Rate-Optimized Constellation Rearrangement for the Relay Channel

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Majid N. Khormuji and Erik G. Larsson

***Abstract*—We study the instantaneous relay channel where the relay's output only depends on the current received signal at the relay. We propose a novel forwarding strategy for this class of relay channels which can outperform amplify-and-forward, detect-and-forward and estimate-and-forward. The proposed scheme is based on a remapping of the signal constellation at the relay.**

***Index Terms*—Relay channel, instantaneous relaying, detect-and-forward, constellation mapping.**

I. Introduction and Preliminaries

THE relay channel [1] has recently received considerable attention due to its potential in wireless applications [2]. Fig. 1 shows a block diagram of the relay channel that we study in this letter. The channel consists of a source, a relay, and a destination with mutually orthogonal channels between them (i.e., the signal transmitted by the source and that transmitted by the relay do not interfere with each other). This model requires the relay to be able to receive and transmit simultaneously. This is possible, for example, if the relay uses different frequency bands for reception and for transmission [3]. However, one can relax the assumption that the relay can transmit and receive simultaneously, by using orthogonal half-duplex transmission. In this case, all rate expressions presented in the sequel must be divided by two.

The signal received at the relay is given by $y_r = ax + z_r$ where $x \in \mathcal{X}$ is the transmitted symbol, a is the channel gain between the source and the relay, and $z_r \sim \mathcal{N}(0, 1)$ is additive Gaussian noise with unit variance. We assume that $\mathcal{X}$ is a finite alphabet associated with a modulation scheme. For example, when the source uses uniform 4-PAM modulation, then $\mathcal{X} = \{-3d, -d, d, 3d\}$ where d is a normalization constant. We assume that all symbols in $\mathcal{X}$ are equally likely to be transmitted. The signal received from the source at the destination is given by $y_1 = x + z_1$ where $z_1 \sim \mathcal{N}(0, 1)$.

We confine the relay to use a *one-dimensional* (possibly complex-valued) mapping. That is, the relay performs memoryless symbol-by-symbol processing. We call this instantaneous relaying. The main motivation for this scenario is relays that can afford very little signal processing. Specifically, upon receiving y_r, the relay transmits $x_r = f(y_r)$, where $f(\cdot)$ is the mapping used at the relay. The received signal from the relay at the destination is then $y_2 = bx_r + z_2 = bf(ax + z_r) + z_2$ where b is the channel gain between the relay and the destination, and $z_2 \sim \mathcal{N}(0, 1)$. We assume that z_r, z_1, and z_2 are mutually independent. We further assume that the source and the relay operate under average power constraints. That is $\mathbb{E}\left[|X|^2\right] \leq P_s$ and $\mathbb{E}\left[|X_r|^2\right] \leq P_r$, for some P_s and P_r.

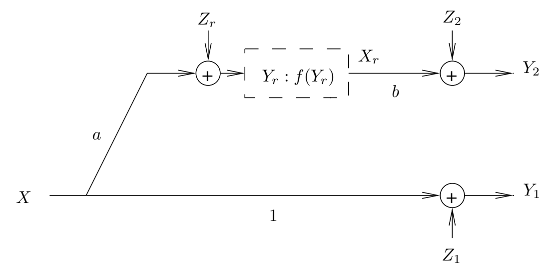

Fig. 1: The instantaneous Gaussian, three-node relay channel with orthogonal receive components.

Contribution: Our main contribution is a new scheme, detect-remap-and-forward (DRF), for the instantaneous relay channel with higher-order modulation schemes. The key idea behind DRF is to let the relay employ a symbol-mapping different from the one used by the source.[1] For coded systems, with mutual information as performance measure, DRF provides a significant gain over schemes known from the literature (see Section II) provided that the relay is able to detect the transmitted symbol reliably. (Table III summarizes the gains.) This paper can be seen as an extension of [9] where we only studied uncoded transmission. Related literature on instantaneous relaying [3]–[6] is discussed briefly in Section II.

II. Instantaneous Relaying: State-of-the-Art

There are three main instantaneous relaying strategies found in the literature: amplify-and-forward (AF), detect-and-forward (DF), and estimate-and-forward (EF).

Amplify-and-Forward (AF): With AF, the relay retransmits the received sample value, normalized so that the power constraint is satisfied. That is,

$$x_r = f(y_r) = \sqrt{\frac{P_r}{\mathbb{E}\left[|y_r|^2\right]}}\, y_r. \tag{1}$$

The main drawback of AF is the amplification of the noise in y_r.

Detect-and-Forward (DF): With DF, the relay detects (with hard decision) the transmitted symbol, and then remodulates

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[1]This must not be confused with schemes in which the relay performs block-wise processing, e.g., decoding and re-encoding with a different channel code (for example, see [7], [8]).

TABLE I: Examples of relay remappings $\pi(\cdot)$ for DRF

		$\hat{x}$			
		$-3d$	$-d$	d	$3d$
x_r	π_1	$-3d$	$-d$	d	$3d$
	π_2	$-3d$	d	$-d$	$3d$
	π_3	$-d$	$-3d$	$3d$	d
	π_4	d	$-3d$	$3d$	$-d$

and retransmits it using the same signal constellation. That is,

$$x_r = f(y_r) = \arg\max_x p(x|y_r). \tag{2}$$

This strategy suffers from error propagation at low signal-to-noise ratio (SNR).

Estimate-and-Forward (EF): Using EF, upon receiving y_r, the relay transmits an estimate of the transmitted symbol X. Commonly the MMSE estimator (conditional mean) is used:

$$x_r = \lambda \mathbb{E}[x|y_r] = \lambda \sum_{x' \in \mathcal{X}} x' \frac{p(y_r|x')p(x')}{\sum_{x'' \in \mathcal{X}} p(y_r|x'')p(x'')} \tag{3}$$

where λ is a constant chosen so that $\mathbb{E}\left[|X_r|^2\right] = P_r$. With EF, the relay provides *soft* information about the transmitted symbol. When the SNR of the source-relay link is high EF behaves as DF; and when the SNR is low EF behaves as AF.

AF, DF and EF have been proposed and extensively analyzed in previous literature. Most notably, for coded transmission, [4] considers instantaneous relaying and shows numerically that EF is optimal for BPSK signaling, provided there is no direct link available. Reference [3] considers AF relaying for Gaussian signal alphabets $\mathcal{X}$. For uncoded instantaneous relaying [5] studies BPSK modulation for the case that there is no direct link. The work of [6] deals with AF, EF, and DF relaying protocols and compares their performances. We now proceed to present our new scheme.

III. Proposed Scheme: Detect-Remap-and-Forward (DRF)

With DRF, the relay takes a hard decision on the transmitted symbol, and then remodulates and transmits it using a different signal constellation than what the source used. More precisely, the signal constellation used for transmission by the relay is a permuted version of the constellation used by the source. That is,

$$x_r = \pi(\hat{x}) = \pi(\arg\max_x p(x|y_r)). \tag{4}$$

where π is a remapping (permutation). Table I shows some possible remappings $\pi(\cdot)$ for uniform 4-PAM modulation. In Table I, the "identity" mapping π_1 (which does not rearrange the points) corresponds to the DF protocol.

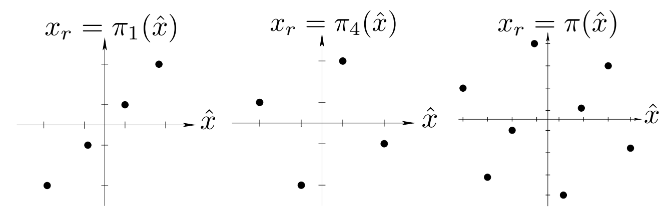
In order to optimize the performance of DRF we look for the mapping $\pi(\cdot)$ that maximizes the mutual information between the transmitted symbol and the received signals at the destination. This mutual information is given by

$$\begin{aligned} I(X;Y_1,Y_2) &= h(Y_1,Y_2) - h(Z_1) - h(Y_2|X) \\ &= -\mathbb{E}\left[\log_2 p(y_1,y_2)\right] + \mathbb{E}\left[\log_2 p(z)\right] + \mathbb{E}\left[\log_2 p(y_2|x)\right]. \end{aligned}$$

Given $p(y_2|x)$ and $p(y_1,y_2)$, one can easily compute $I(X;Y_1,Y_2)$ numerically. For the case of real-valued alphabet

TABLE II: $p(x_r|x)$ for DRF with $\pi_1(\cdot)$

		x_r			
		$-3d$	$-d$	d	$3d$
x	$-3d$	$Q(-ad)$	$Q(ad)$–$Q(3ad)$	$Q(3ad)$–$Q(5ad)$	$Q(5ad)$
	$-d$	$Q(ad)$	1–2$Q(ad)$	$Q(ad)$–$Q(3ad)$	$Q(3ad)$
	d	$Q(3ad)$	$Q(ad)$–$Q(3ad)$	1–2$Q(ad)$	$Q(ad)$
	$3d$	$Q(5ad)$	$Q(3ad)$–$Q(5ad)$	$Q(ad)$–$Q(3ad)$	$Q(-ad)$



(a) $a = 1$, 4-PAM (b) $a = \sqrt{10}$, 4-PAM (c) $a = \sqrt{10}$, 8-PAM

Fig. 2: Mapping rules $\pi(\cdot)$ that maximize $I(X;Y_1,Y_2)$ for uniform 4-PAM and 8-PAM modulation, when $b = 1$ and $P_s = P_r = 5$ dB. (For 4-PAM, $\pi_i(\hat{x})$ are also given in Table I.)

$\mathcal{X}$, we have

$$p(y_2|x) = \sum_{x_r} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_2 - bx_r)^2}{2}\right) p(x_r|x), \tag{5}$$

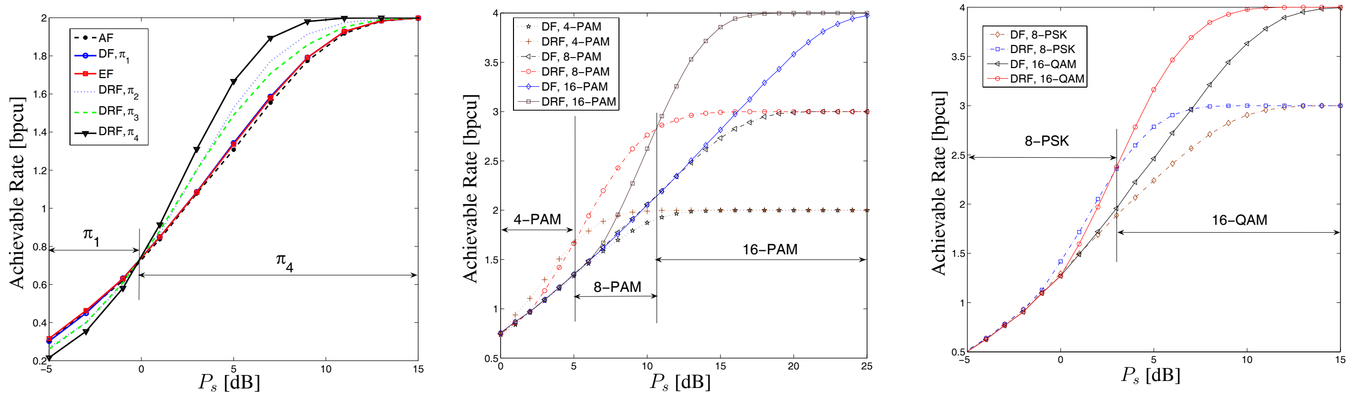
and
$$\begin{aligned} p(y_1,y_2) = \sum_x \sum_{x_r} p(x) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_1 - x)^2}{2}\right) \\ \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_2 - bx_r)^2}{2}\right) p(x_r|x). \end{aligned} \tag{6}$$

(The case of a complex-valued alphabet is treated similarly.) In (5) and (6) $p(x_r|x)$ denotes the transition probability of the source-relay link. As an example, Table II shows $p(x_r|x)$ for uniform 4-PAM modulation and the DRF forwarding strategy with the mapping $\pi_1(\cdot)$. In Table II, $Q(\alpha) \triangleq \frac{1}{\sqrt{2\pi}} \int_\alpha^\infty \exp\left(-\frac{t^2}{2}\right) dt$.

For a given modulation scheme with spectral efficiency q bits per channel use [bpcu], there are $2^q!$ possible mapping rules $\pi(\cdot)$. That is, the number of mapping rules increases exponentially with the size of the constellation. This precludes the use of an exhaustive search to obtain the optimal mapping for large q. As a remedy, for constellations with more than 8 points, we resort to a search method based on simulated annealing [10]. Figure 2 shows optimized DRF mappings that we obtained this way, for uniform 4-PAM and 8-PAM modulation. Note that the optimal mapping will depend on the SNR (more specifically on a and b).

IV. Numerical Examples

Figure 3(a) shows the achievable rates obtained by the AF, DF, EF and DRF strategies with 4-PAM modulation when $a = \sqrt{10}$, $b = 1$ and $P_r = P_s$. For all cases, the constellation used at the source is $(-3d, -d, d, 3d)$ (cf. Table I). At high SNR, AF has the worst performance (as expected) while DF provides somewhat higher rates (this is so since the quality of the source-relay link is good in this example). DRF with



(a) 4-PAM modulation using DF, EF, AF, and DRF with different mappings.

(b) 4-, 8- and 16-PAM modulations using DF and and optimized DRF.

(c) 8-PSK and 16-QAM using DF and optimized DRF.

Fig. 3: Achievable rates for $a = \sqrt{10}$, $b = 1$ and $P_r = P_s$.

mapping $\pi_4(\cdot)$ has the best performance. At low SNR, the performances of AF and EF are slightly better than those of DF and DRF. At low SNR, the highest rates using DRF are achieved when the source and the relay use the same mapping (that is, mapping $\pi_1(\cdot)$ or equivalently, DF). Note that the optimal mapping depends on the quality of the links (i.e., a and b). For large values of $|a|$, the relay can reliably detect the transmitted symbol and the points in the $(\hat{x}, x_r)$-space (cf. Figure 2) are located as far as possible from each other. However, when $|a|$ is small relative to $|b|$, the relay and the source should use the same mapping and the DRF simplifies to the conventional DF. These results are in agreement with those in [9] (for uncoded transmission, with bit-error-rate as performance measure).

Figure 3(b) shows the achievable rates of DF and DRF for 4-PAM, 8-PAM, 16-PAM modulation when $a = \sqrt{10}$, $b = 1$ and $P_r = P_s$. Here we have assumed that the relay chooses a proper mapping based on the quality of the links and informs the destination about this mapping. (This requires forward channel state information, which would be needed anyway to choose an appropriate code-rate.) DRF with the identity mapping (DF) is optimal at low SNR. However, at high SNR, optimization of the mapping brings a power gain. Table III shows the relative power gain of DRF over DF for some target rates. We can see that this gain increases with the size of the constellation. From Figure 3(b) we see that at low SNR, DRF with 4-PAM has superior performance whereas for moderate and high SNRs, 8-PAM and 16-PAM respectively provide the best achievable rate. This is so because at low SNR, the relay can detect the transmitted symbol obtained by 4-PAM more reliably than that obtained by 8-PAM and 16-PAM. A similar optimization can be conducted for complex modulation schemes. Figure 3(c) shows the results for 8-PSK and 16-QAM.

TABLE III: Relative gain of DRF over DF at fixed rate.

	4-PAM	8-PAM	16-PAM	8-PSK	16-QAM
Rate [bpcu]	1.8	2.5	3.5	2.5	3.5
Gain [dB]	3	4.5	6.5	3	3.5

V. Concluding Remarks

The proposed DRF strategy does not add any complexity to neither the source, relay nor the destination. Additionally, it does not impose any additional requirements on synchronization or knowledge of auxiliary parameters. DRF is most beneficial when the source-relay link is strong relative to the other links. This situation is encountered in practice when the source chooses a partner located in its proximity. It should be noted that DRF cannot be used with BPSK modulation (since then the remapping operation is meaningless).

Possible extensions of this work include combinations of DRF with decode-and-forward with repetition coding (e.g., see [2]). However, DRF would not be beneficial if combined with so-called parallel coding (that is, when the relay uses a channel code which is independent of the one used by the source).

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