Transformation of In-Flight Measured Loads to a Fatigue Test Spectrum

Omvandling av uppmätta flygprovlaster till lastspektra för utmattningstests

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Upphovsrätt

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Abstract

Fatigue is a well-recognized issue in lightweight and high-performance aircraft structures. As fatigue failures have led to serious accidents and caused significant economic impact in the past, design against fatigue is crucial. Fatigue testing of full-scale aircraft as well as components is an important tool for the advance identification of potential fatigue issues in both new and operational aircraft. Furthermore, coupon testing is used extensively to obtain allowables for materials and structural details to be used in the design process.

To obtain accurate results from fatigue testing, not only the test object but also the used load spectrum must accurately represent reality. If the aircraft is operational, an accurate load spectrum can be obtained by measuring the loads in-flight during a sufficiently long period of normal operation of the aircraft. However, the in-flight measured loads data contains an extraordinarily large number of cycles, resulting in long and uneconomical test durations.

This thesis aims to propose a method for the selection of an optimal filtering level for fatigue test spectra developed from in-flight measured loads. The thesis also discusses and recommends methods for in-flight measurement of loads, cycle counting as well as damage evaluation using a crack-growth approach. Furthermore, ways to validate the proposed method and its practical application are discussed.

An example filtering study is conducted using four different specimens chosen to represent typical structural details of aircraft. The study uses real in-flight measured loads of a light aircraft and also discusses temperature compensation of the loads data. The effect of filtering on fatigue damage is evaluated using crack-growth simulations conducted at a range of filtering and stress levels.

The results show that a remarkable reduction of testing time is possible and as many as 99% of all cycles in the studied flight load history can be discarded without significantly reducing fatigue damage. The allowable filtering level is shown to differ between the specimens and the different stages of fatigue crack growth. In addition, the applied stress level is found to have a consistent effect on the allowable filtering level.
Acknowledgments

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Patrick Dümig
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## Nomenclature

### Abbreviations and Acronyms

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<th>Description</th>
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<tr>
<td>AFGROW</td>
<td>Air Force Growth (fracture mechanics and fatigue crack growth software)</td>
</tr>
<tr>
<td>CA</td>
<td>Constant Amplitude</td>
</tr>
<tr>
<td>COM</td>
<td>Component Object Model</td>
</tr>
<tr>
<td>FAA</td>
<td>United States Federal Aviation Administration</td>
</tr>
<tr>
<td>FH</td>
<td>Flight Hour</td>
</tr>
<tr>
<td>FNK</td>
<td>Forman-Newman-de Koning (crack-growth model, also known as the NASGRO model)</td>
</tr>
<tr>
<td>FoS</td>
<td>Factor of Safety</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>MATLAB</td>
<td>Matrix Laboratory (programming language and numeric computing platform)</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NASGRO</td>
<td>NASA crack-growth software and model (also known as the FNK model)</td>
</tr>
<tr>
<td>NDT</td>
<td>Nondestructive Testing</td>
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<tr>
<td>USAF</td>
<td>United States Air Force</td>
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<td>VA</td>
<td>Variable Amplitude</td>
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### Greek Symbols

- $\alpha$: Plane strain / stress constraint factor
- $\beta$: Geometry factor
- $\Delta \sigma$: Stress range
- $\sigma$: Stress
\( \sigma_{rc} \) Remote stress
\( \sigma_{ax} \) Axial stress
\( \sigma_{bp} \) Bypass stress
\( \sigma_{br} \) Bearing stress
\( \sigma_{f} \) Fatigue limit
\( \sigma_{max} \) Maximum stress
\( \sigma_{min} \) Minimum stress
\( \sigma_{m} \) Mean stress
\( \sigma_{op} \) Crack-opening stress
\( \sigma_{tu} \) Ultimate tensile stress
\( \sigma_{ty} \) Yield tensile stress
\( \tau \) Shear stress
\( \tau_{sc} \) Remote shear stress

**Latin Symbols**

\( a \) Crack length (through-thickness)
\( C \) Material constant in crack-growth equations
\( c \) Surface crack length
\( D \) Diameter
\( D_{fat} \) Fatigue life utilization ratio
\( F_{bp} \) Bypass force
\( F_{br} \) Bearing force
\( F_{lt} \) Transferred force (e.g. by fastener)
\( F_{sec} \) Section force
\( g \) Gravitational acceleration
\( K \) Stress intensity factor
\( k \) Irregularity factor
\( K_{c} \) Critical stress intensity factor
\( K_{Ic} \) Mode I plane-strain fracture toughness
\( K_{f} \) Fatigue notch factor
\( K_{max} \) Maximum stress intensity during loading cycle
\( K_{op} \) Opening stress intensity factor
\( K_{min} \) Minimum stress intensity during loading cycle
\( K_{OL} \) Stress intensity factor during overload
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$K_t$</td>
<td>Elastic stress concentration factor</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>Stress intensity factor range</td>
</tr>
<tr>
<td>$\Delta K_{th}$</td>
<td>Threshold stress intensity factor range</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift force</td>
</tr>
<tr>
<td>$LT$</td>
<td>Load transfer ratio</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
</tr>
<tr>
<td>$m$</td>
<td>Walker exponent (material constant in Walker equation)</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of cycles to failure</td>
</tr>
<tr>
<td>$n$</td>
<td>Material constant in crack-growth equations</td>
</tr>
<tr>
<td>$n_z$</td>
<td>Load factor</td>
</tr>
<tr>
<td>$q$</td>
<td>Notch sensitivity</td>
</tr>
<tr>
<td>$R$</td>
<td>Stress ratio</td>
</tr>
<tr>
<td>$R_{th}$</td>
<td>Racetrack-filtering threshold range</td>
</tr>
<tr>
<td>$T$</td>
<td>Thickness</td>
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1 Introduction

1.1 Motivation

Fatigue failures in metallic structures have been observed since the 19th century and they have caused serious accidents as well as tremendous economic impact [1]. In the early 1950s, around the time of the catastrophic fatigue failures of two de Havilland Comet airliners, fatigue in aircraft structures was first recognized as a major issue [2]. The development of increasingly large, high performance and pressurized aircraft as well as the advent of stressed-skin construction and ever higher design stress levels were contributing factors [2].

Most in-service failures of aircraft structures occur due to simple design faults and poor detail design [3]. A prerequisite for design against fatigue is extensive fatigue data that is usually obtained by coupon testing. Furthermore, full-scale fatigue testing is used to find possible deficiencies in the complete aircraft structure [4].

To obtain realistic results from fatigue testing, the loads applied to the test object must accurately represent the loads encountered in service [1]. If the test object is an in-service aircraft type, the actual loads on the aircraft structure may be measured in-flight to obtain realistic loads data. However, applying in-flight measured loads to fatigue testing involves certain challenges related to the processing and reduction of the data gathered from flight tests. Such data is recorded at a high sampling rate and contains an extraordinary amount of load cycles, many of which may not contribute to fatigue damage. In order to be able to perform the test economically and in a realistic amount of time, such superfluous data must be identified and removed without altering the damaging effect of the load sequence.
1.2 Aim

The objectives of this thesis project can be summarized as follows:

- Study and compare methods for formation of a fatigue test sequence from in-flight measured loads data.
- Study and compare methods for calculation of damage induced by a load sequence.
- Develop and implement a method for determination of the acceptable level of load spectrum filtering for fatigue testing.
- Perform and evaluate the results of a generalizable filtering study using the proposed method and real in-flight measured loads data.
- Suggest an experimental test program for validation of the method.

1.3 Delimitations

In order to keep the work focused and the scope of the project reasonable, some limitations were introduced. The major delimitations are as follows:

- One metallic material, 7050-T7451 aluminum, is considered in this study. The methods may be applicable to other metallic materials, but this would require further consideration.
- Only high-cycle fatigue is considered and all crack-growth analysis is done using linear elastic fracture mechanics (LEFM). The limitations of LEFM apply.
- The concept of nonproportional loading is introduced in section 2.4.3 but all other comments and analysis are limited to uniaxial proportional loading.
- Only a single load spectrum is applied in this study. While the effect of different load levels (spectrum scaling) is considered, no direct conclusions regarding the sensitivity of the results to the spectrum content are drawn.
2 Theory

2.1 Fatigue Testing

Fatigue testing can be conducted to obtain data on the fatigue properties of materials, structures, joints and other structural details. The tests may also be comparative, i.e. serve the purpose of determining the effect of different surface finishes, production methods, environmental factors or loading on fatigue life. Tests are also conducted to verify developed prediction models and testing may concentrate on fatigue nucleation, crack growth or total life of the test object depending on the nature of the test. [1]

Fatigue tests can be divided into different categories by the scale and scope of the test. Tests commonly used to support aircraft design and continuing airworthiness are coupon tests, component tests and full-scale fatigue tests. The loading used in these tests ranges from simple constant-amplitude loading to flight-simulation loading. Flight-simulation load sequences may be generated using a combination of ground-air-ground cycles and statistically determined gust loads [1], or the load sequence may be based on actual in-flight measured loads data. The latter method is described further in section 2.3.

2.1.1 Coupon Testing

Tests aiming to characterize the properties of a material are generally coupon tests, where a small and often standardized specimen is tested in a fatigue-testing machine. Coupon tests can also be conducted to obtain data and allowances for joints or structural details such as holes, fillets and other stress raisers. Comparative coupon testing is used to gain knowledge on the influence of surface treatments or environmental factors on fatigue life.

If the results of a coupon test program are to be applied to real structure which includes regions of stress concentration, tests with notched specimens are necessary. It was observed long ago that the fatigue limit \( \sigma_f, \text{unnotched} \) of notched specimens is not equal to the fatigue limit of unnotched specimens \( \sigma_f, \text{unnotched} \), leading to the introduction of the fatigue notch factor

\[
K_f = \frac{\sigma_f, \text{unnotched}}{\sigma_f, \text{notched}}.
\]  

(2.1)
2.1. Fatigue Testing

The fatigue notch factor is related to the notch sensitivity $q$ of the material and the size of the specimen. The notch sensitivity is defined as

$$q = \frac{K_f - 1}{K_t - 1},$$

and it may be viewed as a parameter showing to what degree the effect of the theoretical elastic stress concentration is obtained in a ductile material under fatigue loading [5]. Notch sensitivity is higher for larger specimens and stronger materials [1].

Test programs often include different types of standard specimens which produce different elastic stress concentration factors $K_t$. Specimens which produce only a negligible stress concentration $K_t \ll 1$ are referred to as smooth specimens. Specimens with holes produce stress concentrations of $K_t \approx 2...3$ [5] and notched specimens can be used to produce a variety of stress concentrations depending on the notch geometry.

Coupon test programs can also be used to obtain information about the level of scatter and the statistical distribution of fatigue life [1]. Obtaining an accurate distribution of fatigue life however requires a large number of coupons which makes such a test program expensive [1]. Even when only including the minimum amount of coupons required to account for scatter, the test matrix quickly becomes very large for programs that aim to develop design curves by testing specimens at multiple load levels.

2.1.2 Full-Scale and Component Testing

Structural components or even whole structures comprising many components can be fatigue tested. These tests are termed component tests and full-scale tests. The aim of such tests is generally to obtain life estimates for the whole structure under realistic loading, to verify structural design choices and to identify critical locations where fatigue cracks initiate [3]. Historically, full-scale tests have been run to a minimum of twice the design service life goal of the airframe, although at least three times the design life is now recommended [6]. Furthermore, for airframes managed under the safe-life concept, a minimum of five times the expected service life is mandated by defence standards [7]. Full-scale fatigue testing was made mandatory by the USAF on military aircraft in 1969 and by the FAA on civil aircraft in 1998 [6][8].

Component and full-scale fatigue tests generally require a more extensive setup than coupon tests and sample sizes are typically small. These tests are often service-simulation tests, meaning that the applied loading is developed to simulate real operation as closely as possible [1]. The distributed aerodynamic and inertial loads are commonly introduced on the aircraft using hydraulic servo actuators [9]. Electromagnetic shakers may be used to simulate buffet-induced vibrations [9]. Compared to coupon testing, the load introduction on the test object is more complicated and testing is slower as a result of the larger forces and displacements involved.
2.2 Loading of Aerodynamic Surfaces

The loading on various lifting surfaces such as wings or tailplanes of aircraft can be divided into several categories. In addition to the steady pressure loads which cause lift and drag forces during straight-and-level flight, various other variable loads occur during flight. Types of aircraft flight loads that contribute significantly to the fatigue of lifting-surface structures include:

- Maneuver loads
  - Loads caused by maneuvering of the aircraft, i.e. increased and variable aerodynamic forces and inertial forces due to linear and rotational acceleration of the aircraft.

- Gust loads
  - Aerodynamic loads and resulting inertial loads due to momentary changes in aircraft angle of attack, sideslip angle and dynamic pressure caused by wind gusts.

- Buffet loads
  - Loads due to aeroelastic interaction of the structure with unsteady flow [10].

The list is not complete and for some components other loads such as e.g. pressurization or acoustic loads may be significant [1]. The relevance of different loads with regard to fatigue of structure depends on the component, type of aircraft and the aircraft’s mission [1]. For example, maneuver loading contributes significantly to fatigue of wing structures on fighter aircraft, which typically fly missions that include a lot of hard maneuvering. On the other hand, airliners fly very few and only light maneuvers during typical passenger service.

Gust loading is a major fatiguing component in transport aircraft wing load spectra. Such aircraft spend a lot of time in cruise flight where gusts constitute the main fatiguing load component. The severity of gust loading is also dependent on the cruising altitude of the aircraft since the occurrence of gusts in the atmosphere changes with altitude, with gusts being more prevalent at low altitudes [1].

Buffet loading is caused by local unsteady flow affecting a lifting surface of the aircraft. The structural response of the surface to this type of loading can be very damaging as it can result in high oscillatory stresses and strains [11]. Buffeting can be caused by areas of flow separation on the surface or unsteady flow striking the surface. It is commonly experienced by tailplanes as they are affected by the flow that is disturbed by the wings and any possible flow separation occurring at the wings. Buffeting is also well-known to cause fatigue damage to the vertical tails of fighter aircraft flying at high angles of attack [11]. Especially in twin-tail arrangements, the vertical tails may be struck by the vortices generated by upwind surfaces such as wing strakes, causing heavy buffeting [11]. A practical example of this can be seen in figure 2.1.
2.3 Flight Measurement of Loads

Direct measurements can be made to obtain accurate data on airframe loads under real operating conditions. Not only can the measured data be used to verify load predictions made during the design process of an aircraft [3], but it can also be used to track fatigue damage accumulation in operating fleets of aircraft. Load measurements can be made either directly by measuring the strain on the structure or by measuring accelerations, in particular the normal acceleration or load factor $n_z$ of the aircraft.

When in-flight measured loads data is used for fatigue predictions, the data must be recorded at a high sample rate to avoid distortion of the signal [13]. A sampling frequency of at least 10 times the maximum frequency of interest is common practice [14]. For fatigue analyses and fatigue testing the most important features of the load history are the peaks and valleys, which may not be captured correctly if the sampling rate is not sufficiently high [15]. A similar problem can arise if the data is frequency filtered, causing the amplitude of the peaks and valleys to be altered [15]. Thus to downsample or reduce the data and filter out noise, amplitude-based filters such as the racetrack filter introduced in section 2.12 should be used [15].

2.3.1 Acceleration Measurement

Measurement of load-factor exceedances using a fatigue meter was one of the earliest developments allowing more accurate fatigue monitoring than simple flight hour counting [16]. A fatigue meter is a device incorporating an accelerometer which measures the normal acceleration of the aircraft [16]. The acceleration data can be used to estimate the load factor $n_z$ of the aircraft. The load factor is a unitless parameter defined as

$$n_z = \frac{L}{Mg}, \quad (2.3)$$
where $L$ is the total lift force acting on the aircraft, $M$ is the aircraft mass and $g$ is the gravitational acceleration. Despite $n_z$ being unitless, it is commonly expressed in units of “g’s”. The method of counting the load factor exceedances is described in detail in section 2.4.2.

Estimating the loads on e.g. fatigue-critical wing attachment structure using load factor data is relatively straightforward but carries some limitations. The loads can only be estimated for components which are affected by $n_z$, as a transfer function between $n_z$ and the local stress or strain is needed [16]. The transformation coefficients are complicated functions of load level, altitude and airspeed [17], and they are never exact since factors such as e.g. asymmetric loads, changes in aircraft mass [16] or mass distribution as well as the difference in direction of the lift vector and the normal acceleration vector measured by the accelerometer are generally neglected. Nevertheless, successful methods to relate the fatigue meter readings to aircraft life have been developed [18] and have been in use since the 1950s [17].

Accelerometers have also been employed to monitor the fatigue damage on components that are not directly affected by $n_z$ but experience cyclic loading due to e.g. aeroelastic phenomena such as buffeting. For example, accelerometers have been used on the tips of the vertical tails of subscale models of the F/A-18 and F-22 [19][20] as well as on the full-scale F/A-18 aircraft to study vertical tail buffet loads caused by the wing-strake vortices [10].

### 2.3.2 Strain Measurement

A proven method of measuring the loading of an airframe in-flight is by strain-gauge measurements. Strain gauges can be used to obtain the structural loads on a wide variety of different aircraft components, such as lifting and control surfaces, landing gears and empennage assemblies. By combining strain-gauge measurements with knowledge of the weight distribution of the aircraft and flight parameters such as acceleration, it is also possible to derive the aerodynamic loads acting on the structure. [18]

Strain gauges are devices which change their electrical resistance in response to strain [14]. The change in resistance is caused by the change in length, cross-sectional area and resistivity of the conductors on the gauge in response to mechanical strain [14]. The gauges are made of an insulating backing material and an appropriately shaped resistance wire or foil on top [14]. Gauges are attached to the structure using e.g. cyanoacrylate adhesive with the paint and other coatings removed from the material [21]. Different gauge geometries exist for measurement of uniaxial, biaxial, triaxial and shear strain. Figure 2.2 shows an example of a strain gauge for uniaxial strain measurement.

![Figure 2.2: A typical linear strain gauge for uniaxial strain measurement.][22]
A very common source of error in strain-gauge measurements is temperature. If the temperature expansion coefficient of the gauge differs from that of the measured material, a change in temperature will produce an apparent strain in the gauge, causing the output to drift [23]. A similar effect can be observed due to the resistivity of the gauge material changing with temperature [23]. Issues related to the physical installation of the gauge, such as environmental effects and quality of the bond, may also cause drift over time. Taking a zero-reference measurement on the ground before each flight is common practice [18].

Several solutions for temperature compensation exist. Some strain gauges have a temperature sensor attached, which can be used to mathematically compensate for temperature effects [24]. It is also possible to use a dummy gauge of the same type, mounted on an unstressed part of the same material, to provide electrical temperature compensation [14]. Furthermore, self-temperature-compensation gauges may be used [23]. These gauges are manufactured with the resistance temperature coefficient matched to a certain material so that the thermally induced apparent strain is minimal when the gauge is bonded to that particular material [23].

2.4 Properties of Load Spectra

2.4.1 Variable or Constant Amplitude

Load spectra can be divided into constant-amplitude (CA) and variable-amplitude (VA) spectra. As implied by the names, a CA spectrum contains cycles of only one amplitude, whereas a VA spectrum can contain cycles of many amplitudes. In CA loading the mean load may be non-zero, but it must be constant. Under VA loading there are no limitations on the mean load and it may change from cycle to cycle [1].

Cycles in a load sequence are commonly defined by their stress range and stress ratio. The stress range $\Delta \sigma$ is calculated as

$$\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}},$$

where $\sigma_{\text{max}}$ is the maximum stress and $\sigma_{\text{min}}$ is the minimum stress during the cycle. The stress ratio $R$ is defined as

$$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}.$$  \hspace{1cm} (2.5)

When the stress ratio and range are known, the cycle is fully defined and the mean stress $\sigma_m$ can be calculated as

$$\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \Delta \sigma \cdot \frac{1 + R}{2(1 - R)}.$$ \hspace{1cm} (2.6)

2.4.2 Exceedance Diagrams

In some cases the exceedances of a certain load are measured in order to obtain simple information on the fatiguing effects of the load spectrum. A classic application of exceedance counting is the fatigue meter described in section 2.3.1. The fatigue meter counts the normal acceleration of the aircraft in real time using the level-crossing method, which is further described in section 2.5.1. The device records the exceedances of the load beyond preset load levels and omits small cycles that fall in between load levels [16]. The exceedance data can then be used to draw an exceedance diagram similar to the example shown in figure 2.3.
Figure 2.3: A typical exceedance diagram showing load factor exceedances during 1000 flight hours. The diagram was generated using the vertical acceleration history contained in dataset [25].

Functionality of fatigue meters or counting accelerometers has also been extended to rainflow counting, which records both the range and the mean of the normal acceleration cycles [16]. The recorded data can then be used to form a range-exceedance diagram, which shows the exceedances for cycles independent of the mean load.

Exceedance diagrams are also used in the fatigue design process of aircraft. Using a step-wise approximation, the occurrences of different load levels can be calculated from the exceedance diagram, allowing composition of an equivalent load spectrum [26]. Furthermore, the shape of an exceedance diagram gives quick insight into the load spectrum contents and the diagrams can be used to evaluate the effect of different service and mission types on the fatigue of aircraft structure. Spectra, for which the exceedance diagram shows a steep slope around the region of the peak, are termed “steep” and contain a large amount of small cycles which may fall below the fatigue limit of the material [1]. This is typical of gust-dominated load spectra. The opposite of a steep spectrum is a flat spectrum, which contains a large amount of high loads, as is typical of maneuver-dominated load spectra, such as e.g. fighter aircraft wing spectra.

2.4.3 Multiaxial Fatigue and Proportionality

The most common and basic type of fatigue loading is a time-variable uniaxial load, which in simple geometries results in a mostly uniaxial stress field. Alternatively, the loading may consist of combined tension, torsion and/or shear and thus be termed multiaxial [13].

Another important differentiation in fatigue is that between proportional and nonproportional loading. If the time-varying stress state in the material is such that the principal stress axes do not rotate with respect to the component, the loading is termed proportional [27]. Loading which does not fulfill this requirement is termed nonproportional. Nonproportional stress often occurs in notches or joints and can be caused by combined loading where the different load components are nonproportional or out of phase [13].

In practice, multiaxial, nonproportional loading significantly complicates the estimation of fatigue life and crack growth. Starting with stress-history filtering and cycle counting, the standard methods used for uniaxial and proportional load histories such as the rainflow-counting algorithm described in section 2.5.3 and the racetrack filter introduced in section 2.12 are unsuitable as they do not accurately capture the stress and strain path between two
reversals [15]. Furthermore, both algorithms filter out all points that are not load reversals. If these algorithms are applied to the individual stress or strain components, they may filter out points that do not constitute a reversal of a single strain component, but which may make up a reversal according to multiaxial fatigue theory. [15]

The Wang-Brown rainflow algorithm introduced by Wang and Brown in 1996 [28] is a variation of the rainflow algorithm intended for counting nonproportional load histories [29]. It introduces the concept of relative von Mises strain (or stress), which is a differential form of the original von Mises strain (stress) [29]. In contrast to the classic von Mises strain, the relative strain has a sign and can thus be used for cyclic fatigue calculations. A multiaxial racetrack-filtering algorithm based on similar concepts has been proposed by Meggiolaro et al. [15].

2.5 Cycle Counting

To be able to estimate the fatigue effect of complicated VA load histories, the time-series data should first be cycle counted. The aim of cycle counting is to identify individual loading cycles and their amplitudes in the data. Some cycle-counting methods also record the mean or minima and maxima of the cycles, which can later be used to calculate the stress ratio $R$. If time-dependent effects such as loading rate or environmental corrosion are to be taken into account, the start and end times of the counted cycles should also be stored.

The existing cycle-counting methods can be divided into one- and two-parameter methods. Two-parameter methods retain more information about the original load cycles than one-parameter methods and thus allow the actual loading history to be recreated more faithfully. While one-parameter methods record only the load amplitudes, two-parameter methods represent the load cycles as ranges, which consist of either a minimum and a maximum or an amplitude and a mean value. Table 2.1 shows some common cycle-counting methods and the information retained by them. The methods and their limitations are described in greater detail in the following sections.

<table>
<thead>
<tr>
<th>Counting method</th>
<th>Parameters</th>
<th>Retained information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exceedances</td>
</tr>
<tr>
<td>Level crossing</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Peak-valley</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Rainflow</td>
<td>2</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 2.1: Common counting methods and cycle parameters retained by them.

2.5.1 Level-Crossing Method

In the level-crossing method, a reference level is defined and the load range is divided into discrete levels around the reference level [1]. In the case of a symmetric spectrum, the reference level is commonly defined as zero, but different definitions are possible. For nonsymmetric loading it is common to define the reference level as the mean of the load-time history data [14]. Every time the load history crosses a level on the positive side of the reference with a positive slope, the level-crossing count of that particular level is incremented by one. On the negative side of the reference level the method correspondingly counts all level crossings that occur with a negative slope of the load-time history. In this example the load minima and maxima were truncated to the amplitude levels that were defined for counting, but it is also possible to use the middle value of the interval where the peak occurs as suggested in the ASTM E1049-8 standard [30]. The method is visualized in figure 2.4, where the red dots show the registered level crossings.
2.5. Cycle Counting

Using the level-crossing counts, it is possible to create exceedance diagrams as well as a fatigue-loading sequences. To construct a fatigue-loading sequence the counted level crossings are ordered to form cycles starting from the highest peak and most negative valley with the cycles decreasing in amplitude towards the end of the sequence \[30\]. An example sequence constructed using the counts from figure 2.4 is shown in figure 2.5.

Applying the level-crossing method to non-symmetric load-time histories gives rise to a number of problems, the most obvious being that the cycle sequence is lost completely. Although the cycles were ordered from largest to smallest to obtain maximum damage, this may not be a conservative assumption in all cases and may give unrealistically low lifetimes in other cases, depending on what load-interaction and retardation effects are considered. An alternative method that may be more suitable for some load histories is to order the cycles randomly \[30\].

Furthermore, some of the cycles are contained on the positive or negative side of the reference level, meaning that the load path does not cross the reference level between each peak.
and valley. This results in the method counting maxima that in reality occur only due to small variations in load and that cannot be associated with full cycles to the counted load level [1]. An example of such an ambiguous cycle can be observed in figure 2.4 around time 24. The resulting cycle appears as a dashed line in the constructed sequence shown in figure 2.5. The issue is exacerbated if the number of load levels used for counting is increased as the narrower intervals lead to even smaller variations in load, or even noise, being counted as significant cycles. On the other hand, it is desirable to use a narrow spacing of load levels as this increases the accuracy of the counted load minima and maxima. One way to alleviate this problem is to introduce hysteresis in the counting algorithm. This is common in fatigue counting meters and allows accurate exceedance counts while discarding small variations in load.

An irregularity factor \( k \) has been proposed as a way to characterize the load-time history. The irregularity factor can be calculated as

\[
   k = \frac{n_{pk}}{n_{ref}},
\]

where \( n_{pk} \) is the total number of peaks and valleys and \( n_{ref} \) is the number of crossings of the reference level [1].

If \( k \) equals 1, the load-time history exceedances can be calculated using the level-crossing method and the cycle amplitudes counted by the method are valid. A value of \( k > 1 \) implies that the results obtained using the method are not accurate, and a two-parameter counting method should be considered. [1]

### 2.5.2 Peak-Valley Method

The peak-valley method defines a reference level and counts the peaks occurring above this level as well as the valleys occurring below it. Peaks on the negative side and valleys on the positive side of the reference level are ignored [14]. While the method counts the amplitudes of the peaks and valleys accurately, it otherwise has similar limitations as the level-crossing method. The cycle sequence is lost and false cycle counts occur due to load fluctuations unless the irregularity \( k \) of the load-time history equals one or sufficient hysteresis is introduced. Figure 2.6 shows the maxima and minima in the example time history, as identified by the peak-valley method.

![Figure 2.6: Maxima and minima of a load-time history as identified by peak-valley counting.](image-url)
2.5. Cycle Counting

Mean-crossing peak counting is another variation of peak-valley counting that can be used to eliminate the false cycle count caused by small load fluctuations. The method tracks crossings of the reference or mean load level and keeps only the highest peak or lowest valley that occurs between two mean crossings \[31\]. While this method does not count false cycles even in irregular time histories, it has the pitfall of eliminating all but the largest load cycle between mean crossings \[31\]. Depending on the time history, the smaller load fluctuations that are superimposed on this large cycle may cause much of the damage \[1\]. This is especially true for time histories with few mean crossings relative to the total number of cycles.

Range counting also works similarly to the peak-valley method, except that no reference level is used and all peaks and valleys are kept. The ranges between successive peaks and valleys are then counted and given positive and negative signs according to their slopes \[14\]. The range counting method is thus able to accurately capture all cycle halves. These half-cycles can then be used to form full cycles using different methods.

2.5.3 Rainflow-Counting Method

The rainflow-counting method is a two-parameter counting method originally developed by Matsuichi and Endo \[32\] in 1968. This method is considered to give good damage estimates for complex load histories \[33\]. Although the method is not fully based on rational arguments, the handling of small intermediate cycles as well as the improved calculation of ranges between low minima and high maxima values are advantages of this method \[1\]. Variations of the rainflow method are often referred to as range-pair methods \[30\].

The rainflow method defines cycles as closed stress or strain hysteresis loops \[34\]. The algorithm moves over the peak-valley history and keeps track of the peaks and valleys until a pair that forms a closed loop is found \[34\]. The identified range is then counted and the corresponding peak and valley pair is discarded from the history \[34\]. This process is repeated until the algorithm runs out of data and all unpaired peaks and valleys are kept track of until they can be paired to form a rainflow range \[34\]. Due to the paired peaks and valleys not always being subsequent in the load history, some information about the load sequence is lost in this process. How well the load sequence is retained depends on the particular load history. Often the sequence stays relatively intact and rainflow counting is frequently applied to analysis which takes load-sequence effects into account.

When the algorithm runs out of data, it is possible that some peaks and valleys are left as they have not formed closed hysteresis loops \[35\]. These unmatched peaks and valleys are often referred to as “half-cycles” \[35\] or as the rainflow residual. If the loading history is periodic, it is possible to reorder the sequence to start with the largest peak or valley, wrap around the end of the data, and add a peak or valley equivalent to the starting extremum at the end of the reordered sequence \[36\]. This reordered sequence closes all hysteresis loops and thus forms only full cycles when rainflow counted \[36\].

If the loading history is not periodic or cannot be closed e.g. due to real-time counting of cycles, the residual must be handled appropriately. Sutherland \[37\] describes three practical methods of handling the unclosed residual cycles that are not counted by the rainflow algorithm: The cycles may either be omitted, be counted as half-cycles or be counted as full cycles. Counting the residual as half-cycles is considered the recommended practice whereas counting the residual as full cycles is the most conservative approach \[37\]. If there is a roughly equal amount of up and down half-cycles, it is also possible to use the half-cycles to form approximate full cycles \[38\].
2.6 Fatigue Induced by Cyclic Loading

S-N curves, also referred to as Wöhler curves, are a tool used to describe the fatigue properties of a material [1]. An example of typical S-N curves for aluminum is shown in figure 2.7. The curves contain data for maximum stress or stress amplitude against the number of cycles to failure and they are usually generated experimentally by testing smooth or notched specimens under CA loading [1]. S-N curves are often generated for different stress ratios, but the resulting amount of combinations of $R$ and stress level requires a large amount of samples to be tested, which makes the process expensive. Furthermore, tests at the same stress level always show a level of scatter, which increases the number of tests required.

![Figure 2.7: S-N curves for unnotched ($K_t = 1$) and notched ($K_t = 3$) 7050-T7451 aluminum plate tested in the longitudinal direction. Stresses are based on the net section. The best-fit curves are adjusted for the nonconstant variance of the long-life fatigue data using a method described in [39].](image)

The scatter of the data as well as the effect of notch sensitivity and load ratio are well visible in the figure. Although aluminum does not have a true fatigue limit, an effective fatigue limit at which the material withstands a very large number of stress cycles can be defined. On the S-N curve this limit can be observed as the curves nearly leveling out and runout datapoints (denoted by arrows) appearing beyond $10^8$ cycles.

If only a limited number of stress ratios has been tested, a correction can be applied to obtain the life at other stress ratios or levels of mean stress. Some common empirical correction approaches are the Goodman, Gerber and Söderberg models, of which the Gerber and Goodman models have been found to agree best with experiments [40]. Figure 2.8 shows the effect of mean stress on the fatigue limit of 7050-T7451 aluminum material as predicted by these methods.

![Figure 2.8: Effect of mean stress on the fatigue limit of 7050-T7451 aluminum material as predicted by the Gerber and Goodman models.](image)
2.6. Fatigue Induced by Cyclic Loading

The life under CA loading can be directly read from an S-N diagram, but this is not possible under VA loading, as the maximum stress varies from cycle to cycle. To obtain a total-life estimate for a load spectrum containing different cycles, a cumulative damage model can be applied. A simple and practical cumulative damage model is the Palmgren-Miner rule, which states that the damage caused by a cycle of certain stress level is inversely proportional to the cycles to failure at the same stress level. The total damage caused by different cycles is then found as the sum of the damage from the cycles of different stress levels. The Miner rule can be expressed as in equation 2.8, where $D_{fat}$ is the fatigue life utilization ratio, $n_i$ is the number of cycles at stress level $σ_i$, $N_i$ is the number of cycles that causes failure at stress level $σ_i$ and $k$ is the total number of different stress levels considered. [42]

$$D_{fat} = \sum_{i=1}^{k} \left( \frac{n_i}{N_i} \right)$$  \hspace{1cm} (2.8)

In Miner’s rule $D_{fat} = 1$ is defined as the limit for failure. However, in practice it has been found that failure can occur both at $D_{fat} < 1$ and $D_{fat} > 1$ depending on factors such as load sequence [40]. Miner’s rule does not account for any load-sequence effects so the results are only valid if such effects are insignificant. As discussed in sections 2.7 and 2.8, the life of a component can be divided into a crack-initiation and a crack-growth phase. If for example a load sequence consisting of high loads followed by small cycles is applied, the high loads may initiate the crack and the following low loads then grow it to failure at $D_{fat} < 1$ due to the large cycles having a greater effect on the total life than they would if they occurred randomly or at the end of the sequence [40]. Furthermore, Miner’s rule neglects plastic effects, the resulting residual stresses and crack-retardation effects such as those discussed in section 2.10.

A relative Miner rule, where $D_{fat} = 1$ is replaced by a lower value, has also been proposed for use with VA spectra. To obtain good predictions it is necessary to calibrate the value of $D_{fat}$ at failure using test results obtained from realistic tests on components. Alternatively it is possible to think of the lowered value of $D_{fat}$ at failure as a factor of safety (FoS) on the life prediction, but especially in VA loading even very high FoS do not guarantee a safe life as failures have been recorded at values of $D_{fat}$ as low as 0.15.[1]

---

**Figure 2.8**: Effect of mean stress on fatigue limit of 7050-T7451 aluminum as predicted by different correction models. Material properties $σ_{tu} = 510.2$ MPa, $σ_{ty} = 441.3$ MPa and $σ_{f,R=-1} ≈ 137.9$ MPa [41].
2.7 Crack Initiation

Crack initiation is the process of formation of fatigue cracks from microscopic discontinuities. The initiation phase includes growth of microcracks before they reach a macroscopic scale [1]. Cracks always originate at particularly vulnerable locations, such as stress concentrations, inclusions or grains with unfavorable orientations relative to the stress field [43]. The initial formation or nucleation of cracks is caused by cyclic shear strains which cause nucleation of slip bands along the preferred slip planes of the grains in the material [27]. Continued cyclic loading of the material causes development of further parallel slip lines which penetrate the grain boundaries and eventually form a microcrack [43]. Microcracks can also exist in a material before any loading has been applied, as is common in e.g. welds [40].

Plastic deformation is less constrained in surface grains and the surface of a material often has a less homogeneous stress distribution due to notches, surface roughness or corrosion [1]. For these reasons fatigue is a material surface phenomenon during the crack initiation period [1]. The initial orientation of a fatigue crack correlates with the planes of maximum shear stress [43].

2.8 Crack Growth

Crack growth is the phase of fatigue damage occurring after crack initiation. No quantitative definition for the transition from the initiation phase to the growth phase exists, but the transition can qualitatively be defined as the time when the growth of a microcrack no longer depends on the free surface conditions [1].

Crack growth can occur under three different loadings: Mode 1, Mode 2 and Mode 3 loading. These different modes are visualized in figure 2.9. Mode 1 loading, also referred to as the opening mode, imposes an opening load on the material and is the most common mode of loading. Mode 2 or in-plane shear loading is common on surfaces, whereas Mode 3 out-of-plane shear loading occurs at maximum crack depth [27].

![Figure 2.9: The three modes of loading in crack growth. Redrawn from [44].](image)

In linear elastic fracture mechanics (LEFM), the magnitude of the stress around the crack tip is proportional to the stress intensity factor \( K \). The general definition of the stress intensity factor is given in equation 2.9, where \( \sigma_{\infty} \) (or \( \tau_{\infty} \) for mode 2 and 3 loading) is the remote stress, \( a \) is the crack length and \( f(G) \) is a correction factor that depends on the specimen and crack geometry [45].

\[
K = f(G)\sigma_{\infty}\sqrt{\pi a}
\]  

(2.9)

The correction factor \( f(G) \) is dimensionless and is frequently referred to as the geometry factor \( \beta \). The geometry factor depends on geometric ratios, and beta factors for different
2.8. Crack Growth

stress raisers such as holes or notches can be found in literature. The superposition principle can also be applied in calculating stress intensity factors for geometries which are superpositions of cases for which solutions are known [1]. Many practical problems involving the calculation of stress intensity factors are three-dimensional but can be simplified for 2D analysis by considering stresses and deformations in the crack plane. Finite element analysis of the problem is often required in order to obtain the necessary crack-plane stress data [40].

During a loading cycle, the stress intensity factor ranges from the lowest value in the cycle \(K_{min}\) to the highest value \(K_{max}\). The stress intensity factor range can be calculated as

\[
\Delta K = K_{max} - K_{min} \quad (2.10)
\]

When \(\Delta K\) exceeds the threshold stress intensity factor range \(\Delta K_{th}\), an existing flaw propagates in the material [46]. The change in crack length due to a loading cycle can then be calculated if \(\Delta K\) is known and crack growth rate data (\(\frac{da}{dN}\) vs \(\Delta K\)) at the correct stress ratio \(R\) is available. Figure 2.10 shows a typical crack growth rate curve for 7050-T7451 aluminum.

![Crack Growth Rate Curve](image)

Figure 2.10: A typical crack growth rate curve showing the three regions of crack growth. The curve is generated using the NASGRO equation for aluminum 7050-T7451 at stress ratio \(R = 0\).

2.8.1 Region I

The crack growth rate curve shown in figure 2.10 can be divided into three different regions [40]. Region I is the region where threshold effects dominate the crack growth. A variety of factors such as the stress ratio [1] and environment [47] influence the threshold and the crack growth rate around it. Most importantly however, the published \(\Delta K_{th}\) values are generally developed using long-crack data, the validity of which is questionable for short cracks [45]. The growth of short cracks is discussed further in section 2.11.

2.8.2 Region II

The second region is often referred to as the linear region or the Paris region. Many practical engineering structures operate in this region of the crack growth rate curve [40]. It is characterized by an approximately linear growth on the log-log scale which can be described by the widely accepted Paris crack-growth equation proposed in the 1960s [45]. Equation 2.11 is the Paris equation, where \(C\) and \(n\) are material parameters that can be found in literature or by performing tests [45].

![Paris Equation](image)
2.8. Crack Growth

\[
\frac{da}{dN} = C(\Delta K)^n
\]  

(2.11)

The Paris equation does not account for the stress ratio, which has an effect on crack growth similar to that described for the S-N curve approach in section 2.6. In general, a higher stress ratio causes a higher crack growth rate but the effect is strongly dependent on the material [45]. Various models such as the Forman equation and the Walker equation exist to correct this deficiency and these are discussed further in section 2.9.

2.8.3 Region III

Region III is the region of rapid crack growth. In this region the crack growth becomes unstable and once the stress intensity factor reaches the critical value \( K_c \), fracture occurs [45]. The critical stress intensity factor \( K_c \) depends on the material thickness and the loading mode [40]. In LEFM, under mode 1 loading and plane strain conditions, it is equal to the plane strain fracture toughness \( K_{Ic} \), which is a common material parameter [48]. The point of transition from region II to region III is dependent on material properties such as yield strength and loading properties such as \( K \) and \( R \) [45]. In common engineering problems region III does not have a significant effect on the lifetime of a component as crack propagation occurs in a much shorter time than in regions I and II [45].

2.8.4 Crack Propagation Direction and Mixed Mode Loading

The propagation direction of a crack must be estimated in order to be able to calculate the crack-growth life. In real-life components the stress state is often complex and the loading of the crack may be a combination of tension and shear [48]. Under combined mode 1 and mode 2 loading conditions, the crack growth direction can be determined using the maximum principal stress criterion or the strain energy density criterion, which both give essentially the same result [40]. According to the maximum principal stress criterion a fatigue crack will grow perpendicularly to the direction of the maximum principal stress [48].

As a crack extends the loading condition may change due to factors such as change in net section, change in load eccentricity or curving of the crack path. Some experiments conducted under in-phase mixed mode 1 and 2 loading have shown that as fatigue cracks grow, they tend to curve in a way that changes the loading to pure mode 1 [48]. Although the global propagation direction is perpendicular to the maximum principal stress, cracks tend to locally grow in the directions of maximum shear stress [14]. Especially during early crack growth (region I of the crack-growth curve), the growth direction is also affected by the microstructure of the material [14], with the grain boundaries offering some resistance to crack growth [43].
2.9 Crack-Growth Models

2.9.1 Walker Model

The Walker equation, shown here as equation 2.12, is an extension of the Paris equation and was developed to take into account the effect of stress ratio on crack growth. Like the Paris equation, the Walker equation does not accurately model crack growth in the near-threshold region (region I) and unstable region (region III). The material parameters $C$ and $n$ in the Walker equation are equivalent to those found in the Paris equation and the Walker exponent $m$ is an additional material parameter controlling the $R$-dependent shift of the curve. \[49\]

\[
\frac{da}{dN} = C[(1 - R)^{m-1} \Delta K]^n \tag{2.12}
\]

2.9.2 Forman Model

The Forman equation developed in 1967 can be used to model the crack growth in regions II and III as it takes into account the fracture toughness of the material. The model also accounts for the stress ratio $R$, which has the most effect on growth rate in regions I and III. Threshold effects on crack growth are not modeled by the Forman equation. Equation 2.13 is the Forman crack-growth equation, where $C$ and $n$ are material parameters. These material parameters differ from those used in the Paris equation. \[40\]

\[
\frac{da}{dN} = C(\Delta K)^n \frac{1}{(1 - R)K_c - \Delta K} \tag{2.13}
\]

2.9.3 NASGRO (FNK) Model

The Forman-Newman-de Koning (FNK) equation, also known as the NASGRO equation due to its use in the NASGRO crack-growth software developed by NASA \[49\], is a crack-growth equation capable of modeling all three regions of the crack-growth curve \[50\]. The NASGRO equation also includes fatigue crack closure analysis which calculates the effect of $R$ on the crack growth rate \[51\].

Equation 2.14 is the FNK / NASGRO equation, where $K_c$ is the critical stress intensity factor and $C$, $n$, $p$, $q$ are empirical constants \[50\].

\[
\frac{da}{dN} = \frac{C(1 - f)^n(1 - \frac{\Delta K_{op}}{\Delta K})^p}{(1 - R)^n(1 - \frac{\Delta K}{(1 - R)K_c})^q} \tag{2.14}
\]

The function $f$ in equation 2.14 is Newman’s crack opening function which is defined by equation 2.15 \[50\]. This function is responsible for calculating the crack closure due to plasticity \[51\].

\[
f = \frac{K_{op}}{K_{max}} = \begin{cases} 
A_0 + A_1 R + A_2 R^2 + A_3 R^3, & R \geq 0 \\
A_0 + A_1 R, & -1 \leq R < 0
\end{cases} \tag{2.15}
\]

$K_{op}$ is the opening stress intensity factor and $K_{max}$ is the maximum stress intensity factor during a loading cycle. $A_0$, $A_1$, $A_2$ and $A_3$ are coefficients determined from equations 2.16 to 2.19 in which $\sigma$ is the flow stress, $S_{max}/\sigma_0$ is the maximum stress level ratio and $\alpha$ is a constraint factor that varies from 1 (for plane stress) to 3 (for plane strain). \[50\]

\[
A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)\cos(\pi \frac{S_{max}}{2\sigma_0})^{1/\alpha} \tag{2.16}
\]

\[
A_1 = (0.415 - 0.071\alpha) \frac{S_{max}}{\sigma_0} \tag{2.17}
\]
\[ A_2 = 1 - A_0 - A_1 - A_3 \]  
\[ A_3 = 2A_0 + A_1 - 1 \]

The threshold stress intensity factor \( \Delta K_{th} \) for different stress ratios can be approximately calculated by equation 2.20, where \( \Delta K_0 \) is the threshold stress intensity factor at \( R = 0 \) \[50\].

\[ \Delta K_{th} = \Delta K_0 \frac{4}{\pi} \arctan(1 - R) \]  
(2.20)

The critical stress intensity factor \( K_c \) is dependent on the specimen thickness. It can be calculated by equation 2.21, where \( K_{lc} \) is the mode 1 plane strain fracture toughness, \( t \) is the specimen thickness and \( t_0 = 2.5(K_{lc}/\sigma_{ty})^2 \). Here \( \sigma_{ty} \) is the tensile yield strength of the material and \( A_k \) as well as \( B_k \) are constants. \[50\]

\[ \frac{K_c}{K_{lc}} = 1 + B_k \exp\left[-\left(A_k \frac{t}{t_0}\right)^2\right] \]  
(2.21)

2.9.4 Tabular Lookup

Crack growth rate data obtained by testing can also be presented in the form of curves defined by tabulated data. This method represents the most direct way of using test data for crack-growth prediction. The lookup tables usually contain data for curves spanning from the threshold to the critical stress intensity as well as curves for different stress ratios. As the amount of data points along a curve and data for different stress ratios is always limited, the data can be inter- and extrapolated with regards to both. As an example, the AFGROW software, discussed further in section 3.3.5, uses the Walker equation on a point-by-point basis to inter- and extrapolate data for different stress ratios using the nearest two \( R \) curves \[49\].

2.9.5 Short Crack Growth Models

As will be explained in section 2.11, the accurate prediction of short crack growth is difficult. Nevertheless accurate short crack growth models are indispensable in the design and maintenance of modern airframes \[52\]. Accurate models for specific applications can be obtained by using direct fractography data on short crack growth \[52\]. The formation of such models however requires extensive and costly experiments.

Alternatively it is possible to modify existing long-crack data to cover the short crack region. While this approach is not as accurate as full fractography based models, it may still be sufficient for early design phases, especially when supported by some fractography data.

It has been found that short crack growth curves for aerospace aluminum and titanium follow a Paris-like shape in the threshold region with no clear threshold \[53\]. Thus the short crack modifications extend the Paris region (region II) curve further into the threshold region (region I), lowering the threshold in the process. A detailed example of such a modification is presented in section 3.3.4 in the form of the modified NASGRO model which is used in this study. This type of modification is not generally applicable as many materials do not exhibit the Paris-like relationship in the threshold region \[53\].
2.10 Crack-Retardation and Load-Interaction Effects

Load interaction effects occur when subsequent cycles differ in amplitude [48]. Under VA loading these effects complicate the prediction of crack growth considerably [26]. Meggiolaro and de Castro list the main mechanisms involved in crack closure as follows:

- “Plasticity-induced crack closure”
- “Blunting and/or bifurcation of the crack tip”
- “Residual stresses and/or strains”
- “Strain-hardening or strain-induced phase transformation”
- “Crack face roughness and oxidation of the crack faces” [54].

As the crack tip is surrounded by a plastic region, a fatigue crack always propagates by splitting deformed material [54]. This plastic region contains residual strains which act to close the crack after tensile cycles have been applied, which in turn causes an increased resistance to further crack growth [54]. This effect is a form of crack retardation explained by the Elber mechanism (plasticity induced crack closure) and is most significant when large tensile overloads are present in the load spectrum [1]. The Elber mechanism defines a crack-opening stress level $\sigma_{op}$ [1]. If the applied stress is lower than $\sigma_{op}$, the crack tip remains closed due to the plasticity induced stresses caused by previous large load cycles [1]. Only the parts of the stress cycles during which the crack is opened contribute to further crack extension [1]. Similarly, large compressive underloads may cause a crack acceleration effect on subsequent cycles [54]. Under CA loading the same crack-closure mechanism occurs due to plastic deformation near the crack tip, but the effect on crack growth rate is not observable as it is steady [26]. Figure 2.11 shows a simple example of crack retardation in a cycle sequence containing a tensile overload.

![Figure 2.11: Effect of a tensile overload on the crack growth rate. Adapted from [55].](image)

The extent of crack retardation depends on the ratio of the amplitude of the overload to the amplitude of the following cycles, as well the number of subsequent overload cycles or the hold period at load. Sufficiently large overloads may arrest crack growth completely, but the retardation effect may also wear off due to compressive cycles or hold periods at zero stress. [26]
2.10.1 Load-Interaction Models

Various models for estimation of load-interaction effects exist. The models can be divided into three categories: yield-zone models, crack-closure models and strip-yield models [1]. The following introduces some well-known load-interaction models as well as considerations regarding their use in practice.

The Willenborg and Wheeler models are yield-zone models developed to explain crack retardation due to overloads [1]. Both models calculate the size of the plastic zone induced by an overload [54]. During each subsequent cycle, the size of the plastic zone caused by the new cycle is compared to the boundaries of the plastic zone induced by a previous overload [1]. As long as the plastic zones caused by new cycles stay within the plastic zone of a previous overload, retardation of crack growth is applied with the magnitude of the retardation effect decreasing as the crack progresses through the overload zone [54]. The way the strength of the retardation effect is calculated differs between the models and especially the methods used by the Willenborg model are considered to be in disagreement with the present understanding of crack closure [1]. Neither model is capable of predicting crack acceleration [1]. Despite some fundamental limitations both models have in predicting crack growth under VA loading [1], especially the Willenborg model has been applied in aircraft crack-growth studies and software.

Many crack-closure models have been developed for the purposes of aircraft fatigue studies and flight-simulation loading in particular. These models work by predicting a crack-opening stress level \( \sigma_{op} \) for each cycle in the load history and applying retardation according to the Elber mechanism. The different available crack-closure models show good agreement between predictions and testing, but also include empirical parameters and adjustments not completely founded in theory. In practice, the accuracy of these models is often limited to a certain aircraft load spectrum and specific materials. [1]

2.11 Short Cracks

Cracks can be classified as “small” or “short” according to different definitions. The different definitions are generally used to refer to cracks of different lengths, the growth of which is consequently influenced by different factors such as material grain boundaries or plastic zones and notches [27]. Some common definitions of short cracks are shown in table 2.2.

<table>
<thead>
<tr>
<th>Crack type</th>
<th>Size approximately equal to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microstructurally small</td>
<td>material grain size</td>
</tr>
<tr>
<td></td>
<td>interparticle spacing</td>
</tr>
<tr>
<td>Mechanically small</td>
<td>crack tip plastic zone</td>
</tr>
<tr>
<td>Physically small</td>
<td>&lt;1 mm</td>
</tr>
<tr>
<td></td>
<td>NDT detection limit</td>
</tr>
</tbody>
</table>

Table 2.2: Different types of short cracks and their sizes. [27]

In this work the term “short crack” will be used to refer to physically small cracks. The growth of short cracks in general cannot be explained by the same mechanisms that explain long crack growth. The stress and strain fields around short cracks are not solely defined by the stress intensity factor and continuum mechanics assumptions such as homogeneous material are not valid. [27]

Short cracks are known to grow at stress intensities far below the long crack threshold [27]. Although some approximations in form of modified crack-growth curves exist, the prediction of short crack growth especially under VA loading remains a complex problem influenced by many factors [1] [56]. While short cracks generally grow much faster than long cracks under the same loading conditions, the crack extension rate may be unsteady and unexpected due to
Removing undamaging cycles from the load spectrum is a key challenge in preparation of fatigue test spectra. Several common methods exist for this purpose. A very simple approach is the deadband filter, which removes all cycles that occur within a certain amplitude band, set by a minimum and a maximum value. This method is useful for filtering out small cycles in a spectrum with a well-defined mean level, but is difficult to apply to complicated VA load histories such as aircraft flight loading. The method may however be used to filter out small loads e.g. when the aircraft is taxiing on the ground.

The opposite of the deadband filter is a clipping filter, which removes loads above or below certain thresholds. This filter serves a different purpose and can be used to remove very high tensile loads (overloads) or compressive loads (underloads) from the load history. Removing such loads may be necessary in order to keep the test conservative due to the load interaction effects described in section 2.10.

A more sophisticated filtering method is the Racetrack-filtering algorithm originally proposed in 1973 by Fuchs et al. [57]. The racetrack algorithm can be used to reduce the number of cycles in the load history with minimal impact on fatigue damage [35]. Racetrack filtering always retains extreme points in the load history and thus does not truncate cycle ranges. It also keeps the mean and sequence of the retained load cycles intact [35].

The racetrack algorithm first removes all points in the load history that are not load reversals. Subsequently, the algorithm removes all load reversals that form cycles smaller than the filtering threshold range $R_{th}$. The remaining load reversals that were separated by small load fluctuations in the original data are then connected together to form larger cycles, always retaining the most extreme points. This process can be visualized by offsetting the original data in both the positive and negative direction as shown in figure 2.12, so that a “racetrack” of width $R_{th}$ is created. Whenever a car driving on this track has to make a change in heading between northerly and southerly, a point to be kept in the filtered data is identified [35].

---

Figure 2.12: Racetrack filtering of a VA load history, $R_{th} = 0.5$. 
Alternatively, the process can be visualized by a peg that moves freely in a vertical slot of length $R_{th}$ and along the load history, until it reaches the end of the slot [15]. When this occurs, the whole slot moves and the point is kept in the filtered data [15]. Afterwards, all points that are not load reversals are removed. Vertical bars associated with the slot analogy are also shown in figure 2.12.

In addition to the presented amplitude-based filters, frequency-domain filtering utilizing e.g. the wavelet transform has also been proposed for fatigue applications. This type of filtering is of particular interest when measurement-induced noise or fluctuations have to be removed from the signal. Furthermore, it can be used to decompose signals which include load components at different frequencies. [58]
3 Method

3.1 Spectrum Reduction Study

Reducing the in-flight measured loads data is generally required to obtain a practical load test spectrum, yet the damage produced in the test must be kept realistic. To study means of generating a load test spectrum as well as the effect of filtering on the damage caused by the load sequence, a spectrum reduction study was set up. The objective of the study is to find acceptable methods for load-history formation, data reduction and cycle counting. Of particular interest is the level of filtering that can be applied to load spectra without significantly changing the damage caused by the spectra. A simplified overview of the process from in-flight measured loads data to a fatigue test load sequence is shown in figure 3.1.

![Figure 3.1: Overview of fatigue load sequence development from in-flight measured loads data.](image)

The following sections discuss the different methods that were considered for the study, their tradeoffs as well as general best practices. Attention is paid to make the study as generalizable as possible and suggestions for an experimental validation program are provided.
3.2 Dataset

The dataset used in this study contains data gathered during a flight test program containing a total of 22 flights corresponding to a duration of 25.6 hours. The aircraft was a two-seat, piston-powered trainer aircraft built primarily from composite materials. The flights in the dataset contain a wide variety of different aircraft usage, ranging from cruise flight to aerobatics and upset maneuvers. The data consists of strain-gauge measurements taken from multiple locations on the aircraft as well as accompanying flight parameter and temperature information. The strain measurements from the various linear and shear gauges are recorded at a high sampling rate of 1280 Hz.

3.2.1 Determination of Force Components

To derive interface loads from the measured strains, a correlation matrix was used. The correlation matrix supplied with the dataset was formed using the results of ground calibration tests, during which the aircraft components were loaded according to different calibration load cases and their combinations. For this study, the load spectrum of the horizontal stabilizer front right shear pin was used. As shown in figure 3.2, the horizontal stabilizer is connected to the fuselage by four shear pins, which transmit all aerodynamic forces caused by the lift, drag and pitching moment as well as the inertia of the horizontal tail. The bending moment of the horizontal stabilizer is carried by a spar which connects the left and right sides of the stabilizer together. The shear force acting on the shear pin is derived from strain gauges mounted on the fuselage side in the vicinity of the pin as well as on the horizontal and vertical tail surfaces.

3.2.2 Sources of Error

Several possible sources of error in the measured load history were identified. The ground calibration was done using loads relatively small compared to those that are encountered during flight and the relation between the applied loads and measured strains was assumed to be linear. This is a common limitation when the calibration object is an operational aircraft and local application of large loads would risk damaging the structure. Although the accuracy of the calculated loads has not been verified at higher load levels, slight errors are unlikely to have a significant effect on the results of this study as the spectrum will be scaled and applied to standard geometries.
Furthermore, the ground calibration was only done for certain load cases and combinations thereof. Even if the calibration matrix describes the applied test loads and resultant interface loads perfectly, the scope of the calibration is limited. Due to the complex nature of the actual flight loads experienced by the airframe and the indirect way of measuring the shear pin loads, it is conceivable that during flight the loads would go outside the scope of the calibration and some mixing of loads may occur.

A significant error found in the load data was the shifting of the zero-load level between flights. The magnitude of this effect differed between flights, and it can be observed most clearly at the beginning of flight 4b in figure 3.3. This changes the mean stress and stress ratio of the loading cycles, which could significantly alter the calculated damage of the load history due to the effects of stress ratio described in section 2.6.

3.2.3 Temperature Compensation

Since temperature was known to have a significant effect on the measurements of some of the strain gauges used in the load calculation, temperature variations between flights were suspected to be the primary cause of the error in the load data. As the strain gauges are not matched to the material, part of the measurement drift is caused by thermal expansion of the structure. Furthermore, change in resistivity of the gauge as well as real strains induced in the composite structure by temperature changes are expected to contribute to the error. As only the interface loads at the shear pin are used in this analysis, even the effect of real temperature-induced strains on the loads is unwanted.

The zero-load error of some flights was unacceptably large and would have significantly changed the damage caused by the load history. To reduce this error, the load level was zeroed at the beginning of each flight, based on the mean of the data from the first minute of the flight. This reduced the error, particularly during flights that occurred at a very different temperature than the calibration had been done at. However, the load level at the end of each flight was not always zero, and the load graph still appeared to follow the temperature graph when significant changes in temperature occurred during a flight.

To correct the errors due to temperature changes occurring between as well as during flights, a linear temperature compensation was introduced. The temperature compensation factor was obtained from data where the cold aircraft had been pushed into a warm hangar. Strain gauge measurements were thus available for two temperature equilibrium states with zero load on the aircraft, meaning a temperature correction coefficient could be determined. Applying the temperature correction to the dataset, the observed dependence of the load on temperature vanishes and the correlation coefficient calculated between the measured temperature and the load drops from 0.79 (uncorrected data) and 0.54 (zeroed data) to 0.30 (temperature-compensated data).

Nonetheless the compensation is not perfect and some error remains. The source of temperature used for the correction is a sensor placed on the inside of a maintenance hatch at the rear of the fuselage. Since only states where the structure was in temperature equilibrium were considered in calculating the temperature correction coefficient, this has no direct implications on the accuracy of the coefficient. However, during flight the measured temperature sometimes changes rapidly at the sensor location. Due to the thermal capacity and resistance of the structure, the temperature at the actual strain gauge locations may then differ significantly from that at the location of measurement. This effect was also observed in the experiment from which the coefficient was determined as the relation between temperature and measured gauge strains was nonlinear if data points where the structure was not in thermal equilibrium were considered. Nevertheless, the accuracy of the data was considered sufficient for this study and much improved from the uncorrected state.
3.2.4 Overview of Load History

The entire load history consisting of 22 test flights is shown in figure 3.3 in both the original and the temperature-compensated forms. A positive load indicates an upward acting force on the tailplane. As during normal flight the aircraft experiences mostly negative lift on its stabilizer, the mean load in the load history is negative. The magnitude of the mean load during flight depends on factors such as the airspeed, center of gravity and flap setting of the aircraft.

![Graph showing load history](image)

Figure 3.3: The normalized load history shown in original and temperature-compensated form.

Figure 3.4 visualizes the distribution of different cycles in the load history by the range and mean values of the cycles. The spectrum is quite steep meaning that there is a large amount of small cycles. As expected for the horizontal stabilizer, most cycles are located around a slightly negative mean load. There are no cycles occurring around high mean loads, but some cycles do cover large load ranges. These cycles are also seen as large spikes in figure 3.3 and appear to be caused by buffet of the stabilizer during maneuvers such as stalls and spins.
3.3 Selection of Damage-Evaluation Approach

Different approaches for evaluating spectrum damage were considered. These included the stress-based methods such as S-N curves, strain-based fatigue prediction as well as the fracture mechanics approach where crack growth is calculated. As the aim was to obtain results valid for different typical aircraft components, a method capable of predicting fatigue for different geometries was needed. Furthermore, the method should be able to account for all typical load spectrum features such as mean load, stress ratio and possibly load-interaction effects.

With the S-N method, it is possible to account for different geometries and stress concentrations that occur in real components by using curves created using notched specimens \([1]\). While this method can be applied to components with well-defined geometry and known \(K\) values such as holes or fillets \([1]\), the method cannot account for other common structural features such as loaded fastener holes. Furthermore, the S-N approach predicts a total lifetime and does not distinguish between crack-initiation and crack-growth life. This is usually not a serious limitation for engineering-type predictions on typical structures where the crack-growth life is relatively short \([1]\). However, it can produce very conservative life estimates for structures with high residual strength after crack initiation. Furthermore, aircraft are often designed using a damage-tolerance approach where understanding of the crack-growth process is central and testing also focuses on crack growth. Methods that discern between crack initiation and growth are also useful for estimating the safe life and need for maintenance on in-service aircraft.

A crack-growth approach was chosen for this study. In order to account for crack-initiation life as well, a modified version of the NASGRO model (introduced in section 3.3.4) as well as regular long-crack lookup data are used. The inclusion of a crack-retardation model, such as the Wheeler or Willenborg model, was considered but decided against due to the lack of verified input data for these models. As explained in section 2.10.1, these models have some limitations and are not completely founded in theory. While the models have been successfully applied to real crack-growth problems, using them in a theoretical study...
3.3. Selection of Damage-Evaluation Approach

without input data adjusted and validated for the particular problem may lead to misleading results.

3.3.1 Filtering

For filtering of the load history, the racetrack method was selected. As detailed in section 2.12, the racetrack-filtering method is particularly suitable for VA spectra as it keeps the cycle extremes and the sequence intact. Furthermore, the algorithm is relatively simple to implement. For this study, the racetrack filter was implemented as a MATLAB function, which takes a load history as an input and returns the racetrack-filtered history, keeping the original sample rate of the data. Sample points originally located in omitted cycles are not completely removed from the data but instead interpolated using the values of the two adjacent reversal points. The filtering threshold range $R_{th}$ is defined in percent of the maximum tensile load in the load history.

3.3.2 Cycle Counting

The rainflow algorithm was selected for cycle counting due to its superior ability to represent complex and irregular VA load histories. As discussed in section 2.5.3, the method retains all cycle extremes and the mean of each cycle regardless of the shape of the load history. Furthermore, the cycle sequence of the load history is largely retained, making it possible to run analyses with crack-retardation models. An implementation of the rainflow algorithm according to the ASTM-E1049-85 standard is included in Matlab and was used for this study.

The used algorithm presents the residual of the rainflow calculation in the form of unclosed half cycles. Counted this way, the 25-hour-long time history contains only 64 half-cycles, making up only ~0.00042 % of the total cycle count. However, many of the counted half-cycles are very large and they even include the three largest extreme values of the whole spectrum. Thus despite their small amount, the half-cycles should not simply be omitted as they may contribute significantly to the total damage.

AFGROW does not accept partial cycles in the input spectrum [49] and therefore including the counted half-cycles is not possible. However, the full load history is known and it can be considered periodical since it represents a block of typical flights, starting with the aircraft stationary on the ground and ending in the same condition. Thus the load history was rearranged to start and end with the largest extremum point. As explained in section 2.5.3, this allows all hysteresis loops to be closed and causes the rainflow algorithm to return only full cycles with no residue.

3.3.3 Specimens

To make the results of this study as generalizable as possible, multiple different specimen types were selected for analysis. The specimens were selected to represent typical details of aircraft structures. Different possible specimen types were explored by comparing their beta functions. The shape of the beta function combined with the crack length and the stress level have a direct influence on the value of the stress intensity factor, as can be seen from the definition of the stress intensity factor (equation 2.9). The selected specimens were picked to represent different beta function shapes, allowing conclusions to be drawn regarding possible effects of the beta function on spectrum filtering and tuning thereof. Furthermore, specimens of different thicknesses were included. The resulting stress state under crack growth in these specimens can be considered mixed plane strain / plane stress.

Based on these considerations four different specimens were selected. The specimens are presented below and their exact dimensions can be found in table 3.1. The selection of stress levels is discussed in section 3.3.5 with the selected stress levels for each specimen listed
3.3. Selection of Damage-Evaluation Approach

The studied material is 7050-T7451 aluminum, a high-strength alloy used in components of many aircraft [59].

The first specimen geometry is a simple wide and thin sheet with a corner crack starting from the plate corner and propagating in the longitudinal-short (L-S) and longitudinal-transverse (L-T) directions. The specimen is loaded axially in the direction perpendicular to the crack. The lower wing skin or upper fuselage skin of an aircraft are typical examples of aircraft structures that this specimen represents. These skins consist of many pieces of thin aluminum sheet joined together, with their edges being potential locations for crack initiation. The aforementioned structures are also primarily loaded in tension due to the wing lifting loads the aircraft experiences in flight, with the occasional compressive cycle occurring due to e.g. landing loads.

The second specimen is a narrow plate with a hole located in the middle and a corner crack growing from the corner of the hole in a direction perpendicular to the load. The edge distance from the fastener hole was selected as $2D$. This specimen represents the typical geometry of e.g. a flange, stringer or spar cap that is fastened to other structure with a continuous row of fasteners. In such a case, some of the load in the specimen is transmitted through the fastener as a bearing load and some of it flows by the fastener hole and is termed bypass load. The ratio between the load in the section and the load transferred by the fastener can be expressed as the load transfer ratio

$$LT = \frac{F_{lt}}{F_{sec}}, \quad (3.1)$$

where $F_{lt}$ is the shear force transferred through the fastener and $F_{sec}$ is the force in the gross section of the part, before the fastener. The difference between the aforementioned forces is also called the bypass force $F_{bp}$, which is the fraction of the total section force that is not transferred by the fastener and thus remains to be transferred by the section after the fastener, as illustrated in figure 3.6.

Two different load cases were selected for this specimen. In the first case the hole is loaded by a bearing force with the load transfer ratio being 0.25, a typical value for medium load
3.3. Selection of Damage-Evaluation Approach

transfer joints such as doubler and stiffener joints \cite{60}. In the second load case the hole is not
loaded, as would be the case with e.g. lightening holes in a component.

The third specimen represents a typical axially-loaded lug with edge distance $1D$ and a
corner crack initiating on the edge of the lug hole, transverse to the loading direction. The
lug is loaded in tension by a bearing force acting on the hole. This type of lug connection is
common in aircraft components that must be removable and it is used even in highly loaded
and critical connections such as fighter aircraft wing-root joints \cite{42}.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Corner crack in sheet [mm]</th>
<th>Corner crack in open hole [mm]</th>
<th>Corner crack in loaded hole [mm]</th>
<th>Corner crack in lug [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>101.6</td>
<td>25.4</td>
<td>25.4</td>
<td>25.4</td>
</tr>
<tr>
<td>$T$</td>
<td>2.00</td>
<td>4.00</td>
<td>4.00</td>
<td>12.7</td>
</tr>
<tr>
<td>$D$</td>
<td>-</td>
<td>6.35</td>
<td>6.35</td>
<td>12.7</td>
</tr>
<tr>
<td>Load transfer [-]</td>
<td>-</td>
<td>-</td>
<td>25 %</td>
<td>100 %</td>
</tr>
</tbody>
</table>

Table 3.1: Specimens selected for analysis and their dimensions.
3.3. Selection of Damage-Evaluation Approach

Figure 3.9 shows the different beta functions of the selected specimens for both a through crack and a quarter-circular corner crack case. The beta functions are plotted up to the maximum theoretical crack length $c_{\text{max}}$ or $a_{\text{max}}$, meaning the length that the crack can reach before touching a free edge of the specimen. The crack length is denoted $c$ in the width or surface direction and $a$ in the thickness direction. Due to the through-crack beta functions approaching infinity when the through crack approaches a free edge, the graphs were drawn only to $0.95 - 0.99c_{\text{max}}$ for better scaling of the plots.

![Beta functions of the specimens](image)

Figure 3.9: Beta functions of the specimens as output by AFGROW in the surface ($c$) and through-the-thickness ($a$) directions. The surface beta values are generated for a through crack and the through-the-thickness values for a quarter-circular corner crack with an $a/c$ ratio of 1. All beta values are with reference to gross section stress.

The beta functions in AFGROW follow the standard definition shown in equation 2.9. The stress intensity factor solutions for the sheet and hole corner-crack specimens follow the closed form solutions presented in [61]. For the lug specimen, stress intensity factor solutions are interpolated from tabulated data obtained using finite element analysis. [49]

As can be seen in figure 3.9, the shapes of the beta functions differ significantly between the selected specimens. Because there is no stress concentration in the sheet specimen, the stress intensity at the crack initiation location is comparatively low. As the crack propagates, the beta value increases exponentially, causing very high stress intensities when the through crack approaches the free edge of the specimen. The beta function for the thickness direction follows a slight and more linear growth.

In the beta graph for the hole specimens, the strong stress concentration caused by the hole at the initiation location is clearly visible. As the through crack propagates, the effect of the hole diminishes. When approaching the free edge, the beta value again increases exponentially, as already seen with the sheet specimen. It is also noteworthy that for the thickness direction of the corner crack the beta function is decreasing with crack length.

The beta function of the lug could be described as a combination of the sheet and hole cases, with the stress concentration at the crack initiation location being higher than for the hole specimen. The sudden change in shape of the lug beta function can be explained by the
3.3. Selection of Damage-Evaluation Approach

way the function is applied in AFGROW. For a corner crack AFGROW uses two different solutions, a bearing boundary condition solution and a spring condition solution. The bearing solution assumes a steel pin in the aluminum lug and allows the hole to deform, whereas the spring condition constrains the hole. AFGROW uses the bearing solution when $a/T \leq 0.7$ and the spring solution when $a/T \geq 0.8$ and linear superposition in between these crack lengths. [49]

3.3.4 Crack-Growth Model

Two different crack-growth models were used in the study. The first model consists of 7050-T7451 aluminum long-crack lookup data from the AFMAT Fracture Mechanics Database [62]. This data has been generated under laboratory conditions using longitudinally loaded plate specimens with cracks growing in the L-T direction [62]. The initial crack length to be used with the long-crack lookup data was selected as 1.27 mm, which is a common initial flaw size used in damage-tolerance analysis [26].

As in addition to long crack growth the total life including short crack growth was of interest in the filtering study, a second approach covering this regime of crack growth was selected. Main et al. [52] presented the modified NASGRO model, which is a correction of the NASGRO model, aimed at improving the prediction of short and near-threshold fatigue crack growth as well as total-life prediction. The modification was developed for 7050-T7451 aluminum and it extends the threshold of the NASGRO curve into the short crack region ($\Delta K \approx 1 \text{ MPa}$) by extrapolating the shape of region II of the curve ($\Delta K \approx 4 - 10 \text{ MPa}$). [52]

The aim of this modification is to enable the prediction of total life using the fracture mechanics approach and a very short initial crack length. Although the method is supported by fractography results [52], it may be sensitive to the use of different load spectra. Furthermore, it can predict higher than normal crack growth rates for long cracks due to the globally lowered threshold. The strength of this effect depends on the applied load spectrum and does not seem to severely affect the predicted total life.

The NASGRO model is defined by different parameters describing the shape of the curve as explained in section 2.9.3. It was found that the short crack, modified NASGRO model could be approximated by modifying some of these parameters, namely the $q$ and $C$ parameters, while simultaneously setting $p$ to 0, $C_{th}$ to 0.4 and $\Delta K_0$ to 1 MPa$\sqrt{m}$. A code was created to find the values for parameters $q$ and $C$ that produce the best possible fit to the original NASGRO curve in the unmodified region ($\Delta K > 4 \text{ MPa}$) at different stress ratios. Figure 3.10 shows the resulting modified NASGRO curves overlaid on the original curves.

![Figure 3.10: Original and short crack modified NASGRO curves for 7050-T7451 aluminum.](image-url)
The modified NASGRO equation was selected as the second crack-growth model to be used in the study. As the model is intended to be used for estimation of short crack growth and total life, a significantly smaller initial flaw must be used. The initial crack length was thus set to 0.01 mm as recommended in [52]. This initial flaw size corresponds to the average depth of etch pits and other crack-like discontinuities caused by surface treatment processes [63].

3.3.5 Software and Analysis Setup

The damage tolerance analysis software tool AFGROW was used for all crack-growth analyses in this study. AFGROW is a commercial software package originally developed by the United States Air Force Research Laboratory. It can be used to analyze fatigue crack initiation, growth and fracture and is commonly used for life predictions in aerospace applications [64].

The AFGROW software requires a counted spectrum to be input for the analysis [49]. Filtering the time history using the racetrack algorithm as well as counting the filtered data using the rainflow algorithm was accomplished in MATLAB. As for the results presented in this report alone, close to 10 000 simulations had to be run, the decision was made to fully automate the analysis process. AFGROW provides a Component Object Model (COM) interface, through which the analysis can be set up and run. MATLAB was used to control AFGROW through the COM interface as well as for all supplementary calculation tasks. An overview of the analysis setup is shown in figure 3.11.

![Figure 3.11: Analysis flow.](image)

A number of crack-growth analyses were run using the specimens outlined in table 3.1 and the long-crack lookup data as well as the modified NASGRO equation introduced in section 3.3.4. The analyses used the temperature compensated loads data described in section 3.2.4 with the sign of the loads reversed in order to produce a mostly tensile load spectrum for the crack-growth analyses. This corresponds to a component loaded in tension by a negative lift force acting on the stabilizer.

For each specimen a reference stress level $\sigma_{\text{ref}}$ that produces an estimated fatigue life of 20 000 hours was selected. This lies in the typical range of fatigue test life for a light trainer aircraft considering the test is run to between two and four times the design life. The reference stress represents the magnitude of the maximum tensile peak in the spectrum. With the exception of the lug specimen, the stress is calculated as the force applied to the specimen divided by the gross section of the specimen. For the lug specimen, the bearing stress applied
3.4. Validation of Analytical Results

The results of the spectrum reduction study should be validated experimentally. Both the method presented in this work and any application of it to a specific problem should be validated individually. An experimental validation was not conducted during the course of this work, but a proposal for the needed test program is outlined. The experimental validation should be conducted as a coupon test program. The aim of the test program is to validate the developed method for spectrum reduction and to verify the conclusions that are arrived at through the results of this study.

3.4.1 Coupon Test Program

To understand the effect of filtering on specimen life, several identical coupons must be tested using spectra filtered at different levels. For the validation of the complete results of this study, the whole range of filtering levels from 0 % to 30 % should be covered. In practical applications, it may be sufficient to run tests at a subset of these filtering levels or only at 0 % filtering and at the filtering level chosen for the fatigue test based on the spectrum reduction study. When validating a range of filtering levels, uniform spacing of the levels to be tested is generally not efficient. Instead, it is recommended to choose the test points using the graphs of predicted specimen life (e.g. figure 4.3), so that the range of interest in terms of specimen life is covered sufficiently. Specimen life does not change linearly with filtering level and testing at very low filtering levels can be redundant due to specimen life not being affected by such filtering. Depending on the load spectrum type, attention should be paid to

to the hole is used as the reference stress. The reference stress level was found by reverse analysis using the modified NASGRO model which estimates the total life of the specimen.

Furthermore, a small range of stresses around $\sigma_{\text{ref}}$ was included to investigate possible secondary effects of stress level on the results of the spectrum reduction study. The filtering levels (racetrack-filtering range $R_{\text{th}}$) in the study ranged from 0 % to 30 %, defined as a percentage of $\sigma_{\text{ref}}$. Table 3.2 summarizes the complete selection of analyses run.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\sigma_{\text{ref}}$ [MPa]</th>
<th>Stress range</th>
<th>Filtering range</th>
<th>Crack-growth model</th>
<th>Initial crack length [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corner crack in sheet</td>
<td>241</td>
<td></td>
<td>0.8 - 1.2 $\sigma_{\text{ref}}$</td>
<td>0 - 30 %</td>
<td>Modified NASGRO</td>
</tr>
<tr>
<td>Corner crack in open hole</td>
<td>85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corner crack in loaded hole (LT = 25 %, $\sigma_{\text{br}} / \sigma_{\text{bp}} \approx 1.33$)</td>
<td>69</td>
<td></td>
<td></td>
<td>Long crack lookup</td>
<td>1.27</td>
</tr>
<tr>
<td>Corner crack in lug</td>
<td>86</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Analyses and analysis settings for the spectrum reduction study. $\sigma_{\text{br}} / \sigma_{\text{bp}}$ is the bearing-bypass ratio.

The AFGROW analyses were set to use a residual strength check to determine fracture of the specimen. The required residual strength was selected to be equal to the highest tensile load in the spectrum. AFGROW uses this load to check for net section failure as well as brittle ($K_c$) fracture. The fracture toughness value is interpolated between the plane-strain and plane-stress values for the estimated stress state in the specimen. Continuously checking for failure according to the maximum spectrum load allows accurate life estimation independent of the occurrence of large load peaks in the spectrum.
the possible effects of filtering level on crack-interaction and retardation effects. These effects are not considered in the spectrum reduction study conducted in this work.

The coupon test program should also validate the predicted effect of stress level on the allowable filtering level, meaning that each coupon type should be tested for a range of stress levels. The results of the spectrum reduction study again aid in the selection of appropriate stress levels. In practical applications the stress level is often known and in such cases it may be sufficient to run tests only at the known stress level.

Finally, the coupon type(s) must be chosen appropriately. In this work, four different specimens representing a range of typical aircraft geometry are examined. The coupons used to validate this study should be chosen to represent the studied specimens and their differences. As is evident from the results, the majority of the specimen life is spent in crack initiation and short crack growth, meaning that this stage of crack growth has the most effect on the specimen life estimates. As discussed in section 2.1.1 tests with notched coupons must be conducted to obtain correct results for cases where the actual part has stress raisers. As can be seen from figure 3.9 the beta functions of the different specimens used in this study differ significantly in value at the crack initiation locations of the specimens, indicating that the elastic stress concentration at this location differs between the specimens. However, the change in stress concentration from zero crack length to e.g. 1.27 mm crack length is not as significant and relatively similar for all specimens. This suggests that up to a short crack length, the results of the four rather complicated specimens could be validated using tests with simple notched specimens sized for similar stress concentrations as the studied specimens.

At long crack lengths the beta functions of the specimens differ more significantly. While this stage of crack growth does not affect the total life of the specimen as much, the effect of filtering at long crack lengths was found to be different than at short crack lengths, and thus should be validated as well. One option for simplified validation of the long-crack results would be to use only a standard crack-growth specimen and quantify the effect of filtering on crack growth rate at different crack lengths using e.g. fractography. From such a test the effect of filtering on crack growth rate can be deducted at different stress intensities. In region II of the crack growth rate curve (Paris region), the stress intensity factor range determines the crack growth rate. By using information on the effect of filtering at different stress intensity factor ranges the effect on specimens with different beta functions can be determined.

For verification of a reduced load spectrum for a real component test, it is necessary to choose the coupon type to represent the area of interest on the actual test object as well as possible. In such cases, the choice of coupon is not always straightforward. Real components often poorly resemble standard test coupons as their geometry can be complex, the component may be loaded in multiple modes and the crack-propagation path is often difficult to predict. Choosing a suitable test specimen and judging the relevance of the specimen with regard to the actual structure then requires engineering expertise. Furthermore, the effects of scatter on the obtained results must be considered when planning the coupon test matrix.
4 Results and Discussion

4.1 Specimen Life Estimates

For the spectrum reduction study analyses were run as outlined in table 3.2. Figure 4.1 shows the life of the selected specimen types as a function of the applied stress level. The left side of the figure shows the total life as estimated by the modified NASGRO model using a 0.01 mm initial crack length. This life estimate changes similarly for each specimen as a function of relative stress level. On the right side of the figure the long crack growth life calculated using the long-crack lookup data and a 1.27 mm initial crack length is shown. Note that the baseline stress level $\sigma_{\text{ref}}$ is different for each specimen and was set to produce a 20 000 h total life as explained in section 3.3.5.

![Figure 4.1: Total life (mod. NASGRO) and long crack growth life (lookup) of the different specimens for the studied range of stresses. The life-axis uses a logarithmic scale.](image)

As the lines in figure 4.1 appear approximately linear, it is evident that the life of each specimen is exponentially decreasing as a function of stress level. This relationship is similar for all specimens and as could be expected, it resembles the shape of S-N curves as shown...
in section 2.6. At the lowest stress levels ($\sigma < 0.9\sigma_{ref}$) the relation appears to change slightly, with the total-life estimates being moderately higher than an exactly exponentially decreasing relationship would predict. This could be attributed to the general stress level of the fatiguing cycles approaching the threshold for crack growth or the area of the endurance limit on an S-N curve.

The different specimens show markedly different crack-growth lives. The long crack growth analysis was run using the same stress levels which were derived for a 20 000 h total life of each specimen. Effectively this means that the dissimilarities are due to the different distributions of crack-growth life versus total life between the specimens. The sheet specimen in particular has a very short crack-growth life from the 1.27 mm initial crack to failure and accordingly most of the total life is spent at crack lengths less than 1.27 mm. Figure 4.2 shows the crack-growth life from 0.01 mm to 1.27 mm, approximated by subtracting the results of the long crack growth calculations from the total life. Although the effect is slight, it is evident that the fraction of life spent on short crack growth decreases with increasing stress level for all specimens.

![Figure 4.2: Crack-growth life to 1.27 mm crack length as a percentage of total life to failure.](image_url)

The main reason for the differences in distribution of life between the short and long crack growth phases can be found in the beta functions of the specimens, shown in figure 3.9. The sheet specimen has no stress concentration at the crack initiation location, meaning that the effective stress equals the section stress ($\sigma_{ref}$ for this specimen). Consequently, a very high $\sigma_{ref}$ had to be selected in order for the stress cycles to pass the crack-growth threshold at short crack lengths and for obtaining a lifespan of 20 000 h. The high section stress in turn causes rapid crack growth and fracture at longer crack lengths. Much lower section stresses were used in the analyses of the other specimens, which all have clear stress concentrations at the crack-initiation location. In the beta plots of figure 3.9, this can be seen as beta values larger than one at short crack lengths, in both the surface and thickness directions. While the stress concentration reaches far enough from the specimen edge for the long crack growth to be affected as well, its effect on crack-growth life from 1.27 mm to fracture is far smaller than its effect on crack initiation and short crack growth life. Crack growth at long crack lengths is also influenced by other factors such as the overall specimen geometry and residual strength.
4.2 Effect of Spectrum Reduction on Specimen Life

The results of the spectrum reduction study are shown in figures 4.3 to 4.6. These figures show the influence of filtering level on the normalized life of the specimen at different stress levels. Again, stress levels of $0.8 - 1.2 \sigma_{ref}$ where $\sigma_{ref}$ is the reference stress that gives the specimen an approximate life of 20,000 h, are used. The life values shown as contours in the figures are normalized at each stress level by the respective life with no filtering applied. $R_{th}$ is the racetrack-filtering threshold range. The total-life results are obtained using the mod. NASGRO model and the long crack growth life results using the tabular lookup data.

Figure 4.3: Sheet specimen: Relative life at different filtering levels.

Figure 4.4: Open hole specimen: Relative life at different filtering levels.
### 4.2. Effect of Spectrum Reduction on Specimen Life

<table>
<thead>
<tr>
<th>Stress level (x $\sigma_{ref}$)</th>
<th>$R_{th}$ (% of $\sigma_{ref}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>0.9</td>
<td>5</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>1.1</td>
<td>15</td>
</tr>
<tr>
<td>1.2</td>
<td>20</td>
</tr>
<tr>
<td>1.5</td>
<td>25</td>
</tr>
</tbody>
</table>

#### Figure 4.5: Loaded hole specimen: Relative life at different filtering levels.

<table>
<thead>
<tr>
<th>Stress level (x $\sigma_{ref}$)</th>
<th>$R_{th}$ (% of $\sigma_{ref}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>0.9</td>
<td>5</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>1.1</td>
<td>15</td>
</tr>
<tr>
<td>1.2</td>
<td>20</td>
</tr>
<tr>
<td>1.5</td>
<td>25</td>
</tr>
</tbody>
</table>

#### Figure 4.6: Lug specimen: Relative life at different filtering levels.
With the exception of the sheet specimen, the above contour plots show a relatively consistent effect of filtering level on specimen life. The graphs can be used to determine an appropriate level of filtering for different specimens and stress levels. In fatigue testing correct damage over the whole range of crack lengths is important, so results of both the total-life prediction made using the modified NASGRO equation and the long crack growth prediction using the lookup data will be examined. Worth noting is that the total-life prediction is mostly sensitive to damage occurring at short crack lengths since—as visualized in figure 4.2—much of the specimen life is spent in that region. Although the effect on total life is small, the modified NASGRO method may give inaccurate crack-growth rates at longer crack lengths, as explained in section 3.3.4. Effects of spectrum reduction on damage at long crack lengths should therefore be deducted from the analysis done using long-crack lookup data.

An effect that can be observed in all the results is the allowable level of filtering decreasing with increasing stress level. When the general stress level is increased, a larger number of small cycles exceeds the threshold. A higher level of filtering on the other hand causes more small cycles to be removed from the load spectrum. If these removed cycles are situated below or in the threshold region, little damage is removed from the spectrum. This is the case at low spectrum stress levels. Correspondingly, if the stress levels are higher relative to the threshold of the material, more damage is removed at the same relative filtering level (% of $\sigma_{\text{ref}}$) since the removed cycles reach over the threshold for crack growth.

Although the results are rather consistent between the specimens, the sheet specimen forms an exception. The allowable filtering levels for the sheet specimen, as estimated for long cracks using lookup data, are significantly less than for the other specimens. Due to the stress level $\sigma_{\text{ref}}$—for reasons explained in section 4.1—being much higher for the sheet specimen than for any of the other specimens, the long crack growth life for this specimen is very low. As the high loads result in very high stress intensities at long crack lengths, even low levels of filtering remove damaging cycles and affect the long crack growth life of the specimen significantly.

The allowable filtering level for each specimen at different levels of damage reduction can be read from figures 4.7 and 4.8 for the total-life and long crack growth life predictions respectively. As explained previously, there is a relationship between applied stress level and allowable filtering level. It may not be appropriate to compare the differences between the specimens using the results of the long crack growth analysis due to the effect of different stress levels used for the specimens. On the other hand, the results provide insight into the allowable filtering in real-world testing, as the design stress levels are naturally different for distinct components which are designed for the same life.

The total-life or mod. NASGRO prediction results show a slightly different level of allowable filtering for the four specimens. The sheet specimen allows the highest level of filtering, whereas the loaded hole allows the least filtering for a certain relative life. It is noteworthy that the order is reversed with respect to the results obtained in the long-crack analysis and the differences between the specimens are larger. Looking at total life, the allowable level of filtering appears to be higher for the specimens that have slow crack growth at short crack lengths and less severe stress raisers.

### 4.3 Test Duration

Depending on the requirements and aims of the test, different levels of damage reduction may be acceptable. Choosing the correct filtering level is a compromise between test duration and spectrum damage reduction. This tradeoff is visualized in figures 4.7 and 4.8 which show the estimated duration of a fatigue test at different spectrum filtering levels. The test is run to the equivalent of 20,000 flight hours and vertical lines representing different levels of damage reduction (relative life) are shown for each specimen type.
4.3. Test Duration

Figure 4.7: Effect of spectrum filtering on the duration of testing conducted at different frequencies. Prediction done for total life using the mod. NASGRO model and stress level $\sigma_{\text{ref}}$ as defined in table 3.2.

Figure 4.8: Effect of spectrum filtering on the duration of testing conducted at different frequencies. Prediction done for long crack growth life using the lookup method and stress level $\sigma_{\text{ref}}$ as defined in table 3.2.
4.4 Impact on Aviation and Industry

Tension-compression coupon tests are typically run at frequencies ranging from 10 to 100 Hz. The loading frequency of full-scale fatigue tests is more difficult to estimate as the flexibility and mass of the test object have a large influence on the forces and displacements the actuators must produce and the frequency that can be achieved. 1 Hz is likely a conservative estimate in most cases, but higher testing frequencies may be achieved especially when light and stiff structures are tested and electromagnetic shakers are employed. It should also be noted that in the case of VA loading, the test frequency only represents an average of the frequency at which cycles are applied.

A test duration of 10,000 h corresponds to 1250 days or over three years of testing time, if the test is run for 8 hours a day. The duration of the test can be reduced by running the test around the clock or increasing the frequency. The need to supervise the test places a practical limitation on the hours the test can be run each day, and especially in full-scale testing, the need to periodically inspect or even repair the test object and rig further slows down the test.

Observing the testing time at 0 % filtering it becomes evident why reducing the number of load cycles is absolutely necessary when conducting a fatigue test. Filtering the load sequence using the racetrack filter with a filtering range of only 10 % already reduces the number of cycles to less than 1 % of the original number while extending the life by only around 1 % for most specimens. Applying the unfiltered load spectrum is only realistic in coupon tests run at a high frequencies. Also, in full-scale testing, simple cycle filtering may not be enough to reduce the testing time. Accelerating the test by modifying the load spectrum to cause more damage or using e.g. shakers to increase the achievable testing frequency may be necessary.

4.4 Impact on Aviation and Industry

The presented results show how the duration of a fatigue test using in-flight measured loads can be reduced down to just a small fraction of the original duration by filtering out insignificant load cycles. Although the filtering process is simple, it is essential to select the correct level and type of filtering. In this work a decisive path from in-flight measured loads to the reduced fatigue test spectrum has been described and the required steps of analysis have been defined. The developed methodology can be applied to practical problems, reducing the effort needed to develop a fatigue test spectrum and encouraging the selection of an optimal filtering level by analysis.

An optimally filtered load spectrum significantly reduces the amount of testing time compared to unfiltered or conservatively filtered spectra, which especially in large-scale testing can lead to substantial cost savings. Apart from reducing the cost to certify new aircraft, reduced cost and a lower threshold for fatigue testing also benefits programs for operational aircraft. Fatigue testing can aid in extension of aircraft service life and help in maintaining and improving flight safety.

Furthermore, the presented analysis approach also assists in preventing unconservative filtering of flight load spectra. Any excessive filtering of load spectra removes damage from a fatigue test, resulting in potentially unsafe over-estimation of fatigue life. Nevertheless, the results of the presented filtering study or any application of the presented method to a practical problem should be carefully validated to reduce the risk of false conclusions. General suggestions for a validation were made and careful engineering judgement should be used when judging the validity of results.
Methods to measure the operational loads on an airframe and their limitations were explored with emphasis on usage of such data for fatigue testing. The process of transforming in-flight measured loads to a fatigue test was described and recommended methods to process the data were presented. Comparison of different cycle-counting methods resulted in the rainflow-counting method being selected as the most suitable for counting aircraft load spectra. For the purpose of reducing the in-flight measured loads data—a crucial step in applying such data to fatigue testing—the racetrack-filtering algorithm was found best suited.

An analysis was developed to study the effect of spectrum reduction on the damage inflicted by the spectrum. The study used real in-flight measured loads which were compensated for the effect of temperature. A variety of fatigue damage estimation methods were evaluated for the study. The crack growth approach was selected and the (modified) NASGRO- and tabular-lookup methods in particular were found suitable. Four different specimens representing typical aircraft structure were selected and a reference stress level producing a 20 000 h total life was defined for each specimen. The study was automated using MATLAB as well as the crack-growth software AFGROW and a range of filtering and stress levels was run for each specimen.

The results show that an allowable filtering level, which reduces damage only by a certain amount, can be defined for each specimen. The damage reduction due to filtering increases at higher stress levels, an effect consistent across all specimens. Significant differences in allowable filtering level were noted between the specimens as well as between the analyses done for total life and long crack growth life. Contour charts allowing the determination of an appropriate filtering level for each specimen are provided.

Finally, the impact of spectrum reduction on the duration of fatigue testing was studied and the achievable reduction in testing time was determined. Filtering of the load sequence reduces the number of cycles, which has a direct effect on the duration of a fatigue test. Reducing the length of fatigue tests is of great importance to industry as testing is an expensive and slow process. A remarkable result is that in this study the number of cycles in the in-flight measured load sequence could be reduced to just 1 % of the original, while changing the life of the specimens by only around 1 %.

A proposal for experimental validation of the study was also provided, with particular attention paid to means of simplifying the validation program. Aside from validation of the method, possibilities for future work include further studies on a larger amount of speci-
mens. In this study, the choice of realistic specimens and the choice of stress level by resulting specimen life provided results with high real-world relevance. However, in order to better distinguish between the individual factors influencing the allowable filtering level—such as stress level, notch severity and overall specimen geometry—, running a completely parametric study using simple notched specimens would be useful.


