Sensitivity and Uncertainty Analysis Methods,

with Applications to a Road Traffic Emission Model

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Abstract

There is always a need to study the properties of complex input–output systems, properties that may be very difficult to determine. Two such properties are the output's sensitivity to changes in the inputs and the output's uncertainty if the inputs are uncertain.

A system can be formulated as a model—a set of functions, equations and conditions that describe the system. We ultimately want to study and learn about the real system, but with a model that approximates the system well, we can study the model instead, which is usually easier. It is often easier to build a model as a set of combined sub-models, but good knowledge of each sub-model does not immediately lead to good knowledge of the entire model. Often, the most attractive approach to model studies is to write the model as computer software and study datasets generated by that software.

Methods for sensitivity analysis (SA) and uncertainty analysis (UA) cannot be expected to be exactly the same for all models. In this thesis, we want to determine suitable SA and UA methods for a road traffic emission model, methods that can also be applied to any other model of similar structure. We examine parts of a well-known emission model and suggest a powerful data-generating tool. By studying generated datasets, we can examine properties in the model, suggest SA and UA methods and discuss the properties of these methods. We also present some of the results of applying the methods to the generated datasets.

Keywords : Emission, Grid, Model, Pollutant, Response surface, Road traffic, Sensitivity analysis, Simulation, Uncertainty analysis

Contents

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1 Introduction

1.1 Complex systems

Complex input–output systems are found everywhere. A city road network is such a system, in which the output is total travel time, and the inputs are the speed limits of each link, the traffic light settings at each intersection, and the travel demand distributed throughout the network. System complexity may simply be a matter of the sheer size of the system, but may also stem from the output having a complex dependency on the inputs. Some systems also add random variation to their output. Finally, the area of allowed input values may be irregular in shape. Input–output dependency, limits on the inputs, and other conditions can all be grouped in the category of "relations."

In this thesis, we examine systems with some particular properties. We allow a multidimensional input, but the output must be one-dimensional. We only discuss systems of an input–output form without the possibility of the output having influence on the input through some other system. We do not discuss systems, however complex, if they are not subject to input–output relations. In the example of a road network, we have input–output relations with a multidimensional input and a one-dimensional output, but there may be dependency in both directions between travel time and travel demand, meaning that this may not be a system of pure input–output form.

1.2 Modeling systems

When starting to analyze a system, one may find that it is defined by known relations or laws of nature. If these relations are not exactly known, it may still be possible to have reasonably good knowledge of, for example, the input–output structure, which variables have important influences on the output, whether there are random components, and the shape of the input area. Based on what we know, we may build a *model* of the system. The model is a set of relations that we think structurally agree with the real system. Coefficients of the functions as well as distributions of the input and random component, if present, may be known or be otherwise estimated using statistical methods. One reason for modeling may be our desire to understand the system itself and measure the size or importance of the constituent relations. More importantly for the purposes of this thesis, is that with a model that follows the real system sufficiently well, one can examine the properties of the system without the need for expensive full-scale observations. Though we only mention model studies below, these can also be regarded as indirect studies of real systems.

Although above we only mentioned the case of one model for one system, it is common to start with one separate model for each part of a system, combining the parts afterwards to form a model of the entire system. In the case of a city road network, one can imagine a model being developed in steps, with one part for the inner city area and another for each surrounding area, for example. It is also common for an update of the model to be inserted as a separate correction model rather than as an update of the original model *per se*. In the following discussion, we will say "total model" when discussing a model of the entire system and "sub-model" when discussing a part of the total model.

It is almost impossible to determine the output properties of the total model if we only have information regarding each sub-model—however good this information may be. In our example, sub-models may be different links or intersections; when combining them to form a network, however, the output of one sub-model influences the input of another in such a complex way that it is almost impossible to determine exactly, even with good knowledge of each separate sub-model. Therefore, one must combine the sub-models to form a total model and actually study the total model as such. A total model usually comprises a huge set of rather simple relations (our example of a model of a city road network is no exception to this), and is usually suitable for formulation as computer software. One way to perform model studies is to study the output pattern when making many runs with different input settings or, if random components are present, also studying repeated runs with the same settings. We have avoided using the word *simulation*, because we think it may have different meanings in different scientific fields. A real experiment—not necessarily full scale—can be treated in the same way as a model run. It may, however, be a practical limitation, that some analyses of model data demand many experimental runs.

Though model studies offer many possibilities, they are not a general solution to every problem in complex input–output systems. A model is always only an approximation. The importance of checking and validating the model should not be underestimated, but is matter beyond the scope of this thesis. Our discussion of methods and applications continues after a model is defined and properly checked and validated. We are confident, however, that though model studies are not a general solution to every problem in complex input–output systems, they really do open up many possibilities.

1.3 Output properties of a total model

We have argued above as to why it is important to combine sub-models to form a total model. Once a total model is developed, we can then study its properties by studying the output level and many other output properties, such as the variation, maximum, and shape of the output distribution. We may, for example, want to study the output for input combinations we have not been able to observe in the real system, or how the output reacts to various changes in the model itself. In our example, we may want to know how total travel time will change if the speed limits are changed in some road links or if a new link is opened.

In this thesis, we want to focus on two matters that are both output properties of a total model. First, we want to study methods for sensitivity analysis (SA) from different viewpoints, that is, methods for determining how sensitive the output is to changes in input. Second, we want to study methods for uncertainty analysis (UA), that is, methods for determining how uncertain the output is if the input is uncertain. There are many types of SA and UA, and many available methods, so we will not try to give a formal definition here. A discussion of some SA and UA principles will follow in section 3.

We want to study sensitivity and uncertainty as overall properties of a model, but only as functions of a varying or uncertain input to a fixed model. Sensitivity and uncertainty can also be treated as functions of the model structure or of the coefficients in the model, regarded as part of the model validation procedure.

1.4 Two fields for use of models

Research into traffic-related problems traditionally uses models of traffic flow, road surface maintenance, accidents, car fleet renewal, and many other matters. All these cited matters are traffic related, though they differ greatly from each other. Environmental research in general also uses models of emissions, temperature change, pollutant migration in the soil, etc. Not surprisingly, the traffic and environmental fields share an interest in studying road traffic emissions. There is no doubt that road traffic emissions have an impact on the global environment. It is important to find good tools for calculating the emission levels, and how they will change if the traffic situation changes.

We do not have any practical method for actually measuring the emissions of an entire vehicle fleet. Making such measurements is theoretically possible, but would be too expensive and not of practical interest. Our approach must instead be to estimate the emissions as a function of traffic-related inputs of different shapes, such as the distribution of different technology levels in the car fleet, mean speed on the road network, and ambient temperature.

2 The subject of this thesis

Though all the fields, systems, and models mentioned above are important, we cannot consider them all in this thesis. We are particularly interested in examining SA and UA methods that can be applied to a road traffic emission model. Besides our interest in the methods themselves, we would also like to produce numerical results when applying the SA and UA methods to model-generated data. We note that though much has been written about SA and UA, few studies have considered either SA or UA for road traffic emissions together with providing detailed descriptions of the emission model and the statistical analysis methods used. Others have previously noted and commented on this fact (see Kühlwein *et al.*[11]).

We have selected one emission model for study, and suggest SA and UA methods based on the generated datasets. We also present some numerical results when applying the suggested methods. Our discussion of UA and SA methods is general and can be applied to any problem from any scientific field, as long as the model follows the same type of structure. We will now outline the content of this thesis, which comprises a study of sensitivity and uncertainty analysis methods with applications to a road traffic emission model.

1. Select an established road traffic emission model. Try to determine the main structure of the model, but be aware that when discussing analysis methods, the choice of methods may be based on data rather than on detailed model knowledge. Also determine whether the model defines

properties of the inputs, such as limits, variable types, and whether there is any assumption of dependencies between inputs.

2. Suggest SA methods that can be used over a wide input area, based on data for the particular problem and possibly on some knowledge of the model. Both SA methods based on slopes and variance based sensitivity analysis (VBSA) are of interest in this regard. After suggesting methods, try to find a way to apply them efficiently, and then apply them to the selected emission model.

3. Suggest UA methods based on data for the particular problem and possibly on some knowledge of the model. After suggesting methods, try to find a way to apply them efficiently, and then apply them to the selected emission model.

4. Though we can suggest SA and UA methods as well as guidelines for how to apply them practically, we cannot apply them without having data. Therefore, find detailed information about the model and prepare a tool for easily generating data. General results are of the most interest, so we need data for a wide range of input values; that way we need put less effort into finding input relevant to any particular area or road network.

The included papers do not follow the above enumeration. See section 4 for a proper description of the contents and the order of the included papers.

3 Analysis methods and emission models

In this section, we will briefly describe some of the principles of SA and UA and give some examples. This is meant only as an overview, as it is not our intention to describe each existing method or to provide the details of how to perform the analysis. We also wish to present some discussion of emission models. Finally, we describe how we chose between the presented methods and models.

3.1 Sensitivity analysis

The aim of SA is to determine how sensitive the output is to changes in the input. Such sensitivity can be expressed in many ways and for different purposes.

Local and global methods

Local methods aim to study how sensitive the output is to changes in the input when inputs vary around but close to some standard point, called a nominal point. This may be advantageous for models in which a logical nominal point exists. The results of such methods can be used to determine which variables it is more important to control or to determine the possible benefits of upgrading to a better control system.

Many models, on the other hand, do not have a nominal point. We are then more interested in global methods, i.e., methods that study the impact of a changed input anywhere with the possible input areas. This is typically the case in models in which we can study but not control the inputs, or in models that do not have a logical nominal point. The results of global methods are more general, so global methods need more checking to determine whether the results really can be generalized in a particular way; as well, they need data from a wider input area.

Sensitivity measured as slope

One intuitive approach to SA is to study the input–output functions expressed as slopes. How many units will the output change with a one-unit increase in the input? This means studying the derivatives—or partial derivatives in the case of a multidimensional input. In some situations, another somewhat similar approach can be to examine the relative change, for example, how many times the output changes with a 10% increase of the input. The shape of the model, being mostly additive or multiplicative, is also important in determining the choice between absolute and relative measures.

Direct methods, sampling-based methods etc.

If the model is not too complex, it may be possible to use direct methods, to calculate the sensitivity measures directly from the relations by purely mathematical methods.

In many cases, however, there are problems with the direct methods. The model may be too complex, including step functions or other non-linear properties, power transformations, interactions, and any other property that makes direct methods impossible or at least not attractive in practice. A rather straightforward approach is then to let the model generate output for a set of input combinations and to analyze them using regression analysis or any other statistical method. This also gives a rather simple way to handle many extensions, such as multidimensional inputs, power transformations, and interactions. Factorial inputs can be handled by using indicator variables or by using a general linear model. Statistical methods can also be used as a smoother, if, for example, the response has many small steps that can be reasonably approximated by a smooth surface without great loss of accuracy.

As well, some questions need analysis based on observed data, but with no possibility of controlling the input values. In such situations, the input is more sampling based than subject to experimental design. The statistical analysis given a specific dataset may nevertheless be the same.

Variance-based sensitivity analysis

Though slopes were introduced as an intuitive approach, other approaches are also possible. In some problems we want to study the amount of variation in the output and how it can be explained by different sources, rather than to see the outputs slope against the inputs. If the total variation can be apportioned to variation sources, we can determine whether the output can be stabilized by better controlling the inputs without needing to know the exact input–output relations. These so-called variance-based sensitivity analyses can be very useful in case of non-linear response functions and categorical inputs, but also standing for themselves, as measures of variation. Such methods bring other difficulties to the sampling and analysis. One is that it is often difficult to apportion variation to its various sources. Another is that it is more important to choose representative values for the input variables in variance-based than in slope-based SA.

Order importance

For some models, the most important result is the order of the sensitivities rather than the slopes or the sizes of variation components. Ordering by importance can also be a matter of sorting the inputs into importance groups, rather than finding a complete sorting order. When trying to order by importance, one must also choose a proper importance measure. A steeper slope does not automatically indicate greater importance, because different units may make comparison impossible, and a steep slope may be less important if the input variable can only vary over a short interval.

Graphic methods

Graphic methods are sometimes the best way to summarize the sensitivity results. There are also methods in which a graphic illustration also inherently presents the sensitivity measures in itself.

Categorical or continuous input

Many of the above matters require inputs that are continuous on at least interval scale. However, there are also many models with categorical inputs, with or without ordered categories, and it is impossible to define a slope against a categorical input. The analysis can instead be done by comparing the output levels between categories, or by comparing the sensitivity to the distribution of the categories—a distribution that may be continuous. Also, VBSA methods can perform well with categorical inputs.

Other details

Above, we described some principles and what different SA methods attempt to accomplish. The mathematical procedures for the SA methods may in turn differ widely. We have mentioned direct methods that use mathematical calculations applied directly to the model definitions. SA based on datasets may use the least squares method. There are also other types of calculus, which may be based on maximum likelihood, Fourier transforms, Bayesian methods, or neural networks.

3.2 Uncertainty analysis

The aim of UA is to find out how uncertain an output is, by determining how uncertain the input is and how this uncertainty finally produces uncertainty in the output. It is related to SA but is certainly not the same. UA typically entails studying the variation of an output given a known distribution of the input. An intuitive approach is to use a model, generate data by varying the input according to the known distribution, and simply calculating the variance of the output. As with SA, the intuitive approach is not the only one, and the approach need not be simple; we can discuss direct methods, local and global methods, graphic methods, etc.

3.3 Road traffic emission models

There are many road traffic emission models. The wide variety of models may reflect the fact that different data are available in different countries, and that other circumstances may also differ between countries. For example, road gradient can vary markedly from country to country, and is more important to take it into account in some countries. There are also models of emissions for special situations or areas, such as cold starts or near intersections.

Developing an emission model requires techniques for collecting data, possibly from different sources. One technique is to use special cars equipped with emission instruments, drive them through a particular cycle, and study the relationship between emission level and speed, acceleration, etc. Another way to measure emissions throughout a driving cycle is to use a chassis dynamometer, which does not require on-board instrumentation. For other purposes, it may be preferred to use an instrumented car and follow normal traffic flow rather than using a standardized driving cycle.

Data collection for validation may be done as described above, but when discussing the validity of total emissions results, such a level of detail is not always necessary. For one vehicle going through a single driving cycle, it may also be possible to sample emissions from the exhaust but do the chemical analysis elsewhere. If data from individual vehicles are not important, the emissions can be measured in a more or less closed room, after which the results are matched to traffic data (this technique has been used in tunnel studies). A problem with this approach is that the driving conditions may never vary over the whole area of interesting input combinations.

Giving a full summary of emission models is beyond the scope of this thesis. There are many more models available than those mentioned above, and exact methodological descriptions are not always available. The structure of the emission calculations may vary widely depending on vehicle category, fuel used, and specific pollutant.

With this proliferation of models, there may be some models that give different outputs for exactly the same input. This may not be a problem for every research question: two methods producing different results may well give approximately the same degree of change when the input is changed by a certain amount, meaning that they can both be appropriate for different types of sensitivity analysis.

3.4 Some examples of method applications and emission models

Much has been written about SA and UA in general (Saltelli *et al.*[17]—a collection of articles which have not been individually referenced in this thesis, and Saltelli *et al.*[18]), and about methods connected to road traffic emissions (Xiaojin *et al.*[9]). Examples of most of the methods described above, applied to emission problems, vehicle technology or other traffic related problems can be local SA[13], global SA[14], SA based on slopes[10], direct methods[6], order importance[4], graphic methods[16], UA[5] and emission models(COPERT[1], EMV[8], HBEFA[2], MOBILE[3], Coldstart[7] and a model discussed by Matzoros *et al.*[12]).

3.5 Our choice of sensitivity and uncertainty analysis method and emission model

We want to study SA methods expressed as a change in emission per change in mean speed, for example, hence our choice of methods based on slopes. It is in our interest to determine whether methods based on slopes make sense for a model of the same structure as the chosen emission model. We want to use both one-at-a-time methods and response surface methods. We also want to study VBSA methods, though for different reasons. One is that the use of slopes is not suitable for categorical variables. Another is that we found that the usual VBSA methods require a regular input area; because we have an irregular input area, our study may contribute to the development of new methods. Local methods are not of interest when trying to arrive at general measures of traffic. A nominal point may exist locally, but we want more general results and therefore think it is better to use global methods.

We think that a multivariate approach to SA is not appropriate for our problem, because the different emissions behave too differently from each other. Even if a multivariate approach were possible, these differences in behavior mean that a summary of the sensitivities must in any case be written separately for each emission type.

The traffic data for the emissions from a certain road network must be estimated. With uncertain estimates, the emission results also have uncertainty. In view of this, we want to study a UA method suitable for the kind of uncertainty that appears in our estimated traffic data.

We choose the COPERT III road traffic emission model (COmputer Programme to calculate Emissions from Road Transport, called COPERT from here on) because it is widely used and includes many pollutants and vehicle categories. The emission functions for selected pollutants and vehicles are discussed in detail in one of the included papers. We choose to base our SA on generated datasets. The emission model is too complex for any meaningful direct methods, and we have noted that the emission functions are not continuous over the entire allowed input area, making exact derivatives impossible for some points of the input area. Our measures will be smoothed over such points.

In this study, we chose an established model. We then have the possibility to study the model in detail, and some of the above text tends to emphasize the importance of that fact. The detailed knowledge of the model is needed when preparing our alternative software for generating data. However, the SA and UA methods must be usable when only generated data and some structural knowledge of the model are available. SA and UA methods, which are our main interest, are based more on the generated data and not on detailed model knowledge. When discussing SA and UA methods, we simply pretend that we do not know the model in detail.

4 Included papers

This section discusses the subject of each of the included papers. The subsection titles are not the same as the titles of the included papers.

4.1 Paper I, Emission model and a data-generating tool

It is unrealistic to look for datasets consisting of real observations for this study. We do not have tools for measuring the emissions and exact properties of each vehicle, which, for example, may depend on whether the vehicle is currently running with a cold or hot engine. Also, we want to study the sensitivity of things we cannot master, such as ambient temperature. The only realistic way to accomplish this is to use model-generated emission data.

We decided early on to use the COPERT model because it is widely used for modeling road traffic emissions. The main part of Paper I is a detailed study of the structure of COPERT for a selection of covered vehicle types and pollutants. Paper I describes part of COPERT using mathematical notation; the original documentation contains many details but does not use the same mathematical notation and does not show, using straightforward formulas, how to combine the parts to produce final emissions functions. The emissions are calculated as a function of traffic properties such as mileage, mean speed, and trip length. The emissions also depend on variables unrelated to the actual driving, such as fuel composition and vehicle technology, or on variables we can measure but not change, such as ambient temperature. Some papers describe the drawbacks of using models that use mean speed, for example, as an input (see Trozzi *et al.*[19]). Readers familiar with the COPERT model or only interested in SA and UA methods may well be able to understand the following papers without studying Paper I in detail.

The official COPERT computer software, which can be downloaded from the COPERT website [1], has a graphical user interface with easy input via dialog boxes and scroll-down menus. It is suitable for one or a few runs, but not for the thousands of runs we must perform to get the datasets needed for our analysis. Based on the technical description of COPERT[15], we have written alternative software that uses the COPERT model for emission calculations but a totally different way of putting data through the calculations. Paper I briefly describes this alternative program. It also contains a discussion of how the program was validated and a list of mismatches found between the alternative program, the official COPERT software, and he technical description of COPERT. After serious comparing and checking, we feel confident that the program runs correctly. The COPERT model, however, is all-embracing and calculates emissions for cars, trucks, trains, boats, and other vehicles; we have limited ourselves to only end-of-pipe emissions where a detailed methodology exists for gasoline- and diesel-powered private cars. Though fuel is not itself an emission, we were able to analyze fuel consumption the same way as the selected emissions because they all follow the same structure in COPERT.

Generated datasets have some drawbacks. From the point of view of determining emission levels, it is important to feed correct data into the model. This requires correct distributions of vehicles, fuel composition, speeds, trip lengths, etc., some of which are unknown. From the point of view of sensitivity, this information may be less important, both because we do not want to use local methods and because the sensitivity may be measured correctly even if the levels are not. If comparisons between nations are desired, one must check which data are available and whether they are complete for the emission model.

4.2 Paper II, One-at-a-time sensitivity analysis

Our first approach to SA aims at finding derivative-like measures, i.e., how much the emissions change when the input changes, expressed as slopes. Because we cannot suggest a nominal point, we do not use local methods. Instead, we vary every input over a grid, average the output over all dimensions but one, and calculate the sensitivity to changes in the last dimension, thus forming a mean sensitivity when all other variables have a distribution. Regression analysis was used to calculate the slopes.

The basic method was to use straight lines and a thin data grid. As an extension, we studied quadratic lines over a thin grid, and as another extension we also studied straight lines over a fine grid. Fine grids mean, in this case, not only that the grid is fine for the variable studied, but that it is thin for all the others. We also study the difference between categories by using mean values, even when all other variables have a distribution.

The methods used are discussed in both mathematical and statistical terms. The numerical results are given for hot emissions, cold excess and total emissions. As well, the results have been calculated both for all vehicles and for newer vehicles only.

4.3 Paper III, Sensitivity analysis using response surface

This paper aims to find the same kind of sensitivity measures as in Paper II, but by examining all slopes and categorical levels simultaneously using response surface methods. A general linear model was used for the statistical analysis. This can be seen as a third extension of the basic model mentioned in Paper II. Finally, we considered all the extensions mentioned in Paper II and III at the same time, by using a response surface method with squared relations over fine grids.

4.4 Paper IV, Variance-based sensitivity analysis

The above-discussed methods based on slopes comprise one type of SA approach; in Paper IV, we shift to a variance-based SA approach. With VBSA, we can study how much of the variation in output comes from variation in different inputs. We do this without attempting to exactly determine any function for how a particular output change follows from a change in input. This paper discusses three different approaches to estimating variance components or relations between variance components. All our datasets do not have balanced data, but some VBSA methods can not be used unless the design is strictly balanced. We suggest one method that can be used with unbalanced data and discuss what problems the imbalance cause in the other methods.

4.5 Summary of sensitivity analysis methods

The examined SA methods are meant to give average sensitivities over a wide range of possible traffic situations, subject to some control in that such measures should be meaningful. They have all been made on data generated by our alternative software, and the statistical analyzes have been performed using standard software. Paper IV finalizes our set of SA methods. A summary of all SA results of our slopes methods and variance-based methods has been inserted as a section in Paper IV.

4.6 Paper V, Uncertainty analysis

In this paper, we examine the uncertainty in the output of a model as a function of an uncertain input. A typical basis for UA methods is to treat variation in the output given a known distribution of the input. However, we find this to be unsuitable, because it demands a known input distribution; if the input distribution is itself uncertain, this typical approach fails. For many problems, we think that one can never really know the input distribution. For example, for a traffic situation in a certain area, we need to know mean speed as an input to the emission model; although we can measure this at selected locations, this still does not give us knowledge of the mean speed over the entire area. We suggest a method for UA based on incomplete knowledge of the input; to the best of our knowledge, this is a new approach. The uncertainty measures are simple, including variance and the coefficient of variation. As to what concepts of the suggested method are distinctive, it is the design of the input dataset for the calculation; the calculation itself is simple.

5 Results

Our intention was to study SA and UA methods suitable for a road traffic emission model, and also to obtain numerical results when applying the methods to a chosen emission model. The results are divided accordingly, between a first section that discusses methods, and a second section, dealing specifically with COPERT that includes both numerical results and structural or methodological issues specific to it.

5.1 Methods

The methods have been chosen as representing good alternatives for analyzing the COPERT model. They are, however, not applicable only to that model; any model with the same structure of input–output relations can be analyzed using these methods, possibly with minor changes.

The COPERT model and the emission problem gave us four important conditions from which to start the discussion of SA and UA methods.

- A nominal point does not exist.
- The model is too complex to allow the use of direct methods.
- We must have methods able to accommodate both continuous and categorical variables.
- The method must allow an irregular input area, at least for the categorical inputs.

A general finding is that a model may behave mostly additively, even though its definitions also contain many multiplicative components, COPERT being a good example of such a model. For this type of model, all the methods based on slopes produce practically the same results.

Our discussion of VBSA methods should be of methodological interest. We suggest a method for performing VBSA in instances of an irregular input area, and discuss what this irregularity leads to. We have also discussed the difficulties with methods based on mean squares or R^2-like measures, in instances of an irregular input area.

Our suggested UA method is, to the best of our knowledge, a new UA approach. It is simple in theory. We have also applied it, but so far with only a simple and not very efficient algorithm.

5.2 Results specific to COPERT III

It took considerable time to write and validate our alternative program for the COPERT model. While doing this, we had the best possible opportunity to study the model construction from the perspectives of input variable type, input limits, shape of response function, and differences between pollutants. The program is written in C and has shown itself to be a fast and powerful tool for generating datasets.

The parts of COPERT that we have studied displayed almost linear emission functions, even though the model definitions contain other than linear terms.

The SA methods based on slopes do not change when using finer grids, when adding quadratic relations, or when estimating all slopes at once.

We have many numerical results, for example, for various pollutants, approaches, and fuels. These results cannot be summarized here and must be read in the individual papers; here we can only give a very brief overview.

- The emission *sensitivities* to changes in the inputs differ greatly between gasoline- and diesel-powered cars. The emission *levels* have not been compared between the two fuels, but the levels can be found in Paper II and III. The reader is advised to try making such comparisons while reading the papers.

- The difference in emission levels between gasoline-powered cars with and without catalytic converters is huge for most pollutants. Nowadays, the fraction of cars without converter is very small. In terms of the future applicability of our results, it is probably the results for newer vehicles that will be most useful.

- The emission functions are very different for different pollutants.

- Some, but not all, emissions are much lower in newer vehicles. Notably, fuel consumption has not decreased much for newer vehicles.

6 Discussion

The goal, to suggest and apply SA and UA methods for an emission model, has been reached for selected vehicle categories and pollutants.

When we started examining the COPERT model, the problems looked almost overwhelming. The model was huge, there was a mix of additive and multiplicative terms, discontinuities, various transformations, etc., in its submodels. The software could not live up to our expectations of the sort of tool needed for generating data. Now, after having completed the research, we have found that we need not have worried about this after all. The part of the model we studied was mostly linear, the structure of the model could be made clear, and alternative software was written in reasonable time. Our experience indicates that it is worth trying to perform this kind of analysis, even on models that may seem too troublesome at first glance. It also emphasizes, again, that a grasp of the properties of a total model cannot be replaced by only good knowledge of each sub-model.

The different SA methods based on slopes produced almost the same results. We think that the amount of work is smallest with a response surface method, because it gives all SA results in only one statistical analysis. The similarity between the different methods based on slopes is not general. For example, we have used grids with orthogonal covariates, which also make the difference between methods diminish. We think a response-based method is also better in that it takes better care of non-orthogonal covariates, if present. This is, however, not a finished discussion. The interpretation of a regression coefficient is the effect of a one-unit increase of one predictor variable, given that all other predictor variables remain unchanged. For an SA measure, one can discuss the logic of having all other variables unchanged. If we want to study the sensitivity when one variable changes, and all the others are allowed to change at the same time, then averaging the output over all dimensions but one and calculating the sensitivity to changes in the last dimension may be of use if the dataset is generated over representative combinations of input variables. This discussion can be extended in many ways, for example, to deal with interactions. If an interaction is important, must it then be included in a regression analysis, or is it sufficient to use regression without interaction if the dataset is representative? The answer must depend on the aim of the statistical procedure. There is probably a difference between slope methods and VBSA methods, and the importance of a correct statistical model is not the same for SA as for significance tests.

We have unbalanced data in our study, so we immediately have considerable problems with the VBSA methods. We have suggested a method that partly solves these problems, but there may be other solutions as well. Another way is to study variation-based SA, where "variation" does not need to be variance. For variation measures other than variance, there may be other solutions.

When studying UA, we started with what can be a reasonable quality of input data. The method must be able to use a discrete distribution as an input; the COPERT model uses for example legislation class as an input. The distribution must in itself be allowed to be uncertain, because we cannot exactly know the fleet in a given area or road network. Once this idea became clear, our method was found not to look exactly like any other of which we knew; we could start deriving a complete method, and adapt it to all other input variables. Mean speed is, among others, an input to COPERT, but even a mean can have a distribution or a degree of uncertainty. The result is a suggested method for what we think is a reasonable quality of data for this kind of problem.

If SA and UA are to become more commonly used, we believe that model software should be written so as to allow more runs to be made more easily. The price may be a less user-friendly interface, which is a drawback for those who only want to make a few runs. Both ways of using the model can make use of the same calculation module, and the extra work is only a matter of writing another interface.

We have used the COPERT III model, released 1999, for our emission calculations; COPERT version 4 was released 29 November 2006. In Sweden, we saw changes in the use of different fuels over the year 2006, but it was unclear what changes we would see in the future. We have only studied data for a few pollutants for a limited fleet that does not include ethanol- or LPG-powered cars. Other pollutants, vehicles, and fuels may have totally different emission function structures. As can be seen here, there is almost an endless number of problems in which our methods can be applied if emission models for newer vehicles, other fuels, other pollutants, and other vehicle categories are available. Because some of our methods need careful planning of the intervals for the input variables, we also want to continue our research jointly with traffic research groups, who can improve the choice of distribution of input data.

The UA method we have used can be improved. The basic idea is well described and our implementation works, but it is slow. An idea for further research would be to use the same method, but to improve it by finding and implementing a better algorithm.

Finally, there are many other model structures and data designs that we have not discussed here. Further research could study SA and UA methods for models involving interaction, for multiplicative models, and for unbalanced designs.

We feel confident that there will long be a need for better information about emissions from road traffic and other sources. There is no reason to believe that the use of models will generally decrease. We instead expect that their use will grow, and that the models themselves will become better. Faster computers will allow more model runs and more detailed models, and automatic data collection will provide a better basis for building models. We hope that this thesis will contribute to the use of models in various ways, for UA and various kinds of SA, and in emissions and many other fields.

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