Lateral Position Detection Using a Vehicle-Mounted Camera

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Elisabeth Ågren

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Elisabeth Ågren
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Handledare: Gunnar Farnebäck
Examinator: Maria Magnusson Seger
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Avdelning, Institution
Division, Department
Institutionen för Systemteknik
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Författare
Author
Elisabeth Ågren

Sammanfattning
Abstract
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The problem of using computer vision to measure lateral position can be divided into road marking detection and lateral position extraction. Since the strongest characteristic of a road marking image are the edges of the road markings, the road marking detection step is based on edge detection. For the detection of the straight edge lines a Hough based method was chosen. Due to peak spreading in Hough space, the difficulty of detecting the correct peak in Hough space was encountered. A flexible Hough peak detection algorithm was developed based on an adaptive window that takes peak spreading into account. The road marking candidate found by the system is verified before the lateral position data is generated. A good performance of the road marking tracking algorithm was obtained by exploiting temporal correlation to update a search region within the image. A camera calibration made the extraction of real-world lateral position information and yaw angle data possible.

This vision-based method proved to be very accurate. The standard deviation of the error in the position detection is 0.012 m within an operating range of ±2 m from the image centre. During continuous road markings the rate of valid data is on average 96 %, whereas it drops to around 56 % for sections with intermittent road markings. The system performs well during lane change manoeuvres, which is an indication that the system tracks the correct road marking. This prototype system is a robust and automatic measurement system, which will benefit VTI in its many driving behaviour research programs.

Nyckelord
Keywords
Lateral position, road markings, vehicle, TLC, road marking detection, extraction of lateral position, Hough, peak detection, camera calibration.
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1 Introduction

1.1 Background and Motivation

Drivers’ lack of attention or driver impairment is the underlying cause of many accidents, especially in lane departure accidents, which represent around a third of all fatal accidents in Sweden every year [1]. A greater knowledge of road user behaviour can contribute to a safer traffic environment. The Swedish National Road and Transport Research Institute (VTI) regularly contribute with new facts about traffic safety and road user behaviour, based on experimental studies. In many driving behaviour and crash avoidance research programs at VTI, the problem of measuring the position of a vehicle on the road while driving is faced. While there are numerous ways to measure the longitudinal distance the vehicle has driven, the lateral position of the vehicle has been proven to be more difficult to measure. The lateral position refers to the sideways position in relation to the road markings or to another predefined part of the road.

Research based on experimental studies requires robust measurements and accurate measurement systems. In the past, the lateral position of a vehicle has been measured manually by an observer from calibrated video sequences. This method is very time-consuming and the measures are subject to the observer [2].

An early lateral positioning system developed by VTI combined a line scan video camera with electronic components to perform analogue signal processing. The raw video signal was analysed to detect a section in the video signal with a signature corresponding to that of a painted road marking, i.e. a step increase followed by a step decrease in intensity. The processing was performed in real-time and the system detected the position of the road markings with high accuracy. The major drawback of this system was the tendency to find false road markings in the image [3].
Chapter 1 – Introduction

Given the importance of lateral position data together with the lack of a suitable commercial system, the development of an automatic lateral position measurement system is of great interest to VTI. An automatic system would benefit VTI in its many research programs since it would be custom-made to conform to the high requirements on accuracy and would eliminate the drawbacks of manual measuring.

A longer-term goal of this project is an automatic system for detection of lateral position of a vehicle in real-time while driving. There are a number of possible extensions to such a real-time application, the most interesting one being a lane departure warning system.

1.2 Problem Definition

The intention of this study is to develop and implement a prototype system for detection of the lateral position of a vehicle in relation to a reference, specified here to be the centre road marking. The purpose was also to examine the potential and accuracy of a vision-based approach to lateral positioning.

1.3 System Specifications and Requirements

There are a number of requirements on the prototype system. The system should detect the centre road marking. The prototype system must work independently of the vehicle’s position on the road and there must not be any restrictions on the vehicle’s sideway position. The measurement of lateral position must be made with consistency. The distance should be measured perpendicularly from the front left wheel to the centre road marking. The accuracy of the output data should be ±1 cm. Data should ideally be generated with 25 Hz and the goal is to achieve at least 80-90 % valid data.

1.4 Delimitations

The prototype system should be able to determine the lateral position of a vehicle under the following conditions:

- Structured roads with painted centre road markings, such as country roads and motorways. However, the system is not required to produce data in city traffic, complex road scenery, roundabouts, intersections, motorway exits etc.

- Highly visible road markings with high contrast between road and road markings. Unstructured roads, i.e. gravel roads, without road markings will not be taken into consideration here.

- Good weather conditions, such as daylight and clear or cloudy weather. There are no requirements on the system to process data, which has been collected under poor visibility conditions such as heavy snowing, dense fog, rain or during night-time driving.

- There are no real-time considerations, so the collected data can be processed off-line.
1.5 Method

The proposed method is a vision-based approach, where a camera is mounted on a moving vehicle. The images will be analyzed by means of computer vision and the extraction of real-world data from the images will be made possible by a calibration of the camera.

This approach gives rise to an investigation of a variety of vision-based methods for measuring lateral position. Further, it requires a study of computer vision methods for road marking detection in images and an examination of suitable methods for transformation from image pixels to real-world data. These findings will, together with the requirements on the system and the surroundings restrictions, form a foundation for the development of the prototype system, which will be implemented in MATLAB.

The development of the prototype system will include the choice of a camera position. In the design of the road marking detection method, the presence of shadows and different types of road marking characteristics must be accounted for. Geometrical simplifications will make the extraction of real-world data possible.

1.6 System Overview

The images from the image acquisition device are digitized and fed into the computer vision software where the road markings are detected, see Figure 1.1. The location of the detected road markings in the image is then used in the lateral position extraction part where a mapping to real-world coordinates produces a real-world measurement of the position. The output, in ASCII format, from the system should be raw lateral position data in centimetres, the vehicle deviation angle and the image frame number. The latter is necessary for synchronization with the vehicle longitudinal position.

![Figure 1.1 Program flow.](image)

1.7 Outline of the Thesis

The Introduction chapter covers the background and motivation to the problem as well as the system requirements and a proposed system overview. The outline of the rest of the thesis is as follows. In Background and Previous Work an overview of existing technology and previously developed vision-based algorithms to lateral positioning are presented to the reader. A choice of methods for road marking detection and lateral position extraction are examined. A number of applications of lateral position data are also mentioned to motivate the use of a lateral position measurement. The chapter System Design Approach establishes the main ideas of the approach taken here as well as a frame of conditions and potential problems to which the system will be adapted. A description of the structure of the implemented prototype with details of the camera device, camera calibration, and an outline of the method chosen for road marking detection and lateral position extraction is also found here. The chapters Road Marking Detection and System Calibration and Data Extraction...
give detailed descriptions of the functionality of the implemented software system and the camera calibration. In **Experiments and Results** the results from a validation process of the performance of the system is presented. A discussion and conclusions about the performance and limitations of the developed system is found in **Discussion and Conclusions** at the end.
This chapter starts off with a definition of the lateral position. Previously developed vision-based systems are discussed and a background with basic theory and algorithms both for road marking detection and lateral position extraction is presented to the reader.

2.1 Lateral Positioning

Lateral position is defined to be the perpendicular distance from the left front wheel to the road marking on its side, see Figure 2.1. The road marking is used as a reference, since it is the only available fixed feature on the road with well-defined edges. To maintain consistency in data, this distance must always be measured from the same predefined point. Lateral position data is positive when the vehicle is inside the lane, but when the front left wheel crosses the centre road marking the position becomes negative. This can be the case in a curve, where the vehicle takes a shortcut or when it is overtaking another vehicle.

![Figure 2.1 Definition of lateral position](image)
Chapter 2 – Background and Previous Work

2.1.1 Time-to-Line Crossing (TLC) and other Applications

A continuous measurement of lateral position of a vehicle is of interest in many driver behaviour research studies where focus lies on analyzing irregular lane positioning behaviour or position of the vehicle in lane. The position of the vehicle in relation to the road markings is also essential in a number of safe driving applications in the area of Intelligent Transport Systems (ITS). In driver assistance systems the lane position information can be used to support driving by helping drivers maintain lane position. Such systems include lane keeping, steering control systems and driver condition monitoring, for example drowsy driving systems. AssistWare Technology has developed a commercial driver condition monitoring system called SafeTRAC [29]. A driver condition monitoring system for research was developed within the SAVE program. This multi sensor system uses eye motion sensors together with vehicle position and other vehicle data to determine the driver’s state [4].

Time-to-Line Crossing (TLC) is a measurement of the time reserve left until any part of the vehicle crosses one of the road boundaries providing that speed and steering angle are kept constant [5]. TLC is considered to be an important measure of driver performance. An accurate calculation of TLC is based on a trigonometric model that takes curvature, vehicle deviation angle etc into account. Achieving data of all these types in real-world driving is often a problem so approximations are used instead. The simplest approximation, called the first approximation, is calculated from measurements of vehicle lateral speed and lateral position data and is suitable for field studies and in lane departure warning systems. A more commonly used approximation, called the second approximation, also uses lateral acceleration, and is more appropriate for simulator studies where data is more reliable [6]. The accuracy of the lateral position data is crucial for the validity of TLC calculations. TLC is used in a number of research areas including driver behaviour and driver drowsiness research and has successfully been exploited for lane departure detection in warning systems ([10], [14]). Commercial lane departure detection systems developed by Mitsubishi, Mazda and Fuji are available on the market.

In addition to these applications, the research system RALPH, developed by The Robotics Institute at the Carnegie Mellon University, is a lane keeping and autonomous driving system which exploits lateral position information for steering control [7].

2.2 Vision-based Approach

Over the years a number of measuring techniques for lateral position have been developed and these can be divided into infrastructure-based and vehicle-based techniques. One typical infrastructure-based technique is to use magnetic markers embedded in the lane together with sensors on the vehicle which can determine the distance to the marker in the lane. This approach has been proven to be extremely accurate and the electromagnetic system has the advantage of being unaffected by weather conditions. Therefore, many guidance systems for snowploughing, where the viewing conditions are usually very bad, exploit this kind of technology [8]. However, the drawback of infrastructure-based positioning methods is expensive installation and since modifications of the road are required these systems can only be used on stretches of the road that have been previously modified [8].

To eliminate the need for modifications of the road, a vehicle-based technique is to prefer. Vehicle-based systems typically exploit a passive sensor, such as a camera, which has the advantage of using existing infrastructure and can acquire data in a non-invasive way [16].
Passive sensors are to prefer over active sensors such as radar or satellite positioning systems since the latter have a tendency to be blocked out by trees and high buildings or interfere with other sensors of the same type [9]. A camera mounted on a vehicle can acquire visual information about the road and the desired information can be extracted from the acquired images by means of computer vision, in real-time or off-line. Available references in the environment, such as road markings localized within the image, can be used for estimating the vehicle’s lateral position [9].

A great deal of research has been performed in the area of ITS where many applications use forward looking cameras to acquire lane position information. In many lane detection applications focus lies on extracting vision information for steering control, where obstacle detection, such as discovering other vehicles, humans or animals on the road, and curvature estimation of the road ahead are two important factors [9]. The lookahead distance is far, up to 80 meters in front of the vehicle, and the vanishing point is included in the image. The requirements on those applications are quite different from the ones placed on the prototype system developed here. In particular, the real-time aspect of those systems contributes to different requirements and speed-considerations, but the approach taken for lateral position measuring is also different. In lane detection systems the lateral position of the vehicle is often only estimated based on lane region estimation in the image.

In lane departure warning systems, however, focus lies on measuring the distance from the vehicle to the road markings [10]. This is essentially similar to the problem faced in this thesis, where the distance to the road markings closest to the vehicle is more important to determine than the curvature of the road. There are a number of commercial lane departure detection systems available on the market. In many cases, the only output from these systems is the warning signal. In some systems steering control is activated if the driver is about to leave the lane way. These systems however are not suitable for use in research studies since the user usually has no insight into the workings of the system. The main advantage of the system developed in this project is that it gives the user full insight into the system.

2.3 Image Acquisition

2.3.1 Camera

The low-cost and widespread use of video cameras make them very suitable for this type of application. Another type of camera worth mentioning is the infrared (IR) camera, which is sensitive to heat and hence suitable in driver assistance systems, such as night vision applications, because animals and humans are easily detected in the image. However, since the road and road markings emit the same heat, there is no contrast between the road and the road markings in images obtained from IR-cameras [11].

Using a single camera is sufficient for lateral position extraction, although a stereo-vision approach would increase both the performance and the computational complexity of the system [9].

2.3.2 Camera Mounting

There are a number of possible positions where a camera can be mounted on a vehicle and the choice of placement is primarily determined by the requirements of the system. In many safe driving applications and in experimental studies with test drivers, it is important that the
Chapter 2 – Background and Previous Work

The camera is placed where it does not interfere with the driver [12]. An unobtrusive placement could for example be behind the windshield on the rear-view mirror, integrated into the side-view mirrors or on the roof of the vehicle.

2.3.3 Field of View

The field of view (FOV) is the area seen by the camera. According to Gangyi [13], the road markings closest to the vehicle are of greatest interest for lateral positioning of a vehicle. To achieve high accuracy in lateral position data and TLC calculations the focus in the FOV should hence be on the road marking closest to the vehicle. This is obtained by tilting the camera forward.

In AURORA, a lane departure warning system, a wide angle camera has been mounted on the side of the vehicle, facing the ground, to capture a rectangular area just beside the vehicle [14]. The lateral offset of the vehicle can be determined in this way, and a warning is generated if the lane marking is crossed. This is in line with the lane departure and drowsy driver detection system developed by Ziegler et al. [10] where the camera is mounted on the windshield. The authors argue that, when focus lies on measuring lateral position, a very close look ahead is sufficient and the curvature of the road can be ignored. In addition, a number of problems, related to camera perspectives which include the horizon, can be avoided with a tilted camera mounting.

2.4 Road Marking Detection

Detecting white road markings in an image might at first seem to be a trivial task, however a number of problems related to the visibility and potential absence of the road markings need to be faced. A number of assumptions and simplifications can be made which eases the road marking detection further.

2.4.1 General Problems

Shadows projected by trees, buildings, bridges or other vehicles can change information in the image quickly and a number of lighting conditions (glare or reflections off a wet road) can also cause problems. Redundant information in the image can easily be mistaken for road markings, so robustness to shadow and lighting conditions need to be built into the system [9]. Low visibility of the road markings due to age, damage or dirt retention can also make the detection harder [14].

In addition, there is no guarantee that road markings will always be present in the field of view of the camera. This can occur when the vehicle passes intersections where road marking configuration changes or on sections with intermittent road markings. If the road marking is lost from the field of view, it is essential that the system finds the road marking again when it appears in the image.

2.4.2 Typical Assumptions

Processing the whole image is often not necessary and the system can be sped up by processing only a region of interest (ROI), which is a small area within the image assumed to contain relevant information [15]. The interesting region in the image can to some length be predicted and by narrowing the search area to this region computational time is saved. However, if the wrong ROI is chosen, the system must have a flexibility which enables the
search to continue. The search area can be based on results from previously processed frames or assumptions about a priori knowledge about the road [9].

Assumptions about the road geometry and the shape of the road will limit the search for road markings in the image. The assumption of a straight road is common.

2.5 Algorithms for Road Marking Detection

A selection of algorithms that could be suitable for road marking detection will be described below.

2.5.1 The Gradient and Edge Detection

An edge in a greyscale image is a discontinuity in grey-scale values. This is described in for example [15]. A strong edge can be found where the grey-scale value changes the most, i.e. where the rate of change is high. To find edges in an image, the local maximum gradient is detected. This is done by convolving the original image \( f \) with a derivative operator or kernel, one for each direction, to obtain the new gradient images \( f_x \) and \( f_y \). The Sobel-operators are the simplest kernel pairs consisting of two 3x3 filter kernels, see Figure 2.2. The gradient images \( f_x \) and \( f_y \) are found by computing \( f_x = \text{Sobel}_x \ast f \) and \( f_y = \text{Sobel}_y \ast f \).

\[
\begin{bmatrix}
1 & 0 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1 \\
\end{bmatrix} \quad \begin{bmatrix}
-1 & -2 & -1 \\
0 & 0 & 0 \\
1 & 2 & 1 \\
\end{bmatrix}
\]

**Figure 2.2** The two Sobel-operators, \( \text{Sobel}_x \) and \( \text{Sobel}_y \).

Consequently, \( f_x = \text{Sobel}_x \ast f \) and \( f_y = \text{Sobel}_y \ast f \).

The magnitude of the gradient \( g_m \), see Figure 2.2, is defined by:

\[
g_m(x, y) = \sqrt{f_x(x, y)^2 + f_y(x, y)^2} \tag{2.1}
\]

The angular direction of the gradient vector, \( g_d \), is determined by the maximum rate of change of \( f \) at \( (x, y) \). The direction of the edge at \( (x, y) \) is hence perpendicular to the direction of the gradient vector at that point.

\[
g_d(x, y) = \arctan \left( \frac{f_y(x, y)}{f_x(x, y)} \right) \tag{2.2}
\]
The reason for exploiting edge detection to search for road markings in an image is that the edges of the road markings are prominent features in such an image. The edge detection approach was taken by the first generation lane detection and road marking localization systems, where a thresholding technique applied to the image intensity generated potential lane markings [16].

2.5.2 The Hough Transform

The Hough Transform (HT) is a very powerful global pattern detection method, which was first developed by Hough in 1962 [15]. A theoretical description of the HT can be found in a textbook on digital image processing by for example Gonzales et al [15]. The standard HT detects patterns that form straight lines, but the HT can be generalized to a large number of different patterns, ranging from straight lines to circles and curves [17]. The essential idea is that the HT changes the representation of a difficult feature extraction problem to an easier peak detection problem.

A line can be represented by

$$\rho = x \cos \theta + y \sin \theta$$  \hspace{1cm} (2.3)

where $\rho$ is the perpendicular distance from the origin to the line and $\theta$ is the angle that the normal makes with the x axis, see Figure 2.4. Given a set of points on a straight line in the xy-plane, the corresponding curves in the $\theta\rho$-plane all intersect in one point. The coordinates $(\theta', \rho')$ of the intersection reveal the parameters of the equation of the line connecting the points.

![Figure 2.4](image)

**Figure 2.4** The Hough transform represents infinite lines with $(\theta, \rho)$-parameters.
In general terms, the HT can be described as a voting process “where each point belonging to a pattern votes for all possible patterns that passes through that point” [18, p. 1090]. The votes are collected in a $\theta\rho$-accumulator array (or Hough space), where a peak corresponds to the most salient pattern in the image. The resolution of the accumulator array determines the accuracy with which the parameters can be determined.

The original image in Figure 2.5 (a) contains three points. Each point is represented by a curve, according to Equation (2.3), in Hough space in Figure 2.5 (b), with $\rho$ on the vertical axis and $\theta$ on the horizontal axis. A peak can be found in the intersection of the three curves in Figure 2.5 (b). The coordinates of that peak corresponds to a line with $\rho = 0$ pixels and $\theta = 45^\circ$, which is illustrated in Figure 2.6 as a line that goes through the centre of the image with its normal making 45 degrees with the $x$ axis.

Figure 2.5 Transformation from (a) image space to (b) Hough space. The coordinates of the intersection of the curves in (b) correspond to a line with $\rho=0$ and $\theta=45^\circ$. 
Figure 2.6 The coordinates of the intersection of the three curves in Figure 2.5 (b) above corresponds to a line with $\rho=0$ and $\theta=45^\circ$.

The algorithm of the Hough transform is summarized below. Let $f(x,y)$ be the image and $H(\theta, \rho)$ be the Hough transform.

\[
\text{For all } (x,y) \text{ where } f(x, y) \neq 0 \\
\quad \text{For all } (\theta, \rho) \text{ where } x \cos \theta + y \sin \theta = \rho \\
\qquad H(\theta, \rho) := H(\theta, \rho) + f(x,y) \\
\text{End} \\
\text{End} 
\]

The Hough transform is very robust to noise and discontinuities in the pattern, and it appears to be ideal for localizing road markings that are intermittent or partly occluded. Prior to applying the Hough transform for road marking localization on an image, the edges of the road markings must be located.

The Hough Transform approach has been investigated for automated lane detection [19]. A focus of attention is normally applied in the image-space by selecting a ROI, whereas the focus of attention is applied directly in Hough space in [19]. This is done by limiting the search ranges of $\rho$ and $\theta$, which reduces the time and space complexity of the computation.

2.5.3 Colour Image Processing

A colour image contains more information than greyscale images and this can in certain cases be exploited for better road marking detection. One example of this is found in an article by Bin Ran et al. [20], where colour information is used as a clue for shadow elimination. The colour space HSI, composed of the three channels hue, saturation and intensity, is used for image representation. An analysis of the histograms of HSI images with and without shadows is made, and conclusions based on the shadow characteristics in hue, saturation and intensity is drawn. This information is used in the image segmentation step, where the shadow region can be removed from the image.
2.5.4 Temporal Correlation in Image Sequences

A modern video camera captures images with 25 - 30 frames per second, so there will be considerable similarities between images taken in succession. The temporal correlation between consecutive frames can be used either to ease the feature determination or to validate the results from processing [9].

The LOIS-based lane tracker [16] uses parameters from the previous frame as a starting point for the next frame, which speeds up the search algorithm. However, bad estimates in previous frames might multiply the error into consecutive frames. It is important not to restrict the algorithm too much, and in [16] the parameters are only used as an initial guess of the current lane locations.

2.6 Geometrical Model

A number of assumptions of the road geometry and the shape of the road can simplify the extraction of the lateral position. For example, the assumption of a flat road simplifies the mapping to real-world coordinates. This assumption reduces the complexity of the mapping, however the drawback is that the mapping will produce incorrect data if the road is bumpy [9]. The assumption of a straight road is also common, and with this assumption the road markings can be assumed to form straight line segments within a reasonable distance from the vehicle.

\[ \text{Road markings} \]

\[ \text{Lateral position, } l \]

\[ \text{Figure 2.7 A model of the road and vehicle.} \]

In the road model, see Figure 2.7, the lateral position \( l \) can be found perpendicularly to the vehicle. The yaw angle, \( \psi \), describes the angle between the vehicle heading direction and the road marking.

The following two Euclidean spaces, as defined in [21], will be used throughout the report,

\[ W = \{(x, y, z)\} \in \mathbb{E}^3 \text{ representing the 3D world space (world-coordinate system), where the real world is defined.} \]

\[ I = \{(U, V)\} \in \mathbb{E}^2 \text{ representing the 2D image space (pixel coordinates), where the 3D scene is projected.} \]

The relationship between the two coordinate spaces is illustrated in Figure 2.8. The flat earth assumption causes the \( z \)-coordinate in the world space \( W \) to be 0.
2.7 Camera Calibration

The perspective effect of the camera causes pixels to have different meanings depending on their position in the image plane. The aim of the calibration is to find a mapping matrix $C : I \Rightarrow W$ that can be used to extract real-world coordinates from pixel positions. The performance of a computer vision application is therefore largely dependent on the camera calibration – an accurate camera calibration increases the performance considerably [23].

A camera calibration often involves an estimation of the intrinsic camera parameters, such as the optical characteristics of the camera, and/or extrinsic camera parameters, such as the relative position of the camera and a three-dimensional coordinate space. The camera model $C$ is composed of both intrinsic and extrinsic camera parameters. $C$ can be decomposed into the camera matrix $K$ (containing intrinsic camera parameters such as focal length), the rotation matrix $R$ (representing the orientation of the camera in the world) and the translating vector $V$ according to,

$$C = K [R | V]$$

Many calibration methods are based on certain simplifications in the camera model [23], as will be discussed in subsequent sections.

There are numerous ways to perform the transformation of the image pixels to real-world coordinates and the focus here will be on efficient yet practical methods.

2.7.1 Look-up Table

In AURORA, the lane departure warning system mentioned earlier [14], a straightforward camera calibration technique which compensates for lens distortion and perspective effects in the image has been exploited. The idea here is that the lateral position is measured along one predefined row in the image. The aim is to perform an accurate calibration of this single row instead of the entire image. A calibration stripe with markers equally spaced is placed within the field of view and adjusted to a chosen location in the image via a camera display, see Figure 2.9. The mapping between image pixels and real-world data can then be obtained via a look-up table.
2.7.2 Plane Projective Mapping

A mapping of the entire image space requires a camera model $C$, which can be described by a $4 \times 3$ matrix which maps points in the three-dimensional $W$ space to the two-dimensional $I$ space.

The flat earth assumption suggests that the part of the real-world scene that is viewed is planar, and the mapping between the $I$ space and $W$ space can be described by a plane projective mapping. A plane projective mapping is a projection from one plane through a point to another plane, as illustrated in Figure 2.10. The theory in this section is taken from [24].

A point $(U, V)$ in the $I$ space can be written in homogeneous coordinates as $(u, v, t)$ where

$$U = u / t \quad V = v / t$$  \hfill (2.4)

A camera model $C$ is chosen so that for any point $(x, y, z)$ in $W$,

$$(x, y, z, 1) C = (u, v, t)$$  \hfill (2.5)

where
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\[ \begin{align*}
    u &= (x, y, 1) C_1 = xC_{11} + yC_{21} + zC_{31} + C_{41} \\
    v &= (x, y, 1) C_2 = xC_{12} + yC_{22} + zC_{32} + C_{42} \\
    t &= (x, y, 1) C_3 = xC_{13} + yC_{23} + zC_{33} + C_{43}
\end{align*} \]

(2.6)

The camera model \( C \) is now a 3 x 3 matrix. Rewriting (2.4) to \( u - Ut = 0 \) and \( v - Vt = 0 \) yields,

\[ \begin{align*}
   xC_{11} + yC_{21} + zC_{31} + C_{41} - UxC_{13} - UyC_{23} - UzC_{33} - UC_{43} &= 0 \\
    xC_{12} + yC_{22} + zC_{32} + C_{42} - VxC_{13} - VyC_{23} - VzC_{33} - VC_{43} &= 0
\end{align*} \]

(2.7)

In homogenous coordinates \( C \) can only be determined up to scale, so \( C_{43} \) may arbitrarily be set to 1. Suppose that \( n \) points \( 1 \leq i \leq n \) \((x^i, y^i, z^i)\) corresponding to \((U^i, V^i)\) are measured. The equations can now be rewritten as,

\[ \begin{bmatrix}
    x^1 & y^1 & z^1 & 1 & 0 & 0 & 0 & 0 & -U^1x^1 & -U^1y^1 & -U^1z^1 \\
    0 & 0 & 0 & 0 & x^1 & y^1 & z^1 & 1 & -V^1x^1 & -V^1y^1 & -V^1z^1 \\
    x^2 & y^2 & z^2 & 1 & 0 & 0 & 0 & 0 & -U^2x^2 & -U^2y^2 & -V^2z^2 \\
    \vdots & & & & & & & & \ddots \ddots \ddots \ddots \ddots \ddots \ddots \ddots \ddots \\
    0 & 0 & 0 & 0 & x^n & y^n & z^n & 1 & -V^nx^n & -V^ny^n & -V^nz^n \\
\end{bmatrix}
\begin{bmatrix}
    C_{11} \\
    C_{21} \\
    C_{31} \\
    C_{41} \\
    C_{12} \\
    \vdots \\
    C_{13} \\
    \vdots \\
    C_{33}
\end{bmatrix} = \begin{bmatrix}
    U^1 \\
    V^1 \\
    \vdots \\
    U^n \\
    V^n
\end{bmatrix} \]

(2.8)

Each point correspondence generates two linear equations and since there are eleven unknowns in the equation system above. With \( z = 0 \) the number is reduced to eight, so eight such linear equations are necessary to solve \( C \) above. This requires \( n \), the number of points, to be four. That is by placing four marks of known location in the field of view of the camera the solution for \( C \) can be determined. Using \( n > 4 \) points generates an overdetermined equation system which can be solved by using a pseudo-inverse, for details see for example [24].

Once \( C \) has been determined, calculating the inverse perspective makes it possible to find the real-world correspondences of points in the image plane. In fact, a point \((U, V)\) in the image-plane \( I \) corresponds to a line of sight in the real-world, \( W \).

According to (2.6)

\[ \vec{u} = \vec{x}C, \text{ where } \vec{u} = (u, v, t) \text{ and } \vec{x} = (x, y, z, 1) \]

Then

\[ \begin{align*}
    u &= Ut = \vec{x}C_1 \\
    v &= Vt = \vec{x}C_2 \\
    t &= \vec{x}C_3
\end{align*} \]

(2.9)

By substituting the expression of \( t \) into that of \( u \) and \( v \) above the following equations are obtained,
\[ \begin{align*}
U \bar{x} C_3 &= \bar{x} C_1 \\
V \bar{x} C_3 &= \bar{x} C_2
\end{align*} \]  
(2.10)

This can be rewritten as,
\[ \begin{align*}
\bar{x}(C_1 - UC_3) &= 0 \\
\bar{x}(C_2 - VC_3) &= 0
\end{align*} \]  
(2.11)

The equations above are plane equations, which also can be written as
\[ \begin{align*}
a_1 x + b_1 y + c_1 z + d_1 &= 0 \\
a_2 x + b_2 y + c_2 z + d_2 &= 0
\end{align*} \]  
(2.12)

where
\[ \begin{align*}
a_1 &= C_{11} - C_{13} U \\
a_2 &= C_{12} - C_{13} V \text{ etc.}
\end{align*} \]  
(2.13)

The desired line is the intersection between two planes, determined by any point \((U, V)\) in the image and the camera model \(C\). The direction \((\lambda, \mu, \nu)\) of the intersection can be obtained by taking the cross-product of the normal vectors of the two planes,
\[ \begin{align*}
(\lambda, \mu, \nu) &= (a_1, b_1, c_1) \times (a_2, b_2, c_2) = \\
&= (b_1 c_2 - b_2 c_1, a_2 c_1 - a_1 c_2, a_1 b_2 - b_1 a_2)
\end{align*} \]  
(2.14)

By combining (2.12) and (2.14) and if \(\nu \neq 0\) for any particular \(z\), and a point \((U, V)\) in the image,
\[ \begin{align*}
x &= \frac{b_1 (c_2 z + d_2) - b_2 (c_1 z + d_1)}{a_1 b_2 - b_1 a_2} \\
y &= \frac{a_2 (c_1 z + d_1) - a_1 (c_2 z + d_2)}{a_1 b_2 - b_1 a_2}
\end{align*} \]  
(2.15)

By assuming that the world is flat, the point where the line of sight intersects the ground plane will be the point in the world \((x, y, 0)\) corresponding to \((U, V)\) in the image. Hence the equations above can be reduced to,
\[ \begin{align*}
x &= \frac{b_1 d_2 - b_2 d_1}{a_1 b_2 - b_1 a_2} \quad \text{and} \quad y = \frac{a_2 d_1 - a_1 d_2}{a_1 b_2 - b_1 a_2}
\end{align*} \]  
(2.16)
2.7.3 Angle Extraction

A line in the image plane $I$ corresponds to a viewing plane in the real-world $W$. The flat earth assumption causes the intersection of the viewing plane and the ground plane in the world to correspond to a line on the ground plane.

The line on the ground plane can be found by transforming two points on a line in $I$ to their real-world correspondences, i.e. $(U_1, V_1) \rightarrow (x_1, y_1, 0)$ and $(U_2, V_2) \rightarrow (x_2, y_2, 0)$. The line between $(x_1, y_1)$ and $(x_2, y_2)$ makes an angle with the $y$-axis in $W$ space, see Figure 2.11. The angle can be calculated according to,

$$\psi = \arctan \left( \frac{x_2 - x_1}{y_2 - y_1} \right)$$

(2.17)

![Diagram showing angle in the real world](image)

**Figure 2.11** Illustration of angle in the real world.
3 System Design Approach

3.1 Road Attributes

In order to utilize road markings as a reference in a measuring system, information about the functionality, various configuration types, location and quality of the markings must be taken into consideration when designing a suitable system.

3.1.1 Road Marking Characteristics

The term *road marking* includes longitudinal markings, arrows, transverse markings, text and symbols on the surface of the roadway [25]. The purposes of road markings are to guide, regulate and control the traffic. By being a visual source of information they can assist the driver in his or her decision making. The driver’s perception of the road markings and their visibility influences the driver’s performance and positioning of the vehicle on the road [26].

The appearance of road markings is crucial to a system detecting road markings in images. On Swedish roads, the road markings are white with quite high reflective properties, thanks to additives in the paint such as glass beads which increase the luminance. Although the quality of the road markings is hard to influence, the visibility depends on the contrast between the marking and the neighbouring surface of the roadway and can be enhanced by using a camera with a high dynamic range. Weather conditions, such as fog, snow, heavy rain or direct sunshine, together with dirt retention are the major factors decreasing the visibility [26].

Longitudinal markings are either edge markings, which indicate the limits of the roadway, or lane markings on the roadway to keep the traffic within the lane. The standard widths of edge and lane markings are between 0.1 m and 0.3 m, with the most common types being 0.1 m and 0.15 m. The markings can be either continuous or intermittent. The average length of the individual segments of the intermittent markings are specified according to line segment in
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meter plus space in meter, and the combinations currently in use on Swedish roads are 1 m + 2 m, 3 m + 3 m, 3 m + 9 m, 9 m + 3 m. Different types of combinations of these lines are also in use and the configuration of the road markings normally change or cease during intersections [27].

3.1.2 Road Characteristics

In this study the focus will be on country roads with two lanes and motorways in Sweden. In general the lane widths of these roads are within two intervals, 3 - 4.5m and 5.25 -5.75m, depending on the type of road [27].

A curve on a road can be described as an arc segment of a circle, and the curvature can be defined as the radius of the circle. According to the recommendations for the design of Swedish roads in [27], the minimum radius of a horizontal curve should be at least 90 meters.

3.2 Prototype System Set-up

The prototype system set-up will be described in the following sections. The basic functionality of the road marking detection and lateral position extraction will also be described below.

3.2.1 System Overview

The signal from the video camera is recorded onto a DVCam-tape using a video recorder in the vehicle, see Figure 3.1. The video sequence is later recorded onto the hard drive of a stationary PC using video capturing software resulting in an uncompressed AVI file, see Figure 3.2. The AVI file needs to be compressed with a lossless compression technique to reduce storage but without losing information, and this is done in the video editing software. The output AVI file, now reduced in storage size, is ready to be used in the lateral position detection software system, and the output from the software will be an ASCII file with lateral position data.

![Figure 3.1 The in-vehicle video system.](image1)

![Figure 3.2 The post-processing system on a stationary PC.](image2)
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3.2.2 The Image Acquisition System

The motivation for this project was to examine the potential of a vision-based approach for a lateral position application. Therefore, focus does not lie on choosing the best camera for the purpose, but rather to make use of available camera technology at VTI. In this project, the camera to be used is a CCTV camera from Hitachi. The image sensor is of CCD-type with a resolution of 720 x 576 effective pixels. The scanning system of this camera uses 2:1 interlacing with a capturing frequency of 25 Hz. Interlacing means that the camera captures two fields separately (odd and even rows) that will be added together to constitute one image frame. There is a time lapse between the capturing of the two fields. In images with motion the effect of the time lapse can be seen as double edges of objects. This problem can be solved by either duplicating or discarding every odd field. The latter method is chosen here, since it leads to a reduction in resolution and an increase in sharpness. Hence, a single video field of size 720 x 288 pixels is used.

A greyscale image is represented by a two dimensional matrix in MATLAB, whereas a colour image needs a three dimensional array containing each of the red, green and blue channels of the image. Although much useful information can be extracted from colour images according to Bin Ran et al [20], a typical road scene with white road markings is suitable for grey-scale representation. Using greyscale images is a quick and efficient way of reducing the amount of stored data to a third compared to colour images and still maintaining the valuable information in the image.

3.2.3 Camera Mounting and Field of View

A high positioning accuracy will be obtained by capturing images of the road as close as possible to the vehicle.

By capturing a rectangular area beside the vehicle with a wide angle lens, the lane departure warning system AURORA detects the vehicle position with high accuracy. Experiments show that the standard deviation of the error in position estimation is 0.0105 m [14]. AURORA was briefly described in Section 2.3.3. However, with the available camera in this project, the FOV would be too small and the system would miss out on data during intermittent road markings. Another drawback of placing the camera on the side of the vehicle is that it only allows for detecting lateral position while the vehicle is inside the lane. A lane crossing would make the centre lane line disappear from the FOV.

A better solution is to let the camera face in the driving direction of the vehicle. The distance between the vehicle and the FOV of the camera is minimized by mounting the camera on the roof of the vehicle facing backwards and tilted downwards, see Figure 3.3. In order to assure that the intermittent road markings can be seen in the image, the FOV must cover at least 12 m in the longitudinal direction. Since the centre road marking should be detected, the camera should be mounted on the left of the vehicle so that the road markings are in the centre of the image under normal driving. The road markings will still be visible in the FOV during a lane crossing. By placing the camera outside of the vehicle, the camera is unobtrusive to the driver while driving.
Figure 3.3 The camera will be mounted on the roof of the vehicle, facing backwards and tilted. The distance L should be minimized and the FOV should cover at least 12 m in the longitudinal direction.

3.2.4 Road Marking Detection/Functionality

Using either of the computer vision approaches discussed in Section 2.5 alone would not be sufficient in the application developed here. The problem of detecting white road markings in an image is a complex problem, and the approach taken here is to combine a number of computer vision methods to achieve more robust results.

The strongest characteristic of a typical road marking image is the road marking edges, so an edge detection approach is a reasonable starting point. However, the drawback of edge detection is that it can be very sensitive to noise and does not always present satisfying results on images with shadows and grain. By applying noise reduction filters prior to edge detection, the resulting images will be improved. The most likely positions of the road marking edges in the image can also be exploited to restrict the direction of the edges to be found. The temporal correlation between image frames can be exploited to update a search region in the image.

The assumption that the road markings form straight lines leads to the obvious choice of using a Hough-based method for detection of the edge lines. This method is robust towards noise and well-suited for this approach. By selecting pixels having certain local properties the computational complexity and noise level of the HT are reduced. The focus of attention in Hough space can also be limited by knowledge of the position of the previously detected road marking edges.

3.2.5 Camera Calibration

In this project there is a need for a quick and simple yet efficient calibration procedure, because the camera needs to be calibrated every time it is mounted on the vehicle.

The calibration stripe approach, similarly to the look-up table approach in [14], is attractive since it corrects for lens distortion. If only one row in the image need to be calibrated this is a simple and quick approach.

The plane projective mapping technique, described in Section 2.7.2, on the other hand does not correct for camera lens distortion, but it does represent a practical approach which has the advantage of a straightforward implementation. Since the camera in use is not a wide-angle
camera, the lens distortion can be assumed to be low. By using more than four points, the equation system will be overdetermined and yield a more accurate solution.

The calibration stripe and the plane projective mapping are both interesting and practical approaches, and were investigated during the course of the project. They would both be suitable in this application, but the latter method was chosen because of the ease of implementation.

The plane projective mapping is based on placing equally spaced calibration marks of known location in the FOV of the camera, the corresponding pixel positions are then easily obtained by pointing in the image and a camera model can be determined.

3.2.6 Lateral Position and Yaw Angle Extraction

To maintain consistency, the lateral position should be measured in the same way in every image frame. Due to the varying width of road markings, the reference point for measurement is the centre of the road markings. The resulting image from the road marking detection will contain a straight line at a slope representing the road marking in the image. This line will be denoted ‘measuring line’. The measuring line will be placed in the centre of the road marking, and hence in between the two edges of the road marking. The position of the road markings in the lower part of the image is directly affected by the lateral position of the vehicle [13]. The lateral position could hence be extracted in a straightforward manner simply by transforming the pixel position to a real-world position.

![Figure 3.4. The lateral position is measured perpendicularly to the road marking along the x axis.](image_url)

The lateral position should be measured perpendicularly to the vehicle at the front left wheel, as illustrated in Figure 3.4. However, the vehicle and road marking are not always parallel. The yaw angle between the vehicle and the marking can be calculated by mapping two points on the line to their real-world correspondences, see Figure 3.5.
Figure 3.5 By selecting two points on the measuring line in the image, the yaw angle can be determined.

If the yaw angle is not zero, the lateral position in the FOV will differ from the lateral position at the front left wheel. The yaw angle can be used to calculate the position at the wheel, as in Figure 3.6.

Figure 3.6 The yaw angle, $\psi$, can be used to translate the lateral position, $l_{FOV}$, detected in the image back to the front left wheel of the vehicle. The lateral position is zero in this case.
3.2.7 Algorithm

The algorithm of the system can be divided into three steps, road marking detection, camera calibration and data extraction, see Figure 3.7.

![Algorithm Diagram]

Figure 3.7 A flow chart describing the general steps of the algorithm.

3.3 Potential Problems

3.3.1 Dynamic errors

A moving vehicle on an uneven road is not a stationary system and as a consequence the images from the camera, mounted on the vehicle, will be affected by random repositioning. The geometry in the image is affected, which will generate errors in position data. However, the errors are assumed to be small and will therefore be overlooked in this project.

3.3.2 Curves and Perspective effect in image

Due to the perspective effect, the accuracy in the lower part of the image is better than the upper part. Even though it is possible to detect the road marking position in the top part of the image, the risk of introducing large errors when measuring the lateral position from such a pixel location is high.
If the road contains curvature, it will be reflected in the top part of the image where large longitudinal distances correspond only to a few pixels. With the straight road assumption and Hough-based approach made, this part of the image is useless. It is better to use the part of the image where straight lines do occur and the vehicle position can be measured with high accuracy. This will enhance the quality of the data. The ROI will therefore only include the part of the image in which the road markings are most likely to be straight, see Figure 3.8.

![Figure 3.8](image)

**Figure 3.8** The top part, which contains curvature, will be discarded. The box indicates the ROI in the image.

3.3.3 Intermittent road markings

During sections of the road with intermittent road markings, the gaps between available road markings will naturally affect the continual measurement of the lateral position. There might even be images without road markings.
4 Road Marking Detection

4.1 Pre-processing

To get rid of noise and to smooth the image, it is first filtered with an adaptive Wiener filter, implemented in MATLAB as Wiener2 with a 5 x 5 kernel. Wiener2 performs a low-pass filtering in the areas of the image where there are no edges. The method implemented in MATLAB is based on statistics estimated from a local neighborhood of each pixel.

Secondly, a Median filter, implemented in MATLAB as medfilt2, is applied to the image. The median filter is a nonlinear filter which reduces ‘salt and pepper’ noise but preserves edges well. The theory of both Wiener filters and Median filters can be found in for example Gonzales [15]. The effects of these two filters are illustrated in Figure 4.1 and Figure 4.2.

![Figure 4.1](image) (a) The original image and (b) the image after the Wiener filter has been applied.
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4.2 Road Marking Tracking

The initial ROI is chosen to be the entire image except for the top part, see Figure 3.8. Once a road marking has been detected in an image frame, its position determines the ROI for the following image frame, see Figure 4.3. The nature of the Hough transform, which will be discussed in detail below, causes the support to be strongest for diagonal lines in the image [22]. This can be exploited in the selection of the ROI so that the line searched for always is oriented along the diagonal of the search area.

To minimize unnecessary computations, the ideal program would discard image frames without visible road markings at an early stage. The human eye can easily determine if a road marking is visible in the image, it is however more complicated to automate this check. The check has been implemented based on the intensity level of the image and the fact that the presence of a white road marking leads to a higher intensity level than an image without a white marking. If the intensity level does not exceed a certain threshold T, the program moves on to the next frame and the ROI is reset to the initial region.

The high frame rate results in high temporal correlation between image frames taken in succession. See Figure 4.4.
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Figure 4.3 The solid rectangle shows the ROI in the current frame, and the dashed rectangle shows the ROI for the next frame.

Figure 4.4 The temporal correlation is obvious in the image frames (a) - (i) above. The shadow of the vehicle can be seen in the right bottom part of the image.
4.3 Hough Transformation

The following steps constitute the transformation to and from the Hough space and will be described in detail below,

- Characteristic point detection
- Transform mapping
- Peak detection in Hough space
- Back projection to image space

4.3.1 Characteristic Point Detection

Even though the Hough transform is very robust towards noise, the tendency to find false lines in an image can sometimes overshadow the robustness. By ensuring that only the characteristic pixels are extracted and transformed, a gradient analysis should be made prior to the transformation.

The direction of the gradient is a characteristic property. Since the orientation of an edge only span 0 to \( \pi \) radians, the direction of the normal can be expressed in both negative and positive angles. By representing the orientation with a vector that rotates with twice the speed of the orientation, the direction of the normal will span 0 to 2\( \pi \) radians instead [28].

Edge orientation information can be used to eliminate shadows and other edge pixels that are of no interest here. A restriction of edge pixel orientation can be based on the angle of the detected edge pixels in the previous image together with the typical orientations of road marking edges. The latter is based on the fact that the road marking is likely to have a certain orientation in the image. By using the double angle representation the direction of the normal of the edge is allowed to span \(-3\pi/4 \leq \varphi \leq 3\pi/4\), where \( \varphi = 2 \cdot \theta \). See Figure 4.5. For example, all edges oriented horizontally in the image will be excluded. The edge orientation restriction on a gradient image with road markings is illustrated in Figure 4.6.

![Figure 4.5](image)

**Figure 4.5** Illustration of the correspondence between the orientation of a line and the direction of its descriptive vector. The grey area indicates the orientations of the edges in a typical road marking image.
4.3.2 Transform Mapping

Each edge pixel votes for the line it belongs to. The votes are collected in a $(\theta, \rho)$ accumulator array or Hough space. Similar to the focus of attention approach applied in [19], the search range of $\theta$ will be restricted based on detected $\theta$ in the Hough transformation of the previous image frame. The initial search range of $\theta$ is set to (-67.5, 67.5) degrees, see Figure 4.7.

![Figure 4.7 The Hough space of a typical road marking image. The initial range of $\theta$ is in the interval (-67.5, 67.5) degrees.](image)

4.3.3 Peak Detection Methods

A cell in the accumulator array having a significant number of votes is a candidate for a line. Each peak in the intensity level of the Hough space represents one specific line.
The aim is to determine the coordinates of the peaks with as high accuracy as possible. The accuracy with which the line can be determined is limited by the spatial resolution and the step sizes of the \((\theta, \rho)\)-parameters of the accumulator array [18].

The difficulty in implementing the Hough Transform lies in selecting the right coordinates for the peak. The coordinates of the global peak does not always give the correct coordinates of the "true" line.

The quantization of the parameters \(\rho\) and \(\theta\) causes discretization errors to be present. A too small quantization step results in a spreading of the peak over a number of cells, whereas a too large step size leads to inaccuracy in the peak. In the latter case, if the "true" parameters of a line happen to lie close to a boundary in the quantised parameter space, the votes will get spread over two or more cells, and looking at single cells (as in a search for a global maximum value would) may not reveal the peak [18]. The peak spreading in Hough is illustrated in Figure 4.8 and Figure 4.9.

**Figure 4.8** A typical road marking image has two strong edges, which are reflected in the Hough space as two strong peaks. However, the peaks are spread over a number of cells making the detection of the actual peak difficult.
A number of approaches have been investigated during the course of this project. Global thresholding has proven not to be flexible enough, and was dismissed at an early stage. A number of peak detection methods found in the literature are presented below. However, many of the articles on the subject focus on finding only a single peak in Hough space. The typical road marking image contains at least two edges, causing the number of peaks to be at least two. The difficult part is that the peaks are located at a close distance to each other, but this distance is hard to predict. Limiting the algorithm to a fixed distance might hamper it from finding multiple peaks. Detecting multiple peaks is not straightforward. The peaks are often spread in both \( \rho \) and \( \theta \) directions, causing the classification of the peaks to be intriguing. The problem is that the peak can be spread over cells that are not neighbours according to 8-connectivity. The edge image contains scattered points, which do not always lie along one single correct line. This results in many peaks in Hough space, and many lines in image space. Below, the Hough space is denoted by \( H(\theta, \rho) \) and \( \rho_{size}, \theta_{size} \) are the sizes of \( H(\theta, \rho) \).

**Absolute Peak (AP) Method**

\[
(\theta_{peak}, \rho_{peak}) = \{ (k,l) : H(k,l) = \max\{H(\theta, \rho) ; 0 \leq \theta \leq \theta_{size} - 1, 0 \leq \rho \leq \rho_{size} - 1, \} \}
\]

(4.1)

The Absolute Peak Method selects the global maxima of the Hough space as the peak [18]. This is the most straightforward method of all, but does not always present satisfying results.

**\( N \times N \) Window**

The search for the global maximum can be refined by restricting the search for a local maximum value within an \( N \times N \) window [17]. The choice of \( N \) is critical, and finding a suitable and adapting window size is very difficult. If \( N \) is chosen too large, some peaks might be missed, whereas a too small \( N \) will find to false peaks.
Summing over $n_\rho$ Cells (NR) Method

Due to the quantization the peak is likely to spread. Atiquzzaman presents a way to calculate the number of pixels the peak will spread in the $\rho$ direction [18]. The equation yields,

$$n_\rho = \left[ \frac{d \sin(\Delta \theta / 2)}{\Delta \rho} \right] + 2$$  \hspace{1cm} (4.2)

where $d$ is the maximum length of the line in the image and $\Delta \rho$ and $\Delta \theta$ are the step sizes of the $\rho$ and $\theta$-parameters. Using step sizes $\Delta \rho=1$ and $\Delta \theta=1$ on an image of size 720 x 288 pixels yields $n_\rho = 9$.

The spreading of the peak in the $\rho$ direction can be taken into account by summing over a window of $n_\rho$ cells. The peak is chosen to be the peak of the window with the highest sum.

$$c, r = \left\{ (\theta, \rho) : \max \left( \sum_{m=0}^{n_\rho-1} H(\theta, \rho - m) ; 0 \leq \theta \leq \theta_{size} - 1, n_\rho - 1 \leq \rho \leq \rho_{size} - 1 \right) \right\}$$

$$\left( \theta_{peak}, \rho_{peak} \right) = \left\{ (c, k) : H(c, k) = \max \left( H(c, j) ; r - n_\rho - 1 \leq j \leq r \right) \right\}$$  \hspace{1cm} (4.3)

In summary, there are advantages and drawbacks of all these peak detection methods. Finding the global maximum as in the Absolute Peak Method is a good starting point, but a problem is encountered when trying to find the second peak. In many cases, the second highest maximum belongs to the same line as the maximum value. By clearing a window of $N \times N$ pixels around the global peak, the risk of selecting a maximum value the second time that belongs to the same peak is reduced. The size of $N$ is crucial, but using $n_\rho$ as the size of the window will improve the method. Instead of summing over the entire image, as is done in the NR method, a region with a peak value can be located instead. Based on these conclusions together with a thorough analysis of the general appearance of the Hough space and trial and error, a general and flexible peak detection algorithm was developed.

**Thesis Algorithm: Peak Detection with Adaptive Window**

Firstly, find the coordinates of the global maxima of the Hough space. Secondly, sum over a window $W$ of $N \times N$ pixels with the coordinates of the peak in the centre of the window. By setting the size of $N$ to be $n_\rho$, the maximum number of pixels the peak will spread, the window will include the peak spreading. The size of $n_\rho$ will vary due to the size of the selected ROI. Then clear the $N \times N$ window. Start again by finding the global maximum of the Hough space and repeat the steps until the coordinates of four peaks have been detected. When summing over the windows the original values of the Hough space should be used, and not the partly cleared Hough space. The original Hough space is denoted by $H(m,n)$ in the algorithm and $n_\rho$ was specified in (4.2) above.
\[ T(\theta, \rho) = H(\theta, \rho) \]

For \( i = 1, 2, 3, 4 \)

\[
(\theta_i, \rho_i) = \left\{ (k, l) : T(k, l) = \max(T(\theta, \rho); 0 \leq \rho \leq \rho_{\text{size}} - 1, 0 \leq \theta \leq \theta_{\text{size}} - 1) \right\}
\]

\[
W_i = \left\{ \sum_m \sum_n H(m, n) ; \theta_i - \frac{n_{\rho}}{2} - 1 \leq n \leq \theta_i + \frac{n_{\rho}}{2}, \rho_i - \frac{n_{\rho}}{2} - 1 \leq m \leq \rho_i + \frac{n_{\rho}}{2} \right\}
\]

set \( T(m, n) = 0 ; \theta_i - \frac{n_{\rho}}{2} - 1 \leq n \leq \theta_i + \frac{n_{\rho}}{2}, \rho_i - \frac{n_{\rho}}{2} - 1 \leq m \leq \rho_i + \frac{n_{\rho}}{2} \) 

End \( (4.4) \)

When the coordinates of four peaks and four window sums \( W_i \) have been recorded, the window sums are sorted in descending order and the two top windows are chosen. The coordinates of the peak corresponding to these windows are chosen as the two peaks. Consequently, the first peak is chosen to be,

\[
(\theta_{\text{peak}}, \rho_{\text{peak}}) = \left\{ (\theta_j, \rho_j) ; W_j = \max(W_i ; 1 \leq i \leq 4) \right\}
\]

(4.5)

By clearing a window around the global peak, the risk of selecting two local peaks within the same global peak is reduced. The fact that the size of the window \( W \) adapts with the peak spreading is a strong advantage of the algorithm, because it helps locate the correct peak. A sequence of images illustrating the functionality of the algorithm step by step can be found in Appendix A.

4.3.4 Back-projection

Any pair of \((\theta, \rho)\) in the Hough space represents a unique line in the image. The coordinates of the peak \((\theta_{\text{peak}}, \rho_{\text{peak}})\) are used to extract the detected edge lines [18]. A measuring line, that will be used to extract lateral position later on, will be placed in the centre of the road marking edges.

4.3.5 Verification of Road Marking Candidates

The peaks in Hough space do not always correspond to road marking edges, but can sometimes correspond to other strong edges within the image. By verifying that the road marking candidate actually is a true road marking, the risk of using images with false road markings is reduced. This enhances the quality of the output data.

If the correct peaks were picked in the peak detection step, the back-projected detected line should lie along the road marking in the image. The road marking in the image can be assumed to have a higher intensity level than the road itself.

First, select five verification points along the measuring line and obtain the intensity values of these points, see Figure 4.10. At least one of these intensity values should be above a certain
threshold T for the line to be classified as a road marking. If neither one of the intensity values are above the threshold, the road marking candidate is discarded and the system moves on to the next image frame.

![Figure 4.10 Five verification points are selected along the detected line.](image)

### 4.3.6 Radon Transform

The Hough Transform is implemented in MATLAB as the Radon Transform and the function `radon`. The algorithm of the Radon Transform is summarized below. Let $f(x,y)$ be the image and $R(\theta, \rho)$ be the Radon Transform.

For all $(\theta, \rho)$

For all $(x, y)$ where $x \cos \theta + y \sin \theta = \rho$

$R(\theta, \rho) := R(\theta, \rho) + f(x,y)$

End

End

The road marking detection algorithm has been summarized in a flow chart in Appendix B.
5 System Calibration and Data Extraction

5.1 Camera System Calibration

Calibration of the system is a crucial part of the data collection and must be conducted in exactly the same manner every time the system is initiated.

A tarpaulin with equally spaced circular marks was used in the calibration procedure, see Figure 5.1. The number of calibration marks \( n \) is not fixed, but a minimum of four marks are necessary. The camera model can be determined more accurately with a large number of points. In the calibration performed in this project, more than 25 calibration marks were used.

![Figure 5.1](image)

**Figure 5.1** Two calibration stripes with equally spaced white marks are placed in the field of view of the camera. The dashed line represents the extension of the reference point at the front left wheel.
5.1.1 Calibration Measurements

The following measurements need to be made when performing the calibration:

- The longitudinal distance from the position on the ground in the field of view of the camera where the lateral position will be measured to the front left wheel of the vehicle. See distance $d$ in Figure 5.1.

- The real-world coordinates $(x, y)$ of each of the calibration marks. This can be easily done by noting down the coordinates of one calibration mark, and measuring the distances $a$ and $b$ between the calibration marks, see in Figure 5.1.

The image coordinates corresponding to the calibration marks are found by manual clicking in the image. A user interface has been developed to ease the process.

5.2 Camera Model

The camera model $C$ can be determined by finding the image pixel locations corresponding to the calibration marks of known location. Refer to Section 2.7.2 for a description of the plane projective mapping.

The camera model $C$ is obtained by taking,

$$B = AC + E \quad (5.1)$$

where $E$ represents the error between the prediction and the actuality, $B$ contains the image coordinates and $A$ contains the real-world coordinates. Rewriting (5.1) yields,

$$E^T E = (B - AC)^T (B - AC)$$

$$= B^T B - C^T A^T B - B^T AC + A^T C^T CA$$

$$= B^T B - 2 C^T A^T B + A^T C^T CA \quad (5.2)$$

In order to minimize the error, differentiating with respect to the elements of $B$ and setting the derivative to 0 yields,

$$0 = A^T AC - A^T B$$

$$\Rightarrow C = (A^T A)^{-1} A^T B \quad (5.3)$$

where $(A^T A)^{-1}$ is the pseudo-inverse of $A$.

In MATLAB the pseudo-inverse is implemented as `pinv`. $C$ can be determined in MATLAB by typing the simple command,

```matlab
>> C = pinv(A) * B
```

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5.3 Mapping to Real-world Coordinates

The camera calibration gives the ability to transform every pixel value in the image to its corresponding real-world coordinate.

Given the coordinates \((U, V)\) of any point \(Q\) in the \(I\) space, the equation (5.4) below, which is the same as (2.16), will return the \((x, y)\) coordinates of the corresponding point \(P\) in the \(W\) space.

\[
x = \frac{b_1 d_2 - b_2 d_1}{a_1 b_2 - b_1 a_2} \quad \text{and} \quad y = \frac{a_2 d_1 - a_1 d_2}{a_1 b_2 - b_1 a_2}
\] (5.4)

where the parameters are as follows,

\[
\begin{align*}
a_1 &= C_{11} - C_{13} U \\
a_2 &= C_{12} - C_{13} V \\
b_1 &= C_{21} - C_{23} U \\
b_2 &= C_{22} - C_{23} V \\
d_1 &= C_{41} - C_{43} U \\
d_2 &= C_{42} - C_{43} V
\end{align*}
\] (5.5)

5.4 Yaw Angle Extraction

Two pixel positions along the measuring line in the image are selected, and transformed to the real-world coordinates \((x_1, y_1)\) and \((x_2, y_2)\). See Figure 5.2. The yaw angle can then be determined according to Equation (2.17). See also Section 3.2.6 for a general description of the yaw angle extraction and usage.

5.5 Lateral Position Extraction

The lateral position will be extracted by transforming the pixel location of the measuring line at the bottom of the image. This will generate the lateral position in the FOV, \(LP_{FOV}\). The lateral position at the front left wheel of the vehicle, as illustrated in Figure 5.2, is calculated according to,

\[
LP_{left\,\,wheel} = LP_{FOV} - d \cdot \tan(\psi)
\] (5.6)

where \(\psi\) is the yaw angle and \(d\) is the distance from the front left wheel along the y-axis.
Figure 5.2 The lateral position at distance $d$ from the front left wheel needs to be adjusted with the yaw angle $\psi$ to be correct at the front left wheel.
6 Experiments and Results

6.1 Data Extraction Validation

A validation study was carried out in order to determine the accuracy of output data from the prototype system. The camera was mounted on the roof of the vehicle at height $z = 1.55$ m. The camera was first calibrated using the calibration method described in Chapter 5. The vehicle was parked during the validation process so still images were acquired. A white marking of width 0.15 m was used as a road marking and it was placed at different positions in the FOV of the camera. The positions were measured manually. In the subsequent text and in the data sheet in Appendix A, the measured position is denoted ‘actual position’ whereas the output from the system is called ‘detected position’. It should be added that the aim of the validation was not to put the functionality of the road marking detection on trial, but rather to determine the degree of conformity of the detected positions to the actual positions.

In the image in Figure 6.1, three rows are indicated by the letters A, B and C. The perspective effect of the camera causes the pixels to correspond to different real-world measurements depending on their positions in the image. For example, with this camera position and angle, the width of the FOV was around 6 m along row A, 3 m along row B and 2 m along row C. Hence, the position of the road marking can be determined with the highest accuracy along row C. In order to determine the yaw angle as well, the lateral position was measured along row B and row C in the image so two points were obtained.
Chapter 6 – Experiments and Results

Figure 6.1 The FOV covers 6 m along row A, 3.3 m along row B and 2.0 m along row C.

Even though the FOV at the bottom row is only 2 m, information in the upper part of the image contributes to a larger operating range at row C. The valid operating range of the image at row C was found to be ± 2.0 m from the centre of the image. Within this range, the standard deviation of the position detection was 0.012 m and the maximum deviation was 0.037 m, see Table 6.1. By narrowing the operating range to ±1.0 m, which is covered in the image, the standard deviation was 0.004 with maximum error 0.011 m. Along this row in the image, each pixel roughly corresponds to 0.003 m.

<table>
<thead>
<tr>
<th>Operating range (m)</th>
<th>Standard deviation (m)</th>
<th>Max deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2.00, 2.00)</td>
<td>0.012</td>
<td>0.037</td>
</tr>
<tr>
<td>(-1.50, 1.50)</td>
<td>0.004</td>
<td>0.011</td>
</tr>
<tr>
<td>(-1.00, 1.00)</td>
<td>0.004</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 6.1 Accuracy in lateral position measurement at bottom row (row C) in the image.

The second lateral position measurement was made at the centre of the image (row B). Due to the perspective effect, each pixel roughly corresponds to 0.005 m here. The valid operating range is therefore larger here than at the bottom of the image, but the accuracy is lower. Within ± 2 m the standard deviation of the detected position was 0.014 m with maximum deviation 0.024 m, see Table 6.2. A narrower range does not affect the standard deviation and maximum error.

<table>
<thead>
<tr>
<th>Operating range (m)</th>
<th>Standard deviation (m)</th>
<th>Max deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2.00, 2.00)</td>
<td>0.014</td>
<td>0.024</td>
</tr>
<tr>
<td>(-1.50, 1.50)</td>
<td>0.013</td>
<td>0.024</td>
</tr>
<tr>
<td>(-1.00, 1.00)</td>
<td>0.014</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Table 6.2 Accuracy in lateral position measurement at row B in the image.
The validation study also included yaw angle measurements. On images in which the angle of the road marking spanned 0 to 12 degrees (only positive angles were investigated), the standard deviation of the detected angle was found to be 0.5 degrees with maximum deviation 1.3 degrees. Looking only at images with road marking angles that spanned 0 to 4 degrees, the standard deviation was reduced to 0.4 degrees with maximum error 0.7 degrees, see Table 6.3. The yaw angle is obtained from lateral position measurements at row C and row B, so the accuracies of those measurements affect the yaw angle data.

<table>
<thead>
<tr>
<th>Operating range (deg)</th>
<th>Standard deviation (deg)</th>
<th>Max deviation (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0, 11.0)</td>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>(0.0, 8.0)</td>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>(0.0, 4.0)</td>
<td>0.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 6.3 Accuracy in yaw angle measurements.

Lateral position measured at row C was also used together with the corresponding yaw angle to calculate the lateral position of the vehicle at the front left wheel. Lateral position was calculated perpendicularly to the vehicle, according to

\[
LP_{\text{left wheel}} = LP_{\text{FOV}} - d \cdot \tan(\psi) \tag{6.1}
\]

where \(d\) is the distance from the cameras FOV to the front left wheel, in this case \(d = 4.25\) m was used. The calculated position was compared to the actual position at the front left wheel. Standard and maximum deviations for these calculations can be found in Table 6.4 and Table 6.5. The operating ranges refer to row C in the image, and not at the front left wheel. The standard deviations lie around 0.035 m regardless of operating range and size of yaw angle, whereas the maximum deviation goes from 0.105 m for large angles to 0.062 m for small angles.

<table>
<thead>
<tr>
<th>Operating range (m)</th>
<th>Standard Deviation (m)</th>
<th>Max deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2.00, 2.00)</td>
<td>0.042</td>
<td>0.105</td>
</tr>
<tr>
<td>(-1.50, 1.50)</td>
<td>0.035</td>
<td>0.105</td>
</tr>
<tr>
<td>(-1.00, 1.00)</td>
<td>0.036</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Table 6.4 Lateral position measured perpendicularly to vehicle with yaw angles up to 12 degrees.

<table>
<thead>
<tr>
<th>Operating range (m)</th>
<th>Standard Deviation (m)</th>
<th>Max deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2.00, 2.00)</td>
<td>0.043</td>
<td>0.086</td>
</tr>
<tr>
<td>(-1.50, 1.50)</td>
<td>0.027</td>
<td>0.062</td>
</tr>
<tr>
<td>(-1.00, 1.00)</td>
<td>0.031</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Table 6.5 Lateral position measured perpendicularly to vehicle with yaw angles less than 1 degree.
6.2 Road Marking Tracking Experiments

The performance of the road marking detection algorithm was investigated over several different short image sequences and a longer one of 1500 frames. By analyzing sequences of images the functionality of the road marking tracking can be studied. The different sequences included normal country road driving, a number of different road marking configurations, such as continuous, intermittent and combinations thereof, images with shadows and lane change manoeuvres. These will be discussed in detail in the subsequent sections.

With the system set-up described in Chapter 3, images were acquired with the back-ward facing camera. Weather conditions during image acquisition included sunny and overcast. The camera was mounted on the vehicle at height \( z=1.55 \) m. The FOV was 1 m behind the vehicle, and 4.25 m from the front left wheel. The frame rate of the camera used in the experiment was 25 fps, and every image frame was processed.

6.2.1 Rate of valid data

The road marking flag indicates when data has been generated, and it is particularly interesting to look at the rate of data. In total on the sequence with 1500 frames the rate of data was 62.3\%. This low rate was largely due to the long sections of intermittent road markings. On The rate of correctly detected road markings however might be lower since the measuring line might not be placed in the correct position. In image sequences with continuous road markings the rate of data was 96\% on average, whereas on intermittent road marking sequences the rate of data dropped to 56\% on average. Every single road marking segment was successfully detected, but the size of the space between the road marking segments lead to gaps in data.

6.2.2 Continuous road markings

Sections of the road with continuous road markings have the best condition to generate reliable and continual data. In a sequence of 140 images, the system successfully detected the continuous road marking in every image frame. See Figure 6.2 for an example image. Hence, the rate of valid road markings was 100\% on this particular sequence. The lateral position data that was generated can be found in Figure 6.3. The smoothness of the lateral position curve indicates that the quality of data is reasonable.

![Image](image_url)

Figure 6.2 The black rectangle indicates the region that was processed. The measuring line can be seen in the centre of the detected road marking.
Chapter 6 – Experiments and Results

6.2.3 Other types of road markings

Particularly difficult image sequences to handle are the ones in which the intermittent road markings pop up in image frames randomly. Image frames with partly occluded road markings are difficult to classify correctly, see Figure 6.4 for an example image. This leads to an error in lateral position. Even though there will be gaps in data generated on sections with intermittent road markings, the available data form a smooth curve. See Figure 6.5. The outliers in the data set are from image frames with partly occluded road markings, usually the first and last of the image frames with visible a road marking segment.

Figure 6.3 Lateral position data from an image sequence with continuous road markings.

Figure 6.4 The system failed in finding the correct edges of this partly occluded road marking. This leads to an error in lateral position data.
Intermittent road markings generate gaps in data, but the available data form a smooth curve. The outliers in the data set are from image frames with partly occluded road markings. The system handles images with other types of road marking configurations as well. This is much thanks to the region of interest though. By focusing only on a small region within the image many ambiguous situations are avoided. In Figure 6.6 a road marking arrow is present in the image, but the system finds the correct edges thanks to the ROI.

Intermittent markings next to continuous markings represent an ambiguous situation. The continuous road marking is present and detected in every image frame, whereas the intermittent marking pops up randomly in the image. The idea of the road marking algorithm is that it finds the strongest two edges in the every image frame, which has been illustrated in Figure 6.7 with three image frames taken in succession.
6.2.4 Shadows

If the image acquisition is done on a sunny day, shadows in the image can be a difficult problem. By restricting the gradient direction the problem of disturbing shadows is greatly reduced, but it is difficult to fully get rid of. The presence of shadows in the image changes the contrast in the image, and the road and road markings are sometimes hard to distinguish even with the eye. An image with shadows that the system failed to classify correctly is illustrated in Figure 6.8.
Chapter 6 – Experiments and Results

6.2.5 Lane change manoeuvre

To keep track of the road marking during a lane change manoeuvre is of crucial importance. Lateral position data from two test sequences, which included lane change manoeuvres of duration 100 and 180 frames respectively, are presented in Figure 6.9 and Figure 6.10 below. In both sequences, the system only produced reliable data in 57 % of the image frames. The low rate of data is due to intermittent road markings. However, the system does not miss a single intermittent road marking in either of the sequences, and the shape of the lateral position data in the diagrams are reasonable. The gaps in data can easily be interpolated.

![Figure 6.8](image)

*Figure 6.8* The system failed to detect the road marking in favour of the stronger edge of the shadow.

![Figure 6.9](image)

*Figure 6.9* Lateral position data from a lane change sequence of 100 frames.
Figure 6.10 Lateral position data from a lane change sequence of 180 frames.
Chapter 7 – Discussion and Conclusions

7 Discussion and Conclusions

7.1 Discussion

With the set-up in this project, the system was able to detect the position of the road marking within ± 2 m from the image centre. This means that the lateral position data can be generated independently of the position of the vehicle in a typical lane with width of 3.75 m. It should be noted that the accuracy and operating range will vary based on the installation height and the optics of the camera system.

The validation study only included 18 images, which is slightly too few. The small data set makes it difficult to determine weather any systematic errors are present. Hence, the accuracy measurements should not be seen as specification, but rather as an indication, of the system performance. Some conclusions, such that the highest accuracy is obtained in the lower centre part of the image and that the maximum operating range was found to be 4 m, can still be drawn from the validation study in spite of the size of the data set. Measuring lateral position along the bottom row of the image gives the best accuracy.

Even though the validation study proved that the position of the road marking could be determined with a standard deviation of 0.012 m, the accuracy might be lower in reality. Especially, the output data over time while driving has slightly worse precision. The camera is repositioned in an unpredictable way while driving, which may be the reason for the variation in data.

The road marking verification step is crucial, but there are several drawbacks of the implemented verification step. For example, positive road markings may be discarded and false road markings may pass through the system.
The prototype system tracks continuous road markings with high accuracy and good precision, resulting in a high rate of valid data on such sections. However, during sections on the road with intermittent road markings, the rate of data cannot be expected to be higher than 56%. In spite of the low rate of data, the lane change sequences still indicate good performance of the road marking tracking functionality of the system. By exploiting temporal correlation the performance of the road marking tracking part of the algorithm was increased extensively. Thanks to functionality of the ROI, the system can successfully detect the correct road marking even though other road markings types are present in the image.

7.2 Problem Analysis

Even though much effort was put into developing a road marking detection method independently of the intensity level of the image, the problem of determining whether a road marking is present or not was solved with an intensity based method. A large set of images have been investigated during the course of the project, but it is not guaranteed that this intensity check will work on future images.

Even though the road marking that has been detected is verified thoroughly to make sure it is in fact a true road marking, there might still be false road markings that passes through. Wheel tracks and shadows can be mistaken for road markings by the systems.

Using the prototype system on roads with wide lanes introduces the risk of detecting the right instead of the centre road marking.

With the approach taken in this thesis, the system will not work on images with strong curves, i.e. during intersections or roundabouts. In order to make the system work on roads with strong curves, the core algorithm of the systems needs to be reconsidered.

7.2.1 Sources of Error

Much effort was put into the development of the peak detection algorithm. The performance of the algorithm has been studied over a large set of different images to make sure that it performs satisfyingly. Choosing the correct coordinates of the peak is essential for the accuracy of the output data.

The manual measuring performed for calibration can introduce errors. By being careful and accurate while performing the calibration set-up, the risk of introducing errors is reduced. The camera model can also be inaccurate, but by choosing a large number of calibration marks the size of the error in the camera model is reduced.

The unpredictable repositioning of the camera while driving can lead to errors in data.

However, it should be mentioned that the performance of the road marking detection can never be better than the input image. The quality of the input image is crucial. The image should have a high contrast in general, but it is essential that there is a high contrast between road marking and road since the algorithm is based on finding the edges of the road markings.
7.3 Conclusions

The vision-based approach to lateral position measurement proved to be very accurate. It is possible to keep track of the road marking at all available times independently of the vehicle position on the road, but the rate of valid output data is highly dependent on the road marking configuration. The system tracks continuous road markings successfully, whereas the gaps between intermittent markings lead to gaps in data. The operating range of the camera is sufficient for the system to work on country roads and motorways with standard lane widths.

The prototype system developed in this thesis performs automatic data analysis that will benefit VTI in its many driving behaviour research programs. The prototype system is a robust and automatic measurement system, which eliminates the drawbacks of manual measuring. The objective data generated by the system will increase the quality of the research.

The possibilities with computer vision and vehicle-mounted cameras in the traffic field are numerous. The work done in this thesis can be used as a starting point in the development of for example a lane departure warning system. The potential of such a system is further increased by merging information retrieved from images with information from the vehicle such as vehicle speed, steering angle and acceleration.

7.4 Future Improvements

The algorithm can be refined to improve the performance of the detection and tracking functionality. The system would benefit from a more sophisticated road marking verification step, which today suffers from letting too many wrongly detected road markings pass through. One significant problem is posed by shadows. Using a template matching technique which takes the shape of road markings into account would lead to a better performance of images with shadows. The colour information in the image could also be exploited to improve the performance. The tracking functionality works satisfyingly in the prototype system. However, on sections with intermittent road markings the ROI is reset each time a gap is encountered. This is a waste of information since the next road marking segment is likely to appear in the same part of the image. The tracking functionality would benefit from remembering the ROI for more than one frame.

Even though the accuracy of this one camera system is satisfying, adding another camera is a possible extension in the future that would increase the quality and accuracy of output data. By stabilising the camera with a gyro on the vehicle, the unpleasant effects of the random changes in image geometry can be decreased which would lead to a higher data quality.

The camera lens and angle puts limitations on the operating range of the system. Using a wide angle lens and letting the camera face the ground are also interesting ideas for future improvements.
8 References


Chapter 8 – References


Chapter 8 – References
Appendices

A1 Peak Detection Algorithm

Figure 1. The original Hough space
Figure 2. The window of the first peak is indicated by the box.

Figure 3. The window of the second peak is indicated by the box.
Figure 4. The window of the third peak is indicated by the box.

Figure 5. The window of the fourth peak is indicated by the box.
Figure 6. The two peaks that were selected are indicated by the white pixels.
B1 Lateral Position Detection Algorithm

*Initialize variables*
- set ROI = entire image
- set $\theta$ = entire range

*Image frame*

*Check road marking availability*
- Not available
- Available

*Pre processing*
- Set search area = ROI
- Apply Wiener filter

*Gradient Analysis*
- Estimate image gradient
- Restrict gradient orientation

*Hough Transformation*
- Transform image to Hough space
- Set parameter range in Hough space = $\theta$

*Peak detection*
- Find coordinates ($\theta_{peak}$, $\rho_{peak}$) of peak values in Hough space

*Back projection*
- Use ($\theta_{peak}$, $\rho_{peak}$) to create a line with slope and intercept in image space

*Verification of road marking*
- No
- Yes

*Update variables*
- set ROI = around measuring line.
- set $\theta = (\theta_{peak} - 10 : \theta_{peak} + 10)$
- set Road marking flag = 1

*Output variables*
- Lateral position
- Yaw angle
- Road marking flag

*Real-world Transformation*
- Transform image coordinates to real-world correspondences

*Reinitialize variables*
- set ROI = entire image
- set $\theta$ = entire range
- set Road marking flag = 0

*Yaw Angle Extraction*
- Calculate yaw angle

*Lateral position extraction*

*Initialize variables*
- set ROI = entire image
- set $\theta$ = entire range
C1 Validation Data

C1.1 Lateral position along row C

<table>
<thead>
<tr>
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<th>Actual position m</th>
<th>Detected position m</th>
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<th>Actual angle deg</th>
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### Lateral position

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<th>Standard deviation m</th>
<th>Max deviation m</th>
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### Yaw angle

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C1.2 Lateral position along row B

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</table>
C1.3 Lateral position at front left wheel

Lateral position at row C was used together with yaw angle to calculate the lateral position at front left wheel (located 4.25 m in front of the camera FOV). Lateral position measured perpendicularly to the vehicle, according to

\[ LP_{\text{left wheel-vehicle}} = LP_{\text{FOV}} - d \cdot \tan(\psi) \]

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<td>-0.062</td>
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<tr>
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<td>-0.016</td>
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<td>5</td>
<td>-0.075</td>
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<td>-0.006</td>
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Appendices

Lateral position at row C was used together with yaw angle to calculate the lateral position at front left wheel (located 4.25 m in front of the camera FOV). Lateral position measured perpendicularly to the road marking, according to

\[ LP_{\text{left wheel--road marking}} = (LP_{\text{FOV}} - d \cdot \tan(\psi)) \cdot \cos(\psi) \]

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På svenska

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