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Invited Paper

Capacity Considerations for Uncoordinated Communication in Geographical Spectrum Holes

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Abstract

Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner. The motivation for cognitive radio is various measurements of spectrum utilization, that generally show unused resources in frequency, time and space. These ”spectrum holes” could be exploited by cognitive radios. Some studies suggest that the spectrum is extremely underutilized, and that these spectrum holes could provide ten times the capacity of all existing wireless devices together. The spectrum could be reused either during time periods where the primary system is not active, or in geographical positions where the primary system is not operating. In this paper, we deal primarily with the concept of geographical reuse, in a frequency-planned primary network. We perform an analysis of the potential for communication in a geographical spectrum hole, and in particular the achievable sum-rate for a secondary network, to some order of magnitude.

Simulation results show that a substantial sum-rate could be achieved if the secondary users communicate over small distances. For a small number of secondary links, the sum-rate increases linearly with the number of links. However, the spectrum hole gets saturated quite fast, due to interference caused by the secondary users. A spectrum hole may look large, but it disappears as soon as someone starts using it.

Key words: Cognitive radio, spectrum hole, capacity, achievable rate

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⋆ The research leading to these results has received funding from the European

1 Introduction

Spectrum is a scarce resource, and operators have made huge financial investments to buy licensed spectrum. The licensed spectrum is intended for specific communication technologies, and no one but the spectrum owner is allowed to use it. Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner Brodersen et al. (2004); Haykin (2005); Mitola (1993). The motivation for cognitive radio is various measurements of spectrum utilization, that generally show unused resources in frequency, time and space FCC (2002); McHenry (2005). These ”spectrum holes” could be exploited by cognitive radios. The spectrum could be reused either during time periods where the primary system is not active, or in geographical positions where the primary system is not operating. This paper deals primarily with geographical, or spatial, reuse.

The introduction of cognitive radios, sometimes called secondary users, in an existing primary system will create interference and thus a quality degradation of the primary system. In order to reduce the impact on the primary system, cognitive radios have to sense the spectrum and detect whether there are primary users in the vicinity that are currently using the spectrum. The cognitive radios need to be positioned sufficiently far away from the primary users and transmit at very low power levels. This has been analyzed in e.g. Hoven and Sahai (2005) for a single cell, and in Larsson and Skoglund (2008) for a frequency-planned network. It is unavoidable that there has to be some sort of compromise for the primary system to allow secondary users. In Hoven and Sahai (2005); Larsson and Skoglund (2008) this compromise is a reduction of the primary cell radius. The cell radius is decreased by a small amount and it is required that the same signal-to-interference-plus-noise ratio (SINR) is experienced by the primary users as without any secondary users. Hence, the aggregated transmit power of the secondary users must be constrained to keep the interference level low. For example, if the cell radius is decreased by 5%, then the transmitter power for the cognitive radios is constrained such that the primary users experience the same SINR as they did before, without any secondary users.

A similar one-cell model for spatial frequency reuse has also been analyzed in Tandra et al. (2008), together with more general definitions of a spectrum hole and some metrics to quantify the performance of a spectrum sensing

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algorithm. The focus of this work is more on the uncertainty of detection versus the area that could actually be exploited.

The concept of frequency reuse can be seen as a bucket filled with rocks Tandra et al. (2008). Although the bucket is filled there is still plenty of room for sand. However this metaphor requires a large-scale primary system, and a small-scale secondary. This problem has been stated in Mishra et al. (2007), and especially the problem of coexistence of a secondary system with primary systems of different scales. The main issue is the coexistence of a secondary system in the presence of a small-scale primary network. The Part 74 wireless microphone users that are a concern to the IEEE 802.22 WRAN is given as an example. Relating to the bucket metaphor, if the bucket is filled with sand we cannot fit any more rocks nor sand.

In much literature, the main difficulty has been perceived to be the detection of the primary users. Even if that could be solved, we need to know that the spectrum holes can really be exploited and provide some useful data rate. Some studies suggest that the spectrum is extremely underutilized, and that these spectrum holes could provide ten times the capacity of all existing wireless devices together McHenry (2005). The aim of this paper is to analyze the potential for communication in a geographical spectrum hole, and in particular the achievable sum-rate for a secondary network, to some order of magnitude. In Section 2 we describe the system model, and Section 3 shows some numerical results. Section 4 proposes some improvements on the individual secondary links by using multiple devices. Section 5 concludes the work.

2 Model

We consider the downlink in a hexagonal frequency-planned network, shown in Figure 1. We include the main primary base station and the first tier of co-channel interferers. The cell radius is r, and the distance to the first tier of interfering base stations is $D = \sqrt{3n}r$ Rappaport (2001), where $1/n$ is the frequency reuse factor of the primary system (n is the number of frequency groups). The positions of the base stations are denoted by the vectors $\bar{B_0} = \bar{0}$ and $\bar{B_m} = (D\cos((m-1)\frac{\pi}{3}), D\sin((m-1)\frac{\pi}{3}))$, $m = 1, 2, \cdots, 6$.

Following Hoven and Sahai (2005); Larsson and Skoglund (2008) we assume that the cognitive users are permitted to operate only if they are located at a distance at least d from the nearest primary base station. Furthermore, we assume that N cognitive transmitters are spread out uniformly at random in the allowed region, i.e. in the area between the circles of radii $D - d$ and d respectively, as shown in Figure 1. The positions of the cognitive transmitters are denoted by the vectors $\bar{T_i} = (x_{T,i}, y_{T,i})$, $i = 1, 2, \cdots, N$. For each

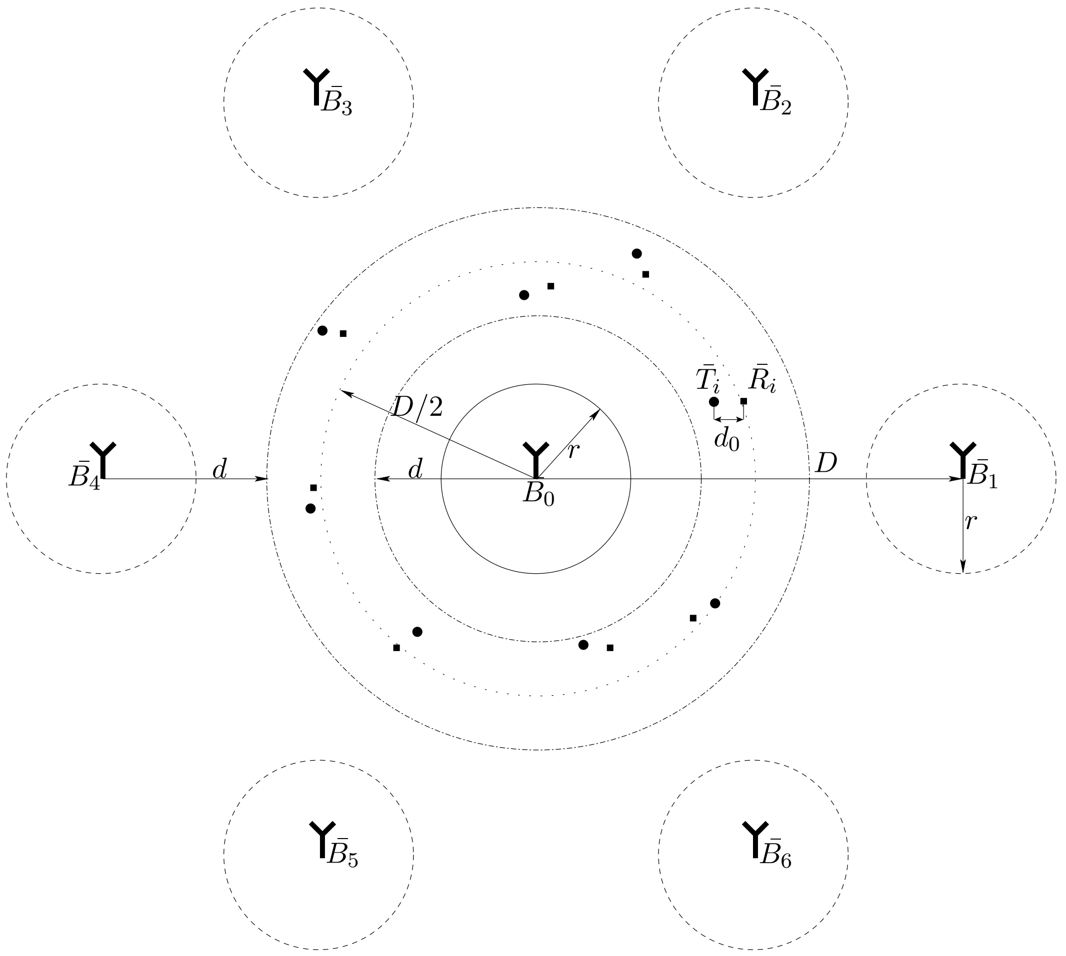

Fig. 1. Model for the cognitive network.

cognitive transmitter there is an associated cognitive receiver at a distance d_0 from the transmitter and at an angle θ_i. The angle θ_i is uniformly distributed in the interval $[-\pi, \pi]$, but such that the receiver is also in the permitted area. The positions of the receivers are denoted by the vectors $\bar{R}_i = (x_{T,i} + d_0 \cos(\theta_i), y_{T,i} + d_0 \sin(\theta_i))$, $i = 1, 2, \cdots, N$.

We consider a log-distance path loss model. Thus, we define the channel gain function at distance x,

$$\rho\left(x\right) = \left(\frac{x}{x_0}\right)^{-\alpha} 10^{\chi/10}, \;\; \chi \sim N(0, \gamma)$$

where α is the path loss exponent, x_0 is a normalization constant and γ is the standard deviation of the lognormal fading in dB. The distance, x, is in general a random variable since we consider random locations for the secondary users. Hence, the received interference is random. However, the distance between a secondary transmitter/receiver pair d_0 is fixed, and the only randomness of the received signal strength is the lognormal fading. The base stations transmit omnidirectionally with power P, and each cognitive user transmits omnidirectionally with power P_c. The transmit power is defined as the power received at the normalization distance x_0.

We assume that the secondary users are uncoordinated and thus transmit simultaneously, using the same channel. Hence, the secondary users will interfere with each other. The interference power experienced by the ith cognitive receiver, and caused by other secondary users can be written as $P_c \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|)$. The secondary users also receive interfering signals from the primary base stations. The interference power from the primary system, for the ith secondary user, can be written as $P \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)$. Now the signal-to-interference-plus-noise ratio (SINR) for the ith cognitive receiver becomes

$$\begin{aligned}\mathrm{SINR}_i &= \frac{P_c \rho(d_0)}{\sigma^2 + P_c \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|) + P \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)} \\ &= \frac{\rho(d_0)}{\frac{\sigma^2}{P_c} + \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|) + \frac{P}{P_c} \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)},\end{aligned}$$

where σ^2 is the receiver noise floor.

The primary system may be either interference limited or noise limited, or somewhere in between. Following Larsson and Skoglund (2008), we can quantify the operating point of the primary system in terms of the expected interference-to-noise ratio at the cell border without secondary users:

$$\psi \triangleq E\left[\frac{P \sum_{n=1}^{6} \rho(|(r\cos(\phi), r\sin(\phi)) - \bar{B}_n|)}{\sigma^2}\right] = \frac{P\overline{\mu}}{\sigma^2},$$

where

$$\overline{\mu} \triangleq E\left[\sum_{n=1}^{6} \rho(|(r\cos(\phi), r\sin(\phi)) - \bar{B}_n|)\right],$$

and the expectation is taken over the lognormal fading and ϕ which is uniformly distributed over $[-\pi, \pi]$. In order to make sure that the cognitive users do not cause too much harmful interference to the primary users, the transmit power P_c must be constrained. We will constrain the aggregate cognitive radio transmit power, such that $NP_c = \epsilon P$, for some $\epsilon > 0$. The choice of ϵ will be discussed later. Thus, we can rewrite the SINR and obtain the following:

$$\mathrm{SINR}_i = \frac{\rho(d_0)}{\frac{N\overline{\mu}}{\epsilon\psi} + \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|) + \frac{N}{\epsilon} \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)}$$

The achievable rate for the ith secondary link is modeled as

$$\begin{aligned}&C_i = \log_2(1 + \mathrm{SINR}_i) = \\ &\log_2\left(1 + \frac{\rho(d_0)}{\frac{N\overline{\mu}}{\epsilon\psi} + \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|) + \frac{N}{\epsilon} \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)}\right) \text{ [bits/s/Hz].}\end{aligned}$$

Hence, the sum-rate offered by the network of all secondary users, C, is

$$C = \sum_{i=1}^{N} C_i =$$

$$\sum_{i=1}^{N} \log_2 \left(1 + \frac{\rho(d_0)}{\frac{N\bar{\mu}}{\epsilon\psi} + \sum_{k \neq i} \rho(|\bar{R}_i - \bar{T}_k|) + \frac{N}{\epsilon} \sum_{n=0}^{6} \rho(|\bar{R}_i - \bar{B}_n|)} \right).$$

We define the relative area of cognitive operation

$$A = \frac{(D-d)^2 - d^2}{D^2} = 1 - 2\frac{d}{D}.$$

This is the fraction of the total system area in which cognitive operation is permitted. Note that the allowed transmit power P_c depends on the permitted area A and on the primary interference-to-noise operating point ψ. The relationship between these parameters was investigated in Larsson and Skoglund (2008). See also Hoven and Sahai (2005) for the special case of only a single primary base station. The primary cell radius was decreased, and the secondary transmit power was constrained such that the SINR for the primary users was not decreased compared to the primary system without any secondary users. The allowed aggregated secondary transmit power $NP_c = \epsilon P$ was computed given a relative area A and a primary interference-to-noise operating point ψ. We will use the values of ϵ, ψ and A obtained from this analysis for our simulations.

3 Simulation Results

In this section we will show some numerical results from Monte-Carlo simulations. For each number of secondary users N, we generated 5000 realizations of the system model. The achievable sum-rate was then calculated as the mean of the sum-rate over all realizations. For each realization we placed N cognitive transmitters uniformly at random in the allowed area between the circles of radii d and $D-d$ respectively. To obtain a uniform distribution over the circular area we created the transmitter positions $\bar{T}_i = (x_{T,i}, y_{T,i})$, $i = 1, 2, \cdots, N$, in polar coordinates. The angle was uniformly distributed over $[-\pi, \pi]$ and the radius, R, was obtained by

$$R = \sqrt{((D-d)^2 - d^2)X + d^2},$$

where X was uniformly distributed over $[0, 1]$. The receiver positions were then created as

$$\bar{R}_i = (x_{T,i} + d_0 \cos(\theta_i), y_{T,i} + d_0 \sin(\theta_i)) \; i = 1, 2, \cdots, N,$$

A [%]	1	25	50
ϵ ($n = 21$) [dB]	0	-3	-12
ϵ ($n = 7$) [dB]	0	-5	-20

Table 1
Parameter values used in the simulations for $n = 7$, 21 and $\psi = -10$ dB, obtained from Fig. 3 in Larsson and Skoglund (2008).

where all θ_i were uniformly distributed over $[-\pi, \pi]$. To make sure that all users were inside the allowed region we simply redraw the angle θ_i whenever a receiver position happened to be outside of the allowed region.

We used $\psi = -10$ dB throughout all simulations. This corresponds to a noise limited primary system. We argue that a practical system where we eventually could make use of this kind of geographical spectrum reuse would typically be noise limited. It could for example be a television network with a very sparse frequency reuse and primary transmitters located far away from each other. In addition, simulations have shown that the value of ψ only has a small impact on the results. The values of ϵ and A that were used in the simulations are shown in Table 1, and obtained from Fig. 3 in Larsson and Skoglund (2008) for $n = 7$, 21 frequency reuses and $\psi = -10$ dB. These values were obtained assuming that the primary cell radius is decreased by 5%, and the primary users experience the same SINR at the 90%-percentile of the distribution, as without any secondary users. Also, in accordance with Larsson and Skoglund (2008), we use the path loss and shadow fading parameters $\alpha = 4$ and $\gamma = 6$ dB throughout the whole paper. We use frequency reuse $n = 21$, except where otherwise stated.

3.1 Achievable Sum-Rate

Figure 2 shows the total system throughput C for $A = 50\%$, 25%, 1% and $\psi = -10$ dB. When increasing N, we observe an increase to some congestion limit. Above this congestion limit, adding more users only causes a throughput degradation due to increased interference. Thus, for a given d, the system throughput is maximized for a certain number of users N. Intuitively we would expect the congestion level to be lower when the allowed region is smaller. The operation region will be saturated for a smaller number of users since the area is smaller. This intuition is confirmed by Figure 2: the number of users maximizing the throughput is higher when the allowed region is larger. Note also that the total throughput is higher for a 25% cognitive area than for 50% or 1%. Hence, the throughput is neither increasing nor decreasing in d. Rather there is some d that maximizes the total throughput. The interpretation of this is that the allowed transmit power P_c is high and the expected interference from the primary system is low for a small cognitive region, but the interference

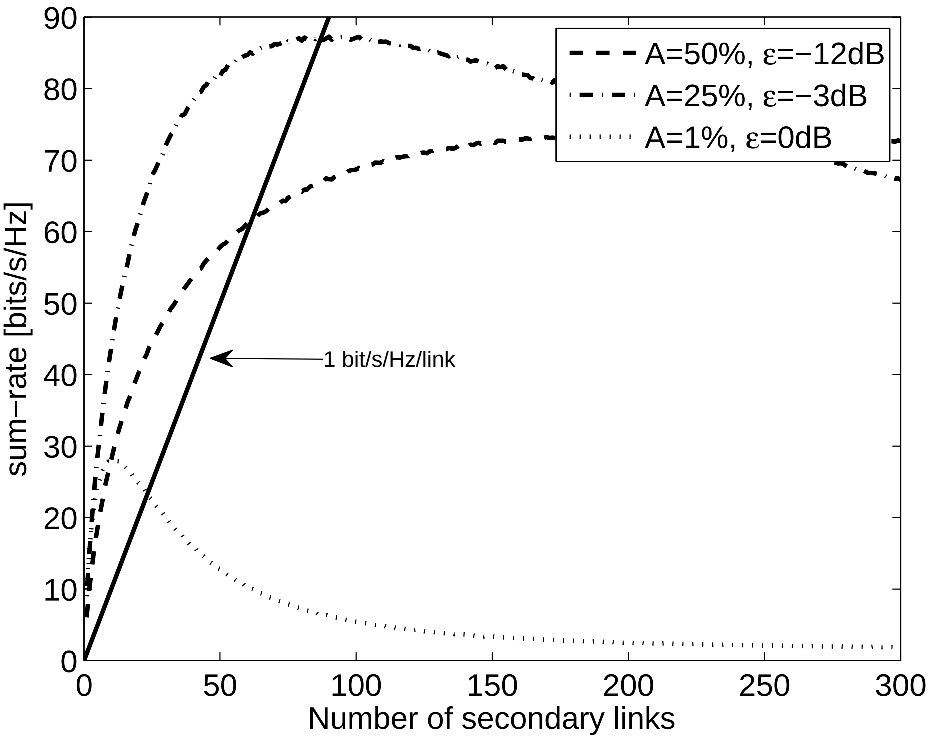

Fig. 2. Total achievable sum-rate for the secondary system for $d_0 = 0.1r$, $\alpha = 4$, $\gamma = 6$ dB, $n = 21$.

from the cognitive users increases fast as the number of users N increases. On the other, hand when the allowed region is large, the allowed transmit power P_c is low and the expected interference from the primary system is higher. The optimum seems to be somewhere in between these two extremes. These results are for frequency reuse $n = 21$, but the same behaviour is seen also for other frequency reuse factors.

As a reference, the solid line shown in Figure 2 is a straight line with slope one, and corresponds to the case where each secondary user gets 1 bit/s/Hz. We consider rates above this line as acceptable whereas rates below the line are less acceptable. Arguably links with less spectral efficiency than 1 bit/s/Hz are not very useful. Note that for a large permitted area ($A = 50\%$) many secondary users can coexist and achieve a quite high sum-rate, but the rate per user is not acceptable.

Figure 3 shows the total achievable rate for different transmitter-receiver distances d_0, both for $n = 7$ and $n = 21$ frequency reuse. Due to the larger operating area for $n = 21$ than $n = 7$ frequency reuse, the secondary users are allowed to use a higher transmit power without causing too much interference for the primary users. Also, since the operating area is larger, the distance between interfering cognitive users is larger on the average. As expected, the

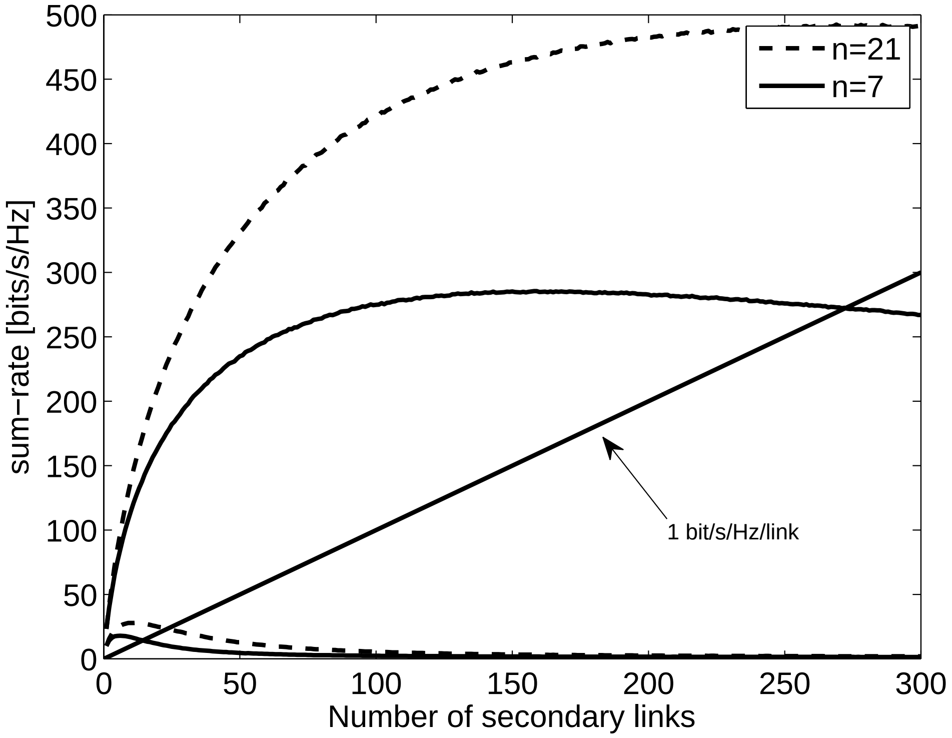

Fig. 3. Comparison of the achievable sum-rates for $d_0 = 0.1r$ and $d_0 = 0.01r$, for $A = 1\%$, $\alpha = 4$, $\gamma = 6$ dB.

performance is better for $n = 21$ than for $n = 7$ frequency reuse, both in terms of the total achievable rate and in terms of the number of users that can be allowed before the congestion limit is hit. We also observe a large improvement from decreasing the communication distance. If we compare the maximum total achievable rates, we note that a distance decrease by a factor 10 yields a throughput increase by a factor 10 for $n = 7$ and a factor 20 for $n = 21$. Thus the order of magnitude of the maximum sum-rate seems to stand in inverse proportion to the communication distance between the secondary users. It is also worth noting that the maximum throughput is attained for a larger number of users when the distance is smaller. Hence, both the total throughput and the number of users can be larger for a smaller communication distance.

3.2 Geographic density

We have seen that the total achievable rate attains a maximum for a certain number of secondary users. If there are more users they will get too close to each other and the interference they generate to each other will increase. The question is then how the users should be distributed to achieve a maximum total system throughput. Is the user distribution dense or sparse at the

A [%]	1	25	50
N_{max}	12	86	220
A_E [%]	19	5	7

Table 2
Effective area for $n = 21$, $d_0 = 0.1r$.

maximum sum-rate operating point?

A reasonable assumption is that the geographic area filled up by each secondary link communicating over a distance d_0 is equal to a circle of radius d_0. Then N secondary links use an area $N\pi d_0^2$. The total area of secondary operation is the area between the circles of radii $D - d$ and d respectively, i.e. $\pi((D-d)^2 - d^2) = \pi AD^2$. We denote by N_{max} the number of users for which the maximum total achievable rate is attained. We define the effective area, A_E, as the ratio of the area used by N_{max} secondary users and the total area of secondary operation, i.e.

$$A_E = \frac{N_{\text{max}} \pi d_0^2}{\pi AD^2} = \frac{N_{\text{max}} d_0^2}{3nr^2 A}.$$

Note that this ratio might actually be greater than one, since the secondary links could overlap. Analyzing the simulation results shown in Figure 2, we obtain the value of N_{max} in various cases. These values and the effective areas are then shown in Table 2. For all sizes of the allowed secondary operation region we see that the effective area is between $5 - 20\%$ (similar numbers have also been observed for $n = 7$). The conclusion is that the users should be quite sparsely distributed to obtain the maximum system throughput. It is also worth noting that for large operating regions, the rate per link for $N = N_{\text{max}}$ users is “non-acceptable” (in the sense defined in Section 3.1) although the sum-rate is maximized. In this case the users have to be even more sparsely distributed in order to get a satisfactory rate per link.

4 Point-to-point improvement

The simulations in Section 3 show that the achievable sum-rate is strongly dependent on the transmission distance d_0. A small distance yields a larger received signal strength for the secondary users and thus a larger achievable rate. This also leads us to another interesting question. Suppose that we want to communicate between a point A and another point B separated by a distance δ, and using power P_{tot}. We neglect shadow fading, i.e. the channel gain

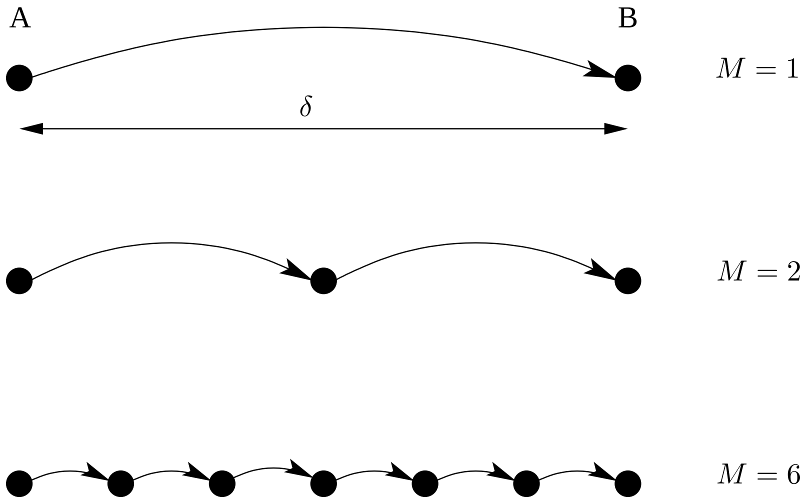

Fig. 4. Point-to-point communication from A to B, either directly or by multihop with time-division multiplexing.

at a distance x is simply $\rho\,(x) = (\frac{x}{x_0})^{-\alpha}$. The achievable rate would then be

$$C = \log_2\left(1 + \frac{P_{tot}\,(\delta/x_0)^{-\alpha}}{J}\right), \tag{1}$$

where J is the received noise plus interference power from the primary system.

Assume further that we can alternatively use in total $M+1$ devices (M links) spread out uniformly on the straight line between A and B, and transmit in a multihop fashion. Then the distance between two neighboring devices is δ/M. We assume that the devices share the channel by time-division multiplexing as shown in Figure 4, i.e. we let the sub-nodes transmit one at a time, starting at A, to the next sub-node until the message reaches B. Since only one device transmits at a time, each device is allowed to use power P_{tot}. The SINR on each link is in this case

$$\mathrm{SINR} = \frac{P_{tot}\left(\frac{\delta}{Mx_0}\right)^{-\alpha}}{J},$$

The received noise plus interference power, J, is assumed to be equal for all devices, since the inter-node distances are small relative to the primary cell radius r. The achievable rate from A to B is

$$\begin{aligned} C =& \frac{1}{M}\log_2\left(1 + \frac{P_{tot}\left(\frac{\delta}{Mx_0}\right)^{-\alpha}}{J}\right) = \\ & \frac{1}{M}\log_2\left(1 + \frac{P_{tot}(\delta/x_0)^{-\alpha}}{J}M^{\alpha}\right)\ \text{[bits/s/Hz]}. \end{aligned} \tag{2}$$

This is also in accordance with (1), for $M = 1$. Note that in this case we are

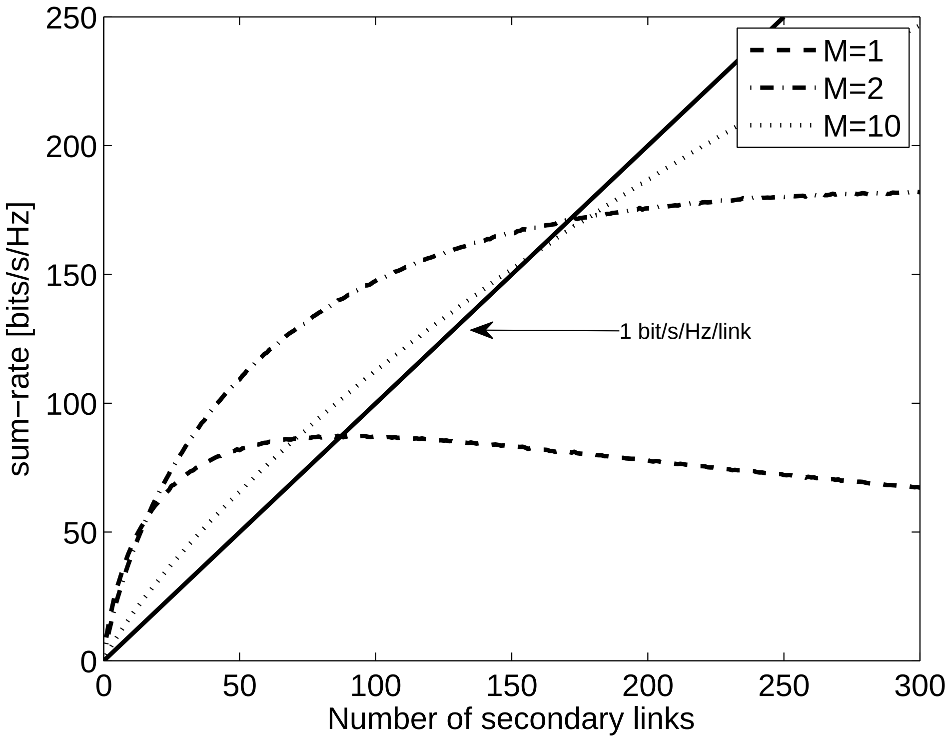

Fig. 5. Comparison of the achievable sum-rate using time-division multiplexed repeater devices between the cognitive receiver and transmitter, for $d_0 = 0.1r$, $\alpha = 4$, $\gamma = 6$ dB, $n = 21$.

not interested in the sum-rate of the "network" consisting of the M links, but in the achievable rate from point A to point B. The information received by the intermediate devices is not useful to them, since they only act as repeaters. The achievable rate for each of the M links will be identical, by symmetry. We have to use M links to communicate from A to B, hence the division by M. We observe from (2) that the achievable rate goes to zero as M goes to infinity, but the optimal strategy actually depends on the SINR.

Figure 5 shows some simulation results of the time-division multiplexing strategy. For simplicity we approximate the interference at all sub-nodes by the interference in between the transmitter and the receiver, i.e. for all sub-nodes associated with link i we use the interference experienced at the position $(\bar{T}_i + \bar{R}_i)/2$. The simulations show that for a small number of secondary users, it makes no big difference what strategy is used. The transmission can be made directly between the transmitter and receiver, or we may use a few intermediate devices. For a large number of secondary users we can achieve some improvement by using intermediate nodes. For example, by simply using one extra device, the rate can be doubled.

More sophisticated strategies are also possible. For example out of M links we could let every other, every third etc., be active in each time slot. Assume that we spread the M links uniformly over T time slots. The transmit power for each link would then be shared between the M/T links in each time slot, i.e. the transmit power is $\frac{P_{tot}}{M/T}$. The achievable rate for the kth active link is then

$$C_k = \frac{1}{T}\log_2\left(1+\frac{\frac{P_{tot}}{M/T}\left(\frac{\delta}{Mx_0}\right)^{-\alpha}}{J+I_k}\right) \text{ [bits/s/Hz]},$$

where I_k is the interference caused by all other sub-nodes transmitting in the same time slot. Due to symmetry, we only need to consider one time slot. The interference I_k is also dependent on M and T. For example if $T = 2$, the interference I will contain one term that is identical to the received signal strength plus other, smaller, terms. This means that the SINR will be smaller than 0 dB. For $T > 2$ however, much greater SINRs can be achieved. The achievable rate from A to B is the mean of the achievable rates of the active sub-links:

$$\begin{aligned} C &= \frac{1}{M/T}\sum_{k:k\equiv 1 \pmod{T}}^{M} \frac{1}{T}\log_2\left(1+\frac{\frac{P_{tot}}{M/T}\left(\frac{\delta}{Mx_0}\right)^{-\alpha}}{J+I_k}\right) \\ &= \frac{1}{M}\sum_{m=0}^{M/T-1}\log_2\left(1+\frac{\frac{P_{tot}}{M/T}\left(\frac{\delta}{Mx_0}\right)^{-\alpha}}{J+I_{mT+1}}\right) \text{ [bits/s/Hz]}. \end{aligned} \tag{3}$$

We will give an example of this for $M = 9$ and $T = 3$. During the first time slot, the first, fourth and seventh devices transmit, and we are interested in calculating I_1, I_4 and I_7. The first active link, where the first device transmits to the second, will experience interference from the fourth and the seventh devices, which are on distance $\frac{2\delta}{M}$ and $\frac{5\delta}{M}$ respectively from the receiver. Hence, the interference power experienced by the receiver of the first link (the second device) is:

$$I_1 = \frac{P_{tot}}{M/T}\left(\frac{2\delta}{Mx_0}\right)^{-\alpha} + \frac{P_{tot}}{M/T}\left(\frac{5\delta}{Mx_0}\right)^{-\alpha}.$$

Similarly

$$I_4 = \frac{P_{tot}}{M/T}\left(\frac{4\delta}{Mx_0}\right)^{-\alpha} + \frac{P_{tot}}{M/T}\left(\frac{2\delta}{Mx_0}\right)^{-\alpha}$$

and

$$I_7 = \frac{P_{tot}}{M/T}\left(\frac{7\delta}{Mx_0}\right)^{-\alpha} + \frac{P_{tot}}{M/T}\left(\frac{4\delta}{Mx_0}\right)^{-\alpha}.$$

Inserting this in (3) yields the achievable rate from A to B for this sub-system. The point-to-point achievable rate of this scheme ($M = 9$, $T = 3$) is shown in Figure 6. It is also compared to the direct transmission ($M = 1$), and the time-multiplexed scheme with 9 transmitting devices ($M = 9$, $T = 9$). We

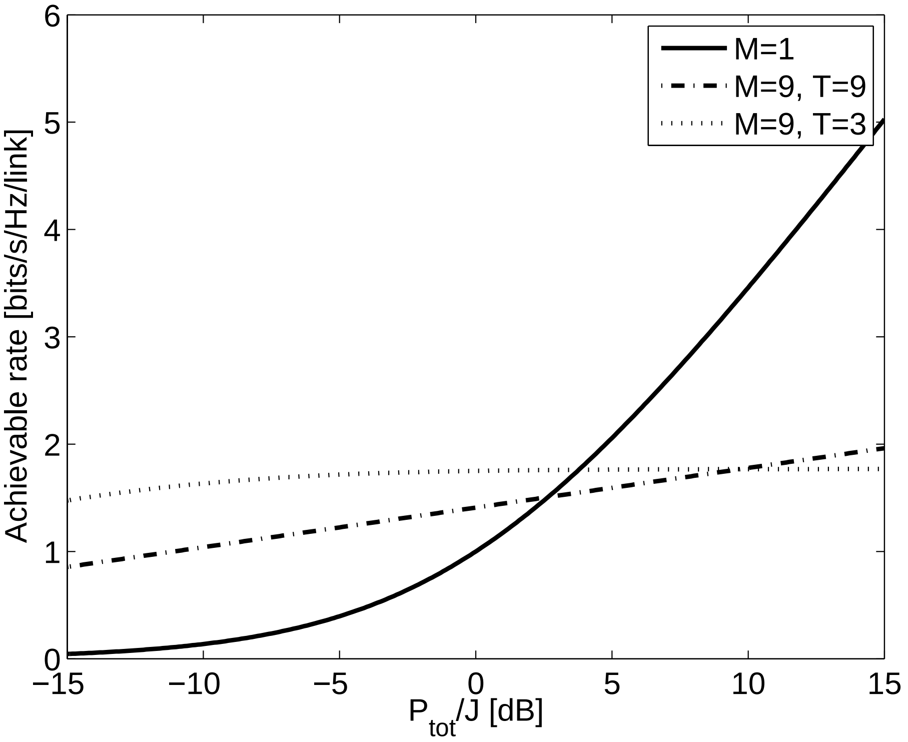

Fig. 6. Comparison of the point-to-point achievable rate using either direct transmission, 9 time division multiplexed repeater devices, or 9 repeater devices of which 3 transmit simultaneously.

note that for high SINR, the best strategy is actually to transmit directly, without using any intermediate devices. It is also clear that for some SINRs, the mixed scheme yields the largest achievable rate. Especially, in accordance with Figure 5, using intermediate repeaters can provide an order of magnitude increase of the achievable rate for low SINR. In general, the gain that can be obtained by a repeater strategy depends much on the P_{tot}/J operating point in Figure 6. The location of this operating point depends on d_0, P_c and N, among others. For the example in Figure 2, the interesting region is arguably around 1 bit/s/Hz/link (cf. the reference line in Figure 2). This paper will not derive the general expression for the achievable rate or go further into detail in this strategy, but we note that it offers one possible way of improving over the results in Figure 5.

5 Conclusions

In this paper we have analyzed the achievable rate of a potential spectrum hole in a frequency planned environment, using spatial frequency reuse. Simulation results show that a substantial sum-rate could be achieved provided that the secondary users communicate over sufficiently small distances. For a small number of secondary links, the sum-rate increases linearly with the number of links. However, the spectrum hole gets saturated quite fast, due to interference caused by the secondary users. A spectrum hole may look large, but it disappears as soon as someone starts using it. We have assumed that the cognitive users can perfectly judge whether it is far enough away from the primary base station, and utilize all of the spectrum holes. This is a rather strong assumption Hoven and Sahai (2005); Larsson and Skoglund (2008), and it remains to analyze the effect of imperfect detection of the primary system.

It is hard to draw strong and general conclusions for the potential capacity of spectrum holes. We have seen that for this kind of a frequency-planned network, cognitive radio may be a solution that could provide a reasonable rate. We have provided some initial reflections on the question whether the utilization of spectrum holes is realistic or not, but further research appears necessary to answer the question conclusively.

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