model parameters are fitted in the range $R \in [-1.0, 0.5]$, the model and its Haigh diagrams can only be trusted for tension-dominated stress fluctuations. The predicted endurance limit of A1.8 and A2.5, respectively, describes a convex curve (Fig. 7), which does not intersect $\sigma_{\text{fl}} = 0$. This suggests that the material behavior in the regime of large mean stresses, near and above the yield limit, is not faithfully reproduced.

4.4. Validation for VA stress

From the literature, we consider the variable-amplitude HCF measurements of Hillbrecht and Hoffer [2], where the Transport Wing StStandard (TWIST) [29] was used. TWIST is a standardized load spectrum developed for the wing root of transport aircraft. Each TWIST sequence (Fig. 8a) includes $N_i = 797330$ load states and corresponds to 4000 flights. The spectrum used herein does not include any on-ground load states; inclusion of on-ground states is not mentioned in Ref. [2].

A spectrum load is defined as a sequence of states $(L_i)_{i=1}^{N_i}$, where $L_i \in [-0.6,2.6]$ for TWIST (Fig. 8a). A time-continuous nominal stress is constructed from $(L_i)_{i=1}^{N_i}$ by cubic interpolation,

$$\sigma_{\text{nom}}(t) = \left[ L_i + \frac{3(t-i)^2 - (2t-i)^2}{6} \right] (L_{i+1} - L_i) \sigma_{\text{ud}},$$

between adjacent states, $t \in [i, i+1]$. Thus, the time derivative is continuous and $\sigma_{\text{nom}}(t)/dt = 0, \forall t$. In Eq. (20), $\sigma_{\text{ud}}$ is the in-flight mean stress corresponding to the aircraft being subjected to a static force of gravity.

Damage is integrated across two sequences, and the damage per sequence $\Delta D_{\text{OSR}}$ is taken as the damage accumulated during the second sequence to reduce transient effects. The number of flights to failure is obtained as $4000/\Delta D_{\text{OSR}}$. A comparison between experimental fatigue data for TWIST [2] and the OSR model prediction is shown by plotting $\sigma_{\text{ud}}$ against flights to failure for P2.5 and P3.4 (Fig. 8d). The OSR model predicts a shorter fatigue life than the experimental observations. This is to some extent due to the hardening at the notch resulting from three singularly severe load states in TWIST, which prolong the fatigue life [2]. The phenomenon is explained by residual, compressive stress created by the overload, in conjunction with the small number of TWIST sequence repetitions before failure. The OSR model does not explicitly consider plasticity, so that beneficial effects of hardening in the low-cycle limit is neglected. As advocated by Hillbrecht and Hoffer [2], the retardation effect of overloads is best addressed by truncating the load in fatigue experiments, because design against fatigue should not rely on fortuitous events. In the case of P3.4, it is likely that the conservative prediction for TWIST fatigue life (Fig. 8d) also derives from the conservative fit of the OSR model to CA fatigue data (Fig. 6d).

Because the damage per cycle, $\Delta D(\sigma, R)$, of CA stress is made available through the OSR model, we can alternatively use a conventional rainflow-counting (RC) method and the Palmgren–Miner rule to calculate the damage per spectrum $\Delta D_{\text{RC}}$. The four-point RC technique [30] yields a number of cycles $n^{(j)}$ corresponding to nominal stress amplitude $\sigma_{\text{ud}}^{(j)}$ and mean stress $\sigma_{\text{m}}^{(j)}$. Thus,

$$\Delta D_{\text{RC}} = \sum_j n^{(j)} \Delta D(\sigma_{\text{ud}}^{(j)}, \sigma_{\text{m}}^{(j)}).$$

(21)
Fig. 6. **abcd**. Comparison between experimental fatigue data for notched specimens and fitted OSR model with stress-gradient correction. **e**. OSR model prediction of nominal stress amplitude against measured stress amplitude at the corresponding experimental life, giving standard deviation \( s = 0.043 \) based on the 10-logarithm of stress, as indicated by dashed lines. **f**. Histogram of residuals based on the 10-logarithm of nominal stress amplitude.

Fig. 7. **ab**. Haigh diagrams for A1.8 and A2.5 with the fatigue limit (curved solid line), \( N_f = 10^6 \) (curved dashed line), \( N_f = 10^5 \) (curved dashed–dotted line) and \( N_f = 10^4 \) (curved dotted line). The dashed lines represents \( (\sigma_a + |\sigma_m|)k_t = R_{P2} \).

The flights to failure, \( 4000/\Delta D_{RC} \), computed using RC with Palmgren–Miner summation deviates little from the flights to failure obtained by integrating the IVPs for the load sequence (Fig. 8d), showing that the OSR model is essentially equivalent to RC for proportional stress in the present case.

With explicit integration, the time complexity of the OSR model is \( O(N_s) \), while memory complexity is \( O(1) \). For the RC method, we observe a time complexity of \( O(N_s^{1.05}) \) in numerical experiments with Gaussian noise input. The memory complexity of the RC algorithm is \( O(N_s) \), which can be prohibitive for long load histories in e.g. complex machinery simulation [12].

5. Conclusions

The OSR model, augmented with a quadratic endurance function and a stress-gradient correction, can be fitted to a wide range of CA, HCF experimental data, spanning different stress ratios, SCFs, and geometries which induce biaxial stress. The resulting fit covers a large regime of HCF for proportional stress, including cases with significant plastic deformation in the notch root.

The OSR model with a Bresler–Pister type endurance surface gives a quantitative, but somewhat conservative, prediction of the fatigue life for proportional, spectrum stress, as demonstrated for the TWIST load spectrum. Its prediction for 7050-T7 is very close to that of rainflow-counting with the Palmgren–Miner rule.

The development and testing of the OSR model is approaching application-readiness, as evidenced herein for 7050-T7. Yet, further investigation of the limits to its predictive capability would be of great interest, including e.g. a plane-bending stress gradient, as previously considered for steel [10]. Moreover, although the OSR model gives a fair to conservative prediction for nonproportional load and unnotched, 7000-series aluminum specimens [11], its predictive capability for nonproportional load with a stress-gradient, e.g. rotary bending, is yet to be demonstrated. Open questions also remain regarding the performance of the OSR model for spectrum loads comprising fluctuations at a large mean stress, and regarding more complex geometries, such as metallic integral structures in air vehicles.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Appendix A. Proportional stress

For proportional stress, we have [7]
\[
\sigma = S(t)e, \quad s = S(t)e - \frac{1}{3}t(\epsilon(t))F, \quad \alpha = a(t)e - \frac{1}{3}t(\epsilon(t))F,
\]
(A.1)
where \( e \) is a constant tensor such that \( |e| = 1 \), \( t(e) \geq 0 \), while \( S(t) = \sigma : e \) and \( a(t) = \epsilon \) are scalar functions. This gives \( I_1 = S(t) e \) and \( \tilde{\sigma} = \gamma(S - a) \), where \( \gamma = \sqrt{\frac{2}{3} - \frac{1}{3}t^2(\epsilon)^2} \), so that
\[
y(S, a) = \frac{\kappa \gamma^2}{E}(S - a) - \frac{t(e)}{E}S^T, \quad \kappa = \text{sgn}(S - a),
\]
(A.2)
We insert Eqs. (A.1) and (A.2) into Eqs. (5a) and (5b) to obtain reduced IVPs
\[
\dot{a} = (S - a)C H(\beta)(a(S, a)), \quad a(0) = 0, \quad \text{(A.3a)}
\]
\[
D = Ke^\epsilon H(\beta)(a(S, a)), \quad D(0) = 0, \quad \text{(A.3b)}
\]
where \( \beta \) is calculated by inserting Eq. (A.2) into Eq. (4), while the expression for \( \nu \) in Eq. (6) simplifies to
\[
\nu(S, a, \alpha) = \gamma \text{sgn}(S - a) \frac{\partial \beta}{\partial \alpha} + t(e) \frac{\partial \beta}{\partial \alpha} \frac{\partial \gamma}{\partial \alpha} S,
\]
(A.4)
where \( \partial \beta / \partial I_1 \) and \( \partial \beta / \partial \theta \) are calculated using Eq. (7).

A.1. Fatigue limit

For cyclic, proportional stress between \( \tilde{S} = \min, S(t) \) and a maximum stress \( \bar{S} = \max, S(t) \), after a transient, the backstress fluctuates between a minimum value \( \tilde{\alpha} \) and a maximum value \( \bar{\alpha} \). The endurance limit is obtained when the amplitude of this fluctuation vanishes, which occurs when \( \beta(\tilde{S}, \tilde{\alpha}) = \beta(\bar{S}, \bar{\alpha}) = 0 \) with \( \tilde{\alpha} \) an unknown, constant backstress of the endurance limit [11]. This corresponds to a system of equations
\[
y(\tilde{S}, \tilde{\alpha})^T A y(\tilde{S}, \tilde{\alpha}) + a^T y(\tilde{S}, \tilde{\alpha}) = 1, \quad k = 1, \quad \text{(A.5a)}
\]
\[
y(\bar{S}, \bar{\alpha})^T A y(\bar{S}, \bar{\alpha}) + a^T y(\bar{S}, \bar{\alpha}) = 1, \quad k = -1, \quad \text{(A.5b)}
\]
We consider a BP type endurance surface, \( A_{11} = A_{12} = 0 \), which makes it possible to eliminate \( \bar{\alpha} \) by adding Eqs. (A.5a) and (A.5b) while using Eq. (A.2), giving
\[
\frac{A_{22} t(e)^2}{E^2}(\tilde{S}^2 + \bar{S}^2) + \frac{a_1^2 r}{E}(\tilde{S} - \bar{S}) + a_2 t(e)(\tilde{S} + \bar{S}) = 2.
\]
(A.6)
This Eq. (A.6) describes the fatigue limit of the BP endurance function.

By introducing a constant mean stress \( S_m = \frac{1}{2}(\tilde{S} + \bar{S}) \) and a stress amplitude \( S_a = \frac{1}{2}(\tilde{S} - \bar{S}) \), Eq. (A.6) is solved for the stress amplitude of the fatigue limit,
\[
S_a^m(S_m) = \left\{ \begin{array}{ll}
E_{S_m,0}(\kappa, 0) \left( -a_1 \gamma + \sqrt{4A_{22} t(e)^2 \left[ 1 - A_{22} t(e)^2 \frac{a_1^2}{E} - a_2 t(e) \frac{a_1^2}{E} + a_2^2 \right]} \right), & A_{22} \neq 0,
E_{S_m,0}(\kappa, 0) \left( -a_1 \gamma \right), & A_{22} = 0.
\end{array} \right.
\]
(A.7)
The physically correct solution was singled out by requiring that \( dS_a^m(0)/dS_m < 0 \), Eq. (A.7) represents the fatigue limit in the Haigh diagram. Similarly, for a constant stress ratio \( R = \tilde{S}/\bar{S} \) and a BP type endurance surface, we substitute
\[
\tilde{S} = (Q_R + 1)S_a, \quad \bar{S} = (Q_R - 1)S_a, \quad Q_R = \frac{1 + R}{1 - R}.
\]
(A.8)
into Eq. (A.6) and obtain

\[
S_n^m(R) = \begin{cases} 
\frac{F}{2Q_{\alpha}} & \frac{1}{1+\alpha_1 r(t)} \times \left( Q_{\alpha}a_2(t + 1) + a_1 r(t) + \text{sgn}(A_{22}) \right) \sqrt{\frac{1}{A_{22}}} 
\end{cases} 
\]

A_{22} \neq 0.

A_{22} = 0.

(A.9)

The correct solution was identified by requiring that \(dS_n^m(−1)/dR < 0\). This relation (A.9) is useful when constructing Wohler curves near the endurance limit.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ijfatigue.2022.106878. It includes FE model validation and stress-field characterization of specimens, as well as the fatigue data of our experiments in tabulated format.

References


