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## GRAVITATION AS A CASIMIR EFFECT

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We describe how dispersion forces between two atoms in ordinary electromagnetism come about and how they may be derived. Then we introduce hypothetical particles interacting with a harmonic oscillator interaction potential. We demonstrate that in a system with this type of interacting particles the retarded dispersion interaction between composite particles varies as  $1/r$ , i.e. just like the gravitational potential does. This demonstrates that gravitation can be a Casimir effect.

*Keywords:* Gravity; Casimir; hypothetical particles.

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### 1. Introduction

J. D. van der Waals found empirically in 1873 that there is an attractive force between all atoms, even between those with closed electron shells. The consensus at that time was that there should not be any long range forces between these type of atoms, but there undoubtedly were. This was a puzzle for a very long time, until 1930, when London<sup>1</sup> gave the explanation in terms of fluctuations, fluctuations in the electron density within the atoms (fluctuating dipoles). This meant the birth of the dispersion force. The force came to be called the van der Waals (vdW) force. The interaction potential varied with distance as  $r^{-6}$  and the force as  $r^{-7}$ . Later Casimir and Polder<sup>2</sup> realized that for large separations the finite speed of light should have the effect that the interaction drops off faster with distance; the potential as  $r^{-7}$  and the force as  $r^{-8}$ . They derived these results using perturbation theory. In this limit the force is usually called the Casimir force or retarded vdW force. The result derived by Casimir and Polder actually covers both the vdW and Casimir ranges and the full result is called Casimir-Polder force. In an alternative description<sup>3</sup> one may, instead of discussing the particles, focus on the electromagnetic fields. Let us study two polarizable atoms in vacuum, atom 1 and atom 2. Let atom 1 at the outset have an induced dipole moment. This dipole moment gives rise to an electric field. Atom 2 gets polarized by this field and attains an induced dipole moment. This dipole moment gives rise to an electric field that polarizes atom 1. If this induced

2 *Bo E. Sernelius*

dipole moment in atom 1 is the same as the one we started out with we have closed the loop and produced self-sustained fields, electromagnetic normal modes. The interaction potential between the atoms is obtained as the sum of the zero-point energy of all all these modes. If one subtracts the corresponding zero-point energies when the atoms are at infinite distance from each other one sets the reference energy to be that at infinite separation. The force is obtained as minus the gradient of the potential.

The two of the fundamental interactions that we experience in our every day life, the Coulomb interaction and the gravitational interaction have the same distance dependence; the potentials vary as  $r^{-1}$  and the forces as  $r^{-2}$ . There are problems attached to both these interactions. The Coulomb interaction has the problem that the energy stored in the electromagnetic fields surrounding an electron is infinite if the electron is a true point particle, which is the general consensus. The Gravitational potential has several problems: One has not been able to detect the graviton; stars on the outskirts of galaxies are moving faster than they should; also galaxies within galaxy clusters are moving faster than they should; an effect noted already by Einstein is that the gravitational interaction is not purely additive – there is a change in inertia of a body when other masses are placed nearby; the Pioneer anomaly.<sup>4</sup> In this work we concentrate on gravitation. The problems just mentioned leads one to the idea that maybe gravitation is not a fundamental interaction after all. The objective of this work is to find out if it is possible that gravitation may be induced from some other and in that case more fundamental interaction.

The article is structured in the following way: Next section, Sec. 2., is intended to familiarize the reader with dispersion forces in electromagnetism. In Sec. 3 we introduce hypothetical particles and describe our basic assumptions. Sec. 4 is devoted to the derivation of the fields from a time dependent dipole, the basic ingredient in the derivation of the dispersion interaction. In Sec. 5 we derive the dispersion potential and finally in Sec. 6 we make a summary and draw conclusions.

## 2. Derivation of Dispersion Forces between Atoms in Electromagnetism

Casimir and Polder<sup>2</sup> derived the dispersion force between atoms using perturbation theory. We follow the more illustrative method of Ref. 3. We start with two atoms in vacuum and assume at the outset that atom 1 is polarized, i.e., has an induced dipole moment,  $\mathbf{p}_1$ , which gives rise to an electric field  $\mathbf{E}_1(\mathbf{r} - \mathbf{r}_1)$  felt by atom 2. Atom 2 becomes polarized and the induced dipole moment,  $\mathbf{p}_2 = \alpha_2 \mathbf{E}_1(\mathbf{r}_2 - \mathbf{r}_1)$  gives rise to an electric field,  $\mathbf{E}_2(\mathbf{r} - \mathbf{r}_2)$ , felt by atom 1. Atom 1 gets polarized by this field and the induced dipole moment is  $\mathbf{p}_1 = \alpha_1 \mathbf{E}_2(\mathbf{r}_1 - \mathbf{r}_2)$ . If now this is the dipole moment we started out with we have closed the loop and obtained self-sustained fields, an electromagnetic normal mode of the two atom system. To

proceed we need the electric field from a time dependent dipole. It is<sup>5,3</sup>

$$\mathbf{E}(\mathbf{r}, t) = -[\hat{\mathbf{p}} - (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \frac{1}{c^2 r} \ddot{p}(t - \frac{r}{c}) - [\hat{\mathbf{p}} - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \left[ \frac{1}{cr^2} \dot{p}(t - \frac{r}{c}) + \frac{1}{r^3} p(t - \frac{r}{c}) \right], \quad (1)$$

for a time dependent dipole at the origin. Fourier transforming this expression with respect to time gives

$$\mathbf{E}(\mathbf{r}, \omega) = - \left\{ [\hat{\mathbf{p}} - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] (1 - i\omega r/c) + [\hat{\mathbf{p}} - (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] (i\omega r/c)^2 \right\} \frac{1}{r^3} p(\omega) e^{i\omega r/c}. \quad (2)$$

It is favorable to use tensor notation. We let the third axis point along the line joining the source and field points. Then the tensors become diagonal. The dipole moments and electric field vectors are now column vectors. We get

$$\mathbf{E}(\mathbf{r}, \omega) = -\tilde{\gamma} \mathbf{p}(\omega) = - \left[ \tilde{\phi} (1 - i\omega r/c) + \tilde{\vartheta} (i\omega r/c)^2 \right] \mathbf{p}(\omega) e^{i\omega r/c}, \quad (3)$$

where

$$\tilde{\phi} = \frac{1}{r^3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}; \quad \tilde{\vartheta} = \frac{1}{r^3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (4)$$

The tensor  $\tilde{\phi}$  is the so-called dipole-dipole tensor. In the non-retarded case  $\tilde{\phi}$  replaces  $\tilde{\gamma}$ .

Now we may write down our coupled equations for the induced dipole moments

$$\begin{aligned} \mathbf{p}_2(\mathbf{r}_2, \omega) &= \alpha_2(\omega) \mathbf{E}_1(\mathbf{r}, \omega) = -\alpha_2(\omega) \tilde{\gamma}(r, \omega) \mathbf{p}_1(\mathbf{r}_1, \omega); \\ \mathbf{p}_1(\mathbf{r}_1, \omega) &= \alpha_1(\omega) \mathbf{E}_2(\mathbf{r}, \omega) = -\alpha_1(\omega) \tilde{\gamma}(r, \omega) \mathbf{p}_2(\mathbf{r}_2, \omega). \end{aligned} \quad (5)$$

Eliminating  $\mathbf{p}_1$  from the equations gives

$$[\tilde{1} - \alpha_1(\omega) \tilde{\gamma}(r, \omega) \alpha_2(\omega) \tilde{\gamma}(r, \omega)] \mathbf{p}_1(\mathbf{r}_1, \omega) = \tilde{A}(r, \omega) \mathbf{p}_1(\mathbf{r}_1, \omega) = 0. \quad (6)$$

The condition for normal modes is that the determinant of the tensor  $\tilde{A}(r, \omega)$  is zero. If we know the analytical expression for the atomic polarizabilities we may find the normal modes. There are two types of mode: modes associated with the atoms, dominating in the vdW range; modes associated with the vacuum, dominating in the Casimir range. In Casimir's famous work<sup>6</sup> on the force between two perfectly reflecting metal plates only vacuum modes entered and there were only a Casimir range. A summation over the zero point energy of the modes obtained here gives us the interaction potential between the two atoms. If we don not know the polarizabilities or if they have complicated analytical forms it is better to use the generalized argument principle<sup>3</sup> and get

$$V(r) = \frac{1}{2\pi i} \oint dz \left( \frac{\hbar z}{2} \right) \frac{d}{dz} \ln |\tilde{A}(r, z)|, \quad (7)$$

where the integration is around a contour in the complex frequency plane enclosing all poles and zeros of the determinant in the right half of the complex

4 *Bo E. Sernelius*

frequency plane. We deform the contour and end up with an integration along the positive part of the imaginary frequency axis

$$V(r) = (\hbar/2\pi) \int_0^\infty d\omega \ln |\tilde{A}(r, z)|. \quad (8)$$

This result is obtained after an integration by parts have been performed. Now with our expression for the determinant we find

$$V_{CP}(r) = -\frac{\hbar}{\pi r^6} \int_0^\infty d\omega \alpha_1(i\omega) \alpha_2(i\omega) e^{-2\omega r/c} \left[ 3 + 6(\omega r/c) + 5(\omega r/c)^2 + 2(\omega r/c)^3 + (\omega r/c)^4 \right]. \quad (9)$$

where we have assumed that the distance is big enough so that the logarithm may be expanded. This is exactly the result obtained by Casimir and Polder.<sup>2</sup> The integrand has interesting property. It can be considered as the product of two factors; the first is the product of the two polarizabilities; the second is the rest of the integrand. Both have a finite range in  $\omega$ . The first has a fixed range while the range of the second depends on the separation,  $r$ . For small  $r$  the second can be considered a constant, for large  $r$  the first can be considered a constant. These two limits are vdW and Casimir limits, respectively. The vdW limit is

$$V_{vdW}(r) \underset{\omega r/c \rightarrow 0}{\approx} -\frac{3\hbar}{\pi} \frac{1}{r^6} \int_0^\infty d\omega \alpha_1(i\omega) \alpha_2(i\omega) = -\frac{3\hbar}{2} \alpha_1(0) \alpha_2(0) \frac{\omega_1 \omega_2}{\omega_1 + \omega_2} \frac{1}{r^6}, \quad (10)$$

where we to get the last equality have used the so-called London approximation<sup>1</sup> for the atomic polarizabilities  $\alpha_j(i\omega) \approx \alpha_j(0) / [1 + (\omega/\omega_j)^2]$ . The frequencies  $\omega_j$ ;  $j = 1, 2$  are characteristic frequencies for the atoms. The Casimir limit is obtained as

$$V_C(r) \underset{r \rightarrow \infty}{\approx} -\frac{23\hbar c}{4\pi} [\alpha_1(0) \alpha_2(0)] \frac{1}{r^7}. \quad (11)$$

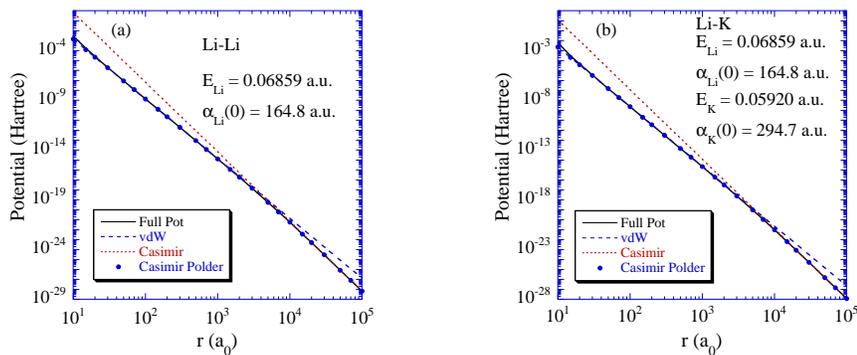


Fig. 1. (a): Fitting of the Casimir Polder potential with London approx. for a  $Li - Li$  dimer. Dashed (dotted) curve is the Casimir (vdW) asymptote. Full circles are the result from the fit. The solid curve is the "exact" result by Marinescu and You; (b): Result for  $Li - K$

Using Eq. (9) together with the London approximation for the atomic polarizabilities works extremely well. We have performed such calculations for all alkali-metal dimers and fitted the results to those from the best available state of the art ab initio calculations.<sup>7</sup> The two parameters for the atomic polarizability were determined through this fit. The obtained static polarizability was in all cases within the spread of experimental values. The characteristic frequencies have no experimental counterpart. We show in Fig. 1a, the results for the  $Li - Li$  dimer. The dashed (dotted) curve is the Casimir (vdW) asymptote. The filled circles are the fitted results. The solid curve is the "exact" results from Ref. 7. The fit is very good except for a small region at closest separation. Here higher order multi-pole contributions show up, not included in the Casimir Polder results. Then we performed calculations for all combination of atom pair with different atoms keeping the originally fixed parameters. All results fitted equally well in this test of the robustness of the calculations. As an example we show in Fig 1b, the results for the  $Li - K$  combination.

### 3. Introducing Hypothetical Particles

In electromagnetism the charged particles, electrons and protons, are responsible for the induced forces – the dispersion forces. Could the gravitational force be the result of a more fundamental interaction between some other particles? This is what we assume in this work. We introduce hypothetical particles with a different fundamental interaction potential. We make the basic assumptions that this interaction travels in vacuum with the speed of light and that Einstein's two postulates in special relativity holds also for this interaction. We will then parallel the derivation of the dispersion forces in electromagnetism.

We have reasons to believe that any fundamental interaction potential should form closed classical particle orbits. There are only two central force fields where all bound orbits are closed: one has the interaction potential  $V \sim -r^{-1}$  and the other has  $V \sim r^2$ . The second type is known as a harmonic oscillator potential and is our obvious choice of candidate.

Our notation throughout this work is in analogy with the electromagnetic case. We put a tilde above the quantities to distinguish them from the electromagnetic counterparts. We assume that the particles have charge  $\pm\tilde{q}$  and postulate that the electric field from a charge  $\tilde{q}$  at the origin is  $\tilde{\mathbf{E}} = \tilde{q}\mathbf{r}$ . This gives the scalar potential the form  $\tilde{\varphi}(r) = -\tilde{q}r^2/2$ . This potential is without bounds. This means that the particles can never be found individually; they are always in pairs. It takes an infinite energy to separate them completely. This type of interaction leads to composite particles that are neutral and have no direct interaction; dispersion interactions due to fluctuations are possible.

To find the effect from fluctuating electric dipoles we start with the field from a dipole with dipole moment  $\tilde{\mathbf{p}} = \tilde{q}\mathbf{d}$ . It is found to be  $\tilde{\mathbf{E}}(\mathbf{r}) = -\tilde{\mathbf{p}}$ . Thus the field from a dipole is just minus the dipole moment. There are no quadrupole or higher

6 *Bo E. Sernelius*

order multi-pole contributions as opposed to in the ordinary electromagnetic theory. Furthermore the field has no spatial dependence. This holds for any static distribution of charges within the composite particle – the field lacks spatial dependence. This implicates that the only static field is a dipole field and this dipole field is constant throughout all space, independent of the position of the particles. There are no other multi-pole fields. This is very encouraging.

From Einstein's two postulates in special relativity follows that there has to be a magnetic field companion to the electric field. Furthermore these two fields obey the two homogeneous Maxwell's equations,  $\nabla \times \tilde{\mathbf{E}} + (1/c) \partial \tilde{\mathbf{B}} / \partial t = 0$  and  $\nabla \cdot \tilde{\mathbf{B}} = 0$ . This means that we may introduce scalar and vector potentials,  $\tilde{\varphi}$  and  $\tilde{\mathbf{A}}$ , respectively, where  $\tilde{\mathbf{B}} = \nabla \times \tilde{\mathbf{A}}$  and  $\tilde{\mathbf{E}} = -\nabla \tilde{\varphi} - (1/c) \partial \tilde{\mathbf{A}} / \partial t$ , all in complete analogy with in electromagnetism. Now we have all we need for our derivations. We start by determining, in the next section, the fields from a time-dependent electric dipole.

#### 4. Fields from a time dependent dipole

We have derived<sup>8</sup> the fields from a time dependent dipole placed at the origin and pointing in the  $z$ -direction in complete analogy with the derivation by Heald and Marion<sup>5</sup> of the field from a Hertzian dipole in electromagnetism. Here we are starting from the retarded potentials that we postulate to be

$$\begin{aligned} \tilde{\varphi}(\mathbf{r}, t) &= -\frac{1}{2} \int d^3 r' \tilde{\rho}(\mathbf{r}', t - R/c) R^2; \\ \tilde{\mathbf{A}}(\mathbf{r}, t) &= \frac{1}{2c} \int d^3 r' \tilde{\mathbf{J}}(\mathbf{r}', t - R/c) R^2; \quad \mathbf{R} = \mathbf{r} - \mathbf{r}', \end{aligned} \quad (12)$$

where  $\tilde{\rho}$  and  $\tilde{\mathbf{J}}$  are the charge and current densities, respectively. We arrive at

$$\begin{aligned} \tilde{\mathbf{E}}(\mathbf{r}, t) &= \left\{ -[\tilde{p}] + 2 \left(\frac{r}{c}\right) [\dot{\tilde{p}}] - \left(\frac{r}{c}\right)^2 [\ddot{\tilde{p}}] \right\} \cos \theta \hat{r} \\ &\quad + \left\{ [\tilde{p}] - \frac{1}{2} \left(\frac{r}{c}\right) [\dot{\tilde{p}}] + \frac{1}{2} \left(\frac{r}{c}\right)^2 [\ddot{\tilde{p}}] \right\} \sin \theta \hat{\theta}; \\ \tilde{\mathbf{B}}(\mathbf{r}, t) &= \left\{ -\left(\frac{r}{c}\right) [\dot{\tilde{p}}] + \frac{1}{2} \left(\frac{r}{c}\right)^2 [\ddot{\tilde{p}}] \right\} \sin \theta \hat{\varphi}. \end{aligned} \quad (13)$$

A dot means the time derivative and square brackets that the function within the brackets is determined at retarded times,  $t - r/c$ . Now we have all we need to calculate the dispersion forces between the composite particles, but before we do so we reformulate the electric field in the equation above to a form similar to that in Eq. (1)

$$\begin{aligned} \tilde{\mathbf{E}}(\mathbf{r}, t) &= (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \left[ -\tilde{p} \left(t - \frac{r}{c}\right) + 2 \left(\frac{r}{c}\right) \dot{\tilde{p}} \left(t - \frac{r}{c}\right) - \left(\frac{r}{c}\right)^2 \ddot{\tilde{p}} \left(t - \frac{r}{c}\right) \right] \\ &\quad + [\hat{\mathbf{p}} - (\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \left[ -\tilde{p} \left(t - \frac{r}{c}\right) + \frac{1}{2} \left(\frac{r}{c}\right) \dot{\tilde{p}} \left(t - \frac{r}{c}\right) - \frac{1}{2} \left(\frac{r}{c}\right)^2 \ddot{\tilde{p}} \left(t - \frac{r}{c}\right) \right]. \end{aligned} \quad (14)$$

We Fourier transform the equation with respect to time and rewrite it on tensor form. We have

$$\begin{aligned} \tilde{\mathbf{E}}(\mathbf{r}, \omega) &= e^{i\omega r/c} \left\{ \tilde{\alpha} \left[ -1 + 2(-i\omega r/c) + (\omega r/c)^2 \right] \right. \\ &\quad \left. + \tilde{\beta} \left[ -1 + \frac{1}{2}(-i\omega r/c) + \frac{1}{2}(\omega r/c)^2 \right] \right\} \tilde{\mathbf{p}}(\omega), \end{aligned} \quad (15)$$

where the tensors are:  $\tilde{\alpha} = r_\mu r_\nu / r^2$ ;  $\tilde{\beta} = \delta_{\mu\nu} - r_\mu r_\nu / r^2 = \tilde{I} - \tilde{\alpha}$ . We let all tensors have double tildes to distinguish them from our fields. Things become very simple if we choose the third principle axis to point along  $\mathbf{r}$ . Then we have

$$\tilde{\alpha} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \tilde{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and  $\tilde{\alpha}^2 = \tilde{\alpha}$ ;  $\tilde{\beta}^2 = \tilde{\beta}$ ;  $\tilde{\alpha} \cdot \tilde{\beta} = 0$ . If we now let

$$\tilde{\gamma} = e^{i\omega r/c} \left\{ \tilde{\alpha} \left[ 1 + 2(i\omega r/c) - (\omega r/c)^2 \right] + \tilde{\beta} \left[ 1 + \frac{1}{2}(i\omega r/c) - \frac{1}{2}(\omega r/c)^2 \right] \right\}, \quad (16)$$

we have  $\tilde{\mathbf{E}}(\mathbf{r}, \omega) = -\tilde{\gamma}(r, \omega) \tilde{\mathbf{p}}(\omega)$ , in analogy with Eq. (3) in the electromagnetic case. Now we are ready to derive the dispersion potential between composite particles.

## 5. Derivation of the Dispersion Potential

We define the polarizability,  $\tilde{\alpha}$ , for a composite particle through:  $\tilde{\mathbf{p}} = \tilde{\alpha} \tilde{\mathbf{E}}$ . The we use the same procedure for finding the normal modes as in Sec. 2 for the two atom case. The derivations are in complete analogy to the electromagnetic case. We arrive at

$$[\tilde{1} - \tilde{\alpha}_1(\omega) \tilde{\gamma}(r, \omega) \tilde{\alpha}_2(\omega) \tilde{\gamma}(r, \omega)] \tilde{\mathbf{p}}_1(\mathbf{r}_1, \omega) = \tilde{A}(r, \omega) \tilde{\mathbf{p}}_1(\mathbf{r}_1, \omega) = 0, \quad (17)$$

and the condition for having normal modes is that the determinant of  $\tilde{A}(r, \omega)$  is zero. The interaction potential between two composite particles becomes

$$V(r) = \frac{1}{2\pi i} \oint dz \left( \frac{\hbar z}{2} \right) \frac{d}{dz} \ln |\tilde{A}(z)|. \quad (18)$$

We deform the contour in the complex frequency plane as before and end up with an integration along the upper part of the imaginary axis. Assuming that the polarizabilities are small enough we may expand the logarithm and finally arrive at

$$V(r) = -\frac{\hbar}{4\pi} \int_0^\infty d\omega \tilde{\alpha}_1(i\omega) \tilde{\alpha}_2(i\omega) e^{-2\omega r/c} [6 - 12(\omega r/c) + 17(\omega r/c)^2 - 9(\omega r/c)^3 + 3(\omega r/c)^4]. \quad (19)$$

This result resembles the Casimir Polder result of Eq. (9) very much but there is one very important difference: There is no  $1/r^6$  factor in front. Two separation limits emerge, the vdW limit for small separations, and the Casimir limit for large. In the vdW limit we have

$$V(r) \approx - (3\hbar/2\pi) \int_0^\infty d\omega \tilde{\alpha}_1(i\omega) \tilde{\alpha}_2(i\omega); \quad r \ll c/\omega_0, \quad (20)$$

where  $\omega_0$  is some characteristic frequency above which the polarizabilities are negligible. Note that the interaction potential lacks an  $r$ -dependence in this range and

8 *Bo E. Sernelius*

consequently there is no force. If we assume that the characteristic frequencies are the same for the two composite particles and that the so-called London approximation<sup>1</sup>,  $\tilde{\alpha}(i\omega) = \tilde{\alpha}(0) / [1 + (\omega/\omega_0)^2]$ , may be used for the polarizabilities the potential in the van der Waals limit becomes  $V(r) = -(3/8) \tilde{\alpha}_1(0) \tilde{\alpha}_2(0) \hbar\omega_0$ .

In the Casimir limit we have

$$V(r) = -(25\hbar c/32\pi r) \tilde{\alpha}_1(0) \tilde{\alpha}_2(0); \quad r \gg c/\omega_0, \quad (21)$$

and the force is

$$F(r) = -(25\hbar c/32\pi r^2) \tilde{\alpha}_1(0) \tilde{\alpha}_2(0). \quad (22)$$

Here, the potential has the  $1/r$ -dependence that we hoped for. The polarizability is unit less and very small so the expansion of the logarithm is well motivated. If the characteristic frequencies for the particles are big enough there are no detectable vdW limit which means that the interaction potential may have the  $1/r$ -dependence at all distances.

## 6. Summary and conclusion

We have introduced hypothetical, fundamental particles with harmonic oscillator interaction potential. We found that these particles induce a dispersion interaction potential between composite particles. The Casimir potential has the desired  $1/r$ -dependence and the attractive force goes like  $1/r^2$ . The pre-factor is a product of one factor depending on one of the particles and one on the other, just as in the gravitational force. In the van der Waals range the potential is constant and the force vanishes. If the characteristic frequency of the composite particles is big enough there is no observable van der Waals range. If the obtained force is identified as the gravitational force the mass gets a new interpretation as a constant times the static polarizability of the composite particle,  $m = \sqrt{25\hbar c/32\pi\gamma\alpha}(0)$ .

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