Coordinated Routing
– applications in location and inventory management

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Abstract

Almost everywhere, routing plays an important role in everyday life. This thesis consists of three parts, each studying different applications where routing decisions are coordinated with other decisions.

In the first part, an application concerning waste management is presented. Many industries generate garbage, and instead of handling the waste disposal themselves, other companies, specialized in garbage collection, handle the disposal. Each industry rents containers from a company to be used for waste, and the garbage collection companies handle the collection. The industries buy a service including one or more containers at the industry and the garbage collection companies are obliged to make sure that the containers never become overfull. The idea is that the industries buy this service and in return, the garbage collection company can plan the collection so that the overall cost and the number of overfull containers is minimized. Two models for the problem facing the garbage collection company are proposed. The first is solved using a Lagrangean relaxation approach on a flow based model, and the second is solved using Benders decomposition on a column based model.

The second part investigates a distribution chain management problem taken from the Swedish pulp industry. Given fixed production plans at the mills, and fixed customer demands, the problem is to minimize the distribution cost. Unlike many other models for marine distribution chains, the customers are not located at the harbors. This means that the model proposed also incorporates the distribution planning from the harbors to the customers. All customers are not served from the harbors; some are served directly from the mills using trucks and trains to distribute the pulp, and these decisions are also included. The problem is modeled as a mixed integer linear program and solved using a branch and price scheme. Due to the complexity of the problem, the solution strategy is divided into two phases, where the first emphasizes the generation of schedules for the vessels operated by the company, while the second deals with the chartering of vessels on the spot market.

In the third part, routing is combined with location decisions in the location-routing problem. Special emphasis is given to strategic management where decision makers must make location, capacity and routing decisions over a long planning period. The studied application comes from strategic school management, where the location and capacity of the schools as well as their catchment areas are under consideration. The problem is modeled as a mixed integer linear program. The computational study shows the importance of incorporating a routing component allowing multiple visits, as well as the danger of having a too short planning period.

A common denominator in all applications is that an intelligent utilization of a fleet of vehicles is crucial for the performance of the system. Another is that the routing decisions must be coordinated with other decisions. In the first part, routing and inventory management decisions are coordinated, in the second part, routing decisions concerning different modes of transportation are coordinated with inventory management, and in the third part, location decision and routing are coordinated.
Acknowledgement

I took my first course in operations research "Introduction to Operations Research" in the fall of 1994. I remember it as a very fun course (I don’t think I would have been here writing this acknowledgement if I hadn’t remembered it that way) where I really could see the practical use of mathematics. I also remember the examiner, Peter Värbrand, and the teaching assistant, Stefan Engevall, for being good teachers, and really enjoying what they were doing. Five years (and four OR courses) later, the same Peter Värbrand offered me a position as PhD-student in Infrainformatics under his supervision. I accepted, and found out that he also is a very good supervisor. With this thesis, journey ends and another one begins. Thank you Peter for guiding me during the first one.

My old friend and master thesis co-worker Tobias Andersson started as a PhD-student one week before me, and May 20 2005 he became the first PhD ever in Infrainformatics. We were roommates for most of the time, and even though I don’t see us as competitors, he made me push myself a little harder than I would have without him. I have a lot to thank you for, both as a co-worker and as a friend. Thanks buddy!

When I started as a PhD-student, I became a member of the Communications and Transport System group. We weren’t many at that time, but we have grown, both in strength and in numbers. I express my gratitude to all current and former members of the group. I also want to thank the whole department including the technical group and the administrative personnel.

I spent six months in Fredericton, Canada, working as a research assistant at the University of New Brunswick. A special thanks goes to Professor H.A. Eiselt. Thank you for a very nice time.

But life is much more than work and long hours at the office, all my fantastic friends in Norrköping, Linköping, Stockholm, Gothenburg, and the rest of the world

    Live long and prosper

Last, but not least, I want to thank the people who always support me, my wonderful family and my beautiful girlfriend Heidi.

Norrköping, December 2005
Henrik Andersson
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Introduction

route /ˈrʌt/; NAmE also rəut / noun, verb

- noun 1 (from A to B) a way that you follow to get from one place to another:  
  Which is the best route to take?  
  Motorists are advised to find an alternative route.  
  a coastal route  
  the quickest route from Florence to Rome  
  an escape route- see also EN ROUTE 2 a fixed way along which a bus, train, etc. regularly travels or goods are regularly sent:  
  The house is not on a bus route.  
  shipping routes  
  a cycle route (= a path that is only for CYCLISTS) 3 ~ (to sth) a particular way of achieving sth:  
  the route to success 4 used before the number of a main road in the US:  
  Route 66

- verb (rout·ing or route·ing, rout·ed, rout·ed) [VN, usually + adv./prep.] to send sb/sth by a particular route:  
  Satellites route data all over the globe.

This definition, taken from Oxford Advanced Learner’s Dictionary, makes it easy to understand the enormous importance routing has in modern society. It is an integrated part of not only everyday life, but also the logistics networks and the information web that cover a large part of the earth.

Knowingly, or without knowing, we make routing decisions every day. For example, when you went to work this morning, why did you take the way you took? Most people will answer something like 'Because it's the shortest way', 'Because it's the most convenient way' or 'I always take that way'. Not many will answer 'Oh, it just happened, I took some right turns and some left turns and suddenly I was here'. This shows that we are decision-makers, or at least most of us are. We have a number of different choices of how to get from our front door to the office, we rank them according to some criteria and we choose the alternative we rank the highest¹. Sometimes we even change our mind while we are on our way. In some way or other, the assumptions change, we reevaluate the alternatives and act accordingly. This often works well, and we seldom need fast computers to help us make the decision, but some days there are just too many options and unknowns for us to cope with, and we wish that something or someone could support us in the decision making process.

This is often the case when decisions are made at companies, by the government or the local government. The problem is too big with complex relations between the decisions and their consequences to be able to survey without a proper decision support tool. Here, Operations Research makes an entrance. Operations Research, OR, can be explained as the art and science of applying advanced analytical methods to help making better decisions. Starting as a distinct branch of science during World War II, the impact of OR can today be seen in many areas as different as logistics, sports and DNA sequencing. OR seeks the best, or optimal, utilization of scarce resources, something that often is done according to the following process

¹The questions of how we find these alternatives and how we rank them will not be answered in this thesis, I gladly leave these questions for other researchers.
Introduction

The OR process

1: Identify the problem  
2: Choose a solution strategy  
3: Construct a model  
4: Solve the model  
5: Analyze the result

Since the last four steps depend on the first, it is not possible to overemphasize the importance of the identification phase. Carefully examining the situation, and really analyze what problem to solve is necessary for the success of the OR process. Once the problem has been identified, the solution strategy best suited for this particular problem is chosen. From the OR tool kit; including simulation, optimization and decision theory among others, one or more tools are selected. After the problem identification and the choice of solution strategy, the next step is to construct a model. In this phase, the identified problem is quantified and simplified to a more stringently stated problem, which in its turn is formulated as a model. What kind of model that is formulated depends on the solution strategy chosen. If optimization is used, a mathematical model with an objective function and constraints may be appropriate, and if a heuristic approach based on local search seems better, the characteristics of a solution and its neighborhood must be specified. The model is then solved using the appropriate tools. These may include commercial software packages, special programs written for the particular problem or a mix of both. In a first analysis phase, the verification phase, the result is compared to the stringently formulated problem to see if the model actually solves that problem. If this phase is successful, the OR process sometimes ends here, and the result is presented as a solution to the real problem identified in Step . This could be devastating for the credibility of the whole process. Instead, a second analysis phase, the validation phase, should start. In this phase, both the result and the stringently formulated problem are evaluated with respect to the identified problem to see if all important aspects of the real problem are accounted for in the formulated problem. If the verification phase fails, the model must be reformulated to better correspond to the formulated problem. If the validation phase fails, the formulated problem must be restated to better agree with the real problem. Once both analysis phases succeed, the result may actually have a chance of being accepted as a solution to the real problem.

Operations research is today extremely powerful in solving problems within its scope. Both the technical development, giving us faster computers with more memory, but especially the development of new solution techniques have enabled us to solve problems that were considered way too big a couple of years back. As much as this is a great achievement and a great help for the researchers and practitioners within the field, there is also a risk of overusing it. Without really thinking of how a particular problem can be solved, we alter it so that it fits our favorite OR tool; not because this solution strategy is best suited for the problem, but because we like it so much. There are many more pitfalls when working with operations research, one of them being the problem of quantifying. When solving real problems, there will always be parameters that cannot easily be measured, or factors that are measured in different units; how do you measure equity when locating a public facility, or how much is it
worth to shorten the travel time to work with ten minutes? Without being aware of the difficulty and danger when quantifying, and the effect the quantified values can have on the overall results, operations research is a quite fragile tool.

In spite of all the problems, pitfalls and difficulties, operations research is a fantastic decision support tool when used in the right way. Reading and hearing about success stories where OR has been an important part is inspiring, and a great help those long, dark hours at the office.

This thesis consists of three parts, each studying different problems where routing plays an important role. The dynamic coordination of deliveries is the main concern of the first part. Combining routing decisions and inventory management in a logistics context leads to a very hard and challenging problem called the inventory routing problem. Paper 1 and 2 deal with this problem.

The second part is a distribution chain management problem. The distribution chain is extracted from a supply chain of a Swedish pulp producing company and involves inventory management as well as multi modal routing decisions. Special emphasis is given to the marine transportation decisions involving routing and scheduling of a fleet owned by the company, but also chartering vessels on the spot market. Paper 3 and 4 are devoted to this problem.

Due to the different planning horizons of location and routing problems, they are seldom combined in the decision making process. In the third part presented in this thesis, a combined problem, called the location-routing problem is discussed. Paper 5 is an annotated bibliography of the problem, and in Paper 6, a dynamic location-routing problem, called the relocation-routing problem, related to the managing of schools and school transports is presented.

A common denominator in all these problems is that an intelligent utilization of a fleet of vehicles is crucial for the performance of the system. Another is that the routing decisions must be coordinated with other decisions. In the first part, routing and inventory management decisions are coordinated, in the second part, routing decisions concerning different modes of transportation are coordinated with inventory management, and in the third part, location decision and routing are coordinated.

The remainder of this chapter is devoted to a survey of areas related to the problems analyzed in the thesis. Since both the inventory routing problem and the location-routing problem have the vehicle routing problem in common, the survey starts with this problem followed by distribution-inventory problems, ship routing problems and location-routing problems. The section about location-routing problems is shorter than the other sections since one of the papers, Paper 5, is an annotated bibliography about this area.


1 Vehicle routing problems

The Vehicle Routing Problem, VRP, is a very well studied problem, and there are many reasons for this. One reason is the large number of applications related to different kinds of vehicle routing, and another reason is that the problem is very easy to state but very hard to solve.

A good starting point for explaining VRP is the Traveling Salesman Problem, TSP. Given a network with a set of nodes, $V$, and a set of edges, $E$, connecting these nodes, TSP is the problem of finding the shortest cycle passing every node exactly once. If not only one, but $M$ cycles are used, the problem is called $M$-TSP. In $M$-TSP, one of the nodes is called the depot, and has to be included in each cycle. The problem is then to find exactly $M$ disjoint, except for the depot, cycles that together contain all nodes such that the sum of the lengths of the cycles is minimized. Instead of minimizing the length of the cycles, a weight can be associated with each edge and the objective is then to minimize the sum of the weights.

In a real-world application, the fairly simple structure of $M$-TSP is often not enough to satisfactorily describe the assumptions of the problem to be solved. To be able to handle more complex situations, different restrictions are added to $M$-TSP, and thus turning it into VRP. The general VRP consists of finding a collection of cycles, henceforth called routes, such that the total cost of the routes is minimized. A common case is that the routes are used by vehicles to visit the customers and collect or deliver goods. The routes should be constructed in such a way that

- All nodes except one, the depot, should be visited exactly once
- All routes start and end at the depot
- A set of restrictions are satisfied

Many different restrictions can be imposed. The most natural constraint to add is to restrict the capacity of the vehicles, hence limiting the number of customers each vehicle can visit. This is done by associating a weight with each customer and then limit the total weight each vehicle can load. The capacity of the vehicles, i.e. the total weight a vehicle can load, can be limited in either a homogeneous way, i.e. all vehicles have the same capacity, or a heterogeneous way, i.e. the vehicles have different capacities. If the weights of all customers are equal, the problem is called the identical customer VRP, and it is possible to restrict the number of visits on a route instead of the capacity. If at least two customers have different weights, the problem is called the Capacitated Vehicle Routing Problem, CVRP. Another common restriction is generalized distance restriction. If a travel time is associated with each pair of customers, it is possible to impose a restriction on the total travel time for each vehicle, as well as it is possible to restrict the length of each route. In this case, the problem is called the Distance-Constrained VRP.
Capacity and/or distance restrictions are seldom enough when trying to model a real-world situation, but other restrictions, called side constraints, are often needed as well. Sometimes there is a restriction on when a customer can be visited, i.e. the customers have an earliest and latest visiting time. In this case, there is a time window restriction, and the problem is called VRP with time windows.

Another class of restrictions has to do with the order in which the customers are visited. In VRP, the problem is either a pure delivery or a pure pickup problem, meaning that the order in which the customers are visited does not matter. In many practical applications, goods are delivered from the depot to some customers and picked up at others and delivered to the depot. In these applications, the order in which customers are visited is crucial. The problem is called VRP with backhauls.

Different side constraints can of course be combined into rather exotic problems such as VRPBTW, the Vehicle Routing Problem with Backhauls and Time Windows. For more information about VRP and many of its variants, the reader is referred to the book by Toth and Vigo [96].

2 Distribution-inventory problems

In a traditional customer-supplier situation, the customer orders goods from the supplier when the amount of goods in the inventory is below a certain level. The supplier collects orders from many customers and then solves the routing problem over the customers who ordered. This means that the traditional distribution-inventory problem is a two-level problem where the customer takes the inventory decision and the routing decision is taken by the distributor, based on the inventory decision. The drawback with this situation is that the possibility to coordinate transportations to different customers is limited.

In the late 80’s, a new concept called vendor managed inventory started to become popular. In a vendor managed inventory situation, the supplier has full control over the replenishment of the inventory at the customers, and thus the transportations could be coordinated and the distribution costs reduced. In this case the supplier takes both the inventory decisions and the routing decision. The only obligation for the supplier is to make sure that the inventory never becomes empty. The fusion of the inventory and routing decisions into one distribution-inventory decision makes the vendor managed inventory problem a one-level problem and by that, it is possible to obtain better solutions.

Vendor managed inventory is an assumption in most studied cases. The supplier can make all decisions about when to deliver and how much to deliver as long as a pre-defined agreement is fulfilled. The agreement often specifies the minimum acceptable inventory level at the customer, but may also include various kinds of compensation agreements, e.g. penalty costs if the inventory level drops below the minimum level, or actions that must be taken by the supplier in emergency situations.
For a general overview of integrated analysis of production-distribution systems, the reader is referred to the excellent review by Sarmiento and Nagi [90].

The number of possible ways to integrate the routing problem and the inventory problem is very large, but mainly three kind of problems have been studied. These are:

- Single period problems where the customers have stochastic demands. The general problem here is to balance the transportation costs, the inventory costs and the shortage costs.

- Multi period problems with either stochastic or deterministic demands. The general problem in this case is often to minimize the transportation costs while maintaining an adequate amount of inventory at each customer. These problems will henceforth be denoted Inventory Routing Problems.

- Problems with an infinite time period with customer specific and deterministic demand rates. The general problem in this case is to determine policies for the replenishment at each customer as well as vehicle routes.

2.1 Single period problem

The probably first paper addressing this problem is Federgruen and Zipkin [47] in which they investigate the combined problem of allocating a scarce resource available at a central depot among several locations using a fleet of vehicles. The problem is a single period problem with stochastic demands. At the beginning of the day, the inventory levels at all customers are known. Based on this, the scarce resource is allocated to the customers and the routes are made. The demands occur at the end of the day and the total cost including routing, inventory and shortage costs is calculated. The model has three different sets of decision variables, the first determines the movements of the vehicles, the second the assignment of customers to routes and the third the amount delivered to each customer. The solution approach is based on the observation that if the second set of variables is fixed, the problem decomposes into an inventory allocation problem and one TSP for each vehicle. The algorithm starts with an initial set of routes, i.e a feasible assignment of customers to vehicles, and then evaluates changes, using r-opt methods, in the assignment. The feasibility check after such a swap, which is crucial in VRP, is not necessary since all assignments are feasible. Instead the inventory allocation problem needs to be reoptimized, something which is undesirable. While the inventory allocation problem could be reoptimized for each potential swap, a better way is to approximate the change and only solve the problem when implementing a switch.

As a second part, Federgruen and Zipkin present an exact algorithm for the problem using generalized Benders decomposition. The problem is somewhat reformulated and projected onto the second set of decision variables, thus forming the master problem. The subproblems in the algorithm are newsboy problems, which are solved and used to find cuts to add to the relaxed master problem.
Chien et al. [30] develop a Lagrangean based procedure to generate heuristic solutions and good upper bounds to a similar problem. There are two main differences between the problems studied by Federgruen and Zipkin and Chien et al. In the latter, the maximum possible demand for each customer is deterministic and known in advance, while in the first, no such bound is known. The second difference is in the objective function. Federgruen and Zipkin use an objective function consisting of a routing cost, an inventory carrying cost and a shortage cost, while Chien et al. use a revenue-penalty cost function. In the revenue-penalty cost function, each unit delivered to a customer earns a revenue and each unit of unsatisfied demand incurs a penalty. In addition to this there is a fixed routing cost and a flow based routing cost. Chien et al. relax the problem using Lagrangean relaxation. For a set of fixed multipliers, the problem decomposes into one inventory allocation problem and one customer assignment/vehicle utilization problem. The inventory allocation problem is a continuous knapsack problem and hence easy to solve. The customer assignment/vehicle utilization problem can be further decomposed into continuous knapsack problems. Both subproblems are solved to optimality and then a subgradient method is used to update the multipliers. Using the solutions from the relaxed problem, a heuristic in two phases is used to find feasible solutions. In the first phase, an initial set of vehicle routes is constructed based on the inventory allocation and the customer assignments and in the second phase the routes are improved in a greedy fashion. Chien et al. also propose an approximate method for solving the multi period problem where the single period problem is a subproblem. The idea is to solve the single period problem for a certain time period and then use this solution to calculate the initial inventory available at the depot and the maximum customer demands for the next time period. The single period problem is then solved for this day and the process is continued until the last time period.

For a more detailed analysis of the single period problem, the reader is referred to Federgruen and Simchi-Levi [46].

2.2 Multi period problem

Both Bell et al. [11] and Golden et al. [56] investigate a distribution problem in the liquid gases industry. Golden et al. do a comparison between the current distribution rule used by the industry and the heuristic algorithm proposed by the authors. An iterative solution process that outperforms the existing rule is developed. First a subset of customers, called potential customers, is identified as candidates for possible delivery using a threshold function. If the inventory level at a customer is less than $\alpha$ % of the inventory capacity, the customer is a candidate, otherwise it is not. In the second step, which customers to select is calculated by solving a time constrained TSP with a modified objective function reflecting the urgency of resupplying the customers. Then a VRP over the selected customers is solved using a Clarke and Wright based method and the routes generated are combined to form day-long work schedules for the vehicles. If it is impossible to form day-long routes, the time constrained TSP is solved once more with a tighter time constraint. When a feasible set of day-long routes has been generated, the amount of gas distributed to each customer is calculated.
Bell et al. formulate the problem as a mixed integer program and use a construct-select approach to solve the problem. First a large set of feasible routes is constructed and then an optimal set of routes is picked from the generated routes. The generated routes only contain information about which customers that are visited and in what order, but not information about when the route starts or the actual amount delivered to each customer. This route generation is possible because the number of customers on a route is small, which makes the number of possible routes reasonable small. A heuristic is used to decide whether or not a particular, technically feasible, route should be included. The planning horizon, between 2 and 5 days, is divided into one hour time periods and a decision is taken if a route, operated by a specific truck, starts in the time period or not. A Lagrangean relaxation approach is used to decompose the problem into knapsack-like problems, one for each vehicle. The solutions to the Lagrangean subproblems are then used in a primal heuristic to obtain feasible solutions to the problem.

Even if the problems in Bell et al. [11] and Golden et al. [56] look similar, there are differences between them. The two most significant differences are the objective functions and that the problem in Bell et al. almost lacks the routing component since the number of customers on a route is small, on average about two.

Another similar problem is investigated by Brenninger-Göthe [22]. Even though this problem is similar to the problems investigated by Bell et al. and Golden et al., there is a big difference in the solution approach. Brenninger-Göthe formulates the problem as a multi period VRP and uses an assignment heuristic and approximation scheme presented by Fisher and Jaikumar [48]. In the original model presented, the objective function consists of a routing cost and a fixed cost for visiting a customer. The idea in the approximation scheme is to reformulate the model into a model where the routing cost is included in the cost for visiting a customer. Using this scheme, the problem decomposes into a cardinality constrained pure fixed charge network flow problem and a number of TSPs. The flow problem gives an assignment of the customers to the vehicles in the different time periods using an approximation of routing cost in the objective function, and the TSPs give the correct objective values. The TSPs are easy to solve since the number of customers visited by each tour is small, so the effort is on the solution of the other problem. Two solution approaches for the cardinality constrained pure fixed charge network flow problem is presented. In the first approach, a Lagrangean relaxation gives subproblems satisfying the integrality property. The problem is strengthened in such a way that the inequalities added are redundant in the original problem but not in the subproblems. In the second approach, the strengthened problem is combined with a constraint generation procedure to produce even better bounds.

The problem of minimizing the annual delivery cost while attempting to ensure that no customer runs out of stock is addressed in Dror et al. [41]. The demand at a customer equals the storage capacity minus the inventory level, and it is assumed that the whole demand is delivered when the customer is served. Another assumption made is that if a customers inventory becomes empty, it is immediately served at a very high cost. Due to the size of the long-term problem, a reduction procedure is needed to be able to solve it. In [41], a procedure consisting of two steps is presented.
In the first step, the planning period of the problem is reduced to a manageable length by approximating the costs for not visiting a customer during the planning period. The second step is the solution of the reduced problem. Starting with the planning period, the customers are divided into two subsets, customers that need to be served in the planning period and customers that do not need to be served. Different costs are associated with each subset. For a customer that needs to be served, the cost reflects the difference in future cost between visiting the customer earlier than the latest possible day and visiting the customer the latest possible day. For the other subset, the cost reflects the difference in future cost between visiting the customer and not visiting the customer. Starting with a deterministic single customer model, Dror and Ball [40] derive the different costs. Their next step is to extend the model to a stochastic single customer model. The main observation in this case is that the expected cost associated with scheduling a delivery on a certain day achieves its minimum at a single point, i.e. the ideal replenishment policy is to serve the customer on this day. The paper closes with the multiple customer problem which is very close to the problem in [41]. Two different approaches to solve the problem are investigated in [41]. Both approaches are assign first-route second approaches following the ideas from Fisher and Jaikumar [48]. In the first, customers are assigned to specific days, and then a VRP is solved for each day. In the second approach, customers are assigned to specific days and specific vehicles, and a TSP is solved for each day and vehicle. Only the first approach is implemented. The reason for this is two-folded, the generalized assignment problem in the first approach is easier than in the second approach, and there does not seem to be a natural surrogate objective that the Fisher and Jaikumar heuristic needs. A third approach, not based on assignments, is also investigated. In this approach, the Inventory Routing Problem is viewed as a modified VRP. There are essentially three modifications. The first is that all customers need not be served, the second is that the customer demand depends on by which vehicle the customer is visited, and the third is that the costs are vehicle dependent. The algorithm can be seen as a drop-algorithm. First a VRP including all customers is solved for each day in the planning period. Here the number of vehicles is sufficiently large so that a feasible solution exists. Based on the costs of these routes, each customer is reassigned for delivery to exactly one day. In the final step, this solution is made feasible by a rather complicated procedure involving interchanges, insertions and deletions.

Campbell [27] and Campbell and Savelsbergh [28] develop a two phase procedure for the Inventory Routing Problem. In the first phase the whole planning period is considered, one month in the specific case, and the problem is to assign the customers to different days and to decide how much to deliver to each customer. In the second phase, only the first few days are examined and the problem is to construct the actual routes and schedules for the vehicles. The first phase is modeled as a huge integer problem resembling a set covering problem, i.e. from the set of all possible routes, choose the cheapest subset fulfilling all constraints. To make the integer program possible to solve, it is reduced in two ways. First the time periods at the end of the planning period are aggregated and second, only a subset of all possible routes is considered. The strategy to select a small but good subset of routes among all possible is based on a concept called clusters. A cluster is a subset of customers that can be served cost effectively by one vehicle for a long period of time. First a large set of different clusters are generated, then the cost of serving each cluster is estimated and finally a set partitioning problem
Distribution-inventory problems

is solved to select clusters covering all customers. During the first phase, reductions
are made to decrease the number of clusters, the number of routes within each cluster
and the number of customers considered in the model. The solution from phase one
specifies the amount of gas to deliver to each customer, but not departure times and
routes for the different vehicles. Although it is the best actions in the long run, it may
not be as good from a short-term perspective. Instead of using the solution from phase
one as an absolute rule and solve the phase two problem as a VRPTW, the solution is
considered only as a recommendation in phase two. An insertion heuristic is used, in
which a move away from the phase one solution can be balanced by a better short-term
solution. When the last insertion is made, the delivered volume is optimized using a
derived optimal policy.

In two recent papers, [91] and [92], Savelsbergh and Song introduce a new version of
the Inventory Routing Problem that addresses some of the complexities not included
in earlier models. These complexities are: limited product availability at facilities, cus-
tomers that cannot be served by single day direct shipping and delivery tours covering
several days. In [91], a randomized greedy heuristic is developed. The heuristic is
based around an urgency measure, which is defined as the remaining time before a cus-
tomer runs out of products. All customers are sorted according to their urgency, and
the heuristic randomly selects a customer. The selected customer is inserted into the
route of the most appropriate vehicle, and the route and urgency measure are updated.
The delivery schedule produced by the heuristic is then improved by optimizing the
delivery volumes without changing the routes. This results in an increase in the volume
delivery per mile, which is an important measure when evaluating solutions to the
Inventory Routing Problem. An extensive computational study testing the heuristic,
the delivery volume optimization and a rolling horizon framework shows good results. A network based mixed integer linear program for the same problem is de-
scribed in [92]. The nodes in the network represent a visit to a customer or facility
at a particular time. A drawback with these kinds of formulations is their size, but
this is handled by a rolling horizon framework, aggregation of time periods at the end
of the planning period and different reduction techniques. With a minimum delivery
quantity defined, it is possible to delete all nodes corresponding to a certain customer
outside the earliest and latest times of delivery. Arcs connecting far away customers
as well as uneconomical facility-customer relations are also removed. To solve the
model, a branch and cut algorithm is proposed. A new valid inequality, called delivery
cover inequality, is derived and added throughout the tree search. A scheme based on
branching over arcs connecting clusters of nodes is used. The branch and cut algo-

Maybe the most important component of the multi period problem is how to handle
the long-term effects. It is often impossible to solve a single problem that includes all
periods because of the explosion of variables and thus the size of the problem. Instead,
a problem with few periods, that in some way reflects the long-term effects, is often
solved. Bell et al. [11] solve the problem every day with a 2–5 days planning horizon
and the schedules for the first day are executed. Golden et al. [56] use a daily customer
selection reflecting the relative difference between the remaining tank level and the
tank capacity. Dror et al. [41] and Dror and Ball [40] reduce the long-term problem
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to a five day planning period by approximating the future cost of visit or not visit a
customer. Brenninger-Göthe [22] decomposes the problem into smaller subproblems
without changing the structure of the long-term effects, problems with planning pe-
riods of 10–23 days are solved. Campbell [27] and Campbell and Savelsbergh [28]
aggregate the one month planning period and start with assigning customers to differ-
ent days. In Savelsbergh and Song, [91] and [92], the five day problem is solved using
a fine time discretization for the first two days, and a coarser for the last three.

2.3 Infinite planning horizon

In the case with finite planning horizon, the demand at the customers is often assumed
to vary. This is not the case when the planning horizon is infinite; instead each cus-
tomer is assigned a demand rate. The introduction of demand rates makes it possible
to derive replenishment strategies, involving a set of fixed routes executed on a regular
basis, instead of changing routes.

In Burns et al. [26], two different distribution strategies are investigated, direct ship-
ping and peddling. Direct shipping is when a truck only delivers goods to one cus-
tomer before returning to the depot and peddling is when the trucks deliver goods to
more than one customer per load. The depot acts as a supplier, producing for each
customer at the same rate as the demand rate of the customer, i.e. the sum of the
demand rates of all customers is the production rate of the supplier. There are three
different costs; a fixed cost of initiating a dispatch, a variable cost associated with the
distribution and an inventory carrying cost associated with the inventory. The in-
ventory includes items waiting to be shipped from the supplier, items in transit to
customers and items waiting to be used by the customers. The goal is to minimize the
total distribution and inventory cost, and to calculate the optimal shipment size and
truck-dispatching rate. Since the number of trucks is assumed to be large, different cus-
tomers can be optimized separately. This means that the optimal shipment size in the
direct shipping strategy is the minimum of the economic order quantity, EOQ, and
the truck capacity. In the peddling case, the customers are divided into subsets where
each subset forms a delivery region. Peddling to a delivery region involves three trans-
portation stages; line-haul, local and back-haul. In the line-haul, a truck travels from
the supplier to the nearest customer, after that the customers in the delivery region
are visited, the local stage, and then the truck returns to the supplier, the back-haul.
One assumption made in [26] is that the truck visits exactly one delivery region and
then returns empty to the supplier. Instead of using coordinates of the customers, only
the customer density is used. Small delivery regions imply small local transportation
costs but a higher inventory cost since more goods are delivered to each customer.
This means that the trade-off between the transportation cost and the inventory cost
is dependent on the size of the delivery region. When a truck visits a certain delivery
region, the load should be distributed among the customers in the region. Burns et al.
assume that not all customers in the region need to be visited. Instead the probability
that any item in the load belongs to a specific customer is a random variable depending
on that customers demand rate relative to the average demand rate. When this relation
is established, it is used to calculate the average number of stops per load.
When deriving an analytical expression for the total peddling unit cost, the fact that the optimal load size equals the truck size is proven and used. The most important result in [26] is that peddling is less expensive than direct shipping when the distributed items are valuable. The advantage increases with the distance from the supplier, the customer density and the inventory carrying charge. It also increases as the average customer demand decreases.

Anily and Federgruen [2] consider a similar problem. The customer demands, as in [26], occur at a deterministic, constant but customer specific rate. The transportation cost consists of a fixed cost per route and a variable cost proportional to the length of the route. The inventory costs are only dependent on the goods stored at the customers, a difference compared to [26]. The objective is to find long-term integrated replenishment strategies that enable all customers to meet their demands while minimizing the long-term inventory and transportation costs. An integrated replenishment strategy is defined as a strategy including both inventory rules and routing patterns. Even for very simple inventory rules, the integrated replenishment strategies are very complex since optimal delivery routes need to be determined and this includes solving VRP. Anily and Federgruen consider a special class of replenishment strategies, called \( \Phi \)-strategies, with the following properties: a strategy specifies a collection of subsets of customers that covers all customers. A customer can belong to different subsets, but it is specified how much of the demand of the customer belongs to each subset. Each time one of the customers in a subset receives a delivery, all other customers in the subset are also visited. Anily and Federgruen derive upper and lower bounds on the over-all costs for the defined strategies, and show that these bounds are asymptotically optimal under certain conditions. In addition to this, three more observations are worth noticing. There exists a uniform upper bound on the total demand rate in a single region, each subset of customers is visited by a vehicle at a constant visiting rate, and the delivered amount of goods is the same every time. But, since each customer can belong to different subsets, the time interval between consecutive deliveries at a specific customer does not have to be constant. The last observation is that there exists a critical distance such that only fully loaded vehicles depart to customers further away from the depot.

In another paper by Anily and Federgruen [3] the analysis in Anily and Federgruen [2] is extended. In [3], central inventories may be kept at the warehouse. This means that in addition to the problem in [2], a replenishment strategy for the warehouse must be determined. The inventory cost is the same for all customers, but different for the warehouse, and there is a fixed ordering cost for the warehouse. The goal is to minimize the system-wide long-run inventory, transportation and ordering costs. Both the uncapacitated case, where there only is a bound on the total demand of a subset, and the capacitated case are examined. In the capacitated case, there is also an upper bound on the frequency a certain route can be driven. Anily and Federgruen [3] show that the gap between their proposed strategy and a lower bound on the minimum cost among all \( \Phi \)-strategies is at most 6 %, if the number of customers is sufficiently large, and that the gap stays small even for problems with a moderate number of customers.

With the same assumptions as in Anily and Federgruen [2], Gallego and Simchi-Levi [54] show that direct shipping is at least 94 % effective whenever the minimal eco-
nomic lot size over all customers is at least 71% of the truck capacity, and that the effectiveness deteriorates with the minimal lot size. They define the effectiveness of a strategy as the ratio of the infimum of the long-run average cost over all strategies to the long-run average cost of the strategy in question.

2.4 Other related problems

When a distribution/routing system is to be developed, not only the inventory and routing decisions are important, but also decisions concerning the fleet of vehicles. This situation is called the strategic inventory routing problem. In the strategic inventory routing problem, the objective is to balance not only the inventory carrying cost and the transportation cost, but also the cost of the fleet of vehicles. Larson [70] presents a strategic inventory routing problem concerning the development of a logistics system to transport municipal sewage sludge from city-operated wastewater treatment plants to an ocean dumping site 106 miles offshore. The heuristic proposed is based on the Clarke and Wright savings algorithm for VRP, which is somewhat changed to account for the special nature of the problem.

In the articles by Bard et al. [7], [8] and Jaillet et al. [60], the Inventory Routing Problem with satellite facilities is addressed. At a satellite facility, the vehicles can be reloaded and the routes can continue until the maximum route time is reached. In [8] the solution approach for the actual Inventory Routing Problem is presented. The algorithm is a decomposition scheme in five steps. In the first step the optimal replenishment day for all customers is calculated using results from [60]. If a customer is not visited on its optimal replenishment day, an incremental cost is added, which is calculated as the difference between the optimal strategy and the non optimal strategy. The customers with an optimal replenishment day within the current planning horizon are then assigned to a given day in the planning horizon by solving an assignment problem, which minimizes all incremental cost while trying to balance the expected total demand per day. In the next step a VRP with satellite facilities is solved for each day. In [8], three different heuristics are proposed and tested; randomized Clarke and Wright, a greedy randomized adaptive search procedure (GRASP), and a modified sweep, while a Branch and Cut algorithm for the same problem is developed in [7]. After VRP is solved heuristically, arc and node interchanges between routes used the same day are performed to improve the solution. At the last step, the trade off between incremental costs and route lengths is examined by swapping customers between different days of the planning horizon.

3 Ship routing problems

Much of the worlds transportations are between continents. Bananas and ore are transported from South America to Europe, coal from Australia to Japan, grain from America to Asia and so forth. The only reasonable way to transport these cargos is by seaborne shipping. Seaborne activities are very dependent on the services offered by the world’s fleet. As with all modes of transportation, the routing and scheduling of
the vessels are crucial as well as complex problems.

On a higher level, the routing and scheduling of ships is not different from routing and scheduling of any other vehicles. The main goal is to utilize the fleet in the best possible way, either by minimizing the cost of performing a set of predefined tasks, or by maximizing the revenue by selecting among a number of different transportation possibilities. Even if the main goal for all routing and scheduling problems may be the same, ship routing and scheduling problems differ from the standard vehicle routing and scheduling problem in some aspects:

- A fleet of vessels is often heterogeneous, and may differ in loading capacity and speed, as well as cost structure. In the standard problems, a homogeneous fleet is often assumed.

- The traveling times between two ports are often long, making it possible, and sometimes even probable, to change the destination while at sea. Changing the destination during a trip is not possible in the standard problems.

- The weather affects the voyages, making the travel time between ports vary. Both heavy weather as well as ocean currents may slow down the vessel, causing the delivery to be delayed. The travel times are considered fixed in the standard problems.

- Vessels are operated around the clock, making the opening hours of the ports important. In the standard problems, the vehicles are often idle during the night.

There are of course many other differences, the interested reader is referred to the surveys by Ronen [86], [88] and the survey by Christiansen et al. [34] for further examples.

Ship operations can be divided into three different categories; liner, tramp and industrial operations. The distinction is made based on the way the vessels are operated, but there are no clear borders between the categories. In the next section, the three modes will shortly be explained. Since this survey focuses on industrial shipping, this mode is given its own extended description in a later section.

3.1 Modes of operation

**Liner:** In liner shipping, the vessels follow published itineraries and schedules. The itineraries and schedules of different ship owners are often coordinated. In the liner shipping industry, many companies operate within conferences, which are international groups of companies that collectively agree on routes, schedules, rates, and other aspects of the liner service between the members of the conference. The conference is based on that the customers, the cargo owners within the conference, undertake to only use the services of the ship owners within the conference. The ship owners, on the other hand, will use the same tariffs for all customers within the conference. This means that the ship owners do not compete with the price, but with service.
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Liner shipping is mainly transporting low volumes of high value cargos, which can be seen by the estimation that liner vessels carry about 20% of the world trade volume, but between 70 and 80% of the value of cargos in sea transportation. Often container vessels and general cargo vessels are used in liner shipping. The main part of the cargo is transported in standardized containers between large container ports, called hubs. Around the hubs, feeder systems are used to supply the hub with containers from smaller ports and to ship containers from the hub to the smaller ports.

The overall goal in liner shipping is to maximize the profit per time period. Planning liner shipping involves strategic, tactical and operational decisions. On the strategic level, the size and mix of the fleet as well as route and schedule design are important decisions. On the tactical level, the assignment of the different vessels in the fleet to the routes is the main decision. Which cargos to accept and reject is the main decision at the operational level.

**Tramp:** Tramp shipping implies that one or many vessels are chartered for a specific assignment of transportation. The chartering is done through contracts of affreightment, which are contracts regulating what product or products to be shipped, between which ports, when the shipping takes place and the price per ton. The contracts can be formulated in many different ways, and may run from a single voyage to as long as the life of the vessel.

Vessels in tramp shipping are directed to the harbors where cargos are. This means that the tramp market becomes a very well functioning market, the slightest change in supply and demand directly affects the price. Typical tramp vessels are tankers, dry bulk carriers and refrigerated vessels.

As for the liner shipping, the overall goal in tramp shipping is to maximize the profit per time period and the planning of tramp shipping involves strategic, tactical and operational decisions. There are always the decisions about fleet size and mix on the strategic level. Which contracts to sign can be seen as decision making on all three levels, a long time charter contract is clearly a strategic decision, while a one voyage contract is an operational decision. A common solution for a ship owner is to long time charter part of the vessel’s capacity, and then voyage charter or short time charter the rest of the capacity. On the operational level, the decisions are not only to accept a cargo or not, but also to avoiding ballast trips.

**Industrial:** In some cases, the same company owns both a fleet of vessels and the cargos. In industrial shipping, the goal is not to maximize the profit, but to minimize the cost for shipping all cargos. If it is not possible to ship all cargos using the fleet, extra vessels may be chartered. Here is a clear example of the floating borders between the different modes, the cargo owner see the chartering of a vessel in the context of industrial shipping, while the ship owner sees it from a tramp shipping point of view.

For an extensive overview of all three modes of operations, as well as many other problems concerning ship routing and scheduling, the presentations by Ronen [86], [88] and Christiansen et al. [34] are recommendable.
3.2 Industrial operations

This chapter will focus on a number of characteristics concerning industrial shipping and try to classify the problems described by those characteristics. In Table 1 on page 26, the papers and their classification is shown. The classification follows the classifications in Bodin et al. [16] of general routing and scheduling problems and Ronen [86] of ship routing and scheduling problems, but is more condensed than the latter. The categories are; number of ports, number of commodities, fleet size and mix and type of demand. Number of ports is a summary of the number of loading ports and the number of discharging ports per vessel voyage, the alternatives are one to one, one to many, many to one and many to many. Depending on the alternative, the routing complexity will change.

Much of what is transported in industrial shipping is bulk cargo, which is cargo carried in loose form without any packing. This means that the products to be shipped must be homogeneous in terms of quality, grade and other classifications. Two cargos from the same origin to the same destination cannot be transported together unless they are indistinguishable. Such products include crude oil, most solid minerals, some chemicals in solid or liquid form and food. To load more than one product on the vessel, the cargos can either be unitized or the cargo hold can be partitioned. Number of commodities is a summary of the number of commodities to be shipped and the number of commodities a vessel can carry at the same time, the alternatives are; one–one, i.e. the problem only concern one commodity, many–one; i.e. it is a multi commodity problem, but each vessels can only load one commodity at a time, and many–many, i.e. it is a multi commodity problem, and each vessel can load more than one commodity. The complexity of the problem usually increases with the number of different commodities.

The category fleet size and mix consists of two parts. The first part, fleet size, describes the number of vessels studied and can be either one or many. In the second part, fleet mix, the type of vessels is described. A homogeneous fleet, where all vessels have the same characteristics, is classified as a one type fleet, while a heterogeneous fleet, with vessels of different types, is classified as a many type fleet. The category fleet size and mix therefore have the alternatives one–one, many–one and many–many. In some applications, each demand is unique, meaning that the demand is specified by a loading port, a discharging port, a time of delivery and so on. In the case when the same product can be loaded at different ports and still meet the same demand, the demand is considered general. This gives that the category type of demand has the alternatives unique and general.

Due to the great resembles between many problems in tramp and industrial shipping planning, and since not many papers are written about tramp shipping routing and scheduling, papers concerning tramp shipping are included in the presentation.

The problem of minimizing the number of vessels given a number of cargos is investigated by Dantzig and Fulkerson [39]. The problem is formulated as a transportation problem. The results are the number of vessels needed as well as schedules for each vessel. Minimizing the sailing time for a given number of vessels is also discussed as
another possible cost function. Mixing strategic decision making, fleet sizing, short-
term planning, and scheduling of the vessels, is not very common, but due to the very
simple assumptions, it is possible in this case.

Flood [51] presents a problem of minimizing the expected total distance to be traveled
in ballast by a fleet of vessels. The problem arises when petroleum is shipped between
designated load and discharge points. Since the fleet is fixed, and the loading and dis-
charging ports are specified for each cargo, the problem is simply to find the cheapest
routing of the empty tankers consistent with the shipping requirements. A computa-
tional study shows a 5% decrease in the total cost compared to if only one-delivery
round-trip voyages are used.

A similar problem to the one discussed in Dantzig and Fulkerson [39] is formulated
by Laderman et al. [66]. A heterogeneous fleet is used to transport contracted cargos
between certain ports on the four great lakes of the USA. Instead of minimizing the
number of vessels, the total operating time required for the vessels to carry out the con-
tracted shipments is minimized. The result is the number of times each vessel should
make a certain voyage. The proposed model is a linear program, but as it turns out,
the non integral values in the solution causes little problem. Another problem with
the formulation is subroutines, but practical considerations often eliminate them. The
model presented in [66] is extended in a note by Whiton [102], where cargo handling
capacity constraints and docking capability and capacity constraints are discussed.

Also Briskin [23] discusses a problem similar to Dantzig and Fulkerson [39], but looks
at it from a more strategic point. In [23], ports demanding less than a full shipload are
grouped into clusters which can accept a full shipload. The problem is to decide when
to visit a cluster, which ports within the cluster to visit when the cluster is visited,
and how much to discharge at each port. A dynamic programming approach is used
to solve the problem. In order to reduce the state space, only combinations of integer
days’ supply are used as decisions. This paper does not fit within the classification and
is therefore excluded from Table 1.

An early example of column generation used for solving a ship routing problem is
presented in Rao and Zionts [85]. The problem discussed is to allocate vessels to
alternative trips at minimum expense while satisfying a set of cargo commitments.
The cargos can be delivered either by a vessel from a predefined fleet or by chartered
vessels. The main focus of the paper is to overcome the problem with large numbers of
variables and constraints. Two different models are presented, the first is a flow based
model resulting in a large number of constraints. The second model is column based
and is solved using column generation.

Appelgren [4], [5] formulates a ship scheduling problem where a set of cargos and
a fleet of vessels are defined. The size of the cargos is comparable to the size of the
vessels, and hence one cargo can be loaded at the same time. The goal is to assign a se-
quence of cargos to each vessel in order to maximize the revenue. In [4], the problem
is formulated and solved using Dantzig-Wolfe decomposition. The solution approach
sometimes gives fractional solutions that cannot be interpreted as feasible ship sched-
ules. In [5], the focus is on the problem with fractional solutions from the master
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problem of the Dantzig-Wolfe decomposition. A cutting plane based method and a branch and bound based method are presented. Only the branch and bound based method guarantees feasible integer solutions. All cargos are not contracted, making the problem a tramp shipping problem.

The papers by Appelgren can be seen as a modeling paradigm shift by the introduction of a sequence of cargos as decision variable. This decision variable makes it possible to divide the ship routing and scheduling problem into two problems, one sequence design problem and one sequence allocation problem. Even though column generation is used in Rao and Zionts [85], which was published before Appelgren [4], the use of a complete cycle of trips as decision variable is not as neat and flexible as the sequence of cargos.

In Bellmore et al. [12], a multi-vehicle tanker scheduling problem is presented. The problem is to route and schedule a fleet of heterogeneous tankers to make a predefined set of shipments. Time windows are defined for the delivery dates, and all shipments must not be made. The objective is to maximize a utility function reflecting the desirability to receive the shipments, the reassignments of the tankers and the services of the tankers. The problem is formulated as a multi commodity flow problem, and extended to incorporate the possibility of partially loaded tankers. A Dantzig-Wolfe decomposition is proposed together with either a cutting plane method or a branch and bound approach to get integral solutions. No results are presented.

McKay and Hartley [74] solve a tanker scheduling problem where a set of assets of different petroleum products and a set of requirements are given. Two different models are proposed, a non linear integer model and a linear integer model. The objective in the first model is to minimize the transportation cost and the purchasing cost while satisfying all requirements. In the second model, a reward is given for satisfying a requirement and for filling the tankers, and the goal is to maximize the rewards minus the transportation cost and the purchasing cost. In the second model, all requirements must not be satisfied. A solution strategy for the second model based on LP relaxation and a rounding scheme is proposed. No results are presented.

Stott and Douglas [62] describe a model based decision support system implemented at an American company. The system consists of a number of subsystems and can handle both the long-term planning as well as decisions concerning the chartering of spot vessels, leading to greater flexibility and effectiveness for the management. Experiments show that using the system can increase the profit.

An interactive decision support system designed to simulate different voyage alternatives for a chemical tanker vessel is described in Boykin and Levary [18]. The system provides a tool to rapidly simulate different itineraries in order to evaluate for example the possibility of accepting an additional load by changing the steaming speed. The decision support system helps the management to save money by reducing the time for planning voyage itineraries, but no savings due to better planning is reported.

The problem of scheduling a fleet of vessels in order to carry a planned set of shipments at minimal cost is discussed by Ronen [87]. Three different solution approaches are
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tested. The first approach is an exact algorithm, based on enumeration. The problem is formulated as a mixed integer non linear problem, and then a set of variables representing port entries are fixed, resulting in a generalized transportation problem. This is repeated for all feasible fixations, and the best solution is saved. In the second approach, a vessel is randomly chosen. A schedule for the vessel is randomly generated and shipments are assigned to the vessel until the vessel is loaded, then a new vessel is chosen and the process is repeated. When all cargos are assigned to the vessels, the solution is evaluated. This is repeated many times and the cheapest solution is selected. The third approach is a heuristic that tries to minimize cost per ton-mile of cargo using a construction scheme. The three approaches are compared to the industry rule of thumb strategy. The proposed algorithms outperform the industry strategy in all cases, but due to long computational times, the exact algorithm is not tested on all instances.

In [24], Brown et al. investigate the problem of transporting crude oil from the Middle East to Europe and North America. The problem is modeled as a set partitioning model, where each column corresponds to a feasible vessel schedule. The only difference between this model and the model presented in Appelgren [4] is the cargo constraints. In [24], all cargos must be loaded, while in [4], some cargos are optional. The proposed algorithm works in four steps, where the first step is a column generator that provides a complete set of feasible schedules. The second step calculates the costs of all feasible schedules. In the third step, different operations are done on the coefficient matrix in order to shorten the solution time of the LP relaxation of the model. The last step is to solve the model. In actual operations, constraints can be violated, but at a cost. This is modeled using an elastic formulation, where penalized variables are added to reflect the infeasibilities.

One of the first papers that combine inventory management and ship scheduling is presented by Miller [76]. A decision support system that consists of four different components is developed. The first component creates a feasible solution using a construction heuristic. This solution is then presented to the scheduler through the report and graph generator. The scheduler can change the solution either manually, or by using the improvement routines of the support system. The fourth component is a schedule evaluator that evaluates the schedule created by the scheduler in terms of its objective value and feasibility. Due to the complexity of the problem, the integer program formulated is not solved, but instead a network model is used. The system is used as a short-time planning tool as well as to monthly update the schedules on a rolling 18 months basis, but other application areas are also mentioned.

A problem similar to the problem discussed in Brown et al. [24] is presented by Fisher and Rosenwein [49], but unlike both this problem and Appelgren [4], all vessels must not be assigned routes. The application investigated by Fisher and Rosenwein is to transport bulk petroleum products worldwide. An fleet owned by the company as well as spot carriers is used to transport a set of cargos. Each cargo is specified by a designated quantity of product to be lifted from one or more load ports to one or more delivery ports, as well as time window constraints on the earliest and latest loading and unloading times. The problem is formulated as a set packing problem. Each column in the model corresponds to a feasible schedule for a vessel in the fleet. Each
Ship routing problems

A schedule consists of a set of cargos that can feasibly be transported by a vessel within the planning horizon. Since the number of cargos lifted by a single vessel within the planning horizon is small, it is possible to enumerate all or almost all feasible schedules. The problem of maximizing gross profit is solved using a Lagrangean based method, and the results are presented through a graphical interface. Results presented show that the cost significantly decreases compared to manual scheduling.

Scheduling the movement of crude oil cargos by tankers is discussed in Perakis and Bremer [82] and Bremer and Perakis [21]. The authors develop ideas from both Appelgren [4], [5] and Fisher and Rosenwein [49], but the proposed model differs in certain respects. In [82], the time horizon is not fixed, but the schedules differ in length, and the objective is to minimize the cost. To handle the problem with schedules of different length, a concept called daily value cost is used. Daily value is defined as the earnings that can be generated by a given vessel in its best alternative use, which often is to out-charter a company owned vessel. The daily value is added to the operational cost of a schedule. The proposed algorithm generates all feasible schedules prior to the integer programming problem, and then solves the problem using commercial software. The algorithm is embedded in a larger decision support system, the implementation of this system together with some results are presented in [21].

Both a different problem and different solution approaches are presented by Vukadinović and Teodorović [98] and Vukadinović et al. [99]. The problem discussed in both papers is the process of loading, transporting and unloading gravel by inland water transportation. The gravel is loaded onto barges at a loading port. Then the barges are transported using pusher tugs to different unloading harbors. When a pusher tug has left the loaded barges at the unloading harbors, empty barges are transported back to the loading port. The key operational problem is the assignment of the barges to pusher tugs. In [98], the problem is solved using fuzzy logic modeling by representing the decisions about waiting time and number of barges left at the unloading harbor as fuzzy sets. Tests show that the results agree with the decisions made by the dispatcher. In [99], a neural network approach is used to solve the problem. Using a two-layered network, results comparable to the current planning are obtained. It is concluded that the proposed neural network can be used as a decision support system for the dispatcher to assign barges to tugs, but it cannot replace human decision making.

How to ship large quantities of bulk commodities under uncertainty is discussed in Kao et al. [63]. Given a system with known demands, but with uncertain shipping times, the problem is to decide on the re-order point and the ordering quantity for each commodity in order to minimize the overall cost. The objective function is the sum of the procurement cost, the holding cost and the shortage cost. Due to limited storage capacity, the total inventory is bounded. The problem is formulated as a non linear program, but since the feasible region as well as the objective function is convex, a method based on the Kuhn-Tucker conditions is proposed. The method is tested on a real-life scenario with good results. This application does not fit into the classification and is therefore excluded from Table 1.
facility and a shipment port, where the different products are loaded onto vessels. The vessels travel from the shipment port to transshipment ports where they discharge the products. There are warehouses at the transshipment ports, and from the warehouses, the products are transported to the customers. To reduce the size of the problem, the customers are grouped into clusters, and each cluster is assigned to the nearest transshipment port, but can be served by other ports when the nearest port is closed. A flow based integer linear model maximizing the revenue of selling the products minus the ocean shipping cost, the loading and discharging costs and the warehousing cost is presented. The model is solved using a branch and bound algorithm. First, a heuristic solution is found by solving the problem with only a minimum number of ports, thus reducing the size of the problem. With this as a starting solution, the whole problem is solved. The solution found is then reviewed by the management to ensure that corporate policies in marketing and transportation were followed. The solution showed that the company should decrease the number of ports used as well as change their distribution policy.

One way of solving multi-commodity multi-vessel scheduling problems is presented in Scott [93]. A scheduling problem usually involves constructing a schedule for a fleet of vessels in order to move a set of commodities. Any limitation or constraint of the problem is a combined vessel and commodity constraint, a vessel only constraint or a commodity only constraint. In [93], the scheduling problem is formulated as a maximization problem using these three sets of constraints. The combining constraint is relaxed using Lagrangean relaxation. The remaining subproblem then decomposes into one vessel scheduling problem and one commodity flow problem. From the relaxation, upper and lower bounds on the objective value of the original problem are derived. The feasible solutions to the relaxed problem with an objective value larger than the lower bound is then evaluated by fixing the vessel part of the solution. It is shown that the optimal solution to the original problem must be among the evaluated solutions. The algorithm is tested on a ship scheduling problem involving the shipping of refined oil products, from a refinery to several depots, and it is proved efficient and finds good solutions quickly.

In [29], Chajakis describes three problems arising in the marine based crude oil transportation area. Following the supply chain of petroleum from the oil fields, via tankers to the refineries where the petroleum is made and then further to the consumers, the first problem described is to generate optimal short-term schedules for the tankers. A system is presented that takes purchasing, tanker chartering, inventory, as well as production costs into account. When the fully loaded tanker arrives at a refinery, draft restrictions may prevent the tanker from approaching the discharge dock. In this case, the tanker can discharge onto smaller vessels until its draft has been adequately reduced. The tanker and the smaller vessel can then approach the discharge dock. The scheduling of the smaller vessels is the second problem described, and a simulation model is developed for this problem. The third problem is to distribute the petroleum from the refineries to tank terminals. A simulation model is developed to analyze this barge scheduling problem.

A decision support system for scheduling a fleet of vessels on the tramp market is presented by Kim and Lee [64]. A set packing model, similar to the one in Appelgren [4],
in which schedules for the vessels are the decision variables, is developed for the sys-
tem. The schedules can be described using a directed acyclic network, and a depth first
graph traversal algorithm generates all feasible schedules. The model is solved using
commercial software and presented using a self-developed graphical user interface.

The distribution of coal in a logistics system consisting of multiple suppliers, one
transshipment port and multiple customers is described in Shih [95]. The coal is
transported from the suppliers using chartered bulk vessels, and is then stored in ware-
houses at the transshipment port. From the warehouses, the coal is distributed to
power plants where it is consumed. A flow based integer linear program minimizing
the costs of inventory, procurement and transportation is developed. The model is
tested on a real-life scenario for a company, and the optimal solution provides a basis
for the planning and scheduling at the company.

Bausch et al. [9] describe a decision support system used to dispatch shipments of
bulk products by vessels and barges among bulk distribution terminals, plants and
industrial customers. A set of cargos is to be transported. Each cargo consists of order
volumes for a number of products, an earliest loading date, a loading location, a latest
discharge date, and one or more discharging locations. Either company owned vessels
or vessels chartered on the spot market can be used to make the deliveries. Besides
the set of cargos that must be transported, optional backhauls may be available. The
problem is formulated as an elastic set partitioning problem, but the formulation is
different from the one presented in Brown et al. [24]. Written in EXCEL, the system
converts inputs from the user to a database. From this database, all feasible schedules
for each vessel are constructed. After the construction, the cost of each schedule is
calculated, and the schedules and costs are fed to the solver. The solution is converted
to a Gantt chart where the recommended activities of each vessel during the planning
period are presented.

In the papers by Christiansen [32] and Christiansen and Nygreen [35], [36] an inven-
tory constrained pickup and delivery problem with time windows is discussed. The
problem is taken from a company producing and consuming ammonia at a number of
sites, or harbors, all around Europe. At each site, the net production or consumption
rates as well as the inventory restrictions are known. A fleet owned by the company,
reserved only for the transportation of ammonia, is to be scheduled in order to keep
the stock levels within their limits. Besides the own production facilities, ammonia
can be bought from a number of external suppliers. For each harbor, the number
of potential visits is calculated, and the problem is presented in a network where the
nodes correspond to a certain harbor and a certain visit. The problem is formulated
as a flow problem in this network. Four main groups of constraints are described.
Network constraints are ensuring feasibility in the movement of the vessels, loading
and discharging constraints regulate the amount of ammonia on board the vessels, and
time window constraints and inventory constraints govern the harbor operations.
Since all ammonia is owned by the company, the ammonia at the external suppliers is
already bought when the scheduling takes place, and hence the only costs are the ves-
sel operating costs. By variable splitting and Dantzig-Wolfe reformulation, the prob-
lem decomposes into one master problem, one vessel subproblem for each vessel and
one harbor subproblem for each harbor. A thorough description of the application
is given in [32], together with the model and the reformulation. The subproblems, formulated as shortest path problems and solved using dynamic programming are described in [36], while implementation issues are discussed in [35]. Scenarios with real data from the company are solved, and the results show that the model can become a tool to assist the company in its planning.

Schedule vessels to transport crude oil from Kuwait to Europe, North America and Japan is discussed in Sherali et al. [94]. Either company owned tankers or contracted tankers can be used to transport the crude oil. There are two available routing possibilities for the tankers, either through the Suez Canal, or around the Cape of Good Hope. Each trip consists of one loading port and one discharging port, resulting in four different routes for each pair of loading and discharging ports. A mixed integer programming formulation is developed, where the movements of the tankers are modeled using the routes instead of schedules for the whole planning period. Unfortunately, this model can only be solved for small instances, so an aggregated reformulation of the model is developed. The aggregation is based on relaxing the individual tankers total usage and approximate the demand and availability constraints. Two rolling horizon algorithms are developed for the aggregated model. A comparison between the current ad-hoc procedure used by the company and the proposed algorithms shows a great improvement in the total cost.

Fox and Herden [53] describe a mixed integer programming model to schedule vessels with a variety of products. The products are transported from a single loading port to a number of discharging ports. Even though the model is not very large, the time needed to solve it speaks for a heuristic approach. Two strategies are used, the first is to introduce a priority order on the integer variables to branch on, and the second is to iteratively relax, fix and introduce binary variables. The proposed algorithm considerably improved the solutions found by manual planning.

A heuristic approach for the problem discussed in Christiansen [32] is developed in Flatberg et al. [50]. The method is based on dividing the decision process into two parts, one part that describes the sequence of ports to call for each vessel, and one part that calculates the arrival time and the quantity to load or discharge for each call. The first part is a combinatorial problem while the second is modeled as a linear program. A greedy algorithm is used to find an initial solution to the first part. The proposed solution is then evaluated, and, if found feasible in the linear program, improved using a local search. The solution method has proven successful on real-world cases, giving good solutions within acceptable time limits.

An extended version of the problem presented in Shih [95] is discussed in Liu and Sherali [71]. Besides a larger scale, with more power plants, discharging ports and coal sources, the possibility of blending coal from different sources is also included in the model. The quality of the coal is measured using a number of different criteria, and by blending, low quality coal can be used in the best possible way. The problem is formulated as a mixed integer program, and a rounding heuristic is used to find near optimal solutions.
Fagerholt and Christiansen [44] describe a method for solving a combined ship scheduling and allocation problem. The vessels to be scheduled are equipped with a limited number of bulkheads that can be placed in a given number of feasible positions, creating different partitions of the cargo space. The allocation part of the problem is to decide on where to place the bulkheads in order to match the size of the cargos lifted. The problem is formulated as a set partitioning problem, similar to the models in Appelgren [4] and Brown et al. [24], and solved using commercial software. The feasible schedules for the vessels are generated beforehand, and for each schedule a TSP with allocation, time windows and precedence constraints is solved using a dynamic programming algorithm, see Fagerholt and Christiansen [45] for further details on the dynamic programming algorithm. A computational study shows good results, even when only a fraction of the feasible schedules are generated.

The work in Fagerholt and Christiansen [44] is further developed in Fagerholt [42] and Christiansen and Fagerholt [33]. In [42], soft time windows are introduced, the motivation is that by allowing small delays in the deliveries to some customers, large savings in the total cost can be achieved. The concept of robust schedules is described in [33]. Here the focus is to find schedules that are less likely to result in vessels staying idle in ports during the weekends. The solution approach resembles the method in [44], but another subproblem is solved to generate the feasible schedules.

The mixed integer non linear model presented in Ronen [87] is reformulated in Cho and Perakis [31]. By introducing a new set of constraints, the problem is formulated as a mixed integer linear problem with less integer variables than the original formulation. It is also shown that the formulation can be seen as a generalized version of the capacitated facility location problem.

An inventory routing problem with point to point deliveries is discussed in Ronen [87]. The problem is to determine the quantity of each product to transport from a certain loading port to a certain discharging port, as well as which vessel to use and the starting time of the shipments. The problem is formulated as a mixed integer linear program, and a computational study shows good results on the tested problem. A heuristic is also developed. The heuristic starts by fulfilling all inventory shortages by creating shipments from the cheapest source available, then, if projected product overflows are found at any source, the cheapest possible shipment from this source is created. Finally, an improvement heuristic is used to combine shipments from the same origins to the same destinations on adjacent days.

In [43], Fagerholt presents a decision support system developed for vessel fleet scheduling. The process of developing the system is thoroughly described, from the skepticism of the industry early in the project to the first success stories a few years later. The system is based around a graphical user interface, where the scheduler manually, or by the use of an optimization algorithm, can schedule the vessels.

Two solution approaches for a distribution chain problem in the pulp industry are presented by Bredström [19]. From pulp mills in Sweden and Norway, the pulp is transported by a fleet of vessels operated by the company, vessels chartered on the spot market and trucks and trains to domestic customers as well as customers in central
Europe and on the British Islands. Each customer can be supplied in a number of
different ways, leading to a very complex distribution situation. A rolling horizon
is used in the first approach. The problem is formulated as a mixed integer linear
program, where routes defined as a number of visits at loading harbors followed by
a number of visits at discharging harbors are used to represent the movement of the
vessels. A subproblem is defined over a shortened planning period divided into three
blocks. In the first block, the routes are fixed, in the second they are represented with
binary variables and in the third the binary restrictions are relaxed. The algorithm
iterates between solving the problem and moving the shortened period forward in
time. The second approach is a genetic algorithm. The chromosome is sequences of
routes for vessels in a fixed fleet, and the offspring are generated by combining routes
with large flows of pulp from different parents. Computational tests on real-world data
show good results with both algorithms, but the results are obtained in shorter time
using the genetic algorithm. The genetic algorithm is further described in Bredström et al. [20].

Persson and Göthe-Lundgren [84] present a shipment planning problem in the oil
refinery industry. The company produces different qualities of bitumen at their re-
fineries, and the bitumen is then transported by tankers owned by the company, char-
tered tankers or barges to depots. To avoid costly production plans due to a skewed
shipment plan, the production planning is included in the shipment planning model.
The problem is formulated as a mixed integer linear program, and valid inequalities
are introduced to strengthen the LP relaxation. Column generation is used to solve
the problem, and a limited tree search is used to find an integer solution. In the tree
search, constraint branching over depot visits is used instead of branching over tanker
schedules. The schedules produced during the computational tests have been studied
and approved as reasonable by the planners at the company.

An interesting thread to follow in the works presented here is the decomposition of
the ship routing and scheduling problem into one master problem where the vessels
are assigned to schedules, and one subproblem where the schedules are generated. The
first papers using this idea are Appelgren [4], [5] where cargo constraints and convex-
ity constraints are in the master problem and the subproblem becomes an ordinary
shortest path problem. In McKay and Hartley [74], the schedules are embedded in
a more complex model, and all schedules are generated beforehand. An interesting
aspect of this paper is that the products to be transported are not specified as cargos,
i.e. with a given loading port, discharging port, time of delivery and so on, but any
supply of the given product can be used to fulfill a demand. Ronen [87], Brown et
al. [24], Fisher and Rosenwein [49], Perakis and Bremer [82] and Bremer and Perakis
[21], Kim and Lee [64] all use cargos and generate the feasible schedules beforehand.

In Christiansen [32] and Christiansen and Nygreen [35], [36] the ship scheduling
problem is one part of a larger distribution chain problem. Besides information about
the visits, the ship schedules also include information about the load quantities and ar-
rival times. The schedules are then matched with harbor visiting schemes in the master
problem. When formulating the subproblem, the load quantities are discretized, and
the network is expanded for each possible load quantity. The formulated problem
is a shortest path problem with time windows. With a small number of products
and harbors, this method works well, but for larger instances, the network will prob-
ably explode in size, especially if more than one product can be transported at the
same time.

Recently, Fagerholt and Christiansen [44], Fagerholt [42] and Christiansen and Fager-
holt [33] all use cargos and generate the feasible schedules beforehand. Persson and
Göthe-Lundgren [84] do not use cargos and do not include information about quan-
tities in the ship schedules. Instead, the loading and discharging decisions are taken in
the master problem.

<table>
<thead>
<tr>
<th>Paper</th>
<th># ports</th>
<th># commodities</th>
<th>fleet size and mix</th>
<th>type of demand</th>
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<td>many–one</td>
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<td>many–many</td>
<td>unique</td>
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<td>Miller (1987)</td>
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<td>Fox, Herden (1999)</td>
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<td>Flatberg et al. (2000)</td>
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<td>Ronen (2002)</td>
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<td>many–many</td>
<td>general</td>
</tr>
<tr>
<td>Persson, Göthe-Lundgren (2004)</td>
<td>many to many</td>
<td>many–many</td>
<td>many–many</td>
<td>general</td>
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</table>

Table 1: Classification of papers on industrial shipping

4 Location-routing problems

Traditionally, location studies have assumed that transportation costs depend on the
radial distance between the customer and the facility. On the other hand, routing
problems have been solved for fixed facilities, with the building and operating costs of
the facilities neglected.

A major difference between the two problem classes is the contexts in which they usu-
ally occur. Location problems are often found on the strategic level. Locating facilities
such as warehouses, plants or hospitals is very costly, affects a large area for a long
time and is difficult to reverse. The effect on the areas surrounding the located facility
make it a decision with many interested parties, which leads to multiple objectives, especially when public facilities are located.

Routing decisions are typically found on the tactical and operational level. Routes of delivery trucks or vessels can be changed on a daily basis, or even more often, they seldom have multiple objectives, and are generally easier to quantify.

In spite of this, there exist many practical situations where the need to simultaneously locate facilities and construct associated routes including more than one customer arises. A few examples are Or and Pierskalla [81]; blood banks, Nambiar et al. [79], [80]; collection stations for natural rubber, Labbé and Laporte [65]; post boxes, Johnson et al. [61]; kitchens for a meals-on-wheels system, and Billionnet et al. [15]; base station controllers in telecommunication.

The class of problems combining routing and location decisions is, quite naturally, called location-routing problems, and this section will cover some aspects of this class of problems. For a thorough exposition and an annotated bibliography, the reader is referred to Andersson and Eiselt [1].

The rest of this section address some of the different solution approaches that have been used to tackle the location-routing problem. A possible approach to solve the location-routing problem is to try and approximate the length of a route, and then use this approximation as the costs in a pure location problem. Bearwood et al. [10] discuss the expected length of a traveling salesman tour and Christofides and Eilon [37] refine the formula for the multi vehicle case. The route approximation method is used by for example Watson-Gandy and Dohrn [101], Wirasinghe and Waters [103], and Daganzo and Newell [38], where the last two papers use the idea to approximate the cost per item delivered, and then propose analytical models.

The location-routing problem can be seen as consisting of three interdependent components; location, allocation and routing. The ideal would be to include all tree components in one model, but this often gives a problem of intractable size and difficulty. In most solution approaches proposed, the problem is solved heuristically using some kind of decomposition approach, see Laporte [67] for a further discussion. Maybe the most natural decomposition approach is location-allocation-routing, which can be described as a sequential approach where facilities are located, customers are allocated to the facilities and finally routes are constructed. Papers where this approach is used are for example Harrison [58] and Jacobsen and Madsen [59]. In [59], a location-allocation problem is first solved, and then the routing is performed. It is common to incorporate more than one component in the subproblems. One sequential approach that uses this is based on solving an allocation-routing problem to form routes and then a location-allocation problem treating the routes as single nodes. For two approaches using this idea, see the third algorithm proposed by Jacobsen and Madsen [59], and more recently Liu and Lee [72].

All approaches mentioned so far are sequential, i.e. they first determine the solution to a location (routing) problem, and then use this information to find a good solution for the routing (location) part. More sophisticated methods are iterative methods,
i.e. methods that iterates between the location problem and the routing problem, and hierarchical methods, where the routing part is treated as a subproblem to a generalized location problem. For a longer discussion about classification of heuristics in the location-routing context, the reader is referred to Wasner and Zäpfel [100].

One of the first and most cited iterative approaches is presented by Perl and Daskin [83]. They decompose the problem into three subproblems. In the first, routes are created assuming that all facilities are open. The routes, except their associated facilities, are then fixed. The second subproblem is a location-allocation problem that locates facilities and allocates the fixed routes to the facilities. The third problem is a routing problem, creating new routes given the facilities located by the location-allocation problem. The algorithm iterates between the second and third subproblem until no cost reduction is found. Different objectives, reflecting the active costs in each subproblem are used. Hansen et al. [57] use the same approach but present improved heuristics for the subproblems. Wu et al. [104] have a slightly different model, but use a similar decomposition. They present a simulated annealing based approach to solve their subproblems.

Nagy and Salhi [77], [78] present an algorithm where the routing part is seen as an inner problem to a generalized location problem. Tabu search is used to solve the problem, and the neighborhood is only defined in the location space. For each move in the location space, the corresponding routing problem is solved using a fast but approximate heuristic in [77], and route length estimation in [78]. Once the most promising move has been found, a more accurate heuristic is used to calculate the real routing cost. A similar idea is also presented by Tuzun and Burke [97].

Some other interesting solution approaches for the location-routing problem are the exact graph transformation based approaches proposed by Laporte et al. [69] and Laporte and Dejax [68], the branch and price method suggested by Berger et al. [14] and the two decomposition based methods developed by Bookbinder and Reece [17] and Bruns and Klose [25].

5 Contributions

5.1 Summary of papers

The thesis is divided into three parts with routing as a common denominator. In the first part, an application concerning waste management is analyzed. Many industries generate garbage, and instead of handling the waste disposal themselves, other companies, specialized in garbage collection, handle the disposal. Each industry rents containers from a company to be used for waste, and the garbage collection company handles the collection. The industries buy a service including one or more containers at the industry and the garbage collection company is obliged to make sure that the containers never become overfull. The idea is that the industry buys this service and in return, the garbage collection company can plan the collection so that the overall cost and the number of overfull containers is minimized. The planning problem facing the garbage collection company is studied in the first part.
In Paper 1, the problem is modeled using a flow based model, and a Lagrangean relaxation scheme combined with local search is used to solve it. Doing a variable split on the inventory variables makes it possible to decompose the model without losing the strength of the inventory balance constraints. The Lagrangean subproblems become vehicle routing problems with variable supply at the nodes. Tabu search is a metaheuristic that has been shown to work well on different routing problems, and an algorithm based on tabu search has been developed to solve the Lagrangean subproblems. Given a routing solution, the problem of choosing the supply at each node in a route is a multiple-choice knapsack problem, and a dynamic programming algorithm has been developed to solve this problem.

The Lagrangean dual problem is solved using a variant of the cutting plane method. Since the problem is initially unbounded, some kind of restriction is needed. A local restriction similar to the Boxstep method, see Marsten et al. [73], is developed, but unlike this method, the size of the box is dynamically updated using subgradient information. The solutions from the Lagrangean subproblems are made feasible through a single customer feasibility algorithm and used as starting solutions in a local search algorithm to generate good feasible solutions.

In Paper 2, another model is proposed. This model is based on representing each route with a variable. Due to the dynamic nature of the problem, it is not always possible to decide which routes that are feasible a certain day and which are not. To model this, the vehicle feasibility constraints can either include a large constant, or, as in this paper, a non-linear formulation can be used. Treating the visit variables, and hence the inventory levels as well, as the complicating variables, a Benders decomposition scheme, see Benders [13], is developed to solve the problem. When solving non-linear problems, generalized Benders decomposition, see Geoffrion [55], is often used. In generalized Benders decomposition, Lagrangean multipliers are used instead of dual variable values when constructing the value and feasibility cuts. An important assumption in generalized Benders decomposition is that the subproblem can be solved essentially independent of the complicating variables. The problem then fulfills the so-called property P. This is unfortunately not the case in the studied problem. Unlike others dealing with problems not fulfilling Property P, see for example Floudas et al. [52], Bagajewicz and Manousiouthakis [6] and Sahinidis and Grossmann [89], and due to the fact that for fixed visit variables, the Benders subproblem becomes linear, LP duality has been used, and the Benders master problem has been extended. With the extension, it is possible to define where the added cuts are not valid and thus make them redundant.

The second part investigates a distribution chain management problem taken from the Swedish pulp industry. Given fixed production plans at the mills, and fixed customer demands, the problem is to minimize the distribution cost. Unlike many other models for marine distribution chains, the customers are not located at the harbors. This means that the model proposed also incorporates the distribution planning from the harbors to the customers. All customers are not served from the harbors; some are served directly from the mills using trucks and trains to distribute the pulp, and these decisions are also included.
In Paper 3, a mixed integer linear model is proposed. To solve the model, a two-phase branch and price algorithm is developed. In the first phase, the focus is on the scheduling problem for the vessels operated by the company, and in the second, given fixed schedules for these vessels, a charting problem is solved. At each node in the search tree in the first phase, new schedules are generated using column generation. To generate the schedules, a network based on clusters of either mills or harbors is constructed, and a modified $k$-shortest path algorithm is developed to solve the problem. The modified algorithm penalizes the schedules already generated, making the generated schedules more diversified than general $k$-shortest path algorithms. An enumeration algorithm based on the same network is used to generate new routes for the chartered vessels.

A similar solution strategy to the one proposed in Paper 3 is developed in Paper 4. Here, a generate and fix approach is developed. Starting from an initial pool of schedules and routes, column generation is used to solve the LP relaxation of the problem. When the relaxed problem has been solved, a modified version of the problem is solved. The solution to this problem is an assignment of one schedule to one of the vessels operated by the company. After the assignment, the column generation is continued. When all company operated vessels have been assigned a schedule, a chartering problem is solved.

In the third part, routing issues are combined with location decisions to the location-routing problem. Special emphasis is given to a dynamic version of the problem.

Paper 5 is an annotated bibliography over location-routing problems. First, the location-routing problem is defined. Due to the difference between the problem formulation and the solution approaches in much of the location-routing literature, the definition takes the formulation as a starting point. This means that a model is considered to be a location-routing problem if it contains both location and routing elements, and in which at least some of the routing is performed not only as simple return trips. After the definition, the objective function of the location-routing problem is discussed. From the location part, there are the famous push/pull/balance objectives, which, in the location-routing problem, are combined with the generalized routing objectives; minimize cost, maximize revenue and balance workload. Each combination gives a certain characteristic to the problem, and situations where the different combinations can be used are identified. Starting with the capacitated fixed charge facility location problem and the vehicle routing problem, these two problems are merged into one generic location-routing problem. An annotated bibliography of major works in the field then follows.

The last paper, Paper 6, presents a model for strategic management where decision makers must analyze location, capacity and routing issues over a long planning period. The model is developed for an application in strategic school management, where the location and capacity of schools as well as their catchment areas are under consideration. The problem is formulated as a mixed integer linear program. In the location part, constraints similar to inventory balance constraints have been used to model both location and capacity, while a flow based approach has been used to model the routing. In the computational study, the importance of incorporating a routing component allowing multiple visits, as well as the danger of having a too short planning period, is shown.
5.2 Main contributions

Paper 1
- A new model for the inventory routing problem.
- A Lagrangean based decomposition scheme where the constraints connecting the different days are relaxed.
- Solution methods for the Lagrangean subproblem, based on tabu search, and the Lagrangean dual problem, based on a dynamic local restriction method.
- A local search based primal heuristic.

Paper 2
- A new model for the inventory routing problem.
- A reformulation of the Benders master problem so that both the master problem and the subproblems become linear.

Paper 3 and Paper 4
- A new version of the distribution chain management problem, incorporating multi modal transports and inventory management.
- A model for the distribution chain management problem.
- Solution methods based on branch and price and partial fixation in combination with column generation.

Paper 5
- A classification scheme based on the characteristics of the objective function.
- An annotated bibliography.

Paper 6
- An analysis of a problem concerning strategic location management in a less-than-truckload environment.
- A model for the relocation-routing problem.
5.3 Future work

In the first part, it would be very interesting to develop the primal heuristic further. Incorporating the heuristic in a meta heuristic framework should have the potential to produce really good solutions. By using a heuristic, it is also possible to incorporate more constraints, and thus decrease the distance between a solution presented by the heuristic and a solution that can be used in a practical application.

Development of the schedule generation in the second part is important if the algorithms are implemented as a part of a larger decision support system. Since the allocation of schedules to vessels and the generation of schedules are two separated activities in the proposed methods, they can be developed independently of each other. There are some modeling issues that would be interesting to investigate further, for example the strengthening of the capacity constraints and its relation to column generation.

There are two major directions for developing the third part. One is to investigate the impact of the proposed model, both analyzing the future, but also from a rearview mirror perspective. To solve the problem using historic data, and then compare the solution with the actual outcome and analyze the differences is an exciting project. The second direction is to develop algorithms for the relocation-routing problem, the brute force method used cannot solve problems of a more realistic size and therefore other approaches are needed. With the heuristics developed for related problems, it is an interesting challenge to develop a heuristic for the relocation-routing problem.

References


