Target recognition by vibrometry with a coherent laser radar

Examensarbete utfört i Reglerteknik och kommunikationssystem

av

Andreas Olsson

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target recognition, vibrometry, coherent laser radar, laser vibration sensing, Doppler technique, time-frequency analysis
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Abstract

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A frequency demodulation method developed at the former FOA, is for the first time validated against real data with turbulence, scattering, rain etc. The issue is to find a robust and reliable system for target recognition and its performance is therefore compared with some frequency distribution methods. The time frequency distributions have got a crucial drawback, they are affected by interference between the frequency and amplitude modulated multicomponent signals. The system requirements are believed to be fulfilled by combining the FOA method with the new statistical method proposed here, the combination being suggested as aimpoint for future investigations.

Keywords:

*target recognition, vibrometry, coherent laser radar, laser vibration sensing, Doppler technique, time-frequency analysis*
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Abbreviations

AF  ambiguity function
AM  amplitude modulation
AS  ambiguity surface
CAF cross ambiguity function
CW  continuous wave
CWD cross Wigner distribution
DFT discrete Fourier transform
FOA Defence Research Establishment, Sweden (now FOI)
FFT fast Fourier transform
FM  frequency modulation
GUI graphical user interface
IF  instantaneous frequency
LADAR laser radar
LO  local oscillator
LP  low pass
PSD power spectral density
SNR signal to noise ratio
SPD statistical peak distribution
SPWD smoothed pseudo-Wigner distribution
STFT short time Fourier transform
SWD smoothed Wigner distribution
TF  time frequency
TFD time frequency distribution
TFR time frequency representation
WD  Wigner distribution
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1.1 Background

Introduction

There is a need for modern weapon systems to be able to identify and classify targets in the battlefield and several methods have been developed for this purpose. Target recognition by vibrometry is one method among others, e.g., 3D laser radar imaging, IR camera and mm-wave radar, all with the same purpose - to identify a target by its signature.

By using doppler technique and an optical laser radar it is possible to measure vibrations from surfaces and analyse their vibration spectra. One can then distinguish between different types of vehicles by their spectra, which indeed can be said to act as a fingerprint.

There are active methods like analysis of the radar echo as well as passive systems such as evaluation of the thermal image. In a sharp attack it is of great importance not to be detected by the enemy and therefore a passive system is preferable. Characterization by vibrometry is however an active method but the signature is very valuable and it is therefore, at least for a short time interval, worth running the risk of being discovered. The disadvantage by the active nature of the system is well compensated by the fact that it is difficult to jam[1].

There are a number of factors which must be considered when using laser vibration sensing to identify airborne and ground based vehicles. The system will operate at long distances and reflections from clouds, rain, turbulence etc have to be taken care of. If the laser radar (LADAR) system is mounted on an fighter aircraft, tank, truck, or any other mobile weapon system, the vibrations from this platform may interfere with the fingerprint from the target if they are in the
same frequency band. Furthermore, as the mobile weapon system is moving a doppler shift will be introduced, along with the one caused by the target, and they need to be separated in the signal processing. Glint or diffuse targets as well as a good camouflage yields different reflections.

1.2 Objectives

A new FM demodulation method called the FOA method was proposed at SPIE Aero Sense in April 1996 by Mille Millnert [2]. It was modified by Hans Nordin to take into account the doppler shift caused by moving targets [3].

The FOA method has previously only been simulated and the issue is to test it on real data and compare the method with various time-frequency analysis methods, e.g., spectrogram and Wigner distribution. The essence of the thesis is to investigate whether or not the FOA method is the best choice in trying to classify vibrating targets. The comparison will be carried out in a statistical manner, i.e., the plots generated from each method when applied to real data will be compared and the method that most often comes up with the same vibrating frequencies will be considered to be the most reliable and robust one. Another task is to see if the FOA method is reliable and robust or if it is possible to come up with a better one.

Data from a NATO sponsored field test in Alabama, USA, has been analysed and the result from a large number of measurements, on five different vehicles, are presented in this thesis. The measurements have been carried out from various angles of aspect, through heavy smoke, of different ranges and in good as well as in bad weather conditions. The types of the vehicles are classified as secret and they will therefore be referred to as vehicle A, B, C, D, and E.
Weather conditions such as rain, snow and fog affects the laserbeam as well as turbulence in the air and speckle patterns due to the surface structure of the vibrating target. The laserbeam, transmitted by the LADAR, propagates through the air, hits the target and a part of the modulated beam is being reflected back to the receiver. In the final section of this chapter the reflected signal with its vibrational and radially moving doppler shifts, acceleration terms and phase shift will be explained in more detail.

2.1 Disturbances on reflected laser radar beam

Gas, dust- and liquid particles disturbs the laser by absorption, refraction, and scattering and causes transmission damping. The atmospheric transmission is wavelength dependent, where some wavelength intervals are better suited for laser use than others.
2.1 Disturbances on reflected laser radar beam

2.1.1 Clutter

Clutter can be described as unwanted reflections from physical objects between the laser radar (LADAR) and the target, or from background objects behind the target, and can be represented by reflections, e.g., from birds, insects, chaff, trees or other kinds of vegetation. The reflections “clutter” the received signal and complicates the target detection.

As if this would not be complicated enough, these disturbances are also always, more or less, in motion which in turn will introduce a clutter doppler frequency. The situation is even more complex if also the LADAR platform itself is in motion, as when mounted on a vehicle, then the clutter will vary with the speed of the platform as well as with the angle to the clutter[4].

2.1.2 Turbulence

The sun heats up the ground and causes temperature variations in the air. Local changes of air density yield random fluctuations of the refractive index of the air. The laser beam changes direction due to these scintillations, see Figure 1.

Small packages of air split the beam into small pieces which interfere and gives a speckle pattern as well as a broadening of the beam[5].

![Diagram of Laser and Clutter](image)

**Figure 1.** *Top:* Large packages of air (larger than the beam diameter) cause random variations of the beam direction. *Bottom:* Small packages of air split the laserbeam into several beams which randomly interfere with each other and cause an intensity distribution with local fluctuations called speckle.
2.1 Disturbances on reflected laser radar beam

2.1.3 Speckle

Not only turbulence, as in Figure 2, gives rise to speckle but also the surface of the target contributes to this phenomena. Due to the short wavelength of the laser radiation even a smooth surface is considered to be rough for a laser and the height variations of the surface will make the reflected laser energy diffuse[6]. The speckle pattern is caused by random interference in the reflected wave from the independent scatterers on the diffuse surface. A speckle pattern will always appear from a diffuse surface and if the laser beam moves over the surface of the target then the reflected intensity will be time-varying.

There will always be a need for a good target tracking system along with the LADAR to make sure that the laser beam hits the same spot of the vehicle during the measurement interval. However, this is very difficult and if a moving target is being tracked, the beam will move around slightly on the target and speckle induces amplitude modulation (AM) of the frequency modulated (FM) signal.

![Figure 2. Intensity distribution from a laserbeam passing through 1 kilometer of turbulent atmosphere and registered with a TV-camera. The two pictures has been taken with 20 ms time delay. The width of the laserbeam is 0.4 m.](image)

2.1.4 Weather conditions

The transmission loss caused by snow and raindrops is more or less independent of the laser wavelength. However, the presence of particles is more important and is the major contribution to the attenuation. The laserbeam is more sensitive to the wetness of the snow than to the thickness[7]. Dry snow has almost no effect on the reflection whereas wet snow decreases the reflection quite a lot. Raindrops and snow will scatter a laser beam and make the laser lobe larger as it reaches the target.

Hazy weather or if it is foggy, is of more concern as well as dust and smoke. The size of the particles are now of about the same size as the laser wavelength and can considerably attenuate the beam. A longer laser wavelength leads to better transmission since the size of the particles becomes smaller than the wavelength.
2.2 The reflected signal

The transmitted laser beam is reflected by the target and received by the detector. The subscripts used in the following section is \( \text{trans} \) for transmitted laser signal, \( \text{ret} \) is returned signal, \( \nu \) vibration, \( r \) represents radial movements and finally \( d \) for doppler shift. Combinations of indices like \( vd \) and \( rd \) refers to instantaneous doppler shift caused by vibration and doppler shift caused by radial motion, respectively.

A laser signal, \( s_{\text{trans}}(t) \), can be described as a sinusoid with a high frequency, \( f_{\text{trans}} \), and amplitude \( A_{\text{trans}} \).

\[
s_{\text{trans}}(t) = A_{\text{trans}} \sin(2\pi f_{\text{trans}} t)
\]  

(1)

The frequency component of the laser signal will be modified as the signal is received after being affected by the movements and vibrations of the target as well as by air conditions. A phase shift will also be present in the returned signal. First of all, the amplitude factor \( A_{\text{trans}} \) becomes time dependent, \( A_{\text{ret}}(t) \), since speckle gives rise to an amplitude modulation of the beam. The target could be moving in a direction straight towards the laser source but more often at an angle \( \phi_r \) to the line of sight and then the radial velocity, \( \nu_r \), becomes

\[
\nu_r = \nu \cos \phi_r \text{ where } \nu \text{ is the speed of the target.}
\]  

(2)

A radial velocity of the target is described with a linear function. The first term is a doppler shift caused by the radial velocity and if the target accelerates or changes its direction of motion there will also be a second, linear frequency modulated term, expressed by the acceleration constant \( \alpha \).

\[
f_{rd} = \frac{2}{\lambda} \nu_r + \alpha t = \frac{2}{\lambda} \nu \cos \phi_r + \alpha t \text{ where } \lambda \text{ is the laser wavelength.}
\]  

(3)

So far the expression for the returned laser signal is \( s_{\text{ret}}(t) = A_{\text{ret}}(t) \sin(2\pi (f_{rd} + \alpha t) t) \) and only the phase shift remains to be defined. Assume that the surface of a stationary target vibrates. The momentary position, \( x \), of the surface can be expressed as a harmonic function, \( x_{\nu}(t) \),

\[
x_{\nu}(t) = x \sin(2\pi f_{\nu} t + \theta_{\nu})
\]  

(4)

where \( f_{\nu} \) is the surface vibration frequency and the term \( \theta_{\nu} \) is an unknown vibrational phase shift due to the deviation from equilibrium position of the target’s surface. The derivative of \( x_{\nu}(t) \) with respect to time, \( v_{\nu}(t) \), will be used in section 2.2.2 where the modulation index is explained and is therefore introduced here.
2.2 The reflected signal

\[ v_v(t) = \frac{d}{dt} x_v(t) = 2\pi f_v x \cos(2\pi f_v t + \theta_v) \]  

(5)

It describes the rapidly changing velocity of the surface at any given time \( t \) [8]. To further understand what happens to the laser beam the term doppler shift has to be brought up for discussion.

2.2.1 Doppler shift

The doppler effect is well known in several fields such as optics and acoustics. In the later case think about how the sound changes as an ambulance passing by. As the ambulance gets closer the sound becomes louder of course but also higher (a brighter tone) and after passing by the sound frequency decreases - all due to the doppler effect. This frequency modulation is also present here, with the laser system. \( R_0 \) is the distance to the target then the beam travels a distance \( 2R_0 \) from the laser to the target and back to the detector. The surface of the target is vibrating and therefore \( R \) becomes time dependent

\[ R(t) = R_0 + x_v(t) \]  

(6)

with the same harmonic function, \( x_v(t) \), as in (4). The total number of wavelengths are

\[ N(t) = \frac{2R(t)}{\lambda} \text{ where } \lambda \text{ is the wavelength.} \]  

(7)

The vibrational contribution is expressed, not as a change in number of wavelengths, but as a phase shift. One wavelength equals a phase shift of \( 2\pi \) radians. Therefore this phase shift can be expressed by multiplying \( N(t) \) with \( 2\pi \)

\[ 2\pi N(t) = \frac{4\pi R(t)}{\lambda} = \frac{4\pi R_0}{\lambda} + \frac{4\pi x}{\lambda} \sin(2\pi f_v t + \theta_v) \]  

(8)

A phase shift, \( \theta_{rd} \), with random behaviour due to scattering, clutter, raindrops, fog or other atmospheric effects completes the expression for the returned laser signal which finally becomes

\[ s_{ret}(t) = \sum_i A_i(t) \sin \left[ 2\pi (f_{rdo} + \alpha t) t + \theta_{rdi} + \frac{4\pi R_{0i}}{\lambda} + \sum_j \frac{4\pi x_{ij}}{\lambda} \sin(2\pi f_{vij} t + \theta_{vij}) \right]. \]  

(9)

The summation index \( i \) refers to the number of returned vibration frequencies, and \( j \) refers to each surface part of the target that the laserbeam covers. One can talk about resolved and unresolved targets. If the target is larger than the laser spot and the whole spot falls within the cross section of the target, then the target is said to be resolved which is desirable. The cross section of a target is defined as a plane projection, perpendicular to the laserbeam, of all surface parts.
2.2 The reflected signal

that can be observed from the laser source point of view at a particular instant of time. However, if the laser spot is larger than the target it will hit also the surrounding environment, e.g., another vehicle standing right behind the first one. The system will then receive vibration information from both targets which may cause some trouble when trying to classify the target. This is called an unresolved target. Index \( j \) might therefore indicate not only vibrations from the target but also from another vehicle nearby, especially for an unresolved target but also for a resolved target in very turbulent atmosphere as the laserbeam can move around quite a lot.

2.2.2 Modulation index

The rate of change in number of wavelengths defines the doppler frequency for a vibrating surface. There is no change in the number of wavelengths for a stationary target with its engine off [9]. The wave just reflects and changes to the opposite direction of motion but the wavelength is the same and the doppler frequency is zero. However, if the surface is oscillating the total number of wavelengths will vary due to the fact that the laser wave fronts get compressed and stretched at reflection and returns in smaller and longer intervals. The derivative of \( N \) with respect to time expresses the rate of change of wavelengths and with the rapidly changing velocity expression, \( v_v(t) \), this yields

\[
f_{vd}(t) = \frac{d}{dt}N(t) = \frac{2}{\lambda} \frac{d}{dt}R(t) = \frac{2}{\lambda} v_v(t) = \frac{4\pi f_v x}{\lambda} \cos(2\pi f_v t + \theta_v)
\]

The definition of modulation index \( \mu \) is the maximum frequency deviation of \( f_{vd}(t) \) divided by the carrier frequency \( f_v \)

\[
\mu = \frac{\max(f_{vd}(t))}{f_v} = \frac{4\pi x}{\lambda}
\]
3.1 Linear time frequency representation

Time domain analysis by itself does not fully describe the nature of signals. Frequency domain analysis is an alternative description. A combination of the two domains has been available and of interest in signal processing for a long time. A signal, $s(t)$, satisfies the superposition principle if it is built up by a linear combination of two signal components, $s_1(t)$ and $s_2(t)$, with constants $c_1$ and $c_2$.

$$s(t) = c_1 s_1(t) + c_2 s_2(t)$$  \hspace{1cm} (12)
3.1 Linear time frequency representation

As the two domains are combined the time-frequency distribution of a signal can be described by a joint function, $T_s(t,f)$, of time $t$ and frequency $f$. $T_s(t,f)$ is called a time-frequency representation of the signal $s(t)$. All linear time frequency representations (TFR), e.g., the wavelet transform and the short time Fourier transform (STFT), satisfy the superposition principle.

$$T_s(t,f) = c_1 T_{s_1}(t,f) + c_2 T_{s_2}(t,f) \quad (13)$$

The STFT is said to be a local spectrum since a time localization is obtained by pre-windowing the Fourier transform of the signal $s(\tau)$ with a shifted analysis window $\gamma(t)$.

$$T_{STFT}(t,f) = \int [s(\tau) \cdot \gamma^*(\tau-t)] e^{-j2\pi f t} d\tau \quad (14)$$

The analysis window, $\gamma(t)$, is centered around $t$ and suppresses all signal features outside a local neighborhood of the time $t$. It is difficult to extract the frequency content of a signal in the time domain, especially for time-varying nonstationary multicomponent signals, perhaps with a combination of amplitude modulation (AM), frequency modulation (FM), and noise.
3.2 Quadratic time frequency representation

An example with two different multicomponent signals (5 and 75 Hz), signal A contains both frequencies during the whole time period whereas signal B starts off with 75 Hz and a quarter of a time period later the frequency change to 5 Hz, see Figure 3. The example shows that they have the same spectrum, the difference is when in time the frequencies appear.

Figure 3. Top left: Signal A consists of two frequencies during the whole time, 5 and 75 Hz, with normalized amplitudes of 1.0 and 0.25 respectively. Top right: Signal B; S1 uses a frequency of 75 Hz, a normalized amplitude of 0.5 and lasts for 0.25 s. S2 is 5 Hz with an amplitude of 1.0 and exists for the last 0.75 seconds. Bottom: Power spectral density plot (PSD) of both signals.

3.2 Quadratic time frequency representation

An energy distributed TFR, $T_3(t,f)$, can be interpreted by using the concepts of the instantaneous power, $p_s(t)$, and the spectral energy density, $P_s(f)$. The instantaneous power is represented by the squared magnitude of the signal,

$$p_s(t) = |s(t)|^2$$

(15)

and the Fourier transform of the signal is being used to express the spectral energy density as
3.2 Quadratic time frequency representation

\[ S(f) = F\{s(t)\} = \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft} dt \]  
(16)

\[ P_s(f) = |S(f)|^2. \]  
(17)

An integration over frequency and time respectively finally gives the energy distributed TFR, \( T_s(t,f) \), as

\[ \int_{\tau} T_s(t,f) \, df = p_s(t) = |s(t)|^2 \]  
(18)

and

\[ \int_{\tau} T_s(t,f) \, dt = P_s(f) = |F\{s(t)\}|^2. \]  
(19)

Note that this is true for an ideal case, an integration over frequency normally cause a loss of time resolution and vice versa. The signal energy, \( E_s \), is obtained by integrating over the entire time-frequency plane

\[ E_s = \int_{t} p_s(t) \, dt = \int_{f} P_s(f) \, df = \int_{t} \int_{f} T_s(t,f) \, df \, dt. \]  
(20)

3.2.1 Spectrogram

The spectrogram is defined as the squared magnitude of the STFT;

\[ T_{SPEC}(t,f) = |T_{STFT}(t,f)|^2 = \left| \int_{\tau} [s(\tau) \cdot \gamma^\tau (\tau-t)]e^{-j2\pi ft} \, d\tau \right|^2 \]  
(21)

The spectrogram does not satisfy a linear superposition principle but a quadratic superposition principle.

\[ s(t) = c_1 s_1(t) + c_2 s_2(t) \]  
(22)

\[ T_s(t,f) = |c_1|^2 T_{s_1}(t,f) + |c_2|^2 T_{s_2}(t,f) + c_1 c_2^* T_{s_1,s_2}(t,f) + c_2 c_1^* T_{s_2,s_1}(t,f) \]  
(23)

Assuming an \( N \)-component signal
3.2 Quadratic time frequency representation

\[ s(t) = \sum_{k=1}^{N} c_k s_k(t) \]  

(24)

and generalizing the quadratic superposition principle will yield a signal term \( |c_k|^2 T_{s_k}(t,f) \) to each component \( c_k s_k(t) \) as well as an interference term \( c_k c_l^* T_{s_k, s_l}(t,f) + c_l c_k^* T_{s_l, s_k}(t,f) \) for each pair of components \( c_k s_k(t) \) and \( c_l s_l(t) \), \( (k \neq l) \). An \( N \)-component signal will have \( N \) signal terms and

\[
\binom{N}{2} = \frac{N(N-1)}{2}
\]

interference terms. A visual analysis of the TFR of a multicomponent signal, as is the case for this thesis, is difficult when there exist several signal components since the number of interference terms grows quadratically with the number of components.

A spectrogram combines the time and frequency domains to visualize when in time a certain frequency appear. The choice of a windowing function is important for the quality of the overall result in spectrum estimation. The main role of the window is to damp out the results from truncation of an infinite series which causes the effects of the so called Gibbs phenomenon. The spectrogram TF method utilizes a short time window where the length of the window is chosen so that the signal can be considered to be stationary over that particular interval of time[10]. The windowed signal is then discrete Fourier transformed (DFT) and a frequency distribution for the time at the center of the window is obtained. Figure 4 illustrates how a spectrogram of signal B from section 3.1 is in principal created. The time signal is divided into sixteen windows and each of them is Fourier transformed and the magnitude of this function builds up the plot. The real spectrogram approach is created in the same way except that a sliding Hanning window function with 50 % overlap is applied before the Fourier transform operation is performed, the result can be seen in Figure 5. The bright area at time interval 0,05-0,25 s is a result of the overlap of the window function. By summing the individual terms to form the window, the low frequency peaks combine in such a way as to decrease the height of the sidelobes.

The spectrogram representation has a crucial drawback. The frequency resolution is directly dependent of the length of the window and to increase the resolution one has to use a longer window, which in turn will smear out the nonstationarities occuring in the interval, in both time and frequency.
3.2 Quadratic time frequency representation

Welch’s periodogram method is a way of estimating the power spectral density (PSD) of a process by averaging modified periodograms[11]. A periodogram is a synonym for the square magnitude of the DFT. Welch divided the samples into segments of equal length and let the segments overlap. Thereafter is a window function applied to each of the segments. By using 50 percent overlap, points near the end of a segment will be near the center of a neighboring segment which in turn makes all samples equally represented on the average. The DFT is computed to the overlapped and windowed segments followed by the last step, averaging the square magnitude of the DFT. The averaging suppresses the randomness in a stationary random signal and yields a relatively smooth spectrum.

Figure 4. A time frequency plot for signal B defined in the example in section 3.1.

Figure 5. Spectrogram for signal A (left) and signal B (right) defined in section 3.1.
3.2 Quadratic time frequency representation

The visibility in a contour plot is often better and has got the ability to clearly present fluctuating frequencies although the cross terms can blur the result if two frequencies has a time overlap, illustrated by Figure 6 and Figure 7.

**Figure 6.** Top left: The two-component signal consists of $f_1=50$ Hz separated in time with $f_2=100$ Hz. Top right: Spectrogram of the two-component signal. Bottom left: A contour plot, as here, surpasses the spectrogram (top right) in presenting a clear and sharp TFR.
If the signal in figure 6 is adjusted so that the two frequencies are mixed during a middle time interval between 0.25 and 0.75 s, as can be seen in the top left plot in Figure 7, cross terms appear for the same time interval which makes it difficult to detect the two frequencies (50 and 70 Hz). It is common to have two or more frequencies at the same time but it cannot be handled satisfactory by the spectrogram method.

![Figure 7. Now the signal terms overlap leading to a cross term in the contour plot. None of the two TFRs can come up with a sensible plot during the time interval where the signals are mixed.](image)

### 3.2.2 Wigner distribution

The Wigner distribution [12] (WD) can be considered as a time frequency distribution (TFD) of the signal energy and is expressed as

\[
W_x(t,f) = \int_{\tau} x\left(t + \frac{\tau}{2}\right) \cdot x^\ast\left(t - \frac{\tau}{2}\right) e^{-j2\pi ft} d\tau
\]  \(\text{(25)}\)

This distribution is often used in experimental applications where linear FM signals can be found, for instance seismic surveying. Hyperbolic FM sonar signals emitted by bats is another conceivable application for the Wigner distribution.
Assuming that the signal $s(t)$ is expressed as a sum of two parts:

$$s(t) = s_1(t) + s_2(t)$$  \hspace{1cm} (26)

the WD can be expressed as

$$W_s(t,f) = W_{11}(t,f) + W_{22}(t,f) + W_{12}(t,f) + W_{21}(t,f)$$  \hspace{1cm} (27)

where

$$W_{12}(t,f) = \int s_1\left(t + \frac{\tau}{2}\right) \cdot s_2^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f \tau} d\tau = \int s_2\left(t - \frac{\tau}{2}\right) \cdot s_1^*\left(t + \frac{\tau}{2}\right) e^{-j2\pi f \tau} d\tau = W_{21}^*(t,f)$$

Since $W_{12}(t,f) = W_{21}^*(t,f)$ then $W_{12}(t,f) + W_{21}(t,f)$ is real valued and the WD becomes

$$W_s(t,f) = W_{11}(t,f) + W_{22}(t,f) + 2 \text{Re}\{W_{12}(t,f)\}. \hspace{1cm} (28)$$

The last term in (28) is called the interference term, or cross term. Note that the interference term is not unique but depends on how the signal is divided into parts, which can be made in an infinite number of ways. With the use of two signals the result is called the cross Wigner distribution (CWD).

The Fourier transform of the CWD yields the cross ambiguity function (CAF).

$$A_{CAF}(\tau, \nu) = \iint_{t,f} W_{12}(t,f) e^{j2\pi (\nu t - \tau f)} df \, dt$$  \hspace{1cm} (29)

where $\tau$ is the Doppler shift and $\nu$ is a time delay. The nature of the interference terms can be described to have an oscillatory structure and because of that they may be attenuated by smoothing (i.e., 2D low pass filtering).

The smoothed WD (SWD) can be derived by convolution of a WD to a signal $s(t)$ with a signal-independent smoothing kernel function[13], $\psi_T(\tau, \nu)$.

$$T_s(t,f) = \iint_{\tau, \nu} \psi_T(t-\tau, f-\nu) W_s(\tau, \nu) d\tau \, d\nu$$  \hspace{1cm} (30)

The result is said to be a member of the Cohen’s class. Every TFR that can be derived from the WD via such a convolution that belongs to the Cohen’s class. They will all have a unique and individual kernel function. The smoothing broadens the WD and the TF resolution decreases. A strong interference attenuation is achieved at the expense of TF resolution. Consider the kernel function;
3.3 Statistical peak distribution

The idea behind the statistical peak distribution method (SPD) is to divide the received signal into short time intervals, apply proper signal processing techniques, and compare the sets in a statistical manner. The benefits of this method is that it can be developed as a mathematical method in the sense that there is no need to analyse a visual 2D-plot but instead some statistical data is delivered by the algorithm. This is useful when it comes to automatization of the target recognition process. The answer to an operator of the system would be a target identification along with a presentation of the probability that the statement is true. The method works better the longer time the laser keeps track of the target since that means more information. The data files analysed here are up to two minutes long with a sample rate of 1 kHz.

The algorithm works in four steps;
- Detection of high amplitude peaks and their corresponding frequency
- Calculation of percentage appearance, or time of existence, for each peak
- Presentation of the time dependent amplitude variations for each specific frequency
- Separation of fundamental tones and overtones

3.3.1 Detection of peaks

The analysed data file (filename 07101204) will serve as an example in all figures throughout this and the following chapters to illustrate the different methods used in the analysis process. This file can be identified as one of the four relating to vehicle A with an aimpoint at the front of the vehicle in Appendix A. The characteristical frequency of vehicle A is presented in Appendix B.
3.3 Statistical peak distribution

First of all, the relevant frequency peaks have to be detected. A FFT is applied to each piece of time interval, or data set. Secondly, the 10 highest peak amplitudes in each data set is identified. This is done for each and every set, here totally 320 values, see Figure 8. In this specific data file the signal is divided into 32 data sets, each with a time interval of about one second.

![Figure 8](image)

**Figure 8.** Top: The FFT from each data set builds up this 3D-plot. Bottom: An averaged FFT based upon the 32 individual FFTs at the top plot. The ten highest peak amplitudes are graphically indicated with dots and frequency located at the left hand side.

3.3.2 Distribution of peaks

The algorithm checks, for each of the 10 highest peaks in every set, whether or not it exists in another data set, iterates through the whole data file and calculates the percentage appearance of that particular peak in the present file, as illustrated in Figure 9. Frequencies that has got a
percentage appearance lower than 10% is excluded in the plot, considered to be noise and not significant enough to be generated by the target. Peaks with a high percentage appearance could be a characteristic frequency for that particular vehicle.

![Percentage amount of peak appearances for all FFT sets](image)

**Figure 9.** A plot of percentage time appearance of peaks for a specific data file with 32 time sets. The ten highest peaks for each time set is registered and if for instance one peak is detected in half of the time sets it would be represented with a 50 percentage appearance in the plot above (see peak 100 Hz). The frequencies 54 and 82 Hz have been detected as one of the 10 highest peaks in all 32 sets and therefore have 100% appearance. 28 Hz can be seen in 90% of the sets and so on.

### 3.3.3 Amplitude variations of peaks

The peak amplitude varies for several reasons and independent measurements show no pattern that can be used for target classification. For instance, if the laser beam is moved along the turret the amplitude changes significantly. This was observed by US Air Force Research Laboratory in the field tests in Alabama. The important thing is that the vibration frequency remains the same.
3.4 The modified FOA method

Figure 10 displays the amplitude variation of the 10 highest peaks from each data set in a specific file. The dots mark the maximum amplitude value. From a target recognition point of view only the frequencies which have got high amplitude values and a lot of dots are of interest.

Figure 10. The 10 highest peak values from each set have been mapped together into one plot and the amount of amplitude variation is typical for all files being analysed here.

3.4 The modified FOA method

The original FOA method [2] is a correlation method which uses the ambiguity function. Some modifications of the method has been made [3] which ended up with a modified FOA method to be able to take care not only of stationary targets but also of radially moving targets, constantly or accelerating.

The FOA method can not demodulate signals with a carrier frequency. Modifying the method to first estimate the carrier frequency and then eliminate the carrier frequency by mixing the signal with a sinusoid, makes it possible to detect moving targets. Mixing the signal with a sinusoid centres the spectrum around zero as if there were no carrier frequency, like for a stationary target.
3.4.1 Algorithm description

The algorithm is able to handle three different kinds of scenarios; stationary targets, constant radially moving and radially accelerating targets. Correspondingly, these cases split the algorithm into three branches; no Doppler shift, constant Doppler shift and linear Doppler shift respectively, as illustrated in Figure 11. A constant Doppler shift means that the signal is modulated with a constant carrier frequency whereas the linear Doppler shift arises from a carrier frequency increasing with time caused by linear FM of the signal. The first branch, no Doppler shift, calls the original FOA method directly without any preprocessing of data. For the last two cases, the algorithm starts off with some signal processing and estimation calculations and thereafter calls back to the original FOA method algorithm.

![Figure 11. Block diagram of the modified FOA method.](image-url)
3.4 The modified FOA method

3.4.2 Acceleration estimation

To be able to demodulate a signal with a carrier frequency the algorithm estimates the Doppler frequency. This can be done by finding the center of the spectrum since the spectrum is composed of a carrier with a set of symmetrically spaced sidebands. To estimate and find the Doppler frequency, \( f_{\text{Dopp}} \), a number of maximum frequencies, \( N \), have to be located along with their values on the frequency axis. It can be explained as a way of locating the centre of gravity of the spectra. The average frequency of two side bands (or the average of all \( N \) values) is the Doppler frequency.

\[
f_{\text{Dopp}} = \frac{1}{N} \sum_{n=1}^{N} f_{\text{max}}(X(f))
\]

(33)

The spectrum has got symmetrically spaced side bands and . Finally, the spectrum is centered according to this peak location.

The centre frequency of the two spectra are compared and if the target is accelerating the frequencies will differ. The acceleration constant \( \alpha \), see (4), is estimated and used to create a linear frequency sinusoid. The algorithm compensates for the effects of an accelerating target (linear Doppler shift) by mixing the laser return signal with the linear frequency sinusoid to get a signal without a carrier frequency.

3.4.3 Mixing by IQ-representation

The surface vibration contributes to the frequency of the return signal as a sum of (9) and (4). This sum expresses the instantaneous frequency (IF) which is defined as the derivative of the signal phase.

It is impossible to distinguish between the two cases \( \cos(\theta) \) and \( \cos(-\theta) \) if only one signal is received. A solution to this problem is to receive two signals with a phase difference of \( \pi/2 \) radians. The I (in phase) and Q (quadrature phase) signals are created by mixing the received signal with two sinusoids, phase shifted \( \pi/2 \) radians relative to each other, the result is finally low pass filtered [14]. Q is delayed \( \pi/2 \) radians relative to the Doppler frequency.

The original FOA method calculates the analytic signal by applying a Hilbert transform [13] to the I as well as the Q signal and implement the result into the discrete AF,

\[
A_{\text{DAF}}(0, \nu) = \sum_{m} s_{i} \left( \frac{m}{2} \right) \cdot s^{*} \left( -\frac{m}{2} \right).
\]

(34)

In the final step the center frequency is eliminated by mixing, in the same way as for the I and Q signals, and the result is again put into the discrete AF. This last step is only necessary if the target is accelerating and the vibration frequencies are to be find either before or after the final step.
3.5 Comparison of different methods

The purpose of this thesis is to compare the modified FOA method with other kind of signal processing techniques. The development of a graphical user interface was necessary to extract the file information needed and control the signal processing in an easy way.

3.5.1 Software development

The graphical user interface (GUI) is designed in MatLab 5.3 and besides choosing signal processing method, the GUI offers an ability to analyse a single data set, a time interval or the complete time sequence of the received signal. The spectra step size can be set by the user, see Figure 12.

![Figure 12. A graphical user interface designed for vibrometry analysis. The filename displayed in the window is the origin to all illustrations and figures of the FOA method and the statistical peak distribution method.](image)

The interface is designed to present results from four spectral analysis methods:
- the modified FOA method
- statistical distribution method
- spectrogram
- time-frequency analysis using the Wigner distribution function

For each and every file being analysed a number of plots take part in the process of finding the characteristic frequencies.
3.5 Comparison of different methods

3.5.2 The modified FOA method

A drawback with the modified FOA method is that the result ($f_{Dopp}$) from the Doppler estimation subfunction is sensitive to small changes of the input parameter $N$ and will give different Doppler frequencies for slightly different values of $N$. Figure 13 illustrates how the Doppler frequency output varies with increasing $N$.

![Figure 13](image1.png)

Figure 13. It would be good to have a Doppler frequency independent of $N$, or at least a linear dependence. $N$ is the number of maximum frequencies.

The fact that $f_{Dopp}$ varies with the number of maxima ($N$) one choose to calculate the average for affects the vibration frequency spectra in a sense that it generates not only different peak amplitudes but also completely different frequency peaks, exemplified by Figure 14.

![Figure 14](image2.png)

Figure 14. Left: $N=5$ Right: $N=25$. There are some common frequencies but 10, 32, 36, 90, 116 and 125 Hz can only be found in one of the two plots as one of the top ten highest peak amplitude values.
3.5.3 Statistical peak distribution

It is preferable to have a long time interval to analyse, the longer time the better statistical accuracy is achieved. A disadvantage compared to the modified FOA method is that only stationary targets can be taken care of.

The noise floor is suppressed by calculating the mean FFT value for all FFTs generated by the original data sets. The statistical peak distribution comes up with stronger signal amplitudes than the FOA method as can be seen in Figure 15.

![Figure 15](image)

**Figure 15.** Left: The mean value of all FFTs from the data sets suppresses the noise and makes the peaks distinct for the statistical peak distribution. Right: The modified FOA method does not fully obtain the same signal amplitude even though the Doppler estimation is manually set to zero (stationary target) for optimal use.
3.5 Comparison of different methods

3.5.4 Spectrogram

The spectrogram lacks for frequency resolution which is the main disadvantage with this method. The contour plot is an alternative presentation which is appealing in some cases, see Figure 16.

![Spectrogram and Contour Plot](image)

**Figure 16.** Top: Spectrogram plot. Bottom: Contour plot with long lasting signals at 28, 54 and 82 Hz.
3.5.5 Wigner distribution

The major disadvantage by this TF method is the cross terms which are significant as it comes to multicomponent signals[17], as in Figure 17. An improvement can be obtained, e.g., by using smoothing kernel functions, see (31). The input parameter $\sigma$ used in (31) controls the degree of smoothing of the TFD. A fixed value of 500 was used throughout the analysis.

![TFD with Wigner distribution](image)

**Figure 17.** Wigner distribution with sigma=500. There should be strong signals at 28, 54 and 82 Hz during the whole time of analysis.

The plot above comes from the same data file as for the other methods being compared in this chapter and it is evident that there are better methods than using the Wigner distribution to analyse this particular file. The long time duration of signals at 28, 54 and 82 Hz that can be seen with, e.g., the statistical peak distribution method yields just small time fluctuations here. In an attempt to achieve a better plot other values of $\sigma$ was also used but with almost the same poor result.

The methods mentioned in this chapter are just a few examples of techniques to analyse a signal with time frequency analysis. A number of conceivable TFR methods, linear and quadratic, for the application studied in this thesis has been developed such as;

- cone kernel distribution
- generalized exponential distribution
- Butterworth distribution
- affine smoothing
- signal-adaptive SWD
- wavelet transform
- higher-order nonlinearity TFR.
4 Target recognition by vibrometry

There are a number of alternatives to choose among when designing a laser radar system. Is it going to be a homo- or heterodyne system, what wavelength and type of laser shall be used and so on. Hardware design is just one half of the issue. An effective software is vital to extract the information needed for recognizing a target signature. In this chapter the data analysis result from a NATO sponsored test will be presented in more detail. First of all the laser system used to collect all data will be explained, target properties will be defined and finally the analysis result is presented.

4.1 Specification of data

This NATO sponsored field test, conducted by the US Army, took place at Redstone Arsenal in Huntsville Alabama during a summer week in 1998, see Appendix A. The available data contains various kinds of information such as time, navigation and scanner information, sample velocity estimate and the corresponding signal-to-noise ratio. One also finds the number of samples in each set and a FFT derived spectra for the current set. The FFT is created by a spectrogram approach, developed by US Air Force Research Laboratory, explained in Section 4.2. Specific information about each file consists of type of target, range to target, frequency step, angle to target, humidity and finally level of smoke.
4.2 US Air Force laser system

The laser system uses a Tm:YALO CW laser that provides the desirable laser energy for both the transmitted laser beam and a reference local oscillator (LO). The laser produces 150 milliwatts and operates at a wavelength, $\lambda$, of 2.02 microns[15].

$$f_{\text{laser}} = \frac{c}{\lambda_{\text{laser}}} = \frac{3 \cdot 10^8}{2.02 \cdot 10^{-6}} = 1.49 \cdot 10^{14} \text{Hz}$$ (35)

The transmit and receive head, where the output of the laser is fiber coupled into, uses a beamsplitter to split out approximately 10% of the beam for the LO signal. The LO signal is then shifted 27 MHz ($f_{\text{eom}}$) by an electro-optic modulator. The remainder is expanded and transmitted through a 50 mm telescope. A similar and co-aligned receive telescope collects the returning beam, backscattered from the target. Finally, the returning beam and the LO beam are combined onto a photo-diode detector.

A traditional FM-discriminator approach can be limited by laser signature characteristics and degrades the performance for two reasons. Firstly, the laser coherence might be poor which results in laser phase noise. Secondly, since the laser wavelength is small compared to the height of the surface variations of the target, the received laser energy is diffused by speckle fading.

Both the amplitude and phase of the reflected laser signal contains target vibration information. The phase imparts a frequency modulation onto the reflected waveform, which require FM demodulation to extract the proper vibration information. However, the time-varying speckle pattern acts as noise in traditional FM demodulation techniques and decreases the performance significantly. A spectrogram approach can process both the amplitude and phase information in the presence of laser speckle and laser phase noise.

The spectrogram processor is built up according to the block diagram in Figure 18. The LO laser signal is optically mixed with the FM-modulated return signal and is therefore said to be offset heterodyne detected. The offset refers to the $f_{\text{eom}}$ shift whereas heterodyne means that there are two light frequencies, in this case coming from the same source but splitted in two, one

---

**Figure 18.** Block diagram of CW spectrogram processor.
of them shifted. Another alternative would be to have a homodyne system where the signal and reference beam has got the same frequency and use two detectors with a $\pi/2$ phase shifter to get the proper sign of the target motion.

The FM signal out of the detector has got a difference frequency of 27 MHz for this heterodyne system and is mixed with an intermediate frequency to generate a low instantaneous frequency of 500 kHz, as indicated in Figure 19.

After LP-filtering the mixer output, the signal is sampled with a 2 MHz digitizer, where the A/D conversion rate is carefully chosen to be consistent with the expected bandwidth of the FM-modulated return signal. N samples, usually 1024, are collected to form the first sample set which is then input into an FFT. A centroid algorithm then estimates the frequency for that particular sample set and this correlates to a velocity. The carrier frequency will be centered around zero since only stationary targets are to be considered here. A time history of such velocities is built up and put into a second FFT where the final output is the vibrational frequency spectrum. The Nyquist frequency is half the sample rate and with velocity estimates at a 2 kHz rate, as this system develops, a frequency capability of up to 1 kHz can be achieved.

The return signal $s_{\text{ret}}(t)$ has to be shifted to a lower frequency region where the signal processing is more suitable. This is done by optically mixing the returned laser signal with the electro-optically frequency shifted LO signal $s_{\text{LO}}(t)$.

$$f_{\text{LO}} = f_{\text{laser}} + f_{\text{eom}}$$  \hspace{1cm} (36)

$$s_{\text{LO}}(t) = A_{\text{LO}} \sin(2\pi f_{\text{LO}} t) = A_{\text{LO}} \sin(2\pi [f_{\text{laser}} + f_{\text{eom}}] t)$$  \hspace{1cm} (37)

Assume a product of two general signals

$$\sin(\omega_1 t) \cdot \sin(\omega_2 t) = \frac{1}{2} [\cos(\omega_1 - \omega_2) t - \cos(\omega_1 + \omega_2) t]$$  \hspace{1cm} (38)

then the product is the sum and difference frequencies and if $\omega_1$ and $\omega_2$ are large with a small difference $(\omega_1 - \omega_2)$ then the first term is a low frequency term which may be extracted by means of a LP-filter.

![Figure 19. Frequency shifting and mixing with the IF frequency.](image-url)
4.3 Target properties

The return signal includes the laser frequency $f_{\text{laser}}$, a doppler shifted frequency, $f_{\text{rd}}$, due to radial target motion and a surface vibrational doppler frequency, $f_{\text{vd}}$, where the two doppler frequencies can be positive or negative. The frequency of the returned signal is within the interval

$$ (f_{\text{laser}} - f_{\text{rd}} - f_{\text{vd}}) < f_{\text{ret}} < (f_{\text{laser}} + f_{\text{rd}} + f_{\text{vd}}) $$

(39)

All of the vehicles analysed in this thesis are stationary and therefore $f_{\text{rd}}$ is zero.

Howcome the LO is shifted with 27 MHz of all frequencies? Consider the equation for the radial doppler shift, where $v$ is radial velocity,

$$ f_{\text{rd}} = \frac{2v}{\lambda_{\text{laser}}} = \frac{2 \cdot 1}{2.02 \cdot 10^{-6}} \approx 1 \text{ MHz} $$

(40)

which means that the radial doppler shift will be about 1 MHz for each meter per second that the vehicle is moving[16]. A vehicle moving with a speed of 27 m/s (97 km/h) would then cause a radial doppler shift of 27 MHz and this is above the maximum speed for the vehicles considered here. In a heterodyne detection system there is a need for a modulation frequency high enough to always get a positive $f_1$ frequency.

$$ f_1 = f_{\text{LO}} - f_{\text{ret}} = (f_{\text{laser}} + f_{\text{eom}}) - (f_{\text{laser}} \pm f_{\text{rd}} \pm f_{\text{vd}}) = $$

$$ = f_{\text{eom}} \pm f_{\text{vd}} = f_{\text{eom}} \pm f_{\text{vd}} = 27 \text{ MHz} $$

(41)

At the last step in the expression for $f_1$ the radial doppler shift is set to zero due to stationary targets and the vibrational doppler shift contribution is small compared to $f_{\text{eom}}$. Finally, $f_1$ is mixed with the intermediate frequency and the result, a low instantaneous frequency, $f_2$, is LP-filtered.

$$ f_2 = f_1 - f_{\text{eom}} = f_{\text{eom}} \pm f_{\text{vd}} - f_{\text{eom}} = \pm f_{\text{vd}} $$

(42)

### 4.3 Target properties

Data from five different kinds of military ground based vehicles has been analysed. All targets were stationary with the engine idle running during the measurements and each of the more than 150 files contains data for a measurement time of about 30-120 seconds. The distance to the target varied between 530 and 3200 meters. A telescopic sight mounted on top of the laser made it possible to adjust the aimpoint to the center, rear or front of the target. Some of the vehicles were tanks and in those cases it was possible to aim at the base of the barrel, on the turret and
even at the small chem-lite markers. White Phosphorus was fired as well as fog oil smoke of different level to see whether or not this would suppress the received vibration signal. Measurements in rainy weather were also performed.

Unfortunately, all five vehicles has not been tested with all aspects and therefore a complete comparation is not possible. A total number of 113 files have been analysed, see Appendix A. It is also desirable to have more than just one file for a specific aspect and vehicle to ensure some kind of statistical certainty.

### 4.4 Different aspects for a certain vehicle

The results are based on a mixture of all four methods, i.e. as one method fails to locate the vibration frequencies in a particular data file another method works better and the result from that method is registered.

Three questions will be answered by this way of studying the result from the signal processing:

1. *Is the vibration frequencies independent of the laser beams point of impact with the targets body?*

2. *Which frequencies are common for a vehicle irrespective of what part of the body you analyse, i.e. which are the characteristic frequencies for a certain vehicle?*

3. *Which method is the most robust and reliable one?*

Often, two or more of the four methods present more or less the same result. Sometimes the methods come up with the same frequency peaks but the peak amplitude is different. Usually one frequency is dominant and much stronger than the other ones and for those cases the strongest peak is the same for all four methods. When the result varies a lot between the methods the result from the statistical peak distribution is registered since it seems to be the most reliable one.

The software has got an ability to deliver a total number of nine plots if all four methods are applied to a data file and typical plots from one measurement can be seen in Figure 20. This is the essential background information that has been used to draw conclusions from.
The basic idea is to find the fundamental tones among all of the significant frequencies that appears and then sort out the overtones to each fundamental tone. Notice that it is possible that an overtone can in fact be a fundamental tone but not vise versa, i.e., a tone sorted out by the algorithm as a fundamental one can never be an overtone. The reason for comparing all tones is to reduce the number of tones into fundamental tones and just overtones. For some fundamental frequencies there might be no significant overtone present and in other cases several overtones are related to the same fundamental frequency. The fundamental tones are always present but their respective overtones can be more or less suppressed.

The spectral resolution varies among the analysed data files, each file with a specific frequency step of 2, 4 or 8 Hz and this has been taken into account when trying to find a fundamental tone and its overtones.

Figure 20. Available plots from all four methods. Top left: modified FOA. Top middle, top right and the whole middle row belong to the statistical peak distribution method. Bottom left and middle is the contribution from a spectrogram with a contour plot in the middle. Bottom right is a pseudoplot of a Wigner distribution applied to the signal. The signal beeing analysed here is the same signal that is exemplified in section 3.3.

4.5 Tone comparisons

The basic idea is to find the fundamental tones among all of the significant frequencies that appears and then sort out the overtones to each fundamental tone. Notice that it is possible that an overtone can in fact be a fundamental tone but not vise versa, i.e., a tone sorted out by the algorithm as a fundamental one can never be an overtone. The reason for comparing all tones is to reduce the number of tones into fundamental tones and just overtones. For some fundamental frequencies there might be no significant overtone present and in other cases several overtones are related to the same fundamental frequency. The fundamental tones are always present but their respective overtones can be more or less suppressed.

The spectral resolution varies among the analysed data files, each file with a specific frequency step of 2, 4 or 8 Hz and this has been taken into account when trying to find a fundamental tone and its overtones.
4.5 Tone comparisons

Assume a frequency step of 2 Hz;

Fundamental tone: \( f = f_0 \pm \text{SpectraStepSize} = f_0 \pm 2 \text{ Hz} \)
1\textsuperscript{st} overtone: \( 2 \cdot f = 2 \cdot f_0 \pm (2 \cdot \text{SpectraStepSize}) = 2 \cdot f_0 \pm 4 \text{ Hz} \)
2\textsuperscript{nd} overtone: \( 3 \cdot f = 3 \cdot f_0 \pm (3 \cdot \text{SpectraStepSize}) = 3 \cdot f_0 \pm 6 \text{ Hz} \)

The frequency interval for the 2\textsuperscript{nd} overtone is \( \pm 6 \text{ Hz} \). If the frequency step is 4 Hz instead of 2 Hz, the interval will be as much as \( \pm 12 \text{ Hz} \) and only an increase of the sample frequency can change this matter.

4.5.1 Vehicle A

As many as seven aspects have been compared for vehicle A. The statistical accuracy is best for this vehicle with 40 files to analyse. However, the quality varies a lot and in some cases is really poor. The frequency step for vehicle A is 2 Hz and the characteristic frequencies are;

<table>
<thead>
<tr>
<th>Fund. tone</th>
<th>1st overtone</th>
<th>2nd overtone</th>
<th>3rd overtone</th>
<th>4th overtone</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 Hz</td>
<td>36 Hz</td>
<td>54 Hz</td>
<td>-</td>
<td>90 Hz</td>
</tr>
<tr>
<td>28 Hz</td>
<td>54 Hz</td>
<td>82 Hz</td>
<td>116 Hz</td>
<td>-</td>
</tr>
</tbody>
</table>

By using these frequencies as a fingerprint for vehicle A and match them with the aspects one by one, one gets a correlation rate table presenting how many of the fingerprint frequencies that are present at a certain angle of aspect, see Appendix B.

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Correlation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>center</td>
<td>4 out of 6</td>
</tr>
<tr>
<td>front</td>
<td>6/6</td>
</tr>
<tr>
<td>rear</td>
<td>6/6</td>
</tr>
<tr>
<td>turret</td>
<td>0/6</td>
</tr>
<tr>
<td>white phosphorus</td>
<td>0/6</td>
</tr>
<tr>
<td>smoke</td>
<td>3/6</td>
</tr>
<tr>
<td>rain</td>
<td>4/6</td>
</tr>
</tbody>
</table>

4.5.2 Vehicle B

The front and rear angle of aspect has got very good signal to noise ratio. In the rear case the time interval is as long as 1.5 - 2 minutes and the seven most frequently occurring peaks defines the signature as seen in Appendix C. There are several files with different levels of smoke and it is interesting to notice that the SNR is remarkably good. The frequency step is 2 Hz for center and front but 4 Hz for the other two aspects.
4.5 Tone comparisons

Table 3. Fundamental- and overtones for vehicle B.

<table>
<thead>
<tr>
<th>Fund. tone</th>
<th>1st overtone</th>
<th>2nd overtone</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 Hz</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28 Hz</td>
<td>56 Hz</td>
<td>116 Hz</td>
</tr>
<tr>
<td>34 Hz</td>
<td>68 Hz, 76 Hz</td>
<td>-</td>
</tr>
<tr>
<td>40 Hz</td>
<td>76 Hz, 86 Hz, 88 Hz</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4. Correlation rate table for vehicle B

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Correlation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>center</td>
<td>4 out of 7</td>
</tr>
<tr>
<td>front</td>
<td>4/7</td>
</tr>
<tr>
<td>rear</td>
<td>7/7</td>
</tr>
<tr>
<td>smoke</td>
<td>6/7</td>
</tr>
</tbody>
</table>

4.5.3 Vehicle C

For this vehicle the files are quite noisy and the time interval is shorter than the average file length. There are a number of frequencies appearing for just a short interval of time but only three significant frequencies could be identified as long lasting ones and they are 6, 30 and 36 Hz. If 6 Hz is assumed to be a fundamental tone then 30 Hz can be considered to be either the 3rd or the 4th overtone with a spectra step size of 2 Hz.

Table 5. Fundamental- and overtones for vehicle C

<table>
<thead>
<tr>
<th>Fund. tone</th>
<th>1st overtone</th>
<th>2nd overtone</th>
<th>3rd overtone</th>
<th>4th overtone</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Hz</td>
<td>-</td>
<td>-</td>
<td>30 Hz</td>
<td>30 Hz, 36 Hz</td>
</tr>
</tbody>
</table>

Appendix D illustrates the lack of long lasting frequencies with just 36 Hz present when the laser is pointed at the rear end of the target.

Table 6. Aspects and correlation rate table for vehicle C

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Correlation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td>3 out of 3</td>
</tr>
<tr>
<td>rear</td>
<td>1/3</td>
</tr>
<tr>
<td>turret</td>
<td>3/3</td>
</tr>
<tr>
<td>barrel</td>
<td>2/3</td>
</tr>
</tbody>
</table>

As will be seen in next section, vehicle C and D has got some common frequencies.

4.5.4 Vehicle D

There are five files to analyse for the center aspect and just one file for rear and turret respectively. The spectra step size is 4 Hz and it is quite noisy perhaps due to high humidity during the measurement. Thus, only two frequencies are worth mentioning, 4 and 36 Hz, according to Appendix E.
4.5 Tone comparisons

Table 7. Correlation rate table for vehicle D

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Correlation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>center</td>
<td>2 out of 2</td>
</tr>
<tr>
<td>rear</td>
<td>0/2</td>
</tr>
<tr>
<td>turret</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Vehicle C and D are similar and they have some common frequencies; 4, 8, 24, 32, 36 and 118 ± 2 Hz.

4.5.5 Vehicle E

Five aspect angles have been compared for vehicle E. The data from this vehicle has got the lowest noise floor of all files, which might be explained by a short distance to the target (550 m). Note that Table 7 contains a pair of frequencies which are close together (∆2 Hz) and not separable due to a frequency step of 4 Hz.

However, it is possible that the true frequencies are e.g. 32 and 38 Hz due to the uncertainty interval set by the frequency step. The two frequencies 68 and 74 Hz listed below are also too close to be separated.

Table 8. Tone comparisons for vehicle E

<table>
<thead>
<tr>
<th>Fund. tone</th>
<th>1st overtone</th>
<th>2nd overtone</th>
<th>3rd overtone</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 Hz</td>
<td>24 Hz</td>
<td>48 Hz</td>
<td>92 Hz</td>
</tr>
<tr>
<td>34 Hz</td>
<td>68 Hz</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>36 Hz</td>
<td>74 Hz</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

There is only one file from the smoke aspect and it is therefore omitted in the process of finding significant frequencies which also explains the poor correlation rate, see also Appendix F

Table 9. Aspects from vehicle E and its correlation with significant spectral tones

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Correlation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>center</td>
<td>4 out of 6</td>
</tr>
<tr>
<td>front</td>
<td>6/6</td>
</tr>
<tr>
<td>rear</td>
<td>3/6</td>
</tr>
<tr>
<td>chem lite</td>
<td>3/6</td>
</tr>
<tr>
<td>smoke</td>
<td>0/6</td>
</tr>
</tbody>
</table>
Each target has got a unique signature which is, with some reservation, possible to identify and classify with the methods examined here. With the exception of 36 Hz, which is present in four out of five aspects irrespective of target model, there seems to be no common frequencies that always arises when a certain aspect is observed. If that was the case a classification would be impossible to make.

Different aspects has got certain influence on the vibration frequencies but the available amount of data is too small to make a correct judgment whether or not it is possible to identify a target from other kind of angles and aspects. At least it seems to be possible to classify vehicles, with the suggested “fingerprints”, for the aspects available.

The modified FOA method has been compared with the statistical peak distribution method and two standard TF methods, the spectrogram and the WD in sense of robustness and reliability.

The modified FOA method is very sensitive to the input parameter value in the Doppler estimation step, as discussed in Section 3.2.2. The sensitivity might be explained by a low SNR and as the algorithm locates a signal peak it is actually coming from noise. With just a few strong signal peaks this could be the case when as many as N=20 peaks are to be located.

A TFD will perform very differently depending on the class of signals it is adopted for and it is vital for the outcome to have access to real data at an early stage. The type and extent of smoothing, window length and the way of graphically presenting the TFD affect the performance. A smoothed WD with $\sigma = 500$ has been used in the kernel function and the result has been analysed with a color image function. There is much more work to do in finding an optimal TFR for the particular type of signals studied here.
4.5 Tone comparisons

It will be easier to distinguish between two or more close frequencies identified by the SPD algorithm if the sample frequency is increased. Thereby it will be possible to specify if two close up frequencies are actually the same or separated which in turn also will affect the identification of the overtones. This phenomena is obvious in Section 4.5.5, where a lower sample frequency has been used than for the other cases in Chapter 4.5. The statistical accuracy for the SPD method will increase along with an increase of the sample frequency and there might be possible to find more overtones, maybe all the way up to 200 Hz. The strength of the SPD method is the use of statistics. The SPD method is reliable in 4 of 5 cases. The outcome of a specific data analysis depends on the SNR and for how long the target has been tracked.

A requirement specification of a robust and reliable system would be a system able to handle all kinds of moving targets and offers a statistical accuracy that the target recognition is indeed true. Such a system can be achieved by combining the modified FOA method with the proposed SPD method or maybe an improved and more sophisticated SPD version with more statistical information.

In a military application a database with all known target signatures has to be maintained and updated. The target will not be tracked a couple of minutes but perhaps only a few seconds and the resulting vibration spectrum shall be compared with the ones in the database. At that point it is necessary to shorten the window length for each data set in the SPD method. The database could contain TF images along with the significant vibration frequencies and the system could then try to match images instead of frequencies. Further investigations in trying to match frequencies rather than images should be made. Think about the complexity of a moving target and all cross terms that arises. The image is complex enough with just a stationary target.
References


4.5 Tone comparisons

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Appendix A

Table 10. List of the total number of files for each vehicle and aspect. There are several aspects that are not present and therefore comparable for all five vehicles. In general, the main data analysis has been focused on different aimpoints since this category of aspect contains most data.

2 Micron Vibration Data
RACE II Tests
Redstone Arsenal in July 1998

<table>
<thead>
<tr>
<th>Aimpoint</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centered at left or right side</td>
<td>3</td>
<td>4</td>
<td>-</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Front of vehicle</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>-</td>
<td>6</td>
</tr>
<tr>
<td>Rear of vehicle</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>On the turret</td>
<td>1</td>
<td>-</td>
<td>3</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Base of barrel</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Left Chem-lite marker</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Distance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>530</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>540</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>550</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>12</td>
</tr>
<tr>
<td>560</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>980</td>
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<td>-</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1000</td>
<td>-</td>
<td>29</td>
<td>18</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>1020</td>
<td>18</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1030</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1035</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3200</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Smoke, fog</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White Phosphorus fired</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Smoke, level 1</td>
<td>1</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Smoke, level 2</td>
<td>-</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Smoke, level 3</td>
<td>4</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Smoke, level 5</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Smoke, level 6</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Rain, 75 deg. F, high humidity</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Sub total: 40 32 18 7 16

Total: 113
**Table 11.** Different aspects from vehicle A. All significant frequencies from each and every data file, with one particular aspect at a time, are put together. Each significant frequency can be detected in different number of data file, for some of the frequencies just in a few files as other frequencies might appear in all files. A percentage representation of data file variation for the detectable frequencies with some frequencies more common than others, the common ones marked with a line.
Table 12. Different aspects from vehicle B. Common frequencies are 8, 56, 68, 76, 86, 88 and 116 Hz irrespective of aspect.
Table 13. Different aspects from vehicle C. For vehicle C only a few frequencies could be detected.
Table 14. Different aspects from vehicle D. A total amount of 7 files has been analysed for vehicle D, which is 3-4 times less than for the other vehicles, see Appendix A. More data files might come up with another result so the frequencies 4 and 36 Hz extracted here could prove to be less significant if they would be compared to more data files.
Table 15. Different aspects from vehicle E. There is a possibility that 12 and 34 Hz are fundamental tones with first the overtone at 24 and 68 Hz, but the accuracy set by the spectral step size is too low to draw such a conclusion.