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The effects of a whole-class mathematics intervention on students' fraction knowledge in primary school

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ABSTRACT

The intention of the study was to examine the effects of a fraction intervention in a whole-class environment. The intervention aimed to enhance students' conceptual fraction knowledge, with a major focus on fraction magnitude understanding. This study included 120 fifth-grade students in standard classroom settings. Utilizing a cluster randomized controlled trial design, students were divided into either an intervention group ($n = 64$) or a control group ($n = 56$). Students in the intervention condition received a series of seven 35-minute lessons. Students in the control condition received "treatment as usual". Both post-test and delayed post-test results revealed that students in the intervention group performed significantly better than those in the control group on fraction concepts, with a stronger effect in measurement aspects compared to part–whole aspects. The intervention group also outperformed the control group on fraction arithmetic on both post-tests, while no significant difference was observed on fraction word problems.

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Intervention; mathematics; fraction learning; fraction magnitude; primary school

Whole-number knowledge, fraction knowledge, and core concepts in measurement and geometry together form the Critical Foundations of Algebra (National Mathematics Advisory Panel [NMAP]), 2008, p. xvii). Competence in fraction knowledge is essential for learning more advanced mathematics, where it serves as a key to developing proficiency in algebra (Booth & Newton, 2012). In addition, proficiency in algebra is an essential element for long-term success in science, technology, engineering, and mathematics (STEM).

Fraction learning is difficult for students in general (Mazzocco & Devlin, 2008) and this is especially true for students with learning difficulties (Tian & Siegler, 2017). Understanding the numerical magnitude of symbolic fractions remains challenging even for adults (Siegler & Braithwaite, 2017), and second graders' estimations of whole numbers on a 0–100 number line often outperform eighth graders' estimation of fractions on a 0–5 number line (Siegler et al., 2011). A study by Jordan et al. (2017) demonstrated that a significant number of students have minimal trajectory growth in fraction knowledge between grades four and six. This is even more evident in students with diagnosed learning disabilities.

The Swedish school system lacks validated instructional programs or research-based instructions to use in fraction intervention programs. One aim of the present study was to contribute to the field of education by expanding knowledge around effective teaching practices in a Swedish context. The

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intention of the study was to implement and evaluate the effectiveness of a mathematical instructional program, in a whole-class setting, designed to increase students' fractional knowledge. We expect that the delivery of a seven-lesson series with a focus on fractions as a number in a whole-class teaching environment will have a positive effect on students' fraction knowledge.

Fraction learning

Fraction learning is based on whole-number knowledge but due to crucial differences between these domains awareness of common pitfalls is essential (Siegler & Braithwaite, 2017). An established difficulty with learning fractions is making assumptions based on whole-number logic (Siegler et al., 2011), which includes the misconception that the greater denominator constitutes the bigger number when the numerator is the same. In addition, fractions are expressed via two numerals, which differs from how other numbers are displayed. Accordingly, the focus should be on fractions as numerical magnitude entities. A conceptual understanding of fractions as one number and the relation between numerator and denominator as a specific magnitude is key fraction knowledge (DeWolf & Vosniadou, 2015; Schumacher et al., 2018; Siegler & Lortie-Forgues, 2017). A lack of focus on this conceptual understanding in instruction during the early stages of student learning could lead to students conceptualizing fractions by their components, that is, as two whole numbers, so-called "whole-number bias" (Ni & Zhou, 2005). As a result, prior knowledge of whole numbers fails to help students understand fraction number concepts. It is the relationship between numerator and denominator that defines the value of a fraction (Malone et al., 2019).

Shifting teaching instruction from whole to rational numbers could create the misconception that there are no numbers between two-unit fractions; for example, between 1/4 and 1/5. Vamvakoussi and Vosniadou (2010) found that, even after several years of mathematics instruction, students continued to have difficulties comprehending that there is an infinite number of numbers between two-unit fractions. Another example is a strategy for solving fraction arithmetic whereby numerator and denominator are added separately (e.g., $2/3 + 1/5 = 3/8$), that is, based on whole-number logic, which leads to an incorrect answer (Tian & Siegler, 2017). The above misunderstandings could refer to problems connected to instruction in mathematics, when shifting from whole numbers to rational numbers (i.e., fractions).

A part–whole interpretation of fractions implies that fractions are parts of wholes or parts of sets of objects; that is, they represent either a continuous or a discrete quantity (Kieren, 1980). Instructions, used in many US schools, are usually based on a part–whole construct, often described as area models (Fuchs et al., 2017). If such a model is over-represented as an interpretation of fractions, it might limit students' understanding of fractions as numbers (Jordan et al., 2017). In principle, the part–whole interpretation of fractions is based on students' experience of sharing and is therefore largely intuitive (Fuchs et al., 2014). Until recently, a part–whole interpretation of fractions has been the focus of US mathematical curricula, as compared to curricula typical of Asian countries, where developing a linear representation of fraction magnitudes—referred to as "measurement of fractions"—is more frequently used (Siegler et al., 2015).

Understanding that fractions reflect the cardinal size of numbers is known as "measurement interpretation". The number line is usually used to underpin an understanding of a fraction as a measure (Gersten et al., 2017; Siegler et al., 2011). There are multiple ways to assess this kind of knowledge, e.g., placing fractions on number lines or comparing or ordering fractions (Fuchs et al., 2014; Siegler & Braithwaite, 2017). Improvement in measurement interpretation is assumed to be more critical in developing fraction understanding than part–whole interpretation (NMAP, 2008). This hypothesis was strengthened by Fuchs et al. (2013), whose results showed that measurement interpretation, not part–whole interpretation, mediated intervention effects on fraction items in the National Assessment of Educational Progress (NAEP) test. The number line, compared to other models of representation, is a successful way of obtaining a deeper conceptual understanding of fractions (Gersten et al., 2017).

Interventions focusing on number lines, accompanied by explicit instructions, provide long-lasting progress in students' conceptual knowledge of fractions (Barbieri et al., 2020). Tian and Siegler (2017) clarify two advantages of using number lines as a didactic tool: first, it reduces difficulties in introducing improper fractions; second, it is more appropriate for implying the continuity of fractions between any two numbers. They also establish that greater use of number lines in a classroom setting potentially helps children, especially children with mathematical difficulties (MDs), to better understand fraction magnitude and fraction arithmetic. Schumacher et al. (2018) emphasizes that number lines could be used both in part–whole (understanding fractions as parts of a whole divided into fair shares) and measurement (understanding fractions as one number, with a specific place on the number line) interpretation. They argue that the number line is an important bridge between part–whole and measurement interpretations. Teaching students to estimate the location of fractions on a number line has been proven effective in improving students' knowledge of magnitude understanding (Fuchs et al., 2017). Flores et al. (2022) also emphasizes that the length model, which is a model based on magnitude, is superior to other models, such as different area shapes or sets of objects used to relate fractions to whole numbers.

Previous research has shown that students' fraction magnitude knowledge promotes fraction arithmetic (Bailey et al., 2017; Fuchs et al., 2013). Intervention studies show that using number lines not only increases students' understanding of fraction magnitudes but also has an effect on students' ability to add and subtract fractions (Fuchs et al., 2017; Siegler & Braithwaite, 2017; Tian & Siegler, 2017). One possible explanation is that increased knowledge of fractions helps students to understand the underlying logic of adding and subtracting, in addition to granting them an extended conceptual understanding that fractions represent a specific magnitude and express one number. Fraction magnitude understanding predicts later fraction arithmetic and one type of knowledge facilitates the other (Bailey et al., 2017). These findings are consistent with the bidirectional model of conceptual and procedural fraction knowledge (Rittle-Johnsson et al., 2015). Bailey et al.'s (2017) research also revealed that transfers between conceptual and procedural knowledge, and vice versa, are expected to appear later in overall fraction knowledge acquisition during schooling, i.e., fifth to sixth grades. However, there is no evidence indicating that development cannot proceed earlier.

Theoretical foundation

To enhance understanding of whole numbers, as well as fractions, an important foundation is understanding of their magnitudes. The integrated theory of numerical development (Siegler & Braithwaite, 2017) states that knowledge proceeds from non-symbolic to symbolic numbers, on to a wider range of numbers (both in magnitude and type). Progression of numerical development is also age-dependent. The age at which a child understands whole numbers between 0–100 ranges approximately from 5–7 years, while the age for interpreting fractions between 0–1 (i.e., proper fractions) is approximately eight years to adulthood.

One key assumption of integrated theory is that increased understanding of numerical magnitude is the core of numerical development and will affect arithmetic as well as more advanced mathematics (Siegler & Braithwaite, 2017). Integrated theory assumes that development of numerical magnitude knowledge also provides a springboard to the development of arithmetic; that is, number magnitude knowledge is crucial for all numerical development. Thus, conceptual understanding of fraction number magnitudes is foundational for overall fraction learning.

Another key assumption is that all numbers are presented along a mental number line. The number line gradually expands to include larger numbers and fractions, and it supports knowledge of different numbers in relation to each other. Along with the fact that all numbers can be represented as magnitudes along a mental number line, the theory also clarifies that learning fractions could be extra challenging if children do not have an essential cardinal understanding of fractions. One important aspect of the theory of numerical development is to understand that several properties that are valid for whole numbers are not valid for fractions. As an example, fractions can be presented in countless ways (e.g., 1/4 = 2/8 = 25/100), unlike whole numbers.

To isolate the importance of the number line compared to area models in fraction learning, Hamdan and Gunderson (2017) examined the benefits of these different representations in second- and third-grade students. Based on findings on both number line estimation and magnitude comparison, they concluded that there is a causal effect between number line and fraction magnitude knowledge, which can reduce whole number bias in the long run.

The relationship between knowledge of numerical magnitude and arithmetic learning, based on the integrated theory of numerical development, is strengthened through interventions focused on developing students' knowledge of numerical magnitudes. Explicit instruction is a multicomponent approach signified by systematic, direct, engaging, and achievement-oriented instruction, which all together forms an effective and efficient method for teaching (Hughes et al., 2017). Explicit instruction promotes activity with frequent and nuanced student responses and provides appropriate feedback from teachers.

Fraction interventions

Prior intervention research targeting Tier 1 is rare; however, to gain a brief though comprehensive overview of prior research, Tiers 2 and 3 are also reviewed. Ennis and Losinski (2019) presented a meta-analysis of 21 fraction interventions with quality elements according to the Council for Exceptional Children Standards for Evidence-Based Practices (CEC EBP) in Special Education. The most frequently used design was randomized controlled trials (RCTs) and the most-frequently used instructional construct was explicit instruction. The interventions were performed in settings ranging from one-to-one instruction to small groups of three to six students. Individual effect sizes across all interventions ranged from Hedges' $g = 0.42$ to $g = 11.51$ and, grouped by instructional effectiveness, the outcomes were: video modeling ($g = 11.51$), graduated instruction ($g = 1.60$), strategy instruction ($g = 1.48$), explicit instruction ($g = 1.25$), and anchored instruction ($g = 0.35$). Ten of the included studies fulfilled 100% of the quality elements targeted by CEC EBP, whereas only explicit instructions showed sufficient evidence to be seen as an evidence-based practice for improving fraction performance, according to CEC EBP standards. Anchored instruction produced only mixed evidence for improving fraction performance, while remaining intervention constructs produced insufficient evidence to determine an evidence base, as a result of not meeting CEC EBP standards. The meta-analysis also showed that the impact was greater when fewer students were involved, namely, a one-to-one or small-group design.

Fraction Challenge, subsequently named Fraction Face Off (FFO!), has been applied as a Tier 2 intervention program in several studies conducted by Fuchs and colleagues (Fuchs et al., 2013, 2014, 2016a, 2016b). In these series of RCTs, the effects of a 12-week, 36-lesson intervention program for fourth graders in the form of small-group instruction were evaluated regarding its effects on promoting students' understanding of fractions. FFO! uses explicit instruction and addresses the measurement interpretation of fractions, that is, the magnitude of fractions. Students at risk of experiencing difficulties in learning about fractions were individually randomized to a control condition, a single intervention group (Fuchs et al., 2013), or a double intervention group (Fuchs et al., 2014, 2016a, 2016b). In each study, results indicated that all intervention conditions outperformed control conditions in measured outcomes: NAEP total score, fraction calculations, and fraction number line (e.g., placing fractions on number lines). Effect sizes ranged from 0.37–2.5 on all fraction outcomes (see Fuchs et al., 2017). Fuchs et al. (ibid) proclaimed that fraction magnitude understanding, and not part–whole interpretation, was a mediator of these effects.

Fraction calculation, as well as word problem skills (e.g., additive) improve when fraction number magnitude understanding grows (Fuchs et al., 2013). To deepen and extend their prior research, Fuchs et al. (2016a, 2016b) placed an additional and specific focus on different word problem strategies and explored the effects of the different forms of reasoning embedded in FFO! The main purpose was to examine the effects of providing high quality explanations to students regarding their mathematical work. The result indicates that, to improve students' fraction word problem ability in general, instructions should emphasize a multiplicative strategy instead of an additive strategy.

Comparable results presented by Malone et al. (2019) displayed effect sizes on all fraction outcomes (NAEP, calculation, and number line), ranging from 0.36–3.14. A significant result was also found for word problem skills, with a mean of ES = 0.73, despite a lack of focus on word problems in the intervention instructions.

Dyson et al. (2020) produced similar results in their RCT study in which they developed and examined the efficacy of a fraction sense intervention. Similar to FFO!, the intervention focused on fraction magnitude understanding and used explicit instruction and a number line approach. Dyson et al. (2020) also used a small-group design; however, it differed from the FFO! in that the intervention consisted of 21 lessons instead of 36, and the targeted students were sixth graders. Their conclusions were in line with previous studies; that is, the intervention group outperformed control groups on all fraction outcomes, described as fraction sense, fraction relations, and fraction calculations. Another contribution to prior intervention studies was a follow-up, seven-week post-intervention, as the intervention group still performed significantly better than the control group on all outcomes, with a comparatively smaller effect size for fraction arithmetic.

In summary, the combined empirical picture of prior intervention research provides strong causal evidence in support of an integrated theory of numerical development (Siegler & Braithwaite, 2017) and the use of explicit instruction (Hughes et al., 2017). It also reveals that general fraction skills (i.e., number line, adding and subtracting fractions, and fraction concept) improve, in favor of the intervention group, when specifically focusing on fraction magnitude understanding (Fuchs et al., 2017) and the use of number lines during instruction (Hamdan & Gunderson, 2017).

However, although multiple studies utilize a small-group design, few prevention studies use a whole-class design. The whole-class design relates to a mathematics lesson in which all students participate in an ordinary classroom setting (Gersten et al., 2009). There is no typical guideline other than a general description of "high quality". Whole-class instruction also encompasses screening of all students, with the purpose of identifying those who are struggling.

As implied by Ennis and Losinski (2019), there is a high demand for evidence-based instruction for a whole-class prevention approach, rather than merely remedial procedures. In a Swedish context, this is essential since both organization of and access to professional special educational support differ among schools (Magnússon et al., 2019). In Sweden, it is highly probable that students in need of special education receive such support in their regular classroom.

The lack of evidence-based instructions and intervention studies in different contexts (i.e., whole class, small group, or one-to-one) is problematic. In light of this, this study was designed to explore the efficacy of a mathematics intervention, and to test whether aspects of a Tier 2 intervention are also valid as a Tier 1 intervention in a Swedish educational context.

Purpose and hypothesis

The purpose of this study was to examine the effects of an intervention in mathematics. The study was theoretically founded on the integrated theory of numerical development (Siegler et al., 2011, 2017) and the intervention program was inspired by the multicomponent fraction program, FFO! (Fuchs et al., 2015). The theory of numerical development provided specific analytical concepts used in instructional elements to enhance important aspects in fraction learning, that is, an explicit transfer from part–whole interpretation to measurement as the intervention progressed. Further, the use of number line as a didactic tool in fraction instruction was another contribution of the foundation provided by the theory of numerical development. The intervention was implemented as a whole-class intervention, unlike many other intervention studies, which have mainly focused on small-group designs. This study included one control group and one intervention group with a specific instructional focus on fraction magnitude knowledge and a specific aim of increasing students' fraction conceptual understanding. In view of this, the study explored three hypotheses related to the integrated theory of numerical development as well as prior intervention studies.

The overall hypothesis was that the intervention group's post-intervention scores would outperform the control group's post-intervention scores on understanding the fraction concept founded on prior research. Therefore, we split the hypothesis in two: (a) the intervention group's improvement in understanding the fraction concept (measurement + part–whole) would be higher than that of the control group; (b) the intervention group's improvement in fraction arithmetic would be higher than that of the control group. Additionally, we wanted to explore the long-term effects of all outcomes and the possibility of transferring effects to fraction word problems.

Method

The overall aim of this study was to evaluate the effects of a Tier 1 intervention, implemented in a whole-class setting, to improve students' conceptual understanding of fractions. Teachers and special educators in south-east Sweden were contacted by e-mail to enquire whether they were interested in participating in the study. In total, five teachers and one special educator expressed an interest, and thereafter proceeded to take part in the study together.

Design and participants

Fifth-grade students were recruited from the schools of participating teachers/special educators, that is, three elementary schools in two different minor cities in south-east Sweden. All schools followed mathematics curricula according to Swedish standards. Students belonging to targeted classrooms (i.e., 165 students) were sent an informed consent letter. In total, 120 (63 male) students and their caregivers gave consent (a 73% response rate), enabling them to participate in the study. Of these students, 64 were assigned to the intervention condition and 56 to the control condition. An RCT was deemed not possible due to logistically challenging aspects relating to participating in schools. Adopting a cluster-randomized controlled trial (C-RCT) approach made it possible to perform the study within the natural settings of the schools. Another factor contributing to the selection of this study's design was that participating teachers were instructing more than one student group. Such a situation would create conflict with possible undesirable transfer effects because the same teacher would take part in both intervention and control conditions. The study comprised of seven clusters, as every school class accounted for one cluster, three of which were assigned to the intervention condition and the remaining four to the control condition. The distribution was as follows: one school with two control groups (one teacher), one school with one intervention group (one teacher), and one school with two control groups and two intervention groups (two teachers).

A baseline assessment was conducted to identify mathematical ability (whole number arithmetic and whole number magnitude comparison), and a cognitive selection bias assessment was also performed (Trailmaking A + B, Mental rotation [symbols] and Raven's BCDE) (see supplemental material). All tests, except whole number arithmetic, were administered by the first author of this article. Apart from the intervention itself, there were no significant differences between the groups (see Table 1, supplemental material). Pretest performance within each fraction outcome (concept [measurement + part–whole], arithmetic, and word problems) were all comparable between the intervention and control groups before intervention.

Ethical approval

The study plan obtained ethical approval from the Swedish Ethical Review Authority (reference number 2020-05495). Both students and caregivers received written information about the planned research, and they returned the signed consent form to the schools. Participants' right to withdraw involvement was upheld throughout the study.

Table 1. Means and standard deviations for all outcome measures.

	Intervention			Control		
	Mean (SD)			Mean (SD)		
Outcome	Pretest	Post-test	Delayed post-test	Pretest	Post-test	Delayed post-test
FCm	10.16 (6.47)	15.88 (5.72)	16.00 (6.06)	10.62 (5.25)	12.72 (5.45)	13.64 (6.16)
FCpw	10.37 (3.06)	11.77 (2.48)	12.56 (2.76)	10.85 (2.68)	11.64 (2.68)	11.85 (2.90)
FA	2.48 (3.24)	5.00 (4.21)	7.72 (3.01)	1.94 (2.50)	2.74 (3.46)	4.58 (4.18)
FWP	4.13 (2.36)	5.13 (2.51)	6.10 (2.09)	4.47 (2.61)	5.74 (2.06)	5.62 (2.46)

Abbreviations: FCm = fraction concept measurement; FCpw = fraction concept part–whole; FA = fraction arithmetic; FWP = fraction word problem.

General procedure

As both the intervention and control conditions were delivered in a whole-class setting by the regular mathematics teacher, instructors were well-known by the students. Consequently, the teacher operated as the intervention agent. The intervention, which was conducted in three to four lessons per week, took place during ordinary scheduled mathematics lessons over a two-week period. The duration of the intervention was determined principally for practical reasons but it also corresponds to the time usually spent on conceptual fraction knowledge during that fifth-grade period.

Description of the intervention condition

A series of seven lessons was constructed, inspired by Fraction Face Off! (FFO!; Fuchs et al., 2015). The seven-lesson series focused on measurement interpretation, with a brief introduction to interpretation changing from part–whole to measurement during the first lesson. This implied introducing the number line as a whole, with the support of rectangular area models. Early in the intervention the principle of equal parts and equivalent fractions to a half and a whole was explained, and later the unit fraction and the application of multiples of unit fractions to express any fraction was introduced. Targeted understanding of fractions as magnitudes was concretized by comparing fractions, ordering fractions, and placing fractions on a number line. FFO! includes a word problem section in the intervention program, which was excluded in this whole-class intervention. Each lesson lasted approximately 30 minutes and was completed by each student and capped by an individual exit-ticket, measuring the outcome of the lesson. Instructions followed an explicit approach and were built around a scripted lesson guide, accompanied by visual support and modeling by the teacher. The teacher provided explicit instructions and drew attention to important features of the fraction concept and students responded to teachers' questions. There was a high level of student involvement in line with the essential components of the explicit instructions (Hughes et al., 2017). The lesson guide included the same sections every session and involved teacher-led explicit instructions, guided practice/working together, and independent/individual practice. Every session or lesson involved a transfer from teacher-led exercises to student independence in line with the instruction model "I do, We do, and You do" (Hudson & Miller, 2006), involving a gradual withdrawal of teacher support. Even if the lessons were scripted, teachers were told to familiarize themselves with the scripted version but that they were not obligated to read from the script during instruction. PowerPoint provided the visual support for use during instruction; which slide to use during instruction and when to use animation for modeling were scripted in the lesson guide. Additional intervention material, such as worksheets for both group and individual practice, was attached to the implementation material delivered by the researchers.

Description of the control condition

The control condition delivered "treatment as usual", and participating teachers designed and provided control conditions using teaching materials that are usually employed to instruct fractions in fifth grade. The control group was active in the way that instruction was focused on conceptual knowledge of fractions, and in the same number of lessons as the intervention. The research team delivered no further information on how to conduct instructions, other than establishing that fraction concept knowledge was at hand for instruction. The mathematical content was delivered over seven lessons lasting the same amount of time as those in the intervention condition; that is, maths instruction time was the same for both groups focusing on fractions.

Outcome measures

The testing procedure was administrated by teachers at the participating schools and followed scripted, written instructions placed on the front page of the test. All tests were paper–pencil, and were implemented in a whole-class environment, ranging from 12–25 students. The test battery was used to collect data consisting of two parts (A + B) assessing measured outcomes: fraction as part–whole, fraction as measurement, fraction arithmetic, and fraction word problems. To index generalized knowledge of fractions, a selection of tasks was made from *Diamant*[1] (Diamond), a national diagnostic test system in mathematics for grades one–nine (Skolverket, 2021). *Diamant* includes, in total, 127 different tests focusing on different mathematical areas, for example: geometry, statistics, and number sense. For the fraction outcome test, problems were chosen from the section involving rational numbers (RB1–RB6 in *Diamant*).

Fraction concepts: part–whole and measurement

This part, referred to as “Part A” in the above section, focused on conceptual knowledge of fractions interpreted as two different aspects: as part–whole or as measurement. Part–whole is usually displayed in a concrete way, using pictures of everyday objects (e.g., pizza, pie) as different area models or as parts of a set of objects, while measurement uses fraction number lines to display information regarding ordinality and cardinality. The total fraction concept test contained 40 items in total, with a maximum score of 40 points. The distribution of problems in each category followed Fuchs et al.’s (2016a) suggestion of tasks in each interpretation, respectively. A total of 16 items were displayed in which different shapes (i.e., triangles, circles, or rectangles) or a number of individual objects were either shadowed to begin with or were supposed to be shadowed by the student. These 16 items together comprise the part–whole aspect of the fraction concept, with a maximum score of 16 points. The measurement aspect comprises 24 items where fractions are presented as numbers. Students were asked to compare, order, write equivalent fractions, or place fractions on a number line. The total score of the measurement interpretation was 24 points. For example, students determined which fraction is greater: 1/2 or 1/3. Cronbach’s alpha was established at $\alpha = 0.90$, suggesting a high internal consistency among the items used to measure the fraction concept.

Fraction arithmetic and fraction word problems

This part, referred to as “Part B” in the fraction measure outcome section above, focuses on fraction arithmetic (14 problems) and fraction word problems (10 problems). Fraction arithmetic problems contained different calculation methods (i.e., addition, subtraction, multiplication, and division). The problems were constructed so that no written arithmetic was necessary. For example, students were asked to give the answer to 1/3 + 1/3, but the test also included more advanced arithmetic, like fraction subtractions with different denominators (e.g., 3/4–1/2). These types of problem are characterized in *Diamant* as conceptualizing problems or mental arithmetic problems.

Implied in fraction word problems were different fractions exposed in the text. The word problems were of two different types: splitting and grouping (see Fuchs et al., 2016a). A splitting problem implies that you are given a certain amount that should be divided into several parts (e.g., Hugo has made two pizzas and cuts them in fourths. How many pizza slices does he have

[1]The *Diamant* material is no longer available on Skolverket’s website.

now?). A grouping problem implies that you are given a certain part that should be grouped into a bigger amount (e.g., Filippa needs 1/5 of a liter of lemonade to fill one glass. How much lemonade does she need to fill up 10 glasses?). In all, this section contained a total of 24 problems. Cronbach's alpha was established at $\alpha = 0.92$, which implies high internal consistency.

Fidelity of implementation

Each lesson was scripted and audiotaped to ensure fidelity of implementation. After finishing the intervention, a member of the research team listened to samples of the collected audio-recordings. Essential points of the intervention were estimated to be addressed by the tutor. Observation as a fidelity check is recommended (Fuchs et al., 2013). Observation and usage of a checklist during the intervention was intended but, due to the COVID-19 pandemic, was not feasible.

Analysis

Prior to analyses, data for all measures was entered in SPSS (IBM SPSS Statistics 28). To test the overall effectiveness of the intervention, a mixed ANOVA 2×3 was used for each outcome (fraction concept measurement, fraction concept part–whole, fraction arithmetic, and fraction word problems) with "group" as between subject and "time" as within subject factors. No covariate was used since the aim was to describe differences at each time-point. Effect sizes were converted from F-values to Cohen's d (1988) using a psychometrica calculator (Lenhard & Lenhard, 2016). For post hoc testing for simple main effects, Sidak comparisons were used.

Results

Means and standard deviations (SDs) at pretest, post-test, and delayed post-test for both groups on all outcome measures are presented in Table 1.

Fraction concept as measurement

ANOVA displayed a non-significant main effect of "group", $F(1, 80) = 2.044$, $p = 0.157$, $d = 0.32$ but a significant "time" effect, $F(2, 160) = 53.960$, $p < 0.001$, $d = 1.65$. ANOVA also revealed a significant time × group interaction effect, $F(2, 160) = 8.294$, $p < 0.001$, $d = 0.65$. Post hoc testing of the simple main effects of the interaction showed no significant results at pretest ($p = 0.731$, $d = 0.08$) and the intervention group scored significantly higher on fraction as measurement at post-test ($p = 0.012$, $d = 0.57$) and with a tendency to a significant effect at delayed post-test ($p = 0.084$, $d = 0.39$) (see Figure 1).

Fraction concept as part–whole

No significant group effect was found for fraction as part–whole, $F(1, 80) = 0.053$, $p = 0.819$, $d = 0.05$. However, there was a significant time effect, $F(2, 160) = 18.726$, $p < 0.001$, $d = 0.97$, and a tendency to an interaction effect, $F(2, 160) = 2.479$, $p = 0.087$, $d = 0.35$ (see Figure 2).

Fraction arithmetic

The ANOVA revealed a significant group effect, $F(1, 58) = 6.664$, $p = 0.012$, $d = 0.68$, a significant time effect $F(2,116) = 47.293$, $p < 0.001$, $d = 1.81$, and an interaction effect, $F(2, 116) = 5.255$, $p = 0.007$, $d = 0.60$ for fraction arithmetic. Post hoc testing of the simple main effects of interaction displayed no significant differences at pretest ($p = 0.47$, $d = 0.19$), but the intervention group

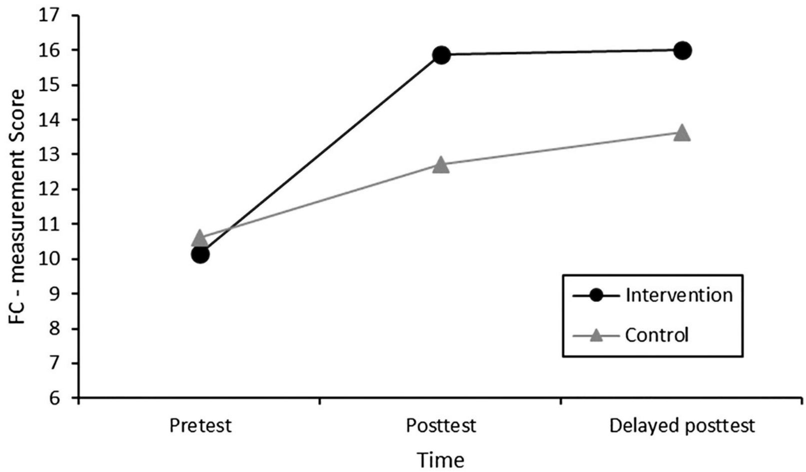

Figure 1. Fraction concept, measurement score: means at post-test and delayed post-test.

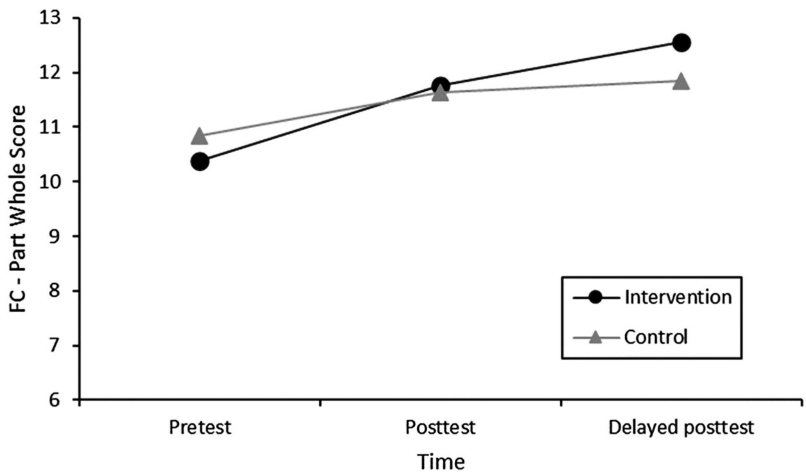

Figure 2. Fraction concept, part–whole score: means at post-test and delayed post-test.

outperformed the control group at post-test ($p = 0.027$, $d = 0.60$) as well as at delayed post-test ($p = 0.002$, $d = 0.87$) (see Figure 3).

Fraction word problem

The main group effect, $F(1, 71) = 0.111$, $p = 0.740$, $d = 0.08$ was not significant though the time effect was, $F(1.84, 142) = 19.309$, $p < 0.001$, $d = 1.05$. The interaction effect were not significant, $F(1.84, 142) = 2.409$, $p = 0.099$, $d = 0.37$ (see Figure 4).

It is important to note the relatively large amount of missing data in the analysis. The measurement and part–whole analysis is based on 68% of the original sample (n = 39 control and n = 43 intervention); the fraction arithmetic analysis is based on 50% of the original sample (n = 31 control, n = 29 intervention); and the fraction word problem analysis is based on 61% of the original sample (n = 34 control, n = 39 intervention). The large amount of missing data is probably due to the pandemic, as the children were required to stay at home if they had even slight symptoms.

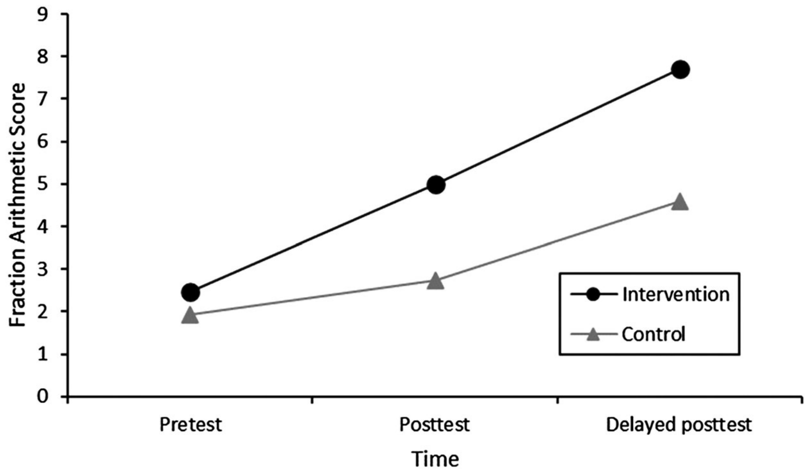

Figure 3. Fraction arithmetic score: means at post-test and delayed post-test.

However, the differences between measurement, part–whole and fraction arithmetic are not accounted for by the pandemic.

Discussion

The purpose of this study was to evaluate a mathematics intervention in a whole-class setting, with an instructional focus on fractions, and particularly fractions as measurements in fifth grade.

For researchers in educational science, adding validated effective teaching instructions in relation to different populations and contexts is valuable. The present study emphasizes the foundation of the integrated theory of numerical development and its overall hypothesis: that instructions and interventions that aim to improve fraction magnitude understanding will improve fraction learning overall, including fraction arithmetic (Siegler et al., 2011, 2017). In this study, fraction knowledge was examined through four different outcomes: fraction concept as measurement, fraction concept as part–whole, fraction arithmetic, and fraction word problems. Overall, the results demonstrate that students who participated in the intervention outperformed the control group on

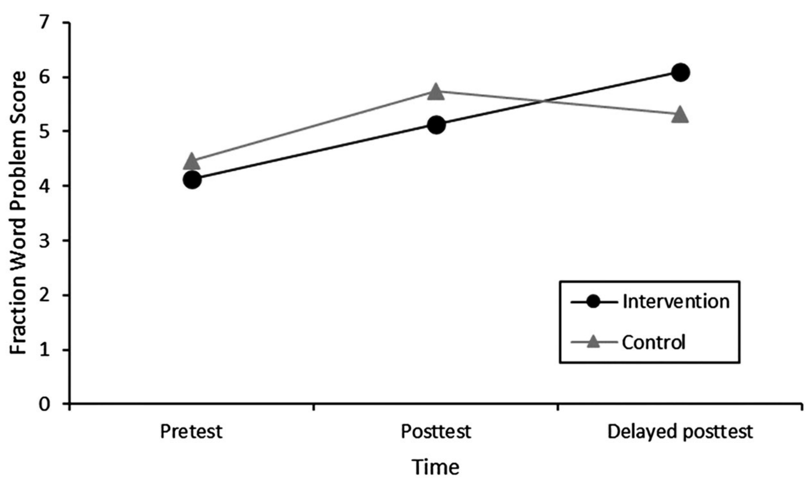

Figure 4. Fraction word problem score: means at post-test and delayed post-test.

fraction concept as measurement, and fraction arithmetic at post-test. Notably, at delayed post-test, performed six months after implementation, the intervention group had a tendency to outperform the control group on fraction as measurement, but significantly outperformed it on fraction arithmetic.

Due to the issue of potentially misleading benchmarks used in educational studies in mathematics, as raised by Bakker et al. (2019), we did not use Cohen's (1988) benchmarks; rather, we call on readers to interpret effect sizes according to other studies with similar characteristics. In our case, a summary of Tier 2 interventions was used for comparison since Tier 1 interventions are not so common.

Hypothesis 1: The intervention group's improvement in fraction concept (measurement + part–whole) will be greater than that of the control group

When separating fraction concepts into two different aspects—measurement and part–whole—it becomes clear that it is within the measurement outcome that the biggest gain in students' improvement is seen. The present findings corroborate prior intervention research by showing that students receiving instructions focusing on measurement interpretation of fraction numbers using number lines as didactic tools improve their conceptual understanding of fraction magnitude (Fuchs et al., 2013, 2014, 2016a, 2016b).

The lack of a significant result regarding students' improvement in understanding the part–whole element of the fraction concept is in line with Fuchs et al.'s (2014) correlational analysis declaring that fraction magnitude understanding is the mediator of the intervention's effects. Thus, a part–whole understanding of fractions does not propel intervention effects. This result strengthens the idea that number line and fraction measurement interpretation play a significant role (Hamdan & Gunderson, 2017) in understanding the fraction concept and, by extension, the importance of using number line as a bridge between part–whole and measurement interpretation (Schumacher et al., 2018).

Hypothesis 2: The intervention group's improvement in fraction arithmetic will be higher than that of the control group

Although fraction arithmetic was not targeted in the intervention, students were instructed during lessons that a fraction, for example 2/3, is equal to 1/3 + 1/3. The core knowledge connected to this example is that every fraction is built from multiple unit-fractions. Bailey et al. (2017) concludes that students generally struggle when applying fraction magnitude knowledge to fraction arithmetic. The hypothesis anticipated that students receiving the intervention should improve fraction arithmetic knowledge; that is students might apply fraction operations displayed during an intervention that could be of help in solving arithmetic problems.

According to the integrated theory of numerical development, instruction based on a measurement interpretation of fractions should improve fraction arithmetic (Siegler et al., 2011, 2017). This is in line with previous results claiming that fraction magnitude knowledge promotes and transfers to fraction arithmetic (Bailey et al., 2017; Fuchs et al., 2013), at least when adding and subtracting fractions (Fuchs et al., 2013). Fuchs et al.'s (2016a) research indicates a correlation between fraction magnitude knowledge and fraction arithmetic. Their results are in line with previous research and indicate that a strong conceptual understanding of fractions-as-measurement is of importance for fraction arithmetic learning. Since no direct instructions in procedural knowledge were included in the present intervention, the effect is considered a far transfer effect. To further explore this, Dyson et al. (2020) proposed that future interventions focus directly on this effect, and scaffolding instructions align fraction magnitude knowledge to arithmetic procedures.

Conceptual fraction knowledge is more strongly correlated to fraction arithmetic than to fraction word problems (Fuchs et al., 2016a). In the light of lack of correlation for word problems, and the fact of an exclusion of an instructional component for word problems in the present intervention, this result was not unexpected. This result is also in line with research that establishes that word problems are less correlated to conceptual knowledge, compared to fraction arithmetic, due to the need of other competencies, for instance reasoning ability and language comprehension (ibid).

Thus, the present study is in line with prior research on fraction concept development, but also contributes to and provides additional knowledge regarding whole-class intervention. Intervention studies referred to earlier mainly focused on small group interventions, whereas experimental studies in whole-class settings are rarer and called for (Ennis & Losinski, 2019).

In addition, effect sizes ranged from $d = 0.57–0.60$ in fraction as measurement and fraction arithmetic, respectively, at post-test even if the intervention was delivered as a seven-lesson series, in a whole-class setting. At delayed post-test, a tendency to a significant effect to use fraction as measurement was detected, as well as an effect size of $d = 0.87$ for fraction arithmetic. Novel findings reveal positive effects on students' fraction arithmetic and a tendency to a significant effect to use the fraction concept as measurement not only on an immediate basis but also long term. This is interesting, especially in relation to Ennis and Losinski's (2019) study, which showed that impact the impact of an intervention increases when fewer students are involved (i.e., one-to-one or small groups). Prior findings also suggest that many interventions experience fadeout effects at delayed post-test (Bailey et al., 2017).

Limitations and future directions

One limitation of this study is that the students were not individually randomized to intervention or control conditions; instead, cluster randomization was used. Considering the small number of classrooms (i.e., clusters), it was not possible to control for nesting effects. The students did not, however, differ in academic or cognitive skills before the intervention phase. The control and intervention groups were similar on relevant measures, which strengthens the validity of the study. As this study explored the efficacy of a predefined Tier 1 intervention, the possibilities of exploring other hypotheses, such as learning processes, or alternative theoretical perspectives on fraction knowledge was limited. Future research should attempt to test the effects of interventions based on a different theoretical foundation of fraction learning (e.g., Steffe & Olive, 2010), as well as include additional intervention groups whereby the efficacy of specific elements of the intervention could be explored. Also, as noted in the results section, there was a great deal of missing data as a result of the pandemic, which is a motivation for replication.

Conclusions and implications for practice

The results of this study provide the basis for two conclusions. First, even a considerable short intervention in time—that is, a seven-lesson series during a two-week period—resulted in a clear effect on both conceptual fraction knowledge as measurement and fraction arithmetical skills. This study also provides experimental evidence of tendency and long-term effects, lasting for up to six months after implementation of the intervention.

Second, instruction focusing on a number line representation of fractions seems more effective for fraction knowledge development overall, not merely conceptual but procedural knowledge also. This could imply that effective teaching instruction should be centered on measurement interpretation supported by number lines, due to advantageous transfer effects on fraction arithmetic (see Bailey et al., 2017; Fuchs et al., 2013, 2017; Tian & Siegler, 2017) and its foundational effects on more advanced mathematics and proficiency in algebra (Booth & Newton, 2012) in the long run.

Disclosure statement

No potential conflict of interest was reported by the authors.

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