Passive Aircraft Altimetry using GPS as a Bistatic Radar - A simulation model

Examensarbete utfört i Datatransmission
vid Tekniska Högskolan i Linköping
av

Anders Andersson och Daniel Hallgren

Reg nr: LiTH-ISY-EX-3384-2003
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Supervisor: Fredrik Neregård
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In this thesis, the use of reflected GPS signals as a bistatic, passive altimeter is examined. A simulation model has been developed and implemented, and simulations using the model have been done. Different types of ground cover have been investigated, both water and land types, with varying reflectivity and scattering behaviour. For larger terrain variations, e.g. mountains and valleys, a ground elevation database has been used. Furthermore, several parameters, like the antenna coverage and the satellite elevation angle, have been varied and the result of this examined.

The results of these simulations show that measuring height is possible for both sea and land surfaces. The accuracy depends on several error factors, like a bias originating from surface roughness and measurement errors due to noise in the receiver. The simulations also show that the most important design parameter is the antenna, which must be designed to give a sufficiently large SNR, capture the specular reflection and avoid unwanted reflections.

**Nyckelord**
Bistatic radar, GPS, altimeter, passive, reflectivity, scattering, altimetry
Abstract

A common way to measure height in aerial vehicles is to use a radar height altimeter (RHM). Since the RHM transmits radar pulses that can be detected, a passive alternative would be desirable in military applications. The idea to use reflected signals from the Global Positioning System (GPS) as a bistatic radar, has been established over the last years. The GPS signals are already present and would not reveal aeroplanes in covert operations.

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The results of these simulations show that measuring height is possible for both sea and land surfaces. The accuracy depends on several error factors, like a bias originating from surface roughness and measurement errors due to noise in the receiver. The simulations also show that the most important design parameter is the antenna, which must be designed to give a sufficiently large SNR, capture the specular reflection and avoid unwanted reflections.

Keywords: Bistatic radar, GPS, altimeter, passive, reflectivity, scattering, altimetry
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Anders Andersson            Daniel Hallgren
Linköping, September 2003
Notation

Symbols

\[ A_e \quad \text{Effective aperture, m}^2 \]
\[ \beta \quad \text{Angle for deciding antenna coverage} \]
\[ c_0 \quad \text{Speed of light in vacuum} \]
\[ \text{dBi} \quad \text{Antenna gain in dB relative to a isotropic antenna} \]
\[ \text{dBW} \quad \text{Power in dB relative to 1 W} \]
\[ \text{dB-Hz} \quad \text{Relative power in dB, per Hz} \]
\[ \text{dBW-Hz} \quad \text{Power in dB relative to 1 W, per Hz} \]
\[ \epsilon \quad \text{Dielectric constant} \]
\[ \epsilon_r \quad \text{Relative permittivity} \]
\[ \varepsilon_0 \quad \text{Permittivity} \]
\[ \eta \quad \text{Loss factor for antenna} \]
\[ G \quad \text{Antenna gain} \]
\[ \gamma^p \quad \text{Radar cross section per unit area, perpendicular to ground} \]
\[ \text{L1, L2} \quad \text{The two carriers of GPS} \]
\[ h_c \quad \text{Critical height for Rayleigh criterion} \]
\[ \kappa \quad \text{Conductivity} \]
\[ \lambda \quad \text{Wavelength, m} \]
\[ \Lambda(\tau) \quad \text{Triangle function} \]
\[ \omega \quad \text{Radian frequency, rad/s} \]
\[ \Omega \quad \text{Scattering area} \]
\[ \phi_h(\tau) \quad \text{Intensity profile} \]
\[ P_t \quad \text{Transmitted power, W} \]
\[ \psi \quad \text{Satellite elevation angle over horizon} \]
\[ r(\tau) \quad \text{Auto correlation function for the C/A code} \]
\[ Q \quad \text{Scattering pattern} \]
\[ R \quad \text{Reflectivity} \]
\[ S_g \quad \text{Power density at ground or target, W/m}^2 \]
\[ S_r \quad \text{Power density at receiver, W/m}^2 \]
\[ \sigma^p \quad \text{Radar cross section per unit area} \]
\[ \sigma_r \quad \text{Standard deviation of measured height, due to receiver noise} \]
\( \theta \) Angle of incidence
\( T_C \) Symbol time of C/A code, s
\( T_d \) Coherence time
\( T_m \) Delay spread
\( \xi(t_e) \) Modeled bias error for the measured height
\( (x, y, z) \) Cartesian coordinates
\( (r, \phi, \theta) \) Spherical coordinates

**Abbreviations**

ADC Analog to Digital Converter
AWGN Additive White Gaussian Noise
C/A Coarse/Acquisition, a pseudo random code used in GPS
CDMA Code Division Multiple Access
DMR Delay Mapping Receiver
GPS Global Positioning System
INS Inertial Navigation System
ISI Intersymbol Interference
LNA Low Noise Amplifier
LFSR Linear Feedback Shift Register
LHCP Left Hand Circularly Polarized
MSL Mean Sea Level
PRN Pseudo Random Noise
RF Radio Frequency
RHCP Right Hand Circularly Polarized
RHM Radar Height Altimeter
RMSE Root Mean Square Error
SNR Signal to Noise Ratio
WSSUS Wide Sense Stationary Un correlated Scattering
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Chapter 1

Introduction

1.1 Background

The height is an important parameter in aerial vehicles, and there are several purposes where it is useful. For example, the height can be used to support navigation systems, like inertial navigation system (INS). For this, the height over mean sea level (MSL) is most commonly used, which can be measured with a barometer. In other applications the height over ground is needed, e.g. in ground collision warning systems or terrain following close to the ground. This is obtained with a radar height altimeter (RHM), which transmits radar pulses. While INS and barometer are passive instruments, the electromagnetic waves transmitted by the RHM can be detected. In military applications it is often desirable with a passive system and such an alternative to the RHM would be beneficial.

One way to accomplish this is to use a bistatic radar altimeter, where a signal is transmitted from a different source, reflected at the ground and then received by the aeroplane. If also a direct signal is received, it is possible for the aircraft to calculate the height over ground by using the difference in arrival time. The radar signals will then not be transmitted by the aircraft and the system will be passive. Besides radar, signals used in radio communications systems are also reflected at the ground, since these signals are transmitted on electromagnetic waves much resembling those of radar. This leads to the idea that these signals can be used for the purpose of a bistatic passive radar system. Signals from satellites in the Global Positioning System (GPS) fulfill the necessary requirements and can be used in such a system.

Research on using GPS signals reflected from ground has been going on for the last decade. In 1993 Martín-Neira proposed the use of GPS signals reflected from
ocean surfaces for satellite altimetry [10]. The main interest was in the sensing of sea surface heights. Experimental proofs that the reflected signals are strong enough were delivered in 1994, when Auber accidently measured the reflections from the ocean [15].

The idea to use the reflected signals for aircraft altimetry was discussed by Katzberg et al. in 1999 [8] and experimental results confirmed this theory. Katzberg et al. also derived some theoretical results concerning accuracy, although the technique to estimate the height from the correlation function was quite simple.

This concept was further developed and examined in studies over both land and sea, by Masters et al. in 2001 [13]. They investigated several methods of estimating the height from the reflected signal, partly with help of a theoretical model for reflected GPS signals [18]. Two experimental flights confirmed the theories and the resulting performance of the bistatic altimeter appeared to be satisfying. Further work in this area is ongoing.

The idea for this thesis originates from Saab Bofors Dynamics (SBD). Many of the military applications developed at this company feature the traditional RHM, e.g. in navigation systems for missiles. It would be interesting to investigate the possibilities of using a passive altimeter instead.

1.2 Purpose

The purpose of this thesis has been to examine and characterize the use of GPS as a bistatic radar altimeter system. Since this idea is new within SBD, focus has mainly been on performing an initial study, and preparing for further research. Furthermore, within the time limits of the thesis, the height measurement method has been examined with respect to different parameters, such as the type of ground cover and terrain variations.

From the original purpose two goals have emerged:

- Develop and implement a simulation model for the reflected GPS signals. The model must be complete enough to make height measurements over varying terrain. Several types of ground covers, e.g. water or forest, with different properties are desirable. Since many simplifications will be done, the simulation model will be designed to be as flexible as possible, i.e. it should be easy to develop and implement e.g. a new model for signal scattering.
• Use the model to perform simulations and evaluate the height measurement method. The method should be evaluated with respect to different parameters, such as terrain variations or width of antenna coverage. The number of parameters and scenarios included in the simulations are decided by the available time. As one evaluation method, the height measured with the simulation model compared to the true height will be used.

The simulation model has become an important part of the thesis, both due to requirements from SBD, and since it is the solution method chosen. The simulation model itself is not supposed to be a part of a larger system, but the final altimeter might be.

1.3 Solution method

The solution method has been to develop a simulation model of the problem, with descriptions of the GPS signal, the ground reflection and the receiver. The benefits with this method are several. To solve the problem analytically would be too difficult and not flexible enough. Some analytical models have been developed, but they are usually only applicable in certain cases, like signals reflected at ocean surfaces. With a general simulation model it is possible to simulate height measurements over several types of ground and terrain, and with special types of terrain variations. It is also easy to change different parameters, like the width of the antenna beam or the power of the signals, and study the effect of this.

Another approach is to actually build a height altimeter and perform real measurements, which also has been done [13], but this would be too costly and also less flexible. However, to validate the simulation model, this is necessary.

1.4 Simplifications

This thesis has mainly been concentrated on developing a simple but fully functional simulation model. The height measurement method has only been examined briefly and there is still much to do here. Many simplifications have been done to the simulation model, due to the limited scope of the thesis. For example, modeling the ground and reflections of the GPS signal are approximated with less complex models. The simplifications of the model are described more in detail in Chapter 5.
1.5 Outline

The outline of the rest of this report is the following:

**Chapter 2** describes the problem in general.

**Chapter 3** gives the necessary theoretical knowledge needed to understand the problem.

**Chapter 4** describes the methods used to estimate the height from the received signal and the effect of noise in the receiver.

**Chapter 5** describes the design and implementation of the simulation model used.

**Chapter 6** contains results from different simulations used to evaluate the model and characterize the height measurements.

**Chapter 7** summarizes the results from the simulations.

**Chapter 8** gives the concluding remarks on the thesis.
Chapter 2

Problem overview

Consider the scenario described in Figure 2.1. An aerial vehicle, e.g., an aeroplane, is flying over the ground at a certain height, \( h \). The GPS signal is transmitted from a satellite with an elevation angle over ground of \( \psi \), and reaches the receiver from two different directions: one direct signal on top of the aeroplane and one reflected signal from the ground. The task for the height measurement system, called the altimeter, is to estimate \( h \) from these signals. The main problem is: how can this be done, and with what uncertainty?

![Figure 2.1. The scenario.](image)
2.1 Measuring height

If the direct signal arrives at time $t_1$ and the reflected signal at time $t_2$, the height over the ground can be calculated from the time delay, $\Delta t = t_2 - t_1$. If the satellite’s elevation angle over ground, relative to the aeroplane, $\psi$, is known, the height can be estimated by the following relationship, assuming that the delay is measured to the shortest reflection [13]

$$h = \frac{c_0 \cdot \Delta t}{2 \sin(\psi)}$$  \hspace{1cm} (2.1)

To obtain the time delay the properties of the GPS signal can be used. A schematic view of the scenario is shown in Figure 2.2. When the signal arrives at the receiver, a so-called correlation function is calculated, which is used to track and locate the signal. The top of the function will approximately correspond to the time of arrival. Hence, if both the correlation function from the direct signal and the correlation function from the reflected signal are calculated, their tops will be at two different times. The difference between these two times will be the delay between the two signals (see Figure 2.3).

In order to compensate for the elevation angle of the satellite, the angle $\psi$ has to be known. If it is not known exactly, the angle must be estimated.

There are other problems that complicate the scenario. In Figure 2.1, the signal from the ground is only reflected at one point, a so-called specular reflection. However, the signal is often reflected at many points, causing a well-known problem called multipath propagation (see Figure 2.4). Depending on how undulating the terrain is and what kind of ground cover there is, the multipath problem can be more or less pronounced. The received correlation function will be a sum of many correlation functions, at different time delays. The shape will change and the top can occur at a later time, introducing an error in the time delay.

Furthermore, the signal will be disturbed by noise in the receiver. This will introduce an uncertainty in the height measurement, which has to be analyzed.
2.1 Measuring height

**Figure 2.3.** Time delay between the two signals, derived from the correlation functions of the signals.

**Figure 2.4.** Multipath propagation of the reflected signal.
Chapter 3

Theory

To be able to create an accurate simulation model it is necessary to understand the components of the problem, e.g. the GPS system and the reflection of the signals. In this chapter, the different items are treated one by one, what existing theories there are and how they work in this case. Readers familiar with the theories can probably read it quickly but should pay notice to the application of them in this thesis.

3.1 Overview

Consider the problem as described in Chapter 2. The model would consist of essentially three parts:

- The transmitting GPS satellite. This includes important issues like signal structure, power and frequency and also position of the satellite.

- The path of the signal traveling to the receiver. The atmosphere will introduce some distortion to the signal, but the main problem is the ground reflection.

- The receiver, including the antenna.

This chapter will treat the different issues and the theory needed to understand each problem in the following way.
The GPS system. To be able to model the GPS signal, it is necessary to understand the signal structure, power and frequency. Besides this, the most important issue is the receiver and the correlator that produces the correlation function. These parts are discussed in Section 3.2 together with other aspects of the GPS system.

The atmosphere. As the signal travels from the satellite to the receiver, it will pass through the layers of the atmosphere. Generally the errors introduced here are modeled as delay in the signal’s path. However, in this thesis the effects of the atmosphere are considered as negligible as a result of the discussion in Section 3.3.

Channel models. A useful tool for describing the propagation of the signal after it has traveled through the atmosphere and is reflected at the ground, is the theory of channel models. This theory will also be necessary when treating other subjects such as fading. Section 3.4 is dedicated to channel models.

Modeling reflected GPS signals. In Section 3.5, a discussion is held on how reflected GPS signals are modeled. It turns out to be more convenient to describe the received signal in terms of power, as a sum of many correlation functions.

The bistatic radar equation. To calculate the correlation function of the reflected signal, the received power as a function of delay from the direct signal, must be determined. This can be done by using the bistatic radar equation from radar theory. A further investigation of this can be found in Section 3.6.

Ground reflection. The most difficult part to calculate in the radar equation is the reflection of the signal at the ground. Several parameters contribute to the reflected power, e.g. the attenuation in the ground and the scattering of the signal power in different directions. This complex issue has to be examined thoroughly, which is done in Section 3.7.

Fading. A common problem in radio communications systems is fading, which is addressed in Section 3.8. There are several types of fading depending on the characteristics of the channel, e.g. if the ground features large variations or the speed which the aeroplane travels with. The main effect of fading will be a randomly varying signal; the amplitude and phase will behave as stochastic variables. However, due to the complex nature of fading, there will only be a discussion about the problem and it is not implemented in the simulation model.

Antennas. The receiving antenna turned out to be of great significance, the width of the antenna coverage being the most important parameter. A narrow antenna coverage has a larger antenna gain and therefore the signal-to-noise ratio is better, but on the other hand low satellite angles tend to cause errors. This is because the specular reflection, which is the shortest traveled path by the reflected signal, may not be captured by the antenna. An introduction on antennas is given in Section 3.9.
3.2 Introduction to GPS

GPS is a system developed by the U.S. Department of Defense (DoD) to give worldwide, accurate positioning. However, in this thesis GPS is not used for that purpose. Instead the characteristics of the GPS signal is used to measure the height above ground for an aerial vehicle, for example an aeroplane. Therefore several details about GPS are left out, but for the interested reader there are many text books on the subject, e.g. [2] and [7].

GPS consists of three segments: the space segment, the user segment and the control segment. The space segment is represented by the satellites orbiting the earth, the user segments are the receivers used to navigate and the control segment deals with the management of the satellites. Only the space and user segment are of interest here.

The following sections describe the space segment, the GPS signal and a typical GPS receiver in general.

3.2.1 Satellite constellation

Figure 3.1 shows a picture of the satellite constellation. The space segment consist of 24 satellites orbiting the earth at a height of 20200 km. Each satellite has a period of 11h 58m which means that they pass the same point twice per day. The constellation has been chosen carefully to make sure that at least four satellites are always visible above the horizon, everywhere on earth where the system is supposed
to function, since that is the minimum number needed to navigate. To achieve this the satellites are distributed in six different orbital planes named A-F, all planes having an inclination angle of 55° relative to the equatorial plane. Each orbital plane contains four satellites distributed unevenly, in case of satellite failure.

3.2.2 GPS signal characteristics

The GPS satellites transmit signals at two different carriers, called L1 (at 1.575 GHz) and L2 (at 1.228 GHz). L1 is used for both civilian and military purposes while L2 is used only by the military. Since the main interest is in the civilian signal, L2 will be left out of the further discussion. Mathematically the signal transmitted on the L1 carrier can be described by (excluding the military part)

\[ s(t) = \sqrt{2P_C} d(t) x(t) \sin(2\pi f_{L1} t + \theta_{L1}) \]  

(3.1)

where \( P_C \) is the signal power, \( \sin(2\pi f_{L1} t + \theta_{L1}) \) the carrier signal, \( d(t) \) the navigation data and \( x(t) \) a code signal. See Figure 3.2. These are the three important parts of Eq. (3.1). The navigation data, \( d(t) \), with a frequency of 50 Hz provides information for positioning. The code signal, \( x(t) \), is a signal of higher frequency, 1.023 MHz, used for the spread spectrum technique, which gives the GPS signal special properties. It is of great importance and is therefore further described below. The carrier is an ordinary RF sinusoidal signal with frequency \( f_{L1} \) (1.575 GHz).

A more thorough description of the GPS signal can be found in e.g. [2].
Spread spectrum

The code signal, \( x(t) \), used in Eq. (3.1) gives the GPS signal properties of a spread spectrum signal. The symbols, or pulses carrying information, of \( x(t) \) are called chips, and are of length \( T_C \) s. Since the frequency of \( x(t) \) is 1.023 MHz, approx. 20000 times higher than the frequency of \( d(t) \), the original signal will be spread over a larger bandwidth when the two signals are multiplied. If \( x(t) \) is a so called pseudo random noise sequence (PRN sequence), which symbols are random-like, it is possible for several transmitters to use the same frequency and yet be able to retrieve the transmitted signal. This is done by multiplying the received signal with the original code, and due to the correlation properties of the PRN sequence the original signal will be the result. Further, if the received signal is multiplied with another code, the original signal is not retrieved and the resulting signal will still be random-like. The concept is called CDMA, code division multiple access, and is utilized by GPS.

There are other advantages with CDMA. For example, no timing among the different transmitters are needed. It is also possible to retrieve the transmitted data signal even if the signal power is below the noise floor. The code is easy to generate, both in hardware and in software.

Correlation properties of the C/A code

Two tools for determining the properties of codes used in CDMA system are the auto correlation and cross correlation functions. The auto correlation function, \( r(\tau) \) of a periodic signal \( c(t) \) describes how \( c(t) \) is correlated with itself at different shifts, \( \tau \). It is calculated in the following way:

\[
r(\tau) = \frac{1}{T} \int_0^T c(t)c(t - \tau)dt \tag{3.2}
\]

where \( \tau \) is the time delay and \( T \) is the length of the signal’s period. For the civilian GPS code, \( r(\tau) \) will be high for \( \tau = 0 \) and low for \( \tau \neq 0 \) (see Figure 3.3). This is an example of good auto correlation properties which makes the code ideal for use in a spread spectrum system.

In a similar way, the cross correlation between two signals, \( c^{(k)}(t) \) and \( c^{(l)}(t) \) describes how these two signals are correlated with each other at different shifts. It is determined by

\[
r^{(k,l)}(\tau) = \frac{1}{T} \int_0^T c^{(k)}(t)c^{(l)}(t - \tau)dt \tag{3.3}
\]

For the civilian GPS code, \( r^{(k,l)}(\tau) \) is approx. 23 dB lower, for all \( \tau \) and pairs of \( (k, l) \), \( k \neq l \), compared to the peak of the auto correlation function for the code \( (k = l) \).
C/A codes and Gold codes

The code used in the civilian GPS signal, is called the C/A code, short for Coarse-Acquisition. Every satellite has its own code which makes it possible for all satellites to use the same frequency, as described above.

To generate the C/A code a family of PRN codes called Gold codes [2] are used. The Gold codes have good auto and cross correlation properties, making them suitable for use in the GPS system. To construct a Gold code, two other carefully chosen PRN sequences of maximal length are needed. These are then added to generate the final sequence. Two maximal length sequences of length N can be used to generate N+2 Gold codes, by shifting one of the original sequences zero or more steps.

These two maximal length sequences can be generated from e.g. linear feedback shift registers (LFSRs). In the C/A code, the sequences are of length 1023 and can thus be generated from shift registers of length 10. The registers are defined by their generator polynomials, here called G1 and G2:

\[
G1 = 1 + X^3 + X^{10} \\
G2 = 1 + X^2 + X^3 + X^6 + X^8 + X^9 + X^{10}
\]

Figure 3.3. Autocorrelation function for one of the C/A-codes.
To produce the final code, the output from register 2 is delayed, depending on the satellite number, a certain number of chips, where one chip corresponds to $T_C$, the symbol time of the C/A code. Different delays yield different codes and these are chosen to have the best cross correlation properties. Instead of delaying the output the same effect can also be accomplished by adding together the values in some of the positions in the shift register; which positions depend on which delay. E.g., to get a delay of 5 chips, or $5T_C$ s, positions 2 and 6 are added to produce the output from register 2, which is then added to the output from register 1 to produce the final output. Figure 3.4 shows a picture of the GPS code generator for the C/A code.

3.2.3 Transmitter and signal power

The GPS satellites transmit the C/A signal with a power of approx. 27 W. An antenna which spreads the signal over the earth is used. To describe the geometric
relation between the satellite and the receiver, the satellite’s elevation angle, $\psi$, is used, see Figure 3.5. Since the distance from the satellite to the receiver is different depending on where on the earth the receiver is, the antenna radiation pattern is not uniform but stronger on the sides. Considering path loss, transmitter antenna gain and other factors the resulting power density at ground level varies from $-132.6$ dBW/m$^2$ to $-134.6$ dBW/m$^2$ [2], depending on the satellite’s elevation angle over the horizon.

A typical GPS receiver antenna will have an effective aperture of around $-25$ dBm$^2$, or $2.87 \cdot 10^{-5}$ m$^2$, which gives a signal power of approx. $-158$ dBW, far below the noise level which is $-140$ dBW in a GPS receiver with a bandwidth of 2 MHz. The ability to collect the data signal despite this low signal power is due to the spread spectrum technique.

### 3.2.4 Receiver

The main task of the GPS receiver is to find the signal and extract the navigation data from the signal. Figure 3.6 shows a simplified GPS receiver.

The received signal is first amplified in a Low Noise Amplifier (LNA) and mixed down to an intermediate frequency. It is then converted to a digital sequence in an analog to digital converter (ADC). In the following signal processing part, the receiver uses the correlation properties of the C/A code to find the signal from a given satellite. This is done by calculating the cross correlating function between the received signal and a replica of the code, for different shifts of the code. When the shift of the replica code is the same as the total delay for the received signal, the cross correlation is at maximum and the peak of the correlation function can be found. Since both the transmitter and the receiver may be in motion it also has to consider the Doppler effect. Once it has found the signal it locks on to it by using a dynamic loop filter. The navigation data is then extracted and passed on to the navigation processing, which is used for positioning.

The receiver used in the application of height altimetry is often a so called Delay Mapping Receiver (DMR), which is described in [3]. The difference between the DMR and an ordinary GPS receiver is that the DMR produces the power of the correlation function, i.e. the received signal correlated with a reference signal. No position data is given, and the power of the correlation function is saved and can be used for processing later.

### 3.3 Atmospheric effects

The atmospheric effects on the signal are mostly due to two layers: the ionosphere and the troposphere (see Figure 3.7). These effects will not be simulated in this thesis and the discussion below shows that they are of minor importance.
3.3 Atmospheric effects

**Figure 3.5.** Geometry of the satellite, earth and receiver. $\psi$ is the satellite's elevation angle.

**Figure 3.6.** Simplified picture of a GPS receiver.

**Figure 3.7.** The two important layers of the atmosphere.
At a height of 50-1000 km, the ionosphere is a result of the sun’s activity, and will therefore vary during the time of the day. In general the effects are least noticeable during the night and will peak during the day. The main effect is that signals with different frequencies will be delayed differently and this will affect the GPS signal since it consists of both carrier and code, where the first one has a significantly higher frequency [2]. The code phase will be delayed while the carrier phase will be advanced by the same amount. The delay is typically in the order of 0.01-0.1 μs, which corresponds to a measured propagation path of 10-30 m.

The troposphere is situated next to the ground (0-50 km) and contrary to the ionosphere all frequencies in the GPS signal are affected in the same way. The main influence from the troposphere is a delay of the signal and a small attenuation, which is typically lower than 0.5 dB. Rain hardly affects the signal at all.

Besides these delays, both layers will also refract the signal and cause a delay due to this. The refraction will bend the signal’s path, which makes it travel a longer way than a direct signal (see Figure 3.7). The error caused by this is generally small, especially for high satellite elevation angles.

Generally, the errors introduced by these two spheres will be larger for small satellite elevation angles, ψ, where the satellite is just over the horizon. There are many models that can be used to estimate the error. However, they are mostly applicable when using GPS the traditional way, for navigation, because the error is often given as a positioning error and not as an error in the correlation function.

In the case of using reflected GPS signals as an altimeter, the relative delay between the direct signal and the reflected signal is used. Both signals will be affected in the same way by the atmosphere, except when the satellite elevation angle is low and the signals travel long through the layers. Hence, these effects can be considered to be negligible and will not be simulated in this model. However, there is a chance that the difference between code and carrier phase introduced by the ionosphere may cause errors, which makes this a subject of further investigation, beyond the scope of this thesis.

### 3.4 Channel models

To be able to analyze the path of the reflected signal it is convenient to describe it in a mathematical way. Most common is the traditional radio communication model as shown in Figure 3.8. The radio channel is modeled as an impulse response, h(t), which can be used to calculate the resulting received signal. In opposite to the transmitter and receiver, it is the only part that cannot be influenced by the system engineer. The following discussion is a short summary of some of the ideas in [1].
3.4 Channel models

In the situation where GPS signals are reflected by the earth’s surface, the transmitter is the GPS satellite and the receiver is the system in the aeroplane measuring the height. The signal processing on both sides prepare and process the signal before and after the channel, making sure the least number of errors occur. It is the signal processing on the receiving side that will produce the correlation function and calculate the height from this.

One of the simplest channel models is composed of two parts; a deterministic time-invariant linear model and stochastic Additive White Gaussian Noise (AWGN). The deterministic model is described by the impulse response $h(\tau)$ while the AWGN, $n(t)$, is characterized by its constant spectral density, $N_0/2$. A further extension to this is the time-variant linear model, where the properties of the channel varies over time. The impulse response can then instead be given by $h(\tau; t)$, which varies much faster in $\tau$ than in $t$. At time $t = t_0$ the channel will have the impulse response $h(\tau; t = t_0)$.

If the input signal is $x(t)$, the output signal $y(t)$ from the channel can then be calculated by the convolution between $h(\tau; t)$ and $x(t)$ and adding the noise:

$$
y(t) = \int_{-\infty}^{\infty} h(\tau; t)x(t - \tau) d\tau + n(t)
$$

The radio signal is transmitted on a RF carrier, e.g. the GPS L1 signal is transmitted at 1.575 GHz. When analyzing signals in the bandpass frequency and also in non-coherent receivers, the signal is often divided into two parts which are orthogonal in phase, an inphase component (described by a cos-term) and a quadrature component (described by a sin-term).

It is not always convenient to analyze signals in the bandpass frequency and it would be preferable to work with signals centered around the zero frequency. Therefore the channel model is often created as an equivalent base-band model. Depending on the problem, in this thesis either the band-pass model or the base-band model will be used to analyze the channel.

\[\text{Figure 3.8. The basic model of a radio communication system.}\]
Often the channel of the signals can be more complex than the deterministic case, as for reflected GPS signals. The behaviour of the channel can be random, changing from time to time. The theory of stochastic channel models [1] can be helpful here, where instead of using an impulse response the auto correlation function of the impulse response (with respect to \( \tau \)) is used. This can be derived from

\[
\phi_h(\tau_1, \tau_2; \Delta t) = E[h(\tau_2; t + \Delta t) h^*(\tau_1; t)]
\]  

(3.5)

Here, a base-band model of the signal is used. \( \phi_h(\tau_1, \tau_2; \Delta t) \) can be thought of as the expected behaviour of a random channel and describes power rather than amplitude, since \( h(\tau; t) \) is squared.

From this function several characteristics about the radio channel can be derived. By setting \( \Delta t = 0 \) and assuming the channel is Wide Sense Stationary Uncorrelated Scattering, WSSUS, we can write [1, Eq. (3.11)]

\[
\phi_h(\tau_1, \tau_2; \Delta t) = \phi_h(\tau_2; 0) \delta(\tau_2 - \tau_1)
\]  

(3.6)

and the so called intensity profile is obtained. For the assumption that the channel is WSSUS to hold, we will assume that the different reflected signals are uncorrelated. The intensity profile is a measure of the expected received power as a function of the delay \( \tau \), compared to the direct signal. The difference in delay between the reflection that arrives first and the reflection that arrives last denotes another important parameter, the delay spread, \( T_m \). This is defined as the width of the region where \( \phi_h(\tau) \) is not zero. The intensity profile and the delay spread describes how much the different reflected signals are scattered in time.

There are other parameters that can be obtained from \( \phi_h(\tau) \), e.g. the coherence time, \( T_d \). For this the Fourier transform of \( \phi_h(\tau) \), called the frequency time correlation function, has to be used. \( T_d \) describes how rapidly the channel changes with time \( t \).

### 3.5 Modeling reflected GPS signals

To model the reflected GPS signal, a function that describes the received power as a function of delay from the direct signal will be derived. The reasons to work with power and not amplitude are two: the phase of the received signal is complicated due to the incoherent nature of the signal, and the receiver is designed to detect power and not amplitude.

In order to model the power of the received signal, first the received electrical field must be studied. The GPS signal is transmitted from the satellite and then reflected...
at the ground, at several points. It can therefore be described as an infinite sum of individual reflections [5, Eq. (6)]:

\[ E = \sum_{k=1}^{\infty} a_k e^{-j\left(\frac{2\pi \tau_k}{\lambda} + \phi_k\right)} \]  

(3.7)

where \( a_k \) is the amplitude, \( \tau_k \) the time delay and \( \phi_k \) the phase of the \( k \)-th reflection. In the same way, the received GPS signal can be described as a sum. If the C/A code on the GPS signal is \( x(t) \) and \( y(t) \) is the received signal after reflection, the following relationship holds:

\[ y(t) = \sum_{k=1}^{\infty} x(t) a_k e^{-j\left(\frac{2\pi \tau_k}{\lambda} + \phi_k\right)} \]  

(3.8)

Here, the data signal and the carrier of the GPS signal have been excluded, since the data signal changes much slower than the code signal, and we work with a baseband model.

The output of the receiver, \( R(\tau) \) is the received signal, \( y(t) \), correlated with the same PRN code, \( x(t) \):

\[ R(\tau) = \frac{1}{T} \int_{0}^{T} y(t)x(t - \tau)dt \]  

(3.9)

where \( \tau \) is the delay compared to the direct signal and \( T \) is the length of the period of \( x(t) \). If Eq. (3.8) is inserted into Eq. (3.9), \( R(\tau) \) can be written as:

\[
R(\tau) = \frac{1}{T} \int_{0}^{T} \sum_{k=1}^{\infty} x(t) a_k e^{-j\left(\frac{2\pi \tau_k}{\lambda} + \phi_k\right)} x(t - \tau)dt
\]

\[
= \sum_{k=1}^{\infty} \frac{1}{T} \int_{0}^{T} x(t) a_k e^{-j\left(\frac{2\pi \tau_k}{\lambda} + \phi_k\right)} x(t - \tau)dt
\]

\[
= \sum_{k=1}^{\infty} a_k e^{-j\phi_k} \frac{1}{T} \int_{0}^{T} x(t - \tau_k)x(t - \tau)dt
\]

\[
= \sum_{k=1}^{\infty} a_k e^{-j\phi_k} r(\tau - \tau_k)
\]  

(3.10)

where \( r(\tau) \) is the auto correlation function of the C/A code. When integrating over a few seconds, \( r(\tau) \) can be approximated with a triangle function, \( \Lambda(\tau) \) [5]:

\[
\Lambda(\tau) = \begin{cases} 
1 - |\frac{\tau}{T_C}|, & -T_C \leq \tau \leq T_C \\
0, & \text{otherwise}
\end{cases}
\]  

(3.11)

where \( T_C \) is the time period of one C/A chip (approx. 1 \( \mu \)s).
As described in [5], by substituting \( r(\tau - \tau_k) \) with \( \Lambda(\tau - \tau_k) \) in Eq. (3.10) and then squaring, taking the ensemble average, accounting for the cancellation of crossterms from different reflections due to the assumption of incoherent scattering, we get

\[
< |R^2(\tau)| > = \sum_{k=1}^{\infty} P_{r,k} \Lambda^2(\tau - \tau_k) \tag{3.12}
\]

where \( P_{r,k} \) is the received power from the \( k \):th reflection.

Note that there is no noise in Eq. (3.12). This is because the noise is not included in [5] and the analysis of receiver noise is performed in another way (see Section 4.3.2).

According to [3] this infinite sum can also be described by the convolution

\[
< |R^2(\tau)| > = \int_{-\infty}^{\infty} P_r(\eta) \Lambda^2(\tau - \eta) d\eta \tag{3.13}
\]

where \( P_r(\tau) \) is the power for a certain delay \( \tau \) from the arrival of the specular reflected signal.

### 3.6 The bistatic radar equation

In the previous section, the radio channel was described with power as a function of delay, \( P_r(\tau) \). The problem is now to calculate this function. Besides radio communication, there is another useful area for this purpose: radar theory. Some radar signals operate in frequencies similar to the GPS signal, with the difference that radar pulses are short while radio communication systems transmit continuous signals carrying data. In other research on reflected GPS signals, radar theory has also been used (see e.g. [18]). The result of this discussion will be an equation that can be used to calculate the received power, \( P_r(\tau) \). A more comprehensive introduction to radar theory can be found in [12].

#### 3.6.1 Basic radar knowledge

The word radar is an abbreviation for radio detection and ranging. The idea is to transmit a short electromagnetic pulse towards a target. The pulse is then reflected back to a receiver. Although the reflected pulse contains only a part of the transmitted energy, it is possible to determine e.g. distance to and size of the target. Two main principles exist, one where transmitter and receiver are at the same location, monostatic radar, and one where transmitter and receiver are
3.6 The bistatic radar equation

Figure 3.9. The two basic radar systems.

located apart, *bistatic radar* (see Figure 3.9). Like radar pulses, GPS signals are also reflected by sea and ground surfaces. Therefore measuring height over ground with GPS can be thought of as a bistatic radar system, where the short radar pulse is replaced by a longer, continuous GPS signal. The transmitter is the satellite, the ground is the target and the receiver is located in the aerial vehicle.

3.6.2 The radar equation

To describe the energy reflected from a target, the radar equation is the main tool in radar theory. This equation is derived in the following way.

To begin with, the target will be viewed as a single point target which reflects the radar pulse. If the radar pulse is transmitted with power $P_t$, the distance between the transmitter and the target is $R_1$ and the distance between the target and the receiver is $R_2$, the received power becomes [12, Section 14.6]

$$P_r = \frac{P_t G_i(\theta_i, \phi_i) G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 \cdot R_1^2 R_2^2} \cdot \sigma \quad [W] \quad (3.14)$$

where $(\theta_i, \phi_i)$ defines the direction from the transmitter to the target and $G_i(\theta_i, \phi_i)$ is the antenna gain in that direction, $(\theta_r, \phi_r)$ and $G_r(\theta_r, \phi_r)$ the corresponding values for the receiving antenna, $\lambda$ the wavelength of the pulse and $\sigma$ is called the *radar cross section*. The radar cross section is a measure of how much power the target captures and radiates back towards the receiver. It has units of area and can be thought of as the size of the target viewed from the receiver.

Eq. (3.14) is called the *bistatic radar equation*. 
Instead of using the transmitted power, $P_t$, the power density at the target before reflection, $S_d$, can be used. The power density is given by [1, Eq. (2.7) and (2.52)]

$$S_d = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2_1} \quad [\text{W/m}^2] \quad (3.15)$$

If the expression in Eq. (3.15) is used to substitute some parts of Eq. (3.14), the simplified expression for the received power becomes

$$P_r = \frac{S_d G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 R^2_2} \cdot \sigma \quad [\text{W}] \quad (3.16)$$

When the target is the ground, the return signal is called ground echo. Since the ground has different properties depending on where the signal is reflected, it is not enough to describe it as a single point target. Instead, to model the reflection of radar pulses from the ground it can be viewed as number of point targets, independent and randomly distributed on the area which returns the echo. These are called scatterers. Eq. (3.14) then has to be extended to cover a whole area (see Figure 3.10). This area, $\Omega$, is defined by the receiving and transmitting antenna and the part of the ground which gives reflections back to the receiver. The size of $\Omega$ is $A = \iint_{\Omega} \, dx \, dy$. The satellite covers the whole earth, so the main parameters deciding $\Omega$ will be either the receiving antenna coverage or the ground reflection properties. Extending the equation can be done by integrating over the area [17]:

$$P_r = \iint_{\Omega} \frac{S_d G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 R^2_2} \cdot \sigma \, dx \, dy \quad [\text{W}] \quad (3.17)$$

Instead of defining $\sigma$ in units of area, the radar cross section has in Eq. (3.17) been normalized with respect to $\Lambda$, radar cross section per unit area, often referred to
3.6 The bistatic radar equation

![Diagram showing bistatic radar configuration with areas A and A']

Figure 3.11. Using the perpendicular area in the radar equation.

as the scattering coefficient:

\[ \sigma^o = \sigma / A \]  \hspace{1cm} (3.18)

The meaning of \( \sigma^o \) is a measure of how well each square meter of the ground reflects the radar pulse in a certain direction, with a certain angle of incidence.

If the reflection area is small, \( G_r(\theta_r, \phi_r) \), \( \sigma^o \) and the distance \( R_2 \) can be regarded as constant over the area. Eq. (3.17) may then be simplified to

\[
P_r = \frac{S_g G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 R_2^2} \cdot \sigma^o \int \int dxdy
\]

\[
= \frac{S_g G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 R_2^2} \cdot \sigma^o \cdot A \quad [W] \]  \hspace{1cm} (3.19)

In some literature, especially in this thesis, instead of \( \sigma^o \) the scattering coefficient per unit area perpendicular to the angle of incidence is used and then denoted \( \gamma^o \) (see Figure 3.11). The total area perpendicular to the angle of incidence is \( A' \), which gives the following relation:

\[ \sigma^o \cdot A = \gamma^o \cdot A' \]  \hspace{1cm} (3.20)

Eq. (3.19) can be rewritten as

\[
P_r = \frac{S_g G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 R_2^2} \cdot \gamma^o \cdot A' \quad [W] \]  \hspace{1cm} (3.21)

Both Eq. (3.19) and Eq. (3.21), which are equivalent, can be used to calculate \( P_r(\tau) \).
3.7 Ground reflection

In the previous section the scattering coefficient, $\gamma^o$, was used in the bistatic radar equation. In this section the theory behind how $\gamma^o$ behaves for different surfaces is discussed. This behaviour is dependent of the electric properties and roughness of the surface.

3.7.1 Overview

When the GPS-signal reaches the earth’s surface a part of the signal power is reflected. The amount reflected depends on the electric properties of the ground and the angle of incidence. For instance, water can at best reflect approximately 65 % of the power, while ground reflects 10-50 %. This applies for relatively small incident angles, $\theta$ (see Figure 3.12). For large $\theta$, more power is reflected, but this is not of interest for measuring height with GPS signals, since small $\theta$ is preferable. Besides attenuation, the signal also experiences scattering. The effect of scattering depends on how rough the ground is. For reflection at a smooth surface the effect of scattering is almost non-existing, i.e. most of the signal power is in the specular direction, but for a rough surface the power is distributed more widely.

The scattering coefficient, $\gamma^o$, gives the amount of power that is reflected in a certain direction. The scattering pattern can analytically be described by the following expression [3, Eq. (8)]

$$\gamma^o = |R|^2 Q$$

(3.22)
where $|\mathcal{R}|^2$ gives the amount of the incident power that is reflected and the scattering pattern, $Q$, describes how the signal is scattered in different directions. $\mathcal{R}$ is dependent on the electrical properties of the ground, which in turn is dependent on the condition of the surface. For example, a wet surface reflects more power than a dry surface. The scattering pattern, $Q$, is dependent on the roughness of the surface. A rough surface scatters the signal more than a smooth surface, for example.

The principles of scattering is shown in Figure 3.13. In radar theory the main interest is the signal that is scattered back to the transmitter, i.e. backscattering. In this thesis scattering in all directions is of interest, which is called forward scattering.

### 3.7.2 Reflectivity

Reflectivity describes how much of the incident electromagnetic wave that is reflected and which phase shift it experiences. It is denoted $\mathcal{R}$ and is used in the expression of the forward scattering coefficient (see Eq. (3.22)). The reflectivity is a complex constant, and therefore handles both the attenuation and the phase shift that the incident wave experiences. The relation between the reflected and incident wave is given by (see Figure 3.12)

$$E_r = \mathcal{R}E_i$$  \hspace{1cm} (3.23)

In terms of power the following applies

$$P_r = |\mathcal{R}|^2 P_i$$  \hspace{1cm} (3.24)
Electric properties for surfaces

To determine the reflectivity, the surface’s dielectric constant, $\varepsilon$, is needed. The dielectric constant is complex and depends on the wavelength and the condition of the surface. The dielectric constant is given by [4]

$$\varepsilon = \varepsilon_r - j \frac{K}{\omega \varepsilon_0} \approx \varepsilon_r - j \varepsilon_0 \lambda K$$ (3.25)

$\varepsilon_r$ is the relative permittivity and $\kappa$ is the conductivity. In Table 3.1 the electric properties for some surfaces in nature are found [6].

<table>
<thead>
<tr>
<th>Surface</th>
<th>Relative permittivity ($\varepsilon_r$)</th>
<th>Conductivity ($\kappa$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry ground</td>
<td>4</td>
<td>$1 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Medium dry ground</td>
<td>7</td>
<td>$4 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>Wet ground</td>
<td>30</td>
<td>$2 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>Fresh water</td>
<td>80</td>
<td>$2 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>Sea water</td>
<td>20</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3.1. Electric properties for some surfaces in nature.

Polarization

The GPS signal’s polarization is of great significance in this thesis, since it is one of the factors which gives the amount of reflected power. Polarization describes the electrical properties of a signal in the atmosphere. The polarization is often described by two orthogonal electrical fields. If only one of the fields exists, the polarization is described as being linear. Often both fields exist, in this case the result is either elliptical or circular polarization. Elliptical polarization is achieved if the amplitude of the orthogonal fields differs and circular polarization is achieved when the amplitudes are the same. For the interested reader more theory about electrical fields can be found in [9].

Reflectivity for linear polarization

The GPS-signal is right hand circularly polarized (RHCP), but in order to get the reflectivity for circular polarization the reflectivity for linear polarization is needed.

Polarization for signals at earth’s surface are described with a vertical and a horizontal component (with respect to the tangent plane), denoted $V$ and $H$. The
Figure 3.14. Reflectivity for linear polarization at dry ground and fresh water for the L1-frequency, f=1575 MHz.

reflectivity for linear polarization is described by $R_{HH}$ and $R_{VV}$, where $R_{HH}$ gives the reflectivity for a horizontally polarized incident wave reflected as a horizontally polarized wave. In the same manner, $R_{VV}$ gives the reflectivity for an incident and reflected vertical wave. I.e., the reflectivity differs, whether the incident electromagnetic wave is horizontally or vertically polarized. The reflectivity is given by the Fresnel equations [4]

\[
R_{HH}(\theta, \epsilon) = \frac{\cos(\theta) - \sqrt{\epsilon - \sin^2 \theta}}{\cos(\theta) + \sqrt{\epsilon - \sin^2 \theta}}
\]

\[
R_{VV}(\theta, \epsilon) = \frac{\epsilon \cdot \cos(\theta) - \sqrt{\epsilon - \sin^2 \theta}}{\epsilon \cdot \cos(\theta) + \sqrt{\epsilon - \sin^2 \theta}}
\]

where $\theta$ is the incident angle (see Figure 3.12) and $\epsilon$ is the dielectric constant for the surface. Figure 3.14 shows the reflectivity for dry ground and fresh water.
These two surfaces show the typical behaviour for many surfaces in nature, and many surfaces are somewhere in between these two. From the plots it is evident that the \( HH \)-component gives a 180° phase shift for all incident angles, while the \( VV \)-component has no phase shift for small \( \theta \), and a 180° phase shift for large \( \theta \). It is also evident that fresh water reflects electromagnetic waves better than dry ground. This exemplifies a typical behaviour, the more moist a surface is the better it reflects electromagnetic waves.

It is important to recognize that the reflectivity given in Figure 3.14 is calculated using the GPS L1-frequency, 1575 MHz, and that a different frequency would give a completely different result.

**Reflectivity for circular polarization**

The GPS signal has right hand circular polarization (RHCP), and to calculate the reflectivity for circular polarization the reflectivity for linear polarization is used.

In the same way as for linear polarization, the reflectivity for circular polarization is described with respect to the incident and reflected wave. The reflectivity is given by \( R_{RR}, R_{LL}, R_{RL} \) and \( R_{LR} \), where \( R \) and \( L \) stand for RHCP and LHCP, \( R_{RR} \), for example, gives the reflectivity for an incident RHCP wave reflected as RHCP. The reflectivity for circular polarized waves is calculated by using the expressions for linear polarization, and is given by \([18, \text{Eq. (36) and (37)}]\)

\[
R_{RR} = R_{LL} = \frac{1}{2}(R_{VV} + R_{HH}) \tag{3.28}
\]

\[
R_{RL} = R_{LR} = \frac{1}{2}(R_{VV} - R_{HH}) \tag{3.29}
\]

Of primary interest is \( R_{RL} \) and \( R_{RR} \) since the GPS signal is RHCP. Figure 3.15 shows the reflectivity for an incident RHCP wave. For small \( \theta \) most of the reflected signal becomes LHCP, but as \( \theta \) increases more of the reflected signal is reflected as RHCP.

The signal from the satellite can be assumed to have a reasonable small incident angle, \( \theta \). From Figure 3.15 it is obvious that most of a RHCP signal, i.e., the GPS-signal, is reflected as LHCP if \( \theta < 45° \), for both dry ground as well as fresh water. Therefore, an antenna suited for a LHCP signal is used to receive the reflected signal.
3.7 Ground reflection

Figure 3.15. Reflectivity for circular polarization at dry ground and fresh water for the L1-frequency, \( f = 1.575 \).

3.7.3 Scattering pattern

In this section the scattering pattern for different surfaces is discussed. The scattering pattern is denoted \( Q \) in Eq. (3.22) and describes how the signal power is scattered in different directions. Depending on the roughness of the surface this pattern varies. It is important to recognize that the scattering pattern distributes a given reflected power (Eq. (3.22)), i.e. no loss is introduced in the scattering pattern. If the reader is familiar with antennas, \( Q \) is treated in the same way as the antenna directivity.

Behaviour of surfaces

Surfaces are often described to be either rough or smooth, although there are different levels of roughness. The Rayleigh criterion is a general rule to determine if a surface is to be considered rough or smooth. For an incident angle, \( \theta \), the critical height is given by [1]

\[
h_c = \frac{\lambda}{8 \cos(\theta)}
\]  

(3.30)

If the minimum to maximum height, \( h \), of the surface is less than \( h_c \) the effect of the surface roughness is considered negligible (see Figure 3.16). In this case specular reflection occurs. For \( h > h_c \) the surface is considered to be rough (at the L1-frequency and \( \theta = 0 \) the critical height is approximately \( 2A \) cm). The larger
Figure 3.16. Surface roughness model for the Rayleigh criterion.

Figure 3.17. Schematic description of the scattering pattern for different roughness.

$h$ gets the less specular the reflection is. The reflection coefficient in the specular direction can be described by [6]

$$R_s = \rho_s R$$

(3.31)

$0 < \rho_s < 1$.

The rougher a surface is the smaller $\rho_s$ becomes, and the more power is reflected in other directions than the specular. A schematic description of the scattering pattern is shown in Figure 3.17. Most of the scattered signal is always in the specular direction and for different degrees of roughness the signal is scattered more or less in other directions.

**Reflection at specular surfaces**

For a perfectly specular surface no scattering occurs, i.e. the receiver experiences the reflected signal as if it originates from the transmitters mirror reflection (see Figure 3.18). With no attenuation, the power density at the receiving antenna is given by

$$S_r = \frac{P_1}{4\pi(R_1 + R_2)^2} = \frac{P_1}{4\pi R_1^2(1 + R_2/R_1)^2} \approx \left\{ R_2 \ll R_1 \right\} \approx \frac{P_1}{4\pi R_1^2}$$

(3.32)
3.8 Fading

![Figure 3.18. Perfectly specular surface.](image)

Taking the surface's electric properties into account, the reflected signal can be regarded as an attenuated direct signal. The shape of the signal will be the same as the direct, but delayed. Making use of the expression for attenuation in Eq. (3.24) it is evident that Eq. (3.32) is substituted by

\[
S_r \approx \frac{|R|^2 P_i}{4\pi R^2} \quad (3.33)
\]

### 3.8 Fading

A common problem in radio communication is a phenomenon called fading. Signals in a radio channel experiences fading when several reflections of the same signal, which have traveled different paths, are added in the receiver. Due to the random phases of the signals, they fully or partly cancel out each other when added. If the receiver is in motion, the effect of fading will vary over time. The same problem happens to cellular phone users when e.g. driving in a car.

When measuring ground height from an aeroplane, using reflected GPS signals, the received signal will be faded. In this section, the different types of fading and their impact in this thesis are examined. However, since time has been limited, fading has not been implemented in the simulation model and therefore this section just features a short discussion on the subject. It also serves as a purpose for further examinations.
3.8.1 Types of fading

Fading is generally categorized into several types, depending on the nature of the radio channel. A channel can be flat or frequency selective fading, which tells us if it affects all frequencies of the signal in the same way. It can also be fast or slow fading, i.e. whether it affects a whole symbol in the same way or changes properties faster than the symbol time. Furthermore, there may be a slowly changing fading effect, often called shadowing, caused e.g. by the receiver moving in behind a mountain. However, when the angle of incidence is small, the risk of shadowing is low, and no explicit modeling is needed.

Flat fading occurs when the data symbols are long compared to the relative delay between two propagation paths. The interference between two symbols will then not be large compared to the symbol length and the effect of this is negligible. The shape of the signal wave form is not a subject to major change, but the received power will vary over time. Frequency selective fading, on the other hand, will attenuate different frequencies of the signal in different ways. This causes a problem known as *intersymbol interference*, or ISI. One symbol affects one or more of the following symbols in the received signal.

**Frequency selective / ISI**

Frequency selective fading, or intersymbol interference, will be investigated from the view of the C/A signal rather than the carrier signal, since the period time of the carrier, \( T_c \) is short and there are not any symbols of that length carrying data.

To decide whether the fading is frequency selective or not, it is necessary to calculate the delay spread, \( T_m \), which describes how the received power is scattered in time. How the delay spread is calculated was described in Section 3.4. If the delay spread is large compared to the symbol time, one symbol will be delayed enough to affect the following symbols. When the signals are added at the receiver, the shape of the resulting signal will be changed (see Figure 3.19).

The delay spread is calculated from the intensity profile, which can be obtained by simulating the channel's response to a short pulse, i.e. instead of using the entire GPS signal as input just a short pulse is used.

Another approach is to make a rough estimate of the delay spread by calculating the maximum differences between two reflected signals. Consider the case in Figure 3.20, with an incident angle of 0°. The reflection which travels the shortest way arrives at \( t_1 \) and the last one arrives at \( t_2 \). The difference, \( \Delta t \), can be calculated in the following way (\( h = 1000, \beta = 15° \)):

\[
R_2 = \sqrt{h^2 + (\sqrt{2h \tan(\pi/12)})^2}
\]

\[
\Delta t = \frac{\Delta s}{c} = \frac{|r_2 - h|}{c} = \frac{1000 - 1000}{3 \times 10^8} = 2.3 \times 10^{-7}
\]

(3.34)
Figure 3.19. Two reflections causing ISI due to the delay.

Figure 3.20. Reflection with an incident angle of $0^\circ$, which does not cause ISI.

Figure 3.21. Ground variations causing large ISI.
In this case, the delay spread would be around 0.2 $\mu$s, and since the symbol time of the GPS C/A code is 1 $\mu$s, the channel can be considered to be flat fading.

On the other hand, if the antenna coverage is wider, the satellite elevation angle is lower and the ground features large variations, $\Delta t$ can be as large as 4-5 chips, which gives much ISI. Figure 3.21 shows a scenario where a sloping ground causes a large $\Delta t$, and the channel could therefore be frequency selective fading.

**Rayleigh clutter**

Due to the short wavelength of the carrier, 19 cm, the different reflections from the ground can be considered to have random phases compared to each other. This causes flat fading; the received signal will be attenuated in a random way and will also experience a random phase shift.

Since the signal is reflected from the ground, and the problem is much like radar echo from a larger surface, the theories in [17, Chapter 3] have been used to analyze the situation. As was stated in Section 3.6, the ground is assumed to return several, independent reflections. Each reflection is considered to originate from a scatterer. If the scatterers fulfill certain assumptions, the amplitude of the received signal can be shown to be Rayleigh distributed. In radar theory this is called Rayleigh clutter. These assumptions are:

1. The scatterers are statistically independent.
2. The number of scatterers is large.
3. The amplitude and phase of the scatterers are independent random variables.
4. The phase $\phi_t$ is uniformly distributed in the range $[0, 2\pi]$.
5. No individual scatterer is significantly stronger than the others.

The properties of the amplitude of the received signal, $a(t)$, are:

$$p(a) = \frac{a}{\sigma^2} e^{-a^2/2\sigma^2}$$  \hspace{1cm} (3.35)

$$E[a] = \sigma \sqrt{\frac{\pi}{2}}$$  \hspace{1cm} (3.36)

where $\sigma$ is standard deviation of the inphase and quadrature components of $a(t)$ (see Section 3.4) and $p(a)$ is the probability density function for $a(t)$. 

3.8 Fading

Since the receiver uses the power of the received signal rather than the amplitude, it is necessary to find the statistical properties of the power, \( P = a^2 \). The power can be shown to be exponentially distributed, i.e.

\[
p(P) = \frac{1}{P_0} e^{-P/P_0} \tag{3.37}
\]

where

\[
P_0 = E[P] = 2\sigma^2 \tag{3.38}
\]

Using the power to detect the symbols is also known as \textit{square law detection}.

If assumption 5 is not valid, i.e. one or a few of the reflections dominate the signal, another distribution has to be used. Rice and Nakagami are two common distributions in this case [17, p. 36].

\textbf{Correlation between flat and frequency selective fading}

One can regard the flat and frequency selective fading as uncorrelated, in the sense that the ISI will not affect the flat fading. The two types of fading can be treated as two separate problems. The motivation for this is the large difference in wavelength between the chip signal, which is affected by ISI, and the carrier, which is faded by e.g. Rayleigh fading. One chip in the C/A-code is 1 \( \mu \)s long which corresponds to approx. 300 m, while the wavelength of the carrier is 0.19 m.

\textbf{Slow and fast fading}

A channel which is considered as slow fading will not change during the time of one symbol, in opposite to a fast fading channel. It is therefore easier to detect the symbols in a slow fading channel. When looking at the GPS C/A code, it is preferable if the channel is slow over a whole C/A period (1 ms), since the correlation is done over this time.

To decide whether the channel is slow or fast fading, the coherence time, \( T_d \), is used. If the symbol time, \( T_s \), is less than \( T_d \), the channel can be considered to be slow fading.

The coherence time is obtained from the following relationship with the \textit{Doppler spread}, \( B_d \): \( T_d \approx 1/B_d \). Assuming that the channel is flat fading, where the receiver is moving with velocity \( v \) and the carrier has a frequency of \( f_c \), the Doppler spread for the channel can be calculated from the the maximum Doppler shift using Eq. (3.38) in [1, p. 144]:

\[
B_d = f_d = \frac{vf_c}{c} \tag{3.39}
\]
For the case of an aeroplane receiving the GPS signal at the L1 carrier frequency, 1.575 GHz, the channel can be considered to be slow fading during a whole C/A-period, if $T_d > 1 \text{ ms}$:

$$T_d \approx \frac{1}{v} > 10^{-3} \Leftrightarrow v \lesssim 200 \text{ m/s} \quad (3.40)$$

Since the typical speed of an aeroplane is 100-300 m/s, the slow or fast fading characteristics of the channel will depend entirely on $v$.

### 3.8.2 Comments

Fading is clearly a noticeable problem in this situation and depending on the type of ground, the effects vary from random attenuation to severe intersymbol interference. In the first case, the received signal will keep its shape but will suffer from low SNR from time to time. In the second case, besides from attenuation the shape of the waveform will also be affected and the correlation function will look differently. Looking at fast or slow fading, it is not certain that the channel can be considered as slow fading all the time, only if the receiver is moving with a speed below 200 m/s.

Depending on how detailed the simulation is, some of these effects will be accounted for automatically. If the signal is processed in the baseband and not in the bandpass frequency, the ISI will happen due to the implementation but the flat fading will have to be modeled and implemented explicitly. To deal with that issue, either the carrier has to be simulated or the fading has to be modeled in another way. However, even if the flat fading is not handled in the simulation, the shape of the resulting signal will still be correct. The only difference between the model and reality will be random attenuation varying over time.

### 3.9 Antennas

The purpose of this section is to give the reader an understanding of why different antenna patterns are used. Depending on how the antenna pattern is designed different results for the altimeter are achieved. Different terrain result in different scattering behaviour, and a certain type of antenna pattern is preferable for a certain terrain. General theory is also discussed, but the reader is assumed to have basic knowledge about antennas.
3.9 Antennas

Figure 3.22. Schematic view of the dependency between the width and gain of an antenna.

3.9.1 Overview

The system for measuring height with reflected GPS signals requires that one antenna is placed on top of the aeroplane and one is placed on the underside. The antenna on top is designed to receive the direct signal only, and the antenna on the underside is designed to receive signals reflected from the ground. Receiving a direct signal is a known problem, and solutions are available in ordinary GPS-technique. Receiving the reflected signal is of greater significance in this thesis, and this is the problem described from now on.

The antenna suited to receive the reflected signal should be designed to receive signals originating from a limited area under the aeroplane (see Figure 3.22). With a wider antenna coverage a larger area is covered, and more reflected signals are available. But a wider antenna coverage also means that the antenna gain is smaller. Hence, there is a trade off between the width of the antenna and the antenna gain. In the same manner as above, a narrow antenna coverage results in a high antenna gain, but less reflected signals are also available.

3.9.2 General theory

In this thesis the only interest is in receiving antennas, but antennas have a reciprocal property, i.e. an antenna works equally well as receiver or transmitter. Because it is easier to describe and understand an antenna as a transmitter, a transmitting antenna is described first, and then the conversion to a receiving antenna is made.
Transmitting antennas

An antenna transmits a certain amount of power, often denoted \( P_t \). Depending on
the design of the antenna this power is distributed unevenly in different directions,
this is described by the antenna pattern. The antenna pattern is described by the
antenna gain, \( G(\theta, \phi) \), which is used in the following way to calculate the power
density at a receiving point

\[
S_r = \frac{P_t G(\theta, \phi)}{4\pi R^2} \tag{3.41}
\]

where \( 0 \leq \theta \leq \pi \) and \( 0 \leq \phi \leq 2\pi \) are spherical coordinates which describe the
direction and \( R \) is the distance to the receiver. The antenna gain has the following
property \cite{1}

\[
\int_0^{2\pi} \int_0^{\pi} G(\theta, \phi) \sin(\theta) d\theta d\phi = 4\pi \tag{3.42}
\]

assuming no losses in the antenna.

Receiving antennas

A receiving antenna is often described to have an effective aperture, \( A_e(\theta, \phi) \), in-
stead of antenna gain, \( G(\theta, \phi) \). The effective aperture gives a measure of the an-
tenna’s ability to capture an incident power field. The relationship between \( G(\theta, \phi) \)
and \( A_e(\theta, \phi) \) is \cite{1}

\[
A_e(\theta, \phi) = \frac{G(\theta, \phi) \lambda^2}{4\pi} \tag{3.43}
\]

and the received power is calculated by

\[
P_r = S_r A_e(\theta, \phi) \tag{3.44}
\]

where \( S_r \) is the power density at the receiving antenna.
Antenna loss

The antenna gain given in the previous section was calculated assuming no power loss. However, all antennas experiences loss due to e.g. heat developed in wires, resistances etc. This affects the amount of power that can be used for processing. The loss factor, $\eta$, affects the antenna gain as

$$G_{\text{loss}} = \eta G \quad 0 < \eta < 1 \quad (3.45)$$
Chapter 4

Measuring height

In this chapter various methods for calculating the height from the correlation function are described. Measuring height in the presence of thermal noise in the receiver is also discussed.

4.1 Calculating the height

The height over ground can be estimated from the correlation function produced by the receiver. Recall Eq. (2.1) on page 6:

\[ h = \frac{c_0 \cdot \Delta t}{2 \sin(\psi)} \]  

(4.1)

where \( h \) is the height over ground, \( \psi \) is the satellite’s elevation angle over ground and \( \Delta t \) is the time delay between the direct and reflected signal. The height \( h \) is not to be mistaken for the altitude, which in this thesis is defined as the height over mean sea level.

The factor \( \sin(\psi) \) is included to compensate for an error due to the satellite’s elevation angle over ground. To be able to use this in the height measurement the angle must be calculated, which can be done if the aeroplane’s and the satellite’s positions are known. To obtain the exact value of \( \psi \) the height of the ground over MSL also has to be known. However, since the distance between the ground and the satellite is much greater than the distance between the ground and the aeroplane, both the direct signal and the signal reflected at the ground can be assumed to be parallel. Therefore, only the position of the aeroplane and the position of the satellite are necessary to calculate \( \psi \).
4.2 Tracking methods

The output from the receiver described in Section 3.2.4, and from the simulation model as well, is the received signal correlated with a code replica and then squared to represent power. This output is also referred to as the received waveform, as in traditional altimetry. There are various methods to find $\Delta t$ from the output. Masters [13] suggests six methods of measuring the time delay, or tracking the delay:

1. The peak point.
2. Half-peak point on the rising edge.
3. Early-late window. This is the same principle used in positioning GPS receivers. Two points, early and late one chip apart, are measured on the correlation function. When the difference between these two points is zero the time delay is measured as the mean of the two points.
4. Power 6 dB above noise floor. The power of the noise is estimated as the mean power in the samples surrounding the actual correlation function.
5. Linear fit. This method assumes that the leading edge of the received correlation function is identical to the leading edge of the auto correlation function of the C/A code. It is further described in [13].
6. Model fit. Compute an analytically modeled correlation function, depending on receiver height and surface roughness, and compare it to the received signal. The model used is described in [18].

Figure 4.1 shows a schematic picture of the time delay estimated by the first five tracking methods. The reason for the sixth method not being shown is that the model described in [18] was to complicated to implement in the time available.

As can be seen these methods do not measure the same time delay, e.g. the peak tracking will measure the peak of the correlation function and the linear fit method will measure the beginning of the correlation function. The output of these methods will therefore have to be adjusted by a constant, which can be measured experimentally and corrected for. This constant is not to be mistaken by the bias depending on the height and the surface roughness, which have to be compensated for separately. In Eq. (4.4), $\Delta t$ is assumed to be measured from the peak of the correlation function.

In this thesis only method 1 has been used to calculate the time delay. This was the least complicated method to implement, and was chosen due to lack of time. However, in [13] this method performed worse than the others. This indicates that the other methods need to be investigated.
4.3 Error sources

In reality, Eq. (4.1) does not give the correct expression for the measured height, since there are several sources of error in the measurement. Due to time limits of the thesis, a thorough error analysis has not been performed. Instead, some of the errors have been identified and examined, namely the error in the measured time $\Delta t$ due to bias effects, and the error due to noise in the receiver. These errors have been examined and characterized separately.

4.3.1 Bias

Several sources, e.g. [13], report an error which increases with the height over ground, called bias. The bias is due to additional reflections, besides the specular reflection, which distort the received waveform and causes the top of the signal to appear later. This gives an error in the measured delay time, which in turn gives an error in the measured height.

If the true time delay is denoted $\Delta t$ and the error in the measured time delay is denoted $t_e$, the relation between $\Delta t$ and the measured time delay, $\Delta \hat{t}$ becomes:

$$\Delta \hat{t} = \Delta t + t_e$$  \hspace{1cm} (4.2)
This gives a measured height, \( \hat{h} \), of

\[
\hat{h} = \frac{c_0 (\Delta t + t_e)}{2 \sin(\psi)} = \frac{c_0 \cdot \Delta t}{2 \sin(\psi)} + \frac{c_0 \cdot t_e}{2 \sin(\psi)}
\]  

(4.3)

The error in measured time delay, \( t_e \), will depend on many factors, including the height over ground and the reflections from the ground. It is difficult to calculate the error analytically, but from Eq. (4.3) it can be seen that it can be modeled with an additive term, \( \xi(t_e) \):

\[
\hat{h} = \frac{c_0 \cdot \Delta t}{2 \sin(\psi)} + \xi(t_e)
\]  

(4.4)

Because the bias error is described by an additive term, it could be compensated for if the error could be modeled.

The effects of the bias depending on the aeroplane’s height and the type of ground cover will be examined in several simulations.

### 4.3.2 Noise

When estimating the time delay from the correlation function the effect of the thermal noise in the receiver must be taken into account. For example, when using the peak tracking method described above, an error can be caused if the noise is strong enough to distort the peak of the signal (see Figure 4.2).

A larger SNR reduces the probability of errors. The noise in the receiver is assumed to be white, and therefore the power of the noise can be calculated by using the equation [2]

\[
N = kTB
\]  

(4.5)

where \( k \) is Boltzman’s constant, \( T \) is the equivalent noise temperature (typically 273 K) and \( B \) is the bandwidth considered. In the front-end of a GPS receiver the noise power density is \( -201 \) dBW-Hz, and with a bandwidth of 2 MHz, this gives a noise power of \( -138 \) dBW. A GPS signal has a power of approx. \( -158 \) dBW for a direct signal measured with an isotropic antenna, well below the noise floor. When the signal is correlated over 1 ms a reduction in bandwidth with 30 dB to 1 kHz is done, which reduces the noise power to \( -171 \) dBW.

Longer integration times reduces the bandwidth even further. A problem is that the data bits of the signal put an upper limit to the time. If integration is done over a data bit transition, the correlation function will have a smaller peak. To be sure to not integrate over a data bit transition, two consecutive correlations over
10 ms can be performed. One of these is assured not to have a data bit transition. It is also possible to integrate over longer periods of time if the data bit transitions are known, e.g. if the receiver has access to a direct signal, which is the case in this thesis. Akos et. al showed that using this technique integration times up to 800 ms are possible [14].

To perform height measurements, the receiver must be able to locate the signal after correlation, i.e. the correlation peak must be visible above the noise floor. This is a well known and examined problem in GPS receivers called acquisition. A common threshold for receivers to acquire the signal is an SNR of at least 35 dB-Hz before correlation [11], which gives an SNR of approx. 5 dB after 10 ms of integration. In this thesis, when the acquisition of the reflected signal is examined, 10 ms will be used as the integration time. Therefore, an SNR of 5 dB will be required as a threshold for locating the correlation peak.

Due to the random nature of the noise, the measured height can be described as a stochastic variable with a mean value and a standard deviation. The following equation from [16] is used:

\[
\sigma_T = \frac{c_0 T_C}{\sqrt{2TP_S/N_0}}
\]  

(4.6)

where \(\sigma_T\) is the standard deviation, or root mean square error (RMSE), of the measured height, \(T_C\) the chip time of the code used, \(T\) the integration time, \(P_S\) the
signal power and $N_0$ the noise power density. Eq. (4.6) holds for the peak tracking method, for the other methods other equations must be used. See e.g. Eq. (4.7) below. Also note that Eq. (4.6) only gives the error due to the receiver noise, and does not include other error sources.

If the parameters are assigned the following values:

\[
\begin{align*}
T_C &= 1 \mu s & \text{C/A code chip length} \\
T &= 10 \text{ ms} & \text{max integration time} \\
P_S &= -160 \text{ dBW} & \text{approx. power of GPS signal reflected in water, measured with an isotropic antenna} \\
N_0 &= -201 \text{ dBW-Hz} & \text{noise power density in a typical GPS receiver}
\end{align*}
\]

\(\sigma_T\) will be 23.8 m. To improve the result several values can be averaged. If 100 samples are averaged, i.e. measurements are done over a period of 1 s, the error is reduced by a factor 10 to 2.38 m. In reality, even better results than this are mostly achieved since the GPS signal is often transmitted with stronger signal power than the values guaranteed. Longer integration times cannot be used if the aeroplane is moving to fast over varying ground, since different heights will be measured for the values. Furthermore, real-time restrictions have to be considered. Depending on the application of the altimeter, producing height measurements at a rate of 1 Hz might be too slow.

In [13] experimental results showed that averaging the values over 1 s when the peak-tracking method was used resulted in a RMSE of 2.76 m, compared to a radar height altimeter which was assumed to give the true height values. This value is in the order of the analytical value calculated above.

Eq. (4.6) is similar to the equation commonly used to calculate the so called pseudorange error when using GPS for positioning (see e.g. [2]):

\[
\sigma_T = \frac{c_0 T_C}{\sqrt{4TP_S/N_0}}
\tag{4.7}
\]

Tracking method 3 is essentially the same as the method used in GPS receivers for positioning which makes Eq. (4.7) probable to use in an error analysis in this case.
Chapter 5

The simulation model

In this chapter a thorough description of the simulation model is made. The objective of the simulation model is to simulate a correlation function of the reflected signal, relative a replica of the GPS-signal. This correlation function gives the time difference between the direct signal and the reflected signal, which is needed to make a height measurement. A short description of how the correlation function is used to measure height is given in Chapter 2.

In order to acquire the correlation function, a model to create the reflected signal is needed. This is achieved by constructing a 3D description of the environment, i.e. aeroplane, satellite and ground. The aeroplane affects the result in two ways, namely the height and the performance of the antenna. The satellite affects the result depending on the angle it has relative the ground. Modeling the GPS signal’s reflection at the ground is more complex. This involves both how uneven the ground is, i.e. hills and other undulations in nature, and the characteristics of the surface, e.g. rough or smooth.

5.1 Simplifications

The following simplifications have been made in the simulation.

- **Baseband model of the GPS signal.** No carrier is modeled since it is not practical to simulate because of the small wavelength (19 cm). This means that no Rayleigh fading or similar fading is modeled. The effect of fading due to the carrier is an important aspect, and could affect the result considerably. The signal power will vary over time, meaning that the effect of noise can increase considerably.
• **Slow fading.** The channel is assumed not to change considerably during the time for one measurement. This means that the signal is affected in the same way during the whole measurement. Slow fading requires that the aeroplane travels with relatively low speed, approximately 200 m/s. This was discussed in Section 3.8.1 on page 37.

• **No data signal.** The 50 Hz data signal carrying the navigation information is assumed compensated for. Only a signal describing a PRN code is therefore modeled. In reality the C/A period (1 ms in duration) is occasionally corrupted due to a change in the data signal (from positive to negative or vice versa). This is not considered to be major problem, and was also discussed in Section 4.3.2.

• **The Doppler effect is assumed compensated for.** This effect occurs primary due to the movement of the satellite. In ordinary GPS receivers this problem is solved, and it is therefore not considered a problem in this simulation.

• **Square antenna coverage with constant gain.** The assumption of constant antenna gain can not be achieved for a real antenna. The gain within the main lobe varies and there are also side lobes. The square form of the coverage is not achievable and has been chosen for simplicity. A circular or elliptical coverage is a more likely coverage. Finally, the antenna has also been assumed to be lossless. The effect of these simplifications is discussed further in Section 5.4.

• **Constant power density at the ground.** The power density has been set to $-135$ dBW/m$^2$ in all simulations, although in reality it varies depending on the location relative to the satellite. This is a pessimistic estimation, and the power density is never below this value. Therefore this simplification introduces no major disadvantages.

• **Simplified model for ground reflection.** This simplification is the most considerable. A theoretical description has been made due to the lack of measured data for forward scattering, and the effect of this simplification is difficult to determine. However, if a more precise theoretical model or measured data becomes available, it could easily be implemented in the simulation model. A description of the model is given in Section 5.3.

• **Horizontal attitude of the aeroplane.** This means that the antenna always is directed vertically. The direction of the antenna will in reality vary when the aeroplane, for example, makes a sharp turn.

• **No interference from other GPS satellites.** Different satellites use different PRN codes for multiple access. But the codes can in practice not be separated entirely and a certain cross correlation exists between them. This cross correlation is at worst approximately $-23$ dB. This effect has not been
modeled in the simulation. The noise that this introduces is very small and does not give considerable errors.

Over all, many simplifications has been made. Many of these can be considered negligible, while some are of more importance. The effect of fading due to the carrier would most likely give a varying behaviour in the received signal and therefore in the correlation function. However, the effect of fading is reduced if more time can be used to make a single height measurement, and the time available is dependent on the speed of the aeroplane. The simplified model of the ground reflection has not been validated with measured data, but the surface types used in the simulation model are designed to cover a wide spectrum of different ground reflection. Therefore, qualitative conclusions can be drawn from the simulations.

5.2 Geometric description of the world

The 3D environment is described by the

- Aeroplane
- Satellite
- Ground

These are described with cartesian coordinates, i.e. \((x, y, z)\). The position of the satellite and aeroplane is easy to describe, while modeling ground variations is more complex.

The coordinates are assigned to represent the directions in a compass. North is given by the positive \(z\)-coordinate and east by positive \(y\)-coordinate. Altitude is described by the negative \(z\)-coordinate. A schematic description of the environment is given in Figure 5.1.

5.2.1 Aeroplane and satellite

The aeroplane describes the location of the receiving antenna and the position is given in cartesian coordinates. The position of the aeroplane is varied in the simulation in order to simulate height measurement for different terrain. The position of the satellite is, in the same way as the aeroplane, given in cartesian coordinates.
5.2.2 Terrain variations

Modeling the terrain is more complex than modeling the position of the aeroplane and satellite. By modeling terrain it is here meant as describing how the ground varies due to e.g. hills and other undulations in nature.

The simulated ground is defined by the area that the antenna covers. The antenna in the simulation is simplified to cover a square area of the ground. This area is divided into ground planes which describe the surface. It is at these ground planes that the GPS signal is reflected. According to Eq. (3.12) on page 22 an
infinite number of reflections at the ground is assumed. In the simulation model this equation is approximated to a finite number of reflections from the ground planes.

However, before the square can be divided into ground planes it is described with a number of ground points which finally construct the ground planes (see Figure 5.2). Depending of the resolution that is needed, different numbers of ground points can be used. Typically 10000 ground points is used to describe the area covered by the antenna. The number of ground points used is dependent of the surface type, e.g. a specular surface requires more ground points than a rough surface to give an accurate simulation. Each ground point is described by a \((x, y, z)\) coordinate and a cover type. The \((x, y)\)-components describe the position of the point north-east and the \(z\)-component gives the height of the ground relative to the mean sea level, MSL (see Figure 5.3). Finally, the cover type gives which type of surface that the point has, e.g. rough and wet. The reason for defining \(x\) and \(y\) with geographic directions is because a database of a real map is used. It is also from this database that ground height and cover for a certain \((x, y)\) position is given.

---

**Example 1**

Describing the ground in the north of Sweden. This region is very hilly and vegetation is quite smooth. Since this region is situated high above the mean sea level the \((x, y, z)\) coordinates could for example be \((5000, 10000, -1000)\) and the cover type would be smooth. Hence, this point is situated 1000 m above MSL. Notice that negative \(z\) values represents height above MSL.

Because of the variation of the terrain with hills and valleys a ground point 1000 m south of the previous ground point could have the coordinates \((4000, 10000, -500)\), i.e. this ground point has a height that is 500 m lower. The cover would most likely be the same as for the previous point.
The ground planes are created by using these ground points, and three ground points construct a triangular ground plane (see Figure 5.4). The number of ground planes can approximately be calculated as $2k$, where $k$ is the number of ground points. The three ground points that the ground plane is created by, typically all have different ground height. Therefore every ground plane has a unique slope that is dependent of the ground points which it consists of. Each ground plane can therefore be described with an area and a normal. This normal and area is used to determine the amount of power that the ground plane captures.

### 5.2.3 Database of ground height and cover

To determine the ground height for the ground points described above a database is needed. This database contains information about both the ground height and cover for a $(x,y)$ coordinate. The database used for the ground height is an authoritative database of Sweden and is provided by “Lantmäteriverket”, while the cover types are provided by “Satellitbild i Kiruna AB” and are also given for the coordinates in Sweden. The resolution of the ground height database is 50 m, i.e. for every 50 m in $x$ or $y$ direction a new ground height is given. The resolution of the cover database is also 50 m. To get the height for a certain ground point, linear interpolation is used. This is because a ground point from the antenna coverage rarely coincide with the coordinates given by the database.
5.3 Ground reflection

In this section a model for the ground reflection is created. A thorough theoretic discussion was made in Section 3.7, and the reader should have understood this well to be able to understand the following discussion. Section 3.6 about the bistatic radar equation must also be clear. With this model the reflection at every ground plane will be calculated in order to get the total reflected signal. The cover type characterizes the reflection at a ground plane, and the term cover is in this thesis used for this purpose.

In Section 3.7 the ground reflection was described to consist of two parts: the reflectivity, \( R \), and a scattering pattern, \( Q \). The scattering coefficient, \( \gamma^o \), is analytically described by

\[
\gamma^o = |R|^2 Q
\]  

(5.1)

where \( |R|^2 \) gives the amount of the incident power that is reflected, and \( Q \) describes how the reflected power is distributed in different directions. The scattering coefficient is used in the bistatic radar equation, which is used to determine the received power, \( P_r \), at the antenna. The bistatic radar equation is given by Eq. (3.21) on page 25:

\[
P_r = \frac{S_g \gamma^o A' G_r(\theta_r, \phi_r) \lambda^2}{(4\pi)^2 \cdot R^2} \quad [W]
\]  

(5.2)

and is used to calculate the received power that each ground plane contributes with. \( S_g \) is the power density at ground, \( A' \) is the area of the ground plane perpendicular to the satellite and \( R \) is the distance to the aeroplane from the ground plane.
5.3.1 Reflectivity

The modeling of $\mathcal{R}$ simply applies the tools that was described in Section 3.7.2. $|\mathcal{R}|^2$ gives how much of the incident power that is reflected and is dependent of the electrical properties of the ground. The electrical properties used in the simulation are, for the GPS L1 frequency 1575 MHz, given by the following surface types

- **Dry ground**
  \[ \epsilon = 4 - j \cdot 1.14 \cdot 10^{-4} \]

- **Medium dry ground**
  \[ \epsilon = 7 - j \cdot 4.56 \cdot 10^{-2} \]

- **Wet ground**
  \[ \epsilon = 30 - j \cdot 2.28 \cdot 10^{-1} \]

- **Water (fresh)**
  \[ \epsilon = 80 - j \cdot 2.28 \cdot 10^{-1} \]

These values were calculated using Eq. (3.25) on page 28, with the values in Table 3.1.

The power reflected at these surfaces, at different incident angles, is given in Figure 5.5. From these plots it is evident that wet ground reflects better than dry, and that water reflects power best. The reflectivity, $\mathcal{R}$, of these surfaces is also described in Section 3.7. In this section it was shown that the signal is reflected with different polarization depending on the incident angle. For relatively small incident angles most of the signal changes polarization from RHCP to LHCP. But as the incident angle increases more power is reflected as RHCP, and at an incident angle of 90° all signal power is reflected as RHCP. However, in between these two extreme cases the signal reflects both LHCP and RHCP.

Assuming that there always is a satellite with a relatively small incident angle, at least smaller than 45°, then most of the GPS signal is always reflected as LHCP. An antenna suited for a LHCP signal must therefore be used, and the plots in Figure 5.5 are for RHCP to LHCP.

The power reflection described above assumes a flat surface. But the surfaces used in the simulation are more or less rough and therefore consist of small slopes. A rough surface is characterized by having large slopes, while for a smooth surface the slopes are very small. This means that the incident angle given by the normal of the ground plane gives an incident angle that does not correspond completely to the slopes. This applies especially for rough surfaces. However, as shown in Figure 5.5 the attenuation is almost constant for angles smaller than 45°. With this in mind the effect of the slopes can be neglected, and using the incident angle to the ground plane gives a good approximation.
5.3 Ground reflection

![Graphs showing the power reflected for different ground conditions.](image)

**Figure 5.5.** Power reflected for an incident RHCP signal reflected as LHCP, f = 1575 MHz.

### 5.3.2 Scattering pattern

While the reflectivity could be modeled according to the theoretical results shown in Section 3.7.2, the modeling of the scattering pattern is more complex. The scattering pattern describes the amount of power that is reflected in a certain direction. According to the theories presented in Section 3.7.3 most of the power is always reflected in the specular direction. For a perfectly specular surface all of the reflected power is in the specular direction. Depending on the level of roughness of the surface more power is reflected in other directions. The rougher a surface is the more power is reflected in other directions than the specular. To model the scattering pattern the following definition has been made:

\[
Q(\alpha) = \begin{cases} 
2(n + 1) \cos^{n}(\alpha) & , \quad 0 \leq \alpha \leq 90 \\
0 & , \quad \text{otherwise}
\end{cases} 
\]  
\tag{5.3}

Figure 5.6. Forward scattering.

where $\alpha$ is the angle relative to the specular direction, which is shown in Figure 5.6. The parameter $n$ gives how narrow the scattering pattern is. A large $n$ corresponds to a specular surface while a small $n$ corresponds to a rough surface. In the extreme case where $n = 0$, the scattered power is equally strong in all direction, this includes the specular direction. The factor $(n + 1)$ is a normalization constant which makes sure that the right amount of power is reflected. The definition in Eq. (5.3) has been made so that different scattering patterns easily could be created, and should be considered as a qualitative model. A validation of the expression is given in appendix A.

Modeling the surfaces with Eq. (5.3) makes it possible to describe different degrees of roughness by varying only $n$. This model is by no means a thorough description of the scattering pattern, and should be regarded as a qualitative model. However, it gives a good idea of the effect of different roughness. Since everything from almost specular to completely rough surfaces can be described, qualitative differences between different levels of roughness can be analyzed.

In the simulation model the following surface types are used (see Figure 5.7):

- **Rough** ($n = 0$)
  Q = 2 for all directions, including the specular.
  Modeled as the extreme case of scattering, where scattering is the same in all directions.

- **Medium rough** ($n = 10$)
  Q = 22 in the specular direction, with significant reflections for $\alpha < 50^\circ$.
  Modeled as a surface with considerable scattering, but with stronger specular reflection than rough.
5.3 Ground reflection

![Graph showing scattering patterns for different surface roughness levels](image)

**Figure 5.7.** Scattering patterns for surfaces. Notice the different magnitudes for small $\alpha$.

- **Small rough** ($n = 100$)
  
  $Q = 202$ in the specular direction, with significant reflections for $\alpha < 15^\circ$. Modeled as a surface with small scattering effects.

- **Smooth** ($n = 1200$)
  
  $Q = 2402$ in the specular direction, with significant reflections for $\alpha < 5^\circ$. Modeled to correspond to an almost specular surface, although some of the reflected power is scattered.

These surfaces are chosen to describe a large spectrum of different surface roughness, with smooth as a near approximation of a specular reflection and rough with a diffuse scatter pattern. These are the two extreme cases of scattering while small rough and medium rough are in between these two. Notice that the defined surface types are used for both land and sea surfaces, except for the smooth which is only used for water surfaces.
5.4 The antenna

The antenna used in the simulation is primary defined by the antenna pattern, and is suited to receive LHCP signals. The antenna has for simplicity been modeled to cover a square area of the ground and to have constant antenna gain. Therefore, reflections from ground planes covering a square area are captured, and the antenna gain is the same for all ground planes.

To calculate the antenna gain, the square area is approximated with a circular area. This is because the mathematical description of a circular antenna coverage is easier to handle than a square. The approximation from square to circular coverage is shown in Figure 5.8. The mathematical description of the circular antenna coverage with constant antenna gain is given by

\[
G(\theta, \phi) = \begin{cases} 
  G_0 & , 0 \leq \theta \leq \beta \\
  0 & , \text{otherwise}
\end{cases}
\]  

(5.4)

where \( \beta \) is the width of the antenna and \((\theta, \phi)\) are spherical coordinates.

From Figure 5.8 it is noticed that the area of the circular approximation gives a smaller area coverage than the square. This means that the antenna gain calculated using Eq. (5.4) results in a greater antenna gain than the square antenna coverage in reality has. If \( r \) is the radius, the ratio between the areas is

\[
\frac{\pi r^2}{4r^2} \approx 0.79
\]  

(5.5)
Therefore, 21% of the square area should in reality not give any contributions at all. This can also be interpreted as a lower antenna gain if the antenna covers the whole square. This simplification does not have a major influence on the height measurement.

As described in Section 3.9 about antennas the antenna gain has the following property

\[
\int_0^{2\pi} \int_0^{\pi} G(\theta, \phi) \sin(\theta) d\theta d\phi = 4\pi
\]  

(5.6)

Using this equation and the approximation of circular coverage then \( G_0 \) can be calculated for a given antenna width, \( \beta \). This yields the following equation

\[
\int_0^{2\pi} \int_0^{\beta} G_0 \sin(\theta) d\theta d\phi = 2\pi(1 - \cos(\beta))G_0 = 4\pi
\]

(5.7)

Extracting \( G_0 \) from this equation gives

\[
G_0 = \frac{2}{1 - \cos(\beta)}
\]

(5.8)

which in terms of effective aperture yields (see Eq. (3.43) on page 40)

\[
A_e = \frac{2\lambda^2}{4\pi(1 - \cos(\beta))}
\]

(5.9)

Observing that \( A_e \) depends on the width \( \beta \), it is obvious that the smaller the antenna width is, the larger \( A_e \) is. This means that a narrow antenna width gives a good ability to receive signals. On the other hand it can only receive signals from a very limited area. The antenna width is one of the parameters that is varied in the simulation to observe the different results it gives.
5.5 The GPS signal

The GPS signal must be modeled so that the reflections at the ground planes can be calculated. The total reflected signal should then be correlated with a replica of the GPS signal to achieve the correlation function needed to make the height measurement. However, in this simulation another method of achieving the wanted correlation function is used. First of all, power has been used instead of amplitude and second, the total reflected signal is never explicitly calculated. Instead, an equivalent expression of achieving the correlation function is used in the simulation. Since the number of planes in the simulation model is finite, Eq. (3.12) on page 22 becomes a finite sum described by

\[ < |R^2(\tau)| >= \sum_{k=1}^{N} P_{r,k} \cdot r^2(\tau - \tau_k) \]  

(5.10)

where \( N \) is the number of ground planes, \( P_{r,k} \) is the power received from the \( k \)th ground plane, \( \tau_k \) is the delay relative the direct signal and \( r^2 \) is the auto correlation function. Notice that \( r^2 \) is used instead of the triangle function \( \Lambda^2 \) in Eq. (3.12), which is an approximation of \( r^2 \) used analytically, for simplicity. In the simulation model this simplification is not necessary and the exact expression can be used. An example of how Eq. (5.10) is used is described in example 2.

---

Example 2

Three ground planes reflect the GPS signal to the receiver, i.e. \( N = 3 \). The following properties for the reflections are given: \( P_{r,1} = 3.0, P_{r,2} = 2.75, P_{r,3} = 2.5 \) and the corresponding delays are \( \tau_1 = 1, \tau_2 = 2, \tau_3 = 3 \). In Figure 5.9 the correlation function originating from each ground plane and the total correlation function is shown.

This example shows the effect of different delays and power from each ground plane. From this it is also obvious that a number of ground planes with the same delay will add their power and create a large peak in the correlation function for that delay. In the simple case of a perfectly specular surface only one peak appears. In contrast, for a rough surface there are many reflections with approximately the same power, but with different delays. This yields a total correlation function which has a broad peak.
5.6 Description of the simulation

Figure 5.9. Summarizing auto correlation functions originating from three ground planes.

Finally, to describe $r^2(\tau)$, which in practice is used as the input signal, an array describing $r^2(\tau)$ is needed. The sample rate used in the simulation model is defined by the number of samples per chip in the GPS signal. Therefore, if the resolution is 100 samples/chip, the sample rate is given by

$$ R \approx \text{resolution} \times 10^{-6} = \frac{100}{10^{-6}} = 10^6 \quad \text{[samples/s]} $$

(5.11)

since one chip is approximately $10^{-6}$ s long. The choice of using the number of samples/chip as definition of the sample rate has been done for practical reasons in the simulation program.

5.6 Description of the simulation

The result from the simulation model is the reflected signal correlated with a replica of the GPS signal. This correlation function is then used to make the height measurement. The simulation model follows the five major steps given below.
1. Create triangular ground planes.
2. Calculate the delay from each ground plane.
3. Calculate the power received from each ground plane.
4. Calculate the correlation function of the total reflected signal.
5. Measure height using the correlation function.

**Create triangular ground planes.** This is done by describing the ground with the number of ground points that is wanted, and then constructing triangular ground planes using the ground points. In the simulations 10000 ground points are used, which yields approximately 20000 ground planes (see Section 5.2.2).

**Calculate the delay from each ground plane.** The delay is given by calculating the different distance that the direct signal and the reflected signal travel. If the coordinates of the aeroplane, satellite and ground plane is given by the vectors \( \bar{a} \), \( \bar{s} \) and \( \bar{g} \) then the delay can be expressed by (see Figure 5.10)

\[
\Delta t = \frac{\Delta d}{c_0} = \frac{|\bar{g} - \bar{s}| + |\bar{a} - \bar{g}| - |\bar{a} - \bar{s}|}{c_0} \tag{5.12}
\]

Since the signal in practice is given by samples at a certain sample rate, small delay differences cannot be registered by the simulation model. This is an error introduced by the simulation model and has no correspondence in a real altimeter. This limitation of detecting delay will therefore introduce an error in the height measurement. The relation between the resolution and the shortest path difference that can be detected is given by

\[
\min\{\Delta d\} > \frac{1}{2} \cdot \frac{c_0 T_c}{\text{resolution}} \approx \frac{150}{\text{resolution}} \tag{5.13}
\]

where \( T_c \approx 10^{-6} \) is the time for one chip and resolution is the number of samples per chip. E.g., with a resolution of 100, the smallest \( \Delta d \) that can be registered is 1.5 m. This makes a measurement with better accuracy than 1.5 m impossible.

**Calculate the power received from each ground plane.** The power reflected from a ground plane is calculated by using the *bistatic radar equation*. This involves calculating the area perpendicular to the satellite, which affects the amount of power that is captured by the ground plane. The captured power is calculated by \( S_\theta A' \), where \( A' \) is the perpendicular area and \( S_\theta \) is the power density at the ground plane. The power density at earth’s surface varies from \(-132.6\) dBW/m\(^2\) to \(-134.6\) dBW/m\(^2\), but in the simulation model the power density \(-135.0\) dBW/m\(^2\)
Figure 5.10. Definition of vectors describing the position of the satellite, aeroplane and ground plane.

is used. The scattering angle, $\alpha$, also affects the received power through the scattering coefficient, $\gamma^o(\alpha)$ (see Figure 5.6 on page 58). The power declines with the distance, $R$, from the ground plane to the aeroplane. Finally, the antennas effective aperture, $A_e$, affects the received power. With these parameters the received power for the $k$th ground plane can be described with the bistatic radar equation in the following way

$$P_{r,k} = \frac{S_d \gamma^o(\alpha_k) A'_e(\theta_k) A_e}{4\pi R_k^2} \, [W]$$

(5.14)

This is the same equation as Eq. (5.2), but with the introduction of $\theta_k$ and $\alpha_k$. Effective aperture, $A_e$, has also been used instead of antenna gain $G$.

Calculate the correlation function. The correlation function of the total reflected signal is calculated by using the calculations of power and delay from each ground plane. These are used in Eq. (5.10) to produce the correlation function that is used to make the height measurement.

Measure height using the correlation function. Several methods can be used to calculate the height from the correlation function, but in this thesis only the so called peak tracking algorithm is used. The methods are described in Chapter 4.
5.7 The simulation program

The simulation program was implemented in ADA 95 using the GNAT compiler. The computer used to make the simulations was a shared server, therefore the time to simulate one measurement varied depending on the work load on the server. With a resolution of 100 samples/chip and 20000 ground planes, one measurement required 2-5 minutes. This made simulations when flying over longer distances very time consuming. For example flying 2000 m with measurements every 50 m took approximately 2 hours. The simulation program can be made faster by either optimizing the program further or making simplifications in the model.

A limitation in the program was also that the resolution and the number of planes could not be chosen arbitrarily. Using too large values resulted in segmentation fault. What this problem originated from has not been examined, but a bug in the compiler is possible.
Chapter 6

Performed simulations

To evaluate the simulation model and the method used to measure the height, several simulations have been done. The simulations and the result of these are discussed in this chapter.

6.1 Overview

In order to characterize the measurement method, simulations have been done where the different parameters in the system have been changed, e.g. the width of the antenna coverage or the type of ground cover. Simulation scenarios have been constructed to cover and isolate the effect for every parameter. Finally, the results have been studied and conclusions have been made about the behaviour of the simulation model and of the height measurement method.

The simulations have been divided into two stages. In the first stage, one parameter has been the subject of change every time and the effect of this has been studied. Only homogeneous surfaces, where the ground consists of one type of ground cover, have been used here. In the second stage, heterogeneous surfaces and more complex scenarios have been used where some parameters have been chosen on basis of the results from the first stage of simulations.

6.1.1 Height estimation method

To calculate the time delay and estimate the height, the peak tracking method has been used (see Section 4.2). This method uses the peak of the correlation function
as the time delay to calculate the height over ground. The reason for choosing this algorithm is primary because it is easy to implement.

The notation altimeter refers to the system that performs the height measurement and the notation waveform refers to the shape of the received correlation function, as in traditional radar altimetry.

6.1.2 Evaluation of the measurement method

To evaluate the height measurement method three methods have been used:

1. Root mean square error (RMSE) of the measured height versus the true height.
2. A plot of the measured height versus the true height.
3. A plot of the error of the measured height.

Conclusions about the height measurement method and the effect of the various parameters have been made based on the results of these evaluation methods.

RMSE

Assume that for one simulation, \( N \) measurements have been made where \( \hat{h}_i \) is the estimated height and \( h_i \) the true height for the \( i \)th measurement. The RMSE for this simulation is then given by

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=0}^{N-1} (\hat{h}_i - h_i)^2}
\]

(6.1)

The RMSE is equivalent to the standard deviation of the measured height.

This can be a good way of estimating the overall quality of a longer simulation, or simulations where trends in the error do not need to be discovered.

Plot of estimated height

The RMSE is not always enough to describe the accuracy of the estimated height. E.g., in Figure 6.1(a) two example simulations have been plotted. The constant error of simulation 1, called bias, gives a large RMSE (20 m) but is in this case easy to compensate for. The estimated height in simulation 2 gives a smaller RMSE (2.5 m) but tracks the ground profile less accurate and the error is harder to compensate for.

Therefore, plotting the measured ground profile will also be used to estimate the quality of the height measurements.
Figure 6.1. Plot of measured ground profile and error in measurement for two example simulations.
Plot of error

If the simulation is done at the same \((x,y)\) coordinate while the aeroplane moves upwards, increasing the height, one way to evaluate the result is to plot the error of the height measurement in every position, i.e.

\[
e = \hat{h}_i - h_i
\]

By examining this plot, e.g., linearly dependencies in the error can be discovered. In Figure 6.1(b) the errors for the two examples simulations used above have been plotted.

6.1.3 General parameters for the simulations

Where otherwise has not been stated, the parameters have been assigned the values stated in Table 6.1. The values are motivated below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>100 samples/chip</td>
</tr>
<tr>
<td>Ground planes</td>
<td>19602</td>
</tr>
<tr>
<td>Power density at ground of GPS signal</td>
<td>(3.16 \times 10^{-14}) W/m²</td>
</tr>
<tr>
<td>Noise power</td>
<td>0 W</td>
</tr>
<tr>
<td>(A_r) for an isotropic antenna</td>
<td>(2.87 \times 10^{-3}) m²</td>
</tr>
<tr>
<td>Satellite elevation angle</td>
<td>90°</td>
</tr>
</tbody>
</table>

Table 6.1. Parameters that have been used in simulations where not otherwise is stated.

**Resolution** was set to 100, which causes an error in the height measurement of 1.5 m. Although a smaller error would be desirable, this value of resolution was chosen due to practical reasons. The execution time of the simulation program was acceptable, and restrictions in the implementation (see Section 5.7) allowed no greater value.

**Ground planes** was set to 19602, as a result of the simulations described in Section 6.2.1.

**Power density** was set to \(3.16 \times 10^{-14}\) W/m² (−135 dBW). Since the power density of the transmitted GPS signal varies from −132.6 dBW/m² to −134.6 dBW/m² (see Section 3.2.3), this assures that the resulting signal power will not be lower in reality.

**Noise power** was set to 0 W. This is because the shape of the received waveform has been of most interest and not the signal to noise ratio (SNR). The effect of noise is discussed in Section 6.3.6.
Effective aperture for isotropic antenna, $A_e$, was set to $2.87 \cdot 10^{-3}$ m$^2$, according to e.g. [2].

Satellite elevation angle $\psi$ was set to 90° since no errors due to the satellite are caused at this angle.

6.2 Implementation parameters

There are essentially two parameters in the simulation model that are implementation specific:

- The resolution of the signals.
- The number of ground planes.

Setting these parameters to too low values may result in errors introduced by the implementation and not by the height measurement method itself. The resolution parameter was discussed in Section 6.1.3, whereas the number of ground planes will be discussed below.

6.2.1 Ground planes

It is necessary to determine the minimum number of ground planes needed. If this value is set too low, factors such as terrain variations can be missed in the simulations, since the planes will be too large. Furthermore, the delay for the signals from two adjacent planes cannot be larger than the time between two samples in the simulated signal. There is also a possibility that the received power will be too small for near-to-specular scattering surfaces.

Simulations

To determine the minimum number of ground planes needed, two aspects were considered: the received power density and the height measurements. Several simulations were performed with approx. 20, 40, 100, 200, 500, 1000, 2000, 4000, 10000, 20000 and 40000 ground planes. No greater amount than 40000 planes was used due to the restrictions in the implementation (see Section 5.7). These numbers are approximate, the reason for this being that the parameter that is varied in the simulation program is the number of ground points used (see Section 5.2.2). E.g., choosing 10000 ground points gives actually a number of 19602 ground planes. If
there was no difference in received power density or measured height between e.g. 4000, 10000, 20000 and 40000 ground planes, 4000 would be selected as a sufficient number.

First, power density was measured. Flat ground and two types of ground cover were used, a rough surface and a smooth surface. The result is showed in Figure 6.2. For a rough surface, the number of planes could almost be arbitrary chosen between 40 and 40000, since there was little difference in received power. For a smooth surface, the received power is the same for 2000 planes and more.

As a second test, simulations were performed with varying terrain. Data from the ground elevation database was used, one simulation with small undulations and one simulation with large undulations. Rough and small rough cover was used in both cases. The result is shown in Figure 6.3.

For rough surfaces, there is essentially no difference in the number of planes. For small rough surfaces, there is a small difference, but when the number of planes exceeds 10000, the result is sufficient.

Finally, the height was measured, which is plotted as a function of the number of planes in Figure 6.4. When the number of ground planes is greater than 4000 there is no significant difference in measured height, although the height is not constant for rough cover. These small differences were considered negligible.

**Comments**

Considering the simulations done, any number of ground planes greater than 10000 seems to be sufficient. To have some extra margin for unaccounted factors, such
6.2 Implementation parameters

**Figure 6.3.** Plot of normalized received power density as a function of number of ground planes. Undulating ground with small variations and large variations has been used.

**Figure 6.4.** Plot of measured height as a function of number of ground planes.

as largely varying terrain, the number of ground planes was chosen to be approx. 20000 in the rest of the simulations, which in reality becomes 19602 ground planes due to implementation issues.
6.3 Homogeneous surfaces

First, homogeneous surfaces where the ground consists only of one type of ground cover, were examined. Six parameters have been varied, one by one, to try to isolate the effect of every parameter on the height measurement method. The parameters were:

- The width of the antenna coverage.
- The type of ground cover, e.g. rough or smooth.
- The terrain variations, e.g. flat or undulating ground.
- The elevation angle of the satellite, $\psi$.
- The height of the aeroplane above ground.
- The presence of additive white Gaussian noise (AWGN) in the receiver.

The parameters have been treated separately as far as possible, i.e. the effect one parameter has on the others has not been taking into account. In reality the situation is more complex, e.g. the effects of the ground cover also depends on the width of the antenna lobe. It is hard to investigate all the cross correlations between the parameters, and when e.g. varying the ground cover in Section 6.3.1 a fix width of the antenna lobe has been used.

6.3.1 Ground cover

In the simulation model four types of ground cover are used with four different types of reflectivity. The reflectivity and scattering pattern for a certain type of ground cover influence the received signal noticeably and it is therefore important to examine the behaviour of this function over different types of ground. A narrow antenna, $\beta = 15^\circ$ has been used, since the satellite has been right above the aeroplane.

Reflectivity

The reflectivity only affects the strength of the received signal and it is therefore probable that the received correlation function will have the same appearance when the scattering pattern is the same but the reflectivity is changed. A simulation was done to examine this, where each type of cover (rough, medium rough and small
6.3 Homogeneous surfaces

![Graph showing power of received correlation function vs time delay for different conditions.]

**Figure 6.5.** Power of received signal for rough terrain with three types of reflectivity. $\beta$ is $15^\circ$.

rough) was used with three reflectivity types (dry, medium dry and wet), i.e. 9 simulations in total were performed.

In Figure 6.5 the resulting received signal for rough terrain is shown. As can be seen these functions only differ in magnitude. This was also confirmed by scaling the functions for dry and medium dry reflectivity by constants, making them fit exactly to wet reflectivity.

**Scattering pattern**

To investigate the effects of ground covers with different scattering patterns, several simulations were done with the following types of ground cover:

- smooth water
- medium rough water
- small rough wet ground
- medium rough wet ground
Figure 6.6. Received signals for ground covers with different types of scattering pattern. \( \theta \) is 15°. The vertical lines are there to show the differences in time delay between the tops of the received signals.

- **rough wet ground**

The received signals were plotted and compared (see Figure 6.6). The measured heights were also compared, the differences between measured and true height can be seen in Table 6.2. The waveform gets wider when the cover is more rough, which is not easily seen from the figure, since the differences are small. In addition to this, the peak of the waveform will occur later and a larger height will be measured. This affects the measured heights and gives a larger error (bias) for more rough cover, which also can be seen from Table 6.2. The received power will also decrease for rougher cover.

**Comments**

Different reflectivity only affects the magnitude of the received power and not the principal appearance. Therefore, in all the following simulations, only wet reflectivity will be used. It is important to remember that a weak signal can suffer from low SNR, but this will be discussed in Section 6.3.6.
6.3 Homogeneous surfaces

<table>
<thead>
<tr>
<th>Ground cover</th>
<th>Difference [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth water</td>
<td>0</td>
</tr>
<tr>
<td>Medium rough water</td>
<td>10.3</td>
</tr>
<tr>
<td>Small rough wet</td>
<td>2.9</td>
</tr>
<tr>
<td>Medium rough wet</td>
<td>10.3</td>
</tr>
<tr>
<td>Rough wet</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Table 6.2. Error in measured height for various ground covers. Aeroplane at a height of 1000 m, $\beta$ is 15° and $\psi$ is 90°.

Considering the scattering pattern of the ground cover, rough cover produces reflections from the entire antenna footprint, in opposite to e.g. small rough which essentially produces reflections from only a smaller area, even if the antenna coverage is widened. As a result of this, the received waveform will be wider with the peak delayed, for rough cover. This will introduce an error in the height measurement.

6.3.2 Antenna

The antenna coverage decides which reflections that will be included in the resulting received signal and how much these will be amplified. The wider the antenna coverage is the more reflections with longer delay will be included. The antenna gain will decrease, and even if these additional reflections contribute with more power, the signal strength can be lower. Certain ground cover types with close-to-specular reflectivity may not be included if the antenna coverage is narrow. It is therefore important to examine which width of the antenna coverage that gives the best result, given a certain satellite elevation angle.

A real antenna will also be more complex, with varying antenna gain and perhaps several side lobes, but this is not examined in this thesis. There is also a possibility to direct the antenna towards the strongest reflections, but that will not be examined either. Instead, the antenna will always be pointed in the nadir direction and the parameter that will be changed is the width of the antenna coverage.

Simulations

Four simulations were done where the width of the antenna coverage was increased from 15° to 63°, over different types of ground cover and terrain. The types of ground cover and terrain were the following:

1. Flat rough wet ground.
2. Flat small rough wet ground.
3. Flat smooth water.
4. Undulating medium rough wet ground.

The greatest effect of varying the width of the antenna coverage could be seen in simulation 1, where the ground cover is rough (see Figure 6.7(a)). Contributions from reflections further away increase as the antenna coverage increase, since reflections exist for all angles between the specular direction and the reflected direction. The typical appearance of the waveform will be a tail due to these reflections, and the peak will be more delayed as the antenna coverage increases. Figure 6.7(a) clearly shows this behaviour. This increases the bias error in the height measurement (see Table 6.3) with wider antenna coverage.

<table>
<thead>
<tr>
<th>( \beta ) for antenna [degrees]</th>
<th>Error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>10.3</td>
</tr>
<tr>
<td>16.4</td>
<td>13.2</td>
</tr>
<tr>
<td>18.0</td>
<td>16.1</td>
</tr>
<tr>
<td>20.0</td>
<td>19.1</td>
</tr>
<tr>
<td>22.5</td>
<td>24.9</td>
</tr>
<tr>
<td>25.7</td>
<td>32.3</td>
</tr>
<tr>
<td>30.0</td>
<td>44.0</td>
</tr>
<tr>
<td>45.8</td>
<td>92.3</td>
</tr>
<tr>
<td>63.0</td>
<td>90.9</td>
</tr>
<tr>
<td>80.2</td>
<td>101.2</td>
</tr>
</tbody>
</table>

Table 6.3. Error in measured height for varying antenna coverage width over rough ground. Aeroplane at a height of 1000 m.

For simulations 2-4, the only difference could be seen in the correlation function, where the maximum value decreases with wider antenna coverage. This is due to less antenna gain for wider coverage. Figure 6.7(b) shows the receiver output for simulation 2, flat small rough wet ground. Comparing to Figure 6.7(a) one can see that the tail is missing for small rough coverage and that the peak is not delayed as much as in the rough case.

The ground covers used in simulation 2-4 are small rough wet ground, smooth water and medium rough wet ground. These covers do not produce reflections from greater delays and a wider antenna coverage will therefore not include more reflections. When estimating height from the peak of the received correlation function, the error will not increase with wider antenna coverage.
Figure 6.7. Power of received correlation functions for different antenna coverage widths.
Comments

A wider antenna coverage will generally produce less received power, since the antenna gain will decrease. This is true even for rough ground cover which captures more reflections when the antenna coverage is wider. For ground cover which gives diffuse scattering, a bias in the height measurement will also be introduced due to an increased number of received reflections. For other ground covers, which are more or less specular reflective, the received power will decrease, but there will be no increased error in the height measurement.

6.3.3 Terrain variations

The ground elevation database and fictive created terrain have been used to simulate height measurements over different types of terrain variations. The received signal can be supposed to vary a lot depending on the variations of the terrain. E.g., a mountainous area will probably behave differently from flat ground or calm sea water. A narrow antenna, $\beta = 15^\circ$ has been used, since the satellite has been right above the aeroplane.

Simulations have been done with the five following terrain variations. Since previous simulations has shown that it is of importance if the ground cover type is rough or have some specular reflection, all simulations have been performed with two ground cover types, rough and small rough.

1. One measure over flat terrain. Aeroplane at an altitude of 1000 m.

2. Flying over flat terrain followed by a steep slope, 1000 m height, followed by flat terrain. The aeroplane followed the ground profile at a constant height over ground of 1000 m.

3. Flying over terrain featuring small variations (max-min = 20 m). Aeroplane at a constant altitude of 500 m, with the height over ground being approx. 400 m due to the terrain variations.

4. Flying over terrain featuring large variations (max-min = 80 m). Aeroplane at a constant altitude of 500 m, with the height over ground being approx. 400 m due to the terrain variations.

5. Flying over a narrow peak, 50 m high and 50 m wide. Three simulations with different heights were used: 100, 500 and 1000 m.
Table 6.4. RMSE for the different simulations with terrain variations. $\beta$ is 15°.

For every simulation, the height has been measured and plotted versus the true height. The RMSE has also been calculated. Table 6.4 shows a summary of the RMSE for the different simulations.

In the case of flat terrain, there is a bias depending on several parameters. The RMSE is higher for rough cover than for small rough, 10.3 m versus 2.9 m. This bias was also discussed in e.g. Section 6.3.2.

In the scenario with a steep slope, the RMSE was larger for small rough cover than for rough cover. Figure 6.8 shows a plot of true ground profile and estimated ground profile for both covers. As can be seen from this figure, the altimeter tracks the ground profile slightly better for rough cover. The large RMSE is probably not due to bias, as the error is smaller for the start and end of the simulation where the ground is flat. Instead, since the error is so large over the slope, one probable
Figure 6.9. Ground profiles, true and estimated, for terrain with small and large variations. $\beta$ is $15^{\circ}$. 
Figure 6.10. True and measured ground profiles for simulation 5, flying over a narrow peak. $\beta$ is $15^\circ$. 
cause is that the sloping terrain causes the specular reflection not to be beneath the aeroplane even when the satellite is right above the aeroplane.

In the other scenarios with small and large terrain variations (number 3 and 4), the ground height was estimated with a RMSE of 2.5 m – 5.8 m. Small rough gave a smaller RMSE in one case; rough gave a smaller RMSE in the other. Looking at the plots of the two simulations, Figure 6.9, it is evident that the ground profile is tracked better for rough terrain, and that the larger RMSE in simulation 3 is due to the bias. The altimeter seems to be more sensitive to terrain variations for small rough cover, which can be seen in Figure 6.9(a).

In the last simulation the aeroplane was flying over a small cube (50x50 m wide, 50 m high), representing a peak. Measurements were done at intervals of 50 m, for 825 m ≤ x ≤ 1175 m. The cube was located at 950 m ≤ x ≤ 1000 m. The true and measured ground profiles are plotted in Figure 6.10. It is important that the altimeter tracks the terrain right, both when the peak is right beneath the antenna and when the peak is inside the antenna coverage but not right beneath the antenna. For the altitude of 100 m, the peak is only inside the antenna coverage when the aeroplane is right above it. For the altitude of 500 m, the peak is inside the antenna coverage for the measurements at 875 m ≤ x ≤ 1125 m. For the altitude of 1000 m, the peak is inside the antenna coverage for all the measurements.

As can be seen from Figure 6.10, the altimeter tracks the terrain reasonably well at the height of 100 m, but does not notice the peak at the other two heights. The ground following and the RMSE are somewhat better for rough terrain at the lowest height.

**Comments**

The estimated height has a RMSE which is due to bias effects and is larger for more rough cover. If this bias could be compensated for the altimeter could probably track the ground profile quite well. The measurements are more sensitive to small terrain variations for small rough cover than for rough cover. This could be because the antenna captures more reflections from the whole covered area for rough cover, while small rough cover only gives reflections from a small area. Finally, a narrow peak within the antenna coverage will only be noticed at small heights, regardless of the ground cover.

**6.3.4 Satellite elevation angle**

In the previous simulations, the satellite had an elevation angle over the ground, ψ, of 90°, i.e. right above the aeroplane. However, in many situations it is not
6.3 Homogeneous surfaces

possible to choose a satellite with such a beneficial elevation angle, even if the GPS system is guaranteed to have at least four satellites over the horizon on a greater part of the earth.

The satellite elevation angle introduces an error in the height measurement when it is not 90°, but this can be compensated for by dividing the measured time delay, \( \Delta t \), by \( \sin(\psi) \), as was described in Section 4.1. This resulted in the following equation:

\[
    h = \frac{c_0 \cdot \Delta t}{2 \sin(\psi)}
\]  

(6.3)

where \( \Delta t \) is the time delay.

Another source of error is that although the correct height is calculated, it is not the height right beneath the aeroplane, but rather the height between the point where the specular reflection occurs and the aeroplane’s z-coordinate. See Figure 6.11 for an example of this. Depending on the application of the measured height, this may have a more or less serious impact.

The simulations performed to examine this parameter were the following:

1. \( \psi \) increasing from 1° to 90° over flat terrain, medium rough cover.
2. \( \psi \) increasing from 1° to 90° over flat terrain, small rough cover.
3. \( \psi \) increasing from 1° to 90° over flat terrain, smooth water cover.
4. \( \psi \) increasing from 1° to 90° over terrain with small variations, (max-min = 2 m), medium rough cover.


Figure 6.11. The height measured at the wrong location due to lower satellite elevation angle.
Figure 6.12. Specular reflection not captured by antenna due to narrow antenna coverage.

5. $\psi$ increasing from $1^\circ$ to $90^\circ$ over terrain with small variations, (max-min = 2 m), rough cover.

6. $\psi$ increasing from $1^\circ$ to $90^\circ$ over terrain with large variations, (max-min = 80 m), medium rough cover.

Different ground covers were chosen to examine if this would have an impact on the height measurement. The reason for $\psi$ to start at $1^\circ$ is that an elevation angle of $0^\circ$ will not give a reflection and is useless for height measurement. For simulation 1-3, the height over ground was 500 m. For simulation 4-6, the height over ground was approx. 900 m.

Furthermore, when $\psi$ decreases the antenna coverage of the aeroplane will be of great importance since the specular reflection may not be captured by the antenna. Figure 6.12 shows an example of this. To examine the impact of the antenna coverage, two sets of simulations were done, one with a narrow antenna coverage ($\beta = 15^\circ$) and one with a wide antenna coverage ($\beta = 63^\circ$). In the case of the narrow antenna, $\psi$ was varied from $1^\circ$ to $90^\circ$. In the second set of simulations (wide) only satellite angles which will give specular reflections to the antenna were used, i.e. $\psi$ was varied from $30^\circ$ to $90^\circ$.

Results

The first set of simulations, with a narrow antenna coverage, produced height measurements with large errors, as expected. These simulations were not examined any further.
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![Graph showing error vs satellite elevation angle]

**Figure 6.13.** Errors for different satellite elevation angles, four types of ground cover. The height is divided by \(\sin(\psi)\). A \(\beta\) of 63° have been used.

The second set of simulations yielded better results. Measurements both with compensating for the elevation angle (dividing by \(\sin(\psi)\)) and not were done. In Table 6.5 results are shown for both cases. As can be seen, the RMSE is much smaller when the measured height is divided by \(\sin(\psi)\), and the larger error for medium rough and rough cover can explained by the bias. In Figure 6.13 the errors for simulation 1,2,3 and 5 are plotted, when the height is divided by \(\sin(\psi)\). The error is larger for smaller angles, especially for more rough cover. This could explain the large RMSE for simulation 5, since the RMSE includes all satellite angles used. The large RMSE in simulation 6 could be explained by the same idea, since the large terrain variations in combination with low satellite angles causes a large measurement error. For smooth cover, there is essentially no error for small angles.

For rough cover, the specular reflection is weaker since all reflection angles give the same reflected power. This can explain the larger error even when the height is divided by \(\sin(\psi)\). It is also worth noticing that the large terrain variations in simulation 6 gave a larger error than the other simulations, despite that medium rough cover and not rough cover was used.

Besides height, also the received correlation function was plotted and the total received power were calculated. Generally, the waveform will be wider for lower \(\psi\) if the cover is rough or medium rough with terrain variations (simulation 4 and 5), otherwise the shape will remain the same. The received power will also be less for more rough cover, while for smooth water the received power will be almost the same, independent of \(\psi\).
### Table 6.5

<table>
<thead>
<tr>
<th>Simulation</th>
<th>RMSE [m] (without $\sin(\psi)$)</th>
<th>RMSE [m] (with $\sin(\psi)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>93.2</td>
<td>20.9</td>
</tr>
<tr>
<td>2</td>
<td>103.2</td>
<td>2.7</td>
</tr>
<tr>
<td>3</td>
<td>104.4</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>155.6</td>
<td>41.9</td>
</tr>
<tr>
<td>5</td>
<td>120.8</td>
<td>103.2</td>
</tr>
<tr>
<td>6</td>
<td>268.1</td>
<td>134.5</td>
</tr>
</tbody>
</table>

RMSE for the different simulations, both with and without $\sin(\psi)$-compensating. A $\beta$ of 63° have been used.

### Comments

The error due to a lower satellite elevation angle ($\psi$) can almost be compensated for by dividing the measured height by $\sin(\psi)$, if the positions of the aeroplane and the satellite are known. There will however always be a larger error for lower satellite elevation angles. It is important that the antenna coverage is wide enough to cover the specular reflection, otherwise the error will be large and cannot be compensated for. For rough cover with little specular reflection, dividing by $\sin(\psi)$ will reduce the error less.

Considering the received waveform and power, lower $\psi$ will give a wider waveform and less power for rough and medium rough cover. For ground covers with more specular reflectivity, on the other hand, the shape of the waveform will remain the same and there will not be a large decrease in power for lower $\psi$.

### 6.3.5 Height of aeroplane

The purpose of these simulations is to determine if the height of the aeroplane over the ground affects the height measurement. Several references (e.g. [13]) report a bias depending on the height of the aeroplane, which is due to the roughness of the surface. As was discovered in Section 6.3.1 it seems like this error increases when the ground’s roughness increases. It is also of interest how well the altimeter tracks the terrain variations at different heights.

Two sets of simulations were done, one to examine the effects of different ground covers when the aeroplane was at different heights over flat terrain, one to examine how well the altimeter could track terrain variations at different heights. A narrow antenna, $\beta = 15^\circ$, has been used, since the satellite has been right above the aeroplane and it would be certain that the specular reflection would be captured.
6.3 Homogeneous surfaces

Figure 6.14. Error for different ground covers with aeroplane rising from 50 m to 9050 m. $\beta$ is 15°.

Ground covers

The aeroplane was set to rise from a height of 50 m to a height of 9050 m in intervals of 200 m, over flat terrain. The following ground covers were used:

1. rough ground
2. medium rough ground
3. small rough ground
4. smooth water

The RMSE of every simulation was calculated and the error was plotted. Figure 6.14 shows the error for the different ground covers. Obviously there seems to be an error depending linearly on the height. The error is also larger when the ground cover gets rougher.

To determine the linear trend in every simulation, the detrend-function in MATLAB was used. Figure 6.15 shows the original error and the detrended error for rough cover. Clearly the linear error has been removed, but since the error was not strictly linear, detrend did not manage to remove the whole error. Table 6.6 shows the RMSE in measured height before and after detrend.
Figure 6.15. Error for rough cover with aeroplane rising from 50 m to 9050 m, without detrend and with detrend. $\beta$ is 15°.

<table>
<thead>
<tr>
<th>Ground cover</th>
<th>RMSE [m] (before detrend)</th>
<th>RMSE [m] (after detrend)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rough</td>
<td>46.6</td>
<td>6.6</td>
</tr>
<tr>
<td>medium rough</td>
<td>39.6</td>
<td>4.8</td>
</tr>
<tr>
<td>small rough</td>
<td>13.4</td>
<td>1.7</td>
</tr>
<tr>
<td>smooth water</td>
<td>1.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 6.6. RMSE for the measured height before and after detrend. $\beta$ is 15°.

Terrain variations

The aeroplane was set to fly over an undulating terrain at heights of 100, 500, 1000, 5000 and 10000 m. The RMSE of every simulation was calculated and the ground profile was plotted. Both rough and small rough cover were used. Figure 6.16 shows a plot of the true and measured ground profile for the simulations for both covers and Table 6.7 shows the RMSE for different heights.

For rough cover, up to heights of 1000 m the altimeter tracks the terrain variations quite well, but at 5000 m the ground profile is not followed very well and at 10000 m the altimeter has lost track completely. For small rough cover the altimeter performs slightly better for the height of 5000 m, but can only be considered to track the ground well up to 1000 m. As can be seen in Table 6.7 the RMSE is much larger for the heights of 5000 and 10000 m.
Figure 6.16. True and measured ground profile for different heights. $\beta$ is 15°.
<table>
<thead>
<tr>
<th>Aeroplane height</th>
<th>RMSE [m] (rough)</th>
<th>RMSE [m] (small rough)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>500</td>
<td>2.7</td>
<td>4.9</td>
</tr>
<tr>
<td>1000</td>
<td>8.4</td>
<td>6.2</td>
</tr>
<tr>
<td>5000</td>
<td>56.3</td>
<td>40.8</td>
</tr>
<tr>
<td>10000</td>
<td>88.9</td>
<td>60.6</td>
</tr>
</tbody>
</table>

Table 6.7. RMSE for the measured height for different aeroplane heights over undulating terrain. $\beta$ is 15°.

Comments

As was expected, there is an error which depends nearly linearly on the height over ground. Using the detrend-function in MATLAB removes most of the error, but not entirely since the error is not strictly linear.

The altimeter manages to track the ground profile quite well on heights up to 1000 m for both rough and small rough cover, but performs much worse for heights of 5000 and 10000 m. For these heights, reflections from a large area are captured in the antenna and will cause an error. The measured height will resemble a mean value of the ground heights within the area giving reflections.

6.3.6 Noise

A discussion on the impact of white noise in the receiver can be found in Section 4.3.2. The noise causes an error in the height measurement depending on the SNR. There are three ways of increasing the SNR and thereby decreasing the error: 1) increase the integration time, 2) increase the signal power or 3) decreasing the noise power in the receiver. It is difficult to influence the noise power in the receiver, so in reality the signal power or the integration time will have to be increased. There is an upper limit on the integration time due to the data bits and the speed of the aeroplane. The signal power depends on several parameters according to the radar equation and the only way for the user to increase the power is to use an antenna with a higher gain.

In these simulations the signal power of reflected signals from the following ground cover types were examined:

1. smooth water
2. small rough wet ground
6.3 Homogeneous surfaces

3. small rough dry ground
4. rough wet ground
5. rough dry ground

Each ground cover type was examined with both flat and undulating terrain. Large terrain variations (max-min = 80 m) were used.

The signal power density at ground level was assumed to be $-135 \text{ dBW/m}^2$ and an integration time of 10 ms is used which gives a noise power of $-181 \text{ dBW}$. Two types of antennas were used; one with a wide coverage, $\beta = 63^\circ$ which gives an antenna gain of 5.6 dBi, and one with a narrow coverage, $\beta = 15^\circ$ which gives an antenna gain of 17.7 dBi. In all simulations the satellite elevation angle was set to $\psi = 90^\circ$. This assured that the specular reflection would always be captured by the antenna.

**Results**

The results from the simulations with a wide antenna can be found in Table 6.8, while the results from the simulations with the narrow antenna can be found in Table 6.9.

<table>
<thead>
<tr>
<th>Cover</th>
<th>Flat terrain</th>
<th>Undulating terrain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR [dB]</td>
<td>$\sigma_r$ [m]</td>
</tr>
<tr>
<td>Smooth water</td>
<td>24.2</td>
<td>13.1</td>
</tr>
<tr>
<td>Small rough wet ground</td>
<td>23.1</td>
<td>14.9</td>
</tr>
<tr>
<td>Small rough dry ground</td>
<td>16.7</td>
<td>31.0</td>
</tr>
<tr>
<td>Rough wet ground</td>
<td>22.5</td>
<td>16.0</td>
</tr>
<tr>
<td>Rough dry ground</td>
<td>16.1</td>
<td>33.2</td>
</tr>
</tbody>
</table>

*Table 6.8. Simulations with wide antenna ($\beta = 63^\circ$): SNR and $\sigma_r$ for different types of ground cover, each cover simulated with flat and undulating terrain.*

As can be seen, the SNR is sufficiently high, well above the 5 dB needed to locate the signal after correlation. On the other hand, the measurement errors due to noise are still quite large for a wide antenna coverage, in the order of 10-30 m. For a narrow antenna coverage the results are better, accuracy of less than 10 m can be achieved in most cases. For rough, on the other hand, SNR is lower and the error is larger for a narrow antenna coverage. This is probably due to the fact that fewer reflections are captured by the antenna and the SNR will be lower, despite the higher antenna gain.
<table>
<thead>
<tr>
<th>Cover</th>
<th>Flat terrain</th>
<th></th>
<th>Undulating terrain</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR [dB]</td>
<td>$\sigma_z$ [m]</td>
<td>SNR [dB]</td>
<td>$\sigma_z$ [m]</td>
</tr>
<tr>
<td>Smooth water</td>
<td>36.2</td>
<td>3.3</td>
<td>22.6</td>
<td>19.7</td>
</tr>
<tr>
<td>Small rough wet ground</td>
<td>35.0</td>
<td>3.8</td>
<td>35.5</td>
<td>3.6</td>
</tr>
<tr>
<td>Small rough dry ground</td>
<td>28.7</td>
<td>7.8</td>
<td>29.2</td>
<td>7.4</td>
</tr>
<tr>
<td>Rough wet ground</td>
<td>21.5</td>
<td>17.9</td>
<td>21.5</td>
<td>17.8</td>
</tr>
<tr>
<td>Rough dry ground</td>
<td>15.1</td>
<td>37.2</td>
<td>15.2</td>
<td>37.0</td>
</tr>
</tbody>
</table>

Table 6.9. Simulations with narrow antenna ($\beta = 15^\circ$): SNR and $\sigma_z$ for different types of ground cover, each cover simulated with flat and undulating terrain.

Comments

For an integration time of 10 ms, the SNR is sufficiently high for finding the correlation peak and performing height measurements. When using a wide antenna coverage, the accuracy is in the range of 10-30 m, which may not be satisfactory in some applications. When using a narrow antenna coverage, the error is reduced to 3-10 m for most covers. This is not true for rough scattering terrain, which will have a lower SNR and therefore a higher error, when using a narrow antenna coverage.

6.4 Heterogeneous surfaces

In the previous simulations only one cover type has been used within the antenna coverage. In these simulations the effect of different cover types within the antenna coverage was investigated. In reality, the ground reflections caught by the antenna is not only reflected from one type of surface. An example of this is when flying from a land surface to a water surface, which gives two types of very different surfaces. The land surface can be rough with poor reflectivity, while the water surface can be smooth and always has good reflectivity. These simulations combine some of the parameters that have investigated one by one in the previous sections.

The scenarios are constructed to have flat terrain with two different covers separated by a sharp line, these are called cover 1 and cover 2. The aeroplane flies at a height of 500 m, over the sharp line separating the two ground covers (see Figure 6.17). Measurements are made every 50 m and the dividing line between the covers is at $x = 1100$ m.

The primary reason for these simulations is to examine how the height measurements are affected by different cover types within the antenna footprint. To analyze this, ground covers that have very different behaviour has been used. First, covers
with only different reflectivity has been examined, then only different roughness and finally both roughness and reflectivity is varied simultaneously.

For each flight, two antenna widths and two satellite positions are used. The satellite positions used have $\psi = 90^\circ$ and $\psi = 45^\circ$ as shown in Figure 6.17. The satellite angle $\psi = 45^\circ$ has been chosen to analyze the effect of reflections from significantly more reflective or specular surface, which are not under the aeroplane. To be able to examine these effects, a narrow and a wide antenna coverage has been used. The narrow antenna coverage uses $\beta = 15^\circ$, and the wide antenna uses $\beta = 60^\circ$. From above it is evident that the specular reflection from the satellite with $\psi = 45^\circ$ is not captured by the narrow antenna. The effect of this is examined in the simulations. The starting point of the aeroplane has been chosen so that the antenna coverage only is over ground cover 1, in the same way, at the aeroplane's end point the antenna only covers ground cover 2. For the wide antenna the first and last 236 m are only over one cover, and for the narrow antenna the corresponding number is 966 m.

A bias error, discussed in Section 6.3.1, is present in all simulations because of the surface roughness. This error is largest for a rough surface and smallest for a smooth surface. In this simulation the bias error is not the main interest, but rather what happens when both cover types are within the antenna coverage. The bias error for cover 1 can be observed at $x = 0$ m, and the bias error for cover 2 at $x = 2200$ m.

### 6.4.1 Different reflectivity

In this simulation the following cover types have been used
cover 1 = medium rough dry ground
cover 2 = medium rough water

This could for example illustrate flying from relatively rough land to a wavy sea surface. The reflectivity for water is much larger than for dry ground. At $\psi = 90^\circ$, water reflects approximately 64%, while dry ground reflects about 11%. Medium rough has been chosen so that reflections from cover 2 is scattered towards the aeroplane when it is still over cover 1.

Wide antenna

The height measurements for the wide antenna are shown in Figure 6.18(a). Starting with a satellite angle of $\psi = 90^\circ$. As always the bias error is present, 16 m for both covers, since they have the same roughness. Putting the bias aside, it is noticed that at approximately $x = 700$ m the measured height increases. This is because the specular reflection, which is straight under the aeroplane, gives a weaker reflection than the non-specular reflections from cover 2. This is due to the better reflectivity at cover 2, and that medium rough scatters the reflected signal widely. The better measurements from approximately $x = 1200$ m is because the specular reflection here is the strongest reflection, since the aeroplane is over the highly reflective water surface.

At a satellite angle of $\psi = 45^\circ$ an improvement around $x = 700$ m is observed. This improvement is because at this point the antenna covers both surfaces. The sought specular reflection originates from cover 2, which has good reflectivity. However, the antenna still captures reflections from cover 1, which has low reflectivity. This effectively means that less delayed signal power is reflected, since cover 1 reflects significantly less power. Therefore, the only delayed reflections that gives noticable power, originates from cover 2. This results in less delayed reflections that disturbs the sought specular reflection.

Narrow antenna

The results for the same scenario as above, but with a narrow antenna, are shown in Figure 6.18(b). The bias error is smaller here because the antenna coverage is narrow. With a satellite angle of $\psi = 90^\circ$, the measured height is near the true height. The reason for the better results is that less reflections from the water is captured when flying over the ground surface. A satellite angle of $\psi = 45^\circ$ on the other hand introduces a large error. This is because the specular reflection is not captured by the narrow antenna, which results in that a longer reflected distance is measured. Therefore, the compensation with $\sin(\psi)$ does not work properly, and introduces an error.
6.4 Heterogeneous surfaces

(a) Wide antenna, $\beta = 60^\circ$.

(b) Narrow antenna, $\beta = 15^\circ$.

Figure 6.18. Different reflectivity. Cover 1 is medium rough dry ground and cover 2 is medium rough water. Satellite angles are $\psi = 90^\circ$ and $\psi = 45^\circ$. 
6.4.2 Different scattering patterns

To examine the effect of different scattering patterns the following covers were chosen.

cover 1 = rough wet ground
cover 2 = small rough wet ground

Wet ground was chosen for both covers to eliminate the effect of reflectivity. This scenario could for example correspond to flying from rough forest to plains.

Wide antenna

Figure 6.19(a) shows the resulting height measurements. For a wide antenna and a satellite angle of $\psi = 90^\circ$ the measured height becomes better as more of the antenna coverage is over small rough. This is due to the fact that rough terrain introduces a large bias, because many reflections with similar power to the specular reflection, are captured from non specular points. For a small rough surface this effect is less. From the plot an improvement is noticed from approximately $x = 400$ m. This is because more and more of the antenna coverage is over cover 2, which at this point gives no reflections. This means that less delayed reflections occur, which decreases the bias error.

For $\psi = 45^\circ$ the results are better over the rough surface. This is because the specular reflection from the small rough surface is captured earlier, and is stronger than the reflections from the rough surface. A small rough surface introduces a smaller bias than a rough surface and the satellite angle is compensated for.

Narrow antenna

The results of using a narrow antenna is shown in Figure 6.19(b). Good results are achieved if the satellite angle is $\psi = 90^\circ$, and only small variations is noticed when the aeroplane is near the dividing line. With a satellite angle of $\psi = 45^\circ$ the results are worse. This is, as was described earlier, because the specular reflection is not captured by the antenna. The sharp peak at approximately $x = 1000$ m appears because at this point the rough surface gives the best reflection, and hence a longer delay is measured. Since cover 2 has a narrow scattering pattern and the satellite angle is $\psi = 45^\circ$, most of the reflected power from the area within the antenna coverage does not reach the antenna.
6.4 Heterogeneous surfaces

(a) Wide antenna, $\beta = 60^\circ$.

(b) Narrow antenna, $\beta = 15^\circ$.

Figure 6.19. Different scattering pattern. Cover 1 is rough wet ground and cover 2 is small rough wet. Satellite angles are $\psi = 90^\circ$ and $\psi = 45^\circ$. 
6.4.3 Different roughness and reflectivity

In this simulation the following covers has been used.

cover 1 = rough dry ground
cover 2 = small rough water

These two surfaces are each others opposites, where cover 1 is a poorly reflective and widely scattering surface, and cover 2 has good reflectivity and a narrow scattering pattern. The corresponding scenario in nature could for example be when flying from a rough land surface to a calm water surface.

Wide antenna

The results of using the wide antenna is shown in Figure 6.20(a). Comparing to the Figure 6.19(a), it is noticed that they have a similar appearance. The rough surface yields a large bias, and as soon as the specular reflection from the small rough surface is captured by the antenna good result is achieved. From this it is understood that the scattering pattern is of more significance than the reflectivity.

Narrow antenna

The plots for the narrow antenna are shown in Figure 6.20(b). This also gives a similar result as in the previous simulation with a narrow antenna (Figure 6.19(b)). This is a more expected result, since using a narrow antenna coverage means that most of the time only one cover type is within the antenna coverage. In this case only the roughness affects the measurement. The peak around $x = 1100$ m is explained as in Section 6.4.2.

6.4.4 Comments

Different reflectivity does not give very different results for either a wide or a narrow antenna. For a narrow antenna the effect is almost non-existing, while it is more noticeable for wide antenna. The scattering pattern proved to be of much more significance. When the antenna covered both a rough and a smooth surface simultaneously large changes were noticed. This is clearly shown in the plots. A narrow antenna proved to give a better result here as well, but this assumes that the specular point is within the antenna coverage. The results for a wide antenna were more varying. When the wide antenna covered both cover types, the effect on the measurement was considerable.
(a) Wide antenna, $\beta = 60^\circ$.

(b) Narrow antenna, $\beta = 15^\circ$.

Figure 6.20. Different reflectivity and scattering pattern. Cover 1 is rough dry ground and cover 2 is small rough water. Satellite angles are $\psi = 90^\circ$ and $\psi = 45^\circ$. 
Figure 6.21. Flight path for the long simulation.

Using different reflectivity and scattering pattern for cover 1 and cover 2 showed that the effect of different scattering patterns was the dominating factor. Finally, the specular point was found to be very important to have within the antenna coverage in order to achieve a good result. This is discussed in more detail in section 6.3.4.

The effect of noise has not been accounted for in these simulations, and could affect the results, especially for surfaces with poor reflectivity.

6.5 Long simulation

Besides the previous simulations, a longer simulation was performed where the aeroplane was set to fly a real flight path over the south-east of Sweden (see Figure 6.21). The ground elevation database and the ground cover databases described in Section 5.2.3 were used. A narrow antenna coverage ($\beta = 15^\circ$) was used and the satellite was positioned approx. in zenith ($\psi \approx 90^\circ$).

6.5.1 Results

The overall RMSE of the simulation was 4.6 m, which is considerably good, especially since the bias has not been compensated for. In Figure 6.22(a) the true
and measured height are plotted. Due to the large scale on the vertical axis and the relatively small error, the measured height appears to be identical to the true height. When the error is plotted, in Figure 6.22(b), the differences can be seen more clearly. During the first time of the flight, the error is low, mainly because a narrow antenna coverage is used and the aeroplane remains at a low height, approx. 100 m above the ground. As the aeroplane increases the height at the end of the flight, the error increases. Some linear trend can be noticed, which coincide with the previous results about the bias. The variations in the error are probably due to the variations of the ground elevation and the ground cover. It would have been interesting to examine the distribution of different types of ground cover during the flight, but unfortunately there was not time enough to do this.
Figure 6.22. Plot of true and measured height and the error in the measurements for the longer simulation.
Chapter 7

Discussion

In this chapter an evaluation of the simulation model and the performed simulations is made.

7.1 The simulation model

In the simulation model several simplifications have been made, due to the complex nature of the problem or because of lack of time. One problem that proved to be complex was simulating the effects of fading. Therefore, a baseband model was implemented and the effects of fading due to the carrier was left out of the simulation. Only the ISI between different reflections was implemented. A simple square antenna coverage was also used, and although a circular form would have been more realistic, this simplification does not introduce large errors.

Another complex and important part of the simulation was the reflection at the ground. The lack of measurements of the scattering coefficient for bistatic radar was solved by creating an analytical model of this. The scattering coefficient was modeled to consist of two parts, reflectivity and scattering pattern. The reflectivity was a moderate problem and the theories from Chapter 3 could be used, except for one approximation; the incident angle was assumed to be the same over the entire ground plane. In reality each ground plane consists of small slopes depending on the roughness. The incident angle should therefore vary over the ground plane. This variation has been assumed to result in small changes in the reflectivity, and has been neglected (see Section 5.3.1). The scattering pattern for different ground types was more complex to model. Four surfaces with different roughness were created. These covered a large spectrum, from totally diffuse to almost specular. The
surfaces were not designed to correspond to actual surfaces in nature, but rather to cover the whole spectrum of roughness. However, there are several similarities, for example, the smooth surface corresponds well to a calm water surface. A simplification in the scattering model is also that the surfaces were modeled to have the same roughness independent of the angle to the satellite. In reality a surface becomes smoother with larger incident angle (see Eq. (3.30) on page 31).

Another simplification was made by neglecting the interference that other GPS satellites introduces because of the cross correlation between codes. This effect also gives small errors. The GPS signal was assumed to consist only of the PRN code, i.e. the 50 Hz data signal was excluded. Since the PRN code repeats itself 20 times over a data bit, errors occur if the data bit changes. The effect of this has not been examined thoroughly in this thesis, but it is likely that the PRN code that is affected by changing a data bit can be foreseen and be discarded for the height measurement. Of more significance are the limitations in describing the simulated world, i.e. the GPS signal and the ground planes. The GPS signal was implemented with 100 samples/chip, which gives an uncertainty of 1.5 m. The number of samples has been chosen to make the simulation model reasonably fast, and due to a bug in the compiler used. The number of planes used to model the ground was based on the total received power. The number of planes was considered sufficient when received power no longer was influenced significantly. At approximately 20000 planes the received power was almost the same as for 40000 planes, therefore 20000 ground planes were used. Due to implementation issues, the number of planes closest to 20000 that could be chosen was 19602, which was the number used in reality.

To validate the simulation model, two factors were examined: the shape of the received waveform and the received power. If these would correspond reasonable well to the available data the simulation model can be considered to be sufficiently accurate.

Regarding the waveform, there are not much experimental data to compare with, especially not for ground reflections. Some conclusions can be drawn from comparing the simulated waveform with the ones shown in [13]. When inspected visually, the appearances seem to correspond quite well. A rougher surface, which in [13] is represented by higher wind speeds, produce a wider waveform and the typical “tail” at the end. This behaviour was noticed in several simulations done with our simulation model, see e.g. Figure 6.7(a) on page 79.

Further investigations of the shape of waveform can be done by e.g. implementing the model for bistatic scattering used in [18]. Due to the complexity of this model and lack of time, this could not be done in this thesis. If the simulation model turns out to be insufficient, the simplified scattering pattern could be modeled with the more complex model. Also, experiments could be performed where data is collected to analyze returned waveforms from different ground covers.
7.2 Performed simulations

It is difficult to validate the received power as well. This can be done for smooth water by using the theories described in Section 3.7.3 on page 31, where the reflection was thought of as originating from the transmitter’s mirror reflection. Calculating the power density analytically for an aeroplane at height 100 m over the sea surface and the satellite in zenith, Eq. (3.33) can be used by adding atmospheric losses and antenna gain to:

\[
S \approx \frac{|R|^2 P_t G_t L_A}{4\pi R^2} \tag{7.1}
\]

where \( P_t \) is transmitting power, 27 W, \( G_t \) the transmitting antenna gain, 10.2 dBi in zenith, and \( L_A \) the losses due to the atmosphere, approx. 2 dB. These numbers are from [2]. The reflectivity, \(|R|^2\), is set to 0.64, which is typical for fresh water. This gives a result of \(-136.5\) dBW/m². The simulation model gives a result of \(-137\) dBW/m², which is close to the analytical value. The difference in 0.5 dBW/m² is due to the fact that the simulation model uses a lower value on the power density at the ground, \(-135\) dBW/m² in opposite to \(-134.5\) dBW/m² which would be the correct analytical value.

For the other covers, it is hard to make any conclusions since there is little experimental data and an analysis would be too complicated. Considering a perfect specular reflection on ground, with an angle of incidence of 0°, the received power density should be in the order of 10 % for dry ground up to 50 % for wet ground, compared to a direct signal. When the surface is not smooth but more rough, these numbers change since more power is reflected in other directions, while the antenna receives more reflections at the same time. Simulations showed the received power density to be from 2 % up to 45% of a direct signal, which are reasonable values.

One important, but missing factor is the effect of flat fading, which has not been implemented. This would reduce the mean received power density and the values produced by this simulation model should be seen as maximum values.

7.2 Performed simulations

The following parameters affects the result of the height measurement.

- Height measurement method
- Antenna coverage
- Ground cover
- Terrain variations
- Satellite angle
• Height of the aeroplane
• Noise in the receiver
• Speed of the aeroplane

The antenna coverage is the only parameter that can be manipulated by the user. Of primary importance is that the antenna covers the specular reflection. Without the specular reflection, the \( \sin(\psi) \)-term which compensates for the satellites elevation angle, will cause errors. A narrow coverage was found to give better results, since the antenna gain is higher and therefore a higher peak for the correlation function is achieved. The narrow antenna works particularly well for surfaces with a narrow scattering pattern, while the rough surface gave slightly less power compared to a wide antenna. Using a narrow antenna and having the specular point within the antenna coverage means that the satellite position must be favourable, with an elevation angle near \( \psi = 90^\circ \).

The ground cover affects the height measurement very differently depending primarily on the roughness. The scattering pattern for different covers gives a bias error. This is because reflections originating from points far from the specular point are delayed relative to the sought specular reflection. This widens the correlation function and the time delay for the peak increases. The bias error is largest for a rough surface and smallest for a smooth surface, while medium rough and small rough have a bias error in between these two. By using a narrow antenna coverage the bias error due to the surface roughness is decreased. The narrow antenna coverage will capture less reflected signals far from the specular point, which yields a better result. The bias error is not a constant error, and increases with height. A linear dependency between the height and the bias was noticed, and is clearly shown in Figure 6.14 on page 89. This error is introduced because a larger height increases the area that gives significant reflections. This effect is largest for rough cover and smallest for smooth cover. The rough surface always gives reflections from the entire antenna coverage, and the antenna coverage becomes larger with increasing height. For a smooth surface the reflections still only originates from a limited area of the antenna coverage, but this area is enlarged with increasing height. Since the area causing reflections does not increase as much for a smooth surface, better results are achieved. The simulations with increasing height was only done using flat terrain, but a similar behaviour is likely for varying terrain.

The simulations performed with different ground covers within the antenna coverage showed that varying the scattering pattern resulted in larger changes than varying the reflectivity. The simulations also showed that using a wide antenna \( (\beta = 60^\circ) \) could result in errors much larger compared to the bias error due to the roughness. Figure 6.19(a) on page 99 with \( \psi = 90^\circ \) shows a clear example of when large errors occur. On the other hand, using a narrow antenna \( (\beta = 15^\circ) \) proved more successful. Observing Figure 6.19(b) on page 99 and \( \psi = 90^\circ \), where
7.2 Performed simulations

A narrow antenna has been used, it is evident that the dividing line between the two cover types introduces no significant error. In the simulation where both the scattering pattern and the reflectivity were varied, a small error could be noticed near the dividing line, but this error is negligible. A narrow antenna has therefore been proven to be the best choice here as well.

How well the altimeter works depending on terrain variations is also of major interest. Two aspects of this are: 1) how well the terrain is tracked for different cover types and 2) the effect of increasing the height of the aeroplane. The satellite elevation angle is also of major significance when the ground varies. With a rough surface it was found that the terrain was followed more smoothly than for small rough, which did not notice changes in the terrain equally well. This is clearly shown in Figure 6.9(a) on page 82. Rough surfaces yield relatively strong reflections from the entire antenna coverage, and the height measurement is therefore affected by the terrain within the whole coverage. This results in that the height measurement will resemble a mean value of the ground. Small rough surfaces on the other hand only give significant reflections for near specular directions, which therefore only originate from a limited area. For flat terrain this is good, and the limited area with reflections yields a smaller bias. For varying terrain this can result in that strong near-specular reflections, from points far from the point beneath the aeroplane, are captured by the antenna. An example of this is given in Figure 6.8 on page 81, where flying over a steep slope was simulated. A way to decrease the number of unwanted specular reflections, which are not beneath the aeroplane, is to use a narrow antenna coverage.

When increasing the height of the aeroplane it was found that the terrain could be followed reasonable well up to approximately 1000 m. This applied for both rough and small rough cover. Considering the satellite elevation angle, large errors can occur, for example if $\psi = 45^\circ$ and the specular reflection originates from a point which is situated at a larger altitude than the point beneath the aeroplane (see Figure 6.11, page 85). In this case the height above the specular point is measured, and not the sought point beneath the aeroplane.

The presence of white noise in the receiver requires a sufficiently high SNR after correlation, otherwise the error in the height measurement will be too high. To begin with, the receiver needs a SNR of at least 5 dB after correlation, to be able to find the correlation peak. By doing simulations over various types of ground cover it was found that this is rarely a problem, if an integration time of at least 10 ms is used.

The SNR depends on three factors: the signal power, the integration time and the noise power. The noise cannot be influenced by the user and if the SNR is not high enough for the desired accuracy, the user has two alternatives: increase the signal power or increase the integration time. The signal power can be increased by either using a more narrow antenna coverage which results in higher antenna
gain, or by choosing ground covers which produces strong reflections. Since it is not always possible to choose the ground cover, the main design parameter is the antenna. Increasing the integration time will also increase the SNR, but there are limits to this. If the integration time is more than 10 ms, there is a possibility that the integration will be performed over a data bit transition and the result will be corrupted. Experimental results by Akos et al [14] show that this can be avoided if the transitions are known, and long integration times can be achieved. The speed of the aeroplane then becomes the most critical factor. If the aeroplane moves too fast, the ground over which the height is measured will change and the true height will not be measured. In the simulations, the effect of noise has been calculated separately and only one period of the PRN code has been used to make measurements.

Simulations with a wide antenna coverage showed that height measurements with errors in the range of 3-30 m can be achieved with an integration time of 10 ms. The error depends mostly on the width of the antenna coverage and the reflectivity of the ground cover. The roughness can also affect the received power, especially for a narrow antenna coverage.

There is a possibility to reduce the measurement error due to receiver noise without increasing the SNR. The peak tracking algorithm has been used to calculate the height from the correlation function. Several methods were described in Section 4.2, but the peak tracking algorithm was the easiest to implement. The other methods are more complex and will probably give a more accurate measurement. Experimental results by Masters [13] showed that there were large differences between the six tracking methods they used.

In this simulation model the impact of fading has not been included. A theoretical discussion in Section 3.8, suggests that the received signal amplitude will be Rayleigh distributed, and that the mean signal power will therefore be lower than the results from these simulations. This means that the resulting errors are optimistic estimates. Results from [13] show that height measurements could be produced at a rate of 1 Hz. An integration time of 100 ms was used to produce one waveform, and 10 waveforms had to be averaged to deal with the fading effects. When these experiments were performed, a hemispherical antenna was used. If a more narrow antenna coverage is used, the SNR would be better and height measurements could probably be performed at a higher rate.
Chapter 8

Concluding remarks

A simulation model for receiving and correlating GPS signals, reflected at the ground, has been developed and implemented. Besides this, a simple algorithm for estimating the height from the received correlation function has been implemented. Using the model several simulations have been performed, in order to characterize the height measurement method.

8.1 Conclusions

The simulation model has given reasonable results, although several simplifications have been done. The shape of the simulated waveform and the magnitude of the received power coincide with experimental results according to many other references. There is an error due to implementation issues, but the maximum size of this error is known and depends on an implementation parameter.

The many simulations performed have given an introducing survey of the height measurement method. The method estimates the height with e.g. an RMSE of 0-100 m at a height of 1000 m. The RMSE is dependent on several parameters, like the width of the antenna coverage and the roughness of the ground. Simulations also show that it is often possible to find the correlation peak in the presence of noise, even for weak signals. The most noticeable error is the bias, which increases with the height, width of antenna coverage and roughness of the ground. The bias is nearly linearly dependent of the height, and it should be possible to reduce this error if this dependency is examined further. There is also a measurement error due to the thermal noise in the receiver. The RMSE of this error can be estimated from the SNR and the integration time.
Another factor is the elevation angle of the satellite used to make the measurement. Over water surfaces, the error due to the angle is easy to model, but on rough land surfaces, it can result in larger errors which are more difficult to compensate for. Also, large terrain variations can cause errors since the height is not measured right beneath the aeroplane for low satellite angles.

The main design parameter is the antenna, which has to cover an area large enough to include the specular reflection. It also has to be wide enough to encounter for movements of the aeroplane, e.g. changes in attitude. On the other hand, the more narrow the antenna coverage is, the more amplified the signal will be, and the bias will be less since less unwanted reflections are captured.

Besides this, there are two main limitations to the aeroplane: the speed and the height. If the aeroplane travels too fast, the integration time cannot exceed a certain time, since the ground will change. Furthermore, simulations have shown that the altimeter does not follow the terrain too well at greater heights, since the covered area will be too large.

### 8.2 Further work

Since research is comparatively new in this area, there are many subjects for further work. Besides the research, also improvements of the simulation model are discussed.

- **The antenna.**
  The antenna was proven to be of great significance to achieve good results. Having the specular point within the coverage was most important. Using a steerable antenna with a narrow coverage would be beneficial and would result in high antenna gain and few unwanted reflections. This could however prove to be too complex for use in practice. A more simple approach is to change the width of the antenna coverage. In this way the width could be chosen to be just wide enough to cover the specular direction.

- **Making use of several satellites.**
  Since the GPS system is a multiple access system, using more than one satellite would be a minor problem. A separate height measurement for each satellite could be done, and then a decision depending on how reliable each measurement is could be made. For example, satellites close to zenith are generally thought of as more reliable. A simple way of using many satellites would be to calculate the mean value of the height measurements, but a more complex algorithm that takes the satellites reliability into account would be better.
8.2 Further work

- **Modeling the bias error.**
  If this error could be modeled in an accurate way, the RMSE of the height measurements could be decreased significantly, even at heights greater than 1000 m. When modeling the bias error, both the height of the aeroplane and the type of ground cover within the antenna coverage have to be considered.

- **Using another height measurement method.**
  The peak tracking algorithm used in this thesis could be replaced with another, more precise algorithm. There are suggestions of other methods, that could reduce the error due to receiver noise. These are more complex to implement and could require a greater amount of calculation time.

- **Low power acquisition.**
  There are many studies on the subject of acquiring low power GPS signals. The SNR has turned out to be significant for the measurement error, and since the signals reflected from the ground are weaker than the ones reflected from water, it would be interesting to investigate if any of these algorithms could improve the result.

- **Improving the simulation model.**
  The scattering pattern has been modeled schematically and developing this is possible. Two possibilities exists: 1) developing the analytical expressions or 2) making use of experimental data of forward scattering.

  The antenna pattern used in the model was simplified, both in its square shape and in the constant antenna gain. A more realistic model could reasonably be implemented in the model.

  Increasing the resolution in the GPS signal would give more accurate simulations. The limitations for using a higher resolution were the increasing simulation time, and also the problem of the bug in the compiler, when using a too high resolution.

  Improvement of modeling the terrain variations is possible. The database for describing the ground height is only given with a resolution of 50 m. In between these points linear interpolation is used. By using a more sophisticated interpolation method a more realistic modeling of the terrain would be achieved.

  Approximating the auto correlation function, \(|r(\tau)|\), by using the triangular \( \Lambda \)-function would result in faster simulations. Since the \( \Lambda \)-function approximates \(|r(\tau)|\) with only the characteristic triangle form, fewer samples describing the input signal would be needed. The \( \Lambda \)-function is often used as a simplification in analytical models, e.g. in [5]. It is also discussed in Section 3.5.
Appendix A

Validation of the modeled scattering patterns

The scattering pattern, denoted $Q_r$, is in the simulation model described by the following expression

$$Q(\alpha) = \begin{cases} 2(n + 1) \cos^n(\alpha) & , 0 \leq \alpha \leq 90 \\ 0 & , \text{otherwise} \end{cases} \quad (A.1)$$

where $\alpha$ is the angle relative to the specular direction, and $n$ is the parameter which characterizes different scattering patterns. The factor $(n + 1)$ in the expression normalizes the power density to the total power. Below a validation is made, which shows that the power density corresponds to the total reflected power.

If the total reflected power is $P$ and $R$ is the distance from the scattering point, the power density, $S$, is given by

$$S = \frac{PQ(\alpha)}{4\pi R^2} \quad (A.2)$$

Since all the reflected power radiates through the upper hemisphere the following must apply

$$P = \int_0^{2\pi} \int_0^{\pi/2} S \sin(\alpha) R^2 d\alpha d\phi = \int_0^{2\pi} \int_0^{\pi/2} \frac{PQ(\alpha) \sin(\alpha)}{4\pi} d\alpha d\phi \quad (A.3)$$

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where $\alpha$ and $\phi$ are spherical coordinates. Using the expression for $Q$ above achieves the criterion in the eq. (A.3).

The integration over the upper hemisphere actually requires that specular direction is vertical, since the integration over $\alpha$ is from 0 to $\pi/2$. It is a satisfactory approximation for other specular directions as well.
Bibliography


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