Signal Processing Aspects of Massive MIMO

Ema Becirovic
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Massive MIMO

Ema Becirovic
This is a Swedish Doctor of Philosophy thesis. The Doctor of Philosophy degree comprises 240 ECTS credits of postgraduate studies.

Cover: Cityscape with massive MIMO base stations on the roofs. The cover is designed by Lisen Hansson.

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Abstract

Massive MIMO (multiple-input-multiple-output) is a technology that uses an antenna array with a massive number of antennas at the wireless base station. It has shown widespread benefit and has become an inescapable solution for the future of wireless communication. The mainstream literature focuses on cases when high data rates for a handful of devices are of priority. In reality, due to the diversity of applications, no solution is one-size-fits-all. This thesis provides signal-processing solutions for three challenging situations.

The first challenging situation deals with the acquisition of channel estimates when the signal-to-noise-ratio (SNR) is low. The benefits of massive MIMO are unlocked by having good channel estimates. By the virtue of reciprocity in time-division duplex, the estimates are obtained by transmitting pilots on the uplink. However, if the uplink SNR is low, the quality of the channel estimates will suffer and consequently the spectral efficiency will also suffer. This thesis studies two cases where the channel estimates can be improved: one where the device is stationary such that the channel is constant over many coherence blocks and one where the device has access to accurate channel estimates such that it can design its pilots based on the knowledge of the channel. The thesis provides algorithms and methods that exploit the aforementioned structures which improve the spectral efficiency.

Next, the thesis considers massive machine-type communications, where a large number of simple devices, such as sensors, are communicating with the base station. This thesis provides a quantitative study on which type of benefits massive MIMO can provide for this communication scenario — many devices can be spatially multiplexed and their battery life can be increased. Further, activity detection is also studied and it is shown that the channel hardening and favorable propagation properties of massive MIMO can be exploited to design efficient detection algorithms.

The third part of the thesis studies a more specific application of massive MIMO, namely federated learning. In federated learning, the goal is for the
devices to collectively train a machine learning model based on their local data by only transmitting model updates to the base station. Sum channel estimation has been advocated for blind over-the-air federated learning since fewer communication resources are required to obtain such estimates. On the contrary, this thesis shows that individually estimating each device’s channel can save a huge number of resources owing to the fact that it allows for individual processing such as gradient sparsification which in turn saves a huge number of resources that compensates for the channel estimation overhead.


Nästa situation handlar om massiv maskintypskommunikation, där ett stort antal enkla enheter, till exempel sensorer, kommunicerar med basstationen. I avhandlingen presenteras en kvantitativ fallstudie som studerar vilka typer av vinsten massiv MIMO kan erbjuda för detta scenario. Resultaten visar att många enheter kan betjänas samtidigt och att deras batteritid kan förbättras. Vidare visas att egenskaperna från massiv MIMO kan utnyttjas för att utforma effektiva algoritmer som kan detektera när olika sensorer är aktiva och vill kommunicera med basstationen.

Det tredje fallet handlar om en specifik tillämpning av massiv MIMO, närmare bestämt federerad maskinlärning. Federerad maskinlärning är
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My deepest gratitude goes to my amazing friends — you know who you are! Thank you for being so supportive and enthusiastic while I was on this journey. I feel so lucky to have such great friends; you are like a family that I chose.

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Finally, to my family. You are so incredibly supportive. Hopefully this thesis answers the question that my mother asked when I started this journey: “What is the point of wireless communication research? It already works!”
Thank you for never doubting me and letting me do my thing, and thank you for being a constant throughout my life.

Ema Becirovic
Linköping, September 2022
List of Abbreviations

5G                  fifth generation of cellular network technology
AWGN               additive white Gaussian noise
eMBB               enhanced mobile broadband
FDD                frequency-division duplex
i.i.d.             independent and identically distributed
LASSO              Least Absolute Shrinkage and Selection Operator
LPWA               low-power wide-area
MAP                maximum a posteriori
MIMO               multiple-input multiple-output
MMSE               minimum mean-square error
mMTC               massive machine-type communication
MR                 maximum ratio
RF                 radio frequency
ROC                receiver operating characteristic
SGD                stochastic gradient descent
SNR                signal-to-noise ratio
TDD                time-division duplex
URLLC              ultra-reliable low-latency communication
ZF                 zero forcing
Chapter 1

Introduction and Motivation

Wireless communication has enabled the increasing globalization of the world. By simply carrying a mobile phone in our pocket, we can, at a blink of an eye, access the Internet and communicate with the whole world. The mobile phone is no longer seen as a device to only communicate one-on-one with our friends and family, instead, it is an extension of ourselves. We use the phones to access social media, news, and for entertainment, with, e.g., video streaming. This use of the mobile phone has been enabled by cellular wireless communication. The generations of cellular wireless communication have reflected this shift in focus: from voice calls in rudimentary cellular systems such as 2G, to high-speed data transmission for wireless Internet access and video streaming in 4G, and more recently 5G.

In wireless communication, the end goal is to share information between a transmitter and a receiver through a wireless channel. The fundamental limit of such a system is the capacity, i.e., how much information that can be shared reliably across the channel. The capacity-limiting factor is the wireless channel. However, the signal processing solutions that we employ will ultimately decide how close to the capacity we can actually operate. We, as a research community and engineers as a whole, have been very good at pushing the communication solutions to approach the capacity limit. The current solutions work very well in most scenarios; however, there will always be cases where the general solutions do not work well. In order to squeeze the very last drop from our communication systems, we also need to adapt and design solutions to the cases where the general solutions lack in performance. This thesis covers a selection of situations where there is room to bridge the gap to the fundamental limits.

Three main use cases were highlighted when the future of wireless com-
Introduction and Motivation

Communication was envisioned. The use cases envisioned for the 5G wireless communication are the trifecta: enhanced mobile broadband (eMBB), ultra-reliable low-latency communication (URLLC), and massive machine-type communication (mMTC) [1]. The three use cases cover different requirements and needs of the future:

**eMBB** is intended to give higher data rates to be used for applications such as virtual reality, augmented reality, and high quality video streaming, among other demanding applications.

**URLLC** is intended for time-sensitive communication such as communication between vehicles [2], remote surgery [2, 3], and envisioned to essentially replace cables in industrial settings for robots and similar [2, 4, 5]. These applications need high reliability and low latency, but the data rates need not be very high.

**mMTC** is intended to cope with the massive increase of low data rate devices such as different types of sensors [6, 7]. While there are many of them, the individual devices do not transmit large amounts of data, they are not time-critical, and they can be assumed to have low mobility.

Together these three use cases cover the anticipated needs of wireless communication for both humans and machines in the coming years.

However, it is important to note that 5G is not “the end of history”. Specifically, these use cases are not an exhaustive list of what we can anticipate in the future. We have already started looking ahead, i.e., what happens beyond 5G, in 6G? The answer is that we need to further specify, and specialize, the wireless communication for different use cases and the demands will be even higher. It is likely that we will have many highly specialized solutions which can co-exist and provide exactly what the specific use case requires [8]. Further, we can expect to see a mix of the previously mentioned use cases, e.g., reliable eMBB [9].

In order to meet the requirements of the use cases, we need to develop technologies and systems that can support these high demands. One such technology, and arguably the most compelling of them all, is multiple-input multiple-output (MIMO).

Massive MIMO is a technology that has been established as one of the most important technologies that made 5G possible [10]. Since it was conceived in [11], it has been prevalent in communications research. Massive MIMO is essentially a communication system, operating in time-division duplex (TDD) mode, where a base station with a massive number of antennas serves multiple
users in the same time- and frequency resource by spatially multiplexing them.\(^1\) The technology then relies on the law of large numbers [13] to produce elegant mathematical expressions which simplify the signal processing. Importantly, massive MIMO technology has been empirically shown to work with the help of test-beds in both academia and in industry [14]. It is also currently being deployed as a 5G technology. The specifics of “conventional” massive MIMO will be discussed further in Chapter 3.

Massive MIMO is not the only multi-antenna technology that will be used in the future [15–17]. More recently, and what will probably be actuated in real-world systems, distributed massive MIMO has been on the rise. In distributed massive MIMO, many access points co-operate to jointly serve the users in the system. A special case of distributed massive MIMO is called cell-free massive MIMO [18–22]. As the name suggests, it erases the cell boundaries by serving one user with multiple single-antenna access points. Cell-free massive MIMO gives benefits by decreasing the probability that a user has a bad channel to the access points.

The main focus of massive MIMO research has been increasing the data rates, i.e., targeted towards the eMBB requirements. It has been very successful in this aspect and can increase the spectral efficiency, i.e., the number of bits that can be transmitted per second in a given frequency band and area [23]. This is achieved by being able to spatially multiplex users in the same time- and frequency resource. To increase the spectral efficiency several aspects might be considered. One aspect that is important is power control [23–32]. The power control can aim to create fairness between the users or increase the sum spectral efficiency in the network. Many more aspects of massive MIMO have been considered, such as pilot assignment [31,33–36], broadcast and multicast [37–42], physical-layer security [43–50], and hardware impairments [51–61].

Massive MIMO is not only beneficial for the eMBB use case — it also provides some benefits for URLLC [62,63]. Using massive MIMO can help reduce the outage probability, and therefore increase the reliability, compared to a system with single-antenna base stations. Due to the increased reliability, there is less need for retransmitting packets, which means that the latency will also reduce. Additionally, the latency for a user can further be improved as the user does not have to wait for other users to be served until it itself can be served. This is due to the base station being able to concurrently serve multiple users. A note of caution for URLLC in massive MIMO systems is that massive MIMO conventionally operates in TDD which delays the

\(^1\)See [12] for a discussion on the definition of massive MIMO.
1 Introduction and Motivation

communication, hence increasing latency. Another metric, related to latency, is age of information [64]. Age of information is (a function of) the difference in time between the last received packet and when it was generated. This metric is closely related to latency but is more suitable when considering the cases where the data is related between different transmissions.

In addition to massive MIMO being beneficial for eMBB and URLLC, massive MIMO is also beneficial for the mMTC use case [63,65–68]. The most straightforward benefit is the increase in signal-to-noise ratio (SNR) due to the beamforming gain [24]. With higher SNR, wireless communication with remote sensors and other mMTC devices will be possible since higher channel attenuations can be tolerated. The increased SNR can also be used to lower the transmit power of the devices and therefore increase the battery life of the devices [69]. Another benefit is that it is easier to separate users in massive MIMO [23]. This helps the base station detect users in the system’s random access phase [70]. For mMTC, it is preferable if the random access is grant-free, meaning that the users send their identification and data without waiting for an acknowledgment from the base station [65,71–73].

The users in a network are not homogeneous. They have different requirements and are in different use cases. We usually study the use cases in isolation — in the paragraphs above, use cases were considered individually. Nonetheless, it is important to show that the users of the different use cases can coexist [74,75] — which they can.

1.1 Contributions of the Thesis

This thesis solves a variety of research problems in wireless communication. The thesis consists of two parts; the first part gives an introduction and some preliminaries of the topics covered in the thesis and the second part is a collection of papers which consider specific problem formulations. Specifically, the papers provide signal processing solutions for massive MIMO systems.

The first two papers, Papers A and B, consider ways of improving the channel estimation quality. This is especially important when the uplink SNR is low, see Section 3.2. In Paper A, low-mobility non-coherent users are considered, and in Paper B, a scenario where the user has access to accurate channel estimates is considered. Papers C, D and E, consider mMTC scenarios. In Papers C and D, we present activity detection algorithms for grant-free random access, and Paper E considers the benefits of massive MIMO for low-power wide-area mMTC users. Finally, Paper F presents some aspects of analog combining for federated learning in massive MIMO systems. The
considered scenarios are of course only a small subset of scenarios that need to be considered for the future of wireless communication and massive MIMO.

The first author is the main contributor of the included papers. The first author has written the papers, derived the analytical results, and implemented the empirical results. The co-authors are acknowledged for contributing to the papers by sharing feedback on the manuscripts, helping with research directions, having insightful discussions, and sharing their professional expertise. The resulting papers are greatly improved by the combined efforts of all the co-authors.

Included Papers

Paper A: Joint Antenna Detection and Bayesian Channel Estimation for Non-Coherent User Terminals

Authored by: Ema Becirovic, Emil Björnson, and Erik G. Larsson

Abstract: In this paper, we propose a method of improving the channel estimates for non-coherent multi-antenna terminals, which are terminals that cannot control the relative phase between its antenna ports, with channels that can be considered constant over multiple time slots. The terminals have multiple antennas and are free to choose whichever antenna they want to use in each time slot. An unknown phase shift is introduced in each time slot as we cannot guarantee that the terminals are phase coherent across time slots. We compare three different clustering techniques that we use to detect the active antenna. We also compare a set of different statistical and heuristic estimators for the channels and the phase shifts. We evaluate the methods by using correlated Rayleigh fading and three different bounds on the uplink capacity. The accuracy of the capacity bounds are verified with bit-error-rate simulations. With our proposed methods we can have an SNR improvement of approximately 2 dB at 1 bit/s/Hz.
1 Introduction and Motivation

Paper B: Combining Reciprocity and CSI Feedback in MIMO Systems

Authored by: Ema Becirovic, Emil Björnson, and Erik G. Larsson

Published in IEEE Transactions on Wireless Communications, 2022, doi: 10.1109/TWC.2022.3182749.

Abstract: Reciprocity-based time-division duplex (TDD) Massive MIMO (multiple-input multiple-output) systems utilize channel estimates obtained in the uplink to perform precoding in the downlink. However, this method has been criticized of breaking down, in the sense that the channel estimates are not good enough to spatially separate multiple user terminals, at low uplink reference signal-to-noise ratios, due to insufficient channel estimation quality. Instead, codebook-based downlink precoding has been advocated for as an alternative solution in order to bypass this problem. We analyze this problem by considering a “grid-of-beams world” with a finite number of possible downlink channel realizations. Assuming that the terminal accurately can detect the downlink channel, we show that in the case where reciprocity holds, carefully designing a mapping between the downlink channel and the uplink reference signals will perform better than both the conventional TDD Massive MIMO and frequency-division duplex (FDD) Massive MIMO approach. We derive elegant metrics for designing this mapping, and further, we propose algorithms that find good sequence mappings.

Paper C: Detection of Pilot-Hopping Sequences for Grant-Free Random Access in Massive MIMO Systems

Authored by: Ema Becirovic, Emil Björnson, and Erik G. Larsson

Published in the proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), 2019.

Abstract: In this paper, we study an active user detection problem for massive machine type communications (mMTC). The users transmit pilot-hopping sequences and detection of active users is performed based on the received energy. We utilize the channel hardening and favorable propagation properties of massive multiple-input multiple-output (MIMO) to simplify the user detection. We propose and compare a number of different user detection methods and find that using non-negative least squares (NNLS) is well suited for the task at hand as it achieves good results as well as having the benefit of not having to specify further parameters.
1.1. Contributions of the Thesis

**Paper D: Activity Detection in Distributed Massive MIMO With Pilot-Hopping and Activity Correlation**

Authored by: Ema Becirovic, Emil Björnson, and Erik G. Larsson

Paper submitted to IEEE Wireless Communications Letters

**Abstract:** Many real-world scenarios for massive machine-type communication involve sensors monitoring a physical phenomenon. As a consequence, the activity pattern of these sensors will be correlated. In this letter, we study how the correlation of user activities can be exploited to improve detection performance in grant-free random access systems where the users transmit pilot-hopping sequences and the detection is performed based on the received energy. We show that we can expect considerable performance gains by adding regularizers, which take the activity correlation into account, to the non-negative least squares, which has been shown to work well for independent user activity.

**Paper E: How Much Will Tiny IoT Nodes Profit from Massive Base Station Arrays?**

Authored by: Ema Becirovic, Emil Björnson, and Erik G. Larsson

Published in the proceedings of the 26th European Signal Processing Conference (EUSIPCO), 2018

**Abstract:** In this paper we study the benefits that Internet-of-Things (IoT) devices will have from connecting to a massive multiple-input-multiple-output (MIMO) base station. In particular, we study how many users that could be simultaneously spatially multiplexed and how much the range can be increased by deploying massive base-station arrays. We also investigate how the devices can scale down their uplink power as the number of antennas grows with retained rates.

We consider the uplink and utilize upper and lower bounds on known achievable rate expressions to study the effects of the massive arrays. We conduct a case study where we use simulations in the settings of existing IoT systems to draw realistic conclusions.

We find that the gains which ultra narrowband systems get from utilizing massive MIMO are limited by the bandwidth and therefore those systems will not be able to spatially multiplex any significant number of users. We also conclude that the power scaling is highly dependent on the nominal signal-to-noise ratio (SNR) in the single-antenna case.
1 Introduction and Motivation

Paper F: Optimal MIMO Combining for Blind Federated Edge Learning with Gradient Sparsification
Authored by: Ema Becirovic, Zheng Chen, and Erik G. Larsson
Published in the proceedings of the 23rd IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2022

Abstract: We provide the optimal receive combining strategy for federated learning in multiple-input multiple-output (MIMO) systems. Our proposed algorithm allows the clients to perform individual gradient sparsification which greatly improves performance in scenarios with heterogeneous (non i.i.d.) training data. The proposed method beats the benchmark by a wide margin.

Excluded Papers
The papers in Table 1 have contributions by the author but are excluded from the thesis due to them being preliminary versions of included papers or out of scope.
1.1. Contributions of the Thesis

Table 1: Excluded papers

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1 Introduction and Motivation
Chapter 2

Preliminaries

This chapter serves to remind the reader of some fundamental concepts and techniques that are used in the thesis. We briefly introduce detection and estimation, the two most important tools for statistical signal processing. Then, we introduce some concepts from machine learning, namely neural networks and clustering. The signal processing solutions introduced here are only stepping stones to the algorithms and solutions developed later in the thesis.

2.1 Detection

Detection theory is a fundamental part of statistical signal processing [76]. It is used when we need to distinguish among two or more hypotheses. Detection theory appears in many different embodiments in communication systems. For example, in communication systems, the hypotheses can be to decide whether a user is present in the system or not, or to decide which message was sent by a transmitter.

In the simplest detection problem, we try to distinguish between two hypotheses; one, $H_0$, where we only observe noise, and the other, $H_1$, where we observe a known signal with noise. Mathematically, we observe $N$ samples of the received signal $y$, whose distribution changes whether we are under $H_0$ or $H_1$:

$$H_0 : y = w,$$  \hspace{1cm}  \text{or}\hspace{1cm}  H_1 : y = A1 + w, \hspace{1cm} (1)$$

where $w$ is the $N$-dimensional noise vector. In this case, the signal is a DC-level with amplitude $A$. In the applications in this thesis, the noise is modeled as independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian, $w \sim CN(0, \sigma^2 I)$, where $\sigma^2$ is the variance.
This model can be generalized into the multi-hypothesis linear model, wherein the goal is to distinguish between $M$ hypotheses. Mathematically, under the $m$:th hypothesis, the received signal can be written as

$$H_m: y = Ax_m + w,$$

(2)

where $A$ is an $N \times p$ known observation matrix, $p$ is the number of parameters, and $x_m$ is the $p$-dimensional signal corresponding to hypothesis $m$.

To minimize the probability of error, the maximum a posteriori (MAP) detector is used [76, Sec. 3.8]. The MAP detector decides $H_m$ if

$$p(H_m \mid y) > p(H_{m'} \mid y), \quad m' \neq m,$$

(3)

i.e., we choose the hypothesis with the largest posterior probability. Further, if all the hypotheses are equally probable, $p(H_m) = \frac{1}{M}$, the maximum likelihood detector [76, Sec. 3.8] decides $H_m$ if

$$p(y \mid H_m) > p(y \mid H_{m'}), \quad m' \neq m,$$

(4)

i.e., we choose the hypothesis with the largest likelihood. In i.i.d. Gaussian noise, the maximum likelihood detector reduces to deciding $H_m$ if

$$\|Ax_m - y\|^2 < \|Ax_{m'} - y\|^2 \quad m' \neq m.$$  

(5)

In some applications it might be of interest to minimize something other than the error probability. In such cases, a cost, $C_{ij}$ is assigned to the decision to choose $H_i$ when $H_j$ is true. The detection criterion is then to minimize the expected cost. Formally, the detector is called the minimum Bayes risk detector [76, Sec. 3.8], and it decides $H_m$ if

$$\sum_{i=0}^{M-1} C_{mi} p(H_i \mid y) < \sum_{i=0}^{M-1} C_{mi'} p(H_i \mid y), \quad m' \neq m,$$

(6)

i.e., we choose the hypothesis that minimizes the cost. The MAP detector in (3) is a special case of the minimum Bayes risk detector, with the following costs:

$$C_{ij} = \begin{cases} 0, & i = j, \\ 1, & i \neq j, \end{cases}$$

(7)

i.e., all errors have the same cost.
2.2 Estimation

In detection problems with two hypotheses, the costs determine the threshold which is then compared to the ratio of the likelihoods under the different hypotheses. We choose $H_1$ if

$$\frac{p(y | H_1)}{p(y | H_0)} > \frac{C_{10} - C_{00} p(H_0)}{C_{01} - C_{11} p(H_1)} = \gamma, \quad (8)$$

where $\gamma$ is the threshold. Instead of determining the costs, we commonly plot the receiver operating characteristic (ROC), where each point in the plot corresponds to a different threshold which gives different false alarm and miss detection probabilities. False alarm probability refers to the probability that the detector decides $H_1$ given that $H_0$ is true and miss detection probability refers to the probability that the detector decides $H_0$ given that $H_1$ is true.

2.2 Estimation

Estimation is another fundamental tool in statistical signal processing [77]. Estimation is very closely related to detection. However, in estimation, rather than making hard decisions, we aim to produce estimates of the parameters that are as close as possible to the true parameters. Both in the context of this thesis and in massive MIMO in general, channel estimation is an important tool.

There are two schools of thought in statistical estimation: the classical approach and the Bayesian approach [77, 78]. In the classical approach, the parameters of interest are assumed to be deterministic but unknown. For simplicity, here we will only consider Bayesian estimation, i.e., where the parameters to be estimated are random and follow a prior distribution, and we estimate one realization of said parameters. A rule in Bayesian estimation is that using more prior knowledge will make the estimate more accurate.

Just as in Section 2.1, we once again consider the linear model, where the received signal is

$$y = Ax + w, \quad (9)$$

where $A$ is an $N \times p$ known observation matrix, and $w$ is the additive noise. The goal is to estimate the $p$-dimensional parameter vector, $x$, from the received signal, $y$.

The simplest estimator of $x$ is the least-squares estimator [77, Ch. 8],

$$\hat{x} = \arg\min_x \| Ax - y \|^2. \quad (10)$$

The least-squares estimator finds the parameters that best describe the observed data by minimizing the squared error between the noiseless data,
Ax and the observed data y. The least-squares estimator dates back to the end of the 18th century [77, Ch. 8] [79], and is very good in many scenarios. Although no guarantees on optimality can be made, since the method is widely used, there are a lot of variations and other results that can be found [77, Ch. 8]. The least-squares estimator does not assume any distribution of the observed data y; it does not utilize the knowledge of the noise distribution, nor the prior of x. Hence, the estimator can be improved by using more information.

In case we know the noise distribution, we can state the maximum likelihood estimator [77, Ch. 7],

$$\hat{x} = \arg\max_x p(y \mid x),$$

(11)
i.e., the estimator that gives the most likely parameters. If the noise is i.i.d. Gaussian, the least-squares estimator and the maximum likelihood estimator coincide.

Further, if the prior of x is known, we can minimize the mean-square error of the parameters. The minimum mean-square error (MMSE) estimator is the mean of the posterior probability density function [77, Sec. 11.4]

$$\hat{x} = \mathbb{E}\{x \mid y\},$$

(12)
which minimizes the Bayesian mean-square error

$$\mathbb{E}\{\|x - \hat{x}\|^2\}.$$  

(13)

Unless the noise distribution and the prior are very well behaved (Gaussian), it is usually hard to find the MMSE estimator in closed form. This makes it hard to design signal processing algorithms around the MMSE estimator.

Another estimator which uses the prior is the MAP estimator [77, Sec. 11.5]. As the name suggests, the MAP estimator chooses the estimate which maximizes the posterior distribution,

$$\hat{x} = \arg\max_x p(x \mid y) = \arg\max_x p(y \mid x) p(x).$$

(14)
In the case of i.i.d. Gaussian noise, the MAP estimator simplifies to a regularized least squares. The regularization depends on the prior of x. If y and x are jointly Gaussian, the MAP estimator and the MMSE estimator are equivalent.
2.2. Estimation

Compressed Sensing

An area of estimation that has gained a lot of traction is compressed sensing [80–83]. Compressed sensing refers to estimation problems where the parameter vector, $\mathbf{x}$, is sparse, meaning that only a few of its $p$ elements are non-zero. Further, the observation matrix, $\mathbf{A}$, is not full rank, i.e., we have more unknown parameters, $p$, than measurements, $N$, i.e., $p \gg N$.

A common approach to compressed sensing is to start with an optimization problem where we find the vector that minimizes the noise under a sparsity constraint

$$
\min_{\mathbf{x}} \| \mathbf{A} \mathbf{x} - \mathbf{y} \|^2 \\
\text{s.t.} \| \mathbf{x} \|_0 \leq S,
$$

where $\| \cdot \|_0$ is the $\ell_0$-norm\(^1\) that counts the number of non-zero components of the vector, and $S$ is an upper bound on the number of non-zero components we allow in the parameter vector. However, this problem generally turns out to be an NP-hard combinatorial problem [85]. Hence, the problem is relaxed. Specifically, the $\ell_0$-norm is relaxed to the $\ell_1$-norm. Further, the problem is usually rewritten as an unconstrained minimization

$$
\min_{\mathbf{x}} \| \mathbf{A} \mathbf{x} - \mathbf{y} \|^2 + \lambda \| \mathbf{x} \|_1,
$$

where $\lambda$ is the Lagrange multiplier. The problem in (16) is called the LASSO (Least Absolute Shrinkage and Selection Operator) [86]. The LASSO will also give sparse solutions [86]. This is relieving since the LASSO can be solved much more efficiently since it is a convex problem. In the noiseless case, i.e., when $\mathbf{w} = \mathbf{0}$, and under certain conditions on $\mathbf{A}$, the LASSO will give the same solution as the original combinatorial problem in (15) [81].

Another approach is to view compressed sensing as a Bayesian estimation problem. The LASSO coincides with the MAP estimate when the noise is i.i.d. Gaussian and the prior of $\mathbf{x}$ is zero-mean Laplace distributed [83, Sec. 2.8]. The Lagrange multiplier will be related to the variance of the noise and the scale parameter of the Laplace distribution.

The field of compressed sensing is much wider than what is mentioned here. An important portion of the research about compressed sensing deals with determining the conditions and design of the observation matrix $\mathbf{A}$ that yield desirable properties for sparse signal processing [83, Ch. 4]. Further,

\(^1\)Formally, it is not a norm since it does not satisfy the absolute homogeneity property [84, Sec. 5.1], i.e., $\| \alpha \mathbf{x} \|_0 \neq |\alpha| \| \mathbf{x} \|_0$, but it is colloquially referred to as such for simplicity.
other structural properties than simple sparsity of the parameter vector can occur that can be further exploited to produce accurate estimates. For example, in group sparsity, the parameters are grouped and only a subset of groups is non-zero at the same time [87].

2.3 Neural Networks

A concept that has taken the world, and the field of computer science, by storm is deep learning, which refers to deep neural networks [88]. It is a technology that is used to solve a large set of difficult signal processing problems. The origin of neural networks stems from an attempt to reverse engineer the way information is processed in the human brain [89]. In simple terms, neural networks are black-box models that are trained to solve problems that are easy for humans to solve manually, but where it is difficult to design signal processing algorithms with the traditional signal processing toolbox. Many popular applications are in the field of image processing. Say, for example, that we want to be able to determine which animal a picture depicts. It is easy for a human to distinguish a cat in a picture, but it is very hard to formally come up with a mathematical model that can describe a general picture of a cat. In the training phase of a neural network, the network is fed with many examples, in this case, of animal pictures. If trained properly, with enough data and a good architecture, the network is then able to distinguish between the different animals. The crux is that in order for the network to perform well, huge amounts of data and computational resources are needed in the training phase. In recent years, improvements in the development of hardware and computational resources have made machine learning and especially neural networks valid and legitimate signal processing solutions. With more resources, the networks can be made more complex, and deep; hence, we have the expression: deep learning.

The goal of the training process is to find an approximation, \( f(x; \theta) \), of the function, \( f^*(x) \), which is parameterized by the parameter vector \( \theta \). Both the original function and the approximation take input values, \( x \), which can be images, or other observed signals, and map them to an output. The output can either be continuous, corresponding to regression, or discrete, corresponding to classification. The network is trained in the supervised learning framework. In supervised learning, there is a training set of \( N \) observed data points, \( (x_i, y_i), i = 1, \ldots, N \). The \( i \)-th data point consists of an input \( x_i \), and an output \( y_i \), where \( y_i \) is a (noisy) observation of the target function: \( y_i \approx f^*(x_i) \). The neural network is represented by some
hyperparameters, e.g., the learning rate, and the number of layers, i.e., the
depth. The trainable parameters in the network are contained in the vector \( \theta \). The goal is to find the optimal parameter vector, \( \theta^* \), that minimizes some
loss function, \( F(\theta) \):

\[
\theta^* = \arg\min_{\theta} F(\theta),
\]

where the loss function is computed over the training data set. Usually, the
parameter vector is found through stochastic gradient descent (SGD).

### Federated Learning

Federated learning is a form of collaborative, distributed deep learning [90,91].
In federated learning, \( K \) clients collaborate to train a machine learning model
in a distributed fashion, without sharing raw data with the other clients or
the central server. In canonical deep learning, the training data is collected at
a central server and trained in one location. On the other hand, in federated
learning, data is not collected centrally, instead each client trains a model
locally using their own local data, and then the local models are aggregated
at a central server. By distributing the learning process, the privacy of the
clients is preserved.

In federated learning, the goal is to find the optimal parameter vector \( \theta^* \),
which minimizes a loss function defined by

\[
F(\theta) = \sum_{k=1}^{K} w_k F_k(\theta),
\]

where \( F_k(\theta) \) is the loss function computed over the local training data set
possessed by client \( k \), and \( w_k \) is the weight factor associated with client \( k \).

Federated learning, specifically Federated Averaging [90], works in an
iterative fashion where in the \( t \):th round, with \( t = 1, 2, \ldots \), the following
steps are executed:

1. The server broadcasts the current global model \( \theta(t) \).
2. Each client \( k \) runs SGD and the update rule follows

\[
\theta_k(t + 1) = \theta_k(t) - \alpha_t \nabla F_k(\theta_k(t)),
\]

where \( \alpha_t \) is the step size at iteration \( t \). Let \( \Delta \theta_k(t) \) represent the local
model update from client \( k \): \( \Delta \theta_k(t) = \theta_k(t + 1) - \theta_k(t) \). Note that,
the clients can run several steps of local SGD within each round. For
simplicity, we omit the local iteration index here.
3. After receiving the local updates from the clients, the server aggregates the received information,

\[
\Delta \theta(t) = \sum_{k=1}^{K} w_k \Delta \theta_k(t),
\]

(20)

and updates the global model as

\[
\theta(t + 1) = \theta(t) + \alpha_t \Delta \theta(t).
\]

(21)

This process continues until convergence or until some stopping criterion has been reached.

2.4 Clustering

Clustering is a form of unsupervised learning. In unsupervised learning, contrary to supervised learning, the training input data is not paired with the corresponding output data; instead the goal is to infer something about the underlying data distribution. In the case of clustering, we try to split the data point into groups, such that all points within each group are similar [92]. The similarity metric can represent different things. There are many types of clustering, and the resulting clusters can be used in different ways. For example, in centroid-based clustering, the goal is to split the data into groups that are all similar to the centroid of each cluster, and in density-based clustering the data is split into clusters where each point in a cluster is close to at least one other point in the cluster. Naturally, in density-based clustering, outliers are easily identified.

In Algorithm 1, we present one of the most popular clustering algorithms, the K-means algorithm [93, Ch. 5] [92, Sec. 4.2]. The K-means algorithm is popular because of its simplicity of implementation. Given \(N\) data points, \(x_n, n = 1, \ldots, N\), and the desired number of clusters, \(K\), the algorithm returns the assignment of each data point, and the centroids of each cluster, \(c_k, k = 1, \ldots, K\). The algorithm tries to find the clustering which minimizes

\[
\sum_{k=1}^{K} \sum_{n \in C_k} \|x_n - c_k\|^2,
\]

(22)

where \(C_k\) is the set that contains the data points that are assigned to cluster \(k\). The first step of the algorithm is to initialize the centroids. The initialization can be made randomly by e.g., picking \(K\) points from the data set, or with
Algorithm 1 K-means clustering algorithm

**Input:** \( \{x_n\}_{n=1}^N \), \( K \)

**Output:** \( \{C_k\}_{k=1}^K \), \( \{c_k\}_{k=1}^K \)

1. Initialize \( \{c_k\}_{k=1}^K \) with appropriate algorithm, e.g., [94]
2. \( C_k = \{ n : \|x_n - c_k\| \leq \|x_n - c_{k'}\| \}, k' = 1, \ldots, K \}, k = 1, \ldots, K \)
3. while \( \{C_k\}_{k=1}^K \) changed do
4. \( c_k = \frac{1}{|C_k|} \sum_{n \in C_k} x_n, k = 1, \ldots, K \)
5. \( C_k = \{ n : \|x_n - c_k\| \leq \|x_n - c_{k'}\| \}, k' = 1, \ldots, K \}, k = 1, \ldots, K \)
6. end while

more sophisticated methods, e.g., [94]. The algorithm then iteratively updates the assignments and centroids until it has converged to a local minimum of (22). Since convergence to a global minimum cannot be guaranteed, the algorithm is often times run with many different initializations and the clustering with the smallest metric (22) is chosen as the final cluster.
2 Preliminaries
Chapter 3

Conventional Massive MIMO

In this chapter, the conventional single-cell massive MIMO concept is presented and discussed. The chapter serves to remind the reader about the benefits of massive MIMO and why it has become a widespread solution for wireless communication.

3.1 System Model

We assume a simple, single-cell massive MIMO system. The base station has $M$ antennas, and serves $K$ single-antenna users. We study this system in a sufficiently small enough time-frequency block, called the coherence block, such that we can consider the channel between any base-station antenna and any user to be constant. This is called the block-fading model. The size of this block is further discussed in Section 3.1.2.

Each of the base-station antennas has one radio frequency (RF) chain such that the base station can digitally control the relative phase and amplitude between all of its antenna elements and which therefore makes the base station capable of spatially directing signals to the users [23].

The channel between a base-station antenna $m$ and a user $k$, which encompasses multipath propagation (i.e., scattering, diffusion, and reflection [95,96]), attenuation, and shadowing, can be characterized by two complex values, one for the case when the base-station antenna is transmitting and the user is receiving – the downlink, and one for the case when the user is

\footnote{Throughout the thesis, the users will be called users, devices, terminals, sensors, and clients interchangeably.}

\footnote{The channel is not actually complex, but is a notational consequence of using complex baseband representation as is common in communication [97,98].}
transmitting and the base-station antenna is receiving – the uplink. We group these values in vectors and denote the uplink and downlink channels of a user $k$ by $\mathbf{g}_{k}^{UL} \in \mathbb{C}^{M}$ and $\mathbf{g}_{k}^{DL} \in \mathbb{C}^{M}$, respectively. Figure 1 shows a small illustration of the system model.

Conventional massive MIMO operates in TDD, i.e., the transmissions in the uplink and downlink are separated in time, as opposed to frequency-division duplex (FDD), where the uplink and downlink are separated in frequency. The reason for TDD operation will become apparent after the discussion in the subsequent sections. Finally, the received signals are affected by additive white Gaussian noise (AWGN).

### 3.1.1 Reciprocity

One key assumption that helps cement massive MIMO as a key technology of 5G is the assumption of *reciprocity*. This means that the uplink and downlink channels of a user and base-station antenna pair are the same. Hence, we drop the superscripts and refer to the channel of user $k$ as

$$\mathbf{g}_{k} = \mathbf{g}_{k}^{UL} = \mathbf{g}_{k}^{DL}$$

regardless of whether the user is transmitting or receiving. Usually, to simplify the notation, we consider the downlink channel to be the conjugate (Hermitian) transpose of the uplink channel without loss of generality. The channels of the users are grouped in the matrix $\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_K]$, for notational purposes.
3.1. System Model

With the reciprocity assumption, we only need to estimate the channel of a user in one direction. This is very important for massive MIMO because if we had to estimate the downlink channel, the time spent on training would increase with the number of base-station antennas, $M$ [23,24].

Note that the physical propagation environment is always reciprocal when TDD is used. However, the transmit and receive RF chains are not generally reciprocal [99]. Therefore, reciprocity is not always assumed. Nonetheless, there exists methods of reciprocity calibrating the RF chains, e.g., see [100]; hence, the reciprocity assumption is reasonable.

3.1.2 Coherence Block

By the block-fading channel assumption we assume that the channel is constant in a time-frequency interval called the coherence block or the coherence interval. The coherence block is the product of two quantities, the coherence time, which is the time interval where we can consider the channel to be time invariant, and the coherence bandwidth, which is the frequency interval where we can consider the channel to be flat [23,24,97].

The coherence time, $T_c$, of a channel depends on the Doppler spread of the channel, which in turn is proportional to the wavelength and inversely proportional to the velocity of the transmitter [24,97]. This means that a user on a moving train will have much lower coherence time than a user that is standing still.

The coherence bandwidth, $B_c$ of a channel will depend on the delay spread of the channel which in turn depends on the difference between the shortest and longest paths in the propagation environment [24,97].

The size of the coherence block is a product of the coherence time and the coherence bandwidth, $\tau_c = T_c B_c$, and is different for each user. Because we need a system-wide coherence block, we refer to the coherence block as the smallest coherence block of all the users. The size of the coherence block is usually on the order of hundreds of symbols. However, the size can increase to several thousands if the users are stationary [24,96].

3.1.3 Channel Model

Since the wireless channel between the transmitter and the receiver ultimately determines the potential of a wireless communication system, channel modeling becomes very important in the design of the communication system. We use channel models as mathematical and simulation tools to e.g., construct estimators and evaluate performance, as we cannot perform experiments in
real-world systems that have not been constructed yet. With channel models that accurately describe the physical propagation, we can design the signal processing algorithms that achieve the best result.

The most tractable channel model is a case where there is no multipath propagation. We call this a line-of-sight channel. In the line-of-sight channel model, the transmitter and receiver are assumed to be in free space. Only the distance between the two will affect the channel [24,96]. The line-of-sight channel model is also used when we have a strong line-of-sight component and the non line-of-sight components have significantly smaller, negligible, power. When the $M$ base-station antennas are placed in uniform linear array, meaning that the antennas are equidistantly placed on a line, with $\lambda/2$ antenna spacing, where $\lambda$ is the wave-length of the carrier, the channel to user $k$, which we assume is placed in the far-field of the array, can be described as [24, Sec.7.2.2]

$$ g_k = \sqrt{\beta_k} e^{j\phi_k} \left[ 1, \ldots, e^{-j(m-1)\pi \sin \theta_k}, \ldots, e^{-j(M-1)\pi \sin \theta_k} \right]^T, \quad (24) $$

where $\beta_k$ describes the attenuation (path-loss), $\phi_k$ describes the phase-shift related to the distance between the user and the base station, and $\theta_k$ is the angle of the user position relative to the array boresight.

As soon as we start mixing in multipath components we will have a more complicated model. If the number of paths which the signal takes is only a handful, we can try to model all the paths with a deterministic geometric model [101,102]. However, this approach quickly becomes intractable and hard to analyze.

In a multipath channel, several attenuated and time-delayed copies of a transmitted signal will arrive at the receiver. As the channel is constantly changing due to objects that move around in the propagation environment, we cannot feasibly track all the time delays and attenuations from multipaths in the channel; hence it is common to employ a statistical model of the channel. The statistics of the channel will be constant over a time that is longer than the coherence time. Hence, they will have to be estimated but can be done less frequently than estimating the instantaneous channel.

One of the most common channel model assumptions, which is also extensively used in this thesis, is the Rayleigh fading channel which is based on the central limit theorem [13], i.e., that the sum of many independent and random paths will approach a normal distribution [23,97]. The Rayleigh fading channel gets its name from that the channel induces an amplitude that follows a Rayleigh distribution [95,96].

In the Rayleigh fading model the channel in each coherence block follows
a circularly symmetric complex Gaussian distribution, so for a user $k$, its channel is distributed as
\[ g_k \sim \mathcal{CN}(0, R_k), \]
where $R_k$ is the (spatial) correlation matrix of user $k$. If there is no spatial correlation the channel is said to be uncorrelated Rayleigh fading and the correlation matrix is equal to a scaled identity matrix, $R_k = \beta_k I$. The large-scale fading coefficient, $\beta_k$, captures the attenuation of the $k$:th channel and depends on the distance between the user and the base station. In the case of correlated Rayleigh fading, there are several different ways of modeling the correlation. Often it is assumed that there is a group of scatterers concentrated in one or more locations. This assumption leads to models such as the local scattering spatial correlation model [23,96]. The large-scale fading coefficient and the spatial correlation matrix can be estimated with different methods [103–106]. However, in the context of this thesis, they are assumed to be known perfectly.

Of course, there are many more different statistical channel models to employ in massive MIMO. Examples of these are the Rician fading channel [96,107], and the keyhole channel [24,96].

With some channel models, including the Rayleigh fading channel model [24, Ch. 7] [23, Sec. 2.5] and the uniformly random line-of-sight channel model [24, Ch. 7], we get two important properties which will make dealing with a massive number of base-station antennas easier. The first property is called channel hardening and is mathematically defined as [23,24,96]
\[ \frac{\|g_k\|^2}{\mathbb{E}\{\|g_k\|^2\}} \xrightarrow{a.s.} 1, \quad \text{as } M \to \infty, \]
for a channel $g_k \in \mathbb{C}^M$, with $M$ antennas. In layman’s terms, the norm of a channel that has the channel hardening property will behave deterministically if the number of the antennas is sufficiently large. This is a property that helps with signal processing and deriving closed-form bounds on the channel capacity.

The other property is called asymptotic favorable propagation and its definition is [23,24,96]
\[ \frac{g_k^H \tilde{g}_k}{\sqrt{\mathbb{E}\{\|g_k\|^2\}} \mathbb{E}\{\|\tilde{g}_k\|^2\}} \xrightarrow{a.s.} 0, \quad \text{as } M \to \infty, \]
where $g_k \in \mathbb{C}^M$ and $\tilde{g}_k \in \mathbb{C}^M$ denote the channels of two different users, $k \neq k'$. This property is more simply stated as: if we have a sufficient number
3 Conventional Massive MIMO

of antennas, the channels of two different users will be almost orthogonal. This helps separate users such that they can be served in the same time- and frequency block.

3.2 Channel Estimation

To be able to effectively direct the signals to the users we need to estimate the channel. Because we have assumed reciprocity, it suffices to estimate the channel in one direction. In massive MIMO, due to having more base-station antennas than users, we estimate the uplink channel. Therefore, the coherence block of a conventional massive MIMO system is split into three parts: the uplink training phase, the uplink data phase, and the downlink data phase, as pictured in Figure 2 [23, 24]. The training phase can be placed anywhere in the coherence block as long as it is placed before the downlink data phase. In fact, one can also superimpose the uplink data and the training symbols [108, 109], however this is not covered in conventional massive MIMO. By studying Figure 2, one notices that the more symbols we spend on training, the fewer data symbols we can transmit. Thus, while increasing the number of symbols used for training improves the channel estimates, the rate might go down as the number of transmitted information bearing bits is smaller.

To help us estimate the channels, we start by assigning a sequence of symbols to each user. This sequence is called a pilot and the pilot of user $k$ is denoted as $\phi_k \in \mathbb{C}^{\tau_p}$. The pilots assigned to the different users are mutually orthogonal, $\phi_k^H \phi_{k'} = 0$, $k \neq k'$, have unit energy, $\|\phi_k\| = 1$, $k = 1, \ldots, K$, 

![Figure 2: A depiction of the TDD coherence block. The coherence block spans $\tau_c = T_c B_c$ symbols of which $\tau_p$ are used for training. The uplink and downlink data transmission parts are divided in time.](image-url)
and have length $\tau_p$. Hence, $K \leq \tau_p < \tau_c$. We group the pilots in the matrix $\Phi = [\phi_1, \ldots, \phi_K]$, for notational purposes. If the pilots are not mutually orthogonal, a phenomenon called pilot contamination occurs and the users cannot be separated. Pilot contamination can in some cases be very degrading for the performance of massive MIMO [11, 23, 110]. All the $K$ users transmit their pilot\(^3\) over their respective channels at the same time and the base station receives

$$Y^p = \sqrt{\rho_{ul} \tau_p} G \Phi^H + W^p,$$

(28)

where $\rho_{ul}$ is the normalized uplink SNR per pilot symbol and $W^p \in \mathbb{C}^{M \times \tau_p}$ is AWGN with i.i.d. $\mathcal{CN}(0, 1)$ elements. A sufficient statistic of the channel of user $k$ is

$$y^p_k = Y^p \phi_k = \sqrt{\rho_{ul} \tau_p} g_k + w^p_k,$$

(29)

where $w^p_k = W^p \phi_k \sim \mathcal{CN}(0, I)$.

We assume that we know that the channels are uncorrelated Rayleigh fading and that we know all the large-scale fading coefficients of all the users. The best we can produce is the MMSE estimator, which for user $k$ is [24, 77]

$$\hat{g}_k = \sqrt{\rho_{ul} \tau_p} \beta_k y^p_k.$$  

(30)

The mean-square of the channel estimate between base-station antenna $m$ and user $k$ is [24, 77]

$$\gamma_k = \mathbb{E}\{|[\hat{g}_k]_m|^2\} = \rho_{ul} \tau_p \beta_k^2 \frac{1}{1 + \rho_{ul} \tau_p \beta_k^2}.$$  

(31)

When the channel estimate is good, the mean-square of the channel estimate will be close to the large-scale fading coefficient, i.e., $\gamma_k \approx \beta_k$.

Other, inferior, estimators exist. One is the least-squares estimator which does not consider any knowledge of the channel. The least-squares estimator is [23, 77]

$$\hat{g}_k = \frac{y^p_k}{\sqrt{\rho_{ul} \tau_p}}.$$  

(32)

A disadvantage of the least-squares estimator is that it will generally not give an estimate that is uncorrelated with the estimation error, which is not the case for the MMSE estimator [23].

\(^3\)In (28) the users are transmitting the conjugate of the pilots to simplify notation.
3 Conventional Massive MIMO

3.3 Uplink Data Transmission

In this section, we introduce the procedure of uplink transmission. Downlink data transmission is not covered since the rest of the thesis only considers uplink communication.

In uplink data transmission, user $k$ transmits a (weighted) complex sample $x_k$. The sample is zero-mean, has an average power constraint $\mathbb{E}\{|x_k|^2\} \leq 1$, and we assume that the data samples are uncorrelated, $\mathbb{E}\{x_k x_{k'}^\ast\} = 0$, for $k \neq k'$. In the case of conventional digital data transmission, these symbols come from a finite alphabet, while in analog transmission the samples can take any complex value. The base station then receives a superposition of these samples at each antenna

$$y = \sum_{k=1}^{K} \sqrt{\rho_{ul}} \eta_k g_k x_k + w$$

$$= \sqrt{\rho_{ul}} GD_\eta^{1/2} x + w$$

$$= \sqrt{\rho_{ul}} \hat{G}D_\eta^{1/2} x - \sqrt{\rho_{ul}} \tilde{G}D_\eta^{1/2} x + w, \quad (35)$$

where $0 \leq \eta_k \leq 1$ is the power control coefficient of user $k$, i.e., a scalar between 0 and 1 that represents how much of the total allowed power that the user should spend, $D_\eta$ is a diagonal matrix with the power control coefficients of the users on the diagonal, $\hat{G} = [\hat{g}_1, \ldots, \hat{g}_K]$ and $\tilde{G} = [\tilde{g}_1, \ldots, \tilde{g}_K]$ are the channel estimates and channel estimation errors, respectively, $x$ is the vector that collects the transmitted symbols of all users, and $w \in \mathbb{C}^M$ is i.i.d. AWGN. The $M$-dimensional received signal is not directly used to decode the transmitted symbols from the users. Instead, the received signal is linearly processed, by applying a receive combining matrix, such that the transmitted symbol from each user experiences a single-input single-output channel. To decode the message from user $k$, the base station multiplies the received signal (35) with the conjugate transpose of the receive combining vector of user $k$, $v_k$, to obtain,

$$y_k = \sqrt{\rho_{ul}} \eta_k v_k^H \hat{g}_k x_k - \sqrt{\rho_{ul}} \eta_k v_k^H \tilde{g}_k x_k + \sum_{k'=1}^{K} \sqrt{\rho_{ul}} \eta_{k'} v_k^H g_{k'} x_{k'} + v_k^H w. \quad (36)$$

The first term in (36) represents the desired signal, and the other terms will describe different types of noise and interference. In Section 3.4, we discuss the choices of the receive combining further. Essentially, we want to make
3.4 Spatial Multiplexing

The magic of massive MIMO comes from that the base station can spatially multiplex users by directing signals using *beamforming*. What is done in the downlink case is that the base station uses the channel estimates of the users and tries to come up with *precoders* that will make the signals constructively or destructively interfere at the position of the users’ antennas. In the uplink this process is called *receive combining* wherein the base station combines the received signals from each antenna with a phase shift and an amplitude scaling.

To make it more clear, we illustrate this process with a small example. We study a case where a single-antenna transmitter sends a sinusoid to a two-antenna receiver. In this example, we omit the effects of the channel. When decoding the transmitted sinusoid, the receiver can choose to add the received sinusoids coherently, and the resulting signal will have a higher amplitude (SNR) than if the received signals are considered individually. Similarly, the receiver can phase shift one of the received signals by half of a period, which will make the two signals cancel out.

These two cases are directly translatable to the receive combining vectors or linear decoders\(^4\). The case where the aim is to maximize the SNR at the user is called maximum ratio (MR) \([24]\); the receive combining vectors (matrix) of the users are

\[
V_{\text{MR,UL}} = \hat{G},
\]

(37)

where \(\hat{G} = [\hat{g}_1, \ldots, \hat{g}_K]\). That is, the \(k\):th column of \(V_{\text{MR,UL}}\) corresponds to the receive combining vector of user \(k\) which in this case is the channel estimate of user \(k\), \(\hat{g}_k\).

In the case that is called zero forcing (ZF) \([24]\) the base station tries to cancel out all the signals from the other users when decoding the intended user. In the ideal case, where the channel is perfectly known there will be no interference from the signals intended to the other users at all. The receive combining vectors for the ZF case is:

\[
V_{\text{ZF,UL}} = \hat{G}(\hat{G}^H\hat{G})^{-1}.
\]

(38)

Ergo, the ZF receive combing vector for a user \(k\) corresponds to projecting

\(^4\)We do not state the precoders here as the thesis will mainly focus on uplink transmission.
the signal onto the null-space of all other channels. Hence, ZF can only be used if there are more antennas at the base station than there are users [24].

### 3.5 Capacity Bounds

One of the most important ways to measure the performance of a communication system is to look at the capacity. The capacity is the maximum amount of information that can be sent over a channel with an arbitrarily small probability of error [97]. The maximum is achieved by changing the distribution of the input. Further, we need capacity-achieving channel codes. In reality, we are also constrained by the finite input alphabet in digital communication. With these two caveats in mind, we can only hope to approach the capacity. The goal is to be as close as possible.

It is in many cases very hard to find an expression for the channel capacity. We can calculate the channel capacity for simple channels such as the AWGN channel or a channel with deterministic fading [97]. However, what we usually rely on are lower bounds on the channel capacity. The lower bounds are often referred to as achievable rates because we can actually achieve the rates that they are predicting!

Perhaps one of the best bounds used is the side-information bound [23, 24]. The side-information refers to that the receiver has some knowledge about the channel, such as the channel estimate, but not necessarily the channel itself. The uplink achievable rate predicted by the side-information bound for user $k$ in a massive MIMO system with uncorrelated Rayleigh fading and MMSE channel estimation is [24]

$$C_k \geq \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho \eta_k |v_k^H \hat{g}_k|^2}{\sum_{k' \neq k}^{K} \rho \eta_{k'} |v_k^H \hat{g}_{k'}|^2 + \|v_k\|^2 \sum_{k' = 1}^{K} \rho \eta_{k'} (\beta_{k'} - \gamma_{k'}) + \|v_k\|^2} \right) \right\} .$$

(39)

The expectation is with respect to the side-information, i.e., the channel estimates. Generally, we cannot find a closed-form expression for this expectation and it is hard to use this bound to construct meaningful signal processing algorithms, such as power-control schemes.

Because of this reason, we often use another bound — the use-and-forget bound [23, 24]. The name refers to that we use the knowledge about the
channel estimates in the receive combining, but then throw the knowledge away when detecting the signal. The uplink achievable rate predicted by the use-and-forget bound for user $k$ in a massive MIMO system with uncorrelated Rayleigh fading and MMSE channel estimation is [24]

$$C_k \geq \log_2 \left( 1 + \frac{\rho_{ul} \eta_k \mathbb{E}\{v_k^H g_k\}^2}{\sum_{k'=1}^{K} \rho_{ul} \eta_{k'} \mathbb{E}\{v_{k'}^H g_{k'}\}^2 - \rho_{ul} \eta_k \mathbb{E}\{v_k^H g_k\}^2 + \mathbb{E}\|v_k\|^2} \right).$$

(40)

By utilizing the use-and-forget bound we can perform very effective power control that can help increase the fairness among the users or the overall spectral efficiency of the whole system [24]. This is actually one of the benefits of massive MIMO; we only need to use the long-term channel statistics in our power control optimization problems. Hence, the power control can be performed less frequently — only when the large-scale fading of the channel has changed.

### 3.6 Other Performance Metrics

Since achievable rate is a lower bound on the capacity, the quality of the achievable rate as a performance metric varies. It can be hard to know how close the bound is to the actual capacity. Hence, other performance metrics need to be studied.

One performance metric that can tell us a lot about the system performance is the bit-error-rate. When simulating the bit error rate, we need to make some design decisions. For example, we need to choose an error correcting code, and the modulation. The choice of modulation, e.g., quadrature phase shift keying (QPSK), will limit which rates we can study. For example, with QPSK, we cannot have a rate that is higher than 2 bits per channel use. Additionally, the error correcting code will also affect the performance. In massive MIMO, due to channel hardening, the channels are approximately AWGN, and hence, we can use codes that are optimized for such channels [111].

The most important key in unlocking the benefits of massive MIMO is the beamforming and receive combining. The quality of the channel estimates dictates how well we can “steer” the signals. Therefore, a popular metric to study is the mean-square error of the channel estimates. Mean-square error has a direct impact on the achievable rate. For example, consider the
use-and-forget bound in (40). If we take the receive combining vector of user \( k \) to be the MMSE channel estimate, i.e., MR combining, \( \mathbf{v}_k = \hat{\mathbf{g}}_k \), the signal-to-interference-plus-noise ratio scales with \( \gamma_k \), i.e., the quality of the channel estimate [24, (3.42)]. However, the mean-square error is not always a good measure. For example, it cannot capture the effect of pilot contamination [112].

Naturally, when we study other scenarios, where the goal is not “regular” digital communication, we use other metrics. In user detection problems, we look at ROC-curves. In communication systems for deep learning, the performance of the learning model is more important. In communication for critical applications, latency is important, and so on.
Chapter 4

Aspects of Massive MIMO

In this chapter we introduce and discuss some situations and use cases that are covered in this thesis. We also briefly introduce the signal processing solutions that aid in achieving the true potential of the communication systems in these scenarios. We stress that these are only a subset of the specialized scenarios that occur and where massive MIMO can give benefits.

4.1 Channel Estimate Improvement for Stationary Users

As mentioned earlier, the quality of the channel estimates is crucial in the performance of massive MIMO. The uplink SNR will affect the quality of the channel estimates, since the channel estimation is performed in the uplink. That means that when the uplink SNR for a user is low, we cannot reap the benefits of massive MIMO. Hence, we should always strive to improve the channel estimates such that a user with low uplink SNR does not suffer as much.

Cellular systems are designed to accommodate users with high mobility and high data rate, such as people sitting on the train while streaming a high-definition video. The system is designed with this use case in mind. The uplink pilots, the uplink data and the downlink data need to fit within the coherence block of a high-mobility user. However, there might be a user somewhere in the system that is not moving at all. It is only the mobility of the surrounding environment that will affect the coherence block of this user. Because the system is designed for the high-mobility user, the low mobility user, with a coherence block much larger than the high-mobility user, will effectively estimate the same channel multiple times, see Figure 3.
4 Aspects of Massive MIMO

Coherence block of
high mobility user

Figure 3: A high-mobility user has a much shorter coherence block than a stationary user. As a consequence, the stationary user will estimate the same channel in many consecutive time slots.

The situation that a user is both stationary and has low uplink SNR is easily imagined for mMTC users. A sensor could be placed in a basement, or another location with a bad channel to the base station, and not be moved for a long time.

The base station can exploit the knowledge of the low mobility user being stationary and combine the estimates in consecutive time slots to improve the overall channel estimate. If we have $T$ times more measurements of the same channel, the SNR in the channel estimation becomes $T$ times larger. This will lead to better channel estimates, and since the channel estimation quality has such a large impact on the achievable rate, the gain will be reflected in the achievable rate as well. This superior channel estimate can then be used to decode data from the low mobility user. We only assume uplink communication and that there are no strict latency requirements; hence, we can wait to decode the uplink data.

Nevertheless, the system is still designed with the high-mobility user in mind which means that the requirements put on the users are designed for the high-mobility case. What might happen, due to a lack of specification in standards and sub-optimal hardware is that the low mobility user will introduce a phase shift or switch antennas in each time slot [113, Ch. 7], [114, Ch. 11], see Figure 4. This is not a problem in the conventional case as the users are re-estimating their channels in each slot. However, it will need to be compensated for to utilize the channel estimate improvements. This is the scenario and problem formulation studied in Paper A.

In Paper A, we jointly estimate the antenna, the channel, and the phase shifts. This is accomplished by modifying clustering algorithms such as K-means clustering that was introduced in Section 2.4. The performance is evaluated by studying three different achievable rate bounds and the bit-error-rate.
4.2 Can Reciprocity be Exploited at Low SNR?

Conventional TDD Massive MIMO heavily relies on reciprocity-based channel estimation from uplink pilots. Therefore, one concern has been whether the performance of the system breaks down when the uplink SNR is low.

The alternative to estimating the channel through uplink pilots would be to estimate the downlink channel through downlink pilots and then feed back the estimated channel on the uplink. The advantage of this process is that the downlink channel oftentimes has a much higher SNR since the base station can transmit with a much greater power. The downside is that, if there is no additional structure of the channel, the number of resources required to feed back the channel estimates scales with the number of antennas, $M$, which is very large, or massive [24, Sec. 1.4].

In Paper B we study a scenario where the downlink channel comes from a finite set of beams. Such a model can approximate line-of-sight channels or Rician fading channels. The goal in this system model is for the base station to detect the correct beam from the set, in order to be able to beamform to the user. The detection is based on an uplink signal.

Specifically, in Paper B we study different scenarios where the user has different amounts of knowledge about the downlink channel, and if reciprocity holds or not. For example, if the user does not know the downlink channel, but reciprocity holds, the user transmits a pilot, similar to conventional massive MIMO, and the base station detects the most likely beam from the set of beams. However, if the user knows the downlink channel, the user transmits an uplink signal that is a mapping from the downlink channel. If mapped correctly, utilizing the downlink channel in the uplink signaling improves the beam detection performance greatly.

In Paper B we determine a metric that defines a good mapping in each of the scenarios, and evaluate the performance based on the probability of error.
4.3 Random Access for mMTC

Random access is the process where the users get access to the base station [114]. In mMTC this becomes a bottleneck due to the vast number of users present [65]; the mMTC use case envisions an enormous number of users in the system. The random access procedure can be grant-based or grant-free. If the random access is grant-based, users that want to connect to the network first send a sequence of symbols. The base station receives these sequences and decides, for each user, if the user should get access to the network or if it should try again. The base station then sends this information to the users. If the random access is grant-free, there is no response from the base station. The users will send their identifying sequence and data, cross their fingers and hope that the base station successfully received it.

As mentioned earlier, we expect there to be a huge number of mMTC users which are not active all the time. Energy efficiency is desirable for these users; hence, scheduling and grant-based random access is undesirable because of the high overhead cost. Therefore, it is very common to propose grant-free random access schemes for mMTC users [65,68]. In the massive MIMO case, this will mean that the users send both their pilot and their data in the same message for the base station to decode.

In conventional massive MIMO the users send pilot signals that help the base station to form the channel estimates. The pilots are commonly assumed to be mutually orthogonal, see Section 3.2. In the mMTC case, due to the number of users being huge — larger than the number of samples in the coherence block, \( K > \tau_c \), it is not possible to assign orthogonal pilots for all the users.

We can approach this issue in two ways. The first is to use non-orthogonal pilots for the users and hope that the pilot contamination is sufficiently small such that the data is still decodable. Then, the fact that there are much fewer active users than total number of users is used to detect users with compressed sensing [71,73]. There is also a possibility of embedding information bits in the pilots, meaning that the data part of the transmission can be eliminated if the information only consists of a few bits [72].

The other option is to use a pilot-hopping pattern, spanning many coherence blocks, to separate users. The pilots are allowed to collide in occasional blocks, but not in all — each user is assigned a unique pilot-hopping sequence, see Figure 5. This scheme relies on the further assumptions that the users are not latency constrained and that there is no downlink. This is the scheme that was investigated in Papers C and D.

In Paper C, we study the user activity detection problem. Two different
approaches are considered. The first is to consider traditional statistical signal processing techniques. Specifically, we formulate the maximum likelihood and MAP detectors. However, since all the $K$ users can be either active or inactive, this can be viewed as a multi-hypothesis detection problem with $2^K$ hypotheses. Therefore, we relax the corresponding optimization problems. Nonetheless, the problems are non-convex, such that when we solve the problems, we might end up in suboptimal solutions. The other approach is based on the received energies of the signals. It turns out that the problem is linear when the number of antennas in the system tends to infinity. However, the number of measurements that we have is far fewer than the number of unknowns. Luckily, as previously mentioned, only a few of the users are active at the same time. Hence, we can solve the user detection problem with compressed sensing techniques.

In Paper D, we consider a similar, but different, problem formulation as in Paper C. First of all, the system model is changed. Instead of considering a single-cell massive MIMO system as in Paper C where there is a single base station serving all the users, we consider a distributed massive MIMO system where there are multiple base stations serving the users. However, the base stations co-operate through a central processing unit.

Further, and more importantly, Paper D considers correlated user activity. The motivation comes from that if we have a sensor network monitoring a physical phenomenon, the activity of the sensors will be correlated with the physical events, and hence, with each other. The user activity detection problem is solved by once again studying the asymptotic system model, but adding regularizers to account for the user activity correlation.
4 Aspects of Massive MIMO

4.4 Increasing Coverage for mMTC Users

In a truly smart city, different types of sensors and other mMTC devices should be able to be placed everywhere. Sometimes they need to be placed in low converge areas, such as basements and remote places. A signal loses much of its power when it needs to go through obstacles, such as walls. The received power affects the receiver’s ability to detect and correctly decode the signal.

Low-power wide-area (LPWA) is the terminology used for the networks that deal with these requirements [115]. Because of the inadequacy of cellular systems that fulfill these requirements, a number of proprietary solutions are available for LPWA networks. Two noteworthy proprietary solutions are Sigfox and LoRa which both operate in unlicensed spectrum [115].

Sigfox is an ultra narrow-band technology and only uses 100 Hz bandwidth [116]. As a consequence, the data rate is extremely low: only 100 bits per second. However, using ultra narrow-band has its benefits: the power consumption and the noise levels are very low. That means that far-away sensors can still communicate and have a long battery life.

LoRa is a spread-spectrum technology which uses a lot more bandwidth than Sigfox [116] but the exact bandwidth changes. LoRa can adaptively change its data rate, which is beneficial if there are devices in different locations. However, Sigfox supports longer ranges than LoRa [117–119].

Massive MIMO will provide a beamforming gain that is proportional to the number of antennas in the base station [24]. This makes the communication link of the remote users stronger and more likely to be stable. By utilizing the beamforming gain, the users can either transmit further with the same power or improve their energy efficiency by lowering the transmit power. Motivated by this, a case study comparing Sigfox, LoRa and a massive MIMO system was made in Paper E.

4.5 Over-the-Air Aggregation for Federated Learning

Communication for federated learning does not focus on the learning process; instead emphasis is on how the clients transmit their gradient updates to the central server. To alleviate the communication resources required to transmit and receive the model updates from all the clients, over-the-air aggregation has been considered [120]. In over-the-air aggregation, the clients share the channel and the superposition property of the multiple access channel is
4.5. Over-the-Air Aggregation for Federated Learning

utilized to aggregate the model updates.

When we have a single-antenna parameter server and single-antenna clients that have perfect channel knowledge, the clients can compensate for the effect that the channel has on the transmitted gradients such that what is received at the parameter server is the aggregated gradients. When the receiver or transmitter have multiple antennas and no channel knowledge, it is not as obvious how to optimally transmit and receive the gradients.

In Paper F, we study how to optimally aggregate the gradients when the clients have no channel knowledge and the parameter server has \( M \) antennas. Specifically, we compare two different channel estimation schemes, and how those affect the aggregation of the model updates. The first estimation scheme is the conventional way, with orthogonal pilots, as described in Section 3.2. In the other scheme, a single pilot is used to estimate the sum of the channels. This scheme was proposed in [121, 122] to save on communication resources. Specifically, the parameter server obtains the MMSE estimate, \( \hat{h}_{\text{sum}} \), of the sum of the normalized channels, \( h_{\text{sum}} = \sum_{k=1}^{K} \frac{g_k}{\beta_k} \), where the channels are i.i.d. Rayleigh fading, i.e., \( g_k \sim \mathcal{CN}(0, \beta_k I) \). The details of the estimation schemes can be found in Paper F.

Say that each client \( k \) possesses a scalar \( x_k \) and that the goal of the central server is to aggregate the \( K \) scalars, i.e., compute

\[
\bar{x} = \sum_{k=1}^{K} x_k. \tag{41}
\]

In the context of federated learning, the scalars are elements in the gradient update vector. For simplicity, we assume that all the scalars are weighted equally. The received signal at the parameter server is

\[
y = \sum_{k=1}^{K} \sqrt{\rho \eta_k} g_k x_k + w, \tag{42}
\]

where \( \rho \) is an SNR related constant, \( w \) is AWGN with i.i.d. \( \mathcal{CN}(0, 1) \) elements and \( \eta_k \) is the power scaling coefficient of user \( k \). When using the second, sum channel estimation scheme, the power scaling is coordinated among the clients through a parameter \( \eta \) in the following way: \( \eta_k = \frac{\eta}{\beta_k} \). We consider the two estimators

\[
\hat{x}_{\text{BLUE}} = \frac{1}{\sqrt{\rho}} \begin{bmatrix} 1 \end{bmatrix} D_\eta^{-1/2} \left( \hat{G}^H \hat{G} \right)^{-1} \hat{G}^H y, \tag{43}
\]

where \( D_\eta \) is a diagonal matrix with the \( \eta_k \)'s on the diagonal, and

\[
\hat{x}_{\text{SUM}} = \frac{1}{M \sqrt{\rho \beta_{\min} \eta}} \hat{h}_{\text{sum}}^H y, \tag{44}
\]

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where $\beta_{\text{min}}$ is the smallest large-scale fading coefficient, i.e., $\beta_{\text{min}} = \min_k \beta_k$, and $\bar{\gamma}$ is the mean-square of the sum channel estimate, i.e., $\mathbb{E}\{\hat{h}_{\text{sum}}\}_{m} = \bar{\gamma}$.

Conditioned on the randomness of the channel estimates and the noise, the expectation of both the estimators is $\bar{x}$, i.e.,

$$\mathbb{E}\{\hat{x}_{\text{BLUE}}\} = \mathbb{E}\{\hat{x}_{\text{SUM}}\} = \bar{x}. \quad (45)$$

However, the estimators have different variances,

$$\mathbb{V}\{\hat{x}_{\text{BLUE}}\} = \left(\sum_{k=1}^{K} \rho(\beta_k - \gamma_k)\eta_k |x_k|^2 + 1\right) \left(\sum_{k=1}^{K} \frac{1}{\rho\eta_k (M - K)\gamma_k}\right) \quad (46)$$

and

$$\mathbb{V}\{\hat{x}_{\text{SUM}}\} = \frac{1}{M\bar{\gamma}} \sum_{k=1}^{K} |x_k|^2 + \frac{1}{M\bar{\gamma}} \frac{1}{\rho\beta_{\text{min}}\eta}. \quad (47)$$

When the SNR is high, i.e., when $\rho$ grows large, the variance of $\hat{x}_{\text{BLUE}}$ will go to zero, since $\gamma_k$ will approach $\beta_k$. However, the variance of $\hat{x}_{\text{SUM}}$ will tend to a non-zero constant. Hence, at high SNR the estimate based on individual channel estimates, with orthogonal pilots, is, from a pure estimation perspective, more desirable. However, at low SNR, $\hat{x}_{\text{SUM}}$ has a lower variance.

It is important to note that the goal of the aggregation is to find the aggregated model updates of a learning model. Hence, the performance metric should be that of the learning model and how well it has been trained.

Further, we know that the model update vectors are sparse. In [123], it was suggested that dropping 99% of the elements in the model update vectors does not degrade the performance of the learning model significantly. From an over-the-air aggregation perspective, this implies that we can save on communication resources by sparsifying the model update vectors, multiplying them by a wide measurement matrix and recovering the sparse model update at the parameter server by employing compressed sensing techniques.

In Paper F, we include gradient sparsification in our study of how to aggregate the model updates in federated learning. The results show that when we have individual channel estimates, based on orthogonal pilots, the performance is superior to when we have the sum channel estimate. The reason is that in the sum channel estimate case, we can only recover the aggregated sparse model updates and if the clients have different sparsification patterns, the aggregated sparse vector will have more non-zero components and hence is harder to recover. On the other hand, with individual channel estimates, we can recover the individual sparse model updates and aggregate them at the parameter server.
Chapter 5

Future Work

Although 5G has not yet achieved its full potential, the research community has started to look beyond 5G to find the next break-through technology for 6G [9,15,124–126]. There needs to be researchers working on far-future technologies, but the more present-day, or near-future, problems must not be forgotten. Moreover, communication technologies for highly specialized use cases, such as federated learning, need to be further studied and analyzed.

Although conventional massive MIMO systems assume single-antenna users, the likelihood that this will be the case is not high. Mobile phone manufacturers will probably not reduce the number of antennas in the phones in the future. Therefore, it is interesting to investigate signal processing algorithms similar to the ones in Paper A that do not assume that the user has a single antenna.

In terms of the system model in Paper B, there is a plethora of research directions that can be explored. In Paper B, the downlink channel comes from a finite set of beams and the performance metric we considered was pairwise error probability between the different beams. This model could be extended to a continuous beam space for the downlink channel. Further, we could study the case where the different beams have different priors and where the error of different beam pairs is not equal. This will change the design metric that we derived in Paper B. Another direction is to study the effect of the terminal having imperfect downlink channel knowledge and how that affects the design of the mapping.

In Papers C and D we considered activity detection when the users transmit pilot-hopping sequences. When two users use similar pilot-hopping sequences, the detection algorithm is more likely to make an error. In Paper C, the users independently activate while in Paper D the user activities
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are correlated. When the activity is correlated, different errors are more severe than others. For example, if the users activate based on a physical phenomenon, falsely detecting a user far away from the event will be more severe than falsely detecting a user that is close by. Therefore, when the user activity is correlated, the assignment of pilot-hopping sequences will be more significant than when the users activate independently.

An apparent problem with the proposed federated learning scheme in Paper F is that it scales poorly when there are many clients in the system since the channel estimation phase will take up more of the coherence interval. A way to improve that scheme would be to combine the two compared schemes such that some clients share pilots. For the best performance in such a case, we would like users that share a pilot to have similar sparsity patterns. However, if the clients have similar sparsity patterns, they probably have similar data. Hence, extra care has to be taken in order to ensure the privacy of the clients.
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