

A multi-strategy enhanced African vultures optimization algorithm for global optimization problems

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Abstract

The African vultures optimization algorithm (AVOA) is a recently proposed metaheuristic inspired by the African vultures' behaviors. Though the basic AVOA performs very well for most optimization problems, it still suffers from the shortcomings of slow convergence rate and local optimal stagnation when solving complex optimization tasks. Therefore, this study introduces a modified version named enhanced AVOA (EAVOA). The proposed EAVOA uses three different techniques namely representative vulture selection strategy, rotating flight strategy, and selecting accumulation mechanism, respectively, which are developed based on the basic AVOA. The representative vulture selection strategy strikes a good balance between global and local searches. The rotating flight strategy and selecting accumulation mechanism are utilized to improve the quality of the solution. The performance of EAVOA is validated on 23 classical benchmark functions with various types and dimensions and compared to those of nine other state-of-the-art methods according to numerical results and convergence curves. In addition, three real-world engineering design optimization problems are adopted to evaluate the practical applicability of EAVOA. Furthermore, EAVOA has been applied to classify multi-layer perception using XOR and cancer datasets. The experimental results clearly show that the EAVOA has superiority over other methods.

Keywords: African vultures optimization algorithm, global optimization, engineering design problems, metaheuristic, exploration and exploitation, multi-layer perception classification

1. Introduction

With the growing volume of data, the improvements in computing power, and computer technology innovations, artificial intelligence techniques can reshape and change our lives (Fathi et al., 2021; Haeberle et al., 2019; Hussien et al., 2022a; Tavasoli et al., 2021). In the technology and computer revolution-3rd the industrial revolution, data can be similar to electricity and stream that can be considered as two valuable energy sources for the industry progress advancement (Hussien & Amin, 2022; Hussien et al., 2017a; Hussien et al., 2020c; Trapp et al., 2019). Nowadays, we are in the fourth industrial revolution era in which optimization algorithms become one of the most important weapons for finding solutions to complicated and high-dimensional optimization problems (Abualigah et al., 2020; Fang et al., 2010; Hussien et al., 2019).

Optimization can be found everywhere and in everything in our life. It can be founded in medicine, science, manufacturing, engineering, and many more (Gao et al., 2020; Hussien, 2022; Wang et al., 2022a). Optimization approaches can be divided into two classes: (i) mathematical approach (linear and non-linear; Boyd

et al., 2004) and (ii) metaheuristic algorithms (MAs). The former class (mathematical techniques) cannot find the optimal solutions in a short time due to the problem's complexity (Wang et al., 2022b; Yang, 2010). MAs are those algorithms which simulate the physical, biological, or foraging behavior of insects, birds, fishes, or other animals. MAs have gained colossal interest and attention widely from scholars since their advantages like simplicity, flexibility, ease-in-use, and much more.

Recently, various MAs have been introduced and have been used in many fields successfully.

Roughly speaking, these algorithms can be classified into various groups, such as physics-based, chemistry-based, evolution-based, mathematics-based, swarm-based, plant-based, musical-based, light-based, social-based, water-based, and sport-based (Akyol & Alatas, 2017; Alatas & Bingol, 2019, 2020; Bingol & Alatas, 2020; Hussien et al., 2020a; Ong et al., 2022). A brief note on each class with some examples is provided below.

The physics-based group includes algorithms such as simulated annealing (Kirkpatrick et al., 1983), gravitational search

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algorithm (Rashedi et al., 2009), Fick's law algorithm (Hashim et al., 2022), magnetic optimization algorithm (Tayarani-N & Akbarzadeh-T, 2008), ions motion algorithm (Javidy et al., 2015), and lightning search algorithm (Abualigah et al., 2021b).

The chemistry-based group contains algorithms such as chemical reaction optimization (Lam & Li, 2010) and artificial chemical reaction optimization algorithm (ACROA; Alatas, 2011).

The evolution-based group contains algorithms such as differential evolution (DE; Das & Suganthan, 2010), genetic algorithm (GA; Holland, 1992), and barnacles mating optimizer (Sulaiman et al., 2020).

The math-based group contains algorithms such as golden ratio optimization method (Nematollahi et al., 2020) and sine-cosine algorithm (SCA; Mirjalili, 2016).

The swarm-based group contains algorithms such as particle swarm optimization (PSO; Kennedy & Eberhart, 1995), whale optimization algorithm (WOA; Mirjalili & Lewis, 2016), grey wolf optimizer (GWO; Mirjalili et al., 2014), aquila optimizer (AO; Yu et al., 2022), COOT (Mostafa et al., 2022), crow search algorithm (Hussien et al., 2020b), firefly algorithm (El-Shorbagy & El-Refaey, 2022), ant lion optimization (Assiri et al., 2020), Harris hawks optimization (HHO; Gezici & Livatyalı, 2022; Heidari et al., 2019), slime mould algorithm (SMA; Li et al., 2020), marine predators algorithm (MPA; Faramarzi et al., 2020), and snake optimizer (Hashim & Hussien, 2022).

The plant-based group contains algorithms such as root mass optimization algorithm (Qi et al., 2013) and flower pollination algorithm (Yang, 2012).

The musical-based group contains algorithms such as harmony search algorithm (HS; Geem et al., 2001) and melody search (Ashrafi & Dariane, 2013).

The light-based group contains algorithms such as ray optimization (Kaveh & Khayatazad, 2013) and optics-inspired optimization (Kashan, 2015).

The social-based group contains algorithms such as teaching learning-based optimization, poor and rich optimization (Moosavi & Bardsiri, 2019), gaining and sharing knowledge (Mohamed et al., 2020), psychology-based optimization (Das et al., 2020), and search and rescue (Shabani et al., 2020).

The water-based group contains algorithms such as water evaporation optimization (Sacco & Oliveira, 2005) and water cycle algorithm (Hussien et al., 2022b).

The sport-based group contains algorithms such as world cup competitions (Razmjoo et al., 2016), golden ball algorithm (Osaba et al., 2014), and soccer game optimization (Purnomo & Wee, 2015).

Balancing between exploration and exploitation is one of the critical factors in designing a new algorithm or enhancing an already existing one. Exploration is the exploring process of the entire search space, whereas exploitation refers to the search process that will be held in the neighborhood area near an already existing solution (Alba & Dorronsoro, 2005; Lynn & Suganthan, 2015).

African vultures optimization algorithm (AVOA) is a novel nature-based algorithm that simulates the unusual behaviors of African vultures in navigation and foraging (Abdollahzadeh et al., 2021). Many versions of modified AVOA have been proposed, and they have been applied to many fields. An enhanced version of AVOA (EAVOA) is proposed based on a time-varying technique and tent chaotic map (Fan et al., 2021). Another work was done by Humar and Mary in which they developed an improved AVOA based on Newton-Raphson to estimate the parameters of the 3-diode photovoltaic model (Kumar & Mary, 2021). A hybrid version between AVOA and support

vector machine to detect pulmonary disease (Manickam et al., 2021).

The above-mentioned algorithms have some disadvantages and limitations as they only consider one problem to solve or one phase to be enhanced (i.e., diversity, exploitation, exploration, and initial population). Moreover, the above techniques contain many parameters that need to be tuned.

However, AVOA still needs more improvements as it may have some limitations in solving complex problems and high-dimensional ones. The paper introduces an EAVOA to solve the original AVOA limitation based on three strategies: (i) representative vulture selection strategy (RVSS), (ii) rotating flight strategy (RFS), and (iii) selecting accumulation mechanism (SAM).

The main contributions of this paper can be summarized as follows:

- (i) An EAVOA is proposed to overcome the original algorithm problems.
- (ii) The proposed algorithm enhances AVOA exploration by using RVSS.
- (iii) The proposed algorithm enhances AVOA exploitation by using RFS and SAM.
- (iv) The developed algorithm is tested on 23 classical functions and other 10 from CEC 2021.
- (v) EAVOA is compared with wild horse optimizer (WHO), MPA, GWO, WOA, PSO, dynamic sine-cosine algorithm (DSCA), hybrid sine-cosine algorithm with harmony search (HSC-AHS), and sine-waves-based grey wolf optimizer (SWGWO).
- (vi) EAVOA shows good results, a high balance between exploration and exploitation, and good convergence in both constrained and unconstrained problems.

The overall structure of this study is organized as follows: Section 2 contains a review of related works. Section 3 gives a brief introduction to the inspiration of the AVOA and its mathematical model, whereas Section 4 shows the proposed algorithm. Section 5 shows the experiment results of EAVOA and other comparative algorithms in solving benchmark functions and five real-world optimization problems. The conclusion and some ideas for future works are given in Section 6.

2. Literature Review

Since the appearance of AVOA, several studies have been devoted to considering its features and characteristics. Many enhanced versions of AVOA have been proposed recently to address many issues in different fields. For example, to reconfigure PV systems, AVOA is used to address such problems (Alanazi et al., 2022). Also, AVOA has been successfully used to optimize tube heat and shell exchanger (Gürses et al., 2022). A multi-objective AVOA version is introduced to handle multi-objective industrial engineering problems (Khodadadi et al., 2022). Another two binary versions of AVOA have been proposed to solve the feature selection (FS) problem using two transfer functions (S and V shaped; Balakrishnan et al., 2022). Other work has been done to solve the FS problem and sentiment analysis on movie reviews (Shaddeli et al., 2022). Also, an ameliorated AVOA version has been suggested to diagnose the defects of the rolling bearing.

In literature, many hybrid versions of AVOA have been introduced. For example, AVOA has been hybridized with AO to tackle optimization problems (Xiao et al., 2022). Likewise, another hybrid algorithm is introduced by hybridizing AVOA with honey badger algorithm to estimate the parameter of photovoltaic (Diab et al., 2022). Also, AVOA has been embedded in DE with

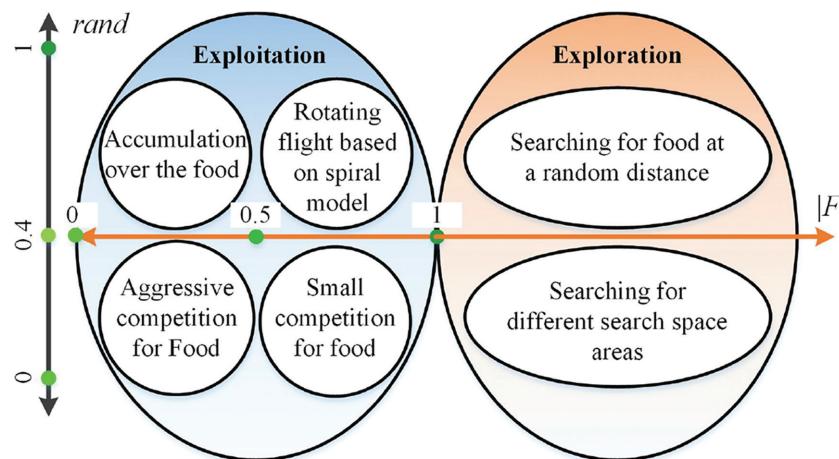


Figure 1: The searching phases of AVOA.

quasi-oppositional (Liu et al., 2022). Likewise, a hybrid version of AVOA and harmony search (Gharehchopogh et al., 2023) is proposed to apply it in clustering.

Multi-layer perceptron (MLP) is a particular type of feedforward neural networks that only contain one hidden layer (Bebis & Georgopoulos, 1994). The performance of MLP mainly depends on the weights between two nodes and the biases of nodes. At first, traditional algorithms like backpropagation algorithm are utilized for training the MLP to obtain optimized weights and biases (Rumelhart et al., 1986). However, when dealing with complex non-linear problems, traditional methods are difficult to find a satisfactory solution. In this case, stochastic search algorithms, such as MAs, are an effective method to optimize the MLP with complex structure (Mirjalili & Sadiq, 2011).

Mirjalili (Mirjalili, 2015) first employed the GWO to train several types of MLP, including five classification (XOR, Balloon, Iris, Breast cancer, and Heart) and three function approximation (Sigmoid, Cosine, and Sine functions) problems. The results show that GWO is more effective in solving this problem compared to PSO and GA. These MLP problems also have been studied by Luo et al. using the spotted hyena optimizer (Luo et al., 2020). Abbas et al. introduced an approach to train MLP based on fitness dependent optimizer (Abbas et al., 2022). The optimized MLP can predict the outcomes of students with high precision.

3. Overview of the AVOA

The AVOA is a newfound metaheuristic proposed in 2021 (Abdollahzadeh et al., 2021). This method mimics the competition and navigation behaviors of African vultures. The African vulture is an intelligent and resilient creature due to its unique physical features. The most impressive thing about African vultures is that they will take appropriate measures in different situations based on the current state of hunger (i.e., the rate of starvation), which is a crucial point for the invented AVOA. The mathematical formulation of the rate of starvation is defined using Equations (1) and (2):

$$F_i(t) = (2 \times rand + 1) \times z \times \left(1 - \frac{t}{T}\right) + dt \quad (1)$$

$$dt = h \times \left(\sin^w \left(\frac{\pi}{2} \times \frac{t}{T}\right) + \cos \left(\frac{\pi}{2} \times \frac{t}{T}\right) - 1\right), \quad (2)$$

where $F_i(t)$ denotes the rate of starvation of i th vulture in the t th iteration, dt denotes the disturbance term of the rate of starvation,

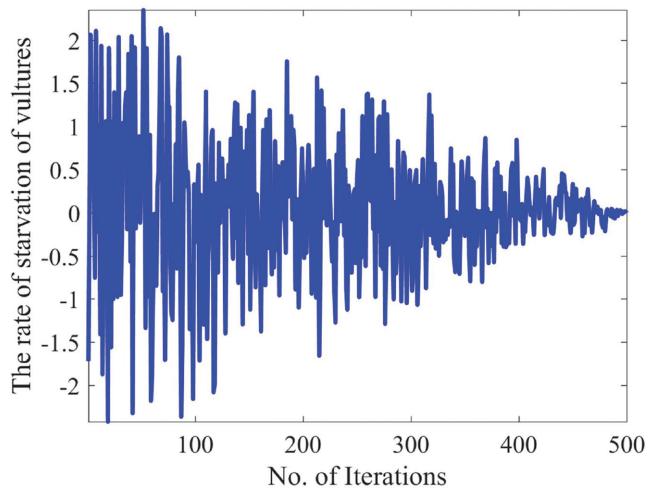


Figure 2: The variation of the rate of starvation of vultures F when the parameter w is 2.5.

h , $rand$, and z are random values range in $[-2, 2]$, $[0, 1]$, and $[-1, 1]$, respectively, w is a fixed number that is set to 2.5 in AVOA, t and T represent the current and maximum number of iterations.

The AVOA will adopt different position updating formulas according to the value of starvation rate (F). Figure 1 presents the exploration and exploitation strategies based on the value of F . Figure 2 shows the varying behavior of the rate of starvation over the course of iterations. The trend of oscillatory contraction shows that vultures emphasize global search in the early stage and local search in the later stage.

To exhibit the crucial characteristics of vultures, the first or second best vulture is selected as the lead vulture. The mathematical equations for this are as follows:

$$R_i(t) = \begin{cases} \text{BestVulture1} & \text{if } p > rand \\ \text{BestVulture2} & \text{Otherwise} \end{cases} \quad (3)$$

where $R_i(t)$ represents the randomly selected lead vulture, BestVulture1 and BestVulture2 represent the first and second best vultures, and p is a constant number that is set to 0.8 in AVOA.

3.1. The exploration phase

When the $|F_i(t)|$ is larger than 1, vultures will randomly search the entire space. Two strategies are used to conduct the exploration

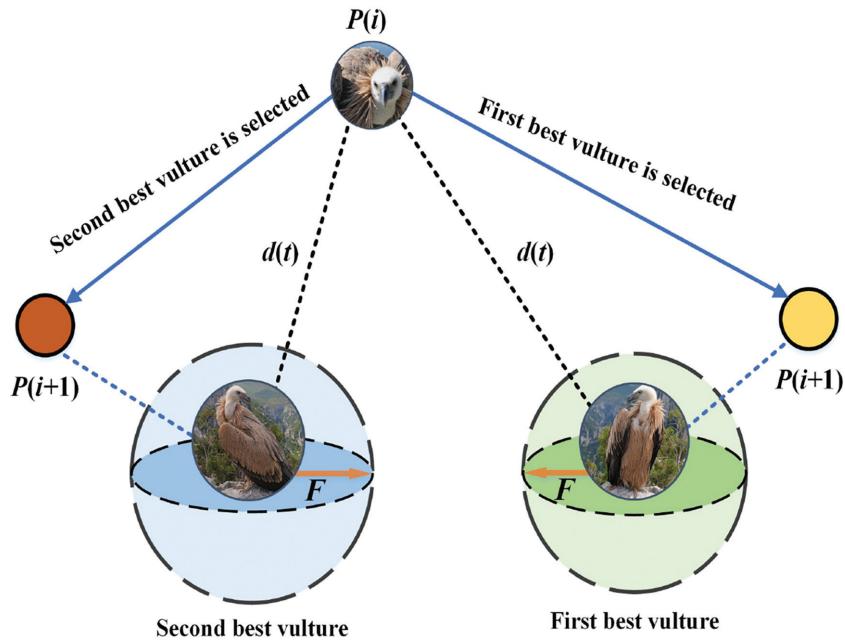


Figure 3: The illustration for the case of competition for food.

Algorithm 1. Pseudo-code of the basic AVOA.

```

1: Initialization
2: Initialize the population size N and maximum iterations T
3: Initialize the positions of all vultures  $P_i$  ( $i = 1, 2, \dots, N$ )
4: Initialize controlling parameters  $P_1, P_2, P_3$ , and  $w$ 
5: while  $t < T$  do
6:   Calculate the vultures' fitness values
7:   Find the first and second best positions of vultures  $P_{BestVulture1}$  and  $P_{BestVulture2}$ 
8:   for each vulture do
9:     Calculate  $F$  using Equation (1)
10:    Determine  $R_i(t)$  using Equation (3)
11:    if  $|F| \geq 1$  then
12:      if  $P_i \geq rand_{P_1}$  then
13:        Update vulture' position using Equation (5)
14:      else
15:        Update vulture' position using Equation (7)
16:      end if
17:    else
18:      if  $|F| \geq 0.5$  then
19:        if  $P_2 \geq rand_{P_2}$  then
20:          Update vulture' position using Equation (9)
21:        else
22:          Update vulture' position using Equation (11)
23:        end if
24:      else
25:        if  $P_3 \geq rand_{P_3}$  then
26:          Update vulture' position using Equation (14)
27:        else
28:          Update vulture' position using Equation (16)
29:        end if
30:      end if
31:    end if
32:  end for
33: end while
34: Return the first best vulture

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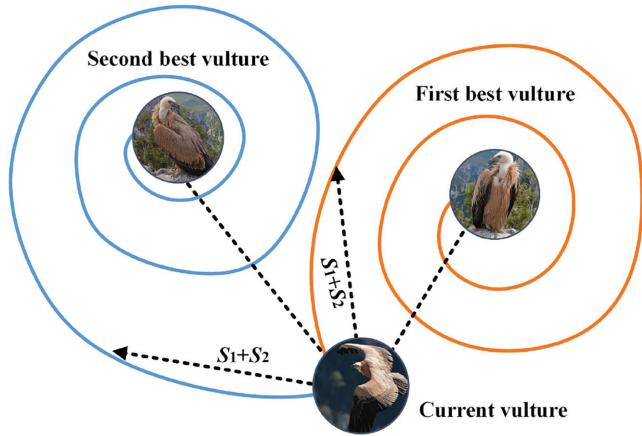


Figure 4: The illustration for the case of rotating flight.

process which is inspired by the vultures' movements for protecting the food. The mathematical model can be expressed in Equations (4–7):

$$P_i(t + 1) = \begin{cases} Eq. (5) & \text{if } P_i \geq rand_{P_1} \\ Eq. (7) & \text{if } P_i < rand_{P_1} \end{cases} \quad (4)$$

$$P_i(t + 1) = R_i(t) - D_i(t) \times F_i(t) \quad (5)$$

$$D_i(t) = |X \times R_i(t) - P_i(t)| \quad (6)$$

$$P_i(t + 1) = R_i(t) - F_i(t) + rand \times ((ub - lb) \times rand + lb), \quad (7)$$

where $P_i(t + 1)$ is newly generated position, P_1 is set to 0.6, $rand_{P_1}$ is a random number between 0 and 1, $D_i(t)$ means the random distance between the current vulture and the selected lead vulture, X is a random number in the range of [0, 2], and ub and lb demonstrate the upper and lower bounds.

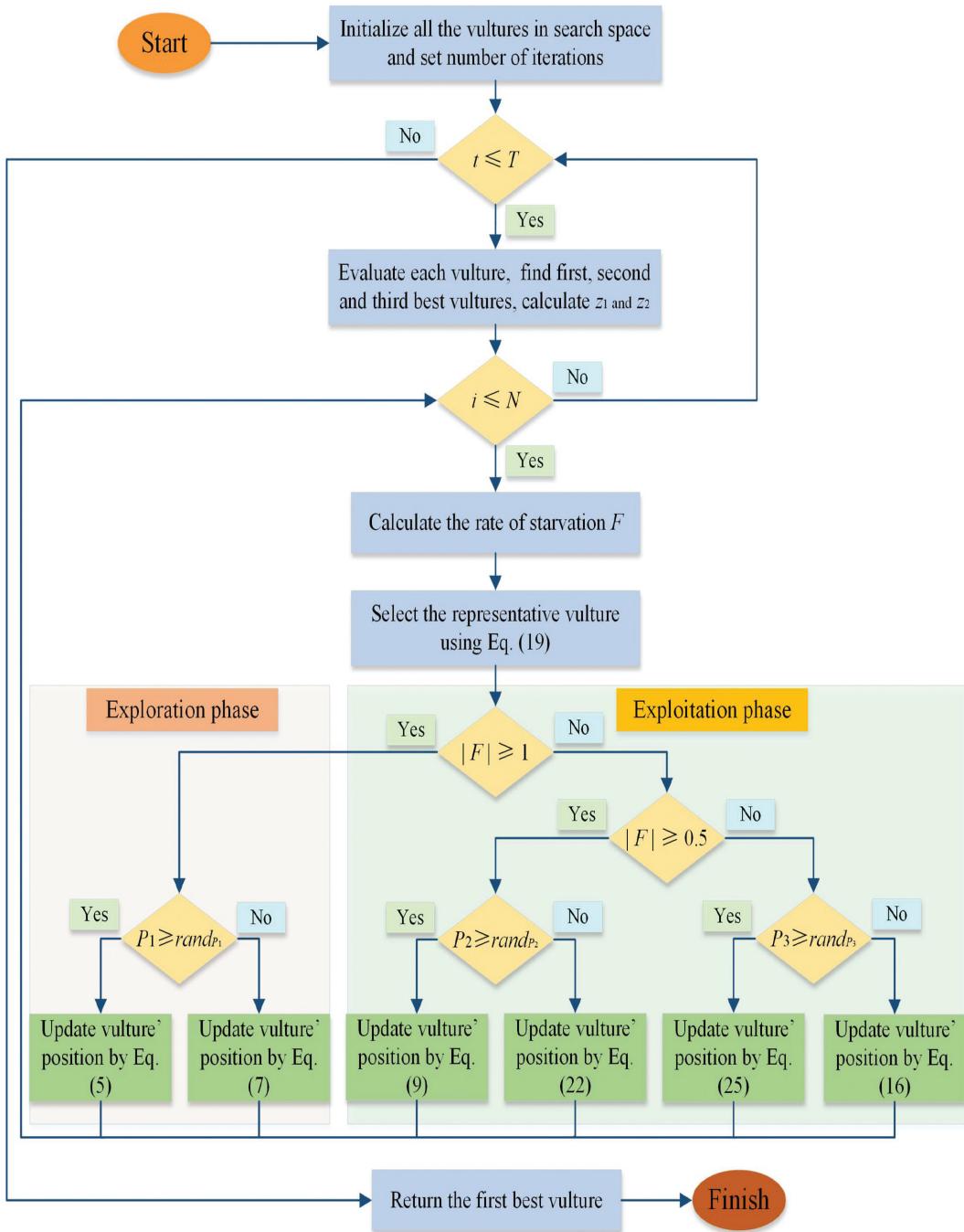


Figure 5: The flowchart of proposed EAVOA.

3.2. The exploitation phase

Vultures start to conduct the exploitation when $|F_i(t)|$ is less than 1. The exploitation phase is divided into two stages. The first stage will be performed when $|F_i(t)|$ is between 0.5 and 1, which is shown in Equation (8):

$$P_i(t+1) = \begin{cases} \text{Eq. (9)} & \text{if } P_2 \geq \text{rand}_{P_2}, \\ \text{Eq. (11)} & \text{if } P_2 < \text{rand}_{P_2}, \end{cases} \quad (8)$$

where P_2 is set to 0.4, and rand_{P_2} is a random number in the range of $[0, 1]$.

Equations (9) and (11) indicate the behaviors of competition for food (Fig. 3) and rotating flight of vultures (Fig. 4), which are modeled as follows.

$$P_i(t+1) = D_i(t) \times (F_i(t) + \text{rand}) - d_i(t) \quad (9)$$

$$d_i(t) = R_i(t) - P_i(t) \quad (10)$$

$$P_i(t+1) = R_i(t) - (S_1 + S_2) \quad (11)$$

$$\begin{cases} S_1 = R_i(t) \times \left(\frac{\text{rand} \times P_i(t)}{2 \times \pi} \right) \times \cos(P_i(t)) \\ S_2 = R_i(t) \times \left(\frac{\text{rand} \times P_i(t)}{2 \times \pi} \right) \times \sin(P_i(t)) \end{cases} \quad (12)$$

In the second stage, $|F_i(t)|$ is less than 0.5. Two vulture behaviors are also modeled, i.e., the accumulation around the food and

Table 1: Definition of the CEC 2005 and CEC 2021 benchmark functions (D denotes the dimensions).

Function type	Fun.	D	Range	Theoretical optimization value
Unimodal test functions	F1	30/100/500/1000	[−100, 100]	0
	F2	30/100/500/1000	[−10, 10]	0
	F3	30/100/500/1000	[−100, 100]	0
	F4	30/100/500/1000	[−100, 100]	0
	F5	30/100/500/1000	[−30, 30]	0
	F6	30/100/500/1000	[−100, 100]	0
	F7	30/100/500/1000	[−1.28, 1.28]	0
Multi-modal test functions	F8	30/100/500/1000	[−500, 500]	−418.9829−D
	F9	30/100/500/1000	[−5.12, 5.12]	0
	F10	30/100/500/1000	[−32, 32]	0
	F11	30/100/500/1000	[−600, 600]	0
	F12	30/100/500/1000	[−50, 50]	0
	F13	30/100/500/1000	[−50, 50]	0
	F14	2	[−65, 65]	0.998 004
Fixed-dimension multi-modal test functions	F15	4	[−5, 5]	0.000 3075
	F16	2	[−5, 5]	−1.031 63
	F17	2	[−5, 5]	0.398
	F18	2	[−2, 2]	3
	F19	3	[−1, 2]	−3.8628
	F20	6	[0, 1]	−3.3220
	F21	4	[0, 10]	−10.1532
CEC 2021 unimodal test function	F22	4	[0, 10]	−10.4028
	F23	4	[0, 10]	−10.5363
	CEC_01	20	[−100, 100]	100
	CEC_02	20	[−100, 100]	1100
	CEC_03	20	[−100, 100]	700
	CEC_04	20	[−100, 100]	1900
	CEC_05	20	[−100, 100]	1700
CEC 2021 hybrid test functions	CEC_06	20	[−100, 100]	1600
	CEC_07	20	[−100, 100]	2100
	CEC_08	20	[−100, 100]	2200
	CEC_09	20	[−100, 100]	2400
	CEC_10	20	[−100, 100]	2500

Table 2: Parameter settings for the EAVOA and other involved methods.

Algorithm	Parameter
EAVOA	$P_1 = 0.6; P_2 = 0.4; P_3 = 0.6; w = 2.5$
AVOA	$P_1 = 0.6; P_2 = 0.4; P_3 = 0.6; w = 2.5$
WHO	$PS = 0.2; PC = 0.13$
MPA	$FADs = 0.2; P = 0.5; CF = [1, 0]$
GWO	$a = [2, 0]$
WOA	$a_1 = [2, 0]; a_2 = [-2, -1]; b = 1$
PSO	$c_1 = 2; c_2 = 2; W \in [0.2, 0.9]; vMax = 6$
DSCA	$w \in [0.1, 0.9], \sigma = 0.1; a_{end} = 0; a_{start} = 2$
HSCAHS	$a = 2; Bandwidth = 0.02$
SWGWO	$A = 2; v_{\alpha} \in [-2, 0]; v_{\beta} \in [0, 1]; v_{\delta} \in [0, 0.5]$

Table 3: Various EAVOAs with three strategies.

Algorithm	RVS	RFS	SA
AVOA	0	0	0
EAVOA1	1	0	0
EAVOA2	0	1	0
EAVOA3	0	0	1
EAVOA4	0	1	1
EAVOA5	1	0	1
EAVOA6	1	1	0
EAVOA7	1	1	1

aggressive competition for food. Similarly, this procedure is shown in Equation (13). Equations (14) and (15) exhibit the movement of vultures around the food source. And Equations (16–18) display the aggressive characteristic of vultures for food:

$$P_i(t+1) = \begin{cases} \text{Eq. (14)} & \text{if } P_3 \geq rand_{P_3} \\ \text{Eq. (16)} & \text{if } P_3 < rand_{P_3} \end{cases} \quad (13)$$

$$P_i(t+1) = \frac{A_1 + A_2}{2} \quad (14)$$

$$\left\{ \begin{array}{l} A_1 = BestVulture1(t) - \frac{BestVulture1(t) \times P_i(t)}{BestVulture1(t) - P_i(t)^2} \times F_i(t) \\ A_2 = BestVulture2(t) - \frac{BestVulture2(t) \times P_i(t)}{BestVulture2(t) - P_i(t)^2} \times F_i(t) \end{array} \right. \quad (15)$$

$$P_i(t+1) = R_i(t) - |d_i(t)| \times F_i(t) \times Levy(d) \quad (16)$$

$$Levy(d) = 0.01 \times \frac{u}{|v|^{\frac{1}{\beta}}}, u \sim (0, \sigma_u^2), v \sim (0, \sigma_v^2) \quad (17)$$

$$\sigma_u = \left(\frac{\Gamma(1+\beta) \times \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2^{(\frac{\beta-1}{2})}} \right)^{\frac{1}{\beta}}, \quad (18)$$

where P_3 is set to 0.4, $rand_{P_3}$ is a random number within [0, 1], u and v satisfy the Gaussian distribution, $\sigma_v = 1$, $\beta = 1.5$, and Γ is the standard gamma function. The pseudo-code of AVOA is presented in Algorithm 1.

Table 4: Sensitivity analysis on the AVOA and various EAVOAs.

Fun.	AVOA	EAVOA1	EAVOA2	EAVOA3	EAVOA4	EAVOA5	EAVOA6	EAVOA7
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F2	1.64E-283	0.00E+00						
F3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F4	5.34E-292	0.00E+00						
F5	4.50E-09	1.52E-09	1.06E-28	1.50E-09	2.47E-25	6.04E-10	1.65E-26	4.92E-31
F6	3.85E-14	3.90E-15	2.62E-39	1.27E-14	3.70E-39	4.79E-15	7.02E-38	1.58E-40
F7	2.43E-08	4.16E-08	6.29E-08	2.55E-08	3.70E-08	2.59E-07	6.75E-08	2.02E-08
F8	1.99E+06	2.36E-05	1.16E+05	6.99E+05	4.17E+05	1.92E-06	1.42E-05	3.14E-06
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	7.89E-31	7.89E-31	7.89E-31	7.89E-31	7.89E-31	7.89E-31	7.89E-31	7.89E-31
F11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F12	3.81E-17	2.49E-17	9.89E-40	1.83E-17	2.12E-37	1.11E-17	2.63E-38	1.96E-37
F13	1.74E-14	6.91E-16	5.01E-33	1.44E-15	8.40E-34	1.66E-15	9.22E-35	6.52E-35
F14	4.46E+00	2.63E-14	4.27E-01	2.38E+00	1.37E+00	2.63E-14	2.63E-14	2.63E-14
F15	9.91E-08	4.02E-08	1.40E-07	7.14E-08	1.14E-07	7.40E-08	1.93E-08	9.41E-09
F16	2.39E-12	2.39E-12	2.39E-12	2.39E-12	2.39E-12	2.39E-12	2.39E-12	2.39E-12
F17	1.27E-08	1.27E-08	1.27E-08	1.27E-08	1.27E-08	1.27E-08	1.27E-08	1.27E-08
F18	1.94E+02	4.86E+01	5.43E-18	7.29E+01	1.56E-22	1.70E+02	2.04E-23	3.59E-25
F19	7.87E-05	1.51E-06	3.20E-10	6.98E-06	3.20E-10	1.31E-06	3.20E-10	3.20E-10
F20	9.13E-03	6.32E-03	8.69E-03	9.18E-03	4.75E-03	5.22E-03	4.34E-03	3.34E-03
F21	1.03E-13	1.03E-13	1.03E-13	1.03E-13	1.03E-13	1.03E-13	1.03E-13	1.03E-13
F22	1.98E-08	1.98E-08	1.98E-08	1.98E-08	1.98E-08	1.98E-08	1.98E-08	1.98E-08
F23	1.21E-08	1.21E-08	1.21E-08	1.21E-08	1.21E-08	1.21E-08	1.21E-08	1.21E-08

4. Proposed EAVOA

In this section, the EAVOA is introduced in details. Three new improvements are utilized to improve the basic AVOA, both in global search and local search, which are RVSS, RFS, and SAM. These three improvements are also inspired based on the vultures' behaviors.

4.1. Representative vulture selection strategy

The first improvement is named as RVSS. The lead vulture is modified by the representative vulture shown in Equations (19–21). Using this method, the representative vulture can be the best, second-best, or randomly generated one.

By introducing the randomly generated vulture, algorithm can effectively avoid falling into the local optimal situation and improve the global exploration ability of the algorithm. The selection of representative vulture is dependent on the rate of starvation and finesse values of best, second-best, and third-best solutions:

$$R_i(t+1) = \begin{cases} \text{BestVulture1} & \text{if } p < z_1 \\ \text{BestVulture2} & \text{if } z_1 \leq p < z_1 + z_2 \\ ((ub - lb) \times rand + lb) & \text{Otherwise} \end{cases} \quad (19)$$

$$p = rand \times |F_i(t)| \quad (20)$$

$$\begin{cases} z_1 = \left(\frac{1}{f(\text{BestVulture2})} \right) / \left(\frac{1}{f(\text{BestVulture1})} + \frac{1}{f(\text{BestVulture2})} + \frac{1}{f(\text{BestVulture3})} \right) \\ z_2 = \left(\frac{1}{f(\text{BestVulture1})} \right) / \left(\frac{1}{f(\text{BestVulture1})} + \frac{1}{f(\text{BestVulture2})} + \frac{1}{f(\text{BestVulture3})} \right) \end{cases}, \quad (21)$$

where the f denotes the calculation of finesse value.

4.2. Rotating flight strategy

The second improvement named as RFS is to increase the exploitation efficiency of rotating flight shown in Equations (22–24). Note that the mathematical models of S_1 and S_2 are modified based on sine and cosine functions in Equation (24) compared to

the basic AVOA. Using S_1 and S_2 , two solutions of rotating flight are generated in Equation (23), respectively. And the greedy selection is completed in Equation (22) to obtain a better performance.

$$P_i(t+1) = \begin{cases} B_1 & \text{if } f(B_1) < f(B_2) \\ B_2 & \text{Otherwise} \end{cases} \quad (22)$$

$$\begin{cases} B_1 = R_i(t) - S_1, B_2 = R_i(t) - S_2, & \text{if } rand < 0.5 \\ B_1 = R_i(t) + S_1, B_2 = R_i(t) + S_2, & \text{Otherwise} \end{cases} \quad (23)$$

$$\begin{cases} S_1 = rand \times ((R_i(t) - P_i(t)) \times F_i(t)) \times \cos(2 \times P_i(t) \times R_i(t)) \\ S_2 = rand \times ((R_i(t) - P_i(t)) \times F_i(t)) \times \sin(2 \times P_i(t) \times R_i(t)) \end{cases} \quad (24)$$

4.3. Selecting accumulation mechanism

To further strengthen the exploitation degree, the SAM is proposed based on the accumulation characteristic of vultures. Equations (25–27) have shown the computational process. Except for the best and second-best vultures, the third-best vulture is also introduced in this mechanism. Similarly, the greedy selection is used to determine better results.

$$P_i(t+1) = \begin{cases} C_4 & \text{if } f(C_4) < f(C_5) \\ C_5 & \text{Otherwise} \end{cases} \quad (25)$$

$$\begin{cases} C_4 = \frac{C_1+C_2}{2} \\ C_5 = \frac{C_1+C_3}{2} \end{cases} \quad (26)$$

$$C_1 = \text{BestVulture1}(t) - \frac{\text{BestVulture1}(t) \times R_i(t)}{\text{BestVulture1}(t) - R_i(t)^2} \times F_i(t) \quad (27)$$

$$C_2 = \text{BestVulture2}(t) - \frac{\text{BestVulture2}(t) \times P_i(t)}{\text{BestVulture2}(t) - R_i(t)^2} \times F_i(t) \quad (27)$$

$$C_3 = \text{BestVulture3}(t) - \frac{\text{BestVulture3}(t) \times P_i(t)}{\text{BestVulture3}(t) - R_i(t)^2} \times F_i(t) \quad (27)$$

4.4. Summary of the proposed method

To improve the performance of basic AVOA, three improvements, i.e., RVSS, RFS, and SAM, are proposed in this work. To be specific,

Table 5: The comparison results of all algorithms on classical test functions (F1-F23) using $D = 30, 200, 500$, and 1000.

Fun.	D	Metric	EAVOA	AVOA	WHO	MPA	GWO	WOA	PSO	DSCA	HSCAHS	SVGWO
F1	30	Mean	0.00E+00	3.36E-294	2.53E-43	8.31E-23	1.00E-27	2.48E-72	1.88E-04	2.36E-103	1.79E-50	9.74E-46
		Std	0.00E+00	9.35E-43	1.01E-22	1.81E-27	1.36E-71	2.63E-04	1.29E-102	5.71E-50	2.43E-45	1.96E-15
		Mean	0.00E+00	9.30E-265	1.54E-32	4.11E-18	9.65E-08	3.44E-70	3.28E+02	4.71E-90	4.95E-34	1.99E-15
	200	Mean	0.00E+00	5.87E-32	3.36E-18	6.18E-08	1.67E-69	3.77E+01	2.56E-89	1.34E-33	1.34E-33	1.99E-15
		Std	0.00E+00	9.71E-282	7.67E-29	7.79E-17	1.57E-03	2.41E-69	5.75E+03	6.26E-89	9.30E-29	8.34E-10
		Mean	0.00E+00	0.00E+00	2.97E-28	5.22E-17	5.84E-04	1.30E-68	3.34E+02	3.43E-88	1.86E-28	5.22E-10
F2	500	Mean	0.00E+00	1.03E-274	3.56E-30	6.21E-16	2.64E-01	1.27E-71	4.11E+04	9.70E-82	4.44E-25	5.65E-07
		Std	0.00E+00	1.00E-00	8.11E-30	4.69E-16	6.78E-02	4.64E-71	2.04E+03	5.31E-81	1.56E-4	3.64E-07
		Mean	7.32E-228	1.40E-157	2.26E-24	2.76E-13	1.09E-16	1.86E-51	9.03E+00	3.98E-63	5.24E-27	4.99E-27
	1000	Mean	0.00E+00	5.38E-157	7.72E-24	2.50E-13	1.11E-16	4.33E-51	1.03E+01	2.09E-62	1.63E-26	1.11E-26
		Std	0.00E+00	1.97E-196	2.52E-148	5.87E-32	7.92E-11	3.12E-05	2.12E-48	4.58E+02	2.43E-52	1.07E-18
		Mean	0.00E+00	1.16E-147	1.92E+02	6.59E-11	6.59E-06	1.08E-47	5.74E+01	1.31E-51	3.71E-18	4.91E-10
F3	500	Mean	5.32E-202	2.94E-146	6.37E+02	6.50E-10	1.09E-02	9.79E-48	1.66E+00	2.95E-46	2.85E-16	1.34E-06
		Std	0.00E+00	1.61E-145	6.55E+02	1.33E-09	1.64E-03	5.08E-47	9.09E+00	1.62E-45	3.61E-16	4.27E-07
		Mean	0.00E+00	3.05E-143	1.79E+03	1.18E+03	3.36E-01	2.34E-49	1.41E+03	INF	2.39E-15	6.30E-04
	1000	Std	0.00E+00	1.67E-142	1.41E+03	2.22E+02	2.72E-01	6.73E-49	6.03E+01	NAN	4.53E-15	5.42E-04
		Mean	0.00E+00	2.16E-275	1.71E-26	2.05E-04	3.53E-05	4.56E+04	8.94E+01	3.49E-57	8.18E-49	5.41E-12
		Std	0.00E+00	0.00E+00	6.98E-26	3.16E-04	8.59E-05	1.33E+04	2.59E+01	1.38E-56	2.34E-48	1.49E-11
F4	200	Mean	2.96E-323	5.56E-252	5.53E-13	3.59E+02	2.08E+04	5.12E+06	8.13E+04	2.51E-48	3.71E-18	4.15E+03
		Std	0.00E+00	0.00E+00	1.47E-12	3.11E+02	1.09E+04	1.65E+06	1.92E+04	1.24E-47	1.07E-30	3.75E+03
		Mean	4.94E-323	6.04E-264	1.10E-07	8.31E+03	3.22E+05	2.98E+07	5.80E+05	3.39E-27	2.30E-26	1.18E+05
	1000	Std	0.00E+00	0.00E+00	5.47E-07	8.31E+03	8.89E+04	9.34E+06	1.48E+05	1.86E-26	4.21E-26	5.72E+04
		Mean	0.00E+00	1.59E-248	4.86E-06	2.36E+04	1.54E+06	1.18E+08	2.26E+06	1.50E-32	6.48E-23	6.62E+05
		Std	0.00E+00	0.00E+00	2.12E-05	1.27E+04	2.38E+05	5.06E+07	6.37E+05	8.22E-32	1.20E-22	1.79E+05
F5	30	Mean	4.04E-226	1.09E-146	1.97E-17	3.46E-09	1.05E-06	4.07E+01	1.15E+00	1.15E+00	7.96E-49	9.70E-13
		Std	0.00E+00	5.98E-146	5.64E-17	1.80E-09	1.68E-06	2.46E+01	1.97E-01	2.83E-48	3.44E-25	2.29E-13
		Mean	1.84E-192	1.34E-142	4.00E-11	1.45E-06	2.62E+01	8.17E+01	1.98E+01	1.12E-31	3.88E-16	3.19E+01
	200	Std	0.00E+00	6.83E-142	1.91E-10	6.14E-07	8.37E+00	2.15E+01	1.50E+00	6.16E-31	5.60E-16	8.39E+00
		Mean	1.42E-184	9.30E-143	2.57E-10	1.95E-05	6.42E+01	8.33E+01	2.76E+01	2.29E-31	3.12E-11	7.10E+01
		Std	0.00E+00	4.28E-142	4.29E-10	2.05E-05	6.41E+00	2.10E+01	1.78E+00	8.73E-31	9.09E-11	5.24E+00
F6	1000	Mean	4.19E-212	1.70E-142	2.12E-09	3.04E-04	7.90E+01	8.00E+01	1.98E+01	4.90E-33	2.08E-07	8.21E+01
		Std	0.00E+00	7.03E-142	3.71E-09	4.28E-04	3.69E+00	9.94E+02	1.58E+00	2.26E-32	3.55E-07	3.70E+00
		Mean	6.66E-14	4.32E-05	3.13E+01	2.52E+01	2.71E+01	2.79E+01	8.93E+01	2.86E+01	2.88E+01	2.75E+01
	200	Std	3.57E-13	3.93E-05	1.56E+01	3.91E-01	6.96E-01	5.65E+01	3.01E-01	5.45E-02	7.66E-01	5.24E+00
		Mean	1.11E-12	2.22E-03	3.01E+02	1.97E+02	1.98E+02	1.98E+02	6.20E+05	1.99E+02	1.99E+02	1.98E+02
		Std	5.86E-12	2.55E-03	3.01E+02	5.03E-01	1.48E-01	1.39E+05	8.33E-02	6.85E-02	3.83E-01	3.83E-01
F7	30	Mean	9.31E-12	6.75E-03	4.98E+02	4.97E+02	4.98E+02	4.96E+02	3.01E+07	4.99E+02	4.98E+02	4.98E+02
		Std	2.89E-11	6.44E-03	6.50E-01	2.32E-01	2.43E-01	3.90E-01	4.49E+06	3.33E-02	3.65E-02	7.07E-02
		Mean	3.81E-11	2.83E-02	9.98E+02	9.97E+02	1.06E+03	9.94E+02	2.76E+08	9.99E+02	9.99E+02	9.98E+02
	1000	Std	1.99E-10	3.03E-02	7.21E-01	1.56E-01	2.41E+01	8.78E-01	2.23E+07	1.74E-02	5.08E-02	9.76E-02
		Mean	9.55E-20	1.87E-07	2.62E-03	4.20E-08	7.33E-01	3.77E-01	2.53E-04	5.66E-00	6.69E+00	1.84E+00
		Std	3.03E-19	1.07E-07	9.59E-03	2.15E-08	3.62E-01	1.95E-01	5.25E-04	2.58E-01	2.22E-01	3.75E-01

Table 5: Continued

Fun.	D	Metric	EAVOA	AVOA	WHO	MPA	GWO	WOA	PSO	DSCA	HSCAHS	SWGWO
200	Mean	2.09E-16	1.41E-02	2.77E+01	1.80E+01	2.88E+01	1.08E+01	3.33E+02	4.80E+01	4.91E+01	3.69E+01	
		Std	7.88E-16	4.20E-02	7.40E+00	1.41E+00	1.03E+00	2.21E+00	4.69E+01	4.69E+01	2.08E-01	1.40E+00
500	Mean	4.10E-17	4.22E-02	1.11E+02	7.62E+01	9.15E+01	3.03E+01	5.76E+03	1.23E+02	1.24E+02	1.08E+02	
		Std	1.77E-16	1.49E-01	3.51E+01	2.52E+00	1.45E+00	6.57E+00	3.77E+02	4.12E-01	2.40E-01	1.40E+00
1000	Mean	8.67E-17	1.20E-01	3.75E+02	1.89E+02	2.02E+02	6.90E+01	4.10E+04	2.48E+02	2.49E+02	2.29E+02	
		Std	2.63E-16	2.82E-01	8.23E+02	2.24E+00	2.59E+00	1.94E+01	1.76E+03	3.81E-01	1.63E-01	1.22E+00
F7	30	1.24E-04	1.21E-04	9.58E-04	1.43E-03	1.90E-03	4.40E-03	2.27E+00	1.34E-03	1.19E-04	7.79E-04	
		Std	1.26E-04	1.90E-04	6.94E-04	8.94E-04	1.13E-03	4.89E-03	3.25E+00	1.18E-03	1.28E-04	6.42E-04
200	Mean	2.57E-04	1.34E-04	2.14E-03	2.10E-03	1.45E-02	4.82E-03	2.81E+03	2.81E+03	2.06E-03	1.06E-04	3.66E-03
		Std	2.43E-04	1.08E-04	1.75E-03	1.12E-03	5.10E-03	6.65E-03	4.45E+02	2.45E-03	1.06E-04	1.93E-03
500	Mean	1.76E-04	1.26E-04	2.23E-03	2.19E-03	4.77E-02	3.63E-03	4.55E+04	3.37E-03	1.24E-04	7.05E-03	
		Std	2.05E-04	8.76E-05	2.11E-03	1.14E-03	1.47E-03	4.28E-03	6.62E+03	6.62E+03	1.16E-04	3.53E-03
1000	Mean	1.47E-04	1.47E-04	1.55E-03	2.19E-03	1.61E-01	4.82E-03	2.43E+05	3.23E-03	9.91E-05	1.26E-02	
		Std	1.76E-04	1.47E-04	1.13E-03	1.16E-03	3.75E-02	5.64E-03	6.10E+03	2.47E-03	8.51E-05	5.91E-03
F8	30	-1.26E+04	-1.17E+04	-9.02E+03	-8.89E+03	-5.84E+03	-1.04E+04	-4.63E+03	-4.45E+03	-2.57E+03	-4.12E+03	
		Std	2.78E-03	7.88E-02	6.04E+02	4.54E+02	9.30E+02	1.81E+03	1.51E+03	5.02E+02	2.88E+02	1.22E+03
200	Mean	-8.38E+04	-7.83E+04	-3.58E+04	-4.28E+04	-2.88E+04	-7.17E+04	-1.39E+04	-1.19E+04	-6.86E+03	-1.29E+04	
		Std	1.25E+01	5.14E+03	2.94E+03	1.79E+03	4.48E+03	1.31E+04	5.97E+03	1.14E+03	1.21E+03	8.36E+03
500	Mean	-2.09E-05	-1.95E+05	-5.81E+04	-8.22E+04	-5.63E+04	-1.73E-05	-2.56E+04	-1.89E+04	-1.02E+04	-2.04E+04	
		Std	1.26E-01	1.65E-04	6.01E+03	3.55E+03	9.04E+03	2.80E+04	8.93E-03	1.93E-03	1.33E+03	1.40E+04
1000	Mean	-4.19E+05	-3.93E+05	-8.18E+04	-1.26E+05	-7.97E+04	-1.26E+05	-3.59E-05	-4.40E+04	-2.71E+04	-4.42E+04	
		Std	3.20E+01	3.14E-04	8.62E+03	5.07E+03	2.47E+04	5.10E+04	1.13E+04	3.02E+03	1.62E+03	1.56E+04
F9	30	0.00E+00	0.00E+00	3.69E-12	0.00E+00	1.95E+00	0.00E+00	1.10E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		Std	0.00E+00	0.00E+00	2.02E-11	0.00E+00	3.09E+00	0.00E+00	2.92E+01	0.00E+00	0.00E+00	0.00E+00
200	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.29E+01	7.58E-15	1.98E+03	0.00E+00	0.00E+00	5.46E-13
		Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.18E+01	4.15E-14	1.30E+02	0.00E+00	0.00E+00	3.41E-13
500	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.04E+03	0.00E+00	6.36E+03	0.00E+00	0.00E+00	4.94E-09	
		Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.09E+01	0.00E+00	1.88E+02	0.00E+00	0.00E+00	1.23E-08
1000	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.06E+02	1.21E-13	1.44E+04	0.00E+00	0.00E+00	1.75E-06
		Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.86E+01	6.64E-13	3.44E+02	0.00E+00	0.00E+00	6.18E-06
F10	30	8.88E-16	8.88E-16	2.43E-15	1.59E-12	1.01E-13	4.09E-15	2.05E-15	4.85E-01	8.88E-16	8.88E-16	9.53E-15
		Std	0.00E+00	0.00E+00	2.02E-15	7.44E-13	1.48E-14	4.09E-14	6.51E+00	0.00E+00	0.00E+00	2.22E-15
200	Mean	8.88E-16	8.88E-16	1.24E-15	1.39E-10	2.19E-05	3.49E-15	6.51E+00	8.88E-16	8.88E-16	8.88E-16	
		Std	0.00E+00	0.00E+00	1.08E-15	7.82E-11	5.39E-06	2.46E-15	4.06E-01	0.00E+00	0.00E+00	1.23E-09
500	Mean	8.88E-16	8.88E-16	1.95E-15	4.07E-10	1.93E-03	4.44E-15	1.18E+01	8.88E-16	8.88E-16	1.34E-06	
		Std	0.00E+00	0.00E+00	1.90E-15	2.11E-10	4.13E-04	2.64E-15	3.75E-01	0.00E+00	0.00E+00	3.57E-07
1000	Mean	8.88E-16	8.88E-16	2.31E-15	8.20E-10	1.85E-02	3.73E-15	1.60E+01	8.88E-16	8.88E-16	6.57E-15	
		Std	0.00E+00	0.00E+00	2.00E-15	2.67E-10	2.69E-03	2.70E-15	3.75E-01	0.00E+00	0.00E+00	2.71E-05
F11	30	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.89E-03	1.39E-02	5.11E-03	0.00E+00	0.00E+00	1.34E-06
		Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.98E-03	5.62E-02	6.50E-03	0.00E+00	0.00E+00	0.00E+00
200	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.39E-06	0.00E+00	2.42E+00	0.00E+00	0.00E+00	0.00E+00	1.44E-15
		Std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.73E-02	0.00E+00	2.59E+00	0.00E+00	0.00E+00	0.00E+00
500	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.59E-02	0.00E+00	8.12E-01	0.00E+00	0.00E+00	0.00E+00	1.24E-10

Table 5: Continued

Fun.	D	Metric	EAVOA	AVOA	WHO	MPA	GWO	WOA	PSO	DSCA	HSCAHS	SWGWO
F12	1000	Std	0.00E+00	0.00E+00	0.00E+00	4.49E-02	0.00E+00	1.27E+01	0.00E+00	0.00E+00	8.17E-11	
		Mean	0.00E+00	0.00E+00	0.00E+00	4.07E-17	5.22E-02	3.70E-18	2.79E+02	0.00E+00	0.00E+00	1.42E-03
		Std	0.00E+00	0.00E+00	0.00E+00	5.44E-17	8.29E-02	2.03E-17	1.55E+01	0.00E+00	0.00E+00	7.77E-03
	30	Mean	3.10E-20	6.18E-09	2.43E-02	1.05E-04	4.86E-02	2.03E-02	2.83E-06	7.95E-01	1.04E+00	1.13E-01
		Std	1.26E-19	6.73E-09	6.49E-02	3.51E-04	2.39E-02	1.24E-02	3.81E-06	8.09E-02	5.05E-02	4.36E-02
		Mean	1.05E-19	1.58E-07	5.33E-01	1.47E-01	5.25E-01	7.04E-02	4.32E+01	1.13E+00	1.19E+00	7.17E-01
F13	200	Std	3.15E-19	1.39E-07	5.51E-01	1.67E-02	5.72E-02	3.78E-02	1.83E+01	3.12E-02	2.61E-02	5.61E-02
		Mean	1.37E-17	2.76E-07	8.42E-01	4.06E-01	7.35E-01	1.07E-01	2.22E+05	1.17E+00	1.19E+00	9.14E-01
		Std	4.58E-17	7.58E-07	3.72E-01	2.90E-02	3.61E-02	5.62E-02	1.12E-02	1.07E-02	9.85E-03	3.18E-02
	1000	Mean	2.34E-17	5.05E-05	9.53E-01	6.49E-01	1.19E+00	1.15E-01	9.51E-01	1.17E+00	1.18E+00	9.88E-01
		Std	9.30E-17	1.25E-04	1.29E-01	2.23E-02	2.16E-01	6.38E-02	2.08E+06	5.31E-03	3.85E-03	1.52E-02
		Mean	7.19E-19	1.02E-07	5.91E-02	1.47E-02	6.64E-01	4.93E-01	3.01E-03	2.82E+00	2.85E+00	1.12E+00
F14	200	Std	2.58E-18	8.44E-08	1.51E-01	2.21E-02	2.66E-01	2.91E-01	4.92E-03	5.86E-02	2.84E-02	1.94E-01
		Mean	2.47E-15	6.71E-03	1.79E+01	1.90E+01	1.69E+01	6.63E+00	6.18E+03	1.99E-01	1.99E+01	1.77E+01
		Std	9.05E-15	3.67E-02	3.52E+00	2.25E-01	5.22E-01	2.23E+00	5.53E+03	5.52E-02	2.67E-02	1.77E+01
	500	Mean	1.06E-14	1.19E-06	5.65E+01	5.65E+01	5.03E+01	1.87E+01	4.61E+06	4.99E-01	4.99E+01	4.77E+01
		Std	4.95E-14	9.78E-07	2.35E-01	1.40E+00	1.40E+00	5.74E+00	1.13E+06	4.91E-02	3.70E-02	4.15E-01
		Mean	2.85E-14	6.78E-03	1.17E+02	9.86E+01	1.24E+02	3.97E+01	8.66E+07	1.00E+02	9.99E+01	9.79E+01
F15	1000	Std	1.17E-13	3.71E-02	1.42E+01	1.17E+02	1.22E+01	1.08E+01	1.19E+07	4.04E-02	3.04E-02	8.45E-01
		Mean	9.98E-01	1.30E+00	1.66E+00	9.98E-01	4.30E+00	2.50E+00	3.10E+00	1.62E+00	2.86E+00	3.49E+00
		Std	2.66E-16	5.92E-01	1.43E+00	1.27E-16	3.89E+00	2.91E+00	3.02E+00	7.95E-01	4.14E-01	3.44E+00
	4	Mean	3.38E-04	5.00E-04	3.49E-03	3.07E-04	5.05E-03	6.69E-04	4.61E-03	1.34E-03	2.26E-03	8.22E-04
		Std	4.95E-05	4.10E-04	6.89E-03	7.21E-15	8.59E-03	4.60E-04	7.50E-03	3.37E-04	1.57E-03	1.47E-03
		Mean	-1.03E+00									
F16	2	Mean	5.08E-16	4.54E-16	5.13E-16	4.74E-16	1.74E-08	6.94E-10	6.45E-16	1.25E-04	4.06E-03	2.93E-05
		Std	3.98E-01	4.00E-01	7.22E-01	3.98E-01						
		Mean	0.00E+00	0.00E+00	0.00E+00	1.47E-06	8.44E-06	8.44E-06	8.00E+00	2.40E-03	2.54E-01	1.09E-05
	2	Std	3.00E+00	1.11E+01	5.70E+00	3.00E+00	3.00E+00	3.00E+00	3.01E+00	3.01E+00	3.01E+00	3.00E+00
		Mean	4.26E-12	1.26E+01	1.48E+01	2.17E-15	4.35E-05	2.68E-04	1.43E-15	8.36E-03	1.27E-02	6.00E-05
		Std	-3.86E+00	-3.86E+00	-3.86E+00	-3.76E-00	-8.52E+00	-3.86E+00	-3.86E+00	-3.84E+00	-3.31E+00	-3.36E+00
F18	2	Mean	7.40E-09	7.54E-03	2.71E-15	2.36E-15	5.22E-01	7.71E-03	2.70E-15	1.94E-02	3.77E-02	4.59E-03
		Std	-3.28E+00	-3.25E+00	-3.28E+00	-3.28E+00	-3.28E+00	-3.28E+00	-3.22E+00	-3.19E+00	-3.02E+00	-3.18E+00
		Mean	5.99E-02	6.19E-02	6.31E-02	1.20E-11	9.84E-02	1.06E-01	3.00E-01	8.49E-02	4.68E-01	1.09E-01
	2	Std	-1.02E+01	-1.02E+01	-8.48E+00	-1.02E+01	-8.26E+00	-8.52E+00	-7.27E+00	-4.39E+00	-6.60E-01	-6.41E+00
		Mean	4.21E-15	2.90E+00	2.90E+00	2.53E-11	2.79E+00	2.52E+00	3.03E+00	2.70E-15	1.94E-02	3.77E-02
		Std	4.20E-15	2.58E-14	3.52E+00	4.27E-11	9.79E-01	2.92E+00	3.10E+00	7.46E-01	3.84E-01	2.29E+00
F20	6	Mean	-1.04E+01	-9.57E+00	-1.04E+01	-1.04E+01	-7.50E+00	-8.74E+00	-3.19E+00	-3.02E+00	-1.66E+00	-3.18E+00
		Std	1.40E-15	1.05E-14	2.16E+00	1.61E-11	1.69E-03	1.04E+01	3.00E-01	8.49E-02	4.68E-01	1.09E-01
	4	Mean	-1.05E+01	-7.53E+00	-1.05E+01	-1.04E+01	-7.87E+00	-8.73E+00	-8.73E+00	-4.31E+00	-8.60E-01	-1.03E+00
		Std	4.20E-15	2.58E-14	3.52E+00	4.27E-11	9.79E-01	2.92E+00	3.10E+00	5.95E-01	3.08E-01	9.85E-01

Table 6: The Wilcoxon signed-rank test results between EAVOA and other algorithms on classical test functions (F1–F23).

Fun.	D	EAVOA vs. AVOA	EAVOA vs. MHO	EAVOA vs. MPA	EAVOA vs. GWO	EAVOA vs. WOA	EAVOA vs. PSO	EAVOA vs. DSCA	EAVOA vs. HSCAHA	EAVOA vs. EAOAs.	EAVOA vs. SWMO
F1	30	1.25E-01	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	9.77E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	9.77E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	7.81E-03	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F2	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	3.91E-02	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F3	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	1.22E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	1.22E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F4	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F5	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F6	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F7	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F8	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	1000	3.05E-04	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
F9	30	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	200	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	500	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	1000	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
F10	30	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05
	200	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00
	500	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	1000	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00

Table 6: Continued

Fun.	D	EAVOA vs. AVOA	EAVOA vs. WHO	EAVOA vs. MPA	EAVOA vs. GWO	EAVOA vs. WOA	EAVOA vs. PSO	EAVOA vs. DSCA	EAVOA vs. HSCAHA	EAVOA vs. SWGWO
F11	1000	1.00E+00	6.10E-05	6.10E-05	6.10E-05	1.00E+00	1.00E+00	1.00E+00	1.00E+00	4.8/14/0 6/0/2/0
F12	30	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	6.10E-05 6.10E-05
F13	200	200	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	6.10E-05 6.10E-05
F14	30	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	4.12/12/0 45/17/0
F15	200	500	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05 6.10E-05
F16	2	2	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F17	2	500	1000	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05	6.10E-05 6.10E-05
F18	2	2	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F19	3	3	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F20	2	2	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F21	500	500	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F22	4	4	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
F23	4	4	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	1.00E-00	6.10E-05 6.10E-05
Overall (+/-)	4	4	4	4	4	4	4	4	4	6.10E-05 6.10E-05

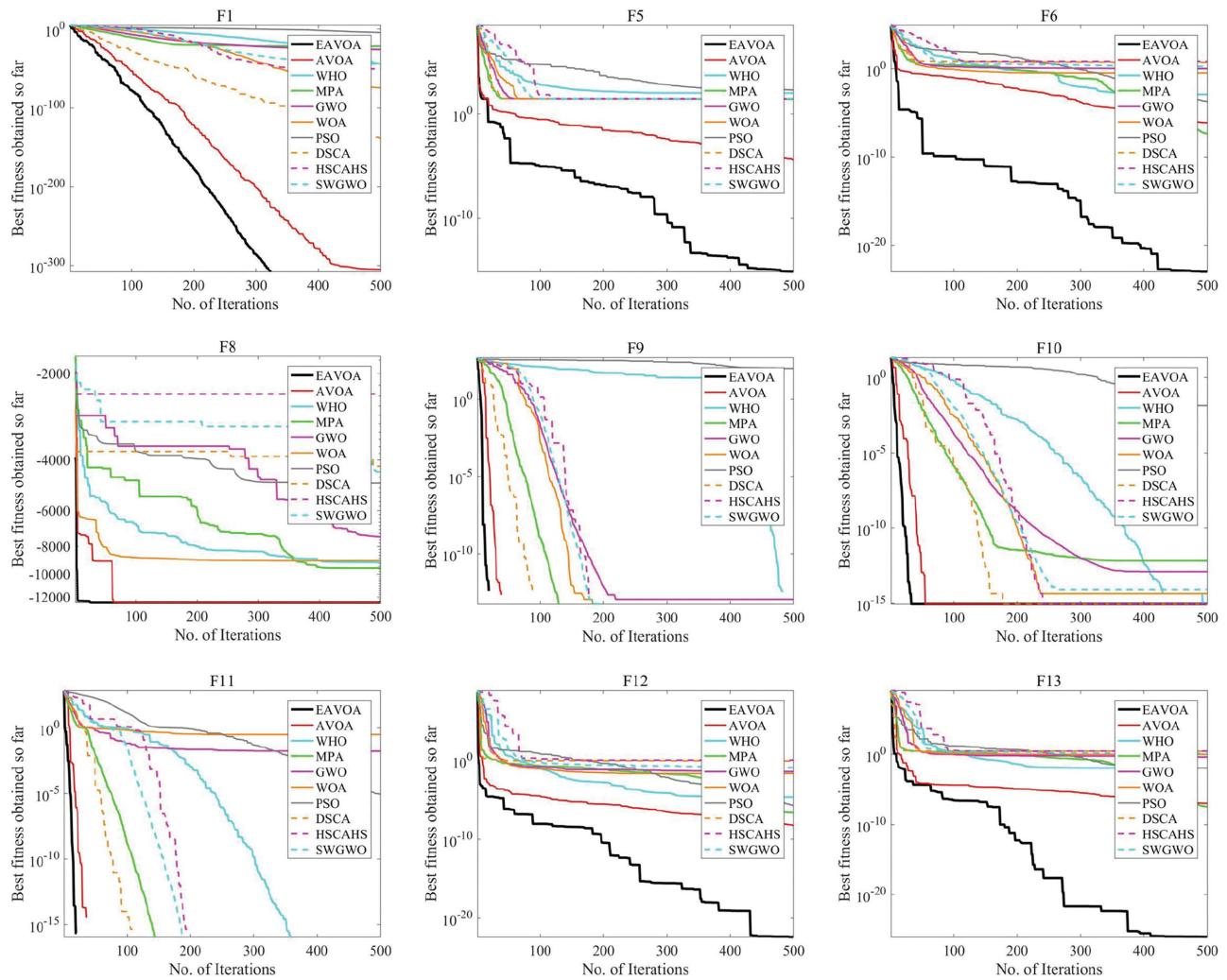


Figure 6: The convergence curves of the EAVOA with $D = 30$ (F1, F5, F6, F8–F10, F11–F13).

RVSS is used to improve the global exploration ability by introducing a random vulture as the lead vulture. It can strike a good balance between global search and local search. RFS and SAM are two methods to improve further the exploitation efficiency of AVOA, which are used to replace the original rotating flight and accumulation around the food behaviors, respectively.

Using the three proposed improvements, the global and local search capability of basic AVOA is effectively strengthened. The pseudo-code of the proposed EAVOA is given in Algorithm 2, and the flowchart is presented in Fig. 5.

4.5. Computational complexity

The computational complexity is associated with the maximum iteration (T), population size (N), and problem's dimension (D), which will affect the efficiency of the optimization algorithm (Tallini et al., 2016). The computational complexity of the basic AVOA is as follows:

- (i) The computational complexity of the initialization phase is $O(N)$.
- (ii) The computational complexity of updating the positions of all vultures is $O(T - N - D)$.
- (iii) The computational complexity of determining the best position is $O(T - N)$.

Therefore, the final computational complexity of AVOA is $O(N(T - D + T + 1))$. For the computational complexity of proposed EAVOA, considering the worst situation, each vulture updates the position using the proposed RFS or SAM throughout the iteration. And then, two candidate positions will be generated for a vulture in each iteration. The computational complexity of updating the positions of all vultures is $O(2T - N - D)$. Therefore, the total computational complexity of EAVOA is $O(N(2T - D + T + 1))$.

Compared with the original AVOA, the computational complexity has been increased to some extent. However, considering that the relatively low probability of actually performing RFS or SAM in the whole process and the better optimization performance obtained, the additional computational complexity is acceptable.

5. Experiment Results

This section discusses the experiment results for the classic CEC 2005 and the latest CEC 2021 test suits (Mohamed et al., 2020; Suganthan et al., 2005). It shows the mean and standard deviation for the 23 functions and the Wilcoxon signed-rank test results. The convergence curves for CEC 2005 and the boxplots for CEC 2021 are also presented in this section. In addition, the impact of RVS, RFS, and SA is also discussed in this section.

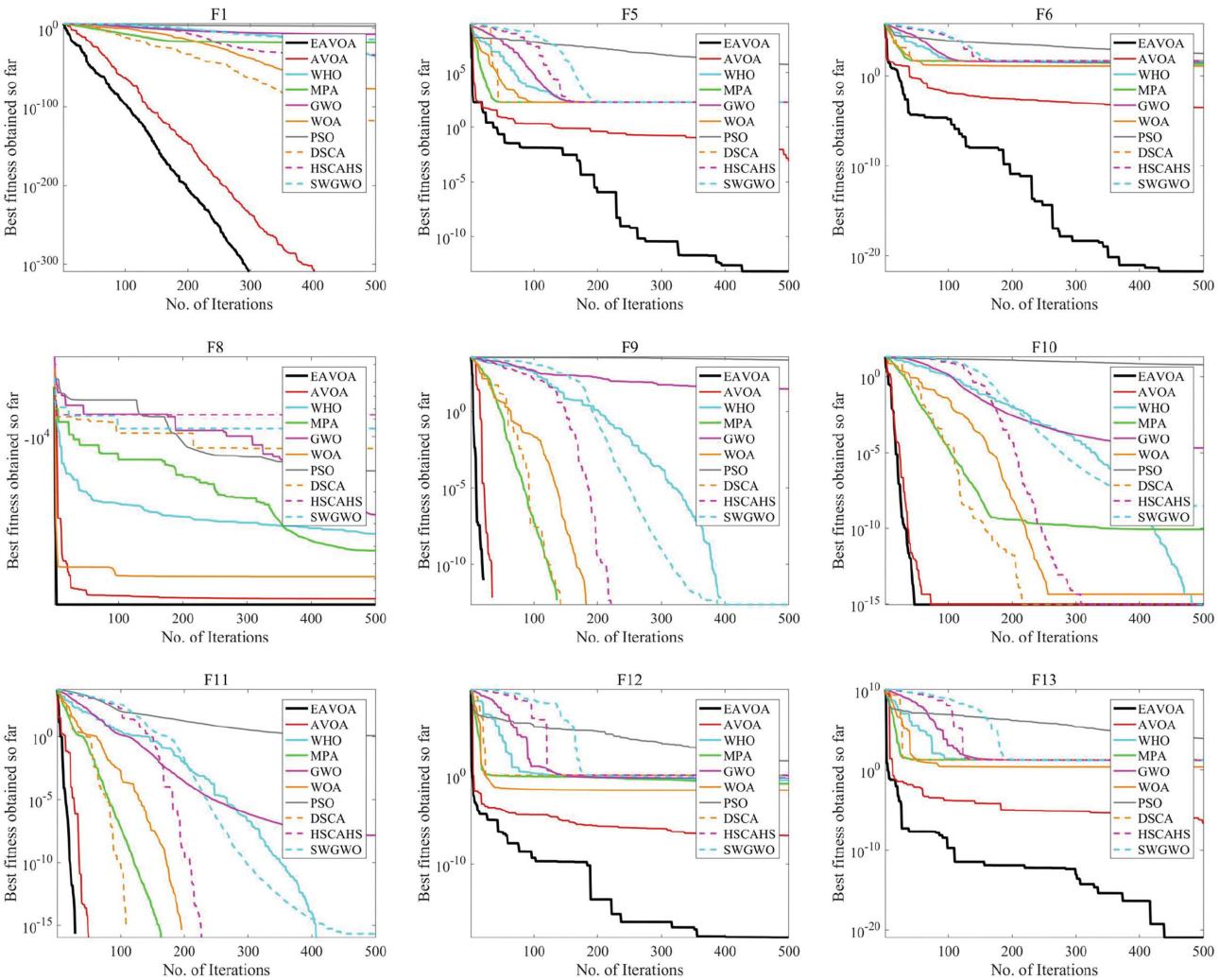


Figure 7: The convergence curves of the EAVOA with $D = 200$ (F1, F5, F6, F8–F10, F11–F13).

Table 1 presents the different functions of CEC 2005 and CEC 2021 by showing the function type, name, dimensions (D), range, and theoretical optimization value. It is observed from the table that the function types include the unimodal, multi-modal, fixed-dimension multi-modal, CEC 2021 unimodal, CEC 2021 basic, CEC 2021 hybrid, and CEC 2021 composition test functions. Different ranges and theoretical optimization values are given by the different functions. The dimensions of 30/100/500/1000 are considered for F1–F13, whereas they vary between 2 and 6 for F14–F23. conversely, a value of 20 is set for the CEC_01–CEC_10 functions.

Different dimensions (F1–F13) are used to perform a scalability test in order to see the efficiency of the suggested optimizer in handling high-dimensional problems. Authors cannot apply the scalability test to fixed-dimension multi-modal as their mathematical models are fixed without more variables.

The proposed EAVOA has been compared to basic AVOA and eight other methods, including WHO (Zheng et al., 2022), MPA (Faramarzi et al., 2020), GWO (Mirjalili et al., 2014), WOA (Hussien et al., 2017b; Mirjalili & Lewis, 2016), PSO (Kennedy & Eberhart, 1995), DSCA (Li et al., 2021), HSCAHS (Singh & Kaur, 2021), and SWGWO (Long et al., 2019).

The experiments were conducted using different parameters for the different algorithms. This is listed in Table 2. For example,

AVOA and AVOA algorithms have values of 0.6, 0.4, 0.6, and 2.5 for P_1 , P_2 , P_3 , and w , whereas WHO has values of 0.2 and 0.13 for PS and PC.

To make a fair comparison, the experiments have been carried out 30 times under the following conditions: the population size is 30 and maximum iteration is 500 for all algorithms during the tests. Moreover, all the experiments were performed on the operating system of windows 10 with Intel (R) core (TM) i5-9500 CPU @ 3.00 GHz and 16 GB RAM. The related algorithms are coded by MATLAB R2016a.

5.1. The impact of RVS, RFS, and SA

Different strategies are applied to the different algorithms, which include the AVOA and EAVOA1–EAVOA7 algorithms. The strategies are RVS, RFS, and SA where the existence of the strategy is represented by one and the absence of the strategy is represented by 0, as presented in Table 3. For example, EAVOA1 considers the RVS strategy only, while EAVOA6 considers the RVS and RFS strategies. On the other hand, EAVOA7 uses all the strategies, including the RVS, RFS, and SA.

Table 4 presents the sensitivity analysis for AVOA and EAVOA1–EAVOA7 algorithms with different functions. The values of the sensitivity analysis show that EAVOA7 has better values than the

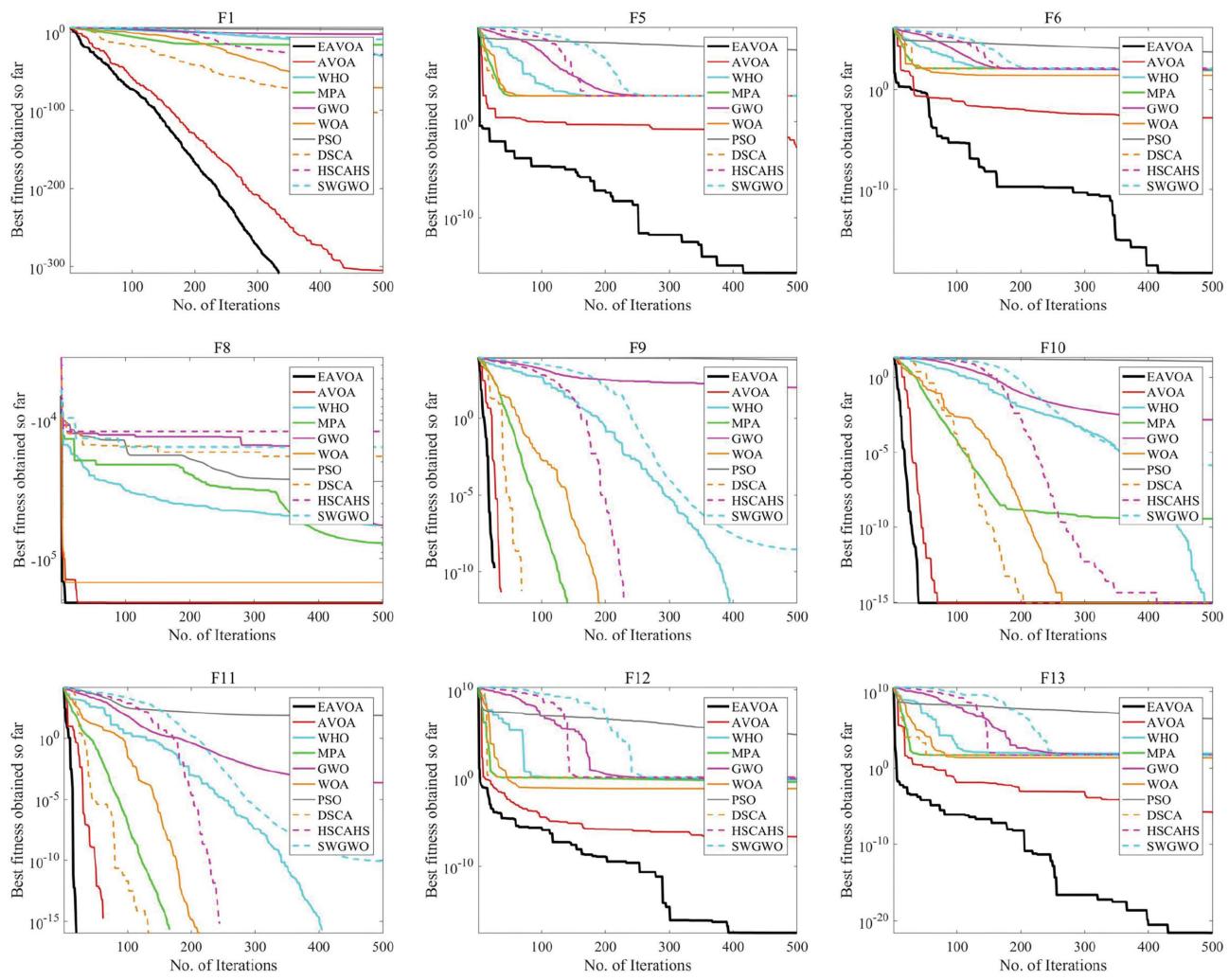


Figure 8: The convergence curves of the EAVOA with $D = 500$ (F1, F5, F6, F8–F10, F11–F13).

other algorithms as it uses three strategies, which are the RVS, RFS, and SA.

5.2. Results for classical CEC 2005 test suit

This section presents the mean values and the standard deviations for each function with different dimensions in the CEC 2005 test suit. This section also displays the Wilcoxon signed-rank test results between EAVOA and other algorithms on classical test functions. In addition, this section presents the convergence curves of the EAVOA with different dimensions, including 30, 200, 500, and 1000 for F1, F5, F6, F8–F10, and F11–F13 along with the convergence curves of the EAVOA for the fixed dimensional test functions for F14, F15, and F17–F23.

Firstly, the mean and the standard deviation values for the CEC 2005 functions are presented in Table 5. The table shows the values for the different dimensions and the different algorithms. It is observed from the table that the proposed EAVOA outperforms the other algorithms in the mean and standard deviation values.

On the other hand, EAVOA has not only striking better values than the AVOA, but also EAVOA outperforms the other algorithms for the different functions. In addition, the Wilcoxon signed-rank test results are presented in Table 6.

The convergence curves for the algorithms for the 30, 200, 500, and 1000 dimension values and the different functions are presented in Figs. 6–9, respectively. It is observed from the figures that the proposed EAVOA has dramatically lower values of the fitness function across the iterations than the other algorithms. It is also observed that the fitness values of the proposed EAVOA have shown gradual descent across the course of iterations.

In addition, Fig. 10 presents the convergence curves for the algorithms on fixed dimensional test functions. It also shows that the proposed algorithm has a predominant convergence process and the best fitness values compared to the other algorithms for the different functions.

Finally, a Friedman test has been carried on to test the rank of EAVOA with other competitors in finding a near-optimal solution when handling different functions (Carrasco et al., 2020; Derrac et al., 2014). Table 7 shows these results in which we can notice that EAVOA ranked first in all functions from F1 to F13 in all dimensions except for F7, where it ranked second after HSCAHS. On the other hand, EAVOA ranked first in functions F14–F23 in all dimensions except for F15 and F20 where it ranked second after MPA algorithm. Overall, EAVOA achieves the first rank in handling all functions.

From the above discussion, we can see that the suggested optimizer is able to find the optimal/near-optimal solutions due to

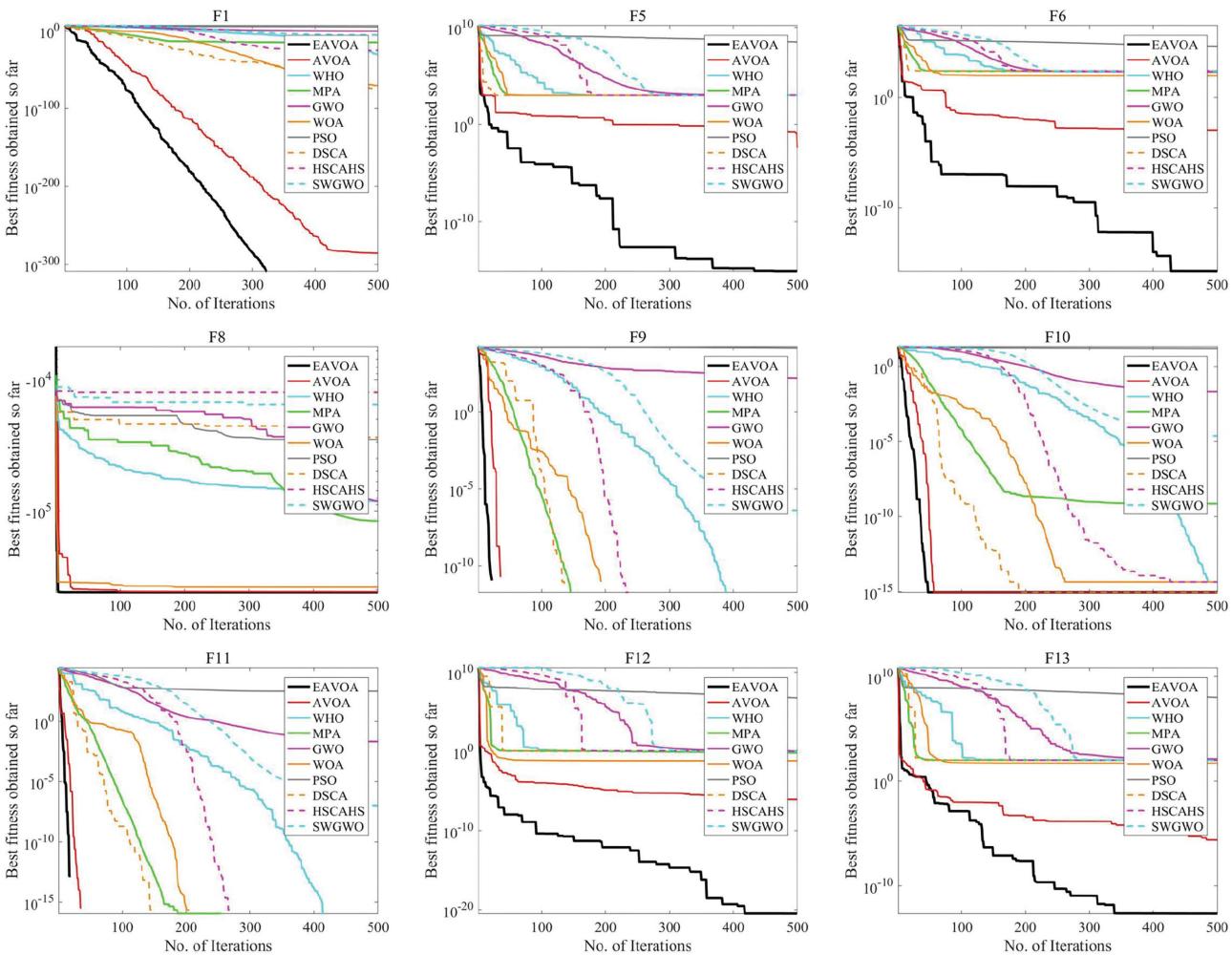


Figure 9: The convergence curves of the EAVOA with $D = 1000$ (F1, F5, F6, F8–F10, F11–F13).

its ability to escape from local optimal regions and achieve a high balance between exploration (diversification) and exploitation (intensification).

5.3. Results for the latest CEC 2021 test suit

This section presents the mean, standard deviation, and ranking for the CEC_01–CEC_10 functions in the CEC 2021 test suit. This section also presents the convergence boxplot of the different algorithms and the different functions. The mean, standard deviation, and ranking for the CEC 2021 functions are presented in Table 8. It is observed from the table that MPA has competitive results with the proposed EAVOA. The proposed EAVOA also has a good ranking compared to the other algorithms. For example, EAVOA ranks second for CEC_01, CEC_03, CEC_08, and CEC_09. The overall ranking values for each algorithm are calculated and the proposed EAVOA has an overall ranking of the second. The boxplots for the algorithms for the different functions are presented in Fig. 11. It is observed from the figures that the proposed EAVOA has lower values of the fitness function for the different runs than most of the other algorithms. It is also observed that the standard deviation values for the proposed EAVOA have shown low values compared to the other algorithms. For example, the standard deviation values for the proposed EAVOA, the WHO, the MPA, and the PSO algorithms are very low for the CEC_01 and CEC_04.

6. Application for Real-World Problems

In this section, the experiments and the results for the proposed EAVOA of the two types of problems, including the engineering design problems and the MLP classification problems are presented.

6.1. Engineering design problems

This section shows the results of three classical engineering design optimization problems, including the three-bar truss, the speed reducer, and the car crashworthiness design problems. The population size and maximum iteration are the same as those in the previous experiments, i.e., 30 and 500.

6.1.1. The three-bar truss design problem

The three-bar truss design optimization problem works by minimizing the weight of the truss $f(\bar{x})$ (Sadollah et al., 2013). The problem includes two parameters which are the x_1 and x_2 , and three functions which are the $g_1(\bar{x})$, $g_2(\bar{x})$, and $g_3(\bar{x})$ as observed from the following equations. Figure 12 shows the three-bar truss representation.

Table 9 lists the optimal values for the x_1 and x_2 , and the optimal weight values for the different algorithms. It is shown that the

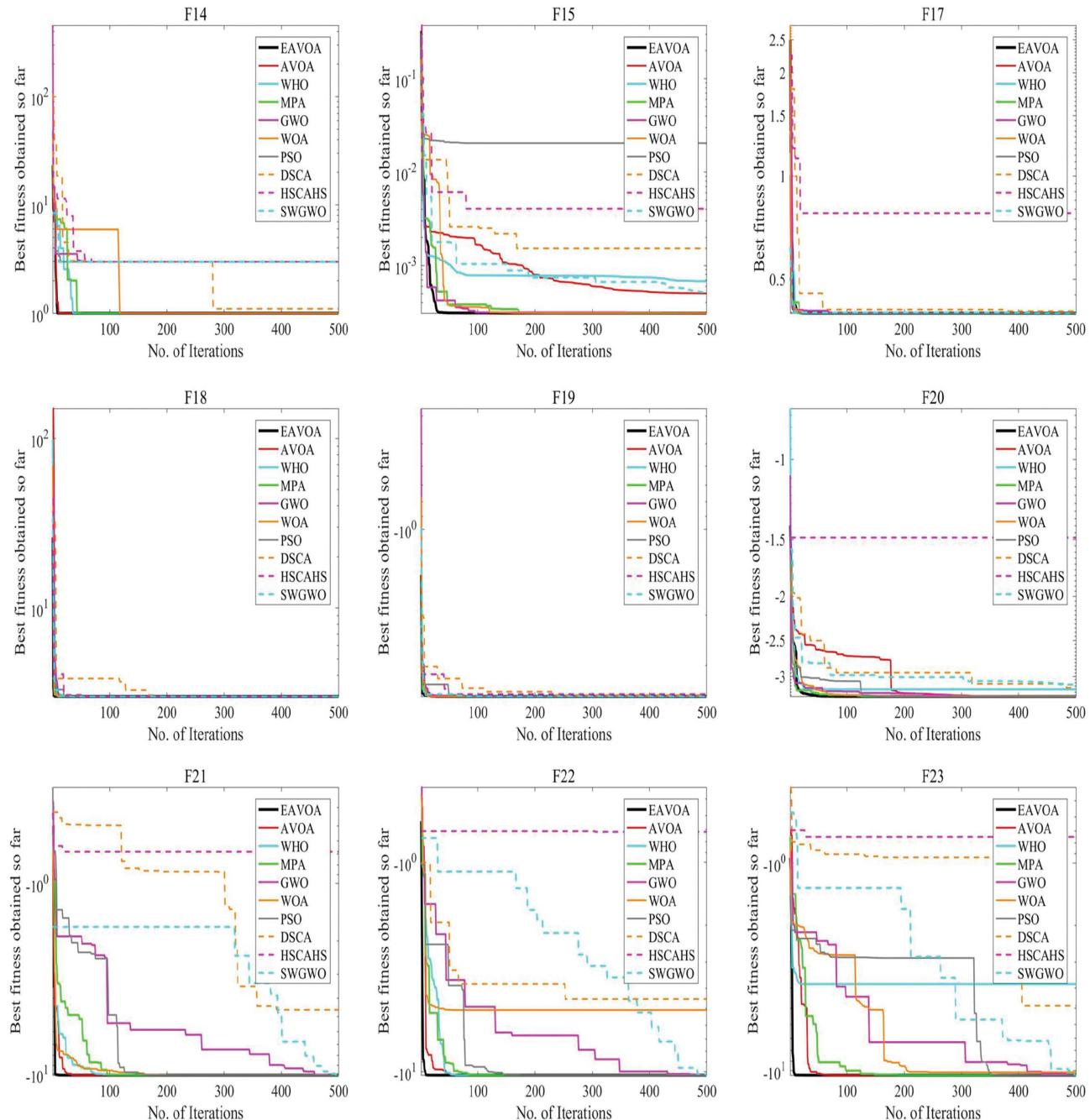


Figure 10: The convergence curves of the EAVOA on fixed dimensional test functions (F14, F15, and F17–F23).

proposed algorithm has obtained the best result with an optimum weight of 263.853 19.

$$g_3(\vec{x}) = \frac{1}{\sqrt{2x_2 + 2x_1}} P - \sigma \leq 0$$

where $l = 100\text{cm}$, $P = 2 \text{ KN/cm}^2$, $\sigma = 2 \text{ KN/cm}^2$, $0 \leq x_1, x_2 \leq 1$

Minimize:

$$f(\vec{x}) = (2\sqrt{2}x_1 + x_2)l$$

Subject to:

$$g_1(\vec{x}) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2x_1^2 + 2x_1x_2}} P - \sigma \leq 0$$

$$g_2(\vec{x}) = \frac{x_2}{\sqrt{2x_1^2 + 2x_1x_2}} P - \sigma \leq 0$$

6.1.2. The speed reducer design problem

The speed reducer design problem is a structural optimization problem for designing a gearbox for a light airplane and minimizing the total weight of the speed reducer $f(\vec{x})$ for eleven constraints $g_1(\vec{x}), g_2(\vec{x}), g_3(\vec{x}), g_4(\vec{x}), g_5(\vec{x}), g_6(\vec{x}), g_7(\vec{x}), g_8(\vec{x}), g_9(\vec{x}), g_{10}(\vec{x})$, and $g_{11}(\vec{x})$ (Abualigah et al., 2021a). These constraints represent the limits on the bending stress of the gear teeth, surface

Table 7: Friedman test results of all algorithms on classical test functions (F1–F23) using D = 30, 200, 500, and 1000.

Fun.	D	Metric	EAOA	AVOA	WHO	MPA	GWO	WOA	PSO	DSCA	HSCAHS	SWGWO
F1	30	1	2	7	9	8	4	10	3	5	6	8
	200	1	2	6	7	9	4	10	3	5	8	8
	500	1	2	5	7	9	4	10	3	6	8	8
	1000	1	2	5	7	9	4	10	3	6	8	5
F2	30	1	2	7	9	8	4	10	3	6	6	8
	200	1	2	5	7	9	4	10	3	6	6	8
	500	1	2	10	6	8	3	9	4	5	7	7
	1000	1	2	9	7	6	3	8	10	4	5	5
F3	30	1	2	5	8	7	10	9	3	4	6	7
	200	1	2	5	6	8	10	9	3	4	7	7
	500	1	2	5	6	8	10	9	3	4	7	7
	1000	1	2	5	6	8	10	9	3	4	7	7
F4	30	1	2	5	7	8	10	9	3	4	6	9
	200	1	2	5	6	8	10	7	3	4	4	9
	500	1	2	5	6	8	10	7	3	4	4	9
	1000	1	2	4	6	8	9	7	3	5	10	10
F5	30	1	2	9	3	4	6	10	7	8	5	5
	200	1	2	9	3	4	4	10	7	7	7	4
	500	1	2	5	4	5	3	10	8	8	5	5
	1000	1	2	5	4	9	3	10	7	7	7	5
F6	30	1	3	5	2	7	6	4	9	10	8	8
	200	1	2	5	4	6	3	10	8	9	9	7
	500	1	2	7	4	5	3	10	8	9	9	6
	1000	1	2	9	4	5	3	10	7	8	8	6
F7	30	3	2	5	7	8	9	10	6	4	7	8
	200	3	2	6	5	9	8	10	4	1	1	1
	500	3	2	5	4	9	7	10	6	1	1	1
	1000	2	2	4	5	9	7	10	6	1	1	1
F8	30	1	2	4	5	6	3	7	8	9	8	10
	200	1	2	5	4	6	3	7	9	9	8	10
	500	1	2	5	4	6	3	7	9	9	8	10
	1000	1	2	5	4	6	3	7	8	8	8	10
F9	30	1	1	8	1	9	1	10	1	1	1	1
	200	1	1	1	1	9	7	10	1	1	1	1
	500	1	1	1	1	10	1	9	1	1	1	1
	1000	1	1	1	1	9	7	10	1	1	1	1
F10	30	1	1	5	9	8	6	10	1	1	1	1
	200	1	1	5	7	9	6	10	1	1	1	1
	500	1	1	5	7	9	6	10	1	1	4	6
	1000	1	1	4	7	9	5	10	1	1	6	1
F11	30	1	1	1	1	8	10	9	1	1	1	1
	200	1	1	1	1	9	1	10	1	1	1	1
	500	1	1	1	1	9	1	10	1	1	1	1
	1000	1	1	1	7	9	6	10	1	1	1	1
F12	30	1	2	6	4	7	5	3	9	8	9	10
	200	1	2	6	4	5	3	10	8	8	9	9
	500	1	2	6	4	5	3	10	7	8	8	6
	1000	1	2	5	4	9	3	10	7	8	8	6
F13	30	1	2	5	4	7	6	3	9	8	8	10
	200	1	2	6	7	4	3	10	5	7	8	5
	500	1	2	8	8	7	3	10	7	5	6	4
	1000	1	2	8	5	9	3	10	7	6	8	5
F14	2	1	3	5	1	10	6	8	4	7	9	9
F15	4	2	3	8	1	10	4	9	6	7	7	5
F16	2	1	1	1	1	1	1	1	1	1	1	1
F17	2	1	1	1	1	1	1	1	1	9	10	1
F18	2	1	10	9	1	1	1	1	1	7	7	1
F19	3	1	1	1	1	9	1	1	8	10	1	1
F20	6	2	4	5	1	2	6	7	9	10	8	8
F21	4	1	1	5	1	6	4	7	9	10	8	8
F22	4	1	1	6	1	1	8	7	9	10	5	5
F23	4	1	1	8	1	4	7	6	9	10	5	5
Avg. ranks	1.14	1.91	5.14	4.354	7.06	4.96	8.33	5.11	5.69	6.40		
Final rank	1		2	6	3	9	4	10	5	7	8	

Table 8: Comparison of optimization results on CEC 2021 test functions between EAVOA and other optimization algorithms (CEC_01–CEC_10).

Fun.	Metric	EAVOA	AVOA	WHA	MPA	GWO	WOA	PSO	DSCA	HSCAHS	SWGWO
CEC_01	Mean	1.14E+05	5.70E+09	2.38E+08	2.75E+03	1.09E+09	1.29E+09	2.91E+07	1.44E+10	3.25E+10	2.02E+09
	Std.	1.62E+05	3.53E+09	1.30E+09	2.87E+03	1.51E+09	7.95E+08	1.60E+08	2.05E+09	2.76E+09	1.18E+09
	Ranking	2	8	4	1	5	6	3	9	10	7
CEC_02	Mean	3.07E+03	3.27E+03	2.74E+03	2.18E+03	2.90E+03	4.32E+03	3.19E+03	5.76E+03	6.33E+03	3.50E+03
	Std.	5.30E+02	4.51E+02	4.08E+02	2.98E+02	4.66E+02	5.34E+02	5.55E+02	2.87E+02	1.76E+02	9.51E+02
	Ranking	4	6	2	1	3	8	5	9	10	7
CEC_03	Mean	7.93E+02	9.04E+02	7.98E+02	7.66E+02	8.96E+02	9.86E+02	7.94E+02	9.81E+02	1.06E+03	8.16E+02
	Std.	2.26E+01	3.53E+01	2.23E+01	1.18E+01	3.80E+01	4.23E+01	2.54E+01	1.89E+01	2.19E+01	3.44E+01
	Ranking	2	7	4	1	6	9	3	8	10	5
CEC_04	Mean	1.92E+03	6.42E+03	1.93E+03	1.90E+03	2.05E+03	2.92E+03	1.90E+03	8.47E+04	4.67E+05	2.20E+03
	Std.	6.56E+00	8.86E+03	1.04E+02	1.05E+00	3.98E+02	1.57E+03	1.20E+00	5.20E+04	1.25E+05	5.78E+02
	Ranking	3	8	4	1.5	5	7	1.5	9	10	6
CEC_05	Mean	6.89E+05	1.49E+06	2.95E+05	2.33E+03	1.06E+06	2.84E+06	2.18E+05	4.39E+06	6.95E+06	1.14E+06
	Std.	5.10E+05	9.80E+05	2.87E+05	1.56E+02	1.10E+06	1.73E+06	1.66E+05	1.90E+06	2.43E+06	9.56E+05
	Ranking	4	7	3	1	5	8	2	9	10	6
CEC_06	Mean	2.08E+03	2.30E+03	2.09E+03	1.73E+03	2.05E+03	2.78E+03	2.28E+03	2.75E+03	3.72E+03	1.99E+03
	Std.	1.99E+02	2.57E+02	1.90E+02	7.57E+01	2.00E+02	2.69E+02	2.84E+02	1.57E+02	2.33E+02	2.12E+02
	Ranking	4	7	5	1	3	9	6	8	10	2
CEC_07	Mean	3.65E+05	1.27E+06	2.43E+05	2.42E+03	5.39E+05	2.24E+06	1.58E+05	2.13E+06	2.99E+06	2.65E+05
	Std.	3.95E+05	1.47E+06	6.31E+05	1.47E+02	8.46E+05	2.14E+06	1.09E+05	9.99E+05	1.77E+06	5.33E+05
	Ranking	5	7	3	1	6	9	2	8	10	4
CEC_08	Mean	2.74E+03	3.39E+03	2.98E+03	2.30E+03	3.66E+03	4.86E+03	3.50E+03	6.25E+03	5.98E+03	3.57E+03
	Std.	1.11E+03	1.25E+03	1.15E+03	1.31E+00	1.21E+03	1.87E+03	1.52E+03	9.73E+02	3.53E+02	1.53E+03
	Ranking	2	4	3	1	7	8	5	10	9	6
CEC_09	Mean	2.87E+03	3.04E+03	2.88E+03	2.83E+03	2.88E+03	3.02E+03	3.08E+03	3.02E+03	3.45E+03	2.90E+03
	Std.	3.57E+02	9.15E+01	3.47E+01	6.19E+01	3.94E+01	6.24E+01	1.03E+02	2.07E+01	7.01E+01	4.50E+01
	Ranking	2	8	3.5	1	3.5	6.5	9	6.5	10	5
CEC_10	Mean	2.99E+03	3.20E+03	2.97E+03	2.92E+03	3.01E+03	3.13E+03	2.96E+03	3.86E+03	5.20E+03	3.04E+03
	Std.	2.46E+01	4.31E+01	1.37E+02	1.43E+01	5.70E+01	8.81E+01	3.46E+01	2.90E+02	4.75E+02	4.81E+01
	Ranking	4	8	3	1	5	7	2	9	10	6
Overall ranking		2	7	3	1	5	8	4	9	10	6

Algorithm 2. Pseudo-code of the proposed EAVOA.

```

1: Initialization
2: Initialize the population size N and maximum iterations T
3: Initialize the positions of all vultures  $P_i$  ( $i = 1, 2, \dots, N$ )
4: Initialize controlling parameters  $P_1, P_2, P_3$ , and  $w$ 
5: while  $t < T$  do
6:   Calculate the vultures' fitness values
7:   Find the first, second, and third best positions of vultures  $P_{\text{BestVulture}1}$ ,  $P_{\text{BestVulture}2}$ , and  $P_{\text{BestVulture}3}$ 
8:   for each vulture do
9:     Calculate F using Equation (1)
10:    Determine R(i) using Equation (19)
11:    if  $|F| \geq 1$  then
12:      if  $P_1 \geq \text{rand}_{P_1}$  then
13:        Update vulture' position using Equation (5)
14:      else
15:        Update vulture' position using Equation (7)
16:      end if
17:    else
18:      if  $|F| \geq 0.5$  then
19:        if  $P_2 \geq \text{rand}_{P_2}$  then
20:          Update vulture' position using Equation (9)
21:        else
22:          Update vulture' position using Equation (22)
23:        end if
24:      else
25:        if  $P_3 \geq \text{rand}_{P_3}$  then
26:          Update vulture' position using Equation (25)
27:        else
28:          Update vulture' position using Equation (16)
29:        end if
30:      end if
31:    end if
32:  end for
33: end while
34: Return the first best vulture

```

stress, transverse deflections of shafts 1 and 2 due to transmitted force, and stresses in shafts 1 and 2 as observed from the following equations. Figure 13 shows the speed reducer representation.

Table 10 lists the optimal values for the x_1-x_7 , and the optimal weight values for the different algorithms. The best result is obtained by the proposed algorithm, which is 2998.74109.

Minimize:

$$f(\bar{\mathbf{z}}) = 0.7854z_1z_2^2(3.3333z_3^2 + 14.9334z_3 - 43.0934) - 1.508z_1 \\ \times (z_6^2 + z_7^2) + 7.4777(z_6^3 + z_7^3) + 0.7854(z_4z_6^2 + z_5z_7^2)$$

Subject to:

$$g_1(\bar{\mathbf{z}}) = \frac{27}{z_1z_2^2z_3} - 1 \leq 0$$

$$g_2(\bar{\mathbf{z}}) = \frac{397.5}{z_1z_2^2z_3} - 1 \leq 0$$

$$g_3(\bar{\mathbf{z}}) = \frac{1.93z_4^3}{z_2z_3z_6^4} - 1$$

$$g_4(\bar{\mathbf{z}}) = \frac{1.93z_5^3}{z_2z_3z_7^4} - 1 \leq 0$$

$$g_5(\bar{\mathbf{z}}) = \frac{1}{110z_6^2}\sqrt{\left(\frac{745z_4}{z_2z_3}\right)^2 + 16.9 \times 10^6} - 1 \leq 0$$

$$g_6(\bar{\mathbf{z}}) = \frac{1}{85z_7^3}\sqrt{\left(\frac{745z_5}{z_2z_3}\right)^2 + 157.5 \times 10^6} - 1 \leq 0$$

$$g_7(\bar{\mathbf{z}}) = \frac{z_2z_3}{40} - 1 \leq 0$$

$$g_8(\bar{\mathbf{z}}) = \frac{5z_2}{z_1} - 1 \leq 0$$

$$g_9(\bar{\mathbf{z}}) = \frac{z_1}{12z_2} - 1 \leq 0$$

$$g_{10}(\bar{\mathbf{z}}) = \frac{1.5z_6 + 1.9}{z_4} - 1 \leq 0$$

$$g_{11}(\bar{\mathbf{z}}) = \frac{1.1z_7 + 1.9}{z_5} - 1 \leq 0$$

Range of variables:

$$2.6 \leq z_1 \leq 3.6$$

$$0.7 \leq z_2 \leq 0.8$$

$$17 \leq z_3 \leq 28$$

$$7.3 \leq z_4 \leq 8.3$$

$$7.8 \leq z_5 \leq 8.3$$

$$2.9 \leq z_6 \leq 3.9$$

$$5 \leq z_7 \leq 5.5$$

6.1.3. The car crashworthiness design problem

The car crashworthiness design problem is a commercial vehicle problem for ensuring the passenger's safety while reducing the weight (Yildiz et al., 2021). The goal is to minimize an objective function $f(\bar{\mathbf{x}})$ with 10 constraints $g_1(\bar{\mathbf{x}}), g_2(\bar{\mathbf{x}}), g_3(\bar{\mathbf{x}}), g_4(\bar{\mathbf{x}}), g_5(\bar{\mathbf{x}}), g_6(\bar{\mathbf{x}}), g_7(\bar{\mathbf{x}}), g_8(\bar{\mathbf{x}}), g_9(\bar{\mathbf{x}})$, and $g_{10}(\bar{\mathbf{x}})$ as observed from the following equations. In addition, Fig. 14 shows the car crashworthiness representation.

Table 11 lists the optimal values for the x_1-x_{11} for the different algorithms. It can be seen that the proposed method has obtained the best result with optimal value 22.8929.

Minimize:

$$f(\bar{\mathbf{x}}) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7$$

Subject to:

$$g_1(\bar{\mathbf{x}}) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_10 - 0.484x_3x_9 \\ + 0.01343x_6x_{10} \leq 1$$

$$g_2(\bar{\mathbf{x}}) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 \\ + 0.0008757x_5x_{10} + 0.080405x_6x_9 + 0.00139x_8x_{11} \\ + 0.00001575x_1x_{10}x_{11} \leq 0.32$$

$$g_3(\bar{\mathbf{x}}) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 \\ + 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 \\ - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} \\ + 0.00121x_8x_{11} \leq 0.32$$

$$g_4(\bar{\mathbf{x}}) = 0.074 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 \\ + 0.227x_2^2 \leq 0.32$$

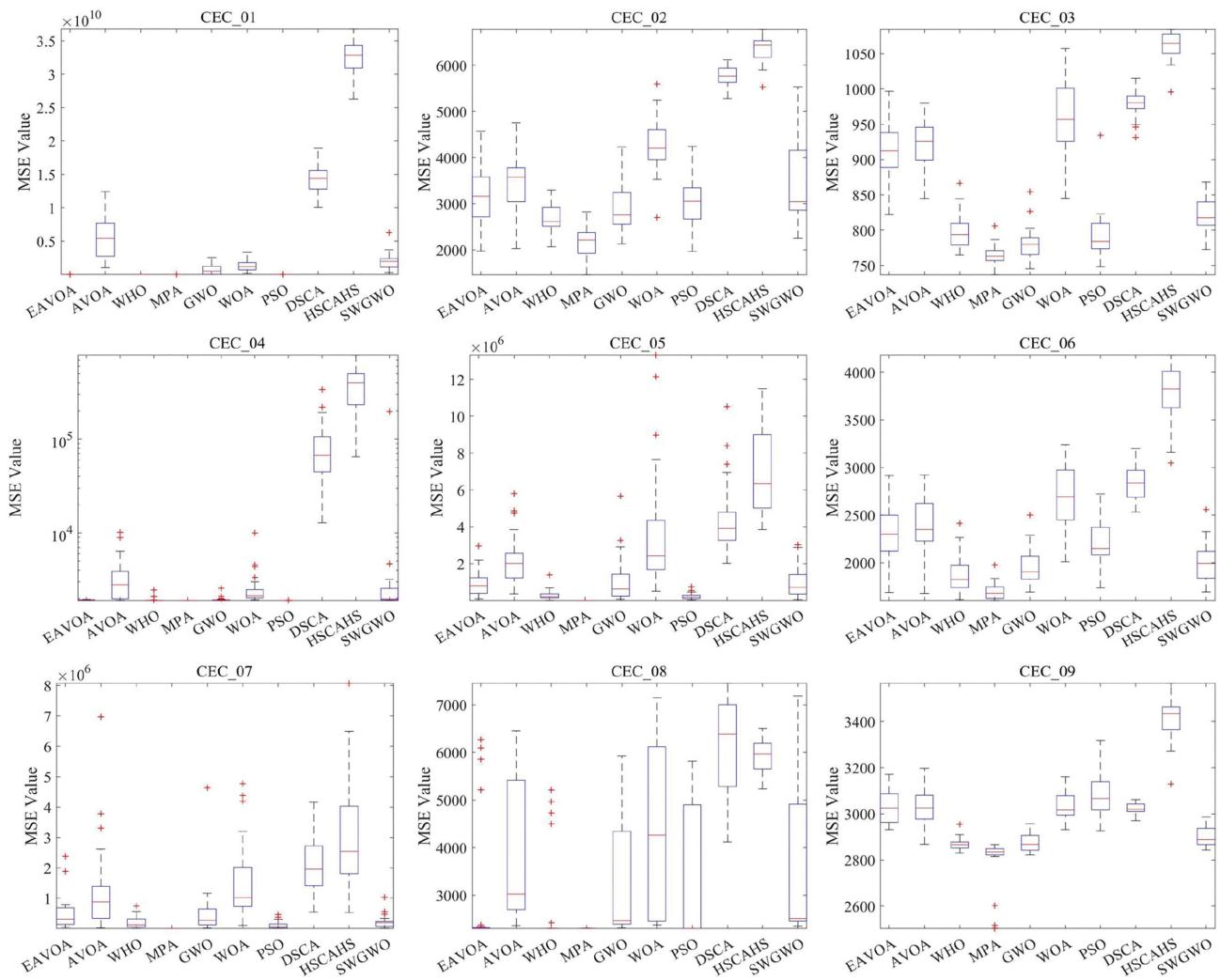


Figure 11: Boxplots for the CEC 2021 test functions (CEC_01–CEC_09).

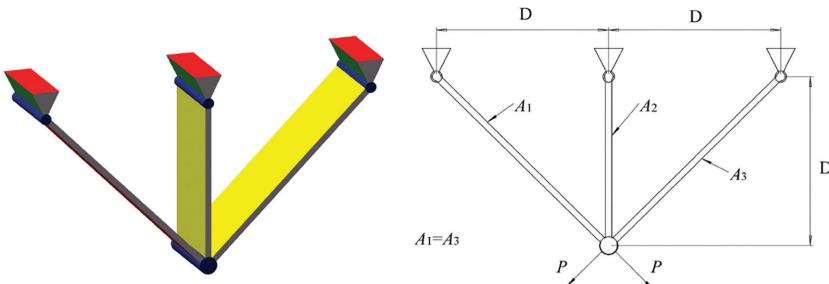


Figure 12: The three-bar truss design problem: three-dimensional (3D) model diagram (left-hand panel) and structural parameters (right-hand panel).

Table 9: Optimization results for the three-bar truss design problem.

Algorithm	Optimal values for variables		Optimum weight
	x_1	x_2	
EAVOA	0.787 346	0.411 149	263.853 19
AVOA (Abdollahzadeh et al., 2021)	0.8129	0.342 88	264.259 02
HHO (Heidari et al., 2019)	0.788 662 816	0.408 283 134	263.895 8434
SSA	0.788 665 41	0.408 275 784	263.895 84
AOA (Singh et al., 2022)	0.793 69	0.394 26	263.9154
MVO	0.788 602 76	0.408 453 07	263.895 8499
MFO	0.788 244 771	0.409 466 906	263.895 9797
GOA	0.788 897 556	0.407 619 57	263.895 8815

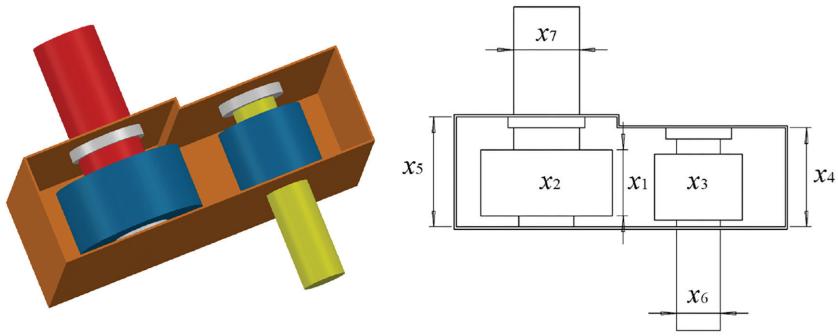


Figure 13: The speed reducer design problem: 3D model diagram (left-hand panel) and structural parameters (right-hand panel).

Table 10: Optimization results for the speed reducer design problem.

Algorithm	Optimal values for variables							Optimum weight
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
EAVOA	3.497 607	0.7	17	7.647 252	7.800 003	3.350 71	5.285 67	2998.741 09
AVOA (Abdollahzadeh et al., 2021)	3.497 62	0.7	17	7.7678	7.8929	3.3509	5.2857	3001.9313
PSO (Kennedy & Eberhart, 1995)	3.5001	0.7	17.0002	7.5177	7.7832	3.3508	5.2867	3145.922
GA (Holland, 1992)	3.510 25	0.7	17	8.35	7.8	3.3622	5.287 72	3067.561
SCA (Mirjalili, 2016)	3.508 75	0.7	17	7.3	7.8	3.461 02	5.289 21	3030.563
HS	3.520 12	0.7	17	8.37	7.8	3.366 97	5.288 71	3029.002
FA	3.507 49	0.7001	17	7.719 67	8.080 85	3.351 51	5.287 05	3010.137 49
MDA	3.5	0.7	17	7.3	7.670 39	3.542 42	5.245 81	3019.583 36



Figure 14: The car crashworthiness design problem: car finite element model diagram (Yildiz et al., 2021).

$$g_9(\bar{x}) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10}$$

$$- 0.0198x_4x_{10} + 0.028x_6x_{10} \leq 9.9$$

$$g_{10}(\bar{x}) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10}$$

$$- 0.0556x_9x_{11} - 0.000786x_{11}^2 \leq 15.7$$

Range of variables:

$$0.5 \leq x_1 - x_7 \leq 1.5$$

$$x_8, x_9 \in (0.192, 0.345)$$

$$-30 \leq x_{10}, x_{11} \leq 30$$

6.2. MLP classification problems

This section shows the results for two MLP classification problems, including the XOR and cancer classification problems (Meng et al., 2021). The population size is set to 50 and 200 for XOR and cancer classification problems, respectively, whereas the maximum iteration is 500 for both problems.

6.2.1. XOR classification problem

The results for the different algorithms of the XOR classification problem are presented in Table 12. It is observed from the table that the proposed EAVOA, WHO, MPA, GWO, and PSO have a rank of first and a classification rate of 100% for this classification problem.

Figures 15 and 16 show the boxplots and the convergence curves for the results, respectively. It is observed from Fig. 15 that the proposed EAVOA has a relatively low fitness value compared to

$$g_5(\bar{x}) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9$$

$$- 7.7x_7x_8 + 0.32x_9x_{10} \leq 32$$

$$g_6(\bar{x}) = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8$$

$$- 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9 \leq 32$$

$$g_7(\bar{x}) = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} \leq 32$$

$$g_8(\bar{x}) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10}$$

$$+ 0.000191x_{11}^2 \leq 4$$

Table 11: Optimization results in car crashworthiness design problem.

Variable	EAVOA	AVOA (Abdollahzadeh et al., 2021)	MPA (Faramarzi et al., 2020)	PSO (Kennedy & Eberhart, 1995)
x_1	0.51	0.5113	0.5	0.5
x_2	1.1605	1.2546	1.2325	1.2267
x_3	0.5468	0.6591	0.5	0.5
x_4	1.161	1.1263	1.1953	1.2267
x_5	0.6182	0.7051	0.5	0.5
x_6	0.6092	0.5995	1.0803	1.5
x_7	0.5044	0.5683	0.5	0.5
x_8	0.3447	0.2966	0.345	0.345
x_9	0.2445	0.2186	0.3387	0.345
x_{10}	1.4227	0.2504	1.2164	0.1755
x_{11}	3.405	3.5056	1.1373	-1.322
Optimum value	22.8929	24.7784	23.1893	23.1951
Variable	DSCA	HSCAHS	SWGWO	
	(Li et al., 2021)	(Singh & Kaur, 2021)	(Long et al., 2019)	
x_1	0.5	0.5	0.5026	
x_2	1.2744	1.2862	1.2296	
x_3	0.5	0.5	0.502	
x_4	1.5	1.5	1.2052	
x_5	0.979	1.2582	0.5041	
x_6	1.5	1.0671	1.4419	
x_7	0.5	0.5	0.5002	
x_8	0.345	0.3287	0.345	
x_9	0.2291	0.345	0.2362	
x_{10}	0.9918	0.1902	0.8438	
x_{11}	28.684	7.9712	1.4873	
Optimum value	25.8671	26.1182	23.2442	

Table 12: The experimental results of XOR classification problem.

Algorithm	Best	Worst	Mean	Std	Classification rate	Rank
EAVOA	8.8683E-07	8.4480E-02	1.6268E-02	5.0138E-02	100%	1
AVOA (Abdollahzadeh et al., 2021)	8.0696E-05	2.5000E-01	5.8270E-02	7.3703E-02	75%	2
WHO (Naruei & Keynia, 2022)	1.1323E-09	2.3694E-05	5.7055E-06	1.0320E-05	100%	1
MPA (Faramarzi et al., 2020)	1.4312E-6	1.2885E-05	3.4816E-06	9.0112E-06	100%	1
GWO (Mirjalili et al., 2014)	5.8297E-05	3.2500E-03	3.3412E-04	7.916E-04	100%	1
WOA (Mirjalili & Lewis, 2016)	3.2781E-04	1.9755E-01	8.4514E-02	6.8825E-02	50%	4
PSO (Kennedy & Eberhart, 1995)	5.5505E-130	3.5434E-41	2.2911E-19	8.7234E-19	100%	1
DSCA (Li et al., 2021)	1.0890E-04	3.7100E-02	1.7334E-02	1.3144E-02	75%	2
HSCAHS (Singh & Kaur, 2021)	1.4947E-01	2.3515E-01	2.0527E-01	2.6747E-02	62.5%	3
SWGWO (Long et al., 2019)	5.6221E-05	2.5160E-02	2.1295E-03	4.9191E-03	25%	5

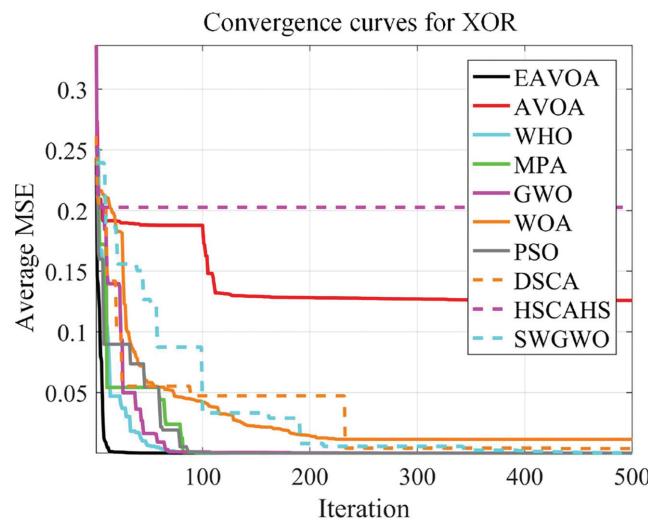
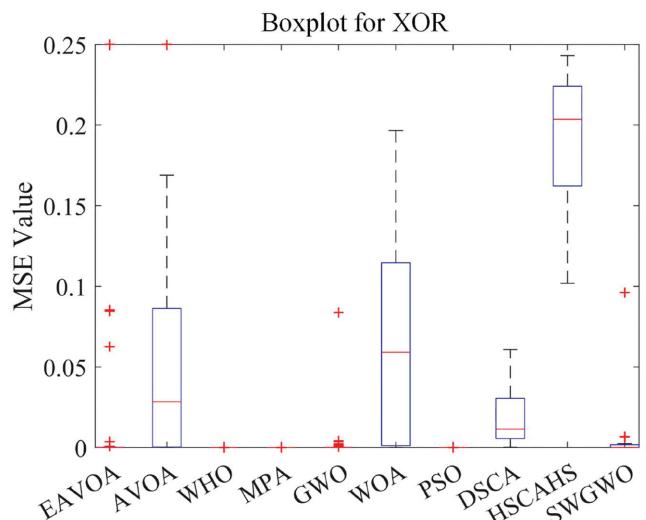
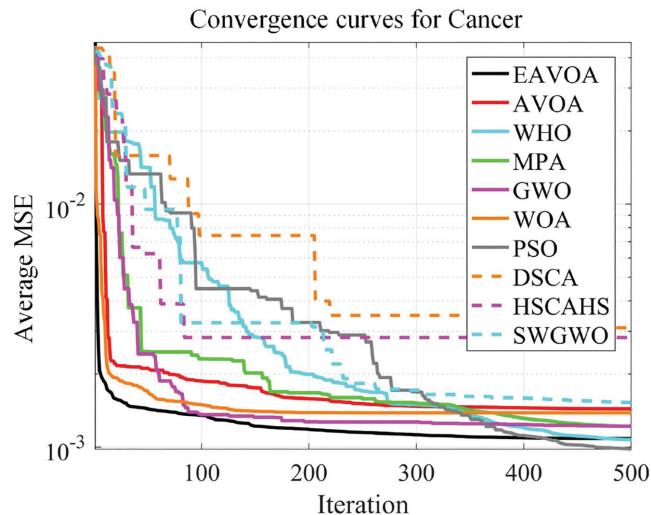
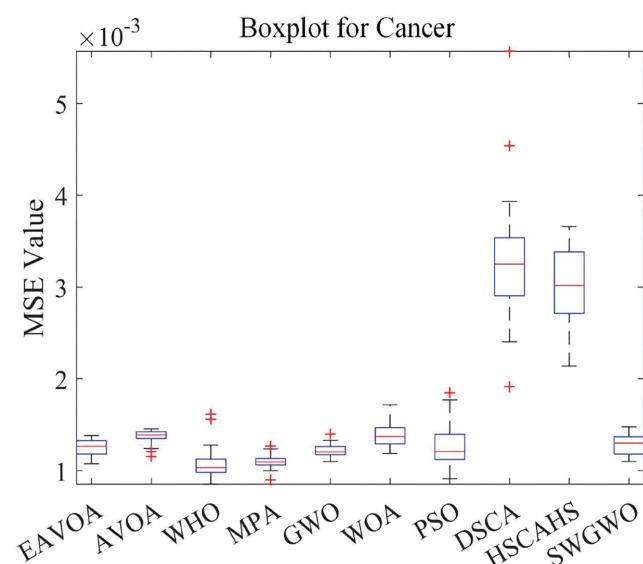
**Figure 15:** The convergence curves of XOR classification problem.**Figure 16:** The boxplot of XOR classification problem.

Table 13: The experimental results of cancer classification problem.

Algorithm	Best	Worst	Mean	Std	Classification rate	Rank
EAVOA	1.0754E-03	1.2843E-03	1.2414E-03	9.0065E-05	99%	1
AVOA (Abdollahzadeh et al., 2021)	1.2074E-03	1.4399E-03	1.3721E-03	7.1572E-05	98%	2
WHO (Naruei & Keynia, 2022)	8.5318E-04	1.0767E-03	1.0789E-03	1.7151E-04	97%	3
MPA (Faramarzi et al., 2020)	9.9971E-04	1.5606E-03	1.1008E-03	6.9695E-05	99%	1
GWO (Mirjalili et al., 2014)	1.1007E-03	1.3995E-03	1.2204E-03	7.0915E-05	98%	2
WOA (Mirjalili & Lewis, 2016)	1.2881E-03	1.7184E-03	1.3907E-03	1.4276E-04	98%	2
PSO (Kennedy & Eberhart, 1995)	9.8199E-04	1.7713E-03	1.2780E-03	2.4909E-04	97%	3
DSCA (Li et al., 2021)	2.4034E-03	5.5679E-03	3.2921E-03	6.7259E-04	91%	4
HSCAHS (Singh & Kaur, 2021)	2.1409E-03	3.6598E-03	3.0228E-03	4.3131E-04	98%	2
SWGWO (Long et al., 2019)	1.1009E-03	1.5273E-03	1.2976E-03	1.1207E-04	98%	2

**Figure 17:** The convergence curves of cancer classification problem.**Figure 18:** The variance diagram of cancer classification problem.

the other algorithms across the course of iterations. The proposed algorithm also shows a low value of the MSE and low standard deviation value as observed in Fig. 16.

6.2.2. Cancer classification problem

The results for the different algorithms of the cancer classification problem are presented in Table 13. It is observed from the table that the proposed EAVOA and MPA have a rank of first and a classification rate of 99% for this classification problem.

Figures 17 and 18 show the boxplots and the convergence curves for the results, respectively. It is observed from Fig. 17 that the proposed EAVOA has a very low fitness value compared to the other algorithms across the course of iterations. The proposed algorithm also shows a low value of the MSE and low standard deviation value as observed in Fig. 18.

7. Conclusions and Future Works

In this paper, we propose an exceptionally efficient modified optimization technique called EAVOA based on the basic AVOA. Three parts of the original method are improved for better optimizing capability. Accordingly, these improvement points are named as RVSS, RFS, and SAM. Using the classical twenty-three benchmark functions, the results of EAVOA and other advanced algorithms demonstrate the superiority of the proposed method. In particular, the proposed EAVOA can obtain significantly better solutions on functions F5, F6, F12, and F13 under different dimensional cases with high stability. Moreover, the results of five real-world problems also make clear the applicability of EAVOA compared to other methods reported in the literature. In the future, we can compare the suggested algorithm with other algorithms such as like monarch butterfly optimization (Wang et al., 2019), earthworm optimization algorithm (Wang et al., 2018), elephant herding optimization (Wang et al., 2015), moth search algorithm (Wang, 2018), SMA (Li et al., 2020), virus colony search (Hussien et al., 2022c), moth-flame optimization hunger games search (Yang et al., 2021), Runge Kutta optimizer (Ahmadianfar et al., 2021), colony predation algorithm (Tu et al., 2021), and HHO (Heidari et al., 2019).

Although, the developed algorithm shows a good performance in handling most of the constrained and unconstrained problems, it may be insufficient to handle all optimization tasks as stated by No Free Lunch (NFL) theory which mentioned that there is no algorithm is able to solve all types of problems. Also, like all other metaheuristic optimizers, it may have a slow convergence in handling complex and high-dimensional issues. This is similar to other MAs.

For further research directions, the EAVOA can be applied to solve other practical issues, such as MLP, FS, image segmentation, engineering design problems, and classification. In addition, the proposed strategies also can be applied to other metaheuristic methods for better optimization performance.

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Conflict of interest statement

None declared.

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