Implementation of Flight Mechanical Evaluation Criteria in an Aircraft Conceptual Design Tool with focus on Longitudinal Motions

Aleksandra Roszkowska (aler997)
Argyro Giota (arggi235)

Academic supervisor: Ludvig Knöös Franzén
Saab supervisors: Kristian Amadori, Peter Furenbäck
Examiner: David Lundström
Abstract

This report focuses on the utilisation of flight mechanics in the context of aircraft conceptual design to assess stability, control, and motion characteristics. The primary objective is to acquire the equations of motion and implement longitudinal stability and control criteria using Pacelab Aircraft Preliminary Design 8.1, a commercial software tool. The equations and criteria employed in this study are derived from an extensive review of relevant literature.

By incorporating a dedicated Flight Mechanics chapter within the software, it becomes possible to evaluate aircraft concepts under varying conditions. To ensure accuracy and validity, DATCOM+ and OpenVSP were employed for testing and verification purposes.

The key aspects covered in this report include flight mechanics, its implementation in Pacelab APD 8.1, determination of aerodynamic derivatives, formulation of equations of motion, and their application to the B747 aircraft model. The emphasis lies in assessing longitudinal stability and control, including specific characteristics such as the phugoid and short period modes.

This report provides valuable insights into the integration of flight mechanics within the Pacelab APD 8.1 software for aircraft conceptual design. The results contribute to a better understanding of stability and control parameters and their impact on aircraft performance.

**Keywords:** flight mechanics, Pacelab APD 8.1, aerodynamic derivatives, equations of motion, B747, stability and control, longitudinal dynamics, phugoid mode, short period mode, DATCOM+, and OpenVSP.
Acknowledgements

We would like to express our gratitude to all those who have contributed to the completion of this thesis.
Nomenclature

Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>AC</td>
<td>Aerodynamic Centre</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>APD</td>
<td>Aircraft Preliminary Design</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of Gravity</td>
</tr>
<tr>
<td>EO</td>
<td>Engineering Object</td>
</tr>
<tr>
<td>EWB</td>
<td>Engineering Workbench</td>
</tr>
<tr>
<td>FO</td>
<td>Functional Object</td>
</tr>
<tr>
<td>KD</td>
<td>Knowledge Designer</td>
</tr>
<tr>
<td>LiU</td>
<td>Linköping University</td>
</tr>
<tr>
<td>MAC</td>
<td>Mean Aerodynamic Chord</td>
</tr>
<tr>
<td>MDT</td>
<td>Multi-Dimensional Table</td>
</tr>
<tr>
<td>MTOW</td>
<td>Maximum Take-Off Weight</td>
</tr>
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</table>

Latin Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>AR</td>
<td>Aspect Ratio of the wing</td>
<td>–</td>
</tr>
<tr>
<td>ARH</td>
<td>Aspect Ratio of the horizontal stabilizer</td>
<td>–</td>
</tr>
<tr>
<td>b</td>
<td>Wing span</td>
<td>m</td>
</tr>
<tr>
<td>bH</td>
<td>Horizontal stabilizer span</td>
<td>m</td>
</tr>
<tr>
<td>c</td>
<td>Mean Aerodynamic Chord of the wing</td>
<td>m</td>
</tr>
<tr>
<td>crH</td>
<td>Horizontal Stabilizer Root Chord</td>
<td>m</td>
</tr>
<tr>
<td>crW</td>
<td>Wing Root Chord</td>
<td>m</td>
</tr>
<tr>
<td>d_f</td>
<td>Equivalent fuselage diameter</td>
<td>m</td>
</tr>
<tr>
<td>d_T</td>
<td>Thrust moment arm relative to CG</td>
<td>m</td>
</tr>
<tr>
<td>e</td>
<td>Oswald Efficiency Factor</td>
<td>–</td>
</tr>
<tr>
<td>FA_X</td>
<td>Aerodynamic Force in x-direction</td>
<td>N</td>
</tr>
<tr>
<td>FA_Y</td>
<td>Aerodynamic Force in y-direction</td>
<td>N</td>
</tr>
<tr>
<td>FA_Z</td>
<td>Aerodynamic Force in z-direction</td>
<td>N</td>
</tr>
<tr>
<td>FX</td>
<td>Thrust Force in x-direction</td>
<td>N</td>
</tr>
<tr>
<td>FY</td>
<td>Thrust Force in y-direction</td>
<td>N</td>
</tr>
<tr>
<td>FZ</td>
<td>Thrust Force in z-direction</td>
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</tr>
<tr>
<td>g</td>
<td>Gravitational Acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>IXX</td>
<td>Airplane Moment of Inertia About x-axis</td>
<td>kg·m²</td>
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<td>Ixz</td>
<td>Airplane Product of Inertia About XZ</td>
<td>kg·m²</td>
</tr>
<tr>
<td>IYX</td>
<td>Airplane Moment of Inertia About y-axis</td>
<td>kg·m²</td>
</tr>
<tr>
<td>IZZ</td>
<td>Airplane Moment of Inertia About z-axis</td>
<td>kg·m²</td>
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<tr>
<td>$K_{AR}$</td>
<td>Coefficient related with wing aspect ratio</td>
<td>–</td>
</tr>
<tr>
<td>$K_{mr}$</td>
<td>Coefficient related with aircraft geometry</td>
<td>–</td>
</tr>
<tr>
<td>$K_{WB}$</td>
<td>Wing-body interference factor</td>
<td>–</td>
</tr>
<tr>
<td>$K_{\lambda}$</td>
<td>Coefficient related with wing taper ratio</td>
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<td>$L_A$</td>
<td>Aerodynamic rolling moment</td>
<td>$N \cdot m$</td>
</tr>
<tr>
<td>$L_T$</td>
<td>Thrust rolling moment</td>
<td>$N \cdot m$</td>
</tr>
<tr>
<td>$m$</td>
<td>Airplane Mass</td>
<td>$kg$</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Air Mass Flow</td>
<td>$kg/s$</td>
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<tr>
<td>$M$</td>
<td>Mach Number</td>
<td>–</td>
</tr>
<tr>
<td>$M_A$</td>
<td>Aerodynamic pitching moment</td>
<td>$N \cdot m$</td>
</tr>
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<td>$M_T$</td>
<td>Thrust pitching moment</td>
<td>$N \cdot m$</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of engines</td>
<td>–</td>
</tr>
<tr>
<td>$n$</td>
<td>Load Factor</td>
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<tr>
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<td>$N \cdot m$</td>
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<td>$N_T$</td>
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<td>$P$</td>
<td>Airplane angular velocity component about x-axis</td>
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<tr>
<td>$\dot{q}$</td>
<td>Dynamic Pressure</td>
<td>$Pa$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Airplane angular velocity component about y-axis</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$R$</td>
<td>Airplane angular velocity component about z-axis</td>
<td>$rad/s$</td>
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<tr>
<td>$S$</td>
<td>Wing Reference Area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$S_C$</td>
<td>Canard Reference Area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$S_H$</td>
<td>Horizontal Stabilizer Reference Area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$T_{av}$</td>
<td>Available thrust of an engine</td>
<td>$N$</td>
</tr>
<tr>
<td>$U$</td>
<td>Linear longitudinal velocity (along the x-axis)</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V$</td>
<td>Linear lateral velocity (along the y-axis)</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$\dot{V}_H$</td>
<td>Horizontal Tail Volume Coefficient</td>
<td>–</td>
</tr>
<tr>
<td>$W$</td>
<td>Airplane Weight</td>
<td>$N$</td>
</tr>
<tr>
<td>$W$</td>
<td>Linear vertical velocity (along the z-axis)</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$\bar{x}_{AC_A}$</td>
<td>Aft shift in airplane AC normalized by MAC (aft shift is counted as positive)</td>
<td>$[-]$</td>
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<tr>
<td>$x_{AC_H}$</td>
<td>Aerodynamic centre of aircraft horizontal stabilizer position in x-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$\bar{x}_{AC_H}$</td>
<td>Aerodynamic centre of aircraft horizontal stabilizer position in x-axis normalized by MAC</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$\bar{x}_{ACWB}$</td>
<td>Aerodynamic centre of aircraft wing and body position in x-axis normalized by MAC</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$x_{CG}$</td>
<td>Centre of gravity position in x-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$\bar{x}_{CG}$</td>
<td>Centre of gravity position in x-axis normalized by MAC</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$x_{csH}$</td>
<td>Beginning of horizontal stabilizer coordinate system position in x-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$x_{csW}$</td>
<td>Beginning of wing coordinate system position in z-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$z_{CG}$</td>
<td>Centre of gravity position in z-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$z_{CG_e}$</td>
<td>Engine centre of gravity position in z-axis</td>
<td>$[m]$</td>
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### Symbol Description Units

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tr>
<td>$z_{csH}$</td>
<td>Beginning of horizontal stabilizer coordinate system position in x-axis</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$z_{csW}$</td>
<td>Beginning of wing coordinate system position in z-axis</td>
<td>$[m]$</td>
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### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Angle of Attack</td>
<td>degree</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Flight Path Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\frac{dc}{d\alpha}$</td>
<td>Downwash gradient</td>
<td>–</td>
</tr>
<tr>
<td>$\frac{dc}{d\epsilon}$</td>
<td>Upwash gradient at the canard</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_C$</td>
<td>Ratio between dynamic pressure on canard and wing</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_H$</td>
<td>Ratio between dynamic pressure on horizontal stabilizer and wing</td>
<td>–</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Airplane pitch attitude angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\Theta_0$</td>
<td>Pitch angle in reference state</td>
<td>rad</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wing Taper Ratio</td>
<td>–</td>
</tr>
<tr>
<td>$\Lambda_{c/2}$</td>
<td>Wing Semi-Chord Sweep Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\Lambda_{c/2H}$</td>
<td>Horizontal Stabilizer Semi-Chord Sweep Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\Lambda_{c/4}$</td>
<td>Wing Quarter-Chord Sweep Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\Lambda_{LE}$</td>
<td>Wing Leading Edge Sweep Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\Lambda_{LEH}$</td>
<td>Horizontal Stabilizer Leading Edge Sweep Angle</td>
<td>degree</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air Density</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\tau_E$</td>
<td>Elevator effectiveness factor</td>
<td>–</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Airplane bank angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Airplane heading angle</td>
<td>rad</td>
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### Subscripts and superscripts

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<tr>
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<tr>
<td>1</td>
<td>Referring to steady state</td>
</tr>
<tr>
<td>A</td>
<td>Referring to airplane</td>
</tr>
<tr>
<td>AC</td>
<td>Referring to aerodynamic centre</td>
</tr>
<tr>
<td>C</td>
<td>Referring to canard</td>
</tr>
<tr>
<td>CG</td>
<td>Referring to centre of gravity</td>
</tr>
<tr>
<td>e</td>
<td>Referring to engine</td>
</tr>
<tr>
<td>E</td>
<td>Referring to elevator</td>
</tr>
<tr>
<td>f</td>
<td>Referring to fuselage</td>
</tr>
<tr>
<td>H</td>
<td>Referring to horizontal stabilizer</td>
</tr>
<tr>
<td>i</td>
<td>Referring to recurring number</td>
</tr>
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</table>
### Abbreviation Meanings

- **LE**: Referring to leading edge of the surface
- **W**: Referring to wing
- **WB**: Referring to wing and body

### Coefficients and dimensionless derivatives

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$</td>
<td>Drag coefficient (airplane)</td>
<td>$-$</td>
</tr>
<tr>
<td>$C_{D1}$</td>
<td>Drag coefficient in steady state (airplane)</td>
<td>$-$</td>
</tr>
<tr>
<td>$C_{D\alpha}$</td>
<td>Variation of airplane drag coefficient with angle of attack</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{D\dot{\alpha}}$</td>
<td>Variation of airplane drag coefficient with dimensionless rate of change of angle of attack</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{D_iH}$</td>
<td>Variation of airplane drag coefficient with stabilizer incidence angle</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{D_{\delta E}}$</td>
<td>Variation of airplane drag coefficient with elevator deflection angle</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{D_q}$</td>
<td>Variation of airplane drag coefficient with dimensionless pitch rate</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{D_u}$</td>
<td>Variation of airplane drag coefficient with dimensionless speed</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient (airplane)</td>
<td>$-$</td>
</tr>
<tr>
<td>$C_{L1}$</td>
<td>Lift coefficient at steady state (airplane)</td>
<td>$-$</td>
</tr>
<tr>
<td>$C_{L\alpha}$</td>
<td>Variation of airplane lift coefficient with angle of attack</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{L\dot{\alpha}}$</td>
<td>Variation of airplane lift coefficient with dimensionless rate of change of angle of attack</td>
<td>$1/\text{rad}$</td>
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<tr>
<td>$C_{L_iH}$</td>
<td>Variation of airplane lift coefficient with stabilizer incidence angle</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{L_{\delta E}}$</td>
<td>Variation of airplane lift coefficient with elevator deflection angle</td>
<td>$1/\text{rad}$</td>
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<tr>
<td>$C_{L_q}$</td>
<td>Variation of airplane lift coefficient with dimensionless pitch rate</td>
<td>$1/\text{rad}$</td>
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<tr>
<td>$C_{L_u}$</td>
<td>Variation of airplane lift coefficient with dimensionless speed</td>
<td>$1/\text{rad}$</td>
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<td>$C_m$</td>
<td>Pitching moment coefficient (airplane)</td>
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</tr>
<tr>
<td>$C_{m1}$</td>
<td>Pitching moment coefficient in steady state (airplane)</td>
<td>$-$</td>
</tr>
<tr>
<td>$C_{m\alpha}$</td>
<td>Variation of airplane pitching moment coefficient with angle of attack</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{m\dot{\alpha}}$</td>
<td>Variation of airplane pitching moment coefficient with dimensionless rate of change of angle of attack</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{m_iH}$</td>
<td>Variation of airplane pitching moment coefficient with stabilizer incidence angle</td>
<td>$1/\text{rad}$</td>
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<tr>
<td>$C_{m_{\delta E}}$</td>
<td>Variation of airplane pitching moment coefficient with elevator deflection angle</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
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</tr>
<tr>
<td>$C_{mq}$</td>
<td>Variation of airplane pitching moment coefficient with dimensionless pitch rate</td>
<td>$1/\text{rad}$</td>
</tr>
<tr>
<td>$C_{mT\alpha}$</td>
<td>Variation of airplane pitching moment coefficient due to thrust with angle of attack</td>
<td>$1/\text{rad}$</td>
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<td>$C_{mTu}$</td>
<td>Variation of airplane pitching moment coefficient due to thrust with dimensionless speed</td>
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<tr>
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</tr>
<tr>
<td>$C_{T0}$</td>
<td>Thrust coefficient in reference state</td>
<td>−</td>
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<tr>
<td>$C_{Ti}$</td>
<td>Thrust coefficient in steady state</td>
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</tr>
<tr>
<td>$C_{Tu}$</td>
<td>Variation of airplane pitching moment coefficient due to thrust with dimensionless speed</td>
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### Dimensional derivatives

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<th>Symbol</th>
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<tr>
<td>$M_{\alpha}$</td>
<td>Pitch angular acceleration per unit angle of attack</td>
<td>$\text{rad/s}^2$ \text{rad}^{-1}$</td>
</tr>
<tr>
<td>$M_{\alpha T}$</td>
<td>Pitch angular acceleration per unit angle of attack due to thrust</td>
<td>$\text{rad/s}^2$ \text{rad}^{-1}$</td>
</tr>
<tr>
<td>$M_{u}$</td>
<td>Pitch angular acceleration per unit change in speed</td>
<td>$\text{rad/s}^2$ \text{m/s}$</td>
</tr>
<tr>
<td>$M_{Tu}$</td>
<td>Pitch angular acceleration per unit change in speed due to thrust</td>
<td>$\text{rad/s}^2$ \text{m/s}$</td>
</tr>
<tr>
<td>$M_{\dot{\alpha}}$</td>
<td>Pitch angular acceleration per unit rate of change of angle of attack</td>
<td>$\text{rad/s}^2$ \text{rad/s}$^{-1}$</td>
</tr>
<tr>
<td>$M_{\delta E}$</td>
<td>Pitch angular acceleration per unit elevator deflection angle</td>
<td>$\text{rad/s}^2$ \text{rad}$^{-1}$</td>
</tr>
<tr>
<td>$X_{\alpha}$</td>
<td>Forward acceleration per unit angle of attack</td>
<td>$\text{m/s}^2$ \text{rad}^{-1}$</td>
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<tr>
<td>$X_{u}$</td>
<td>Forward acceleration per unit change in speed</td>
<td>$\text{m/s}^2$ \text{m/s}$</td>
</tr>
<tr>
<td>$X_{Tu}$</td>
<td>Forward acceleration per unit change in speed due to thrust</td>
<td>$\text{m/s}^2$ \text{m/s}$</td>
</tr>
<tr>
<td>$X_{\delta E}$</td>
<td>Forward acceleration per unit elevator deflection angle</td>
<td>$\text{m/s}^2$ \text{rad}$^{-1}$</td>
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<td>$Z_{\alpha}$</td>
<td>Vertical acceleration per unit angle of attack</td>
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<td>$Z_{u}$</td>
<td>Vertical acceleration per unit change in speed</td>
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<td>$Z_{\dot{\alpha}}$</td>
<td>Vertical acceleration per unit rate of change of angle of attack</td>
<td>$\text{m/s}^2$ \text{rad/s}$^{-1}$</td>
</tr>
<tr>
<td>$Z_{\dot{q}}$</td>
<td>Vertical acceleration per unit pitch rate</td>
<td>$\text{m/s}^2$ \text{rad}$^{-1}$</td>
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<tr>
<td>$Z_{\delta E}$</td>
<td>Vertical acceleration per unit elevator deflection angle</td>
<td>$\text{m/s}^2$ \text{rad}$^{-1}$</td>
</tr>
</tbody>
</table>
## Contents

1 Introduction ................................................. 1  
1.1 Background ............................................ 1  
1.2 Objective ............................................. 1  
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1 Introduction

This Master’s thesis, conducted as part of the Master in Aeronautical Engineering program at Linköping University, is based on a topic provided by the aerospace company Saab AB. The objective of this project is to assess the stability and control derivatives of an aircraft. Subsequently, the focus turns towards formulating the equations of motion. These outcomes serve multiple purposes, including early sizing criteria, stability analysis, and performance evaluation under various conditions.

The thesis comprises both theoretical and practical components. The theoretical aspect involves identifying, testing, and comparing different models available in the literature. In the practical phase, selected models are implemented within the conceptual design software, Pacelab Aircraft Preliminary Design (APD) 8.1 [1], to establish the flight envelope at specific operating points.

This effort involves two teams working together. The present work specifically addresses the longitudinal derivatives for stability and control. By delving into these crucial parameters, this research contributes to enhancing the understanding of aircraft behaviour and performance characteristics.

1.1 Background

The determination of an aircraft’s derivatives is a complex and meticulous task that involves the utilisation of various software, including those that use experimental data and formulas from military articles. Although the US military has developed a comprehensive software tool for stability and control derivatives, called DATCOM+, its user-unfriendly interface and extensive documentation often make it a challenging instrument to navigate.

With the growing popularity of Pacelab Aircraft Preliminary Design (APD) and its adoption within the industry, there has been an increased interest in uncovering the software’s potential.

1.2 Objective

This project leverages the capabilities of Pacelab Aircraft Preliminary Design (APD) 8.1 [1] to model an aircraft and extract critical information for conceptual designers, such as drag polars.

The primary objective of this project is to establish a robust foundation for obtaining stability and control derivatives, as well as formulating the equations of
motion, and obtaining the dynamic and static stability of an aircraft.

The thesis endeavours to address the following questions:

- Can Pacelab APD be effectively utilised to obtain the equations of motion and flight envelope of an aircraft?

- Is it possible to accurately assess the stability of a conceptual aircraft using Pacelab APD?

By investigating these questions, this research aims to explore the capabilities and limitations of Pacelab APD in terms of acquiring essential aircraft dynamics information and evaluating stability.

1.3 Delimitations

Integrating the totality of the stability and control methods is an enormous task. As a long term project, the aim of this thesis is to create a base where more methods can be built on. Technical limitations include the time to work with a new complex software and the integration of the parameters and charts. This thesis work is limited only to flight dynamics of longitudinal motions, lateral and directional analysis is conducted by another team.

Derivatives relying on thrust necessitate additional research and may undergo modifications in the future, as detailed specifications pertaining to engines or wind tunnel data were not available during this study.

Limited data availability poses another constraint for verifying the calculated values. The derivatives’ equations complexity vary depending on the Mach regime, subsonic, transonic, supersonic, and the configuration of the aircraft meaning commercial, delta wing, number and position of engines etc. As a first approach, the coefficients’ forms for the subsonic regime [0-0.6] are calculated for a conventional commercial aircraft. As the entered equations need verification, a model is chosen as the source of data. In this case, the Boeing 747 is selected due to its presence in different software tools. The coefficients and input parameters for the aircraft can be found in the literature [2], and the aircraft has already been modelled in Pacelab APD. The Boeing 747 features a swept wing, four engines, and horizontal and vertical tail surfaces, as depicted in Figure 1.
1.4 Method

In the first stages of the thesis the two teams collaborated to create a document where all derivatives and literature were scripted with comments. Common delimitations were noted.

A thorough literature review was carried out in order to find the needed formulas, methods and conditions. The review showed that the most complete sources for the subsonic regime, for stability and control are Dr. Jan Roskam’s work [4, 5].

The main source for the longitudinal derivatives formulas is reference [5]. For some formulas a clearer explanation was shown in reference [4].

Obtaining thrust coefficients proved to be the most challenging task. Detailed aircraft engine specifications were elusive, and wind tunnel data was scarce. The solution is the use of reference [6]. The approximations of the coefficients depending on thrust are simplified and the formulas are easier to obtain.

In addition to references [4, 5], book [2] was also utilised. The latter being more accessible and compact, which facilitated the first contact with the derivatives and the totality of the calculated derivatives are present in the appendices of book [2] for the B747.

The choice of books and the introduction of the conditions green-lighted the initiation of tutorials, instructed by Pace, for the better understanding of the software.

The code was then first written using another software before it was implemented into Pacelab APD. A first verification of formulas was executed. Once the formulas and functions were implemented, a second verification was made.

Finally, a comparison between the values obtain from the different sources was produced. The schematic workflow of the thesis is shown in Figure 2.
1.5 Structure of the report

The report begins by providing an overview of the relevant theory that was reviewed for this study. It includes a comprehensive description of the derivatives and equations employed in the analysis. Subsequently, the focus shifts to the introduction of Pacelab APD 8.1, outlining its key features and functionalities. As a next step the adopted methodology and the software implementation will be presented, along with the tools used for testing and validation. Afterwards, the results will be stated, before moving on to the discussion. Finally, the report concludes with a summary of the project’s key insights and conclusions.
2 Theory

In this section the theoretical part of the thesis is discussed. The used equations, methods and rules are stated and explained. The principals of Pacelab APD software are also included in this chapter.

The most accurate way of obtaining aerodynamic behaviour of any object is the experimental approach. Such practices, however, can be very costly and time consuming which is why theoretical and empirical methods are valued in preliminary and conceptual design phases. Using a combination of mathematical and empirical models in representing aerodynamic and thrust forces and moments is considered very beneficial if done properly. A correctly modelled system of forces and moments acting on an airplane with certain simplifications can fairly realistically represent those mechanical properties.

The changes in forces acting on an airplane concerning longitudinal stability and control are dependant on many factors including but not limited to: angle of attack, forward velocity, horizontal stabilizer incidence angle or elevator deflection angle. Parameters describing how those changes influence aircraft behaviour in the air are the dimensional and dimensionless derivatives.

2.1 Equations of Motion

The equations of motion are a system of six equations describing the dynamics of an aircraft combined with three kinematic equations. They are shown in equations 1 and equations 2. Those equations are based on Newton’s Second Law of Motion which states that a time derivative of linear and angular momenta of a body is equal in both magnitude and direction to the force imposed on it externally. Conservation of the linear and angular momenta equations along with gravity equations create aircraft equations of motion, given by Eq.(1.51), (1.52) and (1.53) from [4], shown in Equations 1, 2 and 3, respectively.

\[
\begin{align*}
    m(\dot{U} + Q \cdot W - RV) &= -mg \cdot \sin \Theta + F_{Ax} + F_{Tx} \quad (1a) \\
    m(\dot{V} + UR - PW) &= mg \cdot \cos \Theta \cdot \sin \Phi + F_{Ay} + F_{Ty} \quad (1b) \\
    m(\dot{W} + PV - QU) &= mg \cdot \cos \Theta \cdot \cos \Phi + F_{Az} + F_{Tz} \quad (1c)
\end{align*}
\]

\[
\begin{align*}
    \dot{P}I_{XX} - \dot{R}I_{XZ} - PQI_{XZ} + RQ(I_{ZZ} - I_{YY}) &= L_A + L_T \quad (2a) \\
    \dot{Q}I_{YY} - PR(I_{XX} - I_{ZZ}) + (P^2 - R^2)I_{XZ} &= M_A + M_T \quad (2b) \\
    \dot{R}I_{ZZ} - \dot{P}I_{XZ} + PQ(I_{YY} - I_{XX}) + QRI_{XZ} &= N_A + N_T \quad (2c)
\end{align*}
\]
\[ P = \dot{\phi} - \dot{\psi}\sin\Theta \] (3a)
\[ Q = \dot{\Theta}\cos(\Phi) + \dot{\psi}\cos(\theta)\sin\Phi \] (3b)
\[ R = \dot{\psi}\cos\Theta\cos\Phi - \dot{\Theta}\sin\Phi \] (3c)

Where:
- \( m \) is airplane mass \([kg]\),
- \( I_{XX} \) is airplane moment of inertia about x-axis \([kg \cdot m^2]\),
- \( I_{XZ} \) is airplane product of inertia about XZ \([kg \cdot m^2]\),
- \( I_{YY} \) is airplane moment of inertia about y-axis \([kg \cdot m^2]\),
- \( I_{ZZ} \) is airplane moment of inertia about z-axis \([kg \cdot m^2]\),
- \( F_{AX} \) is aerodynamic force in x-direction \([N]\),
- \( F_{AY} \) is aerodynamic force in y-direction \([N]\),
- \( F_{AZ} \) is aerodynamic force in z-direction \([N]\),
- \( F_{TX} \) is thrust force in x-direction \([N]\),
- \( F_{Ty} \) is thrust force in y-direction \([N]\),
- \( F_{Tz} \) is thrust force in z-direction \([N]\),
- \( L_A \) is aerodynamic rolling moment \([N \cdot m]\),
- \( M_A \) is aerodynamic pitching moment \([N \cdot m]\),
- \( N_A \) is aerodynamic yawing moment \([N \cdot m]\),
- \( L_T \) is thrust rolling moment \([N \cdot m]\),
- \( M_T \) is thrust pitching moment \([N \cdot m]\),
- \( N_T \) is thrust yawing moment \([N \cdot m]\),
- \( P \) is airplane angular velocity component about x-axis \([rad/s]\),
- \( Q \) is airplane angular velocity component about y-axis \([rad/s]\),
- \( R \) is airplane angular velocity component about z-axis \([rad/s]\),
- \( U \) is linear longitudinal velocity (along the x-axis) \([m/s]\),
- \( V \) is linear lateral velocity (along the y-axis) \([m/s]\),
- \( W \) is linear vertical velocity (along the z-axis) \([m/s]\),
- \( \Theta \) is airplane pitch attitude angle \([rad]\),
- \( \Phi \) is airplane bank angle \([rad]\),
- \( \Psi \) is airplane heading angle \([rad]\).

### 2.1.1 Steady state

The least complex state of flight is the steady state, which is also a good base for stability analysis. Steady state is the state when all components of linear and angular velocities in all directions are constant. Steady state can be assumed only in conditions where the air density does not change more than 5% during a 30 to 60 seconds time period. After applying steady state definition to Equations 1, 2 and 3, Eq(1.55), (1.56) and (1.57) from [4] were obtained as shown in Equations 4, 5 and 6 respectively.
Another assumption enabling solving problems concerning cruise flight, engine inoperative flight or steady state flight with some failed systems is the case of rectilinear flight in steady state. The flight in a straight line further simplifies the equations of motion as given in Eq.(1.58) and (1.59) from [4], as shown in Equations 7 and 8. Equation 6 becomes trivial after applying rectilinear conditions.

\[0 = -mg \cdot \sin \Theta_1 + F_{A_X 1} + F_{T_X 1}\]  
\[0 = mg \cdot \cos \Theta_1 \cdot \sin \Phi_1 + F_{A_Y 1} + F_{T_Y 1}\]  
\[0 = mg \cdot \cos \Theta_1 \cdot \cos (\Phi_1) + F_{A_Z 1} + F_{T_Z 1}\]  
\[-P_1 Q_1 I_{XZ} + R_1 Q_1 (I_{ZZ} - I_{YY}) = L_{A_1} + L_{T_1}\]  
\[-P_1 R_1 (I_{XX} - I_{ZZ}) + (P_1^2 - R_1^2) I_{XZ} = M_{1} + M_{1}\]  
\[P_1 Q_1 (I_{YY} - I_{XX}) + Q_1 R_1 I_{XZ} = N_{A_1} + N_{T_1}\]  
\[P_1 = \dot{\Phi}_1 - \dot{\Psi}_1 \sin \Theta_1\]  
\[Q_1 = \dot{\Theta}_1 \cos \Phi_1 + \dot{\Psi}_1 \cos \Theta_1 \sin \Phi_1\]  
\[R_1 = \dot{\Psi}_1 \cos (\Theta_1) \cos \Phi_1 - \dot{\Theta}_1 \sin (\Phi_1)\]

Since this project focuses on longitudinal stability, only Equations [7a, 7c] and [8b] are taken into further consideration. Forces and moments used in those equations are presented in Equations [9] and [10] derived in stability axes, as given in Eq.(3.46) from [4].
\[
\begin{align*}
\begin{bmatrix} F_{Tx1} \\ F_{Ty1} \\ M_{T1} \end{bmatrix} = \\ \begin{bmatrix} T \cos(\Phi_T + \alpha) \\ -T \sin(\Phi_T + \alpha) \\ -T d_T \end{bmatrix}
\end{align*}
\]

(10)

After combining Equations 7, 8, 9 and 10, a set of longitudinal equations of motion for steady state in rectilinear motion is obtained as given by Eq.(4.42) from [4], shown in Equation 11

\[
m \sin \gamma_1 = -(C_{D_0} + C_{D_{\alpha}} \alpha + C_{D_{iH}} i_H + C_{D_{\delta E}} \delta_E) \bar{q}_1 S + T_1 \cos(\Phi_T + \alpha_1) \tag{11a}
\]

\[
m \cos \Phi_1 \cos \gamma_1 = (C_{L_0} + C_{L_{\alpha}} \alpha + C_{L_{iH}} i_H + C_{L_{\delta E}} \delta_E) \bar{q}_1 S + T_1 \sin(\Phi_T + \alpha_1) \tag{11b}
\]

\[
0 = (C_{m_0} + C_{m_{\alpha}} \alpha + C_{m_{iH}} i_H + C_{m_{\delta E}} \delta_E) \bar{q}_1 S - T_1 d_T \tag{11c}
\]

In Equation 11 previously used steady state pitch attitude angle \( \theta_1 \) is changed to steady state flight path angle due to expressing these equations in the stability-axis system. The coefficients used in Equation 11 are called dimensionless derivatives. They portray in what way changes in specific conditions affect airplane’s behaviour in the air. Methods of obtaining them are shown in subsection 2.2.

2.1.2 Perturbed state

For the perturbed flight, substitutions are made to motion variables, forces and moments. Each is replaced with variables values on steady state with a small perturbation. After implementing the substitutions, equations of motion 1 and 2 take a form given by Eq.(1.69) and Eq.(1.70) from [4], as shown in Equations 12 and 13. Substitutions applied to the kinematic Equations 3 result in Eq.(1.77) from [4], as shown in Equation 14.

\[
m \left[ \dot{u} - (V_1 + v) (R_1 + r) + (W_1 + w) (Q_1 + q) \right] = 
\]

\[
= -mg \sin (\Phi_1 + \phi) \cos (\Theta_1 + \theta) + F_{Ay1} + f_{Ay} + f_{Ty1} + f_{Sz} \tag{12a}
\]

\[
m \left[ \dot{v} + (U_1 + u) (R_1 + r) - (W_1 + w) (P_1 + p) \right] = 
\]

\[
= mg \sin (\Phi_1 + \phi) \cos (\Theta_1 + \theta) + F_{Ay1} + f_{Ay} + f_{Ty1} + f_{Sz} \tag{12b}
\]

\[
m \left[ \dot{w} - (U_1 + u) (Q_1 + q) + (V_1 + v) (P_1 + p) \right] = 
\]

\[
= mg \cos (\Phi_1 + \phi) \cos (\Theta_1 + \theta) + F_{Ax1} + f_{Ax} + f_{Tx1} + f_{Tx} \tag{12c}
\]

\[
L_{xx} \dot{p} - I_{xx} \dot{r} - I_{xz} (P_1 + p) (Q_1 + q) + (I_{zz} - I_{yy}) (R_1 + r) (Q_1 + q) = 
\]

\[
= L_{Ax} + l_A + L_{Tx} + l_T \tag{13a}
\]

\[
I_{yy} \dot{q} + (I_{xx} - I_{zz}) (P_1 + p) (R_1 + r) + I_{xz} \left[ (P_1 + p)^2 - (R_1 + r)^2 \right] = 
\]

\[
= M_{Ax} + m_A + M_{Tx} + m_T \tag{13b}
\]

\[
I_{zz} \dot{r} - I_{xx} \dot{p} + (I_{yy} - I_{xx}) (P_1 + p) (Q_1 + q) + I_{xz} (Q_1 + q) (R_1 + r) = 
\]

\[
= N_{Ax} + n_A + N_{Tx} + n_T \tag{13c}
\]
After assuming trigonometric approximations for small angle perturbations (up to 15°) and assuming the non-linear terms of the equations to be negligible compared with the linear ones, the Equations 15 and 16 were obtained, as given by Eq.(1.75) and (1.76) from [4]. The same process was performed on the kinematic equations and the result is given by Eq.(1.79) from [3], as shown in Equation 17.

\[
P_1 + p = \left(\dot{\Phi}_1 + \dot{\phi}\right) - \left(\dot{\Psi}_1 + \dot{\psi}\right) \sin (\Theta_1 + \theta) \tag{14a}
\]
\[
Q_1 + q = \left(\dot{\Theta}_1 + \dot{\theta}\right) \cos (\Phi_1 + \phi) + \left(\dot{\Psi}_1 + \dot{\psi}\right) \cos (\Theta_1 + \theta) \sin (\Phi_1 + \phi) \tag{14b}
\]
\[
R_1 + r = \left(\dot{\Psi}_1 + \dot{\psi}\right) \cos (\Theta_1 + \theta) \cos (\Phi_1 + \phi) - \left(\dot{\Theta}_1 + \dot{\theta}\right) \sin (\Phi_1 + \phi) \tag{14c}
\]

Applying those conditions results in equations given by Eq.(1.81), (1.82) and (1.83) from [4]. The flight condition with a relatively small flight path angle“ following conditions can be applied:

- no initial angular velocities: \(P_1 = Q_1 = R_1 = \dot{\Psi}_1 = \dot{\Theta}_1 = \dot{\Phi}_1 = 0\).

According to Dr. Jan Roskam [4] the majority of airplane stability problems are connected to “perturbed motions relative to a wing level, steady state, straight line flight condition with a relatively small flight path angle”. For that flight state, the following conditions can be applied:

- no initial steady state velocity: \(V_1 = 0\),
- no initial bank angle: \(\Phi = 0\),
- no initial angular velocities: \(P_1 = Q_1 = R_1 = \dot{\Psi}_1 = \dot{\Theta}_1 = \dot{\Phi}_1 = 0\).

Applying those conditions results in equations given by Eq.(1.81), (1.82) and (1.83) from [4], as shown in Equations 18, 19 and 20.
\[ m (\ddot{u} + W_1 q) = -mg\theta \cos \Theta_1 + f_{A_x} + f_{T_x} \quad (18a) \]
\[ m (\ddot{v} + U_1 r - W_1 p) = mg\phi \cos \Theta_1 + f_{A_y} + f_{T_y} \quad (18b) \]
\[ m (\ddot{w} - U_1 q) = -mg\theta \sin \Theta_1 + f_{A_z} + f_{T_z} \quad (18c) \]

\[ I_{xx}\dot{p} - I_{xz}\dot{r} = I_A + I_T \quad (19a) \]
\[ I_{yy}\dot{q} = m_A + m_T \quad (19b) \]
\[ I_{zz}\dot{r} - I_{xz}\dot{p} = n_A + n_T \quad (19c) \]

\[ p = \dot{\phi} - \dot{\psi} \sin \Theta_1 \quad (20a) \]
\[ q = \dot{\theta} \quad (20b) \]
\[ r = \dot{\psi} \cos \Theta_1 \quad (20c) \]

Longitudinal equations expressed in the stability-axis system augmented by formulas for the perturbed aerodynamic and thrust forces and moments are given by Eq.(5.1) from [4], as shown in Equation 21, where \( q = \dot{\theta} \) and \( w = U_1 \alpha \).

\[ m \ddot{u} = -mg \cos \theta_1 + \bar{q}_1 S \left\{ - (C_{D_a} + 2C_{D_1}) \frac{u}{U_1} + (C_{T_{sa}} + 2C_{T_{s1}}) \frac{u}{U_1} \right\} + \]
\[ + \bar{q}_1 S \left\{ \frac{C_D}{U_1} - (C_{D_a} - C_{L_1}) \alpha - C_{D_{sc}} \delta_e \right\} \quad (21a) \]
\[ m (\ddot{w} - U_1 q) = -mg \sin \theta_1 + \bar{q}_1 S \left\{ - (C_{L_a} + 2C_{L_1}) \frac{u}{U_1} - (C_{L_a} + C_{D_1}) \alpha \right\} + \]
\[ + \bar{q}_1 S \left\{ -C_{L_a} \frac{\alpha \sigma}{2U_1} - C_{L_{q1}} \frac{q \sigma}{2U_1} - C_{L_{sc}} \delta_e \right\} \quad (21b) \]
\[ I_{yy}\ddot{q} = \bar{q}_1 S c \left\{ (C_{m_u} + 2C_{m_1}) \frac{u}{U_1} + (C_{m_{Tu}} + 2C_{m_{T1}}) \frac{u}{U_1} + C_{m_a} \alpha + C_{m_{Ta}} \alpha \right\} + \]
\[ + \bar{q}_1 S c \left\{ C_{m_u} \frac{\alpha \sigma}{2U_1} + C_{m_{q1}} \frac{q \sigma}{2U_1} + C_{m_{sc}} \delta_e \right\} \quad (21c) \]

Set of equations in [Equation 21] is called fixed control surface equations, they do not include the dynamic behaviour of control systems. That type of equations apply to airplanes with irreversible flight control systems and reversible control systems if the control surfaces stay fixed at an initial position or move in a settled sequence.

To procure a clearer look on the longitudinal equations of motion, dimensional derivatives are utilised. Dimensional derivatives provide a better understanding of the importance of the forces acting upon the aircraft. Methods of obtaining those
derivatives are presented in subsection 2.3. Perturbed longitudinal equations of motion with dimensional stability derivatives are given by Eq.(5.30) from [4], as shown in Equation 22.

\[ \dot{u} = -g \cos \theta_1 + X_u u + X_{\alpha} \alpha + X_{\delta_e} \delta_e \]  
\[ U_1 \dot{\alpha} = -g \sin \theta_1 + Z_u u + Z_{\alpha} \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_{\delta_e} \delta_e \]  
\[ \ddot{\theta} = M_u u + M_T u + M_{\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_{\delta_e} \delta_e \]  

2.2 Dimensionless Derivatives

Formulas from Dr. Jan Roskam’s books [4, 5] were used to calculate the dimensionless derivatives concerning longitudinal stability and control dependent on angle of attack, rate of change of angle of attack, pitch rate, stabilizer incidence angle, forward speed and elevator deflection, while derivatives dependent on thrust were obtained with help of [6]. This thesis focuses on conventional configuration airplanes, parts of the equations referring to canard are not further explained.

The following derivatives are discussed in this subsection:

- \( C_{L\alpha} \) - Lift derivative with respect to angle of attack, given by Equation 23.
- \( C_{D\alpha} \) - Drag derivative with respect to angle of attack, given by Equation 36.
- \( C_{m\alpha} \) - Pitching moment derivative with respect to angle of attack, given by Equation 37.
- \( C_{L\dot{\alpha}} \) - Lift derivative with respect to angle of attack rate, given by Equation 39.
- \( C_{D\dot{\alpha}} \) - Drag derivative with respect to angle of attack rate, equal to 0.
- \( C_{m\dot{\alpha}} \) - Pitching moment derivative with respect to angle of attack rate, given by Equation 40.
- \( C_{L_{iH}} \) - Lift derivative with respect to stabilizer incidence angle, given by Equation 41.
- \( C_{D_{iH}} \) - Drag derivative with respect to stabilizer incidence angle, given by Equation 42.
- \( C_{m_{iH}} \) - Pitching moment derivative with respect to stabilizer incidence angle, given by Equation 44.
- \( C_{L_{\delta E}} \) - Lift derivative with respect to elevator deflection angle, given by Equation 45.
- \( C_{D_{\delta E}} \) - Drag derivative with respect to elevator deflection angle, equal to 0.
• $C_{m_{\delta E}}$ - Pitching moment derivative with respect to elevator deflection angle, given by Equation 46.

• $C_{L_q}$ - Lift derivative with respect to pitch rate, given by Equation 47.

• $C_{D_q}$ - Drag derivative with respect to pitch rate, equal to 0.

• $C_{m_q}$ - Pitching moment derivative with respect to pitch rate, given by Equation 52.

• $C_{L_u}$ - Lift derivative with respect to forward speed, given by Equation 56.

• $C_{D_u}$ - Drag derivative with respect to forward speed, given by Equation 57.

• $C_{m_u}$ - Pitching moment derivative with respect to forward speed, given by Equation 58.

• $C_{m_{T\alpha}}$ - Pitching moment derivative with respect to thrust, given by Equation 59.

• $C_{T_{xu}}$ - Thrust derivative with respect to forward speed, given by Equation 62.

• $C_{m_{T\alpha}}$ - Pitching moment derivative with respect to thrust versus forward speed, given by Equation 65.

2.2.1 Derivatives with respect to angle of attack

Lift derivative with respect to angle of attack

Lift derivative with respect to angle of attack, also referred to as lift-curve slope gradient, is given by Eq.(8.42) from [5] and is shown in Equation 23. The canard part of equation is not used since only standard configuration is considered.

\[
C_{L_{\alpha}} = C_{L_{\alpha WB}} + C_{L_{\alpha H}} \cdot \eta_H \cdot \frac{S_H}{S} \cdot (1 - \frac{d\varepsilon}{d\alpha}) + C_{L_{\alpha C}} \cdot \eta_C \cdot \frac{S_C}{S} \cdot (1 + \frac{d\varepsilon_C}{d\alpha}) \tag{23}
\]

Where:

$C_{L_{\alpha WB}}$ is the wing-body lift curve $[-]$, as stated in Eq.(8.43) in [5] and shown in Equation 24.

\[
C_{L_{\alpha WB}} = K_{WB} \cdot C_{L_{\alpha W}} \tag{24}
\]
Where:

$K_{WB}$ is the wing-body interference factor $[-]$, given by Eq.(8.44) in [5], as shown in Equation 25.

$$K_{WB} = 1 + 0.025 \cdot \frac{d_f}{b} - 0.25 \cdot \left( \frac{d_f}{b} \right)^2$$  \hspace{1cm} (25)

Where:

$d_f$ is equivalent fuselage diameter $[m]$, $b$ is wing span $[m]$.

$C_{L_{\alpha W}}$ is wing lift curve slope $[-]$, given by Eq.(8.22) in [5], as shown in Equation 26.

$$C_{L_{\alpha W}} = \frac{2 \cdot \pi \cdot AR}{2 + \sqrt{\left\{ \frac{AR^2(1-M^2)}{k^2} \cdot \left( 1 + \tan^2(\Lambda_{c/2}) \right) \right\} + 4}}$$  \hspace{1cm} (26)

Where:

$AR$ is wing aspect ratio $[-]$, $M$ is Mach number $[-]$, $\Lambda_{c/2}$ is wing semi-chord sweep angle $[rad]$, $k$ is a function of the wing geometry $[-]$, known as Polhamus formula, taken from [2], shown in Equation 27.

$$k = \begin{cases} 1 + \frac{AR \cdot (1.87 - 0.000233 \cdot \Lambda_{LE})}{100} & \text{for } AR < 4 \\ 1 + \frac{(8.2 - 2.3 \cdot \Lambda_{LE}) - AR \cdot (0.22 - 0.153 \cdot \Lambda_{LE})}{100} & \text{for } AR \geq 4 \end{cases}$$  \hspace{1cm} (27)

Where:

$\Lambda_{LE}$ is wing leading edge sweep angle $[rad]$.

$C_{L_{\alpha H}}$ is the horizontal tail lift curve slope $[-]$, given by Equation 26 with horizontal stabilizer parameters instead of wing parameters, $\eta_H$ is ratio between dynamic pressure on horizontal stabilizer and wing $[-]$, $S_H$ is horizontal stabilizer reference area $[m^2]$, $S$ is wing reference area $[m^2]$, $\frac{d\varepsilon}{d\alpha}$ is downwash gradient $[-]$, given by Eq.(2.27) for subsonic flow and Eq.(2.28) for transonic flow in [4] as shown in Equations 28 and 29.

$$\frac{d\varepsilon}{d\alpha} = \frac{d\varepsilon}{d\alpha}_{M=0} \cdot \sqrt{(1 - M^2)}$$  \hspace{1cm} (28)

$$\frac{d\varepsilon}{d\alpha} = \frac{\varepsilon}{\alpha}_{M=0} \cdot \frac{C_{L_{\alpha W}}}{C_{L_{\alpha W}}}_{M=0}$$  \hspace{1cm} (29)
Where:

\[ C_{L_{MW}} \bigg|_{M=0} \text{ and } C_{L_{MW}} \bigg|_{M=0} \] are wing lift curve slopes for flight Mach number and Mach number equal to 0 respectively \([-\), calculated as shown in Equation 26 with respective Mach numbers,

\[ \left. \frac{d\varepsilon}{d\alpha} \right|_{M=0} \] is downwash gradient for Mach number equal to 0 \([-\), given by \cite{2} (section 2.4), as shown in Equation 30.

\[
\left. \frac{d\varepsilon}{d\alpha} \right|_{M=0} = 4.44 \cdot \left( K_{AR} \cdot K_{\lambda} \cdot K_{mr} \cdot \sqrt{\cos(\Lambda_{c}/4)} \right)^{1.19} \tag{30}
\]

Where:

\( K_{AR} \) is a coefficient related with wing aspect ratio \([-\), represented by Equation 31

\[
K_{AR} = \frac{1}{AR} - \frac{1}{1 + (AR)^{1.7}} \tag{31}
\]

\( K_{\lambda} \) is a coefficient related with wing taper ratio \([-\), represented by Equation 32

\[
K_{\lambda} = \frac{10 - 3 \cdot \lambda}{7} \tag{32}
\]

Where:

\( \lambda \) is wing taper ratio \([-\).

\( K_{mr} \) is a coefficient related with aircraft geometry \([-\), represented by Equation 33

\[
K_{mr} = \frac{1 - \frac{m}{r^{0.333}}}{2} \tag{33}
\]

Where:

\( m \) and \( r \) are geometrical parameters \([-\), shown in Figure 2.22 in \cite{2}, represented by Equations 34 and 35 respectively.

\[
m = \frac{2 \cdot (z_{csH} - z_{csW})}{b} \tag{34}
\]

\[
r = \frac{2 \cdot (x_{csH} - x_{csW} + c_{rH}/4 - c_{rW}/4)}{b} \tag{35}
\]

Where:

\( z_{csH} \) is beginning of horizontal stabilizer coordinate system position in z-axis \([m]\),
\( z_{csW} \) is beginning of wing coordinate system position in z-axis \([m]\),
\( b \) is wing span \([m]\).
$x_{csH}$ is beginning of horizontal stabilizer coordinate system position in x-axis [m],

$x_{csW}$ is beginning of wing coordinate system position in x-axis [m],

$c_{rh}$ is horizontal stabilizer root chord [m],

$c_{rw}$ is wing root chord [m].

$C_{LaC}$ is the canard lift curve slope $[-]$, given by Equation 26 with canard parameters instead of wing parameters,

$\eta_C$ is Ratio between dynamic pressure on canard and wing $[-]$,

$S_C$ is canard reference area $[m^2]$,

$\frac{d\varepsilon_C}{d\alpha}$ is upwash gradient at the canard $[-]$, found from Figure 8.67 from [5].

**Drag derivative with respect to angle of attack**

Drag derivative with respect to angle of attack is given by Eq.(10.18) from [5], as shown in Equation 36

$$C_{Da} = \frac{2 \cdot C_{L1} \cdot C_{La}}{\pi \cdot AR \cdot e} \quad (36)$$

Where:

$C_{L1}$ is airplane lift coefficient in steady state $[-]$,

$C_{La}$ is lift derivative with respect to angle of attack $[-]$, presented in Equation 23,

$AR$ is wing aspect ratio $[-]$,

$e$ is Oswald efficiency factor $[-]$, given in Eq.(4.1.5.2-i) in [7], this formula was used because it was already implemented in Pacelab.

**Pitching moment derivative with respect to angle of attack**

Pitching moment derivative with respect to angle of attack is given by Eq.(3.35) from [4], as shown in Equation 37

$$C_{ma} = C_{LWB} \cdot (\bar{x}_{CG} - \bar{x}_{ACWB}) - C_{LaH} \cdot \eta_H \cdot \bar{V}_H \cdot (1 - \frac{d\varepsilon}{d\alpha}) \quad (37)$$

Where:

$C_{LWB}$ is the wing-body lift curve $[-]$, as stated in Eq.(8.43) in [5] and shown in Equation 24,

$\bar{x}_{CG}$ is centre of gravity position in x-axis normalized by MAC $[-]$,

$\bar{x}_{ACWB}$ is aerodynamic centre of aircraft wing and body position in x-axis normalized by MAC $[-]$,

$C_{LaH}$ is the horizontal tail lift curve slope $[-]$, given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,

$\eta_H$ is Ratio between dynamic pressure on horizontal stabilizer and wing $[-]$,

$\frac{d\varepsilon}{d\alpha}$ is downwash gradient $[-]$, shown in Equations 28 and 29,

$\bar{V}_H$ is horizontal tail volume coefficient $[-]$, given by Eq.(10.23) from [5], shown in Equation 38

$$\bar{V}_H = (\bar{x}_{AC} - \bar{x}_{CG}) \cdot \frac{S_H}{S} \quad (38)$$
Where:
\( \bar{x}_{ACH} \) is aerodynamic centre of aircraft horizontal stabilizer position in x-axis normalized by MAC [\(-\)],
\( \bar{x}_{CG} \) is Centre of gravity position in x-axis normalized by MAC [\(-\)],
\( S_H \) is horizontal stabilizer reference area \([m^2]\),
\( S \) is wing reference area \([m^2]\).

### 2.2.2 Derivatives with respect to angle of attack rate

#### Lift derivative with respect to angle of attack rate

Lift derivative with respect to angle of attack rate is given by Eq.(10.22) from [5] and is shown in Equation 39.

\[
C_{L_\alpha} = 2 \cdot C_{L_{\alpha H}} \cdot \eta_H \cdot \bar{V}_H \cdot \frac{d\varepsilon}{d\alpha} \tag{39}
\]

Where:
\( C_{L_{\alpha H}} \) is the horizontal tail lift curve slope \([-\]), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
\( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-\]),
\( \frac{d\varepsilon}{d\alpha} \) is downwash gradient \([-\]), shown in Equations 28 and 29,
\( \bar{V}_H \) is horizontal tail volume coefficient \([-\]), given by Eq.(10.23) from [5], shown in Equation 38.

#### Drag derivative with respect to angle of attack rate

Drag derivative with respect to angle of attack rate can be neglected according to section 10.2.3 of [5], so it is assumed that \( C_{D_\alpha} = 0 \).

#### Pitching moment derivative with respect to angle of attack rate

Pitching moment derivative with respect to angle of attack rate is given by Eq.(10.24) from [5] and is shown in Equation 40.

\[
C_{m_\alpha} = -2 \cdot C_{L_{\alpha H}} \cdot \eta_H \cdot \bar{V}_H \cdot (\bar{x}_{ACH} - \bar{x}_{CG}) \tag{40}
\]

Where:
\( C_{L_{\alpha H}} \) is the horizontal tail lift curve slope \([-\]), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
\( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-\]),
\( \bar{V}_H \) is horizontal tail volume coefficient \([-\]), given by Eq.(10.23) from [5], shown in Equation 38,
\( \bar{x}_{ACH} \) is aerodynamic centre of aircraft horizontal stabilizer position in x-axis normalized by MAC \([-\]),
\( \bar{x}_{CG} \) is centre of gravity position in x-axis normalized by MAC \([-\]).

### 2.2.3 Derivatives with respect to stabilizer incidence angle

#### Lift derivative with respect to stabilizer incidence angle

Lift derivative with respect to stabilizer incidence angle is given by Eq.(10.91) from
and is shown in Equation 41.

\[ C_{L,H} = C_{\alpha H} \cdot \eta_H \cdot \frac{S_H}{S} \]  

(41)

Where:

- \( C_{\alpha H} \) is the horizontal tail lift curve slope \([-]\), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
- \( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-]\),
- \( S_H \) is horizontal stabilizer reference area \( [m^2] \),
- \( S \) is wing reference area \( [m^2] \).

**Drag derivative with respect to stabilizer incidence angle**

Drag derivative with respect to stabilizer incidence angle is given by Eq.(10.89) from [5] and is shown in Equation 42.

\[ C_{D,H} = \frac{2}{\pi} \cdot \frac{C_L}{AR} \cdot e \cdot C_{\alpha H} \cdot \eta_H \cdot \frac{S_H}{S} \]  

(42)

Where:

- \( C_L \) is the airplane lift coefficient \([-]\), given by Eq.(10.90) from [5], shown in Equation 43

\[ C_L = \frac{n \cdot W}{\bar{q} \cdot S} \]  

(43)

Where:

- \( n \) is the load factor \([-]\),
- \( W \) is the aircraft weight \( [N] \),
- \( \bar{q} \) is dynamic pressure \( [Pa] \),
- \( S \) is wing reference area \( [m^2] \).

- \( AR \) is wing aspect ratio \([-]\),
- \( e \) is Oswald efficiency factor \([-]\), given in Eq.(4.1.5.2-i) in [7], this formula was used because it was already implemented in Pacelab,
- \( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-]\),
- \( S_H \) is horizontal stabilizer reference area \( [m^2] \).

**Pitching moment derivative with respect to stabilizer incidence angle**

Pitching moment derivative with respect to stabilizer incidence angle is given by Eq.(10.92) from [5] and is shown in Equation 44.

\[ C_{m,H} = -C_{\alpha H} \cdot \eta_H \cdot \bar{V}_H \]  

(44)

Where:

- \( C_{\alpha H} \) is the horizontal tail lift curve slope \([-]\), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
- \( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-]\),
- \( \bar{V}_H \) is horizontal tail volume coefficient \([-]\), given by Eq.(10.23) from [5], shown in Equation 38.
2.2.4 Derivatives with respect to elevator deflection angle

Lift derivative with respect to elevator deflection angle
Lift derivative with respect to elevator deflection angle is given by Eq.(3.26) from [4] and is shown in Equation 45.

\[ C_{L_{\delta E}} = C_{L_{\alpha H}} \cdot \eta_H \cdot \frac{S_H}{S} \cdot \tau_E \]  

(45)

Where:
- \( C_{L_{\alpha H}} \) is the horizontal tail lift curve slope \([-]\), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
- \( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-]\),
- \( S_H \) is horizontal stabilizer reference area \([m^2]\),
- \( S \) is wing reference area \([m^2]\),
- \( \tau_E \) is elevator effectiveness parameter \([-]\), taken from Fig.(2.23) from [4].

Drag derivative with respect to elevator deflection angle can be neglected according to section 3.1.2 of [4], so it is assumed that \( C_{D_{\delta E}} = 0 \).

Pitching moment derivative with respect to elevator deflection angle
Pitching moment derivative with respect to stabilizer incidence angle is given by Eq.(3.37) from [4] and is shown in Equation 46.

\[ C_{m_{\delta E}} = -C_{L_{\alpha H}} \cdot \eta_H \cdot \frac{S_H}{S} \cdot \left( x_{AC_H} - \bar{x}_{CG} \right) \cdot \tau_E \]  

(46)

Where:
- \( C_{L_{\alpha H}} \) is the horizontal tail lift curve slope \([-]\), given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
- \( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing \([-]\),
- \( S_H \) is horizontal stabilizer reference area \([m^2]\),
- \( S \) is wing reference area \([m^2]\),
- \( x_{AC_H} \) is aerodynamic centre of aircraft horizontal stabilizer position in x-axis normalized by MAC \([-]\),
- \( \bar{x}_{CG} \) is centre of gravity position in x-axis normalized by MAC \([-]\),
- \( \tau_E \) is elevator effectiveness parameter \([-]\), taken from Fig.(2.23) from [4].

2.2.5 Derivatives with respect to pitch rate

Lift derivative with respect to pitch rate
Lift derivative with respect to pitch rate is given by Eq.(10.69) from [5] and is shown in Equation 47.

\[ C_{L_q} = C_{L_qW} + C_{L_qH} + C_{L_qC} \]  

(47)

Where:
- \( C_{L_qW} \) is the wing contribution to the lift due to pitch rate derivative \([-]\), given by
Eq.(10.70) from [5], shown in Equation 48

\[ C_{LqW} = \frac{AR + 2 \cdot \cos(\Lambda_{c/4})}{AR \cdot B + 2 \cdot \cos(\Lambda_{c/4})} \cdot C_{LqW} \bigg|_{M=0} \]  

(48)

Where:

\( AR \) is the wing aspect ratio [-].
\( \Lambda_{c/4} \) is wing quarter-chord sweep angle [rad],
\( B \) is a coefficient [-], given by Eq.(10.64) from [5], shown in Equation 49.

\[ B = \sqrt{1 - M^2 \cdot \left( \cos(\Lambda_{c/4}) \right)^2} \]  

(49)

Where:

\( M \) is Mach number [-],
\( C_{LqW} \bigg|_{M=0} \) is the wing contribution to the lift due to pitch rate derivative for Mach number equal to 0 [-], given by Eq.(10.71) from [5], shown in Equation 50.

\[ C_{LqW} \bigg|_{M=0} = \left( 0.5 + 2 \cdot \frac{x_w}{\bar{c}} \right) \cdot C_{L_{\alpha W}} \]  

(50)

Where:

\( x_w \) is absolute distance between CG and quarter chord point of wing MAC [m],
\( \bar{c} \) is Mean Aerodynamic Chord of the wing [m],
\( C_{L_{\alpha W}} \) is the wing lift curve slope [-], given by Equation 26.

\( C_{L_{\alpha H}} \) is the horizontal stabilizer contribution to the lift due to pitch rate derivative [-], given by Eq.(10.72) from [5], shown in Equation 51.

\[ C_{L_{qH}} = 2 \cdot C_{L_{\alpha H}} \cdot \eta_H \cdot \bar{V}_H \]  

(51)

Where:

\( C_{L_{\alpha H}} \) is the horizontal tail lift curve slope [-], given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,
\( \eta_H \) is ratio between dynamic pressure on horizontal stabilizer and wing [-],
\( \bar{V}_H \) is horizontal tail volume coefficient [-], given by Eq.(10.23) from [5], shown in Equation 38.

\( C_{L_{qC}} \) is the canard stabilizer contribution to the lift due to pitch rate derivative [-], given by Eq.(10.73) from [5].

**Drag derivative with respect to pitch rate** can be neglected according to Eq.(10.68) of [5], so it is assumed that \( C_{D_q} = 0 \).
Pitching moment derivative with respect to pitch rate

Pitching moment derivative with respect to pitch rate is given by Eq.(10.75) from [5] and is shown in Equation 52.

\[ C_{mq} = C_{mqW} + C_{mqH} + C_{mqC} \]  

Where:
\( C_{mqW} \) is the wing contribution to the pitch moment due to pitch rate derivative, given by Eq.(10.76) from [5], shown in Equation 53,

\[ C_{mqW} = C_{mqW} \bigg|_{M=0} \cdot \left( \frac{AR^3 \tan^2(\Lambda_{c/4})}{AR B + 6 \cos(\Lambda_{c/4})} + \frac{3}{B} \right) \]  

Where:
\( AR \) is the wing aspect ratio [-],
\( \Lambda_{c/4} \) is wing quarter-chord sweep angle [rad],
\( B \) is a coefficient given by Eq.(10.64) from [5], shown in Equation 49,
\( C_{mqW} \bigg|_{M=0} \) is the wing contribution to the pitching moment due to pitch rate derivative for Mach number equal to 0, given by Eq.(10.77) from [5], shown in Equation 54

\[ C_{mqW} \bigg|_{M=0} = -K_W \cdot C_{LaW} \cdot \cos(\Lambda_{c/4}) \cdot \left( \frac{AR \cdot (2 \cdot (x_w/\bar{c})^2 + 0.5 \cdot (x_w/\bar{c}))}{AR + 2 \cdot \cos(\Lambda_{c/4})} + \frac{AR^3 \tan^2(\Lambda_{c/4})}{24 \cdot (AR + 6 \cdot \cos(\Lambda_{c/4})) + 1/8} \right) \]  

Where:
\( K_W \) is correction constant for wing contribution to pitch damping [-], given by Fig.(10.40) in [5],
\( C_{LaW} \) is the wing lift curve slope [-], given by Equation 26,
\( x_w \) is absolute distance between CG and quarter chord point of wing MAC [m],
\( \bar{c} \) is Mean Aerodynamic Chord of the wing [m].

\( C_{mqH} \) is the horizontal stabilizer contribution to the pitch moment due to pitch rate derivative [-], given by Eq.(10.78) from [5], shown in Equation 55

\[ C_{mqH} = -2 \cdot C_{LaH} \cdot \eta_H \cdot \bar{V}_H \cdot (\bar{x}_{ACH} - \bar{x}_{CG}) \]  

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Where:

$C_{L_{\alpha H}}$ is the horizontal tail lift curve slope $[-]$, given by Equation 26 with horizontal stabilizer parameters instead of wing parameters,

$\eta_H$ is ratio between dynamic pressure on horizontal stabilizer and wing $[-]$, 

$V_H$ is horizontal tail volume coefficient $[-]$, shown in Equation 38,

$\bar{x}_{AC_H}$ is aerodynamic centre of aircraft horizontal stabilizer position in x-axis normalized by MAC $[-]$,

$\bar{x}_{CG}$ is Centre of gravity position in x-axis normalized by MAC, $[-]$.

$C_{m_{nc}}$ is the canard contribution to the pitch moment due to pitch rate derivative $[-]$, given by Eq.(10.79) from [5].

### 2.2.6 Derivatives with respect to forward speed

#### Lift derivative with respect to forward speed

Lift derivative with respect to forward speed is given by Eq.(10.11) from [5] and is shown in Equation 56

\[
C_{Lu} = \frac{M^2 \cdot \cos^2(\Lambda_{c/4}) \cdot C_{L_1}}{1 - M^2 \cdot \cos^2(\Lambda_{c/4})}
\]  

(56)

Where:

$M$ is Mach number $[-]$,

$C_{L_1}$ is lift coefficient at steady state $[-]$,

$\Lambda_{c/4}$ is wing quarter-chord sweep angle $[rad]$.

#### Drag derivative with respect to forward speed

Drag derivative with respect to forward speed is given by Eq.(10.10) from [5] and is shown in Equation 57

\[
C_{Du} = M \cdot \frac{\partial C_D}{\partial M}
\]  

(57)

Where:

$M$ is Mach number $[-]$,

$C_D$ is drag coefficient $[-]$.

#### Pitching moment derivative with respect to forward speed

Pitching moment derivative with respect to forward speed is given by Eq.(10.12) from [5] and is shown in Equation 58

\[
C_{m_u} = -M \cdot C_{L_1} \cdot \frac{\partial \bar{x}_{AC\Lambda}}{\partial M}
\]  

(58)

Where:

$M$ is Mach number $[-]$,

$C_{L_1}$ is lift coefficient in steady state $[-]$,

$\bar{x}_{AC\Lambda}$ is aft shift in airplane AC, normalized by MAC (aft shift is counted as positive) $[-]$.
2.2.7 Derivatives with respect to thrust

Pitching moment derivative with respect to thrust versus angle of attack is given by Eq.(10.20) from [5] and is shown in Equation 59.

\[
C_{mT\alpha} = \left( \Delta \left( \frac{dC_m}{dC_L} \right)_T \right) \cdot C_{L\alpha} \tag{59}
\]

Where:
- \(C_{L\alpha}\) is lift derivative with respect to angle of attack \([-]\), calculated in Equation 23,
- \(\Delta \left( \frac{dC_m}{dC_L} \right)_{TL}\) is found in Eq.(8.104) in [5], and shown in Equation 60.

\[
\Delta \left( \frac{dC_m}{dC_L} \right)_{T} = \left( \frac{dC_m}{dC_L} \right)_{TL} + \left( \frac{dC_m}{dC_L} \right)_{N} \tag{60}
\]

Where:
- \(\left( \frac{dC_m}{dC_L} \right)_{TL}\) is the effect of thrustline offset on longitudinal stability \([-]\), (negligible for jet powered aircraft according to section 8.2.8.2 of [4]),
- \(\left( \frac{dC_m}{dC_L} \right)_{N}\) is the effect of inlet normal force on longitudinal stability \([-]\), given by Eq.(8.111) from [5] and shown in Equation 61.

\[
\left( \frac{dC_m}{dC_L} \right)_{N} = \sum_{i=1}^{n} \frac{0.035 \cdot \hat{m}_i \cdot (\frac{d\epsilon_i}{d\alpha}) \cdot l_n}{S \cdot \bar{c} \cdot \rho \cdot U \cdot C_{L\alpha W}} \tag{61}
\]

Where:
- \(\hat{m}\) is air mass flow through the engine \([\text{kg/s}]\),
- \(\frac{d\epsilon}{d\alpha}\) is the effect of nacelle segment location on upwash gradient \([-]\), taken from Fig.(8.115) in [5],
- \(l_n\) is the moment arm of the engine’s inlet lip \([\text{m}]\),
- \(S\) is wing reference area \([\text{m}^2]\),
- \(\bar{c}\) is Mean Aerodynamic Chord of the wing \([\text{m}]\),
- \(\rho\) is air density \([\text{kg/m}^3]\),
- \(U\) is linear longitudinal velocity (along the x-axis) \([\text{m/s}]\),
- \(C_{L\alpha W}\) is wing lift curve slope \([-]\), calculated in Equation 26,
- \(n\) is number of engines \([-]\).

Thrust derivative with respect to forward speed

Thrust derivative with respect to forward speed for aircraft with jet propulsion is given by Eq.(5.3,8) from [6] and is shown in Equation 62.

\[
C_{Tv_u} = -2 \cdot C_{T0} \tag{62}
\]

Where:
- \(C_{T0}\) is thrust coefficient in reference state \([-]\), given by Eq.(5.3,10) from [6] as shown in Equation 63.

\[
C_{T0} = C_{D0} + C_{W0} \cdot \sin(\theta_0) \tag{63}
\]
Where:
- $C_{D_0}$ is profile drag coefficient [-],
- $C_{W_0}$ is weight coefficient in steady state [-], calculated using Equation 64.

\[ C_{W_0} = \frac{W}{\bar{q} \cdot S} \]  \hspace{1cm} (64)

Where:
- $W$ is aircraft weight [$N$],
- $\bar{q}$ is dynamic pressure [$Pa$],
- $S$ is wing reference area [$m^2$],
- $\theta_0$ is pitch angle in steady state [rad].

**Pitching moment derivative with respect to thrust versus forward speed**

Pitching moment derivative with respect to thrust versus forward speed is given by Eq.(10.16) from [5] and is shown in Equation 65.

\[ C_{m_{Tu}} = \frac{dT}{\bar{c}} \cdot C_{T_{xu}} \]  \hspace{1cm} (65)

Where:
- $dT$ is the thrust moment arm relative to CG [$m$], shown in Fig.(8.126) in [5], calculated with Equation 66.

\[ dT = \sum_{i=1}^{n} \frac{dT_i \cdot T_{av_i}}{T_{av_1} + T_{av_2} + ... + T_{av_n}} \]  \hspace{1cm} (66)

Where:
- $T_{av}$ is available thrust of an engine [$N$],
- $n$ is number of engines [-].
- $\bar{c}$ is the mean aerodynamic chord [$m$],
- $C_{T_{xu}}$ is thrust derivative with respect to forward speed [-], given by Equation 62.

### 2.3 Dimensional Derivatives

To better comprehend the relative importance of aerodynamic forces and moments acting on an aircraft dimensional stability and control derivatives are derived. They combine the flight conditions, geometric and inertial characteristics with the aerodynamic modelling. Each obtained derivative illustrates either linear or angular acceleration acting on the airplane.

The following derivatives are discussed in this subsection:

- $X_\alpha$ - Forward acceleration per unit angle of attack [$\frac{m}{s^2 \text{rad}}$], given by Equation 67.
• $Z_\alpha$ - Vertical acceleration per unit angle of attack [$\text{m/s}^2/\text{rad}$], given by Equation 68

• $M_\alpha$ - Pitch angular acceleration per unit angle of attack [$\text{rad/s}^2/\text{rad}$], given by Equation 69.

• $Z_\dot{\beta}$ - Vertical acceleration per unit rate of change of angle of attack [$\text{m/s}^2$], given by Equation 70.

• $M_\dot{\beta}$ - Pitch angular acceleration per unit rate of change of angle of attack [$\text{rad/s}^2/\text{rad}$], given by Equation 71.

• $X_\delta_E$ - Forward acceleration per unit elevator angle [$\text{m/s}^2/\text{rad}$], given by Equation 72.

• $Z_\delta_E$ - Vertical acceleration per unit elevator angle [$\text{m/s}^2/\text{rad}$], given by Equation 73.

• $M_\delta_E$ - Pitch angular acceleration per unit elevator angle [$\text{rad/s}^2/\text{rad}$], given by Equation 74.

• $Z_q$ - Vertical acceleration per unit pitch rate [$\text{m/s}^2/\text{rad/s}$], given by Equation 75.

• $M_q$ - Pitch angular acceleration per unit pitch rate [$\text{rad/s}^2/\text{rad/s}$], given by Equation 76.

• $X_u$ - Forward acceleration per unit change in speed [$\text{m/s}^2/\text{m/s}$], given by Equation 77.

• $Z_u$ - Vertical acceleration per unit change in speed [$\text{m/s}^2/\text{m/s}$], given by Equation 78.

• $M_u$ - Pitch angular acceleration per unit change in speed [$\text{rad/s}^2/\text{m/s}$], given by Equation 79.

• $X_T u$ - Forward acceleration per unit change in speed due to thrust [$\text{m/s}^2/\text{m/s}$], given by Equation 80.

• $M_T u$ - Pitch angular acceleration per unit change in speed due to thrust [$\text{rad/s}^2/\text{m/s}$], given by Equation 81.

• $M_T \alpha$ - Pitch angular acceleration per unit change in angle of attack due to thrust [$\text{rad/s}^2/\text{rad}$], given by Equation 82.

2.3.1 Derivatives per unit angle of attack

Forward acceleration per unit angle of attack represents acceleration imparted to the airplane as a result of a unit change in angle of attack. It is given in Table 5.1 from [4] and is shown in Equation 67.
\[
X_\alpha = \frac{-\bar{q} \cdot S \cdot (C_{D\alpha} - C_{L1})}{m} \left[ \frac{m/s^2}{rad} \right]
\]

Where:
\( \bar{q} \) is dynamic pressure [Pa],
\( S \) is wing reference area \([m^2]\),
\( C_{D\alpha} \) is drag derivative with respect to angle of attack \([-\]), calculated in Equation 36,
\( C_{L1} \) is lift coefficient in steady state \([-\]),
\( m \) is airplane mass \([kg]\).

**Vertical acceleration per unit angle of attack**
Vertical acceleration per unit angle of attack represents acceleration imparted to the airplane as a result of a unit change in angle of attack. It is given in Table 5.1 from [4] and is shown in Equation 68.

\[
Z_\alpha = \frac{-\bar{q} \cdot S \cdot (C_{L\alpha} + C_{D1})}{m} \left[ \frac{m/s^2}{rad} \right]
\]

Where:
\( \bar{q} \) is dynamic pressure [Pa],
\( S \) is wing reference area \([m^2]\),
\( C_{L\alpha} \) is lift derivative with respect to angle of attack \([-\]), calculated in Equation 23,
\( C_{D1} \) is drag coefficient in steady state \([-\]),
\( m \) is airplane mass \([kg]\).

**Pitch angular acceleration per unit angle of attack**
Pitch angular acceleration per unit angle of attack represents acceleration imparted to the airplane as a result of a unit change in angle of attack. It is given in Table 5.1 from [4] and is shown in Equation 69.

\[
M_\alpha = \frac{\bar{q} \cdot S \cdot \bar{c} \cdot C_{ma}}{I_{YY}} \left[ \frac{rad/s^2}{rad} \right]
\]

Where:
\( \bar{q} \) is dynamic pressure [Pa],
\( S \) is wing reference area \([m^2]\),
\( C_{ma} \) is pitch moment derivative with respect to angle of attack \([-\]), calculated in Equation 37,
\( \bar{c} \) is wing MAC \([m]\),
\( I_{YY} \) is airplane moment of inertia about y-axis \([kg \cdot m^2]\).

### 2.3.2 Derivatives per unit rate of change of angle of attack

**Vertical acceleration per unit rate of change of angle of attack**
Vertical acceleration per unit rate of change of angle of attack represents acceleration imparted to the airplane as a result of a unit change in rate of change of angle of attack. It is given in Table 5.1 from [4] and is shown in Equation 70.

\[
M_{\dot{\alpha}} = \frac{\dot{\bar{q}} \cdot S \cdot \dot{\bar{c}} \cdot C_{ma}}{I_{YY}} \left[ \frac{rad/s^3}{rad} \right]
\]

Where:
\( \dot{\bar{q}} \) is dynamic pressure rate \([Pa/s]\),
\( \dot{S} \) is wing reference area rate \([m^2/s]\),
\( \dot{C}_{ma} \) is pitch moment derivative rate with respect to angle of attack \([-\]), calculated in Equation 38,
\( \dot{\bar{c}} \) is wing MAC rate \([m/s]\),
\( I_{YY} \) is airplane moment of inertia about y-axis \([kg \cdot m^2]\).
Pitch angular acceleration per unit rate of change of angle of attack

Pitch angular acceleration per unit rate of change of angle of attack represents acceleration imparted to the airplane as a result of a unit change in angle of attack rate. It is given in Table 5.1 from [4] and is shown in Equation 71.

\[
M_{\dot{\alpha}} = \frac{\bar{q} \cdot S \cdot \bar{c} \cdot C_{m_{\alpha}}}{2 \cdot I_{YY} \cdot U} \left[ \frac{\text{rad/s}^2}{\text{rad/s}} \right]
\]  

(71)

Where:
\(\bar{q}\) is dynamic pressure [Pa],
\(S\) is wing reference area [m²],
\(\bar{c}\) is wing MAC [m],
\(C_{m_{\alpha}}\) is pitch moment derivative with respect to angle of attack rate [−], calculated in Equation 40,
\(m\) is airplane mass [kg],
\(U\) is linear longitudinal velocity (along the x-axis) [m/s].

2.3.3 Derivatives per unit elevator angle

Forward acceleration per unit elevator angle

Forward acceleration per unit elevator angle represents acceleration imparted to the airplane as a result of a unit change in elevator deflection angle. It is given in Table 5.1 from [4] and is shown in Equation 72.

\[
X_{\delta_{E}} = \frac{-\bar{q} \cdot S \cdot C_{D_{\delta_{E}}}}{m} \left[ \frac{\text{m/s}^2}{\text{rad}} \right]
\]  

(72)

Where:
\(\bar{q}\) is dynamic pressure [Pa],
\(S\) is wing reference area [m²],
\(C_{D_{\delta_{E}}}\) is drag coefficient derivative with respect to elevator deflection angle [−].
$C_{D_{\delta E}}$ is drag derivative with respect to elevator deflection angle [−], assumed to be 0, $m$ is airplane mass [kg].

**Vertical acceleration per unit elevator angle**
Vertical acceleration per unit elevator angle represents acceleration imparted to the airplane as a result of a unit change in elevator deflection angle. It is given in Table 5.1 from [4] and is shown in Equation 73.

\[
Z_{\delta E} = \frac{-\bar{q} \cdot S \cdot C_{L_{\delta E}}}{m} \left[ \frac{m/s^2}{\text{rad}} \right]
\] (73)

Where:
- $\bar{q}$ is dynamic pressure [Pa],
- $S$ is wing reference area [$m^2$],
- $C_{L_{\delta E}}$ is lift derivative with respect to elevator deflection angle [−], calculated in Equation 45,
- $m$ is airplane mass [kg].

**Pitch angular acceleration per unit elevator angle**
Pitch angular acceleration per unit elevator angle represents acceleration imparted to the airplane as a result of a unit change in elevator deflection angle. It is given in Table 5.1 from [4] and is shown in Equation 74.

\[
M_{\delta E} = \frac{\bar{q} \cdot S \cdot \bar{c} \cdot C_{m_{\delta E}}}{I_{YY}} \left[ \frac{\text{rad}/s^2}{\text{rad}} \right]
\] (74)

Where:
- $\bar{q}$ is dynamic pressure [Pa],
- $S$ is wing reference area [$m^2$],
- $C_{m_{\delta E}}$ is pitch moment derivative with respect to elevator deflection angle [−], calculated in Equation 46,
- $\bar{c}$ is wing MAC [m],
- $I_{YY}$ is airplane moment of inertia about y-axis [kg · m$^2$].

### 2.3.4 Derivatives per unit pitch rate

**Vertical acceleration per unit pitch rate**
Vertical acceleration per unit pitch rate represents acceleration imparted to the airplane as a result of a unit change in pitch rate. It is given in Table 5.1 from [4] and is shown in Equation 75.

\[
Z_q = \frac{-\bar{q} \cdot S \cdot C_{L_q}}{2 \cdot m \cdot U} \left[ \frac{m/s^2}{\text{rad}/s} \right]
\] (75)
Where:
\( \vec{q} \) is dynamic pressure \([Pa]\),
\( S \) is wing reference area \([m^2]\),
\( \vec{c} \) is wing MAC \([m]\),
\( C_{Lq} \) is lift derivative with respect to pitch rate \([-\]), calculated in Equation 47,
\( m \) is airplane mass \([kg]\),
\( U \) is linear longitudinal velocity (along the x-axis) \([m/s]\).

**Pitch angular acceleration per unit pitch rate**

Pitch angular acceleration per unit pitch rate represents acceleration imparted to the airplane as a result of a unit change in pitch rate. It is given in Table 5.1 from [4] and is shown in Equation 76.

\[
M_q = \frac{\vec{q} \cdot S \cdot \vec{c}^2 \cdot C_{mq}}{2 \cdot I_{YY} \cdot U} \left[ \text{rad/s}^2 \right]
\]  
(76)

Where:
\( \vec{q} \) is dynamic pressure \([Pa]\),
\( S \) is wing reference area \([m^2]\),
\( C_{mq} \) is pitch moment derivative with respect to elevator deflection angle \([-\]), calculated in Equation 52,
\( \vec{c} \) is wing MAC \([m]\),
\( I_{YY} \) is airplane moment of inertia about y-axis \([kg \cdot m^2]\),
\( U \) is linear longitudinal velocity (along the x-axis) \([m/s]\).

**2.3.5 Derivatives per unit change in speed**

**Forward acceleration per unit change in speed**

Forward acceleration per unit change in speed represents acceleration imparted to the airplane as a result of a unit change in forward speed. It is given in Table 5.1 from [4] and is shown in Equation 77.

\[
X_u = \frac{-\vec{q} \cdot S \cdot (C_{D_u} + 2 \cdot C_{D_1})}{m \cdot U} \left[ \text{m/s}^2 \right]
\]  
(77)

Where:
\( \vec{q} \) is dynamic pressure \([Pa]\),
\( S \) is wing reference area \([m^2]\),
\( C_{D_u} \) is drag derivative with respect to forward speed \([-\]), calculated in Equation 57,
\( C_{D_1} \) is drag coefficient in steady state \([-\]),
\( m \) is airplane mass \([kg]\),
\( U \) is linear longitudinal velocity (along the x-axis) \([m/s]\).

**Vertical acceleration per unit change in speed**

Vertical acceleration per unit change in speed represents acceleration imparted to the airplane as a result of a unit change in speed. It is given in Table 5.1 from [4]
and is shown in Equation 78:

$$\begin{align*}
Z_u &= -\bar{q} \cdot S \· \left( C_{L_u} + 2 \cdot C_{L1} \right) \frac{[m/s^2]}{m \cdot U} \\
\end{align*}$$

(78)

Where:
- $\bar{q}$ is dynamic pressure [Pa],
- $S$ is wing reference area [$m^2$],
- $C_{L_u}$ is lift derivative with respect to forward speed [-], calculated in Equation 56,
- $C_{L1}$ is lift coefficient in steady state [-],
- $m$ is airplane mass [kg],
- $U$ is linear longitudinal velocity (along the x-axis) [m/s].

**Pitch angular acceleration per unit change in speed**

Pitch angular acceleration per unit change in speed represents acceleration imparted to the airplane as a result of a unit change in speed. It is given in Table 5.1 from [4] and is shown in Equation 79:

$$\begin{align*}
M_u &= \bar{q} \cdot S \· \bar{c} \· \left( C_{m_u} + 2 \cdot C_{m1} \right) \frac{[rad/s^2]}{I_{YY} \cdot U} \\
\end{align*}$$

(79)

Where:
- $\bar{q}$ is dynamic pressure [Pa],
- $S$ is wing reference area [$m^2$],
- $\bar{c}$ is wing MAC [m],
- $C_{m_u}$ is pitch moment derivative with respect to change in speed [-], calculated in Equation 58,
- $C_{m1}$ is pitch moment coefficient in steady state [-],
- $\bar{c}$ is wing MAC [m],
- $I_{YY}$ is airplane moment of inertia about y-axis [kg $\cdot m^2$],
- $U$ is linear longitudinal velocity (along the x-axis) [m/s].

### 2.3.6 Derivatives depending on thrust

**Forward acceleration per unit change in speed due to thrust**

Forward acceleration per unit change in speed due to thrust represents acceleration imparted to the airplane as a result of a unit change in forward speed caused by thrust. It is given in Table 5.1 from [4] and is shown in Equation 80:

$$\begin{align*}
X_{Tu} &= \bar{q} \cdot S \· \left( C_{Tu} + 2 \cdot C_{T1} \right) \frac{[m/s^2]}{m \cdot U} \\
\end{align*}$$

(80)

Where:
- $\bar{q}$ is dynamic pressure [Pa],
- $S$ is wing reference area [$m^2$],
$C_{Tu}$ is variation of airplane pitching moment coefficient due to thrust with dimensionless speed $[-]$, calculated in Equation 62.

$C_{T_1}$ is thrust coefficient in steady state $[-]$, $m$ is airplane mass $[kg]$, $U$ is linear longitudinal velocity (along the x-axis) $[m/s]$.

### Pitch angular acceleration per unit change in speed due to thrust

Pitch angular acceleration per unit change in speed due to thrust represents acceleration imparted to the airplane as a result of a unit change in speed due to thrust. It is given in Table 5.1 from [4] and is shown in [Equation 81](81):

$$M_{Tu} = \frac{\bar{q} \cdot S \cdot \bar{c} \cdot (C_{m_{Tu}} + 2 \cdot C_{m_{T_1}})}{I_{YY} \cdot U} \left[ \frac{rad/s^2}{m/s} \right]$$

Where:
- $\bar{q}$ is dynamic pressure $[Pa]$,
- $S$ is wing reference area $[m^2]$,
- $\bar{c}$ is wing MAC $[m]$,
- $C_{m_{Tu}}$ is pitch moment derivative with respect to change in speed due to thrust $[-]$, calculated in Equation 65.
- $C_{m_{T_1}}$ is pitch moment coefficient due to thrust in steady state $[-]$, $I_{YY}$ is airplane moment of inertia about y-axis $[kg \cdot m^2]$, $U$ is linear longitudinal velocity (along the x-axis) $[m/s]$.

### Pitch angular acceleration per unit change in angle of attack due to thrust

Pitch angular acceleration per unit change in angle of attack due to thrust represents acceleration imparted to the airplane as a result of a unit change in angle of attack due to thrust. It is given in Table 5.1 from [4] and is shown in [Equation 82](82):

$$M_{Tu} = \frac{\bar{q} \cdot S \cdot \bar{c} \cdot C_{m_{Tu}}}{I_{YY}} \left[ \frac{rad/s^2}{rad} \right]$$

Where:
- $\bar{q}$ is dynamic pressure $[Pa]$,
- $S$ is wing reference area $[m^2]$,
- $\bar{c}$ is wing MAC $[m]$,
- $C_{m_{Tu}}$ is pitch moment derivative with respect to change in speed due to thrust $[-]$, calculated in Equation 59.
- $I_{YY}$ is airplane moment of inertia about y-axis $[kg \cdot m^2]$.

### 2.4 Stability

In order to analyse the stability of an aircraft, first the definitions of different types of stability and its criteria should be introduced. According Dr. Jan Roskam [4]:
“**Static stability** is defined as the tendency of an airplane to develop forces or moments which directly oppose an instantaneous perturbation of a motion variable from a steady-state flight condition.”

Static stability is best illustrated by a picture of equilibria, as shown in Figure 3.

![Figure 3: Static stability equilibria (Source: [4])](image)

“A **static stability criterion** is defined as a rule by which steady state flight conditions are separated into categories of stable, unstable or neutrally stable.”

Static stability criteria are further described in **subsubsection 2.4.1**.

“**Dynamic stability** is defined as the tendency of the amplitudes of the perturbed motion of an airplane to decrease to zero or to values corresponding to a new steady state at some time after the cause of the disturbance stopped.”

An airplane is called dynamically stable if when after a perturbation affects the airplane, it comes back to the initial state, or a state not significantly different from it, in some time. If an aircraft is neutrally dynamically stable, after a perturbation, causing the airplane’s movement, appears, the movement is not damped nor amplified. For an unstable aircraft, the movement caused by an instantaneous perturbation gets magnified and the airplane does not come back to the initial state.

“A **dynamic stability criterion** is defined as a rule by which perturbed motions of airplanes are separated into the categories of stable, neutrally stable and unstable.”

Dynamic stability criteria are further described in **subsubsection 2.4.2**.

### 2.4.1 Static Stability Criteria

The static stability criteria are extracted from [4]. The following arbitrary rules have been applied in order to establish the correct combinations of the forces, moments and perturbations creating the criteria. The rules were presented by Dr. Jan
Roskam in [4].

“1. Linear velocity perturbations are initially opposed only by forces. 
2. Angular velocity perturbations are initially opposed only by moments. 
3. Angle of sideslip and angle of attack perturbations obtained by interpreting the velocity perturbations \( v \) and \( w \) as \( \beta = v/U_1 \) and \( \alpha = w/U_1 \) are initially opposed only by moments.”

**Forward speed stability**
This criterion is very desirable to be fulfilled. It expresses the airplane’s tendency to return to the equilibrium speed after a speed perturbation, either forwards or backwards. That characteristic deems very beneficial especially during approach flight phase. The criterion is expressed by Eq.(4.5) as shown in Equation 83

\[ C_{T_{\alpha u}} - C_{D_{\alpha}} < 0 \] (83)

**Vertical speed stability**
Knowing the shape of the lift-curve slope, it is evident that this criterion given by Eq.(4.13), shown in Equation 84 is fulfilled for the angles of attack lower than the stall angle of attack.

\[ C_{L_{\alpha}} > 0 \] (84)

**Angle of attack stability**
An aircraft fulfilling this criterion has the ability to turn into the direction of the new relative wind as a result of a perturbation in angle of attack. That phenomenon is called ‘weathercocking’, as the airplane acts like a weathercock. That ability to some some extent is desired for an airplane. The requirement is satisfied if the aerodynamic centre is located behind the centre of gravity. The criterion is given by Eq.(4.18) as presented in Equation 85

\[ C_{m_{\alpha}} + C_{m_{T_{\alpha}}} < 0 \] (85)

It is important to point out that, when the sum of \( C_{m_{\alpha}} \) and \( C_{m_{T_{\alpha}}} \) reaches zero, nothing catastrophic happens. Most of the airplanes are still controllable, but the pilot’s input needs to be more significant.

**Pitch rate stability**
The fulfilment of this criterion means that the aircraft generates a pitching moment to oppose the increasing pitching velocity caused by a change in pitch rate. It is given by Eq.(4.30) and expressed by Equation 86

\[ C_{m_{\alpha}} < 0 \] (86)

**Effect of forward speed on pitching moment**
This requirement signifies that when a forward velocity perturbation happens, the airplanes reacts by nose pitching up (positive increase in pitching moment). That results in lowering the forward speed, making the aircraft statically stable. The
criterion is given by Eq.(4.37) as shown in Equation 87.

\[ C_{m_u} + C_{m_T} > 0 \]  

(87)

2.4.2 Dynamic Stability Criteria

According to Dr. Jan Roskam \[4\] for any system which can be modelled with at least one linear differential equation with constant coefficients the following stability criteria are applicable.

“1. A linear system is stable if and only if the real parts of the roots of the characteristic equation of the system are negative.
2. A linear system is convergent (stable) if the roots of the characteristic equation of the system are real and negative.
3. A linear system is divergent (unstable) if the roots of the characteristic equation of the system are real and positive.
4. A linear system is oscillatory convergent (stable) if the real parts of the roots of the characteristic equation of the system are negative.
5. A linear system is oscillatory divergent (unstable) if the real parts of the roots of the characteristic equation of the system are positive.
6. A linear system is neutrally stable if one of the roots of the characteristic equation of the system is zero or if the real parts of the roots of the characteristic equation of the system is zero.”

In order to check if an airplane fulfils the dynamic stability criteria, the characteristic equation is needed. To obtain that, Laplace transform for zero initial conditions is performed upon perturbed longitudinal equations of motion (Equation 22). The result of that operation in matrix form is given by Eq.(5.32) from [4], as shown in Equation 88.

\[
\begin{bmatrix}
(s - X_u - X_{T_u}) & -X_\alpha & g \cos \theta_1 \\
-Z_u & \{s(U_1 - Z_\alpha) - Z_\alpha\} & \{- (Z_q + U_1) s + g \sin \theta_1\} \\
-(M_u + M_{T_u}) & -(M_\alpha s + M_\alpha + M_{T_u}) & (s^2 - M_q s)
\end{bmatrix}
\begin{bmatrix}
u(s) \\
\delta_e(s) \\
\alpha(s) \\
\theta(s)
\end{bmatrix}
= \begin{bmatrix}
X_{\delta_e} \\
Z_{\delta_e} \\
M_{\delta_e}
\end{bmatrix}
\]

(88)

Laplace transform changes the equation of motion from time to frequency domain. Through that transfer functions \( \frac{u(s)}{\delta_e(s)} \), \( \frac{\alpha(s)}{\delta_e(s)} \) and \( \frac{\theta(s)}{\delta_e(s)} \) are obtained. Each transfer function can be written as a ratio of polynomials, as shown in Equation 89, where each function \( f(s) \) is replaced by \( u(s) \), \( \alpha(s) \) or \( \theta(s) \).
\[ \frac{f(s)}{\delta_e(s)} - \frac{N_f}{D_1} = \frac{A_f s^3 + B_f s^2 + C_f s + D_f}{A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1} \]  

Equation (89)

All of the transfer functions have the same denominator. If the denominator is set to be equal to zero, the resulting equation is called the characteristic equation given by Eq.(5.43) from [4], shown in Equation 90

\[ A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1 = 0 \]  

Equation (90)

The longitudinal airplane transfer functions are given by Eq.(5.33), (5.36) and (5.38) from [4], as presented in Equations 91, 94 and 96

\[
\begin{bmatrix}
X_{\delta_e} & -X_\alpha & g \cos \theta_1 \\
Z_{\delta_e} & \{s (U_1 - Z_\alpha) - Z_\alpha\} & \{(Z_q + U_1) s + g \sin \theta_1\} \\
M_{\delta_e} & \{-M_\alpha s + M_\alpha + M T_u\} & \{s^2 - M_\alpha s\}
\end{bmatrix}
= \frac{N_u}{D_1}
\]

Equation (91)

Where denominator is given by Eq.(5.34) from [4], represented by Equation 92

\[ D_1 = A_1 s^4 + B_1 s^3 + C_1 s^2 + D_1 s + E_1 \]  

Equation (92)

Where:

\[
\begin{align*}
A_1 &= U_1 - Z_\alpha \\
B_1 &= - (U_1 - Z_\alpha) (X_u + X_{T_u} + M_q) - Z_\alpha - M_\alpha \cdot (U_1 + Z_q) \\
C_1 &= (X_u + X_{T_u}) \{M_q (U_1 - Z_\alpha) + Z_\alpha + M_\alpha \cdot (U_1 + Z_q)\} + M_q Z_\alpha + Z_u X_\alpha + M_\alpha \cdot g \sin \theta_1 - (M_\alpha + M T_u) (U_1 + Z_q) \\
D_1 &= g \sin \theta_1 \{M_\alpha + M T_u - M_\alpha \cdot (X_u + X_{T_u})\} + \\
&\quad + g \cos \theta_1 \{Z_u M_\alpha + (M_\alpha + M T_u) (U_1 - Z_\alpha)\} + \\
&\quad + (M_\alpha + M T_u) \{-X_\alpha (U_1 + Z_q)\} + Z_u X_\alpha M_q + \\
&\quad + (X_u + X_{T_u}) \{(M_\alpha + M T_u) (U_1 + Z_q) - M_q Z_\alpha\} \\
E_1 &= g \cos \theta_1 \{(M_\alpha + M T_u) Z_u - Z_\alpha (M_\alpha + M T_u)\} + \\
&\quad + g \sin \theta_1 \{(M_\alpha + M T_u) X_\alpha - (X_u + X_{T_u}) (M_\alpha + M T_u)\}
\end{align*}
\]

And nominator is given by Eq.(5.35) from [4], as shown in Equation 93

\[ N_u = A_u s^3 + B_u s^2 + C_u s + D_u \]  

Equation (93)

Where:

\[
\begin{align*}
A_u &= X_{\delta_e} (U_1 - Z_\alpha) \\
B_u &= -X_{\delta_e} \{(U_1 - Z_\alpha) M_q + Z_\alpha + M_\alpha (U_1 + Z_q) + Z_{\delta_e} X_\alpha\} \\
C_u &= X_{\delta_e} \{M_q Z_\alpha + M_\alpha \cdot g \sin \theta_1 - (M_\alpha + M T_u) (U_1 + Z_q)\} + \\
&\quad + Z_{\delta_e} \{-M_\alpha \cdot g \cos \theta_1 - X_\alpha M_q\} + M_{\delta_e} \{X_\alpha (U_1 + Z_q) - (U_1 - Z_\alpha) g \cos \theta_1\} \\
D_u &= X_{\delta_e} (M_\alpha + M T_u) g \sin \theta_1 - Z_{\delta_e} M_\alpha g \cos \theta_1 + M_{\delta_e} (Z_\alpha g \cos \theta_1 - X_\alpha g \sin \theta_1)
\end{align*}
\]
After experiencing a disturbance, short period motion is also an oscillatory motion in which an aircraft moves in an oscillatory manner, changing its pitch angle, with a longer period motions can be introduced to the analysis. Phugoid mode is a motion in which the lift coefficient decreases with a time constant of \( T \).

Having established the transfer functions in the previous section, phugoid and short period motions can be introduced to the analysis. Phugoid mode is a motion in which the lift coefficient decreases with a time constant of \( T \).

Where:

\[
\frac{\alpha(s)}{\delta_c(s)} = \frac{(s - X_u - X_{T_a}) X_{\delta_c} \ g \cos \theta_1}{-Z_u \ Z_{\delta_c} \ \{-(Z_q + U_1) s + g \sin \theta_1\}} \]

\[
\frac{\dot{\theta}(s)}{\delta_c(s)} = \frac{(s - X_u - X_{T_a}) - X_{\alpha} \ \{s (U_1 - Z_{\dot{a}}) - Z_{\alpha}\} \ g \cos \theta_1}{-Z_u \ \{-M_a (X_u + X_{T_a})\} \ \{-(Z_q + U_1) s + g \sin \theta_1\}} \]

\[
N_\alpha = A_\alpha s^3 + B_\alpha s^2 + C_\alpha s + D_\alpha \tag{95}
\]

\[
N_\theta = A_\theta s^2 + B_\theta s + C_\theta \tag{97}
\]

The nominator of the second transfer function is given by Eq.(5.37) from [1], as shown in Equation 95. The denominator is represented in Equation 92.

Where:

\[
A_\alpha = \text{Z}_{\delta_c} \ M_{\dot{a}} + M_{\delta_c} \ (U_1 - Z_{\dot{a}})
\]

\[
B_\theta = X_{\delta_c} \ \{\text{Z}_{\delta_c} \ M_{\dot{a}} + (U_1 - Z_{\dot{a}}) \ (M_a + M_{T_a})\} + Z_{\delta_c} \ \{(M_a + M_{T_a}) - M_{\dot{a}} \ (X_u + X_{T_a})\} + M_{\delta_c} \ \{-Z_{\alpha} - (U_1 - Z_{\dot{a}}) \ (X_u + X_{T_a})\}
\]

\[
C_\theta = X_{\delta_c} \ \{(M_a + M_{T_a}) \ Z_{\alpha} - Z_{\alpha} \ (M_a + M_{T_a})\} + Z_{\delta_c} \ \{-M_a (X_u + X_{T_a}) + X_{\alpha} \ (M_a + M_{T_a})\} + M_{\delta_c} \ \{Z_{\alpha} (X_u + X_{T_a}) - X_{\alpha} Z_{\alpha}\}
\]

### 2.5 Aircraft dynamic modes

Having established the transfer functions in the previous section, phugoid and short period motions can be introduced to the analysis. Phugoid mode is a motion in which an aircraft moves in an oscillatory manner, changing its pitch angle, with a longer period, after experiencing a disturbance. Short period motion is also an oscillatory motion.
motion, but it is heavily damped and has a short period. Two important parameters characterising both of these motions are: natural frequency and damping ratio.

Assuming that phugoid motion appears when the angle of attack is approximately constant, Equations 98 and 99 describe the natural frequency and the damping ratio, respectively, given by Eq.(5.70) and Eq.(5.71) from [4].

\[
\omega_{n_{ph}} = \sqrt{-\frac{\gamma Z_u}{U_1}} \text{[rad/s]} \quad (98)
\]

Where:

- \( g \) is gravitational acceleration \([m/s^2]\),
- \( Z_u \) is vertical acceleration per unit change in speed \([m/s^2]\), given by Equation 78,
- \( U_1 \) is linear longitudinal velocity (along the x-axis) at steady state \([m/s]\).

\[
\zeta_{ph} = -\frac{X_u}{2\omega_{n_{ph}}} \quad (99)
\]

Where:

- \( X_u \) is forward acceleration per unit change in speed \([m/s^2]\), given by Equation 77,
- \( \omega_{n_{ph}} \) is natural frequency of phugoid motion \([\text{rad/s}]\), given by Equation 98.

For a short period motion, an assumption that a short period motion takes place at a relatively constant velocity can be made. Then natural frequency and the damping ratio can be given by Eq.(5.61) and Eq.(5.62) from [4], respectively, as shown in Equations 100 and 101.

\[
\omega_{n_{sp}} = \sqrt{\frac{Z_\alpha M_q}{U_1} - M_\alpha} \quad (100)
\]

Where:

- \( Z_\alpha \) is vertical acceleration per unit change in angle of attack \([m/s^2]\), given by Equation 68,
- \( M_q \) is pitch angular acceleration per unit change in pitch rate \([\text{rad/s}^2]\), given by Equation 76,
- \( M_\alpha \) is pitch angular acceleration per unit change in angle of attack \([\text{rad/s}]\), given by Equation 69,
- \( U_1 \) is linear longitudinal velocity (along the x-axis) at steady state \([m/s]\).

\[
\zeta_{sp} = -\left(\frac{M_q + Z_u}{U_1} + M_\dot{\alpha}\right) \quad (101)
\]

Where:

- \( M_\dot{\alpha} \) is pitch angular acceleration per unit rate of change in angle of attack \([\text{rad/s}^2]\).
given by Equation 71. \( \omega_{nsp} \) is natural frequency of short period motion [rad/s], given by Equation 100.

## 2.6 Software description

Pacelab Aircraft Preliminary Design or Pacelab APD [1] from PACE Aerospace and Information Technology - a TXT company, is a software widely used. The key features of the program are, among others, robust creation of 3D aircraft models, broad library of analysis methods or numeric optimisation possibilities.

The version used in this thesis is Pacelab APD 8.1. Utilised licenses are node-locked, valid only for a certain computer. The student licenses do not warrant access to all of the functionalities and properties, e.g. values of airplanes’ moments of inertia which are not accessible and are implemented as inputs.

Pacelab APD consists of three applications: Pacelab Engineering Workbench (EWB), Pacelab Knowledge Designer (KD) and Pacelab Suite Data Management server. The last app is optional for the usage of the software and was not used in this project. Graphic representation of Pacelab APD system structure is shown in Figure 4.

![Figure 4: Pacelab APD system architecture, taken from [8].](image)

### 2.6.1 Engineering Workbench

Pacelab APD Engineering Workbench (EWB) is the application enabling the user to carry out aircraft design strategies, analyse and optimise created models. It is also a space where the engineering components, created in Knowledge Designer, can
be visualised and tested. All of these features can be performed on a model of an aircraft which can be either taken from a library of preinstalled airplanes or it can be designed by the user.

The opening window of EWB is shown in Figure 5. The airplane can be chosen from library in the Template box. Choosing Empty Project allows the user to design a new aircraft and scrolling down shows a wide range of airplanes ready to work with.

Figure 5: Pacelab APD Engineering Workbench opening layout

Figure 6 shows how the EWB app window looks like after choosing an existing template, in this case it is a model of Boeing 747-400. The project opens at Home ribbon tab and the main window shows 3D view of the aircraft. Structure View on the left hand side, marked with number 5, contains a tree structure of the whole project. All of the created charts, reports or studies can be accessed through there. The view also shows the engineering components created in KD. Red marks visible next to the component symbols inform that the system has not been solved yet or changes influencing these components were applied.

Buttons on the Home ribbon tab in groups Aircraft Configuration, Aircraft Sizing and New are used to create or modify an aircraft design. In order to solve the project, the Solve button, in the Solve group, marked with number 1 on Figure 6 has to be pressed. While solving or calculating anything, EWB prints messages in Output View, marked with number 6. The printed informational messages are black, warnings are blue and if an error appears, it is displayed in red.
If the need to analyse the equations system guarding the project arises, clicking the **Analyze** button in **Solve** group, marked with number 2, provides that feature. Pacelab APD is also equipped with an optimisation tool, accessible through **Optimize** button in **Explore** group, marked with number 4. To gather data for multiple input values a study can be created by clicking **New Study** button in **Explore** group, marked with number 3.

Figure 7 shows the layout of Pacelab APD EWB after solving the project. The **View** ribbon tab is also presented there. The tab contains all of the views able to be displayed. Each of the views can change their locations and hide and appear as the need arises. In the figure, B747 is presented in the three sided **Drawing View**. In **Structure View** on the left side, the red "unsolved" marks are not longer visible. Instead, the Engineering Object (EO) Concept **Aerodynamics**, marked with number 1, was expanded, and **High Speed** EO, marked with number 2, was selected. Properties of the selected EO are displayed in the **Properties View**, on the right hand side, marked with a red rectangle.
In the View ribbon tab in the Unit group units can be converted and display precision can be changed with the use of buttons marked with numbers 3 and 4. A very often utilised view is Parameter view, marked with number 5 on Figure 7. This feature allows the user to see all of the parameters in the considered system, as well as to search for a specific one.

A very convenient way of storing data in Pacelab APD are Multidimensional Data Tables. They can be accessible both through EWB and KD. In EWB the MDTs containing aerodynamic data are found in the Aerodynamics ribbon tab, marked with a red rectangle in Figure 8. A drag polar data table for high speed is also shown in the figure.

### 2.6.2 Knowledge Designer

Knowledge Designer is Pacelab APD application enabling the user to create knowledge components using Methods and Parameters which can be either already existing or newly created. KD uses C# language to program the workspace. The structure of the workspace contains of three types of projects: Pacelab EO Concept Project, Pacelab FO Project and Pacelab Smart Formula Project. In this thesis only the two first project types were used.

When starting the Knowledge Designer application a workspace has to be either opened or created. A workspace contains of multiple projects of each type, in Figure 9 a Functional Object (FO) is shown. Each individual project has its own references. With number 1 a workspace in which the work is performed is marked. In order to check project’s assembly to the repository the button marked with number 7 should be pressed and to check all of the projects to the repositories, the button marked with number 6 should be selected.
Functional Object (FO) is a knowledge component consisting of functions in C# language. The functions can be used internally in the same FO, another FO or an EO. Each function has its own name, description and output type, as marked with red rectangle in Figure 9. A function is marked with number 4 in the figure. Input data used by a function is called Arguments, marked with number 5 in the figure. Each argument has a name and a type. Values of the arguments can be assigned in other Engineering Objects (EOs) or FOs, while employing the function, making sure the type of the argument is matching. FO project items are stored in FO projects in the workspace.

Engineering Object (EO) is an object representing a parametric description of the
physical engineering object which is to be designed, analysed or optimised. Each EO consists of parameters, ports, relations between parameters, formulas, constraints and methods. There are different types of Engineering Objects, all stored in EO projects: EO Concept, EO Category, EO Port Type, EO Port Graph Type. In this thesis only concepts and categories were used. Geometries both in 2D and 3D can be defined for EOs if necessary.

In Figure 10 with number 1 a space where MDTs are stored is marked. Number 2 indicates an EO Concept, 3 and 4 Parameters and Formulas and Constraints, respectively. Each parameter has a name and a type, a display category can also be specified. It defines under which header the parameter appears in the EWB.

A parameter can be used as both input and output, depending on what value was assigned to it. If a parameter does not have an appointed formula it can be treated as an output. To assign a value to a parameter it has to have a corresponding formula. The formula can determine the value or it can call on a method or a function defining the parameter. In that case it is important that both the parameter and the method or the function have the same output type.

Methods used within an EO may be longer than a few lines of code as it is for a formula. A method is marked in Figure 10 with number 5. A method can also navigate for a component of the considered system. To be able to do that it uses an argument of a type INavigationContext, said argument is marked as number 6 in the figure.

2.6.3 Workflow

Pacelab APD is a software used in aircraft conceptual design. It contains two main applications, Engineering Workbench and Knowledge Designer. Engineering Workbench (EWB) is used mostly in the actual aircraft design phase, whereas Knowledge Designer (KD) enables the user to create new parameters and analysis methods. This thesis focuses mostly on the latter - creating a new flight dynamics chapter.

The first step in creating the flight dynamics chapter was implementing dimensionless and dimensional coefficients equations, described in sections 2.2 and 2.3 respectively. The process of equations application is shown in a schematic way in Figure 11.
In order for a new parameter to appear in EWB, a parameter, with its properties, has to be created in the corresponding Engineering Object inside Knowledge Designer. The parameter can be assigned a value through a formula or if the formula is not created, the parameter it becomes an input. The parameter formula can contain either a short C# code inside it or it can call on either a function from any Functional Object or any method from the same or different EO. Methods can call on another method within the same EO, a method from another EO or a function. Each method has properties like name or return type assigned to it. It is important that the output types of the parameters correspond to output types of either methods or functions.
3 Methodology and Software Implementation

In this section, the process that is followed to obtain the different components in Pacelab APD is presented for the longitudinal derivatives. The different software that aided the obtention of the derivatives and their validation will be discussed.

An example of the outline of the FlightMode chapter after merge of the two teams’ work is shown in Figure 12.

![Figure 12: Outline of the FlightMode Chapter.](image)

3.1 Approach

To assure the proper development of this project the tasks were divided into smaller parts. The start of the project was marked by the familiarisation with the Pacelab APD software, with tutorials instructed by the Pacelab Support team. In parallel the derivatives were concentrated into an Excel file along the sub-equations.

Due to the lack of previous use of the Pacelab APD software, an initial conversion of the formulas to C# code was achieved using an intermediate program, while familiarisation with Pacelab was in progress.

The basic steps that were followed to achieve the creation of the Flight Mechanics chapter are explained below. More details about the implementation and the naming will follow in section 3.1.1. The steps and conditions to obtain the point performance through the Study Case is further explained in section 3.1.2.
The first step consisted of creating a Functional Object (FO) where all the equations and figures taken from the literature would be implemented. Subsequently, an Engineering Object (EO) concept project item was created within the StabilityAndControl EO project, where all the input and output parameters were to be implemented alongside the methods.

Once all parameters were implemented, the necessary method selection EO categories were created to make the coefficients available as inputs or calculated by the implemented equations.

With the coefficients available, the stability and control criteria were programmed. The five static stability criteria were entered as a series of conditions returning the appropriate Fulfilled or Unfulfilled expression. The dynamic stability criterion depends on the roots of the characteristic equation, from the equations of motions, and returns the expression Stable or Unstable accompanied by the type of stability.

### 3.1.1 Implementation Pacelab APD KD

The different steps taken to create the FO for the longitudinal derivatives are described below along the procedure for the longitudinal aerodynamic coefficients under the Stability And Control EO. Thereafter, the implementation for the method selection and the stability criteria is shown. The outline of the work is shown in Figure 13.

![Figure 13: Outline of the StabilityAndControl EO concept project.](image-url)

**Derivatives’ Functional Object**
A functional object is created in the APD workspace and it is named *AerodynamicDerivativesMethods_ROSKAM*. The FO contains an FO project item, called *LongitudinalDimensionlessDerivatives*. It has the functions that are needed to obtain the dimensionless derivatives and the graphs taken from the book. Each function is named with the coefficient symbol used in the literature, followed by the acronym of the book they were taken from and finally *Eq* with the equation number:

\[
\text{NAME}_\text{BOOK}_\text{ACRONYM}_\text{EqEquationNumber}.\]

Book acronyms are stated as follows: AFDAFC1 - [4], AD6 - [5], DFSC - [6].

Each function has a certain amount of arguments that need to be stated along with their type, *ParameterType*, which is decided by the person entering the argument. If the function uses any other FO or EO from the APD workspace then their *Namespace* needs to be added and the additional Dynamic Link Libraries (DLL) need to be referenced under *References*. It is possible to use functions from the same or different FO. An example of the derivative \(C_{D_{\alpha}}\) is shown in Figure 14. In this example the book used is [4] and the only additional reference is *AerodynamicMethods_DATCOM.dll*, as the Oswald Efficiency Factor is taken from the FO named *AerodynamicMethods_DATCOM*.

![Figure 14: The structure of the APD workspace (APDComplete) and the Functional Object (AerodynamicDerivativesMethods_ROSKAM).](image)

Inside the function, the arguments are turned into a type *double* and the equation is written in C# language. The *Math.* shortcut cannot be used. A description is added to each individual function and in comments the description of the arguments, pages, equation numbers, book title, author and date of publication are given as to facilitate the readability for third parties.

**Derivatives’ Engineering Objects**

The new EO objects are created in the *StabilityAndControl* engineering object. The first implementation takes place within an EO concept named *LongitudinalAerodynamicCoefficient*. It will later be used inside the *FlightMode* EO category. An EO category is created to share parameters, methods etc. between multiple EO concepts or other categories. The coefficients do not need to inherit any tools from other EOs,
thus a EO concept is used. No geometry is needed, consequently the EO family is changed to Without Geometry.

The EO concept’s tools that are modified are the Namespaces Used, Parameters, Formulas and Constraints, and Methods. The namespaces work in the same way as for the FO. The parameters are the variables that the user will have access to and be able to modify into inputs if needed. The result of a parameter comes from a formula or constraint that has been coded. If that formula or constraint is longer than one line, containing multiple operations, there are two options. The first is to create a method and call the method in the formula, it is the recommended one from Pacelab APD support. The other option is a modification in the code that allows multiple lines to be taken into consideration as one, as shown below.

```csharp
(new [] { 0 }). Select (x =>
{
    // Operations, Equations etc.
}).First()
```

As the first approach was used the EO concept contains methods, where the different arguments are searched within the various EO. The function for each coefficient is called upon. Arguments need to be added in the methods. Depending on what is called and searched for the argument can take multiple forms. The most used ar-
argument in this case is `navigationContext` of type `INavigationContext`. That specific argument references a class, `navigator` to the interface. More details are given in [9] concerning this class.

To keep the names as coherent as possible the methods are named with `Calc_` followed by the coefficient which they calculate as shown below.

```
Calc_CoefficientName
```

When a formula is added to a parameter, the automatic name is the name of the parameter followed by `Formula`. An example is seen in Figure 16 with the coefficient $C_{L\alpha}$.

![Figure 16: Structure of Formulas and Constraints and Methods.](image)

**Method Selection**

The process of creating a method selection for a parameter is described step by step in [10]. In short, a new EO category was created for each coefficient that could be replaced by an input. Inside each of the categories a parameter and one method were created. The parameter properties were altered as seen in Figure 17. The book name and equation number clearly stated, of type Method Selection. The method within the EO category contains one line of code and a singular argument of `INavigationContext` type. The parameters that were changed to selective variables no longer have a formula connected to them. The unnecessary formulas were thus deleted.

![Figure 17: Properties View of a parameter within a Method Selection EO Category.](image)

**Damping and Frequency**
The damping $\zeta_{sp/ph}$ and frequency $\omega_{sp/ph}$ for the short period and phugoid were implemented the same way as the derivatives’ methods. The naming convention changed to Calc_ followed by Phugoid or ShortPeriod, followed by either Damping or Frequency.

\[ \text{Calc}_\text{ModeTypeVariableType} \]

### 3.1.2 Use of Pacelab APD EWB

Pacelab APD EWB was used for searching certain parameters and to execute a study case.

The study case used the given limits on the Altitude and Mach Number to calculate the different derivatives and variables at the desired points.

### 3.2 Tools Used

A variety of different tools were used in the duration of this project and are presented below. The instruments were used to either facilitate the workload or as validation methods.

#### 3.2.1 Visual Studio

Visual Studio is a software created by Microsoft. It is used to create computer programs. It is an integrated development environment.

Visual Studio was downloaded from [11] and used mainly at the beginning of the project to translate the equations into C# language and to validate the results with the literature.

#### 3.2.2 Visual Studio Code

Visual Studio Code is a source-code editor created by Microsoft and can be used with a variety of languages, including C#.

Visual Studio Code [12], was used in regular intervals to validate the implementation of the code and the search of dependencies for the methods.
Instructions on the use of the debugger are found within the KD file found within the DataModels file and can be seen in Appendix A.

### 3.2.3 OpenVSP

OpenVSP [13] is an open source parametric aircraft geometry tool originally developed by NASA. The version of the program used in this thesis is *OpenVSP 3.33.1 64-bit Python 3.9*, it was downloaded from [14]. The software runs in C++ and is used to model 3D aircraft and perform analysis on said models. Running the calculations is a simple process if the geometry is already created. However some problems may occur with specific geometrical shapes. Consequently the models used for calculations are somehow simplified. The used airplane geometry is shown in Figure 18.

The steps needed to get the flight dynamics results from OpenVSP are: degenerating the geometry and running *VSPAERO* from Analysis tab. The method used in this software to obtain the needed data is Vortex Lattice Method so it is important to make sure that that is the selected option in *VSPAERO*. After setting the selected calculation parameters the results are ready to present. The results can be presented as a plot of any two parameters against each other or can be exported in CSV format.

![Figure 18: Airplane geometry for B747.](image)

The geometry presented in Figure 18 offers a very simplified shape of the B747 aircraft. The engines and control surfaces are not included. It may lead to not precise or even inaccurate calculations results as the engines placement influences the airflow around the wing and the control surfaces deflections cannot be modelled.

### 3.2.4 DATCOM+ / MATLAB AID App

DATCOM is the United States Air Force Data Compendium from the 1960’s. Digital Datcom was the first digitisation of the equations from the compendium and DATCOM+ is a the extended computer program that made the software more accessible. Since 2010 DATCOM+ and DATCOM+Pro are supported by Holy Cows,
The software can be downloaded from their website. After downloading, there are three steps to follow in order to get DATCOM running, two of which are also stated on the website. First, all .exe files are to be run in compatibility mode. The windows used was Windows Vista. The steps are shown in Figure 19 as written on the website.

1. After installing the Datcom+ package, Press Windows key + r. Type in “SystemPropertiesAdvanced.exe”.
2. Click on the Environment Variables box.
3. Change the environment variable for DATCOMROOT, removing the leading and trailing double-quote. Save and get out of this.
4. In the Datcom\bin directory on your desktop, edit the Datcom.bat file. On lines 14, 15, and 16, remove the double-quotes (leading and trailing).
5. You should be able to run the programs now.

**Figure 19: Steps to follow before trying to open DATCOM, source: [15]**

The aircraft B737 is one of the models in the pre-installed examples for DATCOM+.

AID stands for Aircraft Intuitive Design. It is an academic MATLAB application intended to assist aircraft design [16]. AID contains different methods for advanced analysis, including Digital DATCOM. The version used is Version 1.2.0.0. One of the pre-loaded models in its library is the B747-400 that can be seen in Figure 20. As Digital DATCOM is an older version of DATCOM+, certain modifications needed to be made for the file to run.

**Figure 20: B747 model taken from the MATLAB AID library.**

### 3.3 Testing and Verification

Testing and validation of the obtained results was performed with the use of literature data [4], [2] and other software, namely OpenVSP [13] and DATCOM+ [15].

Since finding reliable commercial airplanes models is not an easy task, the models were deemed acceptable even if they featured different version of B747 or if the model was not fully complete. Accepting the aforementioned facilitations, the following aircraft models in different sources were approved:
• **Pacelab APD** - Boeing 747-400, available as a template in the program’s library,

• **Literature data** [4], [2] - Boeing 747-200, with data calculated for three specific states of flight,

• **OpenVSP** - Boeing 747 (version not specified), downloaded from OpenVSP-Connect Hangar [17], shown in Figure 18,

• **DATCOM**+ - Boeing 747, modelled on version B747-400, found in the library of Matlab AID [16].

Data gathered from Pacelab APD could be compared easily to the literature data. Since the same books were used to get the needed equations, all of the calculated values were already presented for three different states of flight. Other software, however, have their own limitations and not all of the coefficients could be retrieved. For that reason, only some of the dimensionless derivatives could be compared to all of the available sources. All of the dimensional derivatives, as well as damping and frequencies of phugoid and short period motion could be compared only to Roskam’s results [4].

It is also important to mention that derivatives data presented in Chapter 4 shows Mach values from 0.2 to 0.9, even though equations used in Pacelab are valid only for subsonic regime (from Mach 0.2 to 0.6). Presenting wider velocity range allows to show the deviations of the results values while entering transonic region, or lack thereof.
4 Results

In this section the data from the different software will be presented along the data from the literature. The verification of the results were made on different models of the Boeing 747.

The three flight conditions taken into consideration are:

- **Approach** - Altitude = 0 ft, Mach=0.2, angle of attack = 8.5°,

- **Cruise (Low)** - Altitude = 20000 ft, Mach=0.65, angle of attack = 2.5°,

- **Cruise (High)** - Altitude = 40000 ft, Mach= 0.9, angle of attack = 2.4°.

4.1 Dimensionless Derivatives

The two software programs, as well as the literature, share two common coefficients: $C_{L\alpha}$ and $C_{m\alpha}$. Additionally, the derivatives found in both the literature and OpenVSP include $C_{D\alpha}$, $C_{Lq}$, $C_{m\dot{\alpha}}$ and $C_{mq}$. On the other hand, the derivatives that are present in both the literature and DATCOM+ are $C_{L\dot{\alpha}}$, $C_{D\alpha}$, $C_{Lu}$ and $C_{mu}$.

The plots are colour coded for the different states of flight: Approach [Green], Cruise(low) [Red] and Cruise(high) [Blue]. Different markers have been used to distinguish between the different sources. The results from the Pacelab APD study case have been marked by a singular colour due the minimal variation of the values.

The values shown in Figure 21 for $C_{L\alpha}$ from DATCOM+, literature and Pacelab APD present a linear trajectory up to the limit of the subsonic regime. After the threshold is passed DATCOM+ presents an increase and then abrupt decrease in the transonic regime. The values from OpenVSP show an increase and sudden decline, resembling the beginning of an oscillation. The literature values are following a linear trajectory with an approximate margin of error of 0.5.
Figure 21: Derivative $C_{L_\alpha}$ versus Mach, obtained from DATCOM+, OpenVSP, Pacelab APD and literature [4] for different states of flight.

The results regarding $C_{m_\alpha}$ are presented in Figure 22. OpenVSP oscillates between around 4 and -12, with a decline at the beginning and an incline in the transonic regime. The other sources adhere to a linear progression until the subsonic limit is reached. While DATCOM+ shows a downward trend followed by an upward trend, Pacelab results remain linear as does the literature.

Figure 22: Derivative $C_{m_\alpha}$ versus Mach, obtained from DATCOM+, OpenVSP, Pacelab APD and literature [4] for different states of flight.

Starting with the OpenVSP comparisons, the obtained values for $C_{D_\alpha}$ are portrayed in Figure 23. Data points align in a linear fashion except OpenVSP cruise low and
high, which fluctuates.

Figure 23: Derivative $C_{D_x}$ versus Mach, obtained from OpenVSP, Pacelab APD and literature [4] for different states of flight.

In Figure 24 the data follow similar trajectories with the exception of two decreases for Mach=0.2 in the literature and after the subsonic limit of Mach=0.6 in OpenVSP.

Figure 24: Derivative $C_{m_y}$ versus Mach, obtained from OpenVSP, Pacelab APD and literature [4] for different states of flight.
The derivative $C_{D_u}$ increases with the increase of mach. Oscillations are observed for OpenVSP.

$C_{L_u}$ follows a linear trajectory until the passage to the transonic regime. The values of the derivatives are much higher in the literature.
Figure 27: Derivative $C_{L\dot{\alpha}}$ versus Mach, obtained from DATCOM+, Pacelab APD and literature [4] for different states of flight.

Figure 28 shows the behaviour of $C_{m_{\alpha}}$ with the increase of Mach. Divergence between the models is observed in the transonic regime and difference in pattern as the literature follows a slight decrease, whereas DATCOM+ and OpenVSP a slight increase.

Figure 28: Derivative $C_{m_{\alpha}}$ versus Mach, obtained from DATCOM+, Pacelab APD and literature [4] for different states of flight.

The data sets from the three sources follow similar trajectories to the limits of the subsonic regime for the derivative $C_{Lq}$, as seen in Figure 29.

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Despite having distinct data points, the graphs exhibit a similar pattern within the subsonic regime, as seen in Figure 30.

The behaviour of $C_{D_{iH}}$ versus Mach is shown in Figure 31. The literature and Pacelab follow a decline with an increase of the Mach number.
The $C_{D_{iH}}$ data points from Pacelab seem to follow a linear trajectory. The literature presents a much higher value.

In Pacelab, the values of $C_{L_{iH}}$ exhibit a linear progression, whereas the literature demonstrates variations in these values.
The lift derivative with respect to elevator deflection angle, Figure 34, displays noticeable differences. The derivative depends on the position of the aerodynamic centre and the centre of gravity.

Figure 33: Derivative $C_{L,\alpha}$ versus Mach, from Pacelab APD and literature [4] for different states of flight.

Figure 34: Derivative $C_{m,\alpha}$ versus Mach, from Pacelab APD and literature [4] for different states of flight.
Figure 35: Derivative $C_{m_{Tu}}$ versus Mach, from Pacelab APD and literature \[4\] for different states of flight.

The thrust derivative $C_{m_{Tu}}$ decreases with the increase of Mach as shown in Figure 36.

Figure 36: Derivative $C_{m_{Tu}}$ versus Mach, from Pacelab APD and literature \[4\] for different states of flight.

$C_{Tu}$ increases with Mach as seen in Figure 37. Divergence in pattern is observed after the subsonic limit. The values within the subsonic regime seem to follow a similar trajectory.
4.2 Dimensional Derivatives

Dimensional derivatives are dependant on the corresponding dimensionless ones as can be seen in Section 2.3. The results are compared between Pacelab APD and the literature. The values of the derivatives for the different flight conditions of approach, cruise (low) and cruise (high) are referenced in Table 7 along the margin of error.
Table 7: Obtained values of dimensional derivatives for High Cruise flights state

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Roskam</th>
<th>Pacelab</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_\alpha$</td>
<td>0.372679</td>
<td>4.229</td>
<td>$\frac{m/s^2}{\text{rad}}$</td>
</tr>
<tr>
<td>$Z_\alpha$</td>
<td>-103.327</td>
<td>-92.91</td>
<td>$\frac{m/s^2}{\text{rad}}$</td>
</tr>
<tr>
<td>$M_\alpha$</td>
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<td>0.2991</td>
<td>$\frac{\text{rad}/s^2}{\text{rad}}$</td>
</tr>
<tr>
<td>$Z_\dot{\alpha}$</td>
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<td>-0.5958</td>
<td>$\frac{m/s^2}{\text{rad}}$</td>
</tr>
<tr>
<td>$M_\dot{\alpha}$</td>
<td>-0.1425</td>
<td>-0.1222</td>
<td>$\frac{\text{rad}/s^2}{\text{rad}/s}$</td>
</tr>
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<td>$X_{\delta E}$</td>
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<td>0</td>
<td>$\frac{m/s^2}{\text{rad}}$</td>
</tr>
<tr>
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<td>-2.227</td>
<td>$\frac{m/s^2}{\text{rad}}$</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
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<td>-0.3481</td>
<td>$\frac{\text{rad}/s^2}{\text{rad}/s}$</td>
</tr>
<tr>
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<td>-0.0218</td>
<td>-0.01006</td>
<td>$\frac{m/s^2}{m/s}$</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

4.3 Stability Criteria

Stability criteria were defined in Pacelab APD as conditional loops, implementing inequalities described in Sections 2.4.1 for static stability and 2.4.2 for dynamic stability. Results for static stability are printed in Pacelab APD EWB as shown in Figure 38.

Figure 38: Results of static stability analysis for B747 in Pacelab.

Static stability criteria 3 and 5 are unfulfilled, as shown in Figure 38.

Dynamic stability criteria result is presented in Figure 39. The airplane is not dynamically stable, which is correct, according to [2].
4.4 Aircraft dynamic modes

Values of natural frequency and damping ratio for phugoid and short period motion are calculated and displayed in Pacelab APD EWB. The values calculated for flight state of cruise (low) are presented in Figure 40.

Figure 40: Results of B747’s dynamic modes in Pacelab.
5 Discussion

The data are analysed and explained in this section, along the observations from the method selection.

5.1 Derivatives

In section 4, it was observed that the Pacelab APD software provides approximate values for most of the longitudinal aerodynamic coefficients. The calculated coefficients generally align with the patterns found in the literature and DATCOM+ trajectories for the majority of the derivatives. OpenVSP follows the pattern for the approach and oscillates away from the results for higher mach. However, it is expected that differences exist due to variations in the underlying models and methods employed by each software and the literature.

Significant divergence in results becomes prominent as in the transition regions beyond the subsonic regime or its immediate vicinity, as the transonic region introduces unpredictability, making it challenging to establish a linear mathematical model that accurately predicts aircraft behaviour within this range of Mach.

Derivatives dependent on thrust exhibit more differences since they are based on mathematical approximations rather than actual aircraft engine and thrust data. Consequently, variations arise when comparing these coefficients with aircraft performance.

Additionally, disparities emerge for coefficients that utilise the fuselage diameter as an input. The unique shape of the B747 fuselage, with a wider diameter at the cockpit compared to the rest of the aircraft, contributes to these differences.

To obtain a more comprehensive understanding of coefficient discrepancies, it would be beneficial to utilise a different aircraft model that offers increased reliability. Unfortunately, no other accessible aircraft model was found for comparison in this study.

In summary, while the Pacelab APD software demonstrates consistency with existing literature and DATCOM+ trajectories for longitudinal aerodynamic coefficients, variations arise due to differences in underlying models, the challenges posed by the transonic regime, the mathematical approximation of thrust-related coefficients, and the specific fuselage geometry of the B747. The exploration of alternative aircraft models could offer further insights into the observed coefficient differences and enhance the overall analysis.
5.2 Stability and Control

The results of static stability show that two of the criteria are not met. This instability can be attributed to the positioning of the centre of gravity (CG) behind the aerodynamic centre of the aircraft’s wing and body. The inability of Pacelab to dynamically link the current flight state with inputs to the Flight Dynamics Chapter might be causing this issue. Due to that the position of CG has to be added as a user input.

It is not surprising that the dynamic stability criterion is not met, as many modern aircraft rely on onboard avionic systems to maintain dynamic stability. That is also the case with the commercial airplane Boeing 747.

The obtained values of natural frequency and damping ratio fulfil the longitudinal dynamic requirement. The condition states that damping ratio and frequency of an aircraft’s phugoid motion are much lower than that of the short period motion. This indicates that the aircraft exhibits the characteristics of stable and well-damped longitudinal dynamics.
6 Conclusions

This thesis aimed to explore and evaluate the capabilities and limitations of Pacelab APD 8.1 in acquiring longitudinal aerodynamic coefficients. Through an extensive literature study, the necessary equations were obtained and successfully implemented into Pacelab APD KD, enabling the acquisition of the equations of motion.

To validate the accuracy of the results, a comparison was made with data obtained from DATCOM+ and OpenVSP, using the B747 aircraft as reference. By examining the consistency and disparities among these sources, valuable insights into the performance of Pacelab APD 8.1 and its alignment with established software and literature were gained.

In summary, while the newly implemented flight dynamics chapter in Pacelab APD software demonstrates consistency with the literature and DATCOM+ trajectories for longitudinal aerodynamic coefficients, variations arise due to differences in underlying models, the transonic regime proximity of the data, mathematical approximations of thrust-related coefficients, and the specific fuselage geometry of the B747. Exploring alternative aircraft models could provide further insights into the observed coefficient differences and enhance the overall analysis.

Pacelab APD, with its new implementations, demonstrates its capability to calculate aerodynamic longitudinal derivatives, obtain equations of motion, and determine specific points of the flight envelope. However, it is important to note that further development is necessary to enhance the accuracy of the results.

6.1 Future Work

Determining the stability of an airplane is a very intricate field of flight dynamics. It is not possible to use one version of a set of equations to cover all aircraft configurations or full velocity range. This thesis focuses on formulating and implementing the equations describing flight dynamical behaviour of an airplane in classic configuration in the subsonic speed range. It creates a solid base for further work in this field in Pacelab APD software.

The next steps to expand the usage of newly created chapter could be implementing other aircraft configurations, like canard, delta wing or flying wing. Another way of advancing forward with the development of fully functional flight dynamical tool could be adding a full speed range, from Mach number equal to 0 to even hypersonic values.

In a more software-oriented case of broadening the programs usability, some enhancements can also be made. Due to time limitations plotting of various parameters is
not included in the current code. Being able to present the data in a graphic way would be beneficial to the user. Another advancement would be entering more stability criteria and dynamically connecting values from a specific flight state to the stability calculations, which would surely enrich the software.
References


A Debugging With Visual Studio Code

# Debugging the APD Data Model with Visual Studio Code

## Setup

- Install latest version of Visual Studio Code
- Install the "C#" extension (powered by OmniSharp) – v1.25.0 or higher
- A recommendation is made via a pop-up (right bottom) when the workspace is opened for the first time

## Getting started

- Open the Knowledge Designer application and perform "Check all Project Libraries into the Repository"
- In VS Code, choose File > Open Workspace from File
- Open ‘APDModelDebug.code—workspace’ with VS Code

### 1. Launch EWB with debugger attached

- Go to tab "Run and Debug" (Primary Side Bar)
- Choose "Launch EWB" from the dropdown on the top left
- Click "Start Debugging" or hit F5

### 2. Attach to a running EWB

- Make sure that an instance of EWB is already running, e.g. by starting the EWB externally
- Go to tab "Run and Debug" (Primary Side Bar)
- Choose "Attach to EWB" from the dropdown on the top left
- Click "Start Debugging"
- When you wish to quit debugging, you can disconnect, by clicking "Disconnect" in the floating Debugger panel

## Options when a breakpoint is hit

All the usual actions are possible. You can:

- Continue, Step In/Over/Out (floating debugger panel)
- Inspect the Call Stack (left panel)
- Inspect local variables, watch variables
## Options for creating breakpoints

### 1. Setting a manual breakpoint in KD code

Use these lines to trigger a break only when a debugger is attached:

```cs
if (System.Diagnostics.Debugger.IsAttached)
```

This is especially useful when you want to break in a runtime formulas written in the EWB project, since there is no file where you could set the breakpoint easily yourself (next paragraph).

### 2. Open a KD datamodel XML file and set a breakpoint at runtime

It is easily possible to set breakpoints in KD code at runtime. To do this, just open the respective KD file in VSCode and set the breakpoint inside the C# block (e.g. 'Statements') at the desired line.

Before .NET executes this exact line, the breakpoint is triggered.

You can even set conditions on when you want the breakpoint triggered. To do this, create the breakpoint first, then right-click it, and choose "Edit Breakpoint...".

Now, you can enter a C# expression or set a hit count threshold.

### 3. Break on exceptions

Go to tab "Run and Debug" (floating debugger panel). At the bottom of the left panel, there is a "Breakpoints" section. There, you can configure on which types of exceptions you want the application to break for you to inspect.