Hydro-mechanical forming of aluminium tubes - on constitutive modelling and process design

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Abstract

Tube hydroforming is a forming method which has several advantages. By using pressure in combination with material feeding it is possible to manufacture products with high structural integration and tight dimensional tolerances. The forming method is especially suited for aluminium alloys which have a relatively low ductility. Finite Element simulations are used extensively in the sheet metal stamping industry, where the methodology has contributed to a better understanding of the process and the new prediction capability has significantly reduced costly die tryouts. Similarly, the tube hydroforming industry can benefit from Finite Element simulations, and this simulation methodology is the topic of this dissertation.

Deep drawing and tube hydroforming have a basic difference, namely that the latter process essentially is a force controlled process. This fact, in combination with the anisotropic behaviour of aluminium tubes, enforces a need for accurate constitutive descriptions. Furthermore, the material testing needs to account for the specifics of tube hydroforming. The importance of proper material modelling is in this work shown for hydrobulging and hydroforming in a die with extensive feeding.

The process parameters in hydroforming are the inner pressure and the material feeding, where a correct combination of these parameters is crucial for the success of the process. It is here shown, that Finite Element simulations together with an optimisation routine are powerful tools for estimating the process parameters in an automated procedure.

Finally, the reliability and quality of the simulation results depend on how failure is evaluated, which in the case of hydroforming mainly concerns wrinkling and strain localisation. Since tube hydroforming often is preceded by bending operations this fact also demands the criteria to be strain path independent. In this work, it is shown that the prediction of strain localisation depends on the ability to predict diffuse necking, which in turn is strongly related to the chosen constitutive model.
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I dedicate this work to Camilla and my son Martin for showing absolutely no interest in tube hydroforming. Also, I would like to thank my parents, my brother (my genetic copy), Sara and all my friends for their support. I could not have made it without you!

Mikael Jansson, Linköping August 2006

"I might even find the system which is the basis for how everything works. Then everything will be fine."
Translated from Konstruktören by Sven Yrvind
Dissertation

This dissertation for the Degree of Teknologie Doktor (Doctor of Philosophy) at Linköping University consists of an introductory part and the following appended papers


II. Jansson M; Nilsson L; Simonsson K. *The use of biaxial test data in the validation of constitutive descriptions for tube hydroforming applications*. Accepted for publication in Journal of Materials Processing Technology.

III. Jansson M; Nilsson L; Simonsson K. *Tube hydroforming of aluminium extrusions using a conical die and extensive feeding*. Submitted.

IV. Jansson M; Nilsson L; Simonsson K. *On process parameter estimation for the tube hydroforming process*. Submitted.

V. Jansson M; Nilsson L; Simonsson K. *On strain localisation in tube hydroforming of aluminium extrusions*. Submitted.
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1 Introduction

1.1 Background

Ever since iron was discovered in 500 B.C to this day, the human being has been shaping metals into refined products to use or sell for a profit. The technology evolved through the blacksmiths, steelworks and the powered stamping press. Today, the deep drawing process is the dominating technology for sheet metal forming. However, the automotive industry is facing demands on limiting the environmental impact of their products. One way of increasing fuel economy and hence decreasing emissions, is to increase the use of lightweight materials in order to reduce the mass of the car. The switch from steel to e.g. aluminium is however not an easy task, since the demands on driving performance, crashworthiness, production cost etc. should still be met. The main issues concern the Young’s modulus, which is approximately one third of the Young’s modulus of steel, and the restricted formability of aluminium alloys due to the high percentage of alloy material. These challenges have resulted in alternative lighter design concepts as well as forming and joining methods, and are the reasons to why the industry in recent years has turned its attention to hydroforming as a possible forming method.

1.2 The tube hydroforming process

Hydroforming is a generic term for a forming method where a pressurised fluid is used as the forming media. The method could be applied to either sheets or tubular geometries, where the latter is the topic of this work. Tubular geometries can be used for manufacturing e.g. space frames, camshafts and exhaust systems. Hydroforming of tubes has several advantages:

- It is possible to tailor the properties of the hydroformed tube by varying its initial wall thickness and cross section. The successive forming yields
a product with properties similar to an assembly of parts, and for which the number of secondary operations such as welding or riveting thus are decreased. It is also possible to punch holes during the hydroforming process.

- Hydroforming produces products with high process repeatability due to tight dimensional tolerances and small springback.
- The hydroforming process distributes the straining of the tubular blank and thus effectively utilises the formability of the material.

Tube hydroforming is a unique forming method since during deformation due to inner pressure it is possible to feed material axially by cylinders at the tube ends. If necessary, counter punches can also be used to control the tube expansion, see Figure 1.1. Thus, the process parameters are the inner pressure and the material feeding, see Figure 1.2, and the process can be divided into four phases, namely:

- **Filling/Preforming** The tubular blank, which could be bent from a preceding bending operation, is placed inside a die cavity. The tube is then filled with a fluid, usually water. If preforming is considered during die closure, an inner pressure could be applied to prevent local buckling.

- **Free Forming** During this phase, the tube is expanding using a relatively low pressure. Generally, this is a phase where the the die contact is limited and material feeding is possible.

- **Calibration** During calibration, the tube conforms to the shape of the die cavity. The pressure level is high, and due to material friction and tool geometry, minor feeding is possible and the axial stroke is used to prevent leakage.

- **Pressure relief** The forming is completed and the pressure is released. After this, the dies are opened and the produced part is extracted. During the process, the pressure is continuously monitored. If the maximum pressure is not achieved during the forming operation, this is an indication of a faulty process and the tube is discarded as failed.

Tube hydroforming is still a relatively new forming method in terms of large scale production, and this fact is a drawback when it comes to a general industrial acceptance. The process struggles with long cycle times and expensive equipment, see Ahmetogulo and Altan [1]. These drawbacks, in combination
with a limited knowledge about the process and associated tool design, have made its acceptance in the forming industry lag behind. However, research is performed continuously on the subject and improvements in process control, tooling and lubrication together with advances in simulations, contribute to a more general acceptance and utilisation in industrial production, see Hartl [21].

1.3 Formability in tube hydroforming

There exist several failure modes in tube hydroforming, namely:

1) Pinching during die closure
2) Buckling
3) Wrinkling
4) Bursting
5) Folding back
6) Leakage

Pinching occurs during die closure, where the tube could be squeezed between the upper and lower die. This causes local damage, which may initiate fracture or deteriorate the finish of the final part. Buckling typically occurs during die closure or in the early part of the free forming phase due to excessive feeding. Several authors compare the failure with elastic column
buckling of a thin-walled tube, see e.g. Koç and Altan [30] or Chu and Xu [15]. The critical buckling load thus depends on the free tube length, tube diameter and wall thickness. Another axial stability problem is the development of wrinkles which are symmetrical with respect to the longitudinal tube axis, see Figure 1.3. As opposed to global buckling, wrinkling is not an irreparable problem, since depending on the size and location, the wrinkles can be eliminated during calibration. In fact, the occurrence of wrinkles even may be needed in order to successfully form a part. The main task is then to control the wrinkling by a suitable wrinkling indicator, see e.g. Nielsen et al. [40], Nordlund [42], Strano et al. [47] or Ray and Mac Donald [44].

The failure mode which has been the main topic in research is tube burst. Bursting occurs when the increase in inner pressure cannot be compensated by hardening of the material, which leads to localisation and breakage. Several successful attempts have been made to describe the phenomenon by plastic instability theory, see Hwang et al. [25], Hora et al. [24] or Kulkarni et al. [31]. Boudeau et al. [12] use a local perturbation technique to study the stability of the current equilibrium. By this, they are able to predict both wrinkling and strain localisation. If burst failure is considered to occur due
to a damage process during plastic deformation, then ductile failure criteria could be applicable. The criteria are common in the context of bulk forming, and are based on the state of stress and strain, see e.g. Lei et al. [36] and Kim et al. [29].

Folding back failure occurs when the tube folds back on itself due to excessive feeding, which causes an irreparable failure. This typically occurs as a consequence of wrinkling or if material flowing through a radius releases from the tube wall. Since the inner pressure also acts on the feeding cylinders, the sealing force between the tube and cylinder could be too small and leakage will occur. This problem occurs if the feeding is force controlled. However, leakage can also occur during a stroke controlled feeding if the axial deformation of the tube is larger than the prescribed feeding.

Figure 1.3: Wrinkling and burst failure, from Asnafi and Skogsgårdh [2]
2

FE modelling

Explicit Finite Element (FE) solvers are today widely used for forming simulations, since they have certain advantages compared to implicit solvers. In this work the explicit FE solver in LS-DYNA, Hallquist [19], has been used. Metal forming includes non-linearities due to material behaviour, instabilities and contact constraints. These conditions are effectively managed by the explicit method since it does not suffer from lack of convergence which could be the case with an implicit method. On the other hand, the time integration is conditionally stable, and thus a small time step is needed to guarantee stability. The explicit method also requires less computations per time step and less storage, which makes it suitable for large scale problems.

2.1 Explicit time integration

This section presents a short description of the explicit scheme used in LS-DYNA. For a more extensive discussion on time integration, see Belytschko et al. [10]. The FE discretisation in space of the weak form of the equation of motion yields

\[ Ma + f_{int} = f_{ext} \] (2.1)

More specifically, \( a \) is the acceleration vector and

\[ M = \sum_{e} \int_{\Omega_0} \rho_0 N^T N d\Omega_0 \] (2.2)

where \( M \) is the mass matrix, \( \rho_0 \) is the initial density, and \( N \) is the shape function matrix. The integral is taken over the initial element domain \( \Omega_0 \) and summed for all elements \( e \). The mass matrix is diagonalised, see Belytschko et al. [10], to accommodate for an efficient solution algorithm, where a matrix
inversion is eliminated. The internal force vector is found from

\[ f_{\text{int}} = \sum_e \int_{\Omega} B^T \sigma d\Omega \]  \hspace{1cm} (2.3)

where \( B \) is the strain-velocity matrix and \( \sigma \) is the Voigt vector of the Cauchy stress components. The external force vector consists of body forces and resulting forces from boundary conditions, e.g. in a hydroforming case from material feeding, inner pressure and contact conditions. The external force vector is

\[ f_{\text{ext}} = \sum_e \int_{\Omega} \rho N^T b d\Omega + \sum_e \int_{\Gamma} N^T \bar{t} dA \]  \hspace{1cm} (2.4)

where \( b \) is the body force vector e.g. due to gravity loading, and \( \bar{t} \) is the traction vector acting on the boundary \( \Gamma \). The FE discretisation is solved in time using the Central Difference Scheme, where the internal forces, external forces, velocities \( v^n \), and displacements, \( d^n \), are known at the current state \( n \). The acceleration is then found from Equation 2.1 as

\[ a^n = M^{-1} (f^n_{\text{ext}} - f^n_{\text{int}}) \]  \hspace{1cm} (2.5)

The velocity is evaluated at time \( t^{n+\frac{1}{2}} \), using the time step, \( \Delta t^n \). Accordingly,

\[ t^{n+\frac{1}{2}} = t^n + \frac{\Delta t^n}{2} \]  \hspace{1cm} (2.6)

and

\[ v^{n+\frac{1}{2}} = v^{n-\frac{1}{2}} + a^n \Delta t^n \]  \hspace{1cm} (2.7)

The displacements at time \( t^{n+1} \) is then found from

\[ d^{n+1} = d^n + v^{n+\frac{1}{2}} \Delta t^n \]  \hspace{1cm} (2.8)

To find the internal forces, the stress state of the material has to be updated. This will be covered in detail in Section 3.3. As mentioned above, the Central Difference Scheme is conditionally stable. The largest allowable time step is related to the highest natural frequency of the system, which for an undamped system is

\[ \Delta t \leq \frac{2}{\omega_{\text{max}}} \]  \hspace{1cm} (2.9)

Thus, the critical time step is related to the density and the stiffness of the material.
2.2 Shell elements

The most common element type for tube hydroforming simulations is the plane stress element. By using this element type it is assumed that the stress through the thickness of the tube is negligible in comparison to the in plane stresses. It could however be argued that the through thickness stress is not negligible, implying that a 3D element formulation has to be used. However, to accurately model the through thickness behaviour at least five elements through the thickness have to be used, which would lead to unrealistically small timesteps. In this work, the Belytschko-Tsay (BT) element is used, see Belytschko et al. [9]. The shell element has the following features

- The element is based on the Mindlin plate theory
- The element is under-integrated, thus only one quadrature point is used in the reference plane
- Several quadrature points are used through the thickness, typically 5-9 points
- The element is used in a corotational formulation
- The through thickness deformation is determined by the membrane straining of the element

The BT element is extremely robust and fast, but it suffers from some well known drawbacks. Due to the corotational formulation, the coordinate system of the material rotates with the element. As the element deforms, a difference between the actual material rotation and the local element coordinate system could arise. Further, by using one-point quadrature, the element is sensible to spurious singular mode deformations (hourglass modes), which are deformation modes that results in no straining of the material. These modes are usually suppressed by hourglass stabilisation. More specifically, stiffness is added to the spurious deformation modes such that deformations due to rigid body motion and physical deformation modes are unaffected.

2.3 Contact modelling

The contact problem is modelled using a master and slave formulation, where the slave nodes are checked for penetration of a master segment. If penetration is observed, then contact forces are applied in order to keep the two surfaces apart. The contact condition can be enforced by using a penalty
based or a constraint based algorithm (by Lagrange multipliers). The advantage of a penalty based contact is its straightforward implementation which makes it very efficient and suitable for explicit calculations. On the other hand, the penalty algorithm will always allow for some penetration since the interface forces are based on the penetration depth. Further, if a contact stiffness is used, it may affect the conditional stability of the timestep. If a constraint based contact is used, the contact condition is enforced by implying zero penetration, thus a more accurate solution is obtained. The contact constraint can interfere with other constraints, e.g. rigid body constraints, which renders it less suitable, see Nilsson et al. [41].

2.4 Internal pressure modelling

The internal pressure is applied using a control volume within the tube. The basic assumption of this concept is that the pressure is uniformly distributed. The internal pressure can be applied by prescribing the pressure, which makes the process force controlled. This method has a major disadvantage. Whenever an instability occurs, i.e. a situation where the increase in pressure and deformation of the tube cannot be compensated by material hardening, it will yield instantaneous necking and an unlimited deformation. The method is clearly unphysical, and the process becomes difficult to control. In particular this is the case when the pressure is unknown, e.g. in a process parameter determination problem. If instead the volume of the tube cavity is prescribed, the process becomes deformation controlled. The internal pressure is then a response rather than a prescribed entity. The disadvantage of this method is that the process becomes less intuitive. The fluid is modelled as compressive, and the mass, \( M(t) \), of the fluid in the control volume is evaluated from

\[
M(t) = M(0) + \int F(t)dt - \int G(t)dt
\]  

(2.10)

where \( M(0) \) is the mass of the fluid at time zero, and \( F(t) \) and \( G(t) \) are prescribed mass inflow and outflow, respectively. The volume of the uncompressed fluid, \( V_0(t) \), is found from

\[
V_0(t) = \frac{M(t)}{\rho(0)}
\]  

(2.11)

where \( \rho(0) \) is the initial density (1000 kg/m\(^3\) for water). The current pressure inside the control volume is evaluated from

\[
p(t) = Kln\frac{V_0(t)}{V(t)}
\]  

(2.12)
where $V(t)$ is the current volume of the compressed fluid and $K$ is the bulk modulus of the fluid (2050 MPa for water).
3

Constitutive modelling

This chapter begins with a discussion on general plasticity theory. For more details the reader is referred to e.g. Lubliner [38], Lemaitre and Chaboche [37] and Ristinmaa and Ottosen [46]. Further, the YLD-2000 yield criterion is presented, see Barlat et al. [8]. For further reading on constitutive modelling of aluminium extrusions, see e.g. Lademo [32], Barlat and Lian [6], Barlat et al. [7], Bron and Besson [14] and Karafillis and Boyce [28]. Finally, the stress update algorithm used in this work is presented.

3.1 General plasticity theory

Consider a uniaxial tensile test according to Figure 3.1, where the responses are the engineering stress, $s$, and nominal strain, $e$, according to

$$ s = \frac{P}{A_0} $$

(3.1)

and

$$ e = \frac{\Delta L}{L_0} $$

(3.2)

where $P$ is the applied load, $\Delta L$ is the elongation, and $A_0$ and $L_0$ are the initial cross section area and length, respectively. For small strains, the deformation is reversible and obeys Hooke’s law. Thus,

$$ s = E e $$

(3.3)

where $E$ is the Young’s modulus. At the proof stress, $s_Y$, the material starts to deform plastically, and the deformation becomes irreversible. During plastic deformation, the material hardens to the point $s_u$, which is called the ultimate tensile strength. At this point the hardening of the material cannot
compensate for the increase in load and reduction in surface area, and an instability occurs. This is called diffuse necking, and it is accompanied by an in-homogeneous strain state where a neck is formed on the test specimen, see Figure 3.2. Diffuse necking precedes the localised necking, which is a second instability where the strain is localised through the thickness. Beyond the point of instability, the stress strain curve indicates a softening response. This behaviour is however due to the engineering measures, where stress is related to the undeformed geometry. In reality, the material hardens through the instability to the point of failure. This is evident if true measures of stress and strain are used, where the stress is related to the deformed geometry. The true stress, $\sigma$ and strain, $\varepsilon$, can be evaluated from the engineering stress and strain through
\begin{align}
\sigma &= s(e + 1) \\
\varepsilon &= \ln(e + 1)
\end{align}

To determine the stress at the onset of yielding, a yield criterion is used which combines the current stress components into an equivalent stress, $\bar{\sigma}$. The yield criterion describes a surface in the stress space, where the material is elastic if the stress state is inside the yield surface and plastic if the stress state is on the yield surface. A state of stress outside the yield surface is not
permitted. Further, due to the physical nature of plastic flow in metals, it is assumed that a hydrostatic pressure does not influence the plastic yielding. In the following a corotated reference frame is used. Thus, the corotated Cauchy stress is
\[
\dot{\sigma} = R^T \cdot \sigma \cdot R 
\] (3.6)
where \( R \) is the rotation tensor found from the polar decomposition of the deformation gradient \( F \), i.e.
\[
F = R \cdot U 
\] (3.7)
Similarly, the rotated rate-of-deformation tensor is
\[
\dot{D} = R^T \cdot D \cdot R 
\] (3.8)
where the rate-of-deformation tensor is the symmetric part of the velocity gradient tensor \( L \), i.e.
\[
D = \frac{1}{2} (L + L^T) 
\] (3.9)

The yield criterion, \( f \), is commonly expressed as
\[
f(\dot{\Sigma}_{ij}, \kappa_{ij}) = \bar{\sigma}(\dot{\Sigma}_{ij}) - \sigma_f(\kappa_{ij}) 
\] (3.10)
\[ \Sigma_{ij} = \sigma_{ij} - \alpha_{ij} \]  
(3.11)

where \( \Sigma_{ij} \), \( \sigma_{ij} \) and \( \alpha_{ij} \) are components of the corotated overstress, Cauchy and backstress tensors, respectively, and \( \sigma_f \) is the uniaxial yield stress which depends on the history variables \( \kappa_{ij} \), e.g. plastic work.

It is assumed that the rate-of-deformation, \( \dot{D}_{ij} \), can be decomposed into elastic and plastic parts, i.e.

\[ \dot{D}_{ij} = \dot{D}_{ij}^e + \dot{D}_{ij}^p \]  
(3.12)

where the stress rate, \( \dot{\sigma}_{ij} \), depends only on the elastic part. Thus,

\[ \dot{\sigma}_{ij} = \dot{C}_{ijkl} \dot{D}_{kl}^e \]  
(3.13)

where \( \dot{C}_{ijkl} \) is the corotated hypo-elastic tensor component. The increment in logarithmic strain in a current direction given by the unit vector \( n \) then becomes

\[ d\varepsilon = n \cdot D \cdot n \ dt \]  
(3.14)

The plastic strain rate is given by a flow rule which is commonly expressed in terms of a plastic potential, \( g \),

\[ \dot{D}_{ij}^p = \dot{\lambda} \frac{\partial g}{\partial \Sigma_{ij}} \]  
(3.15)

where \( \dot{\lambda} \) is the rate of the plastic multiplier, which is a positive scalar. In an associated flow rule, the plastic potential \( g \) is equivalent to the yield function \( f \), which is an assumption used in this work. Thus, the plastic flow direction is normal to the yield surface.

During plastic loading, \( \dot{\lambda} \geq 0 \) and \( f = 0 \). During elastic loading, the plastic rate-of-deformation must be zero, i.e. \( \dot{\lambda} = 0 \) and \( f < 0 \). If the direction of loading is tangential to the yield surface, a case of neutral loading occurs, then \( \dot{\lambda} = 0 \) and \( f = 0 \). These three cases are the different situations an elastoplastic model must handle, which can be combined to

\[ \dot{\lambda} \geq 0 \]
\[ f \leq 0 \]
\[ \dot{\lambda} f = 0 \]  
(3.16)

which are the Kuhn-Tucker conditions.
The material plastic hardening is often described by two different models. If the yield surface only expands during plastic deformation ($\alpha_{ij} = 0$ in Equation 3.11), the hardening is called isotropic since the expansion is equal in all directions of the stress space. Isotropic hardening is usually described by the equivalent plastic strain, $\bar{\varepsilon}^p$, which is found by the plastic work relation

$$dW^p = \bar{\sigma} \dot{\bar{\varepsilon}}^p = \Sigma_{ij} D_{ij}^p$$

If $\bar{\sigma}$ is a homogeneous function of the first degree, the following relation holds

$$\frac{\partial \bar{\sigma}}{\partial \Sigma_{ij}} \dot{\Sigma}_{ij} = \bar{\sigma}$$ (3.18)

By combining the relation above with an associated flow rule and Equation 3.17, it is found that

$$\bar{\sigma} \dot{\bar{\varepsilon}}^p = \dot{\Sigma}_{ij} \lambda \frac{\partial f}{\partial \Sigma_{ij}} = \dot{\Sigma}_{ij} \dot{\lambda} \frac{\partial \bar{\sigma}}{\partial \Sigma_{ij}} = \bar{\sigma} \dot{\lambda}$$ (3.19)

Thus, the rate in effective plastic strain is equal to the rate of the plastic multiplier. If the yield surface is considered only to move during plastic loading (i.e. $\kappa_{ij} = 0$ in Equation 3.11), the hardening is denoted kinematic. Such hardening causes a decreased yield-strength in compression as a result of plastic loading in tension, which is called the Baushinger effect. The evolution of the backstress $\alpha_{ij}$ is often assumed to be proportional to the plastic rate-of-deformation

$$\dot{\alpha}_{ij} = c D_{ij}^p$$ (3.20)

which is the linear Prager hardening. In this work, pure isotropic hardening is assumed. Thus, the yield criterion reduces to

$$f(\hat{\sigma}_{ij}, \bar{\varepsilon}^p) = \bar{\sigma}(\hat{\sigma}_{ij}) - \sigma_f(\bar{\varepsilon}^p)$$ (3.21)

More specifically, the yield stress is described by the Multi Component Strain Hardening model (MCSH), see Berstad [11],

$$\sigma_f = \sigma_Y + \sum_{i=1}^{N} Q_i (1 - e^{-C_i \bar{\varepsilon}^p})$$ (3.22)

where $\sigma_Y$ is the initial yield stress, $Q_i$ and $C_i$ are constants, and $N$ is the number of hardening components.

The corotational formulation is used throughout this work. However, to simplify the notations we have left out the $\wedge$ symbol in the following.
3.2 Anisotropy

Due to the extrusion process, aluminium extrusions have different elastic and inelastic properties depending on the choice of axis. The material is thus anisotropic, as opposed to isotropic, which implies equal properties in all directions. Typically, an orthotropic material is produced where the principal material directions are the extrusion, transversal and through-thickness directions. The difference in material properties is characterised by different yield strengths and plastic flow for each direction. Thus, to quantify the anisotropy, material testing is made in different in-plane directions. Consider a material coordinate system according to Figure 3.3, where the in-plane angle from the extrusion direction is denoted $\alpha$. For a uniaxial tensile

![Figure 3.3: Reference material coordinate system](image)

...
by the Lankford coefficient, which is defined as

\[ R_\alpha = \frac{\varepsilon^p_w}{\varepsilon^p_l} = \frac{\varepsilon^p_w}{-\varepsilon^p_w - \varepsilon^p_t} \tag{3.24} \]

where the logarithmic strain indices \( w, l, \) and \( t \) denote the width, longitudinal and thickness directions of the test specimen, respectively. In an isotropic case, the Lankford coefficients become equal to unity. The relation between the logarithmic strain components in the two coordinate systems is found from

\[
\begin{align*}
\{ \varepsilon_l \} &= \left\{ \cos^2(\alpha)\varepsilon_{11} + \sin^2(\alpha)\varepsilon_{22} + 2\sin(\alpha)\cos(\alpha)\varepsilon_{12} \right. \\
\{ \varepsilon_w \} &= \left. \sin^2(\alpha)\varepsilon_{11} + \cos^2(\alpha)\varepsilon_{22} - 2\sin(\alpha)\cos(\alpha)\varepsilon_{12} \right\} \tag{3.25}
\end{align*}
\]

The Lankford coefficients can be related to the yield condition by using equations 3.24-3.25 and the assumption of an associated flow rule, e.g. in the \( 45^\circ \) direction

\[
R_{45} = \frac{\varepsilon^p_w(45)}{-\varepsilon^p_w(45) - \varepsilon^p_l(45)} = \frac{1}{2}(\varepsilon_{11}^p + \varepsilon_{22}^p) - \varepsilon_{12}^p = \frac{1}{2}\left( \frac{\partial f}{\partial \sigma_{\varepsilon^p_{11}}} + \frac{\partial f}{\partial \sigma_{\varepsilon^p_{22}}} \right) - \frac{1}{2}\frac{\partial f}{\partial \sigma_{\varepsilon^p_{12}}} \tag{3.26}
\]

Today, two of the most popular anisotropic yield criteria in metal forming simulations are the criteria according to Hill [22] and Barlat and Lian [6], which henceforth will be referred to as the Hill and the tri-component criteria. The yield criterion by Hill can be seen as an anisotropic extension of the isotropic von Mises yield criterion,

\[
f = F\sigma_{22}^2 + G\sigma_{11}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12}^2 - 2\sigma_f^2(\varepsilon^p) = 0 \tag{3.27}
\]

where \( F, G, H \) and \( N \) are material constants which have to be determined by material testing. The tri-component criterion contains five unknown material parameters

\[
f = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m - 2\sigma_f^m(\varepsilon^p) = 0 \tag{3.28}
\]

where

\[
K_1 = \frac{\sigma_{11} + h\sigma_{22}}{2} \tag{3.29}
\]

\[
K_2 = \sqrt{\left( \frac{\sigma_{11} - h\sigma_{22}}{2} \right)^2 + (h\sigma_{12})^2} \tag{3.30}
\]
Thus, $a$, $c$, $h$ and $p$ needs to be determined from four material tests. The parameter $m$ is assumed to be given by the crystal structure and was suggested to be 6 for BCC metals, and 8 for FCC metals, i.e. aluminium, see Barlat and Lian [6]. These yield criteria, and the criterion according to Karafillis and Boyce [28] were evaluated by Lademo [33], but none of them were found to yield accurate results for the aluminium alloys AA7108-T1, T5 or AA6063-T1. The main problem concerning these yield criteria is the number of available material constants. Usually the yield strength in the extrusion direction $\sigma_{00}$, and three Lankford coefficients $R_{90}$, $R_{45}$ and $R_{90}$, are used to determine the shape of the yield surface. The yield strengths in 45° and 90° directions are then left undetermined. The accuracy in predicting these parameters thus depends on the formulation of the yield criterion, and these predictions have been shown to be inaccurate. To address these drawbacks, Barlat et al. [7] presented a yield criterion which includes seven material parameters. This yield criterion is often denoted YLD96 and has been evaluated, e.g. by Lademo [32], and been found to be quite accurate. However, the yield surface in the YLD96 criterion is not proven to be convex, and its derivatives are not easily found analytically, which is inconvenient for the FE formulations. As a precaution, Barlat et al. [8] proposed the YLD2000 yield criterion which results in a convex yield surface, and which contains up to eight material parameters. A proper constitutive description is more important in tube hydroforming simulations than in e.g. deep drawing simulations since the hydroforming process is close to being a force controlled process. In a deep drawing process, the deformation of the blank is determined by the displacement of the punch, and the predictions in plastic flow is thus of high importance. In a hydroforming process, the deformation of the tube is determined by the equilibrium between the inner pressure and the tube wall, as well as the material feeding. A proper constitutive description of the hardening and the anisotropy both in stress and plastic strain is thus of equal importance.

The basis of YLD2000 is two isotropic convex yield functions formulated in terms of the principal values of the stress deviator, $s_1$ and $s_2$, according to

$$f = \Phi' + \Phi'' - 2\bar{\sigma}_f^m = 0$$

(3.31)

where

$$\Phi' = |s_1 - s_2|^m$$

(3.32)

and

$$\Phi'' = |2s_2 + s_1|^m + |2s_1 + s_2|^m$$

(3.33)
The yield criterion is extended to the anisotropic case by linearly transforming the stress deviator $s$ into $X'$ and $X''$, i.e. using the Voigt notation

$$X' = C's = C'T\sigma$$

$$X'' = C''s = C''T\sigma$$

where

$$C' = \begin{bmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{33} \end{bmatrix}$$

(3.36)

and

$$C'' = \begin{bmatrix} C''_{11} & C''_{12} & 0 \\ C''_{21} & C''_{22} & 0 \\ 0 & 0 & C''_{33} \end{bmatrix}$$

(3.37)

and where

$$\begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} = T\sigma = \begin{bmatrix} 2 & -1 & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$$

(3.38)

The transformation not only preserves the pressure independency, but also the convexity of $\Phi'$ and $\Phi''$. The transformation matrices $C'$ and $C''$ contains ten parameters, but Barlat et al. [8] suggests that $C'_{12} = C''_{21} = 0$ which assumption yields a total of eight parameters. If needed, additional symmetries can be introduced to further reduce the number of material parameters. The resulting YLD2000 yield criterion can thus be formulated as

$$f = |X'_1 - X'_2|^m + 2|X''_1 + X''_1|^m + |2X'_1 + X''_2|^m - 2\overline{\sigma}_m = 0$$

(3.39)

where $X'_1$, $X'_2$, $X''_1$, and $X''_2$ are the principal values of $X'$ and $X''$, according to

$$X'_1 = \frac{1}{2} \left( X'^{i''}_1 + X'^{i''}_2 + \sqrt{(X'^{i''}_1 - X'^{i''}_2)^2 + (2X'^{i''}_2)^2} \right)$$

(3.40)

$$X'_2 = \frac{1}{2} \left( X'^{i''}_1 + X'^{i''}_2 - \sqrt{(X'^{i''}_1 - X'^{i''}_2)^2 + (2X'^{i''}_2)^2} \right)$$

(3.41)
3.3 Stress update

To calculate the internal force vector, $f^{int}$ according to Equation 2.3, the stresses need to be updated. This is done by a hypo-elastic approach, thus

$$\dot{\sigma} = \dot{C}_{el} : \dot{D}$$

(3.42)

and

$$\hat{\sigma}_{n+1} = \hat{\sigma}_n + \hat{\sigma}_{n+1} dt$$

(3.43)

where $\hat{C}_{el}$ is the elastic material stiffness tensor, and $n$ denotes the current state. Thus, the stress update is made in the material coordinate system and then the stress tensor is rotated back to the global coordinate system, i.e.

$$\sigma = R \cdot \hat{\sigma} \cdot R^T$$

(3.44)

It could be argued that if a hypo-elastic approach is used, $C$ needs to be an isotropic tensor function. However, this is not the case in a corotated formulation, since the coordinate system rotates with the material and $C$ is unchanged by the material rotation. Henceforth, the corotational index will be dropped, and it is thus assumed that the calculations occur in the material coordinate system.

The total rate-of-deformation, $D_{n+1}$, as well as the Cauchy stress, $\sigma_n$, and the effective plastic strain, $\dot{\varepsilon}_p^e$, is known at the beginning of the stress update. The task is then to update the latter variables to time $t_{n+1}$. For a yield condition with associated plasticity and isotropic hardening, the problem takes the following form if the rate-of-deformation is decomposed into an elastic and a plastic part

$$D = D^e + D^p$$

$$\dot{\sigma} = C_{el} : (D - D^p)$$

$$D^p = \dot{\lambda} \frac{\partial f}{\partial \sigma}$$

$$\dot{\varepsilon}_p^e = \dot{\lambda}$$

(3.45)

The equations above can be solved by using an operator split technique, thus

$$D_t = D_t^e + D_t^p = D_{n+1}$$

$$\dot{\sigma}_t = C_{el} : (D_t - D_t^p)$$

$$D_t^p = 0$$

$$\dot{\varepsilon}_t^e = 0$$

(3.46)
and

\[ \mathbf{D}_n = \mathbf{D}_n^e + \mathbf{D}_n^p = 0 \]

\[ \dot{\mathbf{\sigma}}_n = C_{el} : (\mathbf{D}_n - \mathbf{D}_n^p) \]

\[ \mathbf{D}_n^p = \dot{\lambda} \frac{\partial f}{\partial \mathbf{\sigma}} \]

\[ \dot{\varepsilon}_p^p = 0 \]

(3.47)

where it should be noted that 3.46 and 3.47 add up to 3.45. The procedure can be viewed as a predictor corrector algorithm, where in the first step \( \mathbf{D}_n^p = 0 \). Thus, a pure elastic stress update is made. If the stress is within the yield surface (\( f_{n+1} < 0 \)), the step is indeed elastic, and the stresses at time \( t_{n+1} \) are equal to the elastic trial stresses. If, on the other hand, the trial stress is outside the yield surface, a plastic corrector is needed. During this step, the total rate-of-deformation is zero, and the plastic rate-of-deformation is evaluated such that \( f_{n+1} = 0 \). The plastic corrector could either be made in an explicit manner, where information in the current point is used, or in an implicit manner where iterations are needed. In this work, an explicit algorithm denoted the cutting plane algorithm is used, see Ortiz and Simo [43]. It should be noted that by this algorithm, the normality to the yield surface is enforced at the current point rather than at the updated point. However, the algorithm was found to yield acceptable accuracy.

Now, consider a case where an elastic step predicts a stress state, \( \mathbf{\sigma}^B \), which penetrates the yield surface, see Figure 3.4. A Taylor expansion is made at this point, where it is enforced that \( f^C = 0 \), thus

\[ f^C = f^B + \frac{\partial f^B}{\partial \mathbf{\sigma}} : d\mathbf{\sigma} + \frac{\partial f^B}{\partial \varepsilon_p^p} d\varepsilon_p^p = 0 \]

(3.48)

and by equations 3.47 and 3.19

\[ f^C = f^B - d\lambda \frac{\partial f^B}{\partial \mathbf{\sigma}} : C_{el} : \frac{\partial f^B}{\partial \mathbf{\sigma}} + \frac{\partial f^B}{\partial \varepsilon_p} d\varepsilon_p \]

(3.49)

since \( \mathbf{D}_n = 0 \). If the derivative of the yield criterion with respect to the effective plastic strain is denoted \( H \), the plastic multiplier can be evaluated according to

\[ d\lambda = \frac{f^B}{\frac{\partial f^B}{\partial \mathbf{\sigma}} : C_{el} : \frac{\partial f^B}{\partial \mathbf{\sigma}} + HB} \]

(3.50)

and

\[ \dot{\lambda} = \frac{d\lambda}{dt} \]

(3.51)
The stress and the effective plastic strain are then updated according to Equation 3.47, and the updated stress state is checked against the yield criterion. If necessary, further iterations are performed to fulfil the yield criterion. In the former situation, the stress, plastic strain, and effective plastic strain become

\[
\dot{\sigma}_{n+1} = C_{el} : D_{n+1} - \dot{\lambda}^B C_{el} : \frac{\partial f^B}{\partial \sigma} - \dot{\lambda}^C C_{el} : \frac{\partial f^C}{\partial \sigma} 
\]

(3.52)

\[
D_{p,n+1} = \dot{\lambda}^B \frac{\partial f^B}{\partial \sigma} + \dot{\lambda}^C \frac{\partial f^C}{\partial \sigma} 
\]

(3.53)

\[
\dot{\varepsilon}_{p,n+1} = \dot{\lambda}^B + \dot{\lambda}^C 
\]

(3.54)

Whenever membrane straining is substantial, such as in a metal forming process, the thickness strain is of great interest. However, the shell elements used in this work are based on the plane stress assumption, and thus the...
increment in thickness strain is not explicitly evaluated. The increment in thickness strain then needs to be evaluated from the stress update routine, which is usually done by enforcing the through thickness stress to be zero ($\sigma_{33} = 0$). If the yield criterion is formulated in 3-D, this is a straightforward operation, but in this case the YLD 2000 is formulated for a plane stress case. This problem is addressed by extending the YLD 2000 according to

$$f(\sigma_{11} - \sigma_{33}, \sigma_{22} - \sigma_{33}, \sigma_{12}) - 2f(\varepsilon^p)^m = 0$$  \hspace{1cm} (3.55)

which reduces to a plane stress formulation when $\sigma_{33} = 0$, see Borrvall and Nilsson [13]. To calculate the through thickness strain, the following algorithm is used

**i)** Assume a plane stress elastic step, $d\varepsilon^e_{33}$

$$d\varepsilon^e_{33} = \frac{-\nu}{1-\nu}(d\varepsilon_{11} + d\varepsilon_{22}) \hspace{1cm} (3.56)$$

where $\nu$ is the Poisson’s ratio. Perform a predictor followed by a plastic corrector step only for the through thickness stress

$$\sigma^e_{33} = \sigma_{33}(d\varepsilon^e) - d\lambda C_{el} \frac{\partial f}{\partial \sigma}$$  \hspace{1cm} (3.57)

**ii)** Assume a purely plastic step $d\varepsilon^p_{33}$

$$d\varepsilon^p_{33} = -(d\varepsilon_{11} + d\varepsilon_{22}) \hspace{1cm} (3.58)$$

and again, a predictor step followed by a plastic corrector for the through thickness stress

$$\sigma^p_{33} = \sigma_{33}(d\varepsilon^p_{33}) - d\lambda C_{el} \frac{\partial f}{\partial \sigma}$$  \hspace{1cm} (3.59)

**iii)** Iterate for the through thickness strain increment

The through thickness strain increment can be estimated through a secant iteration, i.e.

$$d\varepsilon_{33} = d\varepsilon^p_{33} - \frac{d\varepsilon^e_{33} - d\varepsilon^p_{33}}{\sigma^e_{33} - \sigma^p_{33}} \sigma^p_{33} \hspace{1cm} (3.60)$$

If the through thickness stress $\sigma_{33}(d\varepsilon_{33}) = 0$, the iterations have converged and a plastic correction is performed as described previously. Otherwise, further iterations are needed.
3.4 Yield surface calibration

In Section 3.2, it was concluded that test data determined from standard uniaxial tensile tests can be used to calibrate the parameters for the YLD-2000 yield surface. In this work, uniaxial tensile tests were performed in directions 0°, 45° and 90°, which yields three yield strengths and three R-values i.e. a total number of six material data. The test specimens were cut from an extruded sheet. Additional tensile tests in any other direction could have been used for additional material parameters. However, the most information is gained from a test condition which is close to the hydroforming loading process. For this purpose, a hydrobulge test rig according to Figure 3.5 was built. The tube is clamped at both ends, which will yield a state of zero strain in the extrusion direction of the tube assuming small elastic deformations and a sufficiently long tube. Thus, using Hooke’s law

\[ \varepsilon_x = E \left( \sigma_x - \nu \sigma_\phi \right) = 0 \]  

(3.61)

\[ \frac{\sigma_x}{\sigma_\phi} = \nu \]  

(3.62)

\[ \frac{\sigma_x}{\sigma_\phi} = \nu \]  

(3.62)
where $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, and where $\sigma_x$ and $\sigma_\varphi$ are the stresses in the extrusion and the circumferential directions, respectively. A biaxial state of stress is thus found, where the pressure at the onset of yielding can be used to calculate the corresponding stresses. The radial displacement is subsequently measured which yields pressure and radial expansion data to the point of tube burst. This data can then be used for calibrating the hardening behaviour by inverse modelling.

The uniaxial tensile test is a useful test since it is simple, cheap and robust. It is also a standard test, which makes material data easy to obtain if not available. One drawback is that necking occurs in the specimen, which fact makes large strain data difficult to evaluate. By using extruded sheets it became unnecessary to cut out and flatten the test specimen, a process which could alter the properties of the material. It can be argued that material data from uniaxial tensile tests on extruded sheet specimen is not ideal for tube hydroforming. Firstly, since the sheet material properties could differ from the tube material properties due to the different tooling used in the extrusion processes. Secondly, uniaxial test data is of minor importance since the stress state is mainly biaxial during tube hydroforming. Ideally, more material testing should be done with hydrobulging of tubes, e.g. with material feeding, which would give more relevant stress and strain data for calibration. However, by clamping the tube ends, the test becomes well defined and thus repeatable. Further, at least one test which measures the shear properties of the material must be made (e.g. $R_{45}$ and $\sigma_{45}$), and this type of test is not easily obtained with a tubular specimen.
4

Failure Modelling

In this work, material failure is considered to be caused by plastic instability. As described briefly in Section 3.1, plastic instability is twofold. First, the diffuse instability occurs. In a uniaxial tensile test it is accompanied by a strain localisation and a contraction of the specimen width. In the general case, the diffuse instability is not visible. However, it can be identified through an engineering measure, e.g. the pressure in a hydroforming case. Figure 4.1 presents the pressure and displacement response of a hydrobulge test. Diffuse instability is followed by local instability, where the strain localises through the thickness. The width of the localised zone is approximately equal to the thickness of the sheet. Beyond this point, the deformation is concentrated to this area and the strain increases rapidly until rupture.

One classic way to predict diffuse instability is to use the Swift diffuse instability criterion, Swift [48]. Consider a uniaxial tensile test specimen according to Figure 3.1. Diffuse instability is then defined by the point where $dP = 0$. Thus,

$$P = \sigma A \tag{4.1}$$

$$dP = \sigma dA + Ad\sigma = 0 \tag{4.2}$$

By volume constancy

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon \tag{4.3}$$

The point of diffuse instability then becomes

$$\frac{d\sigma}{d\varepsilon} = \sigma \tag{4.4}$$
Swift’s diffuse instability criterion can be extended to a biaxial loading by stating that the condition for diffuse instability is when the in-plane forces reach a maximum point. Thus,

\[ dP_x = 0 \]
\[ dP_y = 0 \]  \hspace{1cm} (4.5)

or

\[ d\sigma_x = \sigma_x d\varepsilon_x \]
\[ d\sigma_y = \sigma_y d\varepsilon_y \]  \hspace{1cm} (4.6)

where it is assumed that the x and y directions of stress are principal directions. Now, consider a thin walled tube which is subject to an inner pressure \( p \). It is assumed that the tube expands with no axial feeding. Thus, the tube deforms under a plane strain condition. If the constitutive behaviour obeys von Mises yield criterion, then

\[ \bar{\sigma} = \sqrt{\sigma_x^2 - \sigma_x \sigma_\varphi + \sigma_\varphi^2} \]  \hspace{1cm} (4.7)

From the flow rule and plane strain condition we get

\[ \sigma_x = \frac{1}{2} \sigma_\varphi \]  \hspace{1cm} (4.8)
where $\sigma_x$ is the axial stress and $\sigma_\varphi$ is the circumferential stress. By differentiating Equation 4.7 and using Equation 4.8 we get

$$d\bar{\sigma} = (2\sigma_x - \sigma_\varphi)d\sigma_x + (2\sigma_\varphi - \sigma_x)d\sigma_\varphi = (2\sigma_\varphi - \sigma_x)d\sigma_\varphi = \frac{3\sigma_\varphi^2}{4\bar{\sigma}}d\varepsilon_\varphi \quad (4.9)$$

The relation between $d\bar{\varepsilon}$ and $d\varepsilon_\varphi$ is found by

$$d\bar{\varepsilon} = \frac{2}{\sqrt{3}}d\varepsilon_\varphi \quad (4.10)$$

By using equations 4.7, 4.9 and 4.10 the condition for diffuse instability at plane strain becomes

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\sqrt{3}}{2} \frac{\bar{\sigma}}{} \quad (4.11)$$

If the hardening of the material is described by a power law, i.e.

$$\bar{\sigma} = K\varepsilon^n \quad (4.12)$$

then by Equation 4.10 the circumferential strain at diffuse necking becomes

$$\varepsilon_\varphi = n \quad (4.13)$$

A tube with an inner pressure $p$ is governed by the following equilibrium equation

$$p = t \left( \frac{\sigma_\varphi}{r_\varphi} + \frac{\sigma_x}{r_x} \right) = t\frac{\sigma_\varphi}{r_\varphi} \quad (4.14)$$

where $r_x = \infty$ and $r_\varphi$ are the longitudinal and circumferential radii, respectively, and where $t$ is the tube thickness, see Figure 4.2. In the case of plane strain, the circumferential and thickness strains become

$$d\varepsilon_\varphi = -d\varepsilon_t = -\frac{dt}{t} = \frac{dr_\varphi}{r_\varphi} \quad (4.15)$$

The diffuse necking occurs when the inner pressure reaches its maximum, thus $dp = 0$, i.e.

$$\frac{dt}{t} + \frac{d\sigma_\varphi}{\sigma_\varphi} - \frac{dr_\varphi}{r_\varphi} = \frac{d\sigma_\varphi}{\sigma_\varphi} - 2d\varepsilon_\varphi = 0 \quad (4.16)$$

By equations 4.7 and 4.10, the point of diffuse necking of a tube with inner pressure becomes

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \sqrt{3}\bar{\sigma} \quad (4.17)$$
The circumferential strain at maximum pressure is thus only half of the limit strain predicted by the Swift criterion. In fact, the maximum pressure defines the actual forming limit during free expansion. Although, the material can be deformed past the maximum pressure, the process is unstable and virtually uncontrollable. It is obvious from the equations above that the instability pressure depends on the hardening of the material and the stress path. It should however be noticed that it, unlike the Swift diffuse instability criterion, depends on the current geometry. That is, if the geometry of the tube changes, the instability could be postponed and further deformation can occur. The point of maximum pressure is accurately predicted by FE analysis, presuming that a proper constitutive description is used.

Beyond the diffuse instability limit, local instability will occur whenever a direction of plane strain exists in the tube wall. This assumption is the basis of the Hill localisation criterion, Hill [23]. Thus, if $\varepsilon_x=0$, the local instability will coincide with the diffuse instability. For a general strain path, a direction
of plane strain is equivalent to

$$\varepsilon(\theta) = \varepsilon_x \cos^2(\theta) + \varepsilon_\phi \sin^2(\theta) = 0$$ \hspace{1cm} (4.19)

where $\theta$ is the angle between the extrusion axis and the direction of plane strain. Further, it is assumed that the $x$ and $\phi$ directions are principal directions, and that $\varepsilon_\phi > \varepsilon_x$. The angle where an instability can occur is thus

$$\tan^2(\theta) = -\frac{\varepsilon_x}{\varepsilon_\phi}$$ \hspace{1cm} (4.20)

By this, it can be noticed that since $\tan^2(\theta) \geq 0$ this type of analysis is not valid for positive strain ratios. Instead, the Marciniak and Kuczyński [39], (MK) model could be applied. In the MK model it is assumed that the instability will originate from a thickness inhomogeneity, see Figure 4.3. A prescribed strain path is applied to the homogeneous part (A), and the stresses and strains are found in the inhomogeneity (B) from equilibrium and compatibility conditions. Whenever, the increment in normal or shear strain in part B is far greater than the corresponding strain increment in part A, the strain is assumed to be localised. This procedure is repeated for various angles $\theta$ until the minimum limit strain is found. There is a definitive resemblance with the criterion according to Hill, where both criteria aims to find the direction of plane strain. However, the MK model is valid for all strain ratios.

![Figure 4.3: Principle model of the MK analysis](image)

In Lademo et al. [34], rectangular FE patches with statistically distributed thicknesses are used to analyse the deformation along different strain paths by prescribing the boundary motions. By a non-local instability criterion, the authors were able to predict the localisation limits for various aluminium
As described in Section 1.2, tube hydroforming consists of several phases. During the free forming phase, the contact with the die wall is limited and feeding is possible. This is the part of the process where the instability pressure is likely to limit the formability. The main task in this step of the process design is thus to avoid prescribing a pressure beyond such an instability. The complex nature of the maximum pressure, hence its dependence both on the material behaviour and the current geometry, makes it difficult to predict for a general case. By FE simulations, it is however possible to apply suitable changes to the process parameters and/or the tool design in order to postpone diffuse necking. As the calibration forming begins, the influence from material feeding will be limited and the strain path will shift towards plane strain. During this phase, the maximum pressure will dramatically increase due to the decrease in section radii, e.g. in the case of corner filling, see Figure 4.4. Due to frictional forces on the tube part in contact with the die, the material adjacent to the radius, still not in contact with the die, is likely to be subjected to high in-plane forces. These circumstances are similar to the conditions assumed in the analytical and experimental Forming Limit Diagrams (FLD).

![Figure 4.4: Corner filling](image_url)
5

Process parameter estimation

The main task when simulating tube hydroforming is to avoid failure by finding an appropriate balance between material feeding and internal pressure, i.e. to identify the working range of the current process, see Figure 5.1. Analytical guidelines on how to choose loading parameters can be found in e.g. Rimkus et al. [45] and Koç and Altan [30]. In Chu and Xu [16], the working range is analytically determined for a free bulge case and compared with experiments. Considering numerical simulations, the Finite Element Method is, as discussed previously, the dominating tool for simulating forming processes. Examples of hydroforming simulations of automotive parts can be found in e.g. Lei et al. [35], Hama et al. [20] and Asnafi et al. [3]. Apart from the process parameters and the working range, the following responses can be found by FE simulations of tube hydroforming:

- **Maximum pressure** This response is important since it determines the necessary clamping force of the hydroform press. More specifically, the clamping force is found by

\[
F_{\text{clamp}} = p_{\text{max}} A_{\text{proj}}
\]

where \( A_{\text{proj}} \) is the projected area of the die cavity. If the sealing force is too low, the upper and lower parts of the die could separate. The maximum pressure depends on the hardening of the material and the smallest radius of the die.

- **Volume increase** The volume increase of the tube and the maximum pressure determine the volume of fluid which has to be pumped into the tube. This determines the capacity of the high pressure pump, which is especially interesting in cases of large geometries with a high degree of expansion.
• *Minimum feeding* The axial stroke of the feeding cylinders is not only used for feeding material into the die cavity, but also for sealing. If the draw in of the tube ends during deformation is larger than the prescribed feeding, usually denoted self feeding, leakage can occur. The minimum amount of feeding can be found by monitoring the reaction force responses corresponding to the prescribed displacements of the tube ends. Also, a so called self feeding simulation can be useful where the tube is pressurised with free tube ends and with no friction between the tools and the tube. This simulation determines the self feed of the process, and thus the minimum amount of feeding.

![Diagram](image)

**Figure 5.1: Working range in a hydroforming process**

In Gao et al. [18], tube hydroforming processes are classified into categories based on their sensitivity to the loading parameters, i.e.

- a) Pressure dominant
- b) Pressure driven
- c) Feeding dominant
- d) Feeding driven

Pressure dominant tube hydroforming processes require none or a limited amount of feeding. This could, e.g. be due to complicated die geometry, limited circumferential expansion or a large amount of preforming during die closure. This type of process requires no loading parameter estimation since it only depends on the internal pressure, and thus it is merely a calibration
forming. The only type of failure which needs consideration is tube burst. If the process is unsuccessful, the only options are either to alter the die geometry and friction conditions or to select another material.

It is most challenging to predict the process parameters in the case of a pressure-driven process. This process type is characterised by a high amount of expansion which requires a large amount of material feeding. Thus, all tube hydroforming failure modes need to be considered. This category of processes benefits most from process parameter optimisation procedures. Together with an optimisation routine, the FE simulation becomes an effective tool for finding optimal load curves. A distinction can be made between iterative and adaptive approaches. In iterative approaches, design sensitivities are used to iteratively find the optimum solution. In Yang et al. [49], direct differentiation and sequential programming are used to determine the optimal load curves for tube expansion of a sub-frame. The conjugate gradient method is used by Fann and Hsiao [17]. However, formulating design sensitivities of objectives and constraints can be difficult in the general case. This difficulty can be avoided by using the Response Surface Method (RSM), which utilises function evaluations to construct polynomial approximations of the objective and constraints, see Imaninejad et al. [26]. Apart from the problem of formulating design sensitivities, the number of iterations could become significant and the method will then not be time efficient.

If instead an adaptive simulation approach is used, a load curve estimation can be found by a limited amount of simulations. The basic idea of the method is to continuously monitor the solution by using indicators on e.g. wrinkling or bursting, and if necessary, apply suitable changes to the process parameters to avoid failure, see e.g. Aue-U-Lan et al. [4], Johnson et al. [27] and Aydemir et al. [5]. The resulting process parameters are however not optimal since tube hydroforming is highly process path dependent.

The task is then to find a loading process which feeds the maximum amount of material into the die cavity while avoiding wrinkling and pressure instability. An algorithm, which finds such a process, requires not only a proper constitutive model, but also a wrinkle indicator which not only detects the occurrence of wrinkles but also rates the severity of the wrinkle. Since the objective of the algorithm usually is to avoid thinning, the resulting process parameters often involve volume expansion during a small, or even no increase, in pressure, see Figure 5.2. The pressure is used only to maintain an acceptable amount of wrinkling, which can be smoothed out during calibration. The process is thus shifted from a pressure dominant process towards
a feeding dominant process.

In the feeding driven processes, the volume of the tube is decreasing dur-

![Graph showing pressure vs. axial feeding](image)

Figure 5.2: A typical optimized process parameter curve

ing tube hydroforming. Thus, the inner pressure is not generated by a high pressure pump but by the feeding itself. In fact, water needs to be let out in order to limit the pressure. Often parts, which are produced with this type of process, have a small bulged area, e.g. as T-shaped parts.
6

Summary of appended papers

6.1 Paper I

In this paper, material modelling of extruded aluminium is studied from a hydroforming point of view. Uniaxial tensile testing together with a hydrobulge test were used to calibrate the YLD-2000 criterion, Barlat et al. [8], for the AA6063-T4. Comparisons were made with the popular Barlat and Lian [6] and Hill [22] yield criteria, which were found to be unable to describe the anisotropic behaviour of AA6063-T4. Further, FE-simulations of the hydrobulge test were performed with the studied yield criteria, and it was concluded that a proper constitutive description is crucial for accurate predictions of hydroforming processes.

6.2 Paper II

The paper concerns hydroforming in a die. Firstly, the hardening behaviour of AA6063-T4 was determined from a hydrobulge test. Secondly, the constitutive behaviour was validated by experiments in a die. By evaluating the circumferential thickness, it was possible to estimate the friction coefficient in dry and lubricated conditions by inverse modelling.

6.3 Paper III

The preceding papers concerned hydroforming without axial feeding, which has been added in this paper. Experiments were carried out with a conical die, where extensive feeding was necessary for a successful result. Since hydroforming, as opposed to conventional deep drawing, is a force controlled process, the deformation is determined by the equilibrium between the inner
pressure and the stresses in the tube wall. This fact raises demands on an accurate constitutive behaviour to predict the strain distribution as well as the corresponding stress distribution. By using interrupted tests it was possible to record the deformation of the tube, i.e. the tube wall thickness and the radial deformation, at several instances during the hydroforming sequence. Further, the Hill yield criterion was included for comparative reasons. It was found that the YLD-2000 material model was able to predict the thickness distribution, radial deformation and tube wall wrinkling with good accuracy.

6.4 Paper IV

This paper concerns the process parameter estimation procedure. Conventionally, the pressure is used as a process parameter in tube hydroforming simulations, since the solution process then becomes intuitive. However, if the process is pressure dominant the deformation becomes very sensitive to the pressure. Thus a deformation controlled process is preferred. Three different estimation procedures are presented from a deformation control point of view and these procedures are applied in the simulation of a hydroforming process with a conical die. Firstly, a self feeding method is presented, which is intended as a method for a first estimate. Secondly, an optimisation setup is presented which uses the Response Surface Methodology. Last, an adaptive procedure is presented, which uses a limited amount of simulations for process parameter estimation.

6.5 Paper V

One of the main concerns, when performing tube hydroforming simulations, is the prediction of strain localisation and tube burst. Several different plastic instability criteria were evaluated, see Swift [48], Hill [23], Marciniak and Kuczyński [39] and Lademo et al. [34]. Contradicting results were found between free bulge cases and hydroforming in a die, where the limiting strains vastly exceeded the ones predicted by e.g. the MK-model. This is believed to be due to the diffuse instability limit which depends not only on the current equilibrium but also on the current geometry. The prediction of the limiting strains thus depends on the prediction of the maximum pressure, which in turn depends on the constitutive description. It was found that an accurate prediction of the strain localisation was possible when a distributed thickness was used which triggered the strain localisation, c.f. Lademo et al. [34].
Outlook

It has been argued that the tube hydroforming process has drawbacks such as long cycle times and expensive tooling. Also, in order to make full use of a hydroformed part, completely new design concepts might need to be considered. Thus, it is not surprising that the use of this forming method has been limited. However, due to continuous improvements in process control, tooling and lubrication, hydroforming is gaining an increasing acceptance in industry. This dissertation concerns FE simulations of the tube hydroforming process, and it is my firm belief that FE simulations will be an important tool in the future development of tube hydroforming. As a matter of fact, FE simulations may be the predicting tool needed for making the hydroforming process generally accepted in industry. Several of the techniques used in deep drawing simulations are applicable also in a tube hydroforming context. However, the user has to be aware of its specifics, such as mechanics, material modelling and testing, and failure prediction.

The topics addressed in this thesis concern the constitutive modelling, failure prediction and process design, but still there is a need for many improvements.

More knowledge about the complex behaviour of the material, which is subjected to hydroforming, may be gained from modelling its micro-structure. With a multi-scaled material modelling approach the details of the constitutive behaviour can be better described, and the plastic flow and hardening might be understood even along complex loading paths. Today multi-scale material models suffer from long computing times. From an industrial application point of view a material model must also efficiently solve large scale problems. Thus there is a need to develop an improved material model, which meets both accuracy and efficiency constraints.
A more complex material model generally requires experimental data from a variety of experimental tests. It is a challenge to develop a minimal set of tests that fully define all parameters needed for the material model in the context of its application.

Material instabilities and rupture are of great concern in the design of all forming processes. Obviously, the utilization of an accurate material model makes the Forming Limit Diagram (FLD) redundant information. There is, however, a need to improve the failure prediction capability both from accuracy and efficiency points of view. Examples of challenging future research needs are the exploration of the acoustic tensor to predict material instabilities and the development of an accurate wrinkling indicator. In a future FE simulation of a hydroforming process, material failure and wrinkling should be predicted "on the fly", and accurate predictions of safety margins to failure should be a part of the simulation results.

The hydroforming of the aluminium alloy AA6063 is performed in an unstable material state (T4). Thus, to establish full strength, the formed part is solution hardened to a stable temper (T6). To be able to accurately predict the behaviour of the finished part in its intended function, the effects from the ageing process on the material properties after forming must be fully understood and a model of the ageing must be developed.
Bibliography


