Strength testing based automatic scaling of muscle-tendon parameters for musculoskeletal models
- An automated method of scaling subject specific muscle-tendon parameters of thigh muscles

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Abstract

A method of estimating subject specific muscle parameters of musculoskeletal models of elite athletes (skiers) was sought. Subject specific models are necessary due to large differences in general anatomy and physical performance of elite athletes relative the general population. Sought muscle parameters concern the force generating capabilities of muscles. The estimation was limited to only include the quadriceps-femoris and hamstring muscle groups due to these muscle having the highest influence on the performance of a skier. A modified interpretation of the method proposed by Heinen et al. [19] was implemented. The method includes experimental strength tests of knee extension and flexion muscles of a test subject, a musculoskeletal model of the experiments coupled with a mathematical optimisation minimisation formulation. The aim of the optimisation was to match the strength of a model to the experimentally obtained strength curve by minimising error between the model and experimental results. The optimisation minimises the error between the model and the experimental data by varying the operating range and strength of the involved muscles. The musculo-tendon parameters are estimated through transformation equations, explicitly related to the design variables. Three healthy and active males were involved in this study. An overall increase of the accuracy of the optimised model relative an unscaled reference model was observed, with the reduction of the objective function in a range of 80.2-92% and a mean absolute error varying between 6.8 to 16.5 Nm. In the case of quadriceps-femoris muscles, the optimised model struggles with incorrect prediction of the peak torque and peak torque angle due to limitations of the muscle model and the distribution of the moment arm. The model predicts both peak torque and peak torque angle with high accuracy in the case of hamstring muscles. In addition, the model struggles with low precision for both knee extension and flexion for all of the involved test subjects. Although great improvement in the accuracy was observed, the model prediction was deemed to have low clinical significance, due to low accuracy and precision. The clinical significance could be improved, for example by a more detailed musculoskeletal model or by modifying the behaviour of the muscle model. Future work should focus on addressing the current issues presented in this study and a further development, as the method still is relatively new and untested. Parallely, the researchers should try to test the method in clinical studies, in order to evaluate the influence on the results by the implementation of this method of parameter estimation.
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Introduction

FIS World Cup Alpine skiers are subjected to injuries due to the high-risk environment of the sport. Knee-related injuries comprise 36% of all injuries [14]. In order to return to active training, the medical staff assess the athletes ability to return to sport (RTS). The RTS decision is based on many factors including the type and history of the injury, the results of physical examinations, type of activity the patient wants to return to etc. [32]. The most common practice of the medical staff is to evaluate the RTS criteria based only on symptoms or diagnostic tests performed by the patients. These tests do not provide the clinician with enough information about the health status of the inner tissue nor the loading of the injured structures during high-intensity activities [32]. Knowledge about these factors is essential in order to make the RTS decision as accurate as possible, ensuring a safe return to the sport for the injured athlete. Currently, it is impossible to investigate the loading of the injured joint-muscle structures in-vivo, and examining the inner tissue requires an invasive biopsy procedure [25], thus a non-invasive method of estimating the loads is in high demand. Musculoskeletal modelling software (AnyBodyModelingSystem™, OpenSim™ etc.) has made it possible to non-invasively make an accurate prediction of the load acting on the joint-muscle structures during different types of motion. Musculoskeletal models are widely used in the fields of orthopaedics [24], sports [31], ergonomics and gait analyses [19] and a demand for highly accurate models has only increased in recent years.

Presently, it is impractical to create a model based solely on medical images of an individual [17]. Musculoskeletal models are therefore based on cadaver examinations which in turn must be scaled to represent the individual. Scaling of the generic model consists mainly of two parts: geometrical scaling of the skeletal structures and scaling of the muscle-tendon unit (MTU) parameters (MTP). Researchers agree that, depending on the field, the level of subject specific details should be adequate for the aim of the study. Heinen et al. suggest that in the case of elite athletes, a high level of subject specific details is required due to high deviations in the bone structure and muscle behaviour in comparison to the generic population [17].
Geometrical scaling has been extensively described and well implemented into most of the musculoskeletal software packages, with methods varying from a simple linear scaling to advanced morphing techniques based on magnetic resonance imaging (MRI). A majority of the geometrical scaling laws make use of easily obtainable anthropometrical measurements such as subject’s height and mass. Alternatively, individual body segments could be measured and used in the scaling laws.

Most of the musculoskeletal models assume a generic Hill-type muscle model which can be customised in order to fit any individual. Such muscle model consists of a contractile element (CE) in addition to serial elastic element (SEE) and parallel elastic element (PEE) [17]. The combination of the properties of these elements form a force-length-speed \( F - l - v \) relationship of the muscles. \( F - l - v \) is particularly important because it pertains to force generating capabilities of the muscles, which in turn control the strength of an individual. Studies have shown that the predictability of such models is highly sensitive to most of their muscle-tendon parameters [8]. However, errors in tendon slack length \( L_s \) (the length at which the tendon starts to generate a resistive force) and optimal fibre length \( L^O \) (the muscle fibre length at which the muscle can generate maximum force) have been proved to have the highest influence on the results [8, 43]. These two parameters in particular are hard to measure in-vivo and do not scale linearly with the skeletal geometry, therefore they need to be obtained analytically, experimentally or by a hybrid analytical-experimental approach.

Most of the current methods of MTP estimation are a combination of experimentally obtained data together with a mathematical optimisation procedure [17]. Heinen et al. proposed a method based on the experimentally obtained strength profiles of a particular athlete, obtained during isometric and dynamic trials [18]. A musculoskeletal model is created to calculate the joint torques of the model under similar conditions as for the experiments. A mathematical optimisation procedure is implemented to minimise the square root difference between the experiments and the model according to:

\[
\min_{\bar{x}} J(\bar{x}) = \sqrt{(T^{exp} - T^{mod})^2},
\]

where \( J(\bar{x}) \) is the error function (or objective function), and \( T^{exp} \) and \( T^{mod} \) are the experimental and modelled torque, respectively. The design variables in \( \bar{x} \) include the operating range of a muscle and a strength scaling factor. The muscle parameters \( L_s \) and \( L^O \) are explicitly expressed in terms of the design variables and eventually computed. The calculated MTP \( (L_s \) and \( L^O) \) may later be used in other models. This method determined the MTP for muscles of the entire lower extremity and proved to greatly improve the predictability of the model.

This master’s thesis is a part of a larger study currently being carried out mainly at Gymnastik- och Idrottsföreningen (GIH). The main objective of that study is to implement musculoskeletal modelling software into the RTS procedure by simulating the diagnostic tests performed by injured alpine skiers. The results of such simulations would provide the medical staff with crucial information about knee joint loading, thus increasing the accuracy of the decision regarding the athletes ability to return to sports. The main focus of this master’s thesis is, to create a generic base for the calculation of MTP of the lower extremity based on the method proposed by Heinen et al [18]. By a generic base it is meant that only the minimum of information about the athlete should be required, in order to calculate the subject specific MTP. Ideally, the bare minimum will consist of the sex, mass, height, anthropometrical measurements of the dominant leg and results of the experiments performed by the individual athlete.
1.1 Purpose and objectives

The objectives are:

• to create a generic base for the calculation of MTP of the lower extremity based on the method proposed by Heinen et al.;

• to implement the method for the knee extension/flexion muscles during isometric (static) performance;

• to propose the experimental methodology that is required in order to obtain all the necessary data;

• to evaluate the predictability of the method in terms of accuracy and precision of the prediction.

1.2 Delimitations

Estimation of the MTP is limited to the quadriceps-femoris and hamstring muscle groups because skiing primarily targets the quadriceps and hamstring muscle groups. The main focus of the study, which this master’s thesis is a part of, is knee injuries. To get accurate prediction of the forces acting on the knee joint, the MTP of muscles pertaining the motion of a knee should be as accurate and as subject specific as possible.

Estimation of the MTP is limited to only isometric trials and the associated MTP. Heinen et al. [18] proposed a method of estimation of MTP related to both force-length (static) and force-speed (dynamic) behaviour of the muscles. The presented results pointed towards improvements in predictability for both cases. The improvements in the predictability are far less in the dynamic cases than in the static due to limitations of the muscle model and potential sources of errors during the experimental procedure. Heinen also emphasise the extra demands on the experiments and analysis that combined approach demands. For this master’s thesis an adequate level of accuracy and workload is covered by isometric trials only.

Predefined scaling laws for the geometrical scaling of skeletal structures will be used instead of the most advanced morphing techniques.
1.3 Abbreviations

- 2D - Two Dimensional
- AMS - AnybodyModelingSystem™
- BMI - Body Mass Index
- CE - Contractile Element
- DOF - Degrees of Freedom
- EMG - Electromyography
- f-l-curve - Force-Length Curve
- f-l-v-curve - Force-Length-Speed Curve
- GIH - Gymnastik och Idrottshögskolan
- MRI - Magnetic Resonance Imaging
- MVC - Maximum Voluntary Contraction
- MAE - Mean Absolute Error
- MTP - Muscle-Tendon Parameters
- MTU - Muscle-Tendon Unit
- PEE - Parallel Elastic Element
- PCSA- Physiological Cross Section Area
- ROM - Range Of Motion
- RTS - Return To Sport (Evaluation)
- RMSD - Root Mean Square Deviation
- SEE - Serial Elastic Element
- TLEM - Twente Lower Extremity Model
### 1.4 List of frequently occurring symbols and muscle-tendon parameters

- $a$ - Activation Level (of a muscle)
- $\bar{f}$ - Force-vector
- $\bar{M}$ - Moment-vector
- $\bar{d}$ - Lever Arm
- $F_{\text{local}}$ - Local Strength Scaling Factor
- $L_f^o$ - Optimal Muscle Fibre Length
- $L_f^s$ - Tendon Slack Length
- $F_{\text{om}}$ - Maximum Isometric Muscle Force
- $L_{\text{max}}^M_T$ - Maximum Length Of a Muscle-Tendon-Unit
- $L_{\text{min}}^M_T$ - Minimum Length Of a Muscle-Tendon-Unit
- $\tilde{L}_{\text{max}}^m$ - Maximum Normalised Muscle Fibre Length
- $\tilde{L}_{\text{min}}^m$ - Minimum Normalised Muscle Fibre Length
- $\bar{x}$ - Vector Of the Design Variables
- $\alpha$ - Pennation Angle of a Muscle
2 Background
2.1 Biomechanics and musculoskeletal system

Biomechanics is the study of the interaction between structures of a biological organism and its mechanical response to external forces or internal nerve impulses. Biomechanics is a broad field of study, with research in biofluid mechanics, musculoskeletal biomechanics of a human body, sports biomechanics and many more [22]. Musculoskeletal biomechanics addresses the interaction between bones and muscles during movement, for example, gait, jumping, or throwing.

The primary function of the musculoskeletal system is to move the body through use of the muscular and skeletal systems. The musculoskeletal system also provides form, stability and support. Musculoskeletal system consists of a set of bones, joints, muscles, tendons and ligaments, treated as a multibody system. Bones are connected to other bones and muscles via joints, tendons and ligaments, and their main purpose is to provide stability. Every bone is modelled as a rigid body with its own unique set of inertial properties. Multiple types of joints connect the bones and allow them to move relative each other. For example, the knee joint allows a leg to extend, flex with a slight medial and transverse rotation. Commonly, every joint in a modeling system is treated as ideal and frictionless. Muscles span from origin to insertion points and act as the main force actuators through their ability to contract. Tendons connect the muscle to bones and allow movement of the bones by transmitting the muscle force. Secondary function of a muscle is to keep the bone in place during movement [16].

2.1.1 Governing equations of motion of a musculoskeletal system and inverse dynamics

A musculoskeletal system is represented by an idealised multibody system. Bones are assumed to be rigid bodies, each with a unique set of inertial properties. Joints act as kinematic constraints by restricting the motion of the bones relative to other bones. Muscles together with the resulting joint reaction forces are represented as constraint forces. Assuming holonomic and scleronomous kinematic constraints together with holonomic and rheanomous driving constraints, the general equations of motion (Newton-Euler) for a dynamic multibody system are [9]:

\[
\ddot{M}\ddot{q} - \ddot{Q}' + \dot{H}^T \lambda = \ddot{0}, \quad (2.1)
\]

\[
\dot{H} (\ddot{q}, \dot{q}) = \ddot{0}, \quad (2.2)
\]

where \(\dddot{M}\) is the inertia matrix, \(\dddot{q}\) is the acceleration vector expressed in terms of absolute coordinates, \(\dot{H}^T\) is the transposed Jacobian of the constraint matrix \(\dot{H}\) and \(\lambda\) is a vector containing the Lagrangian multipliers. \(\dddot{Q}'\) is the vector of external forces. The second equation relates to the assumption of a holonomic multibody system with scleronomous kinematic and rheanomorphic driving constraints. The constraint matrix \(\dot{H}\) is defined according to the equation:

\[
\dot{H} = \begin{bmatrix} \dot{H}^k \\ \dot{H}^d \end{bmatrix}, \quad (2.3)
\]

where \(\dot{H}^k\) and \(\dot{H}^d\) denotes kinematic and driving constraints, respectively. Thus the Jacobian of the constraint matrix \(\dot{H}\) is:

\[
\dot{H}_q = \frac{\partial \dot{H}}{\partial \dot{q}}, \quad (2.4)
\]

The acceleration \(\ddot{q}\) can be obtained by solving the system of equations according to equation (2.5). The acceleration can only be obtained once the velocity is known.

\[
\ddot{H}_q \ddot{q} = \dddot{c}, \quad \dddot{c} = - (\dddot{H}_q \ddot{q}) \ddot{q} - 2\dot{H}_q \dot{q} - \dddot{H}_H, \quad (2.5)
\]
2.1. Biomechanics and musculoskeletal system

where

\[
\left( \ddot{R}_q \dot{q} \right)_q = \frac{\partial}{\partial \dot{q}} \left( \frac{\partial H}{\partial \dot{q}} \dot{q} \right), \quad \dot{R}_{qt} = \frac{\partial^2 H}{\partial q \partial \dot{t}}, \quad \dot{R}_{tt} = \frac{\partial^2 H}{\partial \dot{t} \partial \dot{t}}.
\]  

(2.6)

For detailed derivation of (2.5) refer to [9].

In musculoskeletal systems, equations (2.1) and (2.5) are usually combined, which yields the following system of equations:

\[
\begin{bmatrix}
M & \dot{R}_q^T \\
R_{qt} & 0
\end{bmatrix} \begin{bmatrix}
\dot{q} \\
\lambda
\end{bmatrix} = \begin{bmatrix}
\dot{Q}^c \\
\dot{c}
\end{bmatrix}. 
\]  

(2.7)

In musculoskeletal models, the unknown reaction forces and torques, obtained from $\lambda$, that are acting around the constraints are used to calculate for example muscle activation patterns, muscle work, joint loading etc. AMS uses the inverse dynamics method to calculate the unknown reaction forces. The reaction forces are calculated based on the known kinematics of a body and the inertial properties of body segments.

Nowadays, the kinematics can easily be observed and obtained through use of motion capture. Most musculoskeletal modelling software facilitates the import of the captured data into the software with high precision. This, coupled with reliable and well developed ways to calculate inertial properties of each bone segment, creates an opportunity to calculate internal reaction forces with high accuracy and reliability.

### 2.1.2 Muscle recruitment

In inverse dynamics, muscle recruitment is an algorithm that determines which set of muscle forces will satisfy the equilibrium equations. The process is necessary due to two reasons. Firstly, muscles are unilateral elements which can not push, only pull. This results in muscle forces being restricted in sign, thus only the solutions with zero or positive muscle force are treated as physiologically reasonable. Secondly, the problem of muscle redundancy. Muscle systems tend to have more muscles than is strictly necessary to satisfy the equilibrium conditions, which results in more unknowns than there is equations and therefore also infinite number of solutions [38]. Experiments have shown that the central nervous system activates muscles in a systematic way, which implies that there is some underlying selection process that determines which muscles are activated. Mathematically the muscle recruitment can be formulated as an optimisation problem according to:

\[
\begin{align*}
\min \quad & G \left( \bar{f}^{(M)} \right) \\
\text{s.t.} \quad & C \bar{f} = \bar{r} \\
& 0 \leq f_i^{(M)} \leq N_i \quad i = 1, \ldots, n^{(M)},
\end{align*}
\]  

(2.8)

where the objective function $G$ is intuitively a function of muscle force. The first constraint are equilibrium equations which means whatever solution that is found needs to satisfy the equilibrium. It is basically equation (2.1), where $\bar{f}$ is a vector containing the unknown muscle and joint forces, $\bar{r}$ represents the external and inertial forces and $C$ is the coefficient matrix. The second constraint relates to the aforementioned fact that muscles can only pull and not push, thus only solutions with zero or positive muscle force are considered. The muscle force is also limited by its maximum strength $N$ i.e. a muscle can not generate more force than it is capable of [38].
The polynomial muscle recruitment criterion is the most commonly used type of criterion. The function $G$ that defines the objective function in equation (2.8) for this type of criterion is defined as:

$$G = \sum_{i=1}^{n} \left( \frac{f_i}{N_i} \right)^p,$$

where $f_i$ is the muscle force in the muscle $i$, $N_i$ is the strength of the muscle $i$, and $p$ is the polynomial degree. The term is summed across all muscles, $i = 1, \ldots, n$. The $(f_i/N_i)$ term, also sometimes referred to as muscle activity $a$, quantifies a fraction of a muscle’s strength $N$ being utilised by the current force $f$. Therefore the muscle activity $a$ usually lies in a range of 0 to 1 \[36\]. A polynomial recruitment criterion penalises large terms in the sum, thus distributing the load more evenly across the muscles. Increasing the polynomial degree increases the synergy between the muscles. A polynomial criterion of 2-nd or 3-rd degree is said to be most suitable for explosive types of activities such as sprint, golf swings or jumping. However researchers has not been able to prove which kind of criterion is the most suitable, thus a case to case assessment is often required \[36\].

Theoretically maximum synergy is achieved by letting $p$ tend to infinity. Maximum synergy implies that every muscle capable of contributing to the satisfaction of the equilibrium equation work in such a way that maximum relative load of any muscle in the system is as small as possible. This also means that the load is distributed in such a way that the activation of all the contributing muscles is as close to equal as possible \[37\]. Physiologically this turns out to be a minimum fatigue criterion, since all muscle are equally affected throughout the motion, thus achieving maximum body endurance. AMS allows users to enable such criterion through the Min/Max-strict muscle recruitment criterion. In addition to that, Min/Max-strict offers an attractive linear relationship between muscle activation $a$ and external force, allowing for an effective calculation of model strength \[37\].

\[1\] It is possible for $a$ to go above 1 if the upper bound of muscle activation is disabled, indicating an effort beyond the muscles physiological capabilities. Disabling the upper limit for muscle activation is usually advantageous when calculating the strength of a model. In such case the upper limit for $f_i^{\text{M}}$ in the second constraint of equation (2.8) is disabled.
2.2 Muscles

The muscular system of a human body allow its movement, it is also responsible for posture stabilisation and blood circulation throughout the body. Muscles can be grouped into three distinct categories: skeletal, cardiac and smooth muscles [16].

2.2.1 Skeletal muscle anatomy and function

Skeletal muscles together with the skeletal system form the musculoskeletal system of a human body. It comprise around 600 muscles which accounts for around 40% to 50% of body weight. Skeletal muscles are categorised as voluntary muscles, controlled by the somatic nervous system. Muscle fibres of skeletal muscles are often much longer compared to other types of muscle tissue [28].

A large part of skeletal muscles work in antagonistic pairs, due to an inability of a muscle to generate a pushing force. Thus, one muscle in the pair contracts to move the body part while the other is relaxing (lengthening) and vice versa when a body part is pulled back to its original position. A muscle that contracts during the motion is called an agonist and a muscle that is lengthening is called an antagonist. Biceps and triceps, quadriceps and hamstrings are examples of antagonistic muscle pairs [5].

Muscle fibres generate mechanical tension through muscle contraction. The contractile elements of a muscle are called sarcomers. Muscles are attached to the bones via tendons, which transfer the force generated by the muscle. As the muscle contracts, the generated force pull the bone in the direction of the contraction. There are three distinct types of muscle contraction: concentric, isometric and eccentric. A concentric contraction indicates muscle shortening ($L < 0$), while an eccentric contraction implies muscle lengthening ($L > 0$). Isometric contraction indicates constant muscle length ($L = 0$). Force generating capabilities of a muscle are roughly proportional to the cross sectional area. However factors as pennation angle (angle at which muscle fibres are oriented), gender and training also affect the force generation.

2.2.2 Anatomy of a lower limb and knee joint muscles

Generally speaking, a human leg is the entire lower limb of a human, including the foot, lower leg (shank) and thigh. Sometimes it also includes the hip, however the anatomical term refers only to the section of a lower limb extending from the ankle to the knee, broadly known as the shank. The main function of the lower extremities are all forms of human locomotion and to provide support for the upper body. Mass of the lower extremities account for a significant portion of the total mass of a human. The femur, tibia and fibula comprise the major bones of the leg. Hip, knee and ankle joints connects the the bones and allow for their movement relative each other. Major muscle groups include the gluteus, quadriceps-femoris, hamstring, tibialis and fibularis muscles [16].

The knee joint is categorised as a modified hinge-type joint, which allows for extension and flexion of the lower leg as well as a slight internal and external rotation. It consists of two joints, one between the femur (thigh bone) and tibia and one between the femur and patella. Patella (knee cap) is a rounded triangular bone which primary function is to extend the leg by increasing the leverage produced by extensor muscle tendon, by changes in the angle of patella [16].
Knee flexion defines a bending motion of the lower leg towards the thigh, increasing the angle between the thigh and the shank, see figure 2.1. Knee extension defines a straightening motion of the lower leg, towards its fully extended state at 0° [40], see figure 2.1. The ROM is highly subject specific with large variation between individuals. A common range of motion (ROM) of a knee has been described as 0° of extension to 130° flexion, although hyper-extension/flexion has been frequently observed [13].

Figure 2.1: Flexion and extension of a joint. Knee flexion increases the angle between the lower leg and the thigh. Extension decreases the angle. The observed range of motion of a knee joint is 0° to 130°. The arrow indicates the direction of the motion.
The primary knee extensor muscles are the muscles belonging to the *quadriceps-femoris* group, located on the front of the anterior thigh and inserting into the patella via the quadriceps tendon. The quadriceps-femoris muscle group consist of *rectus femoris*, *vastus lateralis*, *vastus medialis* and *vastus intermedius*. With an exception of *rectus femoris* all quadriceps-femoris muscles arise from the surface of the femur. *Rectus femoris* also spans the hip joint and is thus a biarticular muscle, responsible for both extending the knee and flexing the hip. *Rectus femoris* is located in the centre of the thigh, covering most of the *vastus* muscles, *vastus lateralis* occupies the outer side of the thigh while *vastus medialis* occupies the inner side. *Vastus intermedius* lies on top of the thigh, hidden beneath *rectus femoris* [16], see figure 2.2.

![Quadriceps-femoris muscle group from the anterior point of view. Figure adapted and modified from [40], licensed under Creative Commons license.](image)

Primary knee flexor muscles are the hamstring muscle group which consists of *semitendinosus*, *semitendinosus*, *biceps femoris*-long head and *biceps femoris*-short head. Located in the posterior thigh, all muscles except for *biceps femoris*-short head span both knee and hip joint and are therefore biarticular, thus involved in both knee flexion and hip extension. *Biceps femoris* is located in the centre of the posterior thigh, *semitendinosus* and *semitendinosus* both occupy the inner side of the posterior thigh, with *semitendinosus* muscle covering most of the *semitendinosus* [16], see figure 2.3.
2.2.3 Lower Extremity Model in AMS

AMS allows its users to adopt a lower extremity model based on the desired level of accuracy. The simple leg model with its 35 muscles, hip, knee and ankle joints create a basis for smaller and uncomplicated analyses. For more accurate and sophisticated analyses, AMS allows for implementation of the Twente Lower Extremity Model (TLEM). It contains 55 muscle actuators represented by 166 Hill-type muscle elements. The model comprise eleven joints, resulting in 21 DOF [7]. The main hip, knee and ankle joints are modelled as a spherical joint and hinge joints respectively. The model is based on anatomical dataset of joint and muscle parameters obtained from a cadaver study, published by Horsman [21]. By means of different scaling laws this dataset is adapted to fit any individual. Scaling pertains overall body geometry, muscle insertion points, muscle parameters and wrapping surfaces.
TLEM implements a unique muscle structure with an aim to create a more accurate representation of the natural structure of real muscles. A muscle might be divided in sub-parts based on the differences in morphology. These sub-parts are in turn divided into sufficient number of elements, in order to accurately describe the mechanical effects [21]. TLEM frequently denotes the different muscle sub-parts as the inferior, mid and superior part of a muscle, whereas different sub-part elements are described as Par. Some muscles are only divided into Par-elements, if no morphological differences were observed. Figure 2.4 presents the graphical representation of muscle division in TLEM.

![Figure 2.4: Muscle structure and division implemented in TLEM. Muscles are either divided into sub-parts: inferior, mid or superior and subsequently into muscle elements or directly into muscle elements.](image)

Most of the quadriceps-femoris muscles follow the muscle-sub-part-element division, except for vastus intermedius and rectus femoris muscles, which are divided directly into muscle elements. All hamstring muscles consist only of muscle elements. Muscle division pertaining the muscles involved in the optimisation procedure is presented in table 3.3, in section 3.6. Each muscle element is modelled with an appropriate muscle model selected by the user, thus each element has its own unique set of MTP.
2.2. Muscles

2.2.4 3E-Hill type muscle model

Muscle models can be grouped into two separate types, structural and phenomenological. In structural models the microscopic behaviour inside the sarcomeres is taken into account. The force producing capabilities of a muscle-tendon unit in the structural models are explicitly determined by the relationship between the number of cross-bridges between myofilaments and the involvement of titin. Structural models are not practical to use in musculoskeletal simulations due to their complexity. Phenomenological models implement rheological elements such as springs and dampers to mimic the general biomechanical behaviour of a muscle tendon unit. Such models proved to be far more practical to use in musculoskeletal models [4].

Most of the musculoskeletal models assume a generic, phenomenological 3E-Hill-type muscle model which can be customised in order to fit any individual. Such a muscle model consists of a contractile element (CE) in addition to serial elastic element (SEE) and parallel elastic element (PEE). CE represent the contractile properties of a muscle, PEE represents the elastic properties of the surrounding tissue and SEE represents the elastic properties of a tendon. SEE and PEE are modelled as non linear springs. The pennation angle \( \alpha \) represents the orientation of the muscle fibres with respect to the tendon. Pennation angle influences force generation by allowing larger amount of fibres per cross-sectional area [17]. Figure 2.5 illustrates a 3E-Hill-type muscle model together with all of its components.

![Diagram of 3E-Hill-type muscle model](image)

Figure 2.5: 3E-Hill-type muscle model: contractile element CE, serial elastic element SEE, parallel elastic element PEE, pennation angle \( \alpha \), width \( w \) and length \( L_M \) of CE, length of SEE \( L_T \) and the length of entire muscle-tendon unit \( L_{MT} \).

Muscle force is assumed to be a sum of active and passive muscle force, see equation (2.12). Active force is generated when a muscle is active, meanwhile passive force is associated with a resistive force, generated when a muscle is stretched beyond its optimal length. Hill-type muscle model assume the beginning of the passive force generation exactly at the optimal fibre length, in reality however, this point is slightly translated to the right of the optimum. In the Hill-type muscle model, active-isometric force generation is characterised by \( f - l \) relation of the contractile element CE, see figure 2.6. The force-strain relation of PEE characterises passive force generation of the muscle [17].
2.2. Muscles

Figure 2.6: Normalised $f - l$ curve of the CE in a Hill-type muscle model. Length of the muscle is normalised with respect to optimal fibre length. Muscle force is normalised by peak isometric muscle force $F_{om}$.

As reported by Zajac, the range of normalised muscle length $\tilde{L}_m$ at which muscles develop active force is $0.5\tilde{L}_m$ to $1.5\tilde{L}_m$ [44]. Maximum isometric force is generated when the length of a muscle is equal to the optimal fibre length $L_f^o$. In AMS the $f - l$ relation of CE is controlled by equation (2.10), based on the work done by Otten [29], Daxner [35], and Gloehler et al. [27].

$$\tilde{F}_{CE}(L_{CE}) = \exp \left( - \frac{L_{CE} - L_f^o}{\omega_F L_f^o} \right) \rho_F, \quad \text{where:} \quad \epsilon_F = \frac{L_{CE} - L_f^o}{L_f^o}$$

where $L_{CE}$ is the length of the contractile element ($L_m$) and $L_f^o$ is the optimal fibre length. The parameters $\beta_F, \omega_F$ and $\rho_F$ control roundness, skewness and width of the curve. They are set to values presented by Kaufman et al. [23]. $\tilde{F}_{CE}$ is normalised by the peak isometric muscle force $F_{om}$, thus vary in the range of $0 \leq \tilde{F}_{CE} \leq 1$. 

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The force produced in the muscles is transmitted to the bone via tendons. In the Hill-type model the tendon assumes a non-linear elastic behaviour, characterised by a generic force-strain relationship of the SEE [44], see figure 2.7.

![Figure 2.7: Normalised $f - \varepsilon$ curve of the SEE in a Hill-type muscle model. $L_s$ is the tendon slack length, which is the length at which tendons begin to generate resistive force and $F_{om}$ is the peak isometric muscle force. Strain of 3.3% at $F_t = 1$ is not necessarily true for all muscle-tendon actuators but was suggested as a reasonable assumption by Zajac [44].](image)

The curve is characterised by a non-linear behaviour for low values of the strain, a so called toe-region, and a linear region beyond the initial strain, see figure 2.7 [44]. This indicates an increasing slope of tangent modulus of elasticity $E$ at low strains, which then remains constant in the linear region. Based on experimental data, Zajac suggests $E$ of 1.2 GPa (range: 0.6 to 1.7 GPa) in the linear region and the beginning of linear region at 2% (range: 1.5 to 4%) [44]. Furthermore, Zajac also suggests tendon failure at approximately 10% strain. Strain at which a tendon produces a force equal to maximum muscle force $F_{om}$ varies between different muscle-tendon actuators (range: 2 to 9%), Zajac however, proposes a strain of 3.3% as a reasonable assumption for most of the muscles. A general force-strain relation of a tendon (SEE) can be defined according to the following equation:

$$\tilde{F}_t = \frac{F_t}{F_{om}} = \begin{cases} \frac{A \varepsilon_t^2}{B + C \varepsilon_t} & 0 \leq \varepsilon_t < \varepsilon_{lin} \\ \varepsilon_t \geq \varepsilon_{lin} \end{cases}, \text{ where: } \varepsilon_t = \frac{L_t - L_s}{L_t}$$

(2.11)

where parameters $A, B$ and $C$ are material constants concerning the elastic properties of a tendon. $\varepsilon_{lin}$ corresponds to the strain at which the linear region begins. $L_s$ is tendon slack length, which corresponds to the tendon length at which the tendon starts to generate a resistive force. $\varepsilon_t$ is tendon strain and $L_t$ is tendon length. Tendon force $\tilde{F}_t$ is normalised by peak isometric muscle force $F_{om}$. The tendon has to transfer both the active and passive muscle forces, thus the tendon force $\tilde{F}_t$ may reach values beyond 1.
The governing force equation of the 3E-Hill type muscle model is shown in equation (2.12). For clarity, the dynamic properties are also included [20].

\[
\tilde{F}_t(L_t(L_{CE}(t))) = \begin{bmatrix}
\tilde{F}_{PE}(L_{CE}(t)) + a(t) \cdot \tilde{F}_{CE}(L_{CE}(t), L_{CE}(t)) \\
\tilde{F}_{PE}(L_{CE}(t)) + a(t) \cdot \tilde{F}_{CE}(L_{CE}(t))
\end{bmatrix} \cos \alpha.
\]

(2.12)

For the isometric case, assuming negligible tendon strain, equation (2.12) reduces to:

\[
\tilde{F}_t(L_t(L_{CE}(t))) = \begin{bmatrix}
\tilde{F}_{PE}(L_{CE}(t)) + a(t) \cdot \tilde{F}_{CE}(L_{CE}(t))
\end{bmatrix} \cos \alpha.
\]

(2.13)

where \(a(t)\) is the activation level of a muscle, defined as dimensionless factor \(a = \tilde{F}_{CE}/N\), \(N\) being current strength of the muscle and \(\alpha\) is the pennation angle. Isometric strength \(N\) is defined as:

\[
N = F^0_m \left( \frac{L_{CE}^2}{L_f^2} - 1 \right),
\]

(2.14)

where \(F^0_m\) is the maximum isometric force a muscle can generate, \(L_{CE}\) is the length of the contractile element and \(L_f^2\) is the optimal fibre length.

To scale the muscle model to represent specific muscle-tendon-actuators one need to provide four parameters for each muscle: peak isometric force \(F^0_m\), optimal fibre length \(L_f^2\), tendon slack length \(L_s^t\) and the pennation angle \(\alpha\). In AMS, \(F^0_m\) is defined as: \(F^0_m = K \cdot PCSA = K \cdot Vol/L_f^2\), where \(K\) is the specific tension of a muscle and \(Vol\) is muscle volume. Muscle volume is calculated automatically by AMS and specific tension is assumed to be constant and known for each muscle. Due to this, the number of necessary parameters to estimate is reduced to three. Furthermore, in the case of lower extremity muscles, the pennation angle is often very small and thus neglected, reducing the number of parameters necessary to only two. In AMS, the constants defining force-length and force-strain relations, in equations (2.10) and (2.11) do not need to be provided by the user. However, it is possible to manually adjust the shape of force-strain curves of the SEE and PEE by changing material constants \(f_l\) and \(f_p\) inside the muscle model definitions. In addition to that, the user can manually define the tendon strain at which the tendon produces force equal to maximum muscle force, corresponding to \(\varepsilon_{lin}\) in equation (2.11), via the parameter \(\varepsilon_0\). Changes in \(f_l, f_p\) and \(\varepsilon_0\) are seldom necessary, due to small differences between the individuals and well established methods of estimation of the default values.
2.3 Representing musculo-tendon parameters

Equations derived in this section are based on the work done by Garner and Pandy in [15].

Studying the muscle-tendon parameters \( F_m^o, L_s^o \) and \( L_f^o \) it can be concluded that they all relate to each other in some way. The peak isometric force is related to the optimal fibre length \( (L_f^o) \) through its definition, \( F_m^o = K \cdot PCSA = K \cdot Vol / L_f^o \). In the equation \( F_m^o \) is the peak isometric force, \( K \) is the specific tension of the muscle (maximum muscle stress), PCSA is the physiological cross section area and \( Vol \) is the muscle volume. Tendon slack length \( (L_s^o) \) and optimal fibre length \( (L_f^o) \) are also related. Due to the force-length relationship, there is a limited range of fibre lengths over which a muscle can generate active force, most often referred to as an operating range of a muscle. Muscles with long tendons and short muscle-fibres will have smaller operating ranges and vice versa. A common denominator that relates all of the parameters is needed in order to model these relationships.

Brand et al. investigated the relationship between the operating range and the required excursion of a muscle [6]. They defined excursion of a muscle as the difference between the maximum physiological length \( (L_{MT}^{max}) \) and minimum physiological length \( (L_{MT}^{min}) \) of a muscle-tendon actuator. These extreme lengths correspond to the lengths of an MTU when the joint is moved through its entire range of motion. For example, \( L_{MT}^{max} \) for quadriceps-femoris muscles corresponds to the muscle-tendon length at 0°, while \( L_{MT}^{min} \) is the length at 130° knee flexion. Assuming that a muscle can generate an active force over the entire range of motion, would imply that the optimal fibre length is related to muscle excursion. Figures 2.8 and 2.9 illustrate two cases of a muscle-tendon unit. First with a long tendon and short muscle and second with a short tendon and long muscle.

![Figure 2.8](image-url)

**Figure 2.8:** A muscle-tendon unit with a long tendon and short muscle. The relationship between optimal muscle-fibre length \( L_f^o \), tendon slack length \( L_s^o \), maximum and minimum lengths of a muscle \( (L_{MT}^{min}, L_{MT}^{max}) \) and muscle-tendon \( (L_{MT}^{min}, L_{MT}^{max}) \) unit respectively can also be observed. For simplification the pennation angle \( \alpha \) is not included. Figure adapted and redrawn from "Estimation of Musculotendon Properties in the Human Upper Limb" by Garner and Pandy [15].
2.3. Representing musculo-tendon parameters

Figure 2.9: A muscle-tendon unit with a short tendon and long muscle. The relationship between optimal muscle-fibre length $L^o_f$, tendon slack length $L^s_t$, maximum and minimum lengths of a muscle ($L^{\text{min}}_m$, $L^{\text{max}}_m$) and muscle-tendon ($L^{\text{min}}_{MT}$, $L^{\text{max}}_{MT}$) unit respectively can also be observed. For simplification the pennation angle $\alpha$ is included. Figure adapted and redrawn from "Estimation of Musculotendon Properties in the Human Upper Limb" by Garner and Pandy [15].

Assuming negligible tendon strain compared to muscle strain, indicates that entire muscle excursion can be attributed to changes in muscle length. Muscle-tendon actuators with long tendons and short muscles have relatively small muscle excursion, implying small changes in muscle length and consequently a small operating range and relatively low values for $L^o_f$, see figure 2.8. Conversely, for muscles with high muscle excursion, the operating range is larger, thus one might expect relatively high values for $L^o_f$, see figure 2.9. Tendon slack length $L^s_t$ relates to optimal fibre length and excursion via the definition of muscle-tendon length $L_{MT}$, corrected for pennation angle $\alpha$, $L_{MT} = L_T + \cos \alpha L_M$, where $L_T$ is tendon length and $L_M$ is muscle length. Tendon length will therefore affect the lengths of $L^{\text{min}}_{MT}$ and $L^{\text{max}}_{MT}$, and consequently also the muscle excursion. Relatively high values for both $L^{\text{min}}_{MT}$ and $L^{\text{max}}_{MT}$ imply small excursion which yields high values for tendon length $L_T$ and consequently also high values for $L^o_f$, see figure 2.8 for graphical interpretation. Conversely, low values for $L^{\text{min}}_{MT}$ relative $L^{\text{max}}_{MT}$ yield low values for $L^o_f$.

Garner and Pandy chose to express $L^o_f$ and $L^s_t$ in terms of two normalised variables $\tilde{L}^{\text{max}}_m$ and $\tilde{L}^{\text{min}}_m$, which represent maximum and minimal physiological muscle fibre lengths respectively, both normalised by $L^o_f$. Quantities of these values will establish the operating range of a muscle and will be used as design variables during the optimisation procedure. The derivation of the formulas connecting $L^o_f$ and $L^s_t$ to $\tilde{L}^{\text{max}}_m$ and $\tilde{L}^{\text{min}}_m$ will be presented below.
2.3. Representing musculo-tendon parameters

From figures 2.8 and 2.9 above and accounting for pennation angle, it can be seen that:

\[ L_{MT}^{\min} = L_s + \cos \alpha L_{mt}^{\min}, \]  

(2.15)

\[ L_{MT}^{\max} = L_s + \cos \alpha L_{mt}^{\max}, \]  

(2.16)

where \( \alpha \) is the pennation angle of a muscle. It is assumed that the tendon strain is negligible, thus the all the variations in muscle-tendon length \( L_{MT} \) can be attributed to change in muscle length. Cosinus of the pennation angle \( \alpha \) can be expressed in terms of optimal fibre length \( L_f^o \) and pennation angle at the peak isometric force \( \alpha_o \). For this, consider two cases, one of a muscle with an optimal length \( L_m = L_f^o \), A in figure 2.10 and one of a muscle during contraction with an arbitrary length \( L_m \), B in figure 2.10. It is assumed that all the fibres in a muscle are parallel and of equal lengths and at an angle \( \alpha \) relative to the tendon.

![Figure 2.10: Relationship between optimal fibre length \( L_f^o \), muscle length \( L_m \) and pennation angle \( \alpha \). Muscle length in case A is equal to optimal fibre length, \( L_m = L_f^o \) and pennation angle is equal to pennation angle at optimal fibre length \( \alpha_o \). Case B illustrates a muscle of an arbitrary length \( L_m \) with the current pennation angle \( \alpha \). Figure adapted and redrawn from “Estimation of Musculotendon Properties in the Human Upper Limb” by Garner and Pandy [15].](image)

The width \( w \) of a muscle in the case A is:

\[ w = L_f^o \sin \alpha_o, \]  

(2.17)

where \( \alpha_o \) is the pennation angle at peak isometric force. During contraction, length of muscle fibres decreases and the pennation angle \( \alpha \) increases. However, the width \( w \) of a muscle remains constant throughout the contraction [15], thus:

\[ w = L_m \sin \alpha. \]  

(2.18)

Combining equations (2.17) and (2.18) gives:

\[ \sin \alpha = \frac{L_f^o \sin \alpha_o}{L_m}. \]  

(2.19)

Trigonometric identity together with equation (2.19) gives:

\[ \cos \alpha = \sqrt{1 - \left( \frac{L_f^o \sin \alpha_o}{L_m} \right)^2}. \]  

(2.20)
Rewriting equation (2.20) gives:

\[
\cos \alpha = \sqrt{\left( \frac{L_f}{L_m \cdot L_f} \right)^2 - \left( \frac{L_f \sin \alpha_o}{L_m} \right)^2} = \sqrt{\left( \frac{L_f}{L_m} \right)^2 \left( \left( \frac{L_m}{L_f} \right)^2 \right) - \sin^2 \alpha_o} = \frac{L_f}{L_m} \sqrt{(L_m)^2 - \sin^2 \alpha_o}, \tag{2.21}
\]

where \( \bar{L}_m \) is the aforementioned normalised muscle length. Substituting \( \bar{L}_m \) for \( \bar{L}_{\min} \) and \( \bar{L}_{\max} \), \( L_m \) for \( L_{\min} \) and \( L_{\max} \) in equation (2.21), the equations (2.15) and (2.16) become:

\[
L_{\min}^{\text{MT}} = L_i^f + \cos \alpha L_{\min}^m = L_i^f + L_f \sqrt{(L_{\min}^m)^2 - \sin^2 \alpha_o} = L_i^f + L_f P_{\min}, \tag{2.22}
\]

\[
L_{\max}^{\text{MT}} = L_i^f + \cos \alpha L_{\max}^m = L_i^f + L_f \sqrt{(L_{\max}^m)^2 - \sin^2 \alpha_o} = L_i^f + L_f P_{\max}, \tag{2.23}
\]

where \( P_{\min} \) and \( P_{\max} \) were introduced for convenience. Solving equations (2.22) and (2.23) for \( L_i^f \) and \( L_i^t \) yields the following:

\[
L_i^f = \frac{(L_{\max}^{\text{MT}} - L_{\min}^{\text{MT}})}{(P_{\max} - P_{\min})}, \tag{2.24}
\]

\[
L_i^t = \frac{P_{\min} L_{\min}^{\text{MT}} - P_{\min} L_{\max}^{\text{MT}}}{(P_{\max} - P_{\min})},
\]

where \( P_{\max} \) and \( P_{\min} \) are:

\[
P_{\max} = \sqrt{(L_{\max}^m)^2 - \sin^2 \alpha_o}
\]

\[
P_{\min} = \sqrt{(L_{\min}^m)^2 - \sin^2 \alpha_o}. \tag{2.25}
\]

The unknown values of \( L_{\min}^{\text{MT}} \) and \( L_{\max}^{\text{MT}} \) can be obtained by using a musculoskeletal model. The transformation equations (2.24) and (2.25) are particularly useful during the optimisation process. Because one might expect the values of \( L_{\min}^m \) and \( L_{\max}^m \) to some degree, be similar for all the muscles in a body. As reported by Zajac, \( L_{\min}^m \) should lie in a vicinity of 0.5 and \( L_{\max}^m \) in close proximity to 1.5 [44]. This allows for good initial guesses for the optimisation solver. The optimal pennation angle \( \alpha_o \) in equation (2.25) may be disregarded for quadriceps-femoris and hamstring muscles due to negligible pennation angle \( \alpha \).
2.4 Estimation of musculo-tendon parameters

This section aims to introduce the reader to the fundamental concepts of the estimation method proposed by Heinen et al. in [18].

2.4.1 Maximum isometric torque-angle profile

There exist several ways to express voluntary muscle strength of an individual, this includes maximum weight lifted, angle-specific maximum voluntary isometric torque, non-angle-specific torque etc. Angle specific maximum isometric torque profiles are extensively used in sports for muscle strength measurements of elite athletes [3]. They are easily obtainable and highly subject specific. The existing methods are well established, reliable and provide an isolation of the muscle groups during experimental trials [2].

Angle-specific maximum voluntary isometric torque profile describes the relationship between joint angles and the net maximum voluntary torque that muscles can produce about a particular joint. An isometric torque means static conditions. It is important to point out that the net torque is the sum of all muscle torques contributing to the net torque, not just one single muscle. Therefore, the results should be viewed as strength of the entire muscle group rather than of any particular muscle. An example of the maximum isometric torque profile for the quadriceps femoris muscles during knee extension is presented in the figure 2.11 below. Data is based on the study conducted by Pincivero. et al [30].

Figure 2.11: Average maximum knee extensor torque as a function of knee joint angle for 14 active and healthy males. Redrawn and adapted from Angle- and gender-specific quadriceps femoris muscle recruitment and knee extensor torque by Pincivero. et al [30]. The results were obtained from 14, physically active and healthy males (mean age: 25±4 years, mean height: 1.78±8 cm, mean weight: 79±11.8 kg) Below the graph are the knee joint positions corresponding to a knee angle of 0° (fully extended), 45° and 90° respectively.

Maximum voluntary torque is commonly described as maximum voluntary contraction (MVC).
Figure 2.11 shows a number of important properties of the quadriceps-femoris muscles. The graph indicates that the maximum voluntary torque for the quadriceps-femoris muscles, of this test group, peaks at the knee angle of 70°. In fact, every muscle group in the entire body has an optimal joint angle at which they can produce maximum amount of force, although the angle can vary between individuals. Optimal joint angle relates to the length of a muscle at which it coincides with optimal fibre length. An apparent example of this is the biceps curl exercise. It is hardest to lift the dumbbell when the arm is fully stretched, but the exercise gets much easier when the arm is slightly bent. Such behaviour is a direct consequence of the $f-l$ relationship of the muscles and changes in the lever arm. Muscles will gain strength as they contract, until they reach their optimal fibre length $L_{o}^{f}$, after which the strength decreases. Such behaviour can be observed directly in the plot where maximum torque increases between 0° and 70° and decreases beyond 70°. Notice the broad range of measured maximum torques for angles $\geq 10^\circ$, indicating large differences between the participants.

The lever arm of the muscles is also an important factor to consider while interpreting the results of any torque profile curve. A natural interpretation of the torque profile curve in figure 2.11 would suggest that the lever arms of the muscle group also peak at 70°. However, maximal lever arm does not need to occur at the same angle. This can be explained by studying the moment equation, which for the sake of simplicity is reduced to a 2D case. Recall the moment equation:

$$M = F \cdot d,$$

(2.26)

where $M$ is the moment, $F$ and $d$ is the force and the lever arm, respectively. For a group of muscles, each muscle $i$ has its own force and lever arm contribution, therefore the total torque $M_{tot}$ is:

$$M_{tot} = \sum F_i \cdot d_i.$$

(2.27)

It is quite obvious that $M_{torque}$ increases if both terms increase. However, it is also possible for the moment to increase even though the lever arm is decreasing. If the force output $F_i$ of a muscle increases with varying joint angle, then the total torque $M_{tot}$ could still increase if the increase in muscle force is much greater than the decrease in lever arm $d_i$. All in all, one should be aware of the importance of the lever arms in the calculation of maximal torque, but at the same time, be very careful while drawing any sophisticated conclusions about their quantities.
2.4. Estimation of musculo-tendon parameters

2.4.2 Optimisation procedure for the isometric trials

The main idea behind the optimisation procedure is to match the strength of the musculoskeletal model to the strength of the individual it is based on. Mathematically speaking, it can be seen as minimising the difference between experimentally obtained isometric torque-angle profile and model-predicted torque-angle profile. This is done by modifying the musculo-tendon parameters of the model such that the \( f - l \) relation of the model matches the subjects. The modelled torque-angle profile is obtained by creating musculoskeletal models replicating the experimental trials. This study will use a slightly modified version of the optimisation procedure proposed by Heinen et. al [18]. In this master’s thesis the slack variables were excluded and a different set of constraints was implemented. The implemented mathematical optimisation problem is defined in the equation below.

\[
\min J_{iso}(\vec{x}) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{T_{mod}^i - T_{exp}^i}{T_{exp}^{\max}} \right)^2 \\
\text{s.t.} \quad L_{s}^j \geq 0 \quad j = 1, \ldots, m \\
\bar{L}_{max}^m - \bar{L}_{min}^m \geq 0.1 \quad k = 1, \ldots, o
\] (2.28)

The vector \( \vec{x} \) is a vector of the design variables. The objective function was defined as the root-mean-square-deviation (RMSD), which measured the difference between values predicted by the model and the experimental values. Defining the objective function as RMSD will not only minimise the error but also even out the error distribution. The error terms were summed across the number of different knee angle configurations and divided by the number of trials, \( n \). The squared error is normalised by the maximum torque \( T_{exp}^{\max} \). Normalising the error is beneficial in terms of convenience and comparability. Normalisation also increases the numerical stability of the optimisation process.

As mentioned in section 2.2.3 in AMS some of the muscles are divided into sub-part, which in turn are divided into smaller elements, denoted as “Par”. For example, the vastus lateralis muscle is divided into two sub-parts, vastus lateralis inferior and vastus lateralis superior. Furthermore, vastus lateralis inferior and superior are divided into 6 and 2 Pars respectively. Each Par is modelled with a 3E-Hill type muscle model, thus each Par has its own unique set of MTP that needs to be established. It is important to keep these distinctions in mind while prescribing the design variables.

In this thesis, each sub-part of a muscle has prescribed design variables \( \bar{L}_{max}^m, \bar{L}_{min}^m \). This choice was based on the assumption that the operating range should vary between different sub-parts but not between different Pars. Another reason are the differences in MTP (notably in \( L_f^j \)) between the muscle sub-parts, described and implemented by Horsman inside the generic TLEM model [21]. The relation between \( \bar{L}_{max}^m, \bar{L}_{min}^m \) and MTP \( L_f^s, L_f^p \) was described in the previous section and defined in equation (2.24). \( F_{local} \) is a factor responsible for scaling the maximum force output \( F_m^o \) of a muscle. Physiologically it can be interpreted as a muscle-size scaling factor, which increases the cross-section area of a muscle, thus increasing its maximum force output. \( F_{local} \) is defined in equation below.

\[
F_m^o = K \cdot F_{local} \cdot PCSA,
\] (2.29)

where \( F_m^o \) is the peak isometric force, \( K \) is the specific tension of the muscle (maximum muscle stress) and PCSA is the physiological cross section area. Contrary to \( \bar{L}_{max}^m \) and \( \bar{L}_{min}^m \), \( F_{local} \) was prescribed to each muscle under the notion that an increase or decrease in size of an entire muscle is physiologically more realistic than changes in size of its different sub-parts.
This yields the vector of the design variables $x$ defined as:

$$
\bar{x} = \begin{bmatrix}
\hat{L}_{\text{max},1} \\
\vdots \\
\hat{L}_{\text{max},k} \\
\hat{L}_{\text{min},1} \\
\vdots \\
\hat{L}_{\text{min},k} \\
F_{\text{local},1} \\
\vdots \\
F_{\text{local},r}
\end{bmatrix},
$$

(2.30)

where $\hat{L}_{\text{max},m}$, $\hat{L}_{\text{min},m}$ are the maximum and minimum muscle fibre lengths defined in section 2.3. $F_{\text{local}}$ is the aforementioned local strength scaling factor. $k$ is the number of the muscle sub-parts involved in the optimisation and $r$ is the number of muscles included in the optimisation.

The first linear constraint prevent non-physiological tendon slack lengths by restricting tendon slack length to be greater or equal to 0. The constraint applies to each tendon slack length of a Par, $m$. The second linear constraint guarantees non-negative fibre lengths by constraining $\hat{L}_{\text{max},m}$ to always be greater than $\hat{L}_{\text{min},m}$. This constraint applies to all muscle sub-parts involved in the optimisation process, $o$. 

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2.4. Estimation of musculo-tendon parameters

For the sake of clarity, consider a simple example of the optimisation process. Consider quadriceps-femurs muscles during knee extension. The experimental isometric torque-angle and the model predicted profiles can be seen in figure [2.12]. The total amount of static trials is seven and for the sake of simplicity assume only two muscles, divided into two sub-parts (inferior and superior), \( a \). Each sub-part consists of four muscle-elements, \( P_r, m \). This yields ten design variables, four pairs of \( \tilde{L}_{m}^{\text{max}} \) and \( \tilde{L}_{m}^{\text{min}} \) for each sub-part and two \( F_{\text{local}} \), one for each muscle. The optimisation problem according to equation (2.28) becomes:

\[
\begin{align*}
\min \ J_{\text{iso}}(x) &= \sqrt{\frac{1}{7} \sum_{i=1}^{7} \left( \frac{T_{\text{mod}}^{i} - T_{\text{exp}}^{i}}{T_{\text{max}}^{i}} \right)^{2}} \\
\text{s.t.} \quad &L_{i,j}^{s} \geq 0 \quad j = 1, \ldots, 16 \\
&\tilde{L}_{m}^{\text{max},k} - \tilde{L}_{m}^{\text{min},k} = 0.1 \geq 0 \quad k = 1, \ldots, 4.
\end{align*}
\]

(2.31)

Figure 2.12: Experimental maximum isometric torque-angle profile, together with the model prediction.

It is clear from the figure above that the model underestimates the strength of the muscles and the peak of the model predicted curve is translated to an angle of \( 45^\circ \). The difference \( J_{\text{isom}} \) would potentially lie between 0.3 and 0.6. What the optimisation procedure want to achieve is to translate model predicted curve to the right and upwards, as close as possible to the experimental curve. In terms of physiology, the optimiser will adapt the operating ranges and muscle strength of the model to match the individual. Changes in operating ranges \( \tilde{L}_{m}^{\text{max}} \) and \( \tilde{L}_{m}^{\text{min}} \) will result in a change of the peak angle as well as the slope of the curve. Changes in \( F_{\text{local}} \) will translate the curve upwards, simply by making the muscles stronger. The results of the optimisation might potentially look like as in figure [2.13].
The results would yield $J_{\text{iso}}$ in range between 0.05 to 0.15. Calculated MTP, $L^t$ and $L^o$ are later used as inputs to other models.
2.4.3 Flowchart of the estimation process

Flowchart showing the main phases of the parameter estimation method is shown in the figure below.

The estimation procedure can be summarised to following steps:

1. anthropometric measurements on the participant are performed and maximum voluntary torque is measured during experimental trials for any number of joint angles;
2. a model replicating the experimental trials is created based on a generic human body that is scaled according to measured anthropometrics;
3. joint angles are used as inputs into the model and experimental torques are used as inputs for the optimisation process;
4. optimisation process begins with an initial guess for the design variables and MTP are calculated according to equation (2.24) and provided to the model;
5. model calculates current strength and provides the optimisation with $T_{mod}$;
6. normalised square root difference is calculated and optimisation calculates new values for the design variables;
7. if the error was reduced, the loop starts again, with new values for the design variables and MTP;
8. if the error was not reduced and the optimisation process converged, the process is terminated and the obtained MTP evaluated.
3 Method
3.1 Experimental methodology for extensor/flexor muscles

Three physically active males (mean height 1.82 m, mean weight 82.7 kg) were included in this study. Anthropometrical measurements of the participants were performed. The lengths of shank and thigh were measured in accordance with the general guidelines. Measurements involved only the left leg, assuming similar strength for both legs. The experimental isometric torque profiles of knee flexors and extensors were obtained using an isokinetic dynamometer (IsoMed2000). Before the test sessions, the device was calibrated according to the manufacturer's specifications. The participants were firmly secured by straps around the hips, shoulders and thigh of the measured leg. This prevented any unnecessary movement and allowed for an isolation of the measured muscles. The distal shin pad of the dynamometer lever arm was attached to the shank segment approximately 5 cm above the lateral malleolus, by means of a strap. The lever arm was adjusted to the desired knee joint angles. Anatomical axis of rotation of the knee, at the lateral femoral epicondyle, was matched with the mechanical axis of the dynamometer via a laser pointer. The weight of the tested leg was measured in a fully extended and relaxed state, and was used by the integrated software to account for gravity. Prior to the strength measurements, the participants performed a number of sub-maximal trials as a specific warm-up and to get acquainted with the test-rig. During the isometric test of the extensor muscles, the participants were asked to push as much as possible against the shin pad for approximately three seconds. For the isometric test of the flexor muscles, the participants were asked to pull as much as possible. Six evenly spaced isometric tests were performed throughout the participants range of motion, range: 12°/15° to 90° knee flexion. The participants were offered resting periods between the trials. Verbal encouragement, as well as visual feedback of the test results was provided to ensure maximum effort.
3.2 Data Processing and results of the trials

The obtained torque data was filtered using a software specific, fourth order Butterworth low-pass filter; with the cutoff frequency of 200 Hz. Table 3.1 provides information about the anthropometric measurements of the participants’ bodies. It includes the lengths of shank and thigh segments together with the total height, weight and resulting BMI. Only the left leg was measured.

Table 3.1: Anthropometric measurements of the participants involved in the isometric trials. All measurements are expressed in terms of base SI-units, m, kg and kg/m².

<table>
<thead>
<tr>
<th>Anthropometric Measurement</th>
<th>Subject 1</th>
<th>Subject 2</th>
<th>Subject 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left Shank Length</td>
<td>0.375</td>
<td>0.406</td>
<td>0.397</td>
</tr>
<tr>
<td>Left Thigh Length</td>
<td>0.421</td>
<td>0.470</td>
<td>0.430</td>
</tr>
<tr>
<td>Total Height</td>
<td>1.835</td>
<td>1.840</td>
<td>1.785</td>
</tr>
<tr>
<td>Weight</td>
<td>78.50</td>
<td>82.0</td>
<td>88.0</td>
</tr>
<tr>
<td>BMI</td>
<td>23.29</td>
<td>24.22</td>
<td>27.62</td>
</tr>
</tbody>
</table>

The results of the isometric strength trials for knee extension and flexion are presented in table 3.2.

Table 3.2: MVC obtained during the isometric strength tests of knee extension and flexion for different knee joint angle configurations. All measurements are expressed in terms of base SI-units, degrees and Nm.

<table>
<thead>
<tr>
<th>Knee Joint Angle [°]</th>
<th>MVC Knee Extension [Nm]</th>
<th>MVC Knee Flexion [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subject 1</td>
<td>Subject 2</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>79</td>
</tr>
<tr>
<td>30</td>
<td>140</td>
<td>122</td>
</tr>
<tr>
<td>45</td>
<td>208</td>
<td>178</td>
</tr>
<tr>
<td>60</td>
<td>308</td>
<td>198</td>
</tr>
<tr>
<td>75</td>
<td>287</td>
<td>178</td>
</tr>
</tbody>
</table>

3.3 Musculoskeletal Model Setup

3.3.1 Human model

The musculoskeletal model was based on the generic, standing human model available in the AMMR repository. The lower extremity model was based on TLEMsafe 2.0 data set. Geometrical scaling of the skeletal structures was based on external anthropometric measurements (see table 3.1) and the length-mass-fat scaling law. Arms were excluded, with an intention to simplify the model as much as possible. Only the muscles of the left leg were modelled with the 3E-Hill type muscle model. The specific tension \( K \) was reduced from the default value of 90 to 27 N/m².
3.3.2 Initial position and kinematic constraints

The model was set to an initial, sitting position (Figure 3.1) by setting hip flexion, knee flexion and ankle plantar flexion to $90^\circ$, $90^\circ$ and $15^\circ$ respectively. Remaining joint angles were left unchanged.

![Figure 3.1: Sitting position of the model. Hip flexion was set to $90^\circ$, knee flexion was set to $90^\circ$ and ankle plantar flexion was set to $15^\circ$. Rest of the joint angles were left unchanged.](image)

The model was fixed to the ground via the pelvis. During analysis, the positions of the body segments were driven by kinematic constraints (drivers) for the corresponding joints. It was assumed that the upper body, thigh and foot remained still during the experiment, thus all the DOF of the ankle, hip, pelvis, thorax and neck joints were driven to the same position and joint angle orientations as for the initial position. An exception is the knee joint, which was driven to a knee angle matching the experiment. A kinematic constraint is always accompanied by an unknown and general reaction "force" to sustain the motion/position. In AMS the reaction force can either be an actual force or torque depending on the type of motion that is driven. In the case of knee joint, AMS applies a reaction torque in order to hold the knee in a desired position. This is undesirable for this model, since the knee flexion/extension should be caused entirely by the muscles. This was fixed by turning the default reaction to "off" for the driver of the knee joint. The rest of the drivers did not pose the same problem, as it was reasonable to assume negligible muscle activity of other muscles, thus the reaction was kept at the default option (on). The model can hold its own weight by incorporating gravity forces into constraint reaction forces, which basically mean that the model is floating in the air, held in the air by the pelvic joint. This however pose no real problem for this study since neither joint reaction forces or support forces from the seat were of any interest. The muscle activation patterns during the analysis won't be affected. Also, this model is numerically more stable and simpler than a fully fledged model that includes modelling of the supporting elements.

3.3.3 Muscle Recruitment

In general, it is difficult to find a consensus among the researchers on the topic of selection of a suitable muscle recruitment criterion. There is a lack of knowledge about the function of the central nervous system when controlling muscle activation patterns during different types of motion. In this thesis a min/max recruitment criterion was implemented. The criterion provides an attractive linear relationship between muscle activation and external load.
This proportionality has been shown in a number of EMG based experiments [1, 10]. This linear relation simplifies model strength calculations required to obtain $T^{mod}$ in section 3.4. It would be more difficult to calculate $T^{mod}$ if the polynomial criterion of second or third degree was implemented, due to non-linear relation between muscle activation and external load. Although the min/max criterion is a muscle fatigue oriented recruitment algorithm, more suitable for analysis of body endurance under extended periods of physical activity, its linear properties proved to be crucial for an efficient implementation of the method proposed in this master's thesis. It is also difficult to know for sure the muscle activation patterns during a short-termed maximal effort without EMG based measurements, which in themselves are considered to be somewhat unreliable.

3.4 Model-strength calculation

In AMS, strength is defined as the maximum load a model can carry without exceeding a muscle activation of 1. Strength can be calculated for any given posture and motion. The basis for all strength calculations is the linear relation between applied load and activation level of a muscle. This linear relation can be proved with a simple parameter study. Consider a fully extended leg (angle between shank and thigh is 0°), under an influence of a varying (range: 100 to 200 N) knee extending load, see figure 3.2 and 3.3 below. Figure 3.3 shows activity of quadriceps-femoris muscles as a function of applied load. Due to min/max recruitment criterion, see section 2.1.2, the activity of all muscles of the quadriceps-femoris group is equal.

![Figure 3.2: Model subjected to an extending load. The load is said to be extending, when it generates a torque about positive z-axis.](image)

![Figure 3.3: Muscle activity of quadriceps-femoris as a function of applied load. Muscle activity beyond 1 is regarded as non-physical.](image)

The linear relation can be used to calculate a force at which muscle activity is exactly 1, which implies the maximum external force $F_{max}$ muscles can withstand. Defining the linear function as:

$$a = kx + f_0,$$

where $a$ is the muscle activity, $k$ is the slope of the function and $f_0$ is the intercept with y-axis. The slope $k$ of the function is determined via equation (3.2) together with two arbitrary points (denoted 1,2) located on the curve,

$$k = \frac{\Delta a}{\Delta x} = \frac{a_2 - a_1}{F_2 - F_1}.$$
With the slope $k$ known, determining the maximum force $F_{\text{max}}$ is a matter of substituting $a_2 = 1$ and $F_2 = F_{\text{max}}$. Reformulating equation (3.2) and solving for $F_{\text{max}}$ gives:

$$k = \frac{1 - a_1}{F_{\text{max}} - F_1} \iff F_{\text{max}} = \frac{1 - a_1}{k} + F_1,$$

where $F_{\text{max}}$ is the maximum permissible load, $k$ is the slope of the linear function, $a_1$ is the activity at point 1 and $F_1$ is the force at point 1. This procedure can be repeated for any number of joint angles or postures.

To determine maximum isometric torque of the extensor muscles the model was subjected to two knee-extending loads (see figure 3.4), during two separate trials. The point of application of the load corresponds to the location where the distal shin pad was attached to the shank during the experiments. The load simulates the reaction force from the shin pad during experimental trials. The load was defined in the local coordinate system of the shank, which is rotated $90^\circ$ around the negative z-axis, with respect to the inertial coordinate system. The shank is also slightly adducted, implying the local coordinate system being slightly rotated around the positive y-axis. Direction of the load was always orthogonal to the shank, same as for the experiments. The magnitude of the load had no influence on the process, thus set to arbitrary value of 100 and 250N respectively. The process of determining $F_{\text{max}}$ was iterated for each experimental configuration of the knee angle. For the knee flexors, the direction of the applied load was opposite to the load in extension (see figure 3.5), otherwise the process was the same as for the extensor muscles. Maximum isometric torque ($T^{\text{mod}}$) generated by the extensors/flexors when subjected to $F_{\text{max}}$ was measured via pre-defined in AMS measurement (Net Muscle Moment), and described with respect to the inertial reference system. Net moment muscle calculates net torque around the knee joint generated by the contributing muscles.

![Figure 3.4: Model subjected to an extending load simulating the experimental procedure for knee extensors. The knee angle does not correspond to real analyses. The bright pink regions are muscles activated during extension (quadriceps-femoris).](image1)

![Figure 3.5: Model subjected to a flexing load simulating the experimental procedure for knee flexors. The knee angle does not correspond to real analyses. The bright pink regions are muscles activated during flexion (hamstrings).](image2)

It was necessary to efficiently incorporate the calculations of $T^{\text{mod}}$ into the optimisation loop, due to its changes on every loop iteration. AnyPytools package allows for an interaction between AMS and Python by enabling batch processing and parallelisation of musculoskeletal simulations within a Python environment. Parallelisation provides a possibility to run the processes in parallel, utilising all available processor cores, thus reducing simulation time. This was especially useful in this master’s thesis because of the number of simulations that the optimisation demanded. The necessary operations were defined inside a macro array, which consisted of a number of simple commands, later executed inside AMS. In order to incorporate the calculations of $T^{\text{mod}}$ into the optimisation process, a main function
(Run Model) was created, followed by a number of nested functions. Each nested function consisted of a number of macros that needed to be executed to obtain necessary information. Input to the main function was an array of design variables $\bar{x}$ and the experimental knee angles. The following script presents a general overview of the workflow inside Python environment. For detailed information on how the function Run Model was incorporated into the optimisation loop, see section 3.6.

```python
def Run_Model(x, Experimental_Knee_Angles):
    ROM = [0, 130]
    macro_calculate_max_lengths = [...]  
    execute(macro_calculate_max_lengths)

    L_mt_min = ...
    L_mt_max = ...
    # Calculate extreme lengths of muscle-tendon actuators

    def Calculate_MTP(x, L_mt_min, L_mt_max):
        # Calculate MTP for every muscle involved in the optimisation
        ...
        ...
        return MTP
    MTP = Calculate_MTP(x, L_mt_min, L_mt_max)

    def Calculate_Maximum_Force(MTP, Experimental_Knee_Angles):
        force_array = np.array([100, 250])
        for force in force_array:
            for angle in Experimental_Knee_Angles:
                macro_max_force = [...]  
                execute(macro_max_force)
                # Calculate $F_{\text{max}}$ for each experimental knee angle
                ...
                ...
        return F_max
    F_max = Calculate_Maximum_Force(MTP, Experimental_Knee_Angles)

    for i in range(len(Experimental_Knee_Angles)):
        Experimental_Knee_Angles(i)
        F_max(i)
        macro_max_torque(i) = [...]  
        execute(macro_max_torque)
        # Calculate $T_{\text{mod}}$ for each knee joint angle
        # configuration based on given $F_{\text{max}}$
        return $T_{\text{mod}}$
```

The main function Run Model had two inputs, design variables $\bar{x}$ and experimental knee angles. Max_torque macro applied the calculated MTP and $F_{\text{max}}$ to return $T_{\text{mod}}$ for every experimental knee angle.

The nested function Calculate_MTP had three inputs, design variables $\bar{x}$, maximum, $L_{\text{max}}^{\text{MT}}$ and minimum, $L_{\text{min}}^{\text{MT}}$ physiological lengths. It returned $L_r^p$ and $L_s^p$, calculated according to \( \text{[2.24]} \) and \( \text{[2.25]} \). The nested function Calculate_Maximum_Force had two inputs, MTP and experimental knee angles. Force_array corresponded to the two arbitrary forces simulating
3.5 Calculating maximal and minimal physiological lengths

the experimental trials, necessary in order to calculate \( k \) in equation (3.2). Macro_max_force
is looped across every experimental knee angle, for each force defined in Force_array. Max_force_macro
returns muscle activity for every experimental knee joint angle, which was a necessary component in
the calculations of \( k \) in (3.2). Calculate_Maximum_Force returns \( F_{\text{max}} \) for each experimental knee joint angle, calculated according to (3.3). All macros followed a
simple structure of type:

1. load model,
2. set MTP to the desired values,
3. set knee joint angle equal to experiment,
4. set the desired force,
5. run simulation,
6. dump the necessary information.

3.5 Calculating maximal and minimal physiological lengths

Transformation equations (2.24) that relate \( L_{of}^i \) and \( L_{st}^i \) to \( \tilde{L}_{\text{max}}^m \) and \( \tilde{L}_{\text{min}}^m \) require information
about the maximum and minimum musculo-tendon lengths, \( L_{\text{max}}^\text{MT} \) and \( L_{\text{min}}^\text{MT} \), for each musculo-tendon
actuator involved in the optimisation process. To obtain these extreme lengths, a kinematic
analysis was performed using the musculokseletal model. Assuming that the range
of motion of a knee joint is 0\(^\circ\) (fully extended) to 130\(^\circ\) (fully flexed), these extreme lengths
corresponds to the length of an actuator when these joint angles are reached. For quadriceps-
femoris muscle group (knee extensors) \( L_{\text{max}}^\text{MT} \) is found at 130\(^\circ\) knee flexion, while \( L_{\text{min}}^\text{MT} \) corre-
sponds to 0\(^\circ\) i.e full extension. For the antagonistic hamstring muscles (knee flexors) these
relations are switched, maximum and minimum lengths corresponds to 0\(^\circ\) and 130\(^\circ\) respec-
tively. The results of the analyses were exported in .csv format and subsequently used during
the optimisation process. The extreme lengths are only affected by the geometrical scaling,
thus calculated only once, regardless of the results of the optimisation.
3.6 Isometric Optimisation

Muscles included in the isometric optimisation are presented in table 3.3. The selection of the muscles was based primarily on their contribution to the knee flexion/extension. Primary knee extensor is the quadriceps-femoris muscle group, including vastus and rectus femoris muscles. Primary knee flexor is the hamstring muscle group, including biceps femoris together with semimembranosus and semitendinosus muscles.

Table 3.3: Muscles included in the optimisation, together with their corresponding sub-parts and the number of muscle elements ("Pars").

<table>
<thead>
<tr>
<th>Muscle Group</th>
<th>Muscle Sub-part</th>
<th>Num. of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vastus Intermedius</td>
<td>—</td>
<td>6</td>
</tr>
<tr>
<td>Vastus Lateralis</td>
<td>Inferior</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Superior</td>
<td>2</td>
</tr>
<tr>
<td>Vastus Medialis</td>
<td>Inferior</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mid</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Superior</td>
<td>4</td>
</tr>
<tr>
<td>Rectus Femoris</td>
<td>—</td>
<td>2</td>
</tr>
<tr>
<td>Biceps Femoris Caput Longum</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td>Biceps Femoris Caput Breve</td>
<td>—</td>
<td>3</td>
</tr>
<tr>
<td>Semimembranosus</td>
<td>—</td>
<td>3</td>
</tr>
<tr>
<td>Semitendinosus</td>
<td>—</td>
<td>1</td>
</tr>
</tbody>
</table>

Prescribing a pair of design variables $\tilde{L}_{\text{max}}^m$ and $\tilde{L}_{\text{min}}^m$ to each sub-part of a muscle (muscle, in a case of absent sub-part), and $F_{\text{local}}$ to each muscle resulted in a total of 18 design variables for the quadriceps-femoris and 12 for the hamstring muscles. The number of unique pairs of MTP ($L_1^s$ and $L_1^o$) accounted to 24 and 8 for the quadriceps-femoris and hamstring muscles respectively. Based on the information presented in the table above and the number of the experimental trials, $n = 6$ (see table 3.2), the optimisation problem defined in equation (2.28) was reformulated into two separate problems according to equations (3.4) and (3.5).

For the quadriceps-femoris muscles:

$$
\min J_{\text{iso},Q,F}(x) = \frac{1}{6} \sum_{i=1}^{6} \left( \frac{T_{\text{mod}}^i - T_{\text{exp}}^i}{T_{\text{max}}^i} \right)^2
$$

s.t

$$
L_{s,j}^i \geq 0 \quad j = 1, \ldots, 24
$$

$$
\tilde{L}_{\text{max}}^m - \tilde{L}_{\text{min}}^m \geq 0 \quad k = 1, \ldots, 7
$$

(3.4)
and for the hamstring muscles:

\[
\min J_{iso,H}(x) = \sqrt{\frac{1}{6} \sum_{i=1}^{6} \left( \frac{T_{mod,i} - T_{exp,i}}{T_{max}} \right)^2}
\]

s.t

\[\begin{align*}
L_{i,j} & \geq 0 & j = 1, \ldots, 8 \\
I_{max,k} - I_{min,k} & - 0.1 \geq 0 & k = 1, \ldots, 4
\end{align*}\] (3.5)

The non-linear optimisation problem was solved by means of a sequential linear-quadratic programming method (SLSQP). It was coded using Python and the solver was provided by the open-source package SciPy [41]. The step size used for approximation of the Jacobian, \( \varepsilon \), was increased from \( 1.5 \cdot 10^{-6} \) to \( 5 \cdot 10^{-4} \). The precision was assessed to be sufficient enough to provide a stable Jacobian approximation. Changes to \( \varepsilon \) were made to speed up the iteration process. The default value for the convergence criterion \( ftol \) was changed from \( 1.0 \cdot 10^{-6} \) to \( 1.0 \cdot 10^{-4} \). This was done to speed up the optimisation process, especially towards the end when changes in the objective function are extremely small. It was assessed that higher precision was not necessary. The remaining settings were left unchanged. The following Python script shows the definition of the objective function and how it connects with the \( T_{mod} \) calculating function \( Run\_Model \).

```python
def objfun(x, T_exp, Experimental_Knee_Angles):
    T_mod = []
    T_mod = Run_Model(x, Experimental_Knee_Angles)
    J_isom = np.sqrt((sum(np.power(((T_mod-T_exp)/T_exp_max),2))/len(T_exp)))
    return float(J_isom)
```
Main function `objfun`, initial guesses, constraints and solver settings were used as inputs to the optimisation solver. Every time the objective function was evaluated by the solver, the function `Run_Model` was run for any given array of design variables $\tilde{x}$ to provide the objective function with the modelled torque $T_{mod}$. Table 3.4 provides information about the initial guesses and lower and upper bounds for the design variables. Initial guesses and bounds were equal in both optimisation cases. Upper and lower bounds were somewhat based on the values provided by Garner and Pandy in [15] and Heinen et al. in [18], with modifications made to the upper and lower bounds of $\tilde{L}_{max}$.

Table 3.4: Lower and upper bounds for the design variables, together with initial guesses for the solver.

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Initial Guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{L}_{max}$</td>
<td>0.9</td>
<td>1.65</td>
<td>1.2</td>
</tr>
<tr>
<td>$\tilde{L}_{min}$</td>
<td>0.1</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>$F_{local}$</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Optimisation problems (3.4) and (3.5) were solved independently.
Results
4.1 Results

The optimisation resulted in an overall decrease of $J_{iso}$ relative to the unscaled reference model. The reduction of $J_{iso}$ was observed in all test subjects for both muscle groups, ranging 80.2 to 85.4% and 82.9 to 92% for the quadriceps-femoris and hamstring respectively, see Table 4.1. The optimised values for $J_{iso}$ suggest a much better fit to the experimental data compared to the unscaled model. Mean absolute error (MAE) defined according to equation (4.1) is also presented in the table. MAE is a measurement of the mean of the absolute difference between the optimised torque-profile and the experimental data. The unit of MAE is Nm.

$$MAE = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(T_{mod} - T_{exp})^2} \ [Nm], \quad i = 1, \ldots, n,$$

where $T_{mod}$ is the modelled torque post optimisation and $T_{exp}$ is the experimentally obtained torque. The absolute error is summed across each knee angle configuration $i$ and the sum is divided by the number of trials $n$. MAE proved to be more advantageous to use in terms of interpretability, compared to the objective function $J_{iso}$. However, MAE does not necessarily provide any information about the precision or data fit of the predicted torque.

Table 4.1: The results of the optimisation for the three test subjects involved in this study. The table includes $J_{iso}$ for the unscaled reference model together with $J_{iso}$ obtained post optimisation, the reduction of $J_{iso}$ relative to the reference model and the resulting MAE. Both $J_{iso}$ are dimensionless, while the reduction and the MAE are expressed in % and Nm respectively.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Knee Extension</th>
<th>Knee Flexion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subject 1</td>
<td>Subject 2</td>
</tr>
<tr>
<td>$J_{iso, unscaled}$</td>
<td>0.361</td>
<td>0.314</td>
</tr>
<tr>
<td>$J_{iso, post-opt}$</td>
<td>0.063</td>
<td>0.046</td>
</tr>
<tr>
<td>Reduction [%]</td>
<td>82.6</td>
<td>85.4</td>
</tr>
<tr>
<td>MAE [Nm]</td>
<td>16.5</td>
<td>6.8</td>
</tr>
</tbody>
</table>

The maximum and minimum observed MAE for the quadriceps-femoris amounted to 16.5 and 6.8 Nm respectively and a notable variation between the test subjects was observed. Less variation between test subjects was noticed in the case of the hamstring muscles and MAE revolving around 7 Nm for all test subjects, see Table 4.1.

---

1. An unscaled reference model refers to a model that is geometrically scaled in the same way as the optimised model. However, $L^j_o$ is only scaled based on anthropometry of the individual and $L^j_s$ is scaled through an internal calibration process.
4.1. Knee Extension

The absolute error between the peak of predicted torque and the maximum experimental torque makes the highest contribution to MAE in the case of knee extension, see figure 4.1. This, due to an apparent inability of the model to correctly predict, in some cases, the knee angle at which maximum torque occurs. In the cases of test subject 1 and 2, the model incorrectly predicted 75° as the knee angle at which the maximum occurs. In contrast, the predicted peak torque angle for test subject 3 was correctly predicted, however the maximum torque is greatly underestimated, thus greater MAE. In addition to the predictability issues, the predicted torque curve for the test subject 2 displays a somewhat odd and non-physical behaviour, where the torque remains almost constant between knee angles 50 and 60°, see figure 4.1. For the rest of the knee angles, the model displays an overall good predictability, especially in the ascending and descending regions of the torque curve. The general shape of the predicted torque profile for knee extension is retained and reflects the physical behaviour reported in other studies, see for example figure 2.11.

Figure 4.1: Diagram shows the modelled torque profile curve (knee extension) after the optimisation and the modelled torque profile curve of the unscaled reference model. In addition to that, the experimental data points together with their modelled counterparts after the optimisation are also present.

Figure 4.2 shows the operating range predicted for each quadriceps-femoris muscle involved in the optimisation process. The beginning and the end of each bar indicates the minimum and maximum normalised muscle fibre length of each muscle $\tilde{L}_{m}^{min}$ and $\tilde{L}_{m}^{max}$. The location of each bar represents the part of the force-length curve on which the muscle generates active force.
4.1. Results

Figure 4.2: Diagram illustrating the operating range (knee extension) predicted for each quadriceps-femoris muscle involved in the optimisation process, for each test subject involved in this study. The location of each bar represents the part of the force-length curve on which the muscle generates active force. Muscle symbols are: vastus intermedius (V.I), vastus lateralis inferior (V.L.I), vastus lateralis superior (V.L.S), vastus medialis inferior (V.M.I), vastus medialis mid (V.M.M), vastus medialis superior (V.M.S) and rectus femoris (R.F).

The maximum normalised muscle fibre length $L_m^{max}$ (the end of each bar), for all muscles, lies outside the force-generating range of 1.5 reported by Zajac [44], (Figure 4.2). The estimations of $L_m^{min}$ for some muscles are (V.I, V.L.I, R.F) below 0.5$L_m^{max}$ suggesting that these muscles need to start to generate active force earlier, compared to the suggested values by Zajac. The lengths predicted for most of the muscles, for all the test subjects, indicate that these particular muscles may develop active force over the entire range of their force-length curve, range 0.5$L_m$ to 1.5$L_m$. An exception to that are the V.M.I and to some extent V.L.S muscles which, suggested by the results (Figure 4.2), operates mainly in the descending region of their respective force-length curve. The patterns of results observed are consistent for each test subject indicating a consistent model estimation.

Figure 4.3 shows the distribution of the relative error between the optimised torque curve and the experimental data. Relative error is defined according to equation (4.2) and is a measurement of the prediction error expressed as a fraction of the experimental value.

\[ \eta = 1 - \left( \frac{T_i^{exp} - T_i^{mod}}{T_i^{exp}} \right) \quad i = 1, \ldots n, \quad (4.2) \]

where $T_i^{mod}$ is the modelled torque post optimisation and $T_i^{exp}$ is the experimentally obtained torque. The error is calculated for each data point $i$ for all data points $n$. As per definition (see equation (4.2)), an overestimation of the experimental data yields values above 1, while an underestimation yields values below 1. Goodness of fit of the prediction was not evaluated in terms of the coefficient of determination $R^2$ due to non-linearity of the model (see for example figures 4.1 and 4.4). $R^2$ has been shown to be an inadequate indicator of goodness of fit for non-linear models in [34, 39, 42]. Furthermore, $R^2$ does not provide information about the magnitude of the observed errors, which was found to be a crucial factor during the evaluation of accuracy and precision of the prediction.
4.1. Results

Figure 4.3: Diagram presents the distribution of the relative error for (knee extension) results of the optimisation, for all test subjects. A relative error, $\eta$ of 1 indicates no error, while values below and above 1, indicates an under and overestimation respectively. Each data point corresponds to an experimental data point.

The accuracy of the predicted torque profile vary not only between the test subjects but also within the test subjects, (Figure 4.3). The observed relative error is always kept below 15% ($0.85 < \eta < 1.15$) for all the test subjects, with an error frequently fluctuating between 5-10% ($0.9 \leq \eta \leq 1.1$). Values of $\eta$ switching between $\eta > 1$ and $\eta < 1$ throughout the range of data points, occurring in all of the involved test subjects, indicate that for some points the model overestimates the torque, while for some other the torque is underestimated, which in turn suggests low precision of the results.
4.1. Results

4.1.2 Knee Flexion

The optimised model makes more accurate predictions in case of knee flexion, compared to the extension (Table 4.1). The knee model also, in general, correctly predicts the angle at which peak maximum torque occurs. A small discrepancy could be observed in the case of test subject 3, in which the model incorrectly predicted the angle to be 30°, see figure 4.4. A relatively large error observed in test subject 1 at 30° is most likely attributed to an error in experimental data. Similarly to knee extension the general shape of the predicted torque profile for knee flexion is retained for all subjects and reflects the physical behaviour reported in other studies, see for example [11].

Figure 4.4: Diagram shows the modelled torque profile curve (knee flexion) after the optimisation and the modelled torque profile curve of the unscaled reference model. In addition to that, the experimental data points together with the their modelled counterparts after the optimisation are also present.
4.1. Results

Figure 4.5 shows the operating range predicted for each hamstring muscle involved in the optimisation process. Similarly to the figure 4.2, the position of the bar indicates the part of the force-length curve on which the muscle generates active force.

In contrast to the quadriceps-femoris muscles, the results indicates a much smaller operating range for the included hamstring muscles. Nearly all muscles operate in the ascending region of their respective force-length for practically the entire range of the knee joint. The estimated maximum normalised lengths $\hat{L}_{\text{max}}$ all lie below $\hat{L}_m$, seemingly never developing its peak force, see figure 4.5. The estimated lengths for the semitendinosus muscle lie in a close proximity to its optimal fibre length $\hat{L}_m = 1$, this muscle can thus generate active force when its fibres are close to its optimal values. The estimated lengths of the muscles of test subject 1 differs from the other test subjects, most likely due to the difference in the angle at which peak torque occurs coupled with anthropometrical differences.
Figure 4.6 shows the distribution of the relative error between the optimised torque curve and the experimental data.

![Diagram showing the distribution of relative error for knee flexion](image)

Figure 4.6: Diagram presents the distribution of the relative error for results (knee flexion) of the optimisation, for all test subjects. A relative error, $\eta$ of 1 indicates no error, while values below and above 1, indicates an under and overestimation respectively. Each data point corresponds to an experimental data.

In general, the relative error of the prediction behaves similarly to the one of knee extension. Unusually high overestimation of data point 2 in test subject 1 is most likely a result of the error in the experiments, described earlier. Otherwise the same type of variability between the test subject as well as intra-variability within test subject can be observed, indicating low level of precision of the prediction. Most of the time the relative error varies between $0.9 \leq \eta_i \leq 1.1$ which means an over or underestimation of the torque by 10%.
5.1 Results

5.1.1 Torque prediction and operating range

The apparent inability of the model to correctly predict the peak torque angle for the quadriceps muscles (test subject 1 and 2) pose a number of severe problems. Potential consequences of this include an under/overestimation of the strength of an athlete and an incorrect prediction of the internal loading. This in turn can lead to an incorrect diagnosis and an increased risk for further injury. The prediction inaccuracies for the quadriceps-femoris are consistent with the results presented in figure 3 in Heinen et al. [18]. The authors choose not to address this issue therein, however.

A musculoskeletal model consists of multiple muscle-tendon actuators, each one with its own unique torque profile. Thus, matching the strength of a test subject is a matter of finding a set of MTP that yield a combined torque profile $M_{\text{tot}}$ that fits best to the experimental data. Recall equation (2.27) defining the total torque generated by the muscle-actuators $M_{\text{tot}}$, described in section 2.4.1:

$$M_{\text{tot}}(\theta) = \sum_{i=1}^{n} F_i(\theta) \cdot d_i(\theta) \quad i = 1, \ldots, n,$$

(5.1)

where $F_i$ and $d_i$ is the muscle force and lever arm of the $i$-th muscle-actuator, respectively. Both muscle force and lever arm are functions of the knee joint angle $\theta$. The term is summed across all the contributing muscle-actuators $n$. The lever arm of a muscle-actuator is dependent on muscle and bone geometries, which in turn are related to the anthropometry of the individual. In other words, there is no way for the optimisation solver to control lever arm $d_i$ in equation 5.1. Therefore, the only way to control the total torque $M_{\text{tot}}$ output is to modify the force function $F_i(\theta)$.

The obtained prediction of the torque profile for test subject 1 and 2 points towards a conclusion that the optimiser was unable to modify the force curve $F_i(\theta)$ of each actuator, in such a way that a combined torque $M_{\text{tot}}$ peaks at $60^\circ$. This indicates that regardless of the modifications made to the force curve, changes in the lever arm dominate the behaviour of the torque between $60^\circ$ to $75^\circ$.

Consider the bell-shape of a force-angle curve, similar to the $f-l$-curve (see figure 2.6) and assume that the optimiser will attempt to maximise force output at $60^\circ$. Furthermore, assume a decreasing behaviour of the lever arm (consistent with the findings in [11]), the results obtained for test subjects 1 and 2, suggest that for the muscles with the highest contribution to the total torque:

1. the lever arm has to increase locally, in the range of $60^\circ$ to $75^\circ$;
2. the rate at which the force decreases has to be lower than the rate at which lever arm increases, thus also increasing the generated torque;
3. the torque increase will continue until the rate is reversed or until the behaviour of the lever arm changes.

Slower rate of force decrease can be attributed to the wide plateau region of force-length curve for the quadriceps-femoris muscles, which is a result of wider operating range. This points towards a severe limitation of the method in its current state, at least for the quadriceps-femoris muscles, where an isolation of certain peak angles is proved to be a futile task for the optimiser due to the limitations and internal behaviour of the muscle model. Narrower peaks of the force-length curves could potentially solve these issues, however this
implies modifying the internal coding of the muscle models in AMS, which of course lies outside the scope of this master’s thesis.

In general the maximum physiological lengths $L_{\text{max}}^{m}$ for quadriceps-femoris muscles lie outside the reported force-generating range of $1.5L_{m}^{\text{th}}$. This could potentially be due to the large muscle excursion observed throughout the participants’ ROM of the knee joint. A larger muscle excursion results in a greater region where muscles need to generate active force. The operating range of *vastus medialis inferior* (V.M.I) indicates that V.M.I operates in the descending region of its force-length curve. This can, to some degree, be supported by Slater and Hart in [33]. They showed an increase of muscle activation for *vastus medialis* during the later stages of a squat, which supports the notion of the descending portion of the force-length curve the muscle operates in. However, other parts of *vastus medialis* (V.M.M and V.M.S) have different operating ranges, which is most likely due to the differences in lever arm. Slater and Hart observed similar increase of muscle activation for *vastus lateralis*, which is consistent with the results for *vastus lateralis superior* (V.L.S) for test subject 3. Experimental studies have shown that the length of quadriceps-femoris muscles are most often much greater than their tendons [21]. In section 2.3 it was described that the muscles with greater muscle length relative tendon lengths coupled with high muscle excursion have wider operating ranges. This is consistent with the presented results, further solidifying the validity of the results.

Cadaver studies have shown that a large portion of the length of hamstring muscles consists of tendon length [12, 21] which implies that the operating range should be smaller compared to quadriceps-femoris muscles, see section 2.3. This is consistent with the results obtained for the operating ranges of hamstring muscles, see figure 4.5. In addition to that, Reid et al. [12] showed that during knee flexion *biceps femoris* muscle generates most of the active force, which can be confirmed with the obtained operating ranges of *biceps femoris caput longum* (B.F.C.L) and *femoris caput breve* (B.F.C.B), both being larger compared to other muscles. The differences between the subjects can most likely be attributed to different angles at which peak torque occurs and differences in muscle geometries.

It is noteworthy to mention the low accuracy of the unscaled reference model. Although consistent with some studies [11], the predicted peak torque angle and the peak torque are underestimated by a large margin. Interestingly there seems to be no notable differences between the subjects in regards to the prediction made by the unscaled model, even though great differences in both weight, leg segment lengths and height were noticed.

### 5.1.2 Muscle tendon parameters

It was deliberately decided to not present the results of the estimated musculotendon parameters $L_{s}^{m}, L_{o}^{m}$. This mostly due to the low comparability of such results against other similar studies as well as low interpretability of these numbers without any prior knowledge of muscle geometry in the TLEM model. Low comparability can be attributed to four primary factors.

First, the estimated MTP describe the behaviour of each muscle element of a muscle’s sub-part, see section 2.2.3 and figure 2.4. The estimated MTP are different depending on the position of the element relative to the knee joint. Thus, there is no definitive answer to the question of MTP for the entire muscle. The Hill-type muscle model is a phenomenological model which does not represent a real muscle, rather mimics its ideal behaviour. Thus, one might expect differences in MTP in the model and cadaver studies.
Second, similar studies of MTP estimation, by for example Delp [11] and Nam et al. [43], use different techniques of muscle modelling with different structures, where most of the times the muscles are viewed as holistic entities without any sub-parts or elements. Although the work of Heinen et al. [18] was done in AnyBody™ and their work presented the estimations, they did not disclose any method of calculating the MTP for the muscle sub-parts. Therefore, it is impossible to make any qualitative comparisons to the results therein.

Third, the available information about the quantities of $L^s_t$, $L^o_t$ are based on cadaver studies performed on older subjects, which most likely do not represent the physically active and healthy individuals included in this study.

Fourth, an attempt to calculate the MTP for an entire muscle would most likely involve some kind of a mean of lengths. Such mean would not reflect the observed behaviour of the model and using it directly in the model would yield completely different set of results.

It would of course be possible to disclose the estimated MTP for the muscle elements but the interpretations would most likely consist of a guessing game between the choice of “reasonable” or “not reasonable”, which does not bring any meaningful contribution to the evaluation of the method.

5.1.3 Clinical significance

While evaluating and discussing the predictability of the optimised model and the overall performance of the estimation method, it is important to assess the results from two different perspectives: a perspective of an engineer constructing the model, and a clinician who is performing the analysis and making the final patient diagnosis. Overall, both perspectives should agree on the success of the method proposed by this master’s thesis. The minimum and maximum reduction of the objective function $J_{iso}$ by 80.2 and 92.0%, relative the unscaled reference model, proves that the method does indeed work as intended and the accuracy of the prediction is greatly improved.

At the same time, evaluating the accuracy and precision of the model in terms of clinical significance is crucial in order to make an as in-depth assessment of the estimation method as possible. Especially considering that the aim of this master’s thesis is to suggest a method applicable in clinical studies. With a broad definition of the term, analysing the clinical significance of the prediction in the context of this master’s thesis would imply answering following two questions:

- is the prediction accurate and precise enough to be applied clinically?
- are the differences between the model and experiments important and how would these differences affect the results?

An upper limit of 5% for the relative error was suggested, in order for the prediction to have any clinical significance. Since the aim of the model is to represent elite athletes even the smallest possible error could have severe consequences on the outcome. An inaccurate prediction could potentially lead to an over/under estimation of the strength of an athlete, which in severe cases of overestimation could lead to a wrongful decision and an increased risk for injury. Observing quantities of the relative error for the knee extension (Figure 4.3) indicate that the accuracy of the prediction is too low. Figure 4.6 points towards the same conclusion for knee flexion. Low precision of the prediction pose additional issues. Without high precision there is no way for the clinician to systematically categorise the results. Consider for example a prediction always overestimating the torque, $\eta > 1$, a clinician performing an
analysis with such model could in that case conclude that the results are most likely overestimated and vice versa for consistent underestimation, whereas without such consistency there is no way to draw any conclusions. All in all, the answer to the first question, based on the presented data, is no, the accuracy and precision are not good enough. Summarising the answer to the second question, the differences are extremely important and, depending on the type and magnitude, can lead to detrimental consequences for the examined athlete.

Although the low clinical significance of the prediction points towards low applicability of the model, at least for this target group, it is undeniable that the method does improve accuracy of the prediction, thus also the accuracy of any future diagnosis. An improvement of the accuracy should always be striven for in all cases of modelling. Regardless of clinical significance, the improvement this method achieves should not be disregarded. After all, the Hill-type muscle model is only a phenomenological model, which aims to mimic the ideal behaviour of the muscle. This means that the error will most likely always be present and ideal models, free from any errors are seldom achievable. Therefore, a use of the results based on musculoskeletal models should always be treated as a recommendation to a decision rather than a definitive answer.

5.2 Method

There are limitations of the estimation method. Perhaps most importantly, the solution does not guarantee a unique set of of MTP for the test subject. A musculoskeletal model consists of multiple muscle-tendon actuators, each one with their own unique torque profile. Thus, matching the strength of a test subject is a matter of finding a set of MTP that yield a combined torque profile that fits best to the experimental data. It is possible to satisfy this condition by more than one set of combinations of MTP. This makes a case for a sensitivity study. By modifying the initial values and observing if the solution converges to the same results, more credibility in the results could be gained. Nevertheless, the obtained operating ranges observed in figures 4.2 and 4.5 are at least partially substantiated by the findings of other studies and observations described in the background section 2.3.

The implemented optimisation solver is limited to finding a local minimum, thus the obtained solution may be sub-optimal for the entire domain which could be perceived as a potential limitation. Identifying the global minimum would require a dedicated global optimiser, which in turn requires more iterations and computational power to converge. Implementing a global optimiser would not solve the biggest issues this method currently is struggling with. Potential accuracy improvements imposed by the global optimiser may be disproportional to the costs of the implementation.

Another limitation is that the accuracy of the model prediction is highly dependent on accuracy of the experimental results. Isokinetic dynamometers have been proved to be reliable by studies [2]. Intra-individual variability however, has a potential to negatively impact the reliability of the experimental data. Intra-individual variability means that there is a possibility that a test subject can generate different set of experimental data, even though asked to perform maximally in all cases. The implications of such variability include: low accuracy of the prediction, incorrect model representation of a subject's strength and incorrect MTP. In general, physical tests of peak strength are very susceptible to this kind of variability. This due to the following reasons:

- a perception of maximal activity is different for each individual. Without any visual confirmation or experience it is hard to know whether the performance really is maximal or not. This is especially difficult for individuals without prior experience of maximal effort activities;
5.2. Method

- fatigue affect the performance of an individual. Data points collected at the end of the testing session are most likely affected by muscle fatigue;

- external factors, that lie both outside and within a test subject’s control. These include amount of rest gained by the test subjects, type of diet consumed in close proximity to the tests, motivation etc.

The variability can of course be minimised by following standardised test protocols coupled with rigorous control of the test subjects and large amount of tests. The test subjects involved in this study may suffer less from the variability issues due to their experience, high level of physical activity and standardised test protocols. Nonetheless, the risk of intra-individual variability is always present. This variability is a great source of potential errors.

There are a number of other possible sources of error in the implemented method. A lot of these are connected to the assumptions that were made. Most notably range of motion of the knee joint was assumed to be equal for all involved test subjects and set to 0-130°. An incorrectly assumed range of motion can influence muscle excursion which consequently could lead to incorrect operating ranges and an incorrect estimation of muscle parameters. This could easily have been avoided by examining the range of motion for each test subject. However, it was simply not considered during early stages of this work, when the experimental work was performed.

Another potential source of error is connected to the selection of muscle recruitment algorithm. As explained in the method section the min/max criterion is a fatigue oriented algorithm which tries to distribute muscle force evenly across all the muscles. This results in an equal muscle activation across all muscles. Intuitively speaking, the min/max criterion does not reflect the reality of muscle activation patterns during maximal effort activity. Instead, a more plausible behaviour includes an instantaneous and maximal activation of the strongest muscles supported by the weaker muscles. It is more intuitive for the central nervous system to maximise force generation instead of minimising fatigue during this type of physical activity. This notion is also supported by the AnyBody’s official guide in which it is recommended to use polynomial criterion for explosive physical activities. However, there is no way to know for sure how the muscle behaves without detailed EMG investigations. In addition to that, the proposed method relies heavily on linear relation between external force and muscle activation for calculation of model strength, thus it is currently impossible to implement other types of muscle recruitment algorithms.

Other less significant sources of error include other assumptions such as the assumed quasi static condition during the experiments and no pennation angle.
6 Conclusion

A method of estimating subject specific muscle parameters of the lower extremity was sought. It was required of the method to have a generic, easily modifiable base, adapted to tests of multiple different subjects. The method had to be implemented and applied on the quadriceps-femoris (extensors) and hamstring (flexors) muscles during isometric (static) condition. The method and the definitive strength prediction had to be evaluated.

A method proposed by Heinen et al. [18] was implemented with some minor modifications to the objective function and constraints. The values of MTP were found by matching modelled and experimental torque profiles. The musculoskeletal model that reflects the experiments was based partially on the interpretation of the limited information on the methodology in their paper and partially on my own knowledge. The experimental methodology consisting of a number of knee extension and flexion tests was described, proposed and performed. To gain more confidence in the results and to increase the credibility, three healthy and physically active males were involved in the study. The proposed method resulted in great improvements in the accuracy of the knee joint torque profile with the error reduction of the objective function by 80.2-92% relative the unscaled reference model. Due to the low accuracy and precision, the prediction was deemed as clinically insignificant, which means that the implemented method works as intended but the reduction is not sufficient enough for this type of target group. A number of limitations and potential sources of error have been unveiled. Perhaps the most significant limitation proved to be the inability to isolate the peak torque angle. This, due to a combination of stagnation of force-length curve and changes in the lever arm of the strongest muscles in a region between 60-80°. Main sources of error include intra-individual variability, the assumption of the range of motion of a knee joint and the implemented muscle recruitment algorithm.

Future work should focus on addressing the current issues presented in this study and a further development, as the method still is relatively new and untested. Parallely, the researches should try to test the method in clinical studies, in order to evaluate the influence on the results by the implementation of this method of parameter estimation.


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