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Signals of Opportunity Based Hydrophone Array Shape and Orientation Estimation

Isaac Skog, Senior Member, IEEE, Magnus Lundberg Nordenvaad, and Gustaf Hendeby, Senior Member, IEEE

Abstract—A signal-of-opportunity-based method to automatically calibrate the orientations and shapes of a set of hydrophone arrays using the sound emitted from nearby ships is presented. The calibration problem is formulated as a simultaneous localization and mapping (SLAM) problem, where the locations, orientations, and shapes of the arrays are viewed as the unknown map states, and the position, velocity, etc., of the source as the unknown dynamic states. A sequential likelihood ratio test, together with a maximum a posteriori source location estimator, is used to automatically detect suitable sources and initialize the calibration procedure. The performance of the proposed method is evaluated using data from two 56-element hydrophone arrays. Results from two sea trials indicate that: (a) signal sources suitable for the calibration can be automatically detected; (b) the shapes and orientations of the arrays can be consistently estimated from the different data sets with shape variations of a few decimeters and orientation variations of less than two degrees; and (c) the uncertainty bounds calculated by the calibration method are in agreement with the true calibration uncertainties. Furthermore, the hearing time record from a sea trial with an autonomous mobile underwater signal source also shows the efficacy of the proposed calibration method. In the studied scenario the root mean square bearing tracking error was reduced from 4 to 1 degree when using the calibrated array shapes compared to assuming the arrays’ to be straight lines. Also, the beamforming gain increased by approximately 1 decibel.

Index Terms—Calibration, Hydrophone Array, Tracking, Simultaneous Localization and Mapping, Beamforming

I. INTRODUCTION

For surveillance purposes it is often desirable to be able to quickly deploy a set of hydrophone arrays that can be used to detect, track, and identify marine vessels. If the deployed arrays are of a non-rigid construction, as is the case when hydrophone elements are mounted along a cable, the exact shapes of the arrays after deployment will be unknown. Moreover, the exact locations and orientations of arrays on the sea floor will also be unknown. This will compromise the performance of the surveillance systems [1], [2]. Therefore, it is necessary to estimate the location, orientation, and shape of each array after deployment. This can be done via an active calibration procedure, where sound signals are transmitted at various locations in the vicinity of the arrays [3], [4]. However, this is a time-consuming process that also may reveal the existence of the hydrophone arrays to undesirable counterparts.

To avoid the active calibration phase, different passive calibration methods, where signals of opportunity from nearby vessels are used to estimate the location, orientation, and shape of an array, have been suggested. Maximum likelihood estimators for calibration using both near- and far-field static sources have been presented, and the achievable calibration accuracy has been analyzed using the Cramér-Rao bound [1], [2], [5]–[8]. These maximum likelihood estimation methods have later also been extended to array shape estimation in towed array systems [9], [10]. Other methods for array shape calibration using signals of opportunity that have been suggested include matched-field inversion [11], [12], frequency-wavenumber data processing [13], and ambient noise processing [14]. However, most of the proposed calibration methods have not been fitted into any Bayesian framework, and the inclusion of prior information regarding the locations, orientations, and shapes of the arrays can generally only be done in more or less ad-hoc manners. Nor do they provide any measure of the accuracy of the calibration. Two exceptions are [15] and [16], where the calibration is done using a regularized
non-linear least squares framework and the regularization is used to impose prior information. Furthermore, a majority of the suggested calibration strategies have only been evaluated using simulated data or one-off sea trials, and the consistency of the method across multiple real-world datasets collected at different locations has not been verified; a few exceptions worth highlighting, where the proposed calibration methods are experimentally validated, are [12], [14], [16] and [17]. Moreover, most methods proposed for location, orientation, and shape calibration of non-towed arrays are based upon batch processing, and methods that automatically detect suitable signal sources and sequentially estimate and refine the parameters online as more and more data is collected are lacking.

To that end, the problem of calibrating the geometries of a set of arrays will be formulated as a simultaneous localization and mapping (SLAM) problem [18], where the map states encode the locations, orientations, and shapes of the arrays and the dynamic states encode the position and motion dynamics of the signal source. Time difference of arrival (TDOA) estimates extracted from the received sound signals are then used as observations in the SLAM process; see Fig. 1 for an illustration of the considered setup. The SLAM problem formulation allows prior information about the locations, orientations, and shapes of the arrays, as well as knowledge about the surrounding environment, to be included in the calibration process. Further, it facilitates the use of standard Bayesian inference techniques (both sequential and batch inference methods can be applied), such as the Kalman filter, particle filter, or Markov chain Monte Carlo methods, in the estimation process, as well as in the source detection process. The presented work is an extension of the work in [19] and now includes: (i) a method to automatically detect suitable signal sources and initialize the Kalman filter-based SLAM process; (ii) results from two seatrials with different sound sources; and (iii) a real-world example of the beamforming performance before and after calibration of the arrays.

II. SIGNAL MODEL

A generic SLAM state-space model is given by [20], [21]

\[
\begin{align*}
\mathbf{x}_{k+1} &= f(\mathbf{x}_k, \mathbf{w}_k), \\
\mathbf{m}_{k+1} &= \mathbf{m}_k, \\
\mathbf{y}_k &= h(\mathbf{x}_k, \mathbf{m}_k) + \mathbf{e}_k,
\end{align*}
\]

where \( \mathbf{x}_k \in \mathbb{R}^{N_x} \), \( \mathbf{m}_k \in \mathbb{R}^{N_m} \), and \( \mathbf{y}_k \in \mathbb{R}^{N_y} \) denote the dynamic state, the map state, and the observation vector at time instant \( k \in \mathbb{N} \), respectively. Further, \( \mathbf{w}_k \in \mathbb{R}^{N_w} \) and \( \mathbf{e}_k \in \mathbb{R}^{N_e} \) denote the process and measurement noise, respectively. The process and measurement noise processes are assumed to be mutually independent white noise process normal distributed as \( \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \) and \( \mathbf{e}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \), respectively. The function \( f(\cdot) \) models how the dynamic states evolve with time, and the function \( h(\cdot) \) models the relationship between the observations and the unknown dynamics and map states.

A. Source and map states

In the considered array shape estimation setup, the state \( \mathbf{x}_k \) describes the dynamics of the signal source and is defined as

\[
\mathbf{x}_k \triangleq \begin{bmatrix} \mathbf{s}_k^T & \mathbf{v}_k^T & \mathbf{\xi}_k^T \end{bmatrix}^T.
\]

Here \( \mathbf{s}_k \in \mathbb{R}^2 \) and \( \mathbf{v}_k \in \mathbb{R}^2 \) denote the location and velocity of the source in the horizontal plane, respectively. (Without loss of generality, it has been assumed that the source is close to the surface.) Further, \( \mathbf{\xi}_k \in \mathbb{R}^{L \times (N_r - 1)} \) denotes possible additional auxiliary states needed to describe the motion dynamics. A review of commonly used state dynamics models in target tracking applications can be found in [22]. Within the scope of this paper, a constant velocity model will be used to model the source dynamics. That is, \( \mathbf{x}_k = [\mathbf{s}_k^T \ \mathbf{v}_k^T]^T \) and

\[
f(\mathbf{x}_k, \mathbf{w}_k) \triangleq \begin{bmatrix} \mathbf{I} & \Delta t_k \mathbf{I} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \Delta t_k^2 \mathbf{I} \\ \Delta t_k \mathbf{I} \end{bmatrix} \mathbf{w}_k, \tag{3}
\]

Here \( \Delta t_k = t_{k+1} - t_k \) and \( t_k \) denotes the sample time. Further, \( \mathbf{0} \) and \( \mathbf{I} \) denote all-zeros and identity matrices, respectively.

Assume that the considered system setup consists of \( L \) hydrophone arrays. The map state is then defined as

\[
\mathbf{m}_k \triangleq \begin{bmatrix} (\mathbf{m}_1^T) & \cdots & (\mathbf{m}_L^T) \end{bmatrix}^T, \tag{4}
\]

where the sub map \( \mathbf{m}_l \in \mathbb{R}^{N_l} \), encodes the location, orientation, and shape (geometry) of the \( l \)th array. That is, there exists a function \( g_{j,l} : \mathbb{R}^{N_l} \rightarrow \mathbb{R}^3 \), such that

\[
\mathbf{p}_{j,l} = g_{j,l}(\mathbf{m}_l), \quad j = 1, \ldots, M_l, \tag{5}
\]

where \( \mathbf{p}_{j,l} \in \mathbb{R}^3 \) denotes the location of the \( j \)th hydrophone element in the \( l \)th array. Further, \( M_l \) denotes the total number of hydrophone elements in the \( l \)th array. Note that the dynamics of the source are modeled in two dimensions, whereas the geometries of the arrays are modeled in three dimensions.

B. Hydrophone Array Map Model

Since the goal is to determine the locations of the hydrophone elements in the arrays, one possibility is to directly include the locations of the elements as unknown map states. However, if the hydrophones are mounted on a cable, the maximum distance between hydrophones in the array is
known. One way to incorporate this information is via a set of inequality constraints [23]. However, the computational complexity grows rapidly with the number of imposed constraints, which makes it difficult to apply this method to arrays with many hydrophones. Another way is to use an array shape model similar to that in [24], where the shape of the array is approximated using a set of linked ridge segments; see Fig. 2 for an illustration. Next, such a model for a nested uniform array will be presented.

Consider a nested uniform array designed for the minimum wavelength $\lambda_{\min} \in \mathbb{R}^+$. Define the segment length $r$ as $r = \lambda_{\min} / (2n)$, where $n \in \mathbb{N}$. Then the maximum distance between any hydrophone pair $i$ and $j$ in the $l$th array is an integer multiple $l_{j,i}^l \in \mathbb{N}$ of $r$. That is, $||p_{i,j}^l - p_{i,j}^l|| \leq l_{j,i}^l$, with equality only if the array is stretched and perfectly straight. Next, denote the relative inclination and azimuth angle between link segment $i$ and segment $i + 1$ in the array model as $\psi_{i,j}^l$ and $\phi_{i,j}^l$, respectively. The position $p_{i,j}^l$ of the $j$th hydrophone element in the $l$th array can then be modeled as

$$p_{i,j}^l = p_{i,j}^{l-1} + r \sum_{i=1}^{l-1} u(\psi_{i,j}^l, \phi_{i,j}^l)$$

(6a)

where

$$u(\psi_{i,j}^l, \phi_{i,j}^l) = \begin{bmatrix} \sin(\psi_{i,j}^l) \cos(\phi_{i,j}^l) \\ \sin(\psi_{i,j}^l) \sin(\phi_{i,j}^l) \\ \cos(\psi_{i,j}^l) \end{bmatrix}$$

(6b)

and

$$\psi_{i,j}^l = \psi_{i,j}^{l-1} + \delta \psi_{i-1,j}^{l-1}, \quad \phi_{i,j}^l = \phi_{i,j}^{l-1} + \delta \phi_{i-1,j}^{l-1}$$

(6c)

Here $\psi_{i,j}^l \in [-\pi/2, \pi/2)$ and $\phi_{i,j}^l \in [-\pi, \pi)$ are the inclinational and azimuthal angles, respectively, of the first segment in the array model. To that end, the sub-map vector $m^{(i)}$ that encodes the location, orientation, and shape of the $l$th array becomes

$$m^{(i)} = [(\psi_{i,j}^l)^\top, \phi_{i,j}^l, \psi_{i,j}^{l-1}, \phi_{i,j}^{l-1}, \delta \psi_{i,j}^l, \delta \phi_{i,j}^l, \delta \psi_{i,j}^{l-1}, \delta \phi_{i,j}^{l-1}, \ldots]$$

(7)

And the corresponding function $g_j(\cdot)$ in (5) that maps the sub-map vector $m^{(i)}$ to hydrophone element positions in Cartesian coordinates is given by (6). It is worth noting that the number of parameters in the model, which determines its flexibility, is inversely proportional to the segment length $r$. Therefore, the choice of the segment length $r$ is a trade-off between the model flexibility and calibration accuracy. Theoretically, an appropriate segment length can be found from the model evidence calculated via the employed inference algorithm [25]. In practice, it is easier to select it based on knowledge about the flexibility of the arrays, the expected distortion of the ideal array shapes, etc.

### C. TDOA observation model

Assume that TDOA estimates are calculated from the signals received by adjacent hydrophone elements in each array. Let the vector of TDOA estimates for each array be defined as

$$y_k^l = [\hat{\tau}_k^l, \ldots, \hat{\tau}_k^{M-1,l}]^\top, \quad l = 1, \ldots, L$$

(8)

where $\hat{\tau}_k^{l,j} \in \mathbb{R}$ denotes the TDOA estimate obtained by correlating the signal from the $j$th hydrophone element with the signal from the $(j+1)$th element [26]. Further, assume that no TDOA estimates are calculated across the different arrays. The general observation model in (1c) then becomes

$$y_k^{\text{DOA}} = h^{\text{DOA}}(x_k, m_k) + e_k^{\text{DOA}}$$

(9a)

where

$$y_k^{\text{DOA}} = [y_k^1 \ldots y_k^L]^\top$$

(9b)

$$e_k^{\text{DOA}} = [e_k^1 \ldots e_k^L]^\top$$

(9c)

$$h^{\text{DOA}}(x_k, m_k) = \begin{bmatrix} h^1(x_k, m_k) \\ \vdots \\ h^L(x_k, m_k) \end{bmatrix}$$

(9d)

and

$$|h^l(x_k, m_k)|_j = c^{-1}(||s_k^l - g_{j+1,l}(m^l)|| - ||s_k^l - g_{j,l}(m^l)||)$$

(9e)

Here $|a|_l$ and $c \in \mathbb{R}^+$ denote the $j$th element of the vector $a$ and the sound propagation speed, respectively. Further, the TDOA estimation error $e_k^{\text{DOA}}$ is assumed white and normal distributed with covariance $R^{\text{DOA}} = \sigma_e^2 I$, where $\sigma_e^2 \in \mathbb{R}^+$ denotes the variance of the error in the TDOA estimates. Moreover, $s_k^l \in \mathbb{R}^3$ denotes the location of the source when the sound was emitted and is approximated as $s_k^l = [s_k^l \ 0]^\top \ \forall l$.

In the presented TDOA observation model (8)–(9) several design choices and approximations have been made. The appropriateness of the design choices, and the validity of these approximations, will depend on the condition under which the system is used, and constitute a trade-off between model accuracy and complexity. The most important design choices and approximations are as follows.

First, in (8) only TDOA estimates from neighboring hydrophones are used as observations. If the errors in TDOA estimates calculated from different combinations of hydrophone observations are independent and the probability distributions of the errors are known, then the optimal thing to do from an information theoretical perspective is to use the estimates from all possible combinations of hydrophones [27, p.35]. However, for arrays with many hydrophones, the computational complexity of calculating all of these TDOA estimates can be unmanageable and it is common to only calculate TDOA estimates relative to a single reference hydrophone. But since the coherency between the signal that is observed by the hydrophones decreases with the distance between the hydrophones, it may be difficult to calculate reliable TDOA estimates from hydrophones far apart, especially in shallow waters [28]. For that reason, TDOA estimates from neighboring hydrophones are used as observations.
hydrophones are used here. Other hydrophones combinations can of course be used to calculate the TDOA estimates, and the selected combination will affect the calibration accuracy of the inter-hydrophone distance and total array shape. For example, the use of neighboring hydrophones results in a better inter-hydrophone distance calibration than using a single reference hydrophone, but at the cost of a worse total array shape calibration.

Second, in (9e) it has been assumed that the sound propagation speed $c$ is known and constant throughout the whole water volume. If the sound propagation speed is uncertain, it should be included as an unknown parameter in the map vector $m_k$. The calibration problem then becomes more complex, but the description of the uncertainties in the measurements and the estimated calibration parameters more consistent with reality.

Third, it has been assumed that the TDOA estimates are uncorrelated, which in reality they are not [29]. However, relative to other modeling errors related to sound propagation, background noise, etc., these correlations can often be neglected in practice.

Finally, if the source is moving or the arrays are not collocated, the location of the source when the sound was emitted and when it was received at the $l$th array will be different [30]. For slow-moving sources and when the arrays are closely spaced, the approximation $s^e_k = [s^e_k, 0]^T$ is valid. However, for fast-moving sources or when calibrating multiple arrays that are far apart, the difference in the location of the source when the sound was emitted and when it was received is non-negligible and methods such as that described in [31] may have to be used to compensate for this effect.

D. Source location information

In certain situations, information about the location of the signal source is available via some external source. One such case is when the source is equipped with an automatic identification system (AIS) transponder, which broadcasts the location and course of the source [32]. This information can be included in the state-space model as additional observations in (9), the problem of estimating the source states according to the assumed signal model must be present and an initial state estimate and covariance must be provided. To start the SLAM process a source emitting sound at the AIS reference points must be included when initializing the calibration process.

E. Observability

It is noteworthy that, given only TDOA observations, the SLAM state-space model is not observable and there will be translational and rotational ambiguities in the estimated locations and orientations of the arrays. These ambiguities can be resolved by: (i) assuming the location of one hydrophone element, as well as the direction from this hydrophone element to another hydrophone, to be known; or (ii) including additional measurements from, e.g., an AIS transponder or radar station, that provides an absolute location or orientation measurements with respect to a known frame of reference. Note that, to get a correct description of the uncertainty in the calibration result, the uncertainty in these reference measurements or reference points must be included when initializing the calibration process.

III. State Estimation

Given the state-space model defined by (1a), (1b), and (9), the problem of estimating the source states $x_k$ and $y_k$ is a non-linear problem. There is a plethora of algorithms, such as the extended Kalman filter (EKF), the Unscented Kalman filter, and the particle filter, for solving non-linear filtering and smoothing problems; each algorithm has its pros and cons in terms of estimation performance and computational complexity. Several recursive and batch algorithms specially tailored to utilize the mathematical structure of the SLAM problem have also been developed, see, e.g., the FastSLAM algorithm in [33], the Expectation-Maximization algorithm in [34], and the graph-based SLAM algorithms in [35]. Here, the EKF SLAM algorithm will be used due to its simplicity, while more advanced SLAM algorithms may be better suited in cases where the state space model contains highly nonlinear components. Pseudocode for one iteration of the EKF SLAM algorithm is shown in Alg. 1.

In Alg. 1 the quantities $X_k$ and $\hat{m}_k$ denote the estimated source state and map state, respectively, at time instant $k$. Further, $P_k$ denotes the covariance of the estimated states, i.e.,

$$P_k \triangleq \begin{bmatrix} P_{xx} & P_{xb} \\ P_{bx} & P_{bb} \end{bmatrix}$$

where $P_{xx} \triangleq \text{Cov}\{\hat{x}_k, \hat{x}_k\}$, $P_{bb} \triangleq \text{Cov}\{\hat{m}_k, \hat{m}_k\}$, and $P_{xb} = (P_{bx})^T \triangleq \text{Cov}\{\hat{x}_k, \hat{m}_k\}$.

A. Acceptance and rejection of a potential source

To start the SLAM process a source emitting sound according to the assumed signal model must be present and an initial state estimate and covariance must be provided. To automate this process, a method for automatically identifying suitable signal sources and initializing the SLAM algorithm will be presented next. The method is inspired by a sequential hypothesis test method commonly used for track initiation, confirmation, and deletion, in target tracking applications (see [36, p. 325–335]) and takes its starting point in a
Algorithm 1 One iteration of the EKF SLAM algorithm.
1: function EKF SLAM($y^{\text{DOA}}_{k+1}, \hat{x}_k, \hat{m}_k, P_k$)  
2:  % Time update  
3: \( \hat{x}_{k+1|k} = f(\hat{x}_k, 0) \)  
4: \( \hat{m}_{k+1|k} = \hat{m}_k \)  
5: \( P_{k+1|k} = \left[ \begin{array}{c} P_k \rho_0 P_k F_k \bar{G}_k Q_k F_k^\top + G_k Q_k G_k^\top \end{array} \right] P_k \rho_0 \)  
6: \( F = \nabla_x f(x, w) \big|_{x = \hat{x}_k}, \quad G = \nabla_w f(x, w) \big|_{w = 0} \)  
7: % Measurement update  
8: \( \hat{y}^{\text{DOA}}_{k+1|k} = h(\hat{x}_{k+1|k}, \hat{m}_{k+1|k}) \)  
9: \( \hat{x}_{k+1} = \left[ \begin{array}{c} \hat{x}_{k+1|k} \\ \hat{m}_{k+1|k} \end{array} \right] + K_{k+1} (y^{\text{DOA}}_{k+1|k} - \hat{y}^{\text{DOA}}_{k+1|k}) \)  
10: \( P_{k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} \)  
11: \( K_{k+1} = P_{k+1|k} H_{k+1}^\top S_{k+1}^{-1} \)  
12: \( S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^\top + R^{\text{DOA}} \)  
13: \( H_k = \nabla_m h^{\text{DOA}}(x, m) \big|_{x = \hat{x}_{k+1|k}} \bigg|_{m = \hat{m}_{k+1|k}} \)  
14: \( \nabla_m h^{\text{DOA}}(x, m) \big|_{x = \hat{x}_{k+1|k}} \bigg|_{m = \hat{m}_{k+1|k}} \)  
15: return \( \hat{x}_{k+1}, \hat{m}_{k+1}, P_{k+1} \).

repeatedly used sequential likelihood ratio test (SLRT) [37, pp. 767]. To that end, introduce the two hypotheses
\( H_0 \): Observations do not fit the signal model, i.e., no suitable calibration source exists.
\( H_1 \): Observations fit the signal model, i.e., a suitable calibration source exists.
and the sequentially calculated log-likelihood ratio
\[ T_{k+1} = T_k + \eta_k, \quad T_k = 0, \quad k = k_o, k_o + 1, \ldots \] (12b)
where
\[ \eta_k = \begin{cases} \log \frac{p(y_k | x_k, m_k, H_1) \rho_0}{p(y_k | x_k, m_k, H_0)} & k > k_o \\ \log \frac{p(y_k | x_k, m_k, H_1) \rho_0}{p(y_k | x_k, m_k, H_0)} & k = k_o. \end{cases} \] (12c)
and \( k_o \) denotes the time instant when the test is started. Then the stopping rule of the test is given by
\[ \begin{cases} T_k \leq \gamma_0 : \quad \text{Accept } H_0 \text{ and restart test} \\ \gamma_0 < T_k < \gamma_1 : \quad \text{Continue to monitor} \\ T_k \geq \gamma_1 : \quad \text{Accept } H_1 \text{ and start calibration} \end{cases} \] (12d)
where the thresholds \( \gamma_0 \) and \( \gamma_1 \) depends on the desired probability of detection and false alarm rate. Note that if the \( H_0 \) hypothesis is accepted, i.e., no suitable calibration source exists, then the test is restarted to continue the search for a suitable calibration source. Hence, the first time the SLRT is started \( k_0 = 1 \), whereas later \( k_o \) will depend upon when the test is restarted. Further, note that if the threshold \( \gamma_0 = 0 \) the SLRT becomes equivalent to the Page’s test for detecting signals with unknown start times [37, pp. 781]. Next, methods for calculating the likelihoods \( p(y^{\text{DOA}}_k | y^{\text{DOA}}_{k_0:k-1}, H_0) \) and \( p(y^{\text{DOA}}_k | y^{\text{DOA}}_{k_0:k-1}, H_1) \) will be presented.

Algorithm 2 Automated array calibration.
1: \( \triangleright \) Set initial array states \( \hat{m}_0 \) and covariance \( \hat{P}^{\text{mm}}_0 \) using existing knowledge about the arrays’ shapes, orientations, and locations.  
2: \( k \leftarrow 0 \)  
3: repeat  
4: % Source state and hypothesis test initialization  
5: \( \triangleright \) Estimate position \( \hat{x}_k \) of a potential source from \( y^{\text{DOA}}_k \) using (17) with \( \hat{m}_k \) and \( \hat{P}^{\text{mm}}_k \).  
6: \( \triangleright \) Set initial state \( \hat{x}_k \) and covariance \( P_k \) using (18)-(20).  
7: \( \triangleright \) Reset SLRT statistics in (12b) and then initialize the test using (13) and (21).  
8: \( \triangleright \) Reset the source existence indicator, i.e., \( \text{Source exist} \leftarrow false \)  
9: % Acceptance or rejection of a potential source  
10: while Source exist = false do  
11: \( k \leftarrow k + 1 \)  
12: \( \triangleright \) Run one step of the EKF SLAM algorithm in Alg. 1, i.e., \( x_k, m_k, P_k \leftarrow \text{EKF SLAM}(y^{\text{DOA}}_k, \hat{x}_{k-1}, \hat{m}_{k-1}, P_{k-1}) \).  
13: \( \triangleright \) Calculate the likelihood for observing \( y^{\text{DOA}}_k \) under hypothesis \( H_0 \) using (13).  
14: \( \triangleright \) Calculate the likelihood for observing \( y^{\text{DOA}}_k \) under hypothesis \( H_1 \) using (14).  
15: \( \triangleright \) Update SLRT statistics \( T_k \) using (12b).  
16: if \( T_k \leq \gamma_0 \) then  
17: \( \triangleright \) Source exist = true  
18: \( \triangleright \triangleright \) Source exist = false  
19: else if \( T_k \geq \gamma_1 \) then  
20: \( \triangleright \) Source exist = true  
21: \( \triangleright \triangleright \) Source exist = false  
22: \( \triangleright \triangleright \triangleright \) Store current array state and covariance, i.e., \( \hat{m}_k \leftarrow \hat{m}_k, \hat{P}^{\text{mm}}_k \leftarrow \hat{P}^{\text{mm}}_k \) for later use.  
23: until Arrays shapes are sufficiently calibrated  
24: return Array states \( \hat{m} \) and covariance \( \hat{P}^{\text{mm}} \)

Under the \( H_0 \) hypothesis, which includes the cases with no signal source or multiple signal sources, the behavior of the TDOA estimates is typically unknown, except for the mini-
um and maximum values they can take. Therefore, since the uniform distribution is the least informative distribution with bounded support [25], the TDOA estimates are modeled as statistically independent and uniformly distributed. Hence, the likelihood for the observation \( y_{k}^{\text{TDOA}} \) under the \( \mathcal{H}_0 \) hypothesis becomes

\[
p(y_{k}^{\text{TDOA}} | \mathcal{H}_0) = \prod_{l=1}^{L} \prod_{j=1}^{M_{l}-1} p_{\mu}(\tau_{j,l}^{H}; -\tau_{j,l}^{\text{max}}, \tau_{j,l}^{\text{max}}),
\]

which is independent of the previous observations \( y_{k_{0},k-1}^{\text{TDOA}} \). Here \( p_{\mu}(\cdot; a, b) \) denotes the uniform probability density function with lower and upper bound \( a \) and \( b \), respectively.

Further, \( \tau_{\text{max}}^{j,l} \equiv c/(r_{j+1,l}^{l+1,j}) \) is the maximum possible sound propagation time between hydrophone \( j \) and \( j+1 \) in array \( l \), which is a constant that can be calculated approximately from the assumed ideal array shapes. Note that the use of independent uniform distributions to model the distribution of TDOA estimates under the \( \mathcal{H}_0 \) hypothesis may be far from optimal. If needed and suitable data is available, a better distribution model that increases the detection performance can be learned using e.g., the kernel density method [25].

Under the \( \mathcal{H}_1 \) hypothesis, where the TDOA observations are assumed to fit the SLAM signal model (1), the likelihood of the observations can be calculated using the EKF SLAM algorithm. More specifically, for \( k > k_0 \), the likelihood of the observation \( y_{k}^{\text{TDOA}} \) given the previous observations \( y_{k_{0},k-1}^{\text{TDOA}} \) can be calculated as [38, p. 60]

\[
p(y_{k}^{\text{TDOA}} | y_{k_{0},k-1}^{\text{TDOA}}, \mathcal{H}_1) \approx p_{\mathcal{N}}(y_{k}^{\text{TDOA}}; \tilde{y}_{k|k-1}, S_{k}).
\]

Here \( p_{\mathcal{N}}(\cdot; \mu, \Sigma) \) denotes multivariate normal density function with mean \( \mu \) and covariance matrix \( \Sigma \). Further, \( \tilde{y}_{k|k-1}^{\text{TDOA}} \) and \( S_{k} \) denote the one-step ahead TDOA prediction and innovation covariance, respectively, calculated via the EKF SLAM algorithm in Alg. 1.

To use the EKF SLAM algorithm to calculate the likelihood \( p(y_{k}^{\text{TDOA}} | y_{k_{0},k-1}^{\text{TDOA}}, \mathcal{H}_1) \) an initial source state estimate \( \hat{s}_{k_{0}} \), map estimate \( \hat{\mathbf{m}}_{k_{0}} \), and covariance matrix \( \mathbf{P}_{k_{0}} \) must be provided to the EKF SLAM algorithm when the SLRT is started at time instant \( k_0 \). These quantities, as well as initial likelihood \( p(y_{k_{0}}^{\text{TDOA}} | \mathcal{H}_1) \), can be found from the TDOA observation \( y_{k_{0}}^{\text{TDOA}} \) as presented next.

### B. Initial source state estimate and covariance

An initial estimate of the location of a potential source can be found in multiple ways. Two ways are triangulation from bearing observation and multilateration from TDOA observations. Next, a multilateration scheme that takes into account the uncertainties in the locations, orientations, and shapes of the arrays is presented.

Assume the a priori source location and map state to be independent and distributed as \( p_{\mu}(s_{k_0}; \Omega) \) and \( p_{\mathcal{N}}(\mathbf{m}_{k_0}; \hat{\mathbf{m}}_{k_0}, \mathbf{P}_{k_0}^\text{cov}) \), respectively. Here \( \Omega \) is the potential region in which a source can be located. Next, do a first-order Taylor expansion of the observation model in (9) around the expected value of the map state \( \mathbf{m}_{k_0} \). That is,

\[
y_{k_{0}}^{\text{TDOA}} = h_{k_{0}}^{\text{TDOA}}(\mathbf{x}_{k_{0}}, \mathbf{m}_{k_{0}}) + \epsilon_{k_{0}}^{\text{TDOA}}
\approx h_{k_{0}}^{\text{TDOA}}(\mathbf{x}_{k_{0}}, \hat{\mathbf{m}}_{k_{0}}) + \epsilon_{k_{0}}^{\text{TDOA}} + \mathbf{H}_{m}(\mathbf{m}_{k_{0}} - \hat{\mathbf{m}}_{k_{0}}).
\]

where

\[
\mathbf{H}_{m} \triangleq \nabla_{m} h_{k_{0}}^{\text{TDOA}}(\mathbf{x}_{k_{0}}, \mathbf{m}_{k_{0}})|_{\mathbf{m}_{k_{0}} = \hat{\mathbf{m}}_{k_{0}}}. \tag{15b}
\]

Then it holds that

\[
p(y_{k_{0}}^{\text{TDOA}} | \mathcal{H}_1) \approx p_{\mathcal{N}}(y_{k_{0}}^{\text{TDOA}}; \mu_{k|k-1}, \Sigma_{k_{0}})
\]

where

\[
\mu_{k|k-1} = h_{k}^{\text{TDOA}}(\mathbf{x}_{k}, \hat{\mathbf{m}}_{k_{0}}) \tag{16a}
\]

and

\[
\Sigma_{k_{0}} = \mathbf{H}_{m} \mathbf{P}_{k_{0}}^\text{cov} \mathbf{H}_{m}^{T} + \mathbf{R}_{k_{0}}^{\text{TDOA}}. \tag{16c}
\]

Next, note that the TDOA observations in (9) only depend on the source location and not the other states in \( \mathbf{x}_{k} \). Hence, it holds that \( p(y_{k_{0}}^{\text{TDOA}} | \mathcal{H}_1) = p(y_{k_{0}}^{\text{TDOA}} | s_{k_{0}}, \hat{\mathbf{m}}_{k_{0}}) \).

Finally, exploiting Bayes’ theorem and that the prior probability \( p_{\mu}(s_{k_{0}}; \Omega) \) is constant over the region \( \Omega \) results in the maximum a posteriori (MAP) estimator

\[
\hat{s}_{k_{0}} = \arg\max_{s} p(s | y_{k_{0}}^{\text{TDOA}}, \mathcal{H}_1) = \arg\max_{s} \frac{p(y_{k_{0}}^{\text{TDOA}} | s, \mathcal{H}_1) p_{\mu}(s; \Omega)}{p(y_{k_{0}}^{\text{TDOA}} | \mathcal{H}_1)} \tag{17a}
\]

\[
= \arg\max_{s} p(y_{k_{0}}^{\text{TDOA}} | s, \mathcal{H}_1) \tag{17b}
\]

\[
= \arg\min_{s \in \Omega} V_{k_{0}}(s).
\]

Here \( p(s | y_{k_{0}}^{\text{TDOA}}, \mathcal{H}_1) \) is the posterior probability of the source location given the TDOA observations at time \( k_0 \). Further,

\[
V_{k_{0}}(s) \triangleq \| y_{k_{0}}^{\text{TDOA}} - h_{k_{0}}^{\text{TDOA}}(s, \hat{\mathbf{m}}_{k_{0}}) \|_{\mathbf{P}_{k_{0}}^{-1}}^{2} \tag{17b}
\]

and \( s^{*} = [s^{T} \ 0]^{T} \). It is noteworthy that the MAP estimator is equivalent to a constrained least squares estimator. Thus, if the region \( \Omega \) is large enough so that the constraints are basically non-effective, then the covariance of the MAP estimate is equivalent to the covariance of the least squares estimator. Hence, the covariance of the estimate is approximately

\[
\mathbf{P}_{k_{0}}^{\text{ss}} \triangleq \text{Cov} \{ \hat{s}_{k_{0}}, \hat{s}_{k_{0}} \} \approx (\mathbf{H}_{s}^{T} \Sigma_{k_{0}}^{-1} \mathbf{H}_{s})^{-1}. \tag{18}
\]

To summarize, the EKF-SLAM algorithm can at time \( k_0 \) be initialized using the state estimate and covariance matrix

\[
\hat{s}_{k_{0}} = [\hat{s}_{k_{0}}^{T} \ 0]^{T} (\hat{\mathbf{m}}_{k_{0}})^{T} \tag{19}
\]

and

\[
\mathbf{P}_{k_{0}} = \mathbf{P}_{k_{0}}^{\text{ss}} \oplus \mathbf{P}_{k_{0}}^{\text{cov}} \oplus \mathbf{P}_{k_{0}}^\text{cov}, \tag{20}
\]

respectively. Here \( \oplus \) denotes the matrix direct sum operator and \( \mathbf{P}_{k_{0}}^{\text{cov}} \) denotes the covariance of the source velocity state. Note that if the arrays’ locations, orientations, and shapes are assumed to be perfectly known and only the source location unknown, then the presented method reduces to the standard TDOA multilateration solution. Also note that, before the first source has been detected and used for calibration, then \( \hat{\mathbf{m}}_{k_{0}} \) should be set according to the ideal locations, orientations, and shapes of the arrays aimed at during the deployment process. And the covariance matrix \( \mathbf{P}_{k_{0}}^{\text{cov}} \) should reflect the uncertainties associated with the locations, orientations, and shapes of the arrays. Later, when searching for new calibration sources, then \( \hat{\mathbf{m}}_{k_{0}} \) and \( \mathbf{P}_{k_{0}}^{\text{cov}} \) should be set based upon the map.
state and covariance calculated using the previously observed calibrations source.

Finally, the marginal likelihood \( p(y_{k|k}^{\text{true}}|\mathcal{H}_1) \) needed in the first time step of the SLRT can be calculated as

\[
p(y_{k|k}^{\text{true}}|\mathcal{H}_1) = \int_{\mathbf{s} \in \Omega} p(y_{k|k}^{\text{true}}|\mathbf{s}, \mathcal{H}_1)p_{\text{SLRT}}(\mathbf{s}; \Omega)d\mathbf{s} \\
\approx \frac{1}{N} \sum_{n=1}^{N} p(y_{k|k}^{\text{true}}|s^{(n)}, \mathcal{H}_1)
\]

where \( s^{(n)} \) denote the \( n \)-th Monte Carlo sample drawn from the distribution \( p_{\text{SLRT}}(\mathbf{s}; \Omega) \).

### C. Outlier rejection and source disappearance detection

Due to reverberation effects, noise, and other distortions caused to the source sound signal, the TDOA estimates will be subjected to outliers. To reject the outliers, an outlier test can be included in the EKF algorithm [39]. Under the assumption that the TDOA estimates only include inliers, the normalized residual is

\[
\varepsilon_k \equiv (\mathbf{s}_k)^{-1/2} (y_{k|k}^{\text{true}} - \tilde{y}_{k|k}^{\text{true}}) \sim \mathcal{N}(0, \mathbf{I})
\]

Hence, outliers can be detected by checking for which entries \( j \) that \( |\varepsilon_k| \geq \gamma^o \) and then removing the corresponding TDOA estimates from the measurement update of the EKF. The outlier rejection can also be used to detect when a source has disappeared or is about to disappear by monitoring the total number of outliers in the TDOA measurements. The threshold \( \gamma^o \) determines the rejection probability and can be calculated from the inverse cumulative normal distribution function.

### D. Summary of the proposed automatic calibration method

Bringing it all together, the proposed method for automated array calibration is summarized in Alg. 2. The method consists of three distinct parts. Part one is the source state and hypothesis test initialization, where the SLRT is initialized using current knowledge about the arrays’ shapes, locations, and orientations, i.e., the state \( \hat{\mathbf{m}}_0 \) and its covariance \( \mathbf{P}_{0|0}^{\text{true}} \). Part two is accepting and rejecting the potential source using the SLRT. If the potential source is accepted, part three is started where the EKF SLAM algorithm is used refine the estimate of the arrays’ shapes, locations, and orientations; the refined estimate is stored in the state \( \hat{\mathbf{m}}_n \) and its covariance \( \mathbf{P}_{n|n}^{\text{true}} \). If the potential source is rejected, or when the source disappears, then part one is executed again. The process is continued until a sufficient number of sources has been observed to obtain the desired calibration accuracy.

### IV. Experimental Evaluation

The proposed calibration method has been evaluated with data recorded from two sea trials. Both sea trials were conducted in the Baltic Sea, or more precisely, in the outer archipelago outside of Stockholm, Sweden. The measurements were performed at two different locations (see Fig. 3) and more than one year apart. The environment in both cases can be characterized by a high degree of spatial variation, both in terms of bathymetry but also in terms of sea bottom properties. For the considered locations the sea depth varied from approximately 30 m to approximately 50 m. Furthermore, the measurements were conducted in two different seasons, rendering completely different sound speed profiles, as shown in Fig 5. During sea trial #1, the sound propagation environment had downward refracting properties, while during sea trial #2 the sound speed was close to constant. Moreover, the sound sources in the two sea trials were distinctly different. During sea trial #1, data sets 1 and 2 were collected while the 5000-ton passenger ferry M/S Gotland passed by the arrays. During sea trial #2, data sets 3 and 4 were collected while a pilot boat and a recreational boat passed by the arrays. In Fig. 4 the estimated spectrograms of the received sound signals are shown. The sound sources in sea trial #1 (see Fig. 4a and Fig. 4b) generates mainly broadband signals, whereas the sound sources in sea trial #2 (see Fig. 4c and Fig. 4d) contains both broadband and tonal signal components.

During both sea trials, two PASS-2447 Omnitech Electronics Inc. hydrophone arrays were used. These are 112.5 m long nested arrays whose hydrophones can be combined to create six different 16-element uniform linear arrays half-wavelength spaced for the signal frequencies 100, 200, 400, 800, 1600, and 3200 Hertz; all 56 elements can also be used for broadband beamforming. From the recorded hydrophone data, sampled at 30 kHz, TDOA estimates were calculated using 5 s batches of data. The TDOA estimates were then used as observations in the proposed calibration method. The sound propagation speed was assumed unknown and included as a state to be estimated. The a priori positions of the endpoints of the two arrays were calculated from the position measurements of the arrays’ endpoints taken during the deployment process. Further, the segment length \( r \) in the array model (6) was selected as \( r = \frac{\lambda_{\text{min}}}{2} = 0.23 \) meters, based upon the rigidity of the array cable. Moreover, the sequential likelihood ratio test in Sec. III-A was then used to detect when a source suitable for array calibration was present and initialize the SLAM process; the TDOA estimates were processed both with and without position observations from the ships’ AIS transponders. The sections of the trajectories where the algorithm has detected the sources and used the TDOA data to estimate the shapes and orientations of the arrays are highlighted by solid lines in Fig. 3. Further, the SLRT statistics \( T_k \) along with time instants when a tentative sources were detected, the source existence is confirmed, and the source disappeared are shown in Fig. 4.

#### A. Array shape, orientation, and location estimates

The array shapes estimated during sea trial #1 and #2 are shown in Fig. 6 and Fig. 7, respectively. To be able to compare the estimated array shapes, the estimated shapes have been translated and rotated so that the least square error of the orientation and location between them is minimized. From the figures, the following things can be observed. Firstly, it can be observed that the estimated horizontal shapes are consistent across the data sets, as well as with the inclusion and exclusion of AIS source location information; the difference between the estimates with and without AIS source location information
(a) Sea trial #1.
(b) Sea trial #2.

Fig. 3: Array placements and source trajectories during the two sea trials. The dashed trajectory lines indicate the path of the sources and the solid lines indicate when the sources were tracked, i.e., the data used to estimate the shape, orientation, and location of the arrays. Also shown are zoomed-in cutouts of the array placements where the black dots indicate the reference location $p_{1,i}$ for each array.

(a) Spectrogram (left) and SLRT statistics (right) for data set 1.
(b) Spectrogram (left) and SLRT statistics (right) for data set 2.
(c) Spectrogram (left) and SLRT statistics (right) for data set 3.
(d) Spectrogram (left) and SLRT statistics (right) for data set 4.

Fig. 4: Estimate power spectrums of the four data sets together with the calculated SLRT statistics $T_k$ and time instants when: (A) a tentative sources were detected, i.e., $\gamma_0 < T_k < \gamma_1$ and the SLRT continue to monitor the two hypotheses; (B) the source existence is confirmed, i.e., $T_k \geq \gamma_1$ and hypotheses $H_1$ is accepted; and (C) the source disappeared, i.e., there are too many outliers, and the SLRT is restarted. Also shown are the thresholds $\gamma_0$ and $\gamma_1$ for accepting hypotheses $H_0$ and $H_1$, respectively. Note, that once the source has been confirmed the SLRT is turned off and is only turned on again once the source has disappeared. A source is assumed to have disappeared when the number of outliers in the TDOA measurements exceeds 50%.

is hardly visible. Hence, the TDOA estimates are, together with coarse knowledge about the locations and orientations of the arrays, sufficient to estimate the array shapes. This since the TDOA estimates provide direct information regarding the relative location of the hydrophone elements. However, AIS data is needed to refine the location and orientation estimates of the arrays.

Secondly, it can be observed that the spread between the estimated array shapes is well inside the $\pm 3\sigma$ uncertainty bounds, which indicates that the uncertainty estimates are somewhat too large. Further, the $\pm 3\sigma$ uncertainty bounds obtained in sea trial #2 are slightly larger than those in sea trial #1, which is in agreement with the fact that the source trajectories during sea trial #2 are much shorter and are of
less-favorable geometries from a calibration perspective. Note that the increasing uncertainty in the array shape relative to the reference point does not mean that the absolute location uncertainty of the array elements grows. Only the uncertainty of the shape relative to the reference point grows. Any other hydrophone could have been used as the reference point and the uncertainty about the array shape then had grown with the distance from that reference point.

During sea trial #1 the depths of hydrophones 1, 9, and 56 of both arrays were measured. Fig. 6 shows that there is a good match between the estimated array shapes and the measured depths of the hydrophones. However, the multilateration conditions for estimating the array shapes in the depth direction are poor. This can be seen from the fact that the ±3σ uncertainty bounds of the estimated array shape are almost identical to the a priori ±3σ uncertainty bounds. Further, the proposed observation model (9) assumes that the sound propagation speed is constant throughout the water volume, which disagrees with the measured sound speed profile in sea trial #1. This will introduce model errors in the vertical direction that depend on the distance to the source. Hence, the results regarding array shapes in the depth direction should be interpreted with care.

The estimated orientations of the arrays, defined as the direction from the first to the last element in the array, are shown in Tab. I. Since the absolute orientation can only be estimated if source location information is included in the estimation process, only the results from when both TDOA and AIS observations are used are shown. From the table, it can be seen that the orientation estimates vary by less than two degrees between the data sets. The ±3σ uncertainties calculated for the azimuth estimates are a bit optimistic, indicating that the algorithm is too confident about the orientation of the arrays in the horizontal plane. Further, the estimated array inclinations with respect to the horizontal plane and the calculated uncertainties in the inclinations are of the same order. This is another indication that little information is gained from the calculated TDOA observations about the shapes and orientations of the arrays in the depth direction. Thus, the parameters encoding the array shape in the depth direction and the array inclination should either be removed from the model or source trajectories passing closer to the arrays (ideally directly above the arrays) should be used in the calibration process.

The estimated array positions in the horizontal plane, i.e., $p_{1,i}^1$, in (6a), are shown in Fig. 8. The a priori position estimate of $p_{1,i}^1$ for array #1 is used as the reference point and all positions are relative to this location. Also shown are the 99.7% confidence intervals. Since the absolute location can only be estimated if source location information is included in the estimation process, only the results from when both TDOA and AIS observations are used are shown. The a priori array locations were calculated from the GPS measurements of the anchors located at both ends of each array. And the GPS measurement error was assumed to have a standard deviation of 10 meters. From Fig. 8, it can be seen that the two data sets collected at each sea trial give similar location estimates. The largest difference is in the estimated locations during sea trial #2, where the source from a calibration perspective is moving in a less favorable trajectory. For better calibration, the source should move in a path that gives measurements at a large variety of angles as seen from the array. Nevertheless, as seen from the 99.7% confidence intervals, the positional uncertainty is reduced from several tens of meters to around 10 meters, even with the trajectories in the two sea trials.

B. Sound propagation speed estimates

The sound propagation speed estimates, together with associated ±3σ uncertainty bounds, calculated from the different data sets are shown in Fig. 9. These should be compared with measured sound speed profiles shown in Fig. 5. From these figures, a few things may be noted. First, information about the sound speed is only gained when the source is close to the end-fire directions of the arrays. This can be seen by observing that the uncertainty bounds only significantly decrease when the source passes the end-fire directions of the arrays. This can be seen by observing that the uncertainty bounds only significantly decrease when the source passes the end-fire directions of the arrays in data sets 1 and 2. Second, the estimated sound speed converges toward the measured sound speed at the depth the arrays are located at. This is because the sound propagation speed in the water volume closest to the arrays has the largest effect on the estimated array shapes. Variations in the sound propagation speed in the water volume farther away from the array affect the direction of the incoming sound waves through horizontal refraction, mainly affecting the estimated array orientation.

C. Bearing estimation accuracy before and after calibration

To illustrate the effect of the array calibration on the bearing estimation accuracy, the beamforming accuracy before and after calibration was evaluated on data recorded during sea trial #2 using a Saab AUV62 autonomous underwater vehicle. The Saab AUV62 is an anti-submarine warfare training system that can be configured to emit sound signals corresponding to a submarine sound signature. During the sea trial a signature consisting of a mixture of tonal and broadband noise components was used. From the recorded data the bearing time records (BTR) of the AUV62 before and after calibration of array 1 were calculated. The results are shown in Fig. 10a and Fig. 10b. Also shown is the true bearing and the root mean square error (RMSE) of the bearing estimate. As seen, compared to the bearing estimates calculated using the uncalibrated array, the bearing estimates after calibration match the true bearing to within one degree. Fig. 10c and
Fig. 6: Estimated array shapes relative to the location of the first hydrophone and associated ±3σ uncertainty intervals (99.7% confidence intervals) calculated using data sets 1 and 2 collected in sea trial #1. Estimates calculated both with and without source location information are shown (black dots), but the difference between them is hardly visible. Also shown is the mean shape (red line) calculated as the mean of all estimated array shapes.

Fig. 7: Estimated array shapes relative to the location of the first hydrophone and associated ±3σ uncertainty intervals (99.7% confidence intervals) calculated using data sets 3 and 4 collected in sea trial #2. Estimates calculated both with and without source location information are shown (black dots), but the difference between them is hardly visible. Also shown is the mean shape (red line) calculated as the mean of all estimated array shapes.

TABLE I: Estimated azimuth and inclination of the arrays for the different data sets and sea trials when using both TDOA and AIS observations. Also shown are associated uncertainty estimates in terms of the three standard deviation values.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Array #1</th>
<th>Array #2</th>
<th>Array #1</th>
<th>Array #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth (±3σ) [°]</td>
<td>-100.0 (±0.2)</td>
<td>-99.5 (±0.2)</td>
<td>24.5 (±0.2)</td>
<td>24.3 (±0.2)</td>
</tr>
<tr>
<td>Inclination (±3σ) [°]</td>
<td>-0.1 (±0.4)</td>
<td>-0.1 (±0.4)</td>
<td>0.1 (±0.4)</td>
<td>0.1 (±0.4)</td>
</tr>
</tbody>
</table>

Fig. 10d show slices of the BTR at time $t = 350$ s. From these figures it can be seen that not only is the bearing error reduced, but also the beamforming gain increased by approximately one decibel (dB). It is noteworthy that thanks to the nonlinear shape of the arrays, it is possible, after the calibration, to resolve the bearing (direction of arrival) ambiguity typically associated with linear arrays [40].
V. CONCLUSIONS AND OUTLOOK

By viewing the problem of estimating the geometries of a set of hydrophone arrays as a SLAM problem, a framework for signals-of-opportunity-based hydrophone array calibration has been proposed. An associated sequential likelihood ratio test for the detection of suitable calibration sources has also been proposed. The likelihood ratio test makes the calibration process fully automatic, removing the need for the user to identify suitable signal sources. The formulation of the calibration problem as a SLAM problem enables standard Bayesian inference methods to be used in the calibration process. This in turn allows prior knowledge about the array geometries to be included in the calibration process and quantification of the expected accuracy of the calibration. Thus, the need for additional calibration data can be assessed, and the expected accuracy of the calibration can be considered when using the arrays for target tracking.

The evaluation of the proposed framework on data collected during two sea trials shows that suitable signal sources can be detected automatically and that the different data sets provide similar estimates of the shape, orientation, and location of the arrays. The estimated azimuth of the arrays varies by less than two degrees, whereas the inclination estimates only vary by less than a degree. However, the uncertainty bounds calculated by the algorithm show that little information is obtained regarding the array geometries in the vertical direction given the geometrical conditions between the arrays and sources in the two seatrials. Hence, if the arrays are to be used under similar geometrical conditions the vertical geometry of the array could be considered fixed and removed from the calibration procedure. Regarding the estimated array shapes and orientations in the horizontal plane, the results from a test with a Saab AUV 62 anti-submarine warfare training system indicate that the array shapes are correctly estimated since the beamforming gain increase. The offset between the true and estimated bearing is also reduced, which indicates that the orientations are correctly estimated.

Though the proposed method has been framed and used for hydrophone array geometry calibration, it is straightforward to extend the method to location, orientation, and shape estimation in other applications such as acoustic and electromagnetic sensor networks. Further, by substituting the static map model with a dynamic model such as the water electromagnetic sensor networks. Further, by substituting the static map model with a dynamic model such as the water

REFERENCES

Bearing [deg]
-15
-10
-5
0
Relative energy [dB]
-15
-10
-5
0
Time [s]
RMSE = 4 [deg]
-20
-15
-10
-5
Relative energy [dB]
-15
-10
-5
0
Bearing [deg]
-150 -100 -50 0 50 100 150
Bearing [deg]
260
280
300
320
340
Time [s]
RMSE = 1 [deg]
-20
-15
-10
-5
Relative energy [dB]
-15
-10
-5
0
[0x0](b) BTR after calibration and the true bearing (dashed line) of the source.

Fig. 10: Example of the BTR before and after calibration of array 1 in sea trial #2. The source is a Saab AUV62 anti-submarine warfare training system, and the location, orientation, and shape of the array were estimated using TDOA data from data set 3. The relative energy is calculated based on the peak energy in the entire BTR after calibration.

[34] Z. Sjanic, M. A. Skoglund et al., “EM-SLAM with inertial/visual
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