Efficient Simulation and Optimal Control for Vehicle Propulsion

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Linköping 2008
Efficient Simulation and Optimal Control for Vehicle Propulsion

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ISBN 978-91-7393-904-1        ISSN 0345-7524

Printed by LiU-Tryck, Linköping, Sweden 2008
To my dear family, Carolina, Viktor and Arvid
Abstract

Efficient drive cycle simulation of longitudinal vehicle propulsion models is an important aid for design and analysis of power trains. Tools on the market today mainly use two different methods for such simulations, forward dynamic or quasi-static inverse simulation. Here known theory for stable inversion of non linear systems is used in order to combine the fast simulation times of the quasi-static inverse simulation with the ability of including transient dynamics as in the forward dynamic simulation. The stable inversion technique with a new implicit driver model together forms a new concept, inverse dynamic simulation. This technique is demonstrated feasible for vehicle propulsion simulation and specifically on three powertrain applications that include important dynamics that can not be handled using quasi-static inverse simulation. The extensions are engine dynamics, drive line dynamics, and gas flow dynamics for diesel engines, which also are selected to represent important properties such as zero dynamics, resonances, and non-minimum phase systems. It is shown that inverse dynamic simulation is easy to set up, gives short simulation times, and gives consistent results for design space exploration. This makes inverse dynamic simulation a suitable method to use for drive cycle simulation, especially in situations requiring many simulations, such as optimization over design space, powertrain configuration optimization, or development of powertrain control strategies.

Optimal vehicle propulsion control is developed with special focus on heavy trucks used for long haulage. The power to mass ratio for a typical heavy duty truck makes even moderate road slopes significant in the sense that it is impossible to keep a constant cruising speed. This gives an interesting problem how to control vehicle speed such that fuel consumption is minimized. Today’s telematic systems together with three dimensional road maps can provide the vehicle control system with information of the road topography. This enables intelligent cruise controllers that utilize this information to control engine fueling and gear shifting such that an optimal speed trajectory is obtained.

First the optimal control problem is solved numerically by dynamic programming, giving a controller with real time capabilities that can be used on-line in the vehicles control system. Simulations of such a system on authentic road profiles show that it has potential for significant fuel savings. To achieve knowledge about the underlying physics that affects the optimal solution, the optimal control problem is solved in detail and analytical expressions for the conditions of optimality are derived. Those expressions are then used to find optimal solutions on constructed test road profiles. Such test cases point out the typical behavior of an optimal solution and also which parameters that are decisive for the fuel minimization problem, and also how they quantitatively influence the behavior. It is for example shown that small non-linearities in the engine torque characteristics have significant effect on the optimal control strategy. The solutions for the non linear engine model have a smoother character but also require longer prediction horizons. For optimal gear ratio control it is shown that the maximum fueling function is essential for the solution. For example, in the case of a continuously variable transmission it is shown that the gear ratio never is chosen such that engine speed exceeds the speed of maximum engine power. For a discrete step transmission the gear shifting losses are essential for the optimal shift positions, but over all the solutions are close to continuous solutions.
Sammanfattning


Optimal styrning av fordsensframdrivning utvecklas med särskilt fokus på landsvägskörning med tunga lastbilar. Förhållandet mellan motoreffekt och fordonsmassa för en typisk tung lastbil gör att det inte är möjligt att hålla en konstant marschfart ens i relativt små väglutningar. Därför är det ett intressant reglerproblem att styra fordonets hastighet så att bransleförbrukningen minimeras. Dagens telematiksystem tillsammans med tredimensionella vägkartor kan levera information om vägtopografi till fordonets styrsystem. Denna information ger möjligheten att skapa intelligenta farthållare för att styra branslemission och växling så att en optimal hastighetstrajektoria erhålls.

Acknowledgments

This work has been carried out under the guidance of Professor Lars Nielsen at Vehicular Systems, Department of Electrical Engineering, Linköpings Universitet, Sweden.

The Swedish Energy Agency through the Center for Automotive Propulsion Simulation CAPSIM, and the Swedish Foundation for Strategic Research, through the VISIMOD project and the Excellence Center in Computer Science and Systems Engineering ECSEL, are gratefully acknowledged for their funding.

I would like to express my gratitude to a number of people:
Professor Lars Nielsen for letting me join this group, for supervision and guidance of this work, and for the many interesting discussions along the way.

My assistant supervisors Jan Åslund and Lars Eriksson for interesting research discussions. My colleague Erik Hellström for his contributions to our collaborative work and for his many interesting ideas on the fuel optimal control problem. All other colleagues at the division of vehicular systems for making it a nice place to work at.

Most of all I would like to thank my wife, Carolina and our sons, Viktor and Arvid, for sharing life with me and making me happy. You mean everything for me. I love you!
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For road vehicles performance, cost, and safety have traditionally been important factors to optimize. Also environmental aspects has emerged as a top priority, where one main focus of the industry has been to reduce fuel consumption and thereby CO₂ emission. In this strive, the power trains and the vehicle’s control systems have become more and more complex. One example is hybrid vehicles where two or more power sources, in a coordinated fashion, propels the vehicle. Another example is advanced traffic information systems that provides information to the driver and/or the vehicle control system. Then the current road and traffic situation can be used in an intelligent way, such that the vehicle can be driven more fuel efficiently. Systems that can adapt to current situation and operate in the most fuel efficient way for that situation are for example cruise controllers, gear shifting programs for automatic transmissions, and energy management systems for hybrid vehicles.

In the development of such complex systems simulation and optimization have become necessary tools when designing a competitive product that is optimized with respect to many criteria. The way to handle this is to use mathematical models of the vehicle, and simulation of such models can to a high extent replace physical prototypes when testing different design choices. To find an optimal design of the physical vehicle and/or its control system, simulation of the mathematical models can be used to evaluate different criteria. This process can be automatized by coupling the simulation with an optimization algorithm. Simulation and optimization also shortens the product development cycle which is necessary to cut development costs. A recent case where simulation studies replaced most prototype testing is the new Fiat 500 with a total development time of 18 months. Simulation is also an important part of many control algorithms where predictions of future vehicle states are made by simulation, and then used to find the optimal control signals.

In this thesis simulation methods and optimization of vehicle propulsion has been studied with the main focus on fuel consumption. The first part of the thesis treats ef-
ficient vehicle propulsion simulation methods that are suitable to for example parameter optimization and control strategy evaluation. The second part treats fuel optimal driving of heavy trucks, and special attention is given to optimal control of engine fueling and optimal gear shifting strategies under the assumption that the road topography ahead of the vehicle is known. The results of the thesis can for example be used to design fuel optimal control strategies, but the methods presented are also applicable for other purposes where one important example is emission minimization.

1.1 Contributions

The contributions of the thesis will here shortly be summarized for each appended paper. A more detailed description is given in the introduction for the respective parts of the thesis.

Efficient Drive Cycle Simulation, Anders Fröberg and Lars Nielsen, IEEE Transactions on Vehicular Technology, vol 57, no 2, 2008. The paper proposes a new method for inverse dynamic vehicle simulation. The new method is compared to forward dynamic simulation regarding for example simulation setup effort, consistency for parameter exploration, and simulation time. Also, a new driver model for inverse dynamic simulation has been developed that makes it easy to define drive cycle tracking that is independent of vehicle properties.

Inverse Dynamic Simulation of Non-Quadratic MIMO Powertrain Models - Application to Hybrid Vehicles, Anders Fröberg, IEEE Vehicle Power and Propulsion Conference 2006. Extending the previous paper, it is demonstrated how typical non-quadratic MIMO power train models can be reformulated enabling inverse dynamic simulation. It is also demonstrated how time variant system order and time variant relative degree is handled.

Controlling Gear Engagement and disengagement on heavy trucks for minimization of fuel consumption, Anders Fröberg, Lars Nielsen, Lars-Gunnar Hedstrom, and Magnus Pettersson, IFAC World Congress 2005. This paper treats fuel optimal use of neutral gear using preview information of road topography. The contribution is to show the magnitude of possible fuel savings by making the correct decision in steep downhill slopes whether to disengage the gear or to cut the fuel injection.

A Real-Time Fuel-Optimal Cruise Controller for Heavy Trucks using Road Topography Information, Erik Hellström, Anders Fröberg, and Lars Nielsen, SAE World Congress 2006. It is shown how a predictive cruise controller with real time performance can be designed using dynamic programming, and the magnitude of possible fuel savings is demonstrated.

Explicit Fuel Optimal Speed Profiles for Heavy Trucks on a Set of Topographic Road Profiles, Anders Fröberg, Erik Hellström, and Lars Nielsen SAE World Congress 2006. To gain knowledge of decisive parameters affecting fuel consumption, fueling control is here studied on constructed road profiles. The simple test cases together with analytical solutions to vehicle motion gives valuable insight into the properties of the optimal control.

Optimal Control Utilizing Analytical Solutions for Heavy Truck Cruise Control, Anders Fröberg and Lars Nielsen, technical report that is an extended version of the paper Optimal fuel and gear ratio control for heavy trucks with piece wise affine engine charac-
teristics, Anders Froberg and Lars Nielsen, Fifth IFAC symposium on advances in automotive control, California, 2007. The fuel optimal control problem treated in the previous paper is solved in more detail. Engine torque is a piece wise affine function of fueling, and optimal gear choice is presented both for a continuously variable transmission as well as for a discrete stepped transmission. The theoretical results are used in a simple rule based predictive cruise controller and the possible fuel savings for that method is demonstrated in simulations on authentic road profiles.

The following work have also been published by the author, but are not included here:


A Method to Extend Inverse Dynamic Simulation of Powertrains with Additional Dynamics , Anders Froberg and Lars Nielsen in 1:st IFAC symposium on Advances in Automotive Control.

Extending the Inverse Vehicle Propulsion Simulation Concept-To Improve Simulation Performance, Anders Froberg, Licentiate thesis.

1 Introduction
Part I

Efficient vehicle propulsion simulation
Modeling and simulation are today widely used tools when designing new power trains and control systems. In for example optimization of a power train, a candidate design is evaluated by use of an objective function. When optimizing complex systems the calculation of the objective function can not always be done by calculation of analytical expressions, instead a simulation of the model has to be done to calculate the objective function. Using simulation in this way possibly a large number of simulations have to be performed of the same model, where some parameters are varied from simulation to simulation. For these situations computational efficiency and consistency between simulations are important properties for the simulation method. The aim in this part of the thesis is to find a simulation method that has good behavior with respect to these properties.

When certifying a vehicle with respect to fuel consumption and emission levels, the vehicle is driven according to a given speed profile, a drive cycle. Hence, a typical task for vehicle propulsion simulation is drive cycle simulation which is the main topic of this part of the thesis. Another typical task for vehicle propulsion simulation is performance simulations which are done to test for example acceleration performance or the vehicles ability to keep speed in steep grades. Although it is not exemplified here, the methods presented in this part can be used for such simulations as well.

Mainly two different methods have been used for vehicle propulsion simulation. Forward dynamic simulation and quasi-static inverse simulation. The forward dynamic simulation typically uses models that consist of a set of ordinary differential equations, ODEs, that uses the drivers input, e.g. throttle, brakes, and steering input, to calculate the vehicles states and speed. Since the method is capable of handling dynamic systems, the prediction of for example fuel consumption and emissions can be accurate. Quasi-static inverse simulation uses speed and acceleration given from the speed profile to calculate the required torque and speed at the wheels. The computation then goes backward through the driveline to compute the generating variables to produce the given torque and speed. Each component uses a static model and hence the prediction ability for this type of sim-
ulation is not as good as for the forward dynamic simulation. On the other hand, due to
the static models the simulation time for these models are very short, which makes it a
suitable method for initial concept studies and parameter optimizations. This first part
of the thesis suggests that known theory for inversion of non-linear systems is used to
combine the merits of forward dynamic simulation and quasi-static inverse simulation.

2.1 Overview and contributions of simulation

This thesis part on simulation consists of two papers. Efficient Drive Cycle Simulation,
Anders Froberg and Lars Nielsen, IEEE Transactions on Vehicular Technology, vol 57, no
2, 2008, and Inverse Dynamic Simulation of Non-Quadratic MIMO Powertrain Models
-Application to Hybrid Vehicles, Anders Froberg, IEEE Vehicle Power and Propulsion
Conference 2006. In the first paper a new inverse dynamic simulation method is pro-
aposed. A comparison of forward and inverse simulation of vehicle propulsion models
is presented. The comparison is done in order to evaluate how well different simulation
methods are suited for different tasks. For example how well the method can capture tran-
sients, how suitable it is for optimization, and how computationally efficient the method
is. A new driver model for inverse dynamic simulation has been developed that makes it
easy to define drive cycle tracking that is independent of vehicle properties. This work has
also been presented more thoroughly in Extending the Inverse Vehicle Propulsion Simu-

The second paper is an extension to the first and deals with some practical issues
for vehicle propulsion simulation. For example, inverse dynamic simulation in general
requires a quadratic system, i.e. a system with equally many inputs and outputs. This
paper demonstrates how this requirement can be relaxed for typically non-quadratic vehi-
cle propulsion models. It is also demonstrated how vehicle propulsion models with time
variant system order and time variant relative degree can be simulated.

2.1.1 Related publications

The following publications by the author also treats the subject of this part, but are not
included here.

Dynamic Vehicle Simulation -Forward, Inverse and New Mixed Possibilities for Op-
timized Design and Control, Anders Froberg and Lars Nielsen in Modeling: Diesel En-
gines, Multi-Dimensional Engine, and Vehicle and Engine Systems. Volume 2002-01-
1619 of SAE Technical paper series SP-1826.

A Method to Extend Inverse Dynamic Simulation of Powertrains with Additional
Dynamics , Anders Froberg and Lars Nielsen in 1:st IFAC symposium on Advances in
Automotive Control.

Extending the Inverse Vehicle Propulsion Simulation Concept-To Improve Simulation
Performance, Anders Froberg, Licentiate thesis.
Part II

Optimal control of vehicle propulsion
Look ahead powertrain control

The problem studied in this part is how to drive a given distance at a given time in the most fuel efficient way. Only highway-like driving for heavy trucks has been studied, but the results should be easily transferred to other driving missions and other vehicles by changing the problem parameters.

Driving a given distance with an average speed is equivalent to drive a given distance at a given time. The problem of minimizing fuel consumption under such conditions can be formulated as follows. Let $m_f(t)$ be the fuel flow into the engine, let $v(t)$ be vehicle speed, and $v_{avg}$ a desired average speed. Then minimization of fuel consumption over the time $t_f$ is

$$\min \int_0^{t_f} m_f(t)dt \quad \text{(3.1)}$$

such that

$$\frac{1}{t_f} \int_0^{t_f} v(t)dt = v_{avg} \quad \text{(3.2)}$$

This problem is studied with slightly different view points in the following papers. First some background to the problem is presented, and then some theory on optimal control is summarized in a perspective used in the papers. Finally the papers are summarized and contributions are indicated.

3.1 Background

For heavy trucks even moderate slopes become significant. For a typical heavy truck, with weigh up to 60 metric tons, the mass of the vehicle makes it impossible to keep a constant cruising speed at most roads due to road slope. A typical road in Sweden has variations in slope between approximately $-5\%$ to $5\%$, see Figure 3.1. A negative slope here defines a downhill slope and a positive slope defines an uphill slope. As shown in Paper E the power to mass ratio for a typical vehicle that weighs 60 tons is too small for
the vehicle to keep cruising speed in uphill slopes with an inclination larger than about 1%. Further, in down hill slopes where the engine does not produce any work, the mass of the vehicle will make it accelerate if no brakes are applied, if the slope is steeper than about −1%. For driving missions on such roads there are several possible speed profiles that have the same average speed but with different fuel consumption. These facts make it interesting to study optimal speed profiles that minimizes fuel consumption on a given driving mission while keeping a desired average vehicle speed.

3.2 Optimal control theory

To solve the problem defined by Equations (3.1) and (3.2) optimal control theory is used. A short review of optimal control as described in the classical textbook [4] will now be given.

Let \( t_0, t_f \) be the initial and final time respectively. Let \( x(t) \) be the state vector and \( u(t) \) be the controls of the system defined by

\[
\dot{x}(t) = f(x(t), u(t), t), \quad x(t_0) = x_0
\]  

(3.3)

Let \( \varphi(x_f, t_f) \) and \( L(x(t), u(t), t) \) be functions that are differentiable sufficiently many times. The optimal control problem is then to find the control law \( u(t) \) that minimizes the performance index

\[
J = \varphi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt
\]  

(3.4)
3.2 Optimal control theory

Now, adjoin the system dynamics, i.e. the constraint (3.3), to the performance index with multipliers $\lambda(t)$

$$\tilde{J} = \varphi(x(t_f), t_f) + \int_0^{t_f} \left( L(x(t), u(t), t) + \lambda^T(t) \left( f(x(t), u(t), t) - \dot{x}(t) \right) \right) dt$$  \hspace{1em} (3.5)

Define a scalar function called the Hamiltonian as:

$$H(x(t), u(t), \lambda(t), t) = L(x(t), u(t), t) + \lambda^T(t) f(x(t), u(t), t)$$  \hspace{1em} (3.6)

In Chapter 2.3 in [4] the necessary optimality conditions are stated as follows: The adjoint dynamics for an optimal solution is

$$\lambda^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \lambda^T \frac{\partial f}{\partial x}$$  \hspace{1em} (3.7)

with boundary conditions

$$\lambda^T(t_f) = \frac{\partial \varphi}{\partial x(t_f)}$$  \hspace{1em} (3.8)

and for an extremum it must hold that

$$\frac{\partial H}{\partial u} = 0, \ t_0 \leq t \leq t_f$$  \hspace{1em} (3.9)

Equations (3.7), (3.8) and (3.9) are often referred to as the Euler-Lagrange equations.

A more general result than (3.9) for optimal control is the maximum principle as stated in [7]. For a minimization problem as here the maximum principle becomes a “minimum principle”. Let the set of allowed controls be $u \in U$, let the optimal control be $u^*$, and let the optimal solution be $x^*$. Then the optimal control is found from

$$\min_{u \in U} H(x^*(t), u(t), \lambda(t)) = H(x^*(t), u^*(t), \lambda(t))$$  \hspace{1em} (3.10)

---

**Example 3.1**

Consider a vehicle with air and rolling resistance forces $F_i$ of the form $F_i = a + bv^2$ where $v$ is vehicle speed. Let the resistance due to road slope be $c \sin \alpha(s)$, the total vehicle inertia $J$, the propulsive force $F$, and the distance traveled $s$. The system dynamics can then be written as

$$\dot{v} = \frac{1}{J} \left( F - a - bv^2 - c \sin \alpha(s) \right)$$  \hspace{1em} (3.11)

$$\dot{s} = v$$  \hspace{1em} (3.12)

The system state vector is thus $x = [v, s]^T$ and the control is $u = F$. If the total propulsive work for the driving time $t_f$ is to be minimized, i.e.

$$\min \int_0^{t_f} Fv dt$$  \hspace{1em} (3.13)
i.e. $L = Fv$. The Hamiltonian for this problem is

$$H = Fv + \lambda_v \frac{1}{J} (F - a - bv^2 - c \sin \alpha(s)) + \lambda_n v$$

(3.14)

and the adjoint dynamics is given by (3.7), i.e.

$$\lambda_v = - \frac{\partial H}{\partial v} = -F + \frac{2}{J} \lambda_n bv - \lambda_s$$

(3.15)

$$\lambda_n = - \frac{\partial H}{\partial s} = \frac{1}{J} \lambda_v c \cos \alpha(s) \frac{d \alpha}{ds}$$

(3.16)

with $\lambda_v(t_f) = \lambda_n(t_f) = 0$. The optimal propulsive force is found by minimizing the Hamiltonian with respect to the control $u$. Since the Hamiltonian is linearly dependent on $F$, the result obtained by using (3.9),

$$\frac{\partial H}{\partial u} = v + \frac{\lambda_v}{J}$$

(3.17)

does not give the optimal control directly. However, this function plays an important role and will be referred to as the control switching function. Using the maximum principle (3.10) it is seen that when the control switching function is negative maximum propulsive force is used and when the control switching function is positive minimum propulsive force is used. When

$$\frac{\partial H}{\partial u} = v + \frac{\lambda_v}{J} = 0$$

(3.18)

for finite periods of time further reasoning needs to be done in order to find the optimal control.

Another way of approaching the problem is as follows. Since road slope $\alpha$ is a function of position it is convenient to change independent variable according to

$$\frac{d}{ds} = \frac{1}{v} \frac{d}{dt}$$

(3.19)

The natural choice of states are then vehicle speed $v$ and traveled time $T$, and the system dynamics is

$$\frac{dv}{ds} = \frac{i}{Jv} (F - a - bv^2 - c \sin \alpha(s))$$

(3.20)

$$\frac{dT}{ds} = \frac{i}{v}$$

(3.21)

The objective function is then

$$\min \int_{0}^{s_f} F ds$$

(3.22)

i.e. $L = F$, and the Hamiltonian is

$$H = F + \lambda_v \frac{1}{Jv} (F - a - bv^2 - c \sin \alpha(s)) + \lambda_n \frac{i}{v}$$

(3.23)
The adjoint dynamics becomes

\[
\begin{align*}
\frac{d\lambda_v}{ds} &= -\frac{\partial H}{\partial v} \lambda_v \left(F - a + b v^2 - c \sin \alpha(s)\right) + \frac{\lambda_f}{\dot{v}}^2 \\
\frac{d\lambda_x}{ds} &= -\frac{\partial H}{\partial T} = 0
\end{align*}
\]  

(3.24)  

(3.25)

Note that the second adjoint variable \(\lambda_f\) now is constant.

### 3.2.1 Optimal control with specified final states

Now, consider the optimization problem studied above but with some of the states specified at the final time \(t_f\). If \(x_i\) is specified at the terminal time the boundary condition \(\frac{\partial \phi}{\partial \lambda_i(t_f)} = \lambda_i(t_f)\) is exchanged to \(x_i(t_f) = x_i^f\). If a system with \(n\) states has states \(j = 1, \ldots, q\) specified at the final time, see Chapter 2.4 in [4], the constraint (3.8) is exchanged to the following, where \(\bar{\theta}_j\) are associated multipliers to be decided by the problem:

\[
\lambda_j(t_f) = \begin{cases} 
\bar{\theta}_j, & j = 1, \ldots, q \\
\frac{\partial \phi}{\partial x_j(t_f)}, & j = q + 1, \ldots, n
\end{cases}
\]

(3.26)

#### Example 3.2

Consider again Example 3.1. For that problem to be of any interest a constraint on the traveled distance at the time \(t_f\) has to be imposed. For the formulation using time as independent variable this means that \(\lambda_v(t_f)\) no longer is zero, but instead that value has to be chosen such that the optimality conditions are satisfied. If also \(\nu(t_f)\) is specified, \(\lambda_v\) is treated similarly. If \(s\) is the independent variable instead of time, the same reasoning is applied to \(\lambda_f(s_f)\) and \(\lambda_v(s_f)\).

### 3.2.2 Restrictions on control variables and states

In some problems it is interesting to study optimal solutions under constraints on functions of the control and state variables. Such a constraint is written

\[
C(x, u, t) \leq 0
\]

(3.27)

This problem is handled by adjoining the constraint to the Hamiltonian with a multiplier \(\mu\)

\[
H = L + \lambda^T f + \mu C
\]

(3.28)

where

\[
\mu \begin{cases} 
> 0, & C = 0 \\
= 0, & C < 0
\end{cases}
\]

(3.29)
Necessary optimality conditions are discussed in Chapter 3.10 in [4], and is as follows.

The adjoint dynamics is

$$\dot{\lambda}^T = -H_e = \begin{cases} -L_x - \lambda^T f_x - \mu C_x, & C = 0 \\ -L_x - \lambda^T f_x, & C < 0 \end{cases}$$

(3.30)

and the optimal control is found from

$$H_u = L_u + \lambda^T f_u + \mu C_u = 0$$

(3.31)

When \( C = 0 \), i.e. \( \mu \neq 0 \), (3.27) and (3.31) together determine \( u(t) \) and \( \mu(t) \).

### 3.2.3 Singular solutions

In many applications the system is in the form

$$\dot{x} = f(x) + g(x)u$$

(3.32)

Also assume that \( L \) is linearly dependent on \( u \), i.e.

$$L = l(x)u$$

(3.33)

The Hamiltonian is then

$$H = l(x)u + \lambda^T (f(x) + g(x)u)$$

(3.34)

and

$$\dot{\lambda}^T = -l_u - \lambda^T (f_x + g_xu)$$

(3.35)

If \( u \) is bounded the minimum of \( H \) may occur on the boundary of the set of all \( u \). It is however possible that there are intervals where a function \( u(t) \) will yield \( x(t) \) and \( \lambda(t) \), such that

$$H_u = l(x) + \lambda^T g(x) = 0$$

(3.36)

A convexity condition for a local minimum is \( H_{uu} \geq 0 \). When (3.36) is fulfilled \( H_{uu} = 0 \) and such solutions are referred to as singular solutions. For such sections (3.36) does not directly determine \( u(t) \), but it must also hold that

$$\frac{d}{dt} H_u = l_x \dot{x} + \dot{\lambda}^T g + \lambda^T g_x \dot{x} = 0$$

(3.37)

Using (3.32) and (3.35) in (3.37) gives

$$\frac{d}{dt} H_u = l_x (f + gu) - (l_u + \lambda^T (f_x + g_xu))g + \lambda^T g_x (f + gu) = 0$$

(3.38)

Note that the terms in the control variable \( u \) cancels out in this expression. Equation (3.38) hence does not help directly in finding \( u \), but it may give valuable insight in the dependence between \( \lambda \) and \( x \), which in turn can give information about \( u \). This method will be used later in paper F. In Chapter 8.3 in [4] Equation (3.38) is differentiated once again with respect to time to get an expression that determines \( u \).
3.3 Dynamic programming

Example 3.3

Consider again the problem of Examples 3.1 and 3.2 formulated with position as independent variable. With \( u = F \) the partial derivative of the Hamiltonian (3.23) with respect to the control is

\[
\frac{\partial H}{\partial F} = 1 + \frac{\dot{\lambda}_v}{Jv} \tag{3.39}
\]

and

\[
\frac{d}{ds} \frac{\partial H}{\partial F} = \frac{1}{Jv} \frac{d\dot{\lambda}_v}{ds} - \frac{\dot{\lambda}_v}{Jv^2} \frac{dv}{ds} = \frac{2\lambda_v b}{Jv^2} + \frac{\lambda_T}{Jv^3} \tag{3.40}
\]

For singular arcs \( \frac{\partial H}{\partial F} = 0 \) for finite distances which from (3.39) means that \( \lambda_v = -Jv \). Using this the following dependency between \( v \) and \( \lambda_T \) is obtained

\[
\frac{d}{ds} \frac{\partial H}{\partial F} = \frac{\lambda_T}{Jv^3} - \frac{2b}{J} = 0 \tag{3.41}
\]

Since \( \lambda_T \) is constant \( v \) is also constant during singular arcs. From that information the optimal control is found from the vehicle dynamics.

3.3 Dynamic programming

Dynamic programming became popular after the works [1, 2], and a more recent textbook on the subject is [3]. In the previous section optimal control was discussed given an initial state and time. In control applications it is often desired to know optimal control solutions from many initial states and times in order to implement feedback control. In [4] it is discussed how this is treated for continuous time problems, and a short review will be given here.

Typically, only one optimal path passes through a point \((x(t), t)\) which means that a unique optimal control \( u^o \) is associated with it. An optimal feedback control law can then be expressed,

\[
u^o = u^o(x, t) \tag{3.42}
\]

Also, for each point \((x(t), t)\), following the optimal path to the surface of the terminal boundary, there is a unique optimal value of the performance index, \( J^* \). Hence, \( J^* \) can be regarded as a function of the starting point, i.e.

\[
J^* = J^*(x, t) \tag{3.43}
\]

This is referred to as the optimal return function.

For an arbitrary initial point \((x, t)\) the performance index is

\[
J = q(x(t_f), t_f) + \int_t^{t_f} L(x(\tau), u(\tau), \tau) d\tau \tag{3.44}
\]
with terminal conditions \( \psi(x(t_f), t_f) = 0 \). The optimal return function (3.43) is then

\[
J^0(x,t) = \min_{u(t)} \{ \psi(x(t_f), t_f) + \int_t^{t_f} L(x(\tau), u(\tau), \tau) d\tau \}
\]  
(3.45)

with boundary condition that \( J^0(x,t) = \varphi(x,t) \) on the hypersurface \( \psi(x,t) = 0 \). In [3] there is a thorough description how this problem is solved iteratively by discretizing time. There, time is divided in \( N + 1 \) stages and the system is described as

\[
x_{k+1} = f(x_k, u_k)
\]  
(3.46)

The return function, or cost-to-go as it is called by Bertsekas [3], in the last stage is

\[
J_N(x_N) = g_N(x_N)
\]  
(3.47)

where \( g_N \) is the same function as \( \varphi \) used in [4], see (3.4). Now, let the cost at time instant \( k \) be \( g_k(x_k, u_k) \). Then, from the value of the end cost, (3.47), the optimal control policy is found by iterating backward in time according to

\[
J_k(x_k) = \min_{u_k} \{ g_k(x_k, u_k) + J_{k+1}(f_k(x_k, u_k)) \}, \ k = 0, 1, \ldots, N - 1
\]  
(3.48)

Let the optimal value of the cost-to-go function at stage \( k \) be \( J^*_k(x_k) \).
Let \( \pi^k = \{ \mu_k, \mu_{k+1}, \ldots, \mu_{N-1} \} \). Then the optimal value of the cost to go function is

\[
J^*_k(x_k) = \min_{\pi^k} \{ g_N(x_N) + \sum_{i=k}^{N-1} g_i(x, \mu(x_i)) \}
\]  
(3.49)

This expression can be compared to (3.45). In [3] it is shown that the function \( J^*_k(x_k) \) are equal to the functions \( J_k(x_k) \) generated by the dynamic programming algorithm (3.48).

When solving an optimal control problem it is also practical to discretize the state space. The dynamic programming problem can then be described by a transition graph. In a deterministic graph the same optimal solution is found independent if the algorithm is run backward or forward in the graph.

---

**Example 3.4**

Consider again example 3.1 but now to be solved with the discrete dynamic programming algorithm. The vehicle dynamics using the Euler approximation with step length \( h \) is

\[
v_{k+1} = f(v_k, F_k) = \frac{h}{\mu} (F_k - a - bx_k^2 - cx_k \sin(\alpha)) + v_k
\]  
(3.50)

Let the final state cost be \( g_N(v_N) = 0 \) and the cost for a transition from state \( v_i \) to \( v_{i+1} \) be the propulsive work \( hF_i \). Then the state transition cost is

\[
g_k(v_k, F_k) = hF_k, \ k = 0, \ldots, N - 1
\]  
(3.51)

The optimal control is then found from the backward iteration, with \( J_N = 0 \),

\[
J_k(x_k) = \min_{\pi_k} \{ g_k(v_k, F_k) + J_{k+1}(f_k(v_k, F_k)) \}, \ k = 0, \ldots, N - 1
\]  
(3.52)

The dynamic programming method described is used in Paper C and D. An example simulation is seen in Figure 3.2, which also will be the starting point of the next section.
Figure 3.2: A simulation example of an authentic road from Paper D. Two simulations are depicted, one using a standard cruise controller (PI) and one with a predictive controller using dynamic programming (MPC). The top plot shows the road topography, the second vehicle velocity, the third normalized accelerator and brake levels, and the fourth gear selection.
3.4 Towards practical rule based control

In Figure 3.2, part of an example simulation from Paper D is presented. In that case the dynamic programming algorithm has been used in each sample to calculate the optimal accelerator level, i.e. fueling, optimal brake level, and optimal gear choice. Some typical behavior can be observed in the figure, e.g., prior to a significant incline the vehicle is accelerated which can be seen at 1, 4 and 6 km, and prior to a decline the vehicle is retarded which can be seen at 5 and 7 km. Inspired by such observations it is interesting to see if simple rules can be found that can be used in an intuitive and more computationally efficient control system, and to what extent such a controller is able to save fuel.

To quantify when and how to take actions such as acceleration before inclines and retardation before declines, the optimality conditions from Section 3.2 can be used.

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Example 3.5

Again consider Example 3.1. Recall the control switching function (3.17), and that if \( \frac{dv}{dt} > 0 \) minimum tractive force is used, and that if \( \frac{dv}{dt} < 0 \) maximum tractive force is used. It is seen that vehicle inertia, which mostly depends on vehicle mass, is a decisive parameter for the optimal control, but also that the relation between vehicle speed \( v \) and the adjoint variable \( \lambda_v \) is important.

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Those equations also gives a possibility to find the decisive parameters for the optimal control. It is natural that for example engine torque, vehicle mass, and road inclination are decisive but it is not clear in what relation they will enter into practical rules. By using simple yet descriptive models and formulating the analytical optimality conditions a lot of insight can be obtained. The results point toward a practical rule-based controller as is described in Paper F.

3.5 Overview and contributions of the papers

A brief overview of the papers will here be given and the contributions will be stated.

Controlling Gear Engagement and disengagement on heavy trucks for minimization of fuel consumption, Anders Fröberg, Lars Nielsen, Lars-Gunnar Hedstrom, and Magnus Pettersson, IFAC World Congress 2005. In steep downhill slopes a heavy truck will accelerate even though there is no fuel injected to the engine, i.e. the engine produces negative work due to friction. A possibility to reduce the total powertrain friction is to engage neutral gear. This will increase the vehicle acceleration and the gain in kinetic energy will increase. However, to drive systems like power steering etc., the engine has to be run in idle conditions and thus consuming some amount of fuel. The contribution of this paper is to show the magnitude of possible fuel savings by making the correct decision in significant downhill slopes whether to disengage the gear or to cut the fuel injection.

A Real-Time Fuel-Optimal Cruise Controller for Heavy Trucks using Road Topography Information, Erik Hellström, Anders Fröberg, and Lars Nielsen, SAE World Congress 2006. If knowledge of road profile ahead of the vehicle is known that information can be used to control engine fueling and gear choice in a fuel optimal way. This paper shows how a predictive cruise controller with real time performance can be designed using dynamic programming, and the magnitude of possible fuel savings is demonstrated through
simulations on authentic road profiles. This work is based on the master thesis by Erik Hellström [6] which was supervised by Anders Fröberg.

Explicit Fuel Optimal Speed Profiles for Heavy Trucks on a Set of Topographic Road Profiles, Anders Froberg, Erik Hellstrom, and Lars Nielsen, SAE World Congress 2006. To gain knowledge of decisive parameters affecting fuel consumption fueling control is here studied on constructed road profiles. The simple test cases together with analytical solutions to vehicle motion gives valuable insight into the properties of the optimal control. The results can also be used to validate the behavior of numerical predictive controllers such as presented in Paper D.

Optimal Control Utilizing Analytical Solutions for Heavy Truck Cruise Control, Anders Fröberg and Lars Nielsen, technical report that is an extended version of the paper [5]. Optimal fuel and gear ratio control for heavy trucks with piece wise affine engine characteristics. Again solutions on constructed road profiles are studied, but now by solving the optimal control problem in more detail. The analytical expressions that are derived for the necessary optimality conditions provide insight in how each parameter affects the optimal solution.

Optimal control solutions for affine engine torque modeling are compared to solutions for piece-wise affine models, and it is shown that even small non-linearities have significant effect on optimal control switch points. Solutions on optimal gear ratio control for both a continuous variable transmission and a discrete stepped transmission show that the maximum fueling function and the gear shifting losses are important for the optimal control behavior.

The theoretical results are used in a simple rule based predictive cruise controller and the possible fuel savings for that method is demonstrated in simulations on authentic road profiles showing promising results.

Bibliography


