

Linköping University Post Print

Competition Versus Cooperation on the MISO Interference Channel

Erik G. Larsson and Eduard Jorswieck

N.B.: When citing this work, cite the original article.

©2009 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

Erik G. Larsson and Eduard Jorswieck, Competition Versus Cooperation on the MISO Interference Channel, 2008, IEEE Journal on Selected Areas in Communications, (26), 7, 1059-1069.

<http://dx.doi.org/10.1109/JSAC.2008.080904>

Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-22003>

Competition Versus Cooperation on the MISO Interference Channel

Erik G. Larsson and Eduard A. Jorswieck

Abstract—We consider the problem of coordinating two competing multiple-antenna wireless systems (operators) that operate in the same spectral band. We formulate a rate region which is achievable by scalar coding followed by power allocation and beamforming. We show that all interesting points on the Pareto boundary correspond to transmit strategies where both systems use the maximum available power. We then argue that there is a fundamental need for base station cooperation when performing spectrum sharing with multiple transmit antennas. More precisely, we show that if the systems do not cooperate, there is a unique Nash equilibrium which is inefficient in the sense that the achievable rate is bounded by a constant, regardless of the available transmit power. An extension of this result to the case where the receivers use successive interference cancellation (SIC) is also provided.

Next we model the problem of agreeing on beamforming vectors as a non-transferable utility (NTU) cooperative game-theoretic problem, with the two operators as players. Specifically we compute numerically the Nash bargaining solution, which is a likely resolution of the resource conflict assuming that the players are rational. Numerical experiments indicate that selfish but cooperating operators may achieve a performance which is close to the maximum-sum-rate bound.

Index Terms—multiple-input single-output channel, interference channel, non-cooperative game theory, cooperative game theory

I. INTRODUCTION

A. Background

WE ARE CONCERNED with the following scenario: Two independent wireless systems operate in the same spectral band. The first system consists of a base station BS₁ that wants to convey information to a mobile MS₁. The second system consists of another base station BS₂ that wants to transmit information to a mobile MS₂. The systems share the same spectrum, so the communication between BS₁ → MS₁ and BS₂ → MS₂ is going to take place simultaneously on the same channel. Thus MS₁ will hear a superposition of the signals transmitted from BS₁ and BS₂, and conversely MS₂ will also receive the sum of the signals transmitted by both base stations. This setup is recognized as an interference

Manuscript received August 15, 2007; revised March 10, 2007. This work was supported in part by the Swedish Research Council (VR). E. Larsson is a Royal Swedish Academy of Sciences Research Fellow supported by a grant from the Knut and Alice Wallenberg Foundation. Parts of the material in this paper were presented at the Allerton conference in September 2007.

E. Larsson is with Linköping University, Dept. of Electrical Engineering (ISY), Division of Communication Systems, 581 83 Linköping, Sweden (e-mail: erik.larsson@isy.liu.se).

E. Jorswieck is with the Dresden University of Technology, Communications Laboratory, Chair of Communication Theory, D-01062 Dresden, Germany (e-mail: jorswieck@ifn.et.tu-dresden.de).

Digital Object Identifier 10.1109/JSAC.2008.080904.

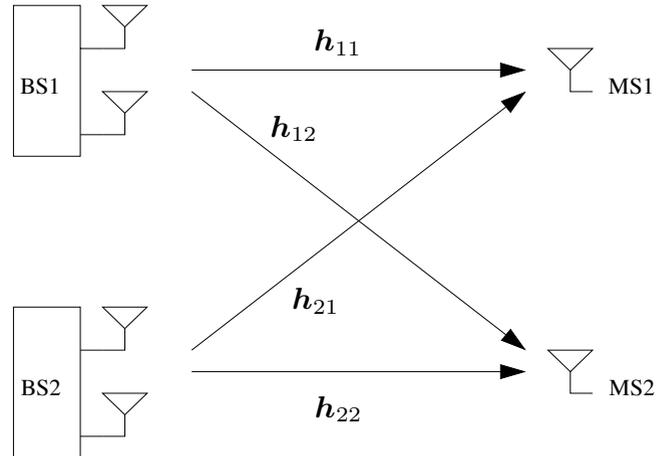


Fig. 1. The two-user MISO interference channel under study (illustrated for $n = 2$ transmit antennas).

channel (IFC) [1]–[3]. In the setup we consider, BS₁ and BS₂ have n transmit antennas each, that can be used with full phase coherency. MS₁ and MS₂, however, have a single receive antenna each. Hence our problem setup constitutes a multiple-input single-output (MISO) IFC. See Figure 1.

We shall assume that transmission consists of scalar coding followed by beamforming,¹ and that all propagation channels are frequency-flat. This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at MS₁ and MS₂:

$$y_1 = \mathbf{h}_{11}^T \mathbf{w}_1 s_1 + \mathbf{h}_{21}^T \mathbf{w}_2 s_2 + e_1$$

$$y_2 = \mathbf{h}_{22}^T \mathbf{w}_2 s_2 + \mathbf{h}_{12}^T \mathbf{w}_1 s_1 + e_2$$

where s_1 and s_2 are transmitted symbols, \mathbf{h}_{ij} is the (complex-valued) $n \times 1$ channel-vector between BS _{i} and MS _{j} , and \mathbf{w}_i is the beamforming vector used by BS _{i} . The variables e_1 , e_2 are noise terms which we model as i.i.d. Gaussian with zero mean and variance σ^2 . We assume that each base station can use the transmit power P , but that power cannot be traded between the base stations. Without loss of generality, we shall take $P = 1$. This gives the power constraint

$$\|\mathbf{w}_i\|^2 \leq 1, \quad i = 1, 2 \quad (1)$$

Throughout, we define the signal-to-noise ratio (SNR) as $1/\sigma^2$. Various schemes that we will discuss require that the transmitters (BS₁ and BS₂) have different forms of channel

¹Single-stream transmission (scalar coding followed by beamforming) is optimal under certain circumstances, for example provided that BS _{i} knows \mathbf{h}_{ii} and MS₁, MS₂ treat the interference as Gaussian noise [4].

state information (CSI). However, at no point we will require phase coherency between the base stations.

The fundamental question we want to address is the following. If BS₁ and BS₂ operate in an uncoordinated manner, how should they choose their beamforming vectors w_1, w_2 ? There is an obvious conflict situation associated with this choice since a vector w_1 which is good for the link BS₁ → MS₁ may generate substantial interference for MS₂ and vice versa. The main contribution of this work is to discuss this conflict situation in a game-theoretic framework. In the course of doing so, we will also present an achievable rate region for the MISO IFC and a characterization of this region.

We stress that in our setup there is no central controller that can dictate what beamforming vectors the two systems (BS₁ and BS₂) should use. That is, we assume that the two systems belong to different infrastructure (for example, are owned by different operators) and hence that they have fundamentally conflicting interests. This stands in sharp contrast to the case (not considered in this paper) when the systems belong to the same infrastructure and are connected via a central controller that has the authority to determine how resources are shared. In the case with a central controller, there would be no conflict between the systems (in a game-theoretic sense), as the controller can decide on any arbitrary operating point at its choice. Notwithstanding this, even if there is no central controller with authority to dictate the resource use, BS₁ and BS₂ may still communicate with each other in order to negotiate how resources should be split. Such communication may take place directly by using a carefully crafted, possibly standardized protocol for this, or indirectly (by using iterative punishment schemes as discussed in [12]). To summarize, the two basic points are: (i) the absence of a central controller does not mean that the systems cannot talk (negotiate) with one another; and (ii) having a central controller or not is not an issue of performance, rather, the cases with and without a central controller are two fundamentally different problems.

B. Related work

Information-theoretic studies of the IFC have a long history [1]–[3], [5]. These references have provided various achievable rate regions, which are generally larger in the more recent papers than in the earlier ones. However, the capacity region of the general IFC channel is still an open problem. For certain limiting cases, for example when the interference is weak or very strong, respectively, the sum capacity is known [6]. For weak interference the interference can simply be treated as additional noise. For very strong interference, successive interference cancellation (SIC) can be applied at one or more of the receivers. Multiple-input multiple-output (MIMO) IFCs have also recently been studied in [7], from the perspective of spatial multiplexing gains.

Recently an increasing body of literature has looked at resource conflict problems in wireless communications using tools from game theory (see, for example [8]). Most of this work deals with networking aspects of communications. There is some available work, however, that studies the IFC from a game-theoretic perspective. In what follows, we summarize the relevant literature that we are aware of. Distributed algorithms

for spectrum sharing in a competitive setup (using noncooperative game theory [9]) were developed in [10] and [11]. A more general analysis of the spectrum sharing problem was performed in [12]. All three [10]–[12] dealt with single-antenna transmitters and receivers, and looked at the problem from a noncooperative game-theoretic point of view. The MIMO IFC has also been studied from a noncooperative game-theoretic perspective in [13] and [14], which presented results on equilibrium rates and proposed distributed algorithms. These noncooperative approaches [10]–[13] generally lead to decentralized schemes for computing stable operating points, so-called Nash equilibria. Unfortunately, these equilibria are often rather inefficient outcomes, as measured by the achievable sum-rate, for example. Less work is available on *cooperative* game theory for IFCs, especially for multiple-antenna IFCs. Some results can be found in [15] which treated the spectrum sharing problem using cooperative (bargaining) game theory and [16] which proposed a decentralized algorithm for finding the bargaining solution. Both [15], [16] considered the case of single antennas at the transmitter and at the receiver. Apart from this, the area of cooperative strategies for the IFC appears largely open. (We shall note [17] that deals with the multiple-access channel (MAC) using coalitional game theory. However the MAC differs fundamentally from the IFC.)

Contributions: We study the MISO IFC both from a noncooperative (competitive) game theoretic perspective, and from a cooperative (bargaining) point of view. We show that the outcome of the noncooperative game is a unique Nash equilibrium but that this is rather bad from an overall system perspective (see Section III-A). We then consider the same problem using cooperative (Nash axiomatic bargaining) theory and show that this can significantly improve the outlook of the problem (see Section III-B). Before we embark on this, we present in Section II some preliminaries, and various other interesting results related to the MISO IFC. This paper is reproducible research [18] and the software needed to generate the numerical results can be obtained from www.commsys.isy.liu.se/~egl/rr.

II. ACHIEVABLE RATES AND OPERATING POINTS

A. An Achievable Rate Region

In what follows we will assume that all receivers treat co-channel interference as noise, i.e., they make no attempt to decode and subtract the interference. (This assumption will be relaxed in Section V.) The main justification for this assumption is that in most envisioned applications, MS_{*i*} would use receivers with a simple structure. Additionally, one can argue that interference cancellation is difficult in an environment where the receivers do not know the coding and modulation schemes used by the interfering transmitters. For a given pair of beamforming vectors $\{w_1, w_2\}$, the following rates are then achievable, by using codebooks approaching Gaussian ones:²

²Strictly speaking, only rates $R_i - \epsilon$ are achievable, for some ϵ . Since the main purpose of this paper is to explain fundamental limitations and possibilities associated with spectrum conflicts, rather than to develop coding theorems for IFCs, we shall say (with some sacrifice of rigor) that the rates R_i are “achievable”.

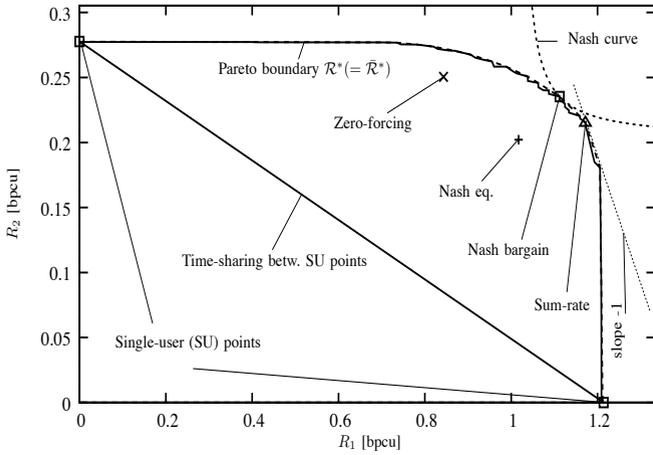


Fig. 2. Rate region, example 1. The channels were chosen at random but such that \mathcal{R} is convex. Here the signal-to-noise-ratio is 0 dB.

$$R_1 = \log_2 \left(1 + \frac{|\mathbf{w}_1^T \mathbf{h}_{11}|^2}{|\mathbf{w}_2^T \mathbf{h}_{21}|^2 + \sigma^2} \right) \quad (2)$$

for the link $\text{BS}_1 \rightarrow \text{MS}_1$, and

$$R_2 = \log_2 \left(1 + \frac{|\mathbf{w}_2^T \mathbf{h}_{22}|^2}{|\mathbf{w}_1^T \mathbf{h}_{12}|^2 + \sigma^2} \right) \quad (3)$$

for $\text{BS}_2 \rightarrow \text{MS}_2$. For fixed channels $\{\mathbf{h}_{ij}\}$, we define the achievable rate region as

$$\mathcal{R} = \bigcup_{\mathbf{w}_1, \mathbf{w}_2, \|\mathbf{w}_i\|^2 \leq 1} (R_1, R_2).$$

We stress that this is *not* the capacity region, because it does not take into account the possibility of performing interference cancellation at the receivers, and it does not take into account the possibility of going beyond Gaussian signaling. However the rates in \mathcal{R} are achievable with simple receiver signal processing, that treats interference as noise. (Extensions to interference cancellation are discussed in Section V.) The outer boundary of \mathcal{R} is called the Pareto boundary, because it consists of Pareto optimal operating points. A Pareto optimal point is a point at which one cannot improve the rate of one link without simultaneously decreasing the rate of the other. We denote the Pareto boundary by \mathcal{R}^* . Note that for fixed \mathbf{h}_{ij} , the region \mathcal{R} is compact, since the set $\{\mathbf{w}_1, \mathbf{w}_2\}$ subject to the power constraint (1) is compact and the mapping from $\{\mathbf{w}_1, \mathbf{w}_2\}$ to $\{R_1, R_2\}$ is continuous. However, the region \mathcal{R} is in general not convex.

We define the convex hull of \mathcal{R} as follows:

$$\bar{\mathcal{R}} = \bigcup_{\substack{0 \leq \tau \leq 1 \\ (R_1, R_2) \in \mathcal{R} \\ (R'_1, R'_2) \in \mathcal{R}}} (\tau R_1 + (1 - \tau)R'_1, \tau R_2 + (1 - \tau)R'_2).$$

Also we denote the Pareto boundary of $\bar{\mathcal{R}}$ with $\bar{\mathcal{R}}^*$. The region $\bar{\mathcal{R}}$ can be interpreted as the set of achievable outcomes if the two systems $\text{BS}_1 \rightarrow \text{MS}_1$ and $\text{BS}_2 \rightarrow \text{MS}_2$ are allowed to split the available degrees of freedom (time or bandwidth in practice) offered by the channel in two parts, and use the beamforming vectors $\{\mathbf{w}_1, \mathbf{w}_2\}$ (corresponding to a rate point (R_1, R_2)) during a fraction τ of the time, and another set of

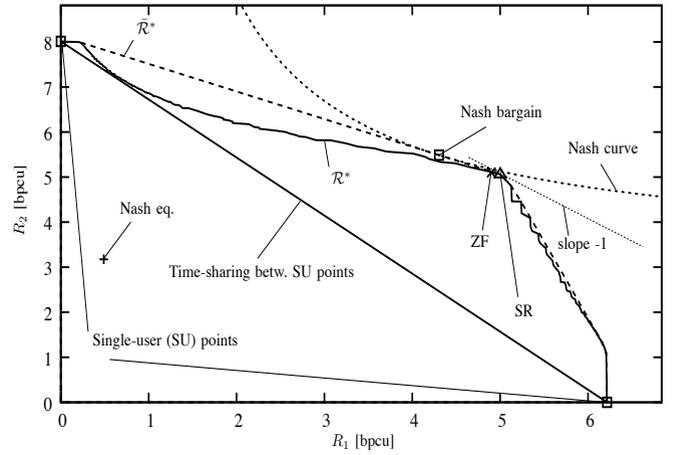


Fig. 3. Rate region, example 2. The channels were chosen randomly, but such that \mathcal{R} was non-convex. Here the signal-to-noise-ratio is 20 dB.

beamforming vectors $\{\mathbf{w}'_1, \mathbf{w}'_2\}$ (corresponding to a different rate point (R'_1, R'_2)) during the rest of the time (i.e., during a fraction $1 - \tau$ of the total time). Implicit in this interpretation is the assumption that the power constraint (1) is unchanged, i.e., the constraint is on the peak power rather than on the long-term average of power. Another interpretation of $\bar{\mathcal{R}}$ is in terms of correlated mixed strategies: $\bar{\mathcal{R}}$ is the set of *average* rates that can be achieved if the two systems decide on two arbitrary rate points in \mathcal{R} and then flip a synchronized coin to decide which one of these two points to operate at. The importance of working with $\bar{\mathcal{R}}$ instead of \mathcal{R} will become clear when we formulate the beamforming problem as a bargaining problem (Section III-B).

Figures 2 and 3 show two examples of the rate region \mathcal{R} . In the first example the channels are chosen so that \mathcal{R} is convex; in the second example \mathcal{R} is nonconvex. The figures also show the convex hull $\bar{\mathcal{R}}$. (The other rate points in the figures will be explained in what follows.) These figures were generated by computing (R_1, R_2) over a grid of beamforming vectors, as explained in more detail in Section IV.

B. Characterization of the Pareto Boundary

A first question to ask is whether any point on the Pareto boundary of \mathcal{R} (or $\bar{\mathcal{R}}$) can be reached unless both BS_1 and BS_2 spend the maximum allowable power, i.e., whether $(R_1, R_2) \in \{\mathcal{R}^*, \bar{\mathcal{R}}^*\}$ always requires $\|\mathbf{w}_1\|^2 = \|\mathbf{w}_2\|^2 = 1$. We will show that the only points on the Pareto boundary which can be achieved without having $\|\mathbf{w}_1\|^2 = \|\mathbf{w}_2\|^2 = 1$ are points where the tangent to the Pareto boundary is either strictly vertical or strictly horizontal. The importance of this observation is that apart from pieces of the Pareto boundary that are strictly vertical or horizontal, it is enough to consider parameterizations of the boundary for which $\|\mathbf{w}_1\|^2 = \|\mathbf{w}_2\|^2 = 1$. More precisely we have the following proposition.

Proposition 1: a) Consider a point (R_1, R_2) in the rate region \mathcal{R} which corresponds to a set of beamforming vectors $(\mathbf{w}_1, \mathbf{w}_2)$ for which $\|\mathbf{w}_1\|^2 < 1$ and $\|\mathbf{w}_2\|^2 \leq 1$. Then there exists a beamforming vector $\hat{\mathbf{w}}_1$, such that the rate operating point (\hat{R}_1, \hat{R}_2) associated with $(\hat{\mathbf{w}}_1, \mathbf{w}_2)$ satisfies

$1 \geq \|\hat{\mathbf{w}}_1\|^2 > \|\mathbf{w}_1\|^2$, $\hat{R}_1 > R_1$ and $\hat{R}_2 = R_2$. In other words, it is possible to improve the rate of system 1 by changing \mathbf{w}_1 in a way so that BS₁ uses more power, and simultaneously keep \mathbf{w}_2 unchanged.

b) The result in (a) holds also if \mathcal{R} is replaced by $\bar{\mathcal{R}}$.

Proof: See the Appendix. \square

Note that the converse is not true. Many points in the interior of \mathcal{R} and $\bar{\mathcal{R}}$ correspond to beamforming vectors for which both base stations use full power.

C. Some Special Operating Points

Some points in the rate region are especially interesting, and we discuss them as follows (in no particular order).

1) The **single-user (SU) points** $(R_1^{\text{SU}}, 0)$ and $(0, R_2^{\text{SU}})$ are the rate points that result if only one user transmits, assuming the base station has full channel knowledge and performs maximum-ratio transmission beamforming (i.e., $\mathbf{w}_1 = \mathbf{h}_{11}^*/\|\mathbf{h}_{11}\|$ and $\mathbf{w}_2 = \mathbf{h}_{22}^*/\|\mathbf{h}_{22}\|$, respectively, as in single-user MISO transmission [4], [19]). The associated rates are

$$R_1^{\text{SU}} = \log_2 \left(1 + \frac{\|\mathbf{h}_{11}\|^2}{\sigma^2} \right), R_2^{\text{SU}} = \log_2 \left(1 + \frac{\|\mathbf{h}_{22}\|^2}{\sigma^2} \right).$$

Note that all convex combinations of the points $(R_1^{\text{SU}}, 0)$ and $(0, R_2^{\text{SU}})$ lie on a straight line (see Figures 2–3). The points on this line correspond to orthogonal multiple access via time-sharing.

We can characterize the average rates associated with the single-user points as follows. Define

$$\mathcal{G}(\sigma^2, n) \triangleq \log_2(e) \exp(\sigma^2) \sum_{k=0}^{n-1} \sigma^{2k} \Gamma(-k, \sigma^2) \quad (4)$$

where

$$\Gamma(a, x) \triangleq \int_x^\infty t^{a-1} \exp(-t) dt$$

is the incomplete Gamma function. Then, if $\{h_{ii}\}$ have independent zero-mean Gaussian elements with unit variance (i.e., the fading is i.i.d. Rayleigh) we have

$$E[R_i^{\text{SU}}] = \mathcal{G}(\sigma^2, n) \quad (5)$$

This follows by applying the results presented in [20, Section IV.B].

Equation (4) shows that the average single-user rate grows logarithmically with the SNR. Unfortunately, this is not of much interest since the single-user points are unstable outcomes of the resource conflict in the sense that *if the systems operate at one of these points, then any of the systems can improve its rate by unilaterally changing its beamforming vector*. This goes also for convex combinations of the single-user points: they are not stable operating points unless the systems have pre-agreed to use orthogonal time-sharing.

2) The **best-user (BU) point** $(R_1^{\text{BU}}, R_2^{\text{BU}})$ is the rate point which is achieved if the system with the best channel (in the sense of largest channel norm) uses all resources and the other

system stays quiet. More precisely we have

$$R_1^{\text{BU}} = \begin{cases} R_1^{\text{SU}}, & R_1^{\text{SU}} \geq R_2^{\text{SU}} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad (6)$$

$$R_2^{\text{BU}} = \begin{cases} R_2^{\text{SU}}, & R_2^{\text{SU}} \geq R_1^{\text{SU}} \\ 0, & \text{otherwise} \end{cases}. \quad (7)$$

It is clear that the average rate associated with the best-user point is at least as good as any of the single-user rates. However, like the single-user points, the best-user operating point is unstable as well.

3) The **sum-rate (SR) point** $(R_1^{\text{SR}}, R_2^{\text{SR}})$ is the point at which $R_1 + R_2$ is maximized. Geometrically, this the point where the Pareto boundary of \mathcal{R} osculates a straight line with slope -1 . (This point is also shown in Figures 2–3.)

The expected sum-rate grows logarithmically with the SNR. This is clear by considering the following chain of inequalities:

$$E[R_i^{\text{SR}}] \geq E[\max(R_1^{\text{SU}}, R_2^{\text{SU}})] \geq \frac{1}{2}(E[R_1^{\text{SU}}] + E[R_2^{\text{SU}}]) \quad (8)$$

and using (5). In (8), the first inequality follows because the line with slope -1 which touches \mathcal{R}^* must lie to the upper right of both the points $(R_1^{\text{SU}}, 0)$ and $(0, R_2^{\text{SU}})$. The second inequality in (8) is immediate. However, a more precise analytical characterization of the sum-rate point appears nontrivial. Fortunately, this is not of much interest anyway, because like the two other rate points discussed above, the sum-rate operating point is also unstable.

4) The **zero-forcing (ZF) point** $(R_1^{\text{ZF}}, R_2^{\text{ZF}})$ is the rate pair which is achieved if BS₁ chooses a transmit strategy that creates no interference at all for MS₂, and vice versa. If we assume that both base stations use the maximum permitted power, then BS₁ should use a unit-norm beamforming vector \mathbf{w}_1 which is orthogonal to \mathbf{h}_{12} and which at the same time maximizes $|\mathbf{w}_1^T \mathbf{h}_{11}|$. This beamformer is uniquely defined and is given by

$$\mathbf{w}_1^{\text{ZF}} = \frac{\Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11}^*}{\|\Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11}^*\|} \quad (9)$$

where $\Pi_{\mathbf{X}}^\perp = \mathbf{I} - \mathbf{X}(\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H$ denotes projection onto the orthogonal complement of the column space of \mathbf{X} . (Among all unit-norm vectors \mathbf{z} for which $\mathbf{X}^H \mathbf{z} = \mathbf{0}$, $\mathbf{z} = \Pi_{\mathbf{X}}^\perp \mathbf{y}$ maximizes $|\mathbf{z}^H \mathbf{y}|$. To see why this is so, let $\Pi_{\mathbf{X}}^\perp = \mathbf{U}\mathbf{U}^H$ where $\mathbf{U}^H \mathbf{U} = \mathbf{I}$ and let $\mathbf{z} = \mathbf{U}\mathbf{p}$ for some \mathbf{p} . Then $|\mathbf{z}^H \mathbf{y}| = |\mathbf{y}^H \mathbf{U}\mathbf{p}|$ and $\|\mathbf{z}\| = \|\mathbf{p}\|$. Clearly, $|\mathbf{y}^H \mathbf{U}\mathbf{p}|$ is maximized, subject to the constraint $\|\mathbf{p}\| = 1$, for $\mathbf{p} = \mathbf{U}^H \mathbf{y} / \|\mathbf{U}^H \mathbf{y}\|$. That is, $\mathbf{z} = \mathbf{U}\mathbf{p} = \Pi_{\mathbf{X}}^\perp \mathbf{y} / \|\Pi_{\mathbf{X}}^\perp \mathbf{y}\|$.) Similarly, BS₂ uses

$$\mathbf{w}_2^{\text{ZF}} = \frac{\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}^*}{\|\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}^*\|}$$

The corresponding rates are

$$R_1^{\text{ZF}} = \log_2 \left(1 + \frac{|\mathbf{w}_1^{\text{ZF}T} \mathbf{h}_{11}|^2}{\sigma^2} \right) \quad \text{and} \quad (10)$$

$$R_2^{\text{ZF}} = \log_2 \left(1 + \frac{|\mathbf{w}_2^{\text{ZF}T} \mathbf{h}_{22}|^2}{\sigma^2} \right). \quad (11)$$

We can characterize the performance with ZF as follows.

Proposition 2: Suppose the fading is i.i.d. Rayleigh and that all channels are independent. Then the average achievable rates if both users perform ZF are given by

$$\begin{aligned} E[R_i^{\text{ZF}}] &= E \left[\log_2 \left(1 + \frac{|\mathbf{w}_i^{\text{ZF}T} \mathbf{h}_{ii}|^2}{\sigma^2} \right) \right] \\ &= \mathcal{G}(\sigma^2, n-1) \end{aligned} \quad (12)$$

where $\mathcal{G}(\cdot, \cdot)$ is defined in (4).

Proof: See the Appendix. \square

Comparing Proposition 2 with the single-user rates (see (5)), we see that the transmitter interference cancellation offered by ZF costs precisely one degree of freedom (since n is reduced to $n-1$ in the argument of $\mathcal{G}(\cdot, \cdot)$). For a small number of antennas, e.g., $n=2$, this will have a major impact on performance. For a large number of antennas, however, the reduction in the number of degrees of freedom associated with ZF is negligible, and the ZF rates will be close to the single-user rates on the average.

The rate in (12) grows with increasing SNR without bound. It is also clear that at high SNR, the ZF point will not lie far away from the Pareto boundary. (This can also be seen in Figure 3.) The reason for this is that the rates associated with ZF and the sum-rate point (which is located at the Pareto boundary, by definition) both grow logarithmically with SNR; hence the normalized difference between these rates does not increase. The operational implication of this is not very important, however, because unless the systems have established a binding agreement to use ZF, then ZF is not going to be a stable outcome of the spectrum resource conflict. (This is so for the same reasons as the sum-rate point was not stable.)

Note that ZF transmission can be implemented without requiring the base stations to cooperate on channel estimation. Namely, ZF only requires that BS₁ knows the channels \mathbf{h}_{11} and \mathbf{h}_{12} (and that BS₂ knows \mathbf{h}_{22} and \mathbf{h}_{21}). In a time-division multiplexing system, BS₁ could directly measure the channels of interest without the help of BS₂, and vice versa.

III. BEAMFORMING AS A GAME THEORETIC PROBLEM

In this section we will treat the beamforming problem in a game-theoretic framework. We will separately discuss the two cases that the systems can cooperate, respectively not cooperate, in choosing their beamforming vectors. Whenever we refer to “cooperation” in this paper, we mean cooperation in the sense of the theory for non-zero-sum games [21].

A. Competitive (Non-cooperative) Solution

If BS₁, BS₂ do not cooperate then the only reasonable outcome of the spectrum conflict will be an operating point which constitutes a Nash equilibrium. This is a point where none of the base stations can improve its situation by *unilaterally* changing \mathbf{w}_i , subject to the power constraint [21]. It is clear (and more generally shown in [12]) that at a Nash Equilibrium both users must use the entire available bandwidth and time, so we make that assumption for the rest of this subsection. A Nash equilibrium is then a pair of vectors $\mathbf{w}_1^{\text{NE}}, \mathbf{w}_2^{\text{NE}}$ such that

$$\log_2 \left(1 + \frac{|\mathbf{w}_1^{\text{NE}T} \mathbf{h}_{11}|^2}{|\mathbf{w}_2^{\text{NE}T} \mathbf{h}_{21}|^2 + \sigma^2} \right) \geq \log_2 \left(1 + \frac{|\mathbf{w}_1^T \mathbf{h}_{11}|^2}{|\mathbf{w}_2^{\text{NE}T} \mathbf{h}_{21}|^2 + \sigma^2} \right)$$

for all \mathbf{w}_1 with $\|\mathbf{w}_1\|^2 \leq 1$ and

$$\log_2 \left(1 + \frac{|\mathbf{w}_2^{\text{NE}T} \mathbf{h}_{22}|^2}{|\mathbf{w}_1^{\text{NE}T} \mathbf{h}_{12}|^2 + \sigma^2} \right) \geq \log_2 \left(1 + \frac{|\mathbf{w}_2^T \mathbf{h}_{22}|^2}{|\mathbf{w}_1^{\text{NE}T} \mathbf{h}_{12}|^2 + \sigma^2} \right)$$

for all \mathbf{w}_2 with $\|\mathbf{w}_2\|^2 \leq 1$. We have the following result.

Proposition 3: There is a unique, pure Nash equilibrium corresponding to the maximum-ratio transmission beamforming vectors

$$\mathbf{w}_1^{\text{NE}} = \frac{\mathbf{h}_{11}^*}{\|\mathbf{h}_{11}\|} \quad \text{and} \quad \mathbf{w}_2^{\text{NE}} = \frac{\mathbf{h}_{22}^*}{\|\mathbf{h}_{22}\|}.$$

Proof: The proof is immediate: if BS_{*i*} uses \mathbf{w}_i^{NE} then there is no other \mathbf{w}_i that satisfies the power constraint and which could yield a larger R_i ; hence \mathbf{w}_i^{NE} must be a Nash equilibrium and it must be unique. \square

The corresponding rates at the equilibrium are

$$\begin{aligned} R_1^{\text{NE}} &= \log_2 \left(1 + \frac{\|\mathbf{h}_{11}\|^2}{\frac{|\mathbf{h}_{22}^H \mathbf{h}_{21}|^2}{\|\mathbf{h}_{22}\|^2} + \sigma^2} \right) \quad \text{and} \\ R_2^{\text{NE}} &= \log_2 \left(1 + \frac{\|\mathbf{h}_{22}\|^2}{\frac{|\mathbf{h}_{11}^H \mathbf{h}_{12}|^2}{\|\mathbf{h}_{11}\|^2} + \sigma^2} \right). \end{aligned} \quad (13)$$

Note that by using \mathbf{w}_1^{NE} , BS₁ can guarantee the rate R_1^{NE} regardless of what beamforming vector BS₂ is using, and vice versa. (A discussion of Nash equilibria for the more general case of a MIMO IFC is given in [13, Proposition 3.1].)

The Nash equilibrium is contained in \mathcal{R} , but in general it does not lie on the Pareto boundary. At low SNR, the Nash equilibrium is not a bad outcome since σ^2 will dominate over the interference terms in (13). Hence using \mathbf{w}_i^{NE} (which amounts to maximum-ratio beamforming) will maximize the rate of each user. However, at high SNR, the equilibrium outcome of the game is generally poor for both systems. This observation is made precise in the following result.

Proposition 4: Suppose the systems operate in i.i.d. Rayleigh fading (the entries of \mathbf{h}_{ij} independent and complex Gaussian with zero mean and unit variance). Then the average Nash equilibrium rates are bounded by

$$\begin{aligned} E[R_i^{\text{NE}}] &= E \left[\log_2 \left(1 + \frac{\|\mathbf{h}_{11}\|^2}{\frac{|\mathbf{h}_{22}^H \mathbf{h}_{21}|^2}{\|\mathbf{h}_{22}\|^2} + \sigma^2} \right) \right] \\ &\leq \frac{\Psi(n) + \gamma + 1/n}{\log(2)}. \end{aligned} \quad (14)$$

for $i=1, 2$, where $\Psi(x)$ is the Psi (DiGamma) function and γ is Euler’s constant. The upper bound in (14) is tight for high SNR ($\sigma^2 \rightarrow 0$).

Proof: See the Appendix. \square

The basic implication of Proposition 4 is that to achieve high rates in unlicensed bands, the systems need somehow to cooperate. For two transmit antennas ($n=2$), the upper bound in (14) is given by

$$\frac{\Psi(n) + \gamma + 1/n}{\log(2)} \approx 2.16 \text{ bpcu}$$

This result corresponds well with the numerical result that we will present in Section IV (Figure 4). Another consequence of Proposition 4 is that since $\Psi(x) = O(\log(x))$, the equilibrium

rate can grow at most logarithmically with the number of transmit antennas per base station, n . This means that adding more antennas help only marginally, if the systems compete with each other. (By using SIC, a higher average sum rate could be achieved. We discuss this briefly in Section V.)

Note that generally a Nash equilibrium is not an optimal, or even desirable, solution in any sense (although misconceptions around this appear to exist). Rather, the equilibrium is a point where one is likely to end up operating if BS₁ and BS₂ compete with each other. All we can say of this outcome is that the result can become no worse, if the base stations choose beamformers by *unilateral* action. In fact, in many games (including the one studied here) the Nash equilibrium is unique but it corresponds to an outcome which is bad for all players. See, the famous example of prisoner's dilemma, for example [21]. The inefficiency of Nash equilibria in a general system-wide context is further discussed in [22].

B. Cooperative (Nash Bargaining) Solution

If BS₁ and BS₂ were able to cooperate, they could achieve rates higher than $(R_1^{\text{NE}}, R_2^{\text{NE}})$, say $(R_1^{\text{NB}}, R_2^{\text{NB}})$ (NB as in Nash Bargaining, to be defined). By reaching an appropriate agreement they could achieve any point in $\bar{\mathcal{R}}$ or on the Pareto boundary. It is clear that if an agreement could be reached then $R_i^{\text{NB}} \geq R_i^{\text{NE}}$, since otherwise at least one of the base stations would resort to the competitive (noncooperative) solution, which we know guarantees each link a rate of at least R_i^{NE} . Thus we may restrict the search for cooperative solutions to the subregion $\bar{\mathcal{R}}^+$ that consists of all points of $\bar{\mathcal{R}}$ for which $R_i \geq R_i^{\text{NE}}$, i.e. the set of points located to the upper right of the Nash equilibrium.

For example, the base stations could agree to operate at the sum-rate or ZF point. However, such an outcome is not likely to occur in practice, unless it is imposed by regulation. Additionally, if such a regulation is imposed it would be very hard to check whether the base stations comply with it. The reason is that generally one of the base stations would have to "give in" more than the other in order to agree on a specific operating point (such as the ZF or the sum-rate point). The basic issue is that if the players try to agree on a point on the Pareto boundary, then any incremental improvement for one leads to a reduction for the other.

We will examine this problem by using the axiomatic bargaining theory developed by economist John Nash in the 1950's (and who was subsequently awarded the Nobel prize in economics) [21], [23]. Nash considered the general problem of establishing an agreement between players with conflicting objectives, under the assumption that there exists no utility (in our case "rate", but in general, money for example) that one player could pay to the other in order to compensate the other for a non-favorable outcome.³ Thus, the players must agree on an outcome $(R_1^{\text{NB}}, R_2^{\text{NB}})$. If they fail, they will resort to playing non-cooperatively which generally results in an

³If we could transfer rate between the systems, then any point $(r_1 + \delta, r_2 - \delta)$, where (r_1, r_2) is an arbitrary point in $\bar{\mathcal{R}}$ would be achievable. This however would require that there is a mechanism so that the two systems can borrow capacity from each other, and a way of paying for that. This is an assumption that we shall not make, since the systems operate in unlicensed spectrum and do not belong to the same set of infrastructure.

operating point no better than $(R_1^{\text{NE}}, R_2^{\text{NE}})$. This fallback point is generally called a "threat point" in bargaining theory, because it represents the outcome in the event the players would realize their threat not to cooperate.

Nash showed that under certain conditions there is a unique mapping between the convex hull of the achievable region ($\bar{\mathcal{R}}$), the threat point (which we take to be $(R_1^{\text{NE}}, R_2^{\text{NE}})$), and the cooperative (bargaining) outcome $(R_1^{\text{NB}}, R_2^{\text{NB}})$. The conditions stated by Nash are a set of axioms. Apart from technicalities, these essentially say that the cooperative outcome must lie on the Pareto boundary, and that the solution should be independent of irrelevant bargaining alternatives in the sense that if the solution is contained in a subset of $\bar{\mathcal{R}}$, say $\bar{\mathcal{R}}'$, then the same bargaining solution would have been obtained if the feasible set had been $\bar{\mathcal{R}}'$ at the outset. Additionally, invariance to linear transformations is required (see [21] for details).

The Nash solution for the two player game at hand can be explicitly computed as follows:

$$(R_1^{\text{NB}}, R_2^{\text{NB}}) = \max_{(R_1, R_2) \in \bar{\mathcal{R}}^+} (R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}}).$$

In other words, the outcome of the bargaining is going to be the point where the Pareto boundary $\bar{\mathcal{R}}^*$ has exactly one intersection point with a curve of the form $(R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}}) = c$ where c is a constant (chosen such that there is precisely one intersection point). Thus, given $\bar{\mathcal{R}}$ and $(R_1^{\text{NE}}, R_2^{\text{NE}})$ the Nash solution can in principle be found graphically. This is illustrated in Figures 2–3. In these figures we can also see that the competitive solution $(R_1^{\text{NE}}, R_2^{\text{NE}})$ is generally much inferior to the Nash bargaining solution $(R_1^{\text{NB}}, R_2^{\text{NB}})$.

Achieving the Nash bargaining solution will require the two systems to communicate in one way or another. (See also the paragraph of discussion at the end of Section I-A.) In this work we assume that there is a vehicle that facilitates such communication, for example via a standardized protocol specifically developed for resource bargaining. Note also that the Nash bargaining solution is only defined for convex outcome regions. Thus, it requires that the systems are willing to perform time-sharing using a pair of two different beamforming vectors as explained in Section II-A (see the discussion where $\bar{\mathcal{R}}$ was defined). However, a negotiation about time-sharing does not appear to be substantially more difficult than bargaining about beamforming vectors.

We finally remark on the notion of fairness versus Nash bargaining. If the Nash bargaining theory is applied with a threat point at the origin (corresponding to zero rates if no cooperation is reached, rather than using the equilibrium rates), then the NB solution will coincide with the outcome of the following maximization problem

$$\max_{(R_1, R_2) \in \bar{\mathcal{R}}^+} \log_2(R_1) + \log_2(R_2). \quad (15)$$

The solution to (15) is sometimes called a "proportional fair" allocation. (See [24] for an extensive discussion of the relation between Nash bargaining and proportional fair allocation.) However, it is important to stress that the Nash bargaining solution has nothing to do with "fairness" in general. Rather it is an attempt to predict what will happen if the players act strict rationally, i.e., they want to cooperate but nevertheless act with self-interest. Generally, a player who is already in a

TABLE I
BIMATRIX OF A TWO-PERSON MISO IFC GAME WITH A BINARY STRATEGY SPACE CONSISTING OF THE NE AND ZF.

Player 2 vs 1	NE ₁	ZF ₁
NE ₂	$(R_1^{\text{NE}}, R_2^{\text{NE}})$	$(\underline{R}_1, \underline{R}_2)$
ZF ₂	$(\overline{R}_1, \overline{R}_2)$	$(R_1^{\text{ZF}}, R_2^{\text{ZF}})$

TABLE II
REALIZATION OF MISO INTERFERENCE GAME IN NE OPTIMAL CONFIGURATION.

$(4, 5)$	$(1, 6)$
$(5, 2)$	$(3, 4)$

good position will gain more because he can be stronger in a negotiation (his threat is more effective). There are numerous examples in economics where the Nash bargaining solution would be considered unfair for most human observers [25].

C. A Reduced Game

To obtain some additional insight we consider the following two-person general-sum game in which the two systems can choose between playing the NE solution and the ZF solution.

In Table I, the rate \overline{R}_i corresponds to the outcome in which one system plays its minimax-optimal single-user strategy (NE) and the other system performs ZF:

$$\overline{R}_i = \log_2 \left(1 + \frac{\|h_{ii}\|^2}{\sigma^2} \right). \quad (16)$$

Similarly, the rate \underline{R}_i corresponds to the case in which system i performs ZF but the other system performs NE:

$$\underline{R}_1 = \log_2 \left(1 + \frac{h_{11}^H \Pi_{h_{12}}^\perp h_{11}}{\sigma^2 + \frac{|h_{22}^H h_{21}|^2}{\|h_{22}\|^2}} \right). \quad (17)$$

and similarly for \underline{R}_2 . Hence, we have the following inequality chain:

$$\overline{R}_i \geq \{R_i^{\text{NE}}, R_i^{\text{ZF}}\} \geq \underline{R}_i \quad \text{for } i = 1, 2.$$

We can distinguish between the following fundamentally different cases:

- 1) The ZF rates in Table I are lower than the NE rates for both systems. This is illustrated in Table II. Here the first row dominates over the second row, and the first column dominates over the second column. This means that the NE strategy is the optimal strategy, regardless of whether the systems want to cooperate.
- 2) The ZF rates of both systems are larger than the NE rates (see Table III for an example). This corresponds to the classical ‘‘prisoner’s dilemma’’ situation [21]. Here the NE strategy is the only stable outcome, but the ZF rates are better.
- 3) The ZF rate is larger than the NE rate for one of the systems, but not for the other one. This configuration is a mix of the two scenarios above. The NE is better for one player whereas the ZF is better for the other.

We can quantitatively characterize the high-SNR performance of the rate points in the reduced game, using the high-SNR offset concept from [26]. Denote the average throughput

TABLE III
REALIZATION OF MISO INTERFERENCE GAME IN ‘‘PRISONER’S DILEMMA’’ CONFIGURATION.

$(2, 4)$	$(1, 6)$
$(5, 2)$	$(4, 5)$

(as a function of the SNR, $\rho \triangleq \frac{1}{\sigma^2}$) by $C(\rho)$. Following [26], introduce the following two high SNR measures:

$$\begin{aligned} \mathcal{S}_\infty &= \lim_{\rho \rightarrow \infty} \frac{C(\rho)}{\log_2(\rho)} \quad \text{and} \\ \mathcal{L}_\infty &= \lim_{\rho \rightarrow \infty} \left(\log_2(\rho) - \frac{C(\rho)}{\mathcal{S}_\infty} \right). \end{aligned} \quad (18)$$

The measure \mathcal{S}_∞ is called the *high-SNR slope* and \mathcal{L}_∞ is called the *high-SNR power offset*. At high SNR, the average throughput behaves like

$$C(\rho) = \mathcal{S}_\infty \left(\frac{\rho|_{\text{dB}}}{3 \text{ dB}} - \mathcal{L}_\infty \right) + o(1)$$

We have the following results.

Corollary 1: The average achievable rate if both systems perform ZF has the following high-SNR characteristics:

$$\mathcal{S}_\infty^{\text{ZF}} = 1 \quad \text{and} \quad \mathcal{L}_\infty^{\text{ZF}} = \frac{\gamma}{\log(2)}.$$

Proof: Follows directly from Proposition 2 and the definition in (18) or from [26, Proposition 1] with $n = 1$. \square

Corollary 2: An upper bound on the average achievable rate if one system performs NE and the other performs ZF, i.e. \overline{R}_i , has the following high-SNR characteristics:

$$\overline{\mathcal{S}}_\infty = 1 \quad \text{and} \quad \overline{\mathcal{L}}_\infty = \frac{\log(n) - \Psi(n)}{\log(2)}.$$

Proof: The corollary is another application of [26, Proposition 1]. \square

Corollary 3: A lower bound on the average achievable rate in the ‘‘worst’’ case, when the user performs ZF but the other user performs NE satisfies

$$\lim_{\sigma^2 \rightarrow 0} E[\underline{R}_i] = \frac{1}{\log(2)}.$$

Proof: This follows from Proposition 4 and the observation that exponentially distributed random variables occur in the numerator as well as in the denominator of (17). \square

The high-SNR analysis in this section is summarized in Table IV. The analysis, together with the result in Proposition 4, indicate that the reduced two-person MISO IFC game with a binary strategy space ends up in the Prisoner’s dilemma configuration. This observation is interesting since it shows that efficient resource allocation on the MISO IFC will require cooperative strategies, even if the problem is simplified to comprise only the two (highly practical) beamforming modes ZF and NE.

IV. NUMERICAL RESULTS

We performed numerical experiments to gain insight into the phenomena analyzed in Sections II–III and the corresponding scaling laws. For a fixed channel realization, the rate region \mathcal{R} was approximated by setting $\mathbf{w}_i = [\alpha_i, \sqrt{1 - \alpha_i^2} e^{j\phi_i}]^T$ and then varying $\{\alpha_1, \alpha_2, \phi_1, \phi_2\}$ over the grid $[0, 1] \times [0, 1] \times$

TABLE IV
SUMMARY OF HIGH-SNR SLOPES \mathcal{S}_∞ , AND HIGH-SNR POWER OFFSETS \mathcal{L}_∞ (IF $\mathcal{S}_\infty > 0$), OR HIGH-SNR UPPER BOUNDS (IF $\mathcal{S}_\infty = 0$) FOR THE REDUCED MISO IFC GAME.

NE		
NE	Sys. 1	Sys. 2
	$\mathcal{S}_\infty = 0$ $E[R_1] \leq \frac{\Psi(n)+\gamma+1/n}{\log(2)}$	$\mathcal{S}_\infty = 0$ $E[R_2] \leq \frac{\Psi(n)+\gamma+1/n}{\log(2)}$
ZF	Sys. 1	Sys. 2
	$\mathcal{S}_\infty = 1$ $\mathcal{L}_\infty = \frac{\log(n)-\Psi(n)}{\log(2)}$	$\mathcal{S}_\infty = 0$ $E[R_2] \leq \frac{1}{\log(2)}$
ZF		
NE	Sys. 1	Sys. 2
	$\mathcal{S}_\infty = 0$ $E[R_1] \leq \frac{1}{\log(2)}$	$\mathcal{S}_\infty = 1$ $\mathcal{L}_\infty = \frac{\log(n)-\Psi(n)}{\log(2)}$
ZF	Sys. 1	Sys. 2
	$\mathcal{S}_\infty = 1$ $\mathcal{L}_\infty = \frac{\gamma}{\log(2)}$	$\mathcal{S}_\infty = 1$ $\mathcal{L}_\infty = \frac{\gamma}{\log(2)}$

$[-\pi, \pi] \times [-\pi, \pi]$ (in total 40^4 points were searched for each channel realization).⁴ The Nash equilibrium and the rate points discussed in Section II-C are easy to compute. We found the Nash bargaining solution numerically by an interval-halving type search.

Illustrations of typical regions were given in Figures 2–3. We also computed the average rates in i.i.d. Rayleigh fading. The computation was accomplished by numerical averaging over 2000 channel realizations. The result is shown in Figure 4. This figure confirms the conclusion of Proposition 4, regarding the high-SNR behavior of the Nash equilibrium. We can also see that all other rates grow with the SNR. Most interestingly, the Nash bargaining solution is about as good as zero-forcing, and it is not far from the sum-rate point. (Neither of the two latter points would be achievable by voluntary bargaining, unless enforced by regulations.) This observation forms one of our major empirical conclusions.

V. EXTENSION TO SIC UNDER STRONG INTERFERENCE

If the received interference is strong at one of the receivers, this receiver may use successive interference cancellation (SIC). More precisely, it can decode the message intended for the other user first and then subtract it from the received signal before decoding the information of interest [5]. Arguably schemes based on SIC are somewhat impractical since even if implementation issues such as timing- and frequency synchronization (at very low signal-to-interference ratios) could be solved, they would require that the systems know the coding and modulation formats of each other. Nevertheless, it is interesting to investigate whether application of SIC would fundamentally change the conclusions of Sections III–IV. To explore this, we make the following two observations.

Proposition 5: The Nash equilibrium in Proposition 3 is unchanged if SIC is used.

Proof: If a mobile performs SIC, then this will only affect the noise-and-interference term of its received signal (i.e.,

⁴Note that \mathbf{w}_i can be rotated by an arbitrary complex phase at no change in rate, so it is enough to use two real-valued parameters to parameterize each beamforming vector.

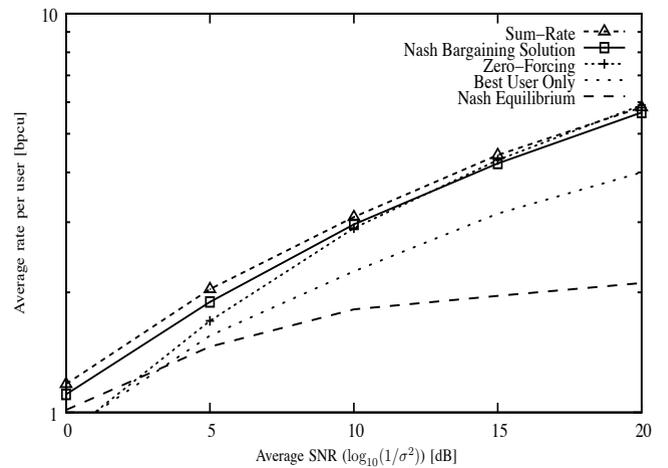


Fig. 4. Average rates for the 2-user MISO IFC with $n = 2$ transmit antennas at the base stations, in the symmetric scenario (all channels i.i.d. Rayleigh fading).

the terms in the denominator in (2) and (3)). Therefore the arguments in the proof of Proposition 3 will directly translate. \square

Proposition 6: Consider the two-user MISO IFC in Figure 1 and define the squared effective channel gain a_{ij} from BS_{*i*} to MS_{*j*}, after power allocation and beamforming, as $a_{ij} = |\mathbf{w}_i^T \mathbf{h}_{ij}|^2$. (That is, $a_{11} = |\mathbf{w}_1^T \mathbf{h}_{11}|^2$, $a_{22} = |\mathbf{w}_2^T \mathbf{h}_{22}|^2$, $a_{12} = |\mathbf{w}_1^T \mathbf{h}_{12}|^2$, and $a_{21} = |\mathbf{w}_2^T \mathbf{h}_{21}|^2$.) Then:

- (a) If $\frac{a_{21}}{a_{11} + \sigma^2} > \frac{a_{22}}{a_{12} + \sigma^2}$ then MS₁ can perform SIC, i.e., MS₁ can decode the message intended for MS₂ and subtract this perfectly (but MS₂ cannot do this in general). The following rates are achievable:

$$R_1 = \log_2 \left(1 + \frac{a_{11}}{\sigma^2} \right) \quad \text{and} \quad R_2 = \log_2 \left(1 + \frac{a_{22}}{a_{12} + \sigma^2} \right).$$

- (b) If $\frac{a_{12}}{a_{22} + \sigma^2} > \frac{a_{11}}{a_{21} + \sigma^2}$ then MS₂ can perform SIC (but MS₁ cannot in general). The following rates are achievable:

$$R_1 = \log_2 \left(1 + \frac{a_{11}}{a_{21} + \sigma^2} \right) \quad \text{and} \quad R_2 = \log_2 \left(1 + \frac{a_{22}}{\sigma^2} \right).$$

- (c) If $\frac{a_{21}}{a_{11} + \sigma^2} > \frac{a_{22}}{\sigma^2}$ and $\frac{a_{12}}{a_{22} + \sigma^2} > \frac{a_{11}}{\sigma^2}$ then both MS₁ and MS₂ can simultaneously perform SIC. The following rates are achievable:

$$R_1 = \log_2 \left(1 + \frac{a_{11}}{\sigma^2} \right) \quad \text{and} \quad R_2 = \log_2 \left(1 + \frac{a_{22}}{\sigma^2} \right),$$

respectively.

If none of the conditions in (a)–(c) are satisfied, then the achievable rates are given by (2)–(3): $R_1 = \log_2 \left(1 + \frac{a_{11}}{a_{21} + \sigma^2} \right)$ and $R_2 = \log_2 \left(1 + \frac{a_{22}}{a_{12} + \sigma^2} \right)$.

Proof: Case (a) is clear because MS₁ must be able to decode the signal intended for MS₂ in the presence of noise with power σ^2 , treating the signal intended for MS₁ as interference (this has power a_{11}). Case (b) follows similarly. Case (c) also follows by similar reasoning, but here, the interfering signals have much higher rate since it is assumed that both MS₁ and MS₂ can do SIC. \square

Note that the conditions (a) and (b) in Proposition 6 do not generally imply the condition (c). That is, the conditions (a) and (b) may be satisfied, but this does not mean that

both MS₁ and MS₂ can do SIC simultaneously. If both (a) and (b) are satisfied, then one must choose whether MS₁ or MS₂ should be allowed to do SIC (and communicate with the correspondingly higher rate). This leads to a new type of conflict situation, since it is not clear whether the systems would easily agree on who will get the benefit from the SIC. Thus the achievable rates, in the event that both conditions in (a) and (b) are satisfied, are not well defined. Based on this observation, we construct the following bounds on the achievable rate (from the perspective of MS₁):

- *Upper bound* (from MS₁'s perspective): If condition (c) is satisfied, then both MS₁ and MS₂ do SIC. Otherwise, check if (a) is satisfied, and if so let MS₁ do SIC (but not MS₂). If not, then MS₁ does not perform SIC. (Whether MS₂ performs SIC, i.e., whether condition (b) is satisfied, is irrelevant here since the rate bound concerns only MS₁.)
- *Lower bound* (from MS₁'s perspective): If condition (c) is satisfied, then both MS₁ and MS₂ do SIC. Otherwise, MS₁ does not perform SIC. (If (b) is satisfied, then MS₂ may perform SIC, but this does not affect the bound for MS₁.)

We illustrate the bounds on the achievable equilibrium rates with SIC in Figure 5 (average over 10⁵ channel realizations). We can see that if the systems agree to let MS₁ perform SIC whenever it is possible (hence forcing MS₂ to sacrifice any possible benefit of SIC), then the achievable rate for MS₁ indeed appears to grow unbounded with the transmit power. (This is the “upper bound” in the figure.) Conversely, if the agreement instead is to let MS₂ perform SIC whenever possible but deprive MS₁ of this possibility, then MS₁ will have the opportunity to do SIC only if condition (c) is satisfied (i.e., when both mobiles can simultaneously do SIC). This is not enough to bring a significant increase in the achievable rate; in fact, the rate appears to be bounded regardless of power. (This is the “lower bound” in the figure.)

To conclude, application of SIC can potentially provide a significant increase in the equilibrium rates. But this requires that the systems can agree on who (MS₁ or MS₂) should be allowed to do SIC. This conflict situation might be modeled and analyzed as a game in itself. However, even the most positive outcome of this conflict game (corresponding to the upper bound in Figure 5) stands short of the bargaining solution to the original beamforming problem (see the “Nash bargaining” curve in Figure 4). Based on this one might conjecture that SIC will not make a fundamental difference to the way one should view conflict games on the MISO interference channel.

VI. CONCLUSIONS

In this paper we have considered the conflict situation that arises when two multiple-antenna systems must share the same (unlicensed) spectrum band. We have made two central points. First, we showed that if the systems do not cooperate, then the corresponding equilibrium rates are bounded regardless of how much transmit power the base stations have available. The important consequence of this is that there is a fundamental need for base station (system) cooperation in spectrum sharing

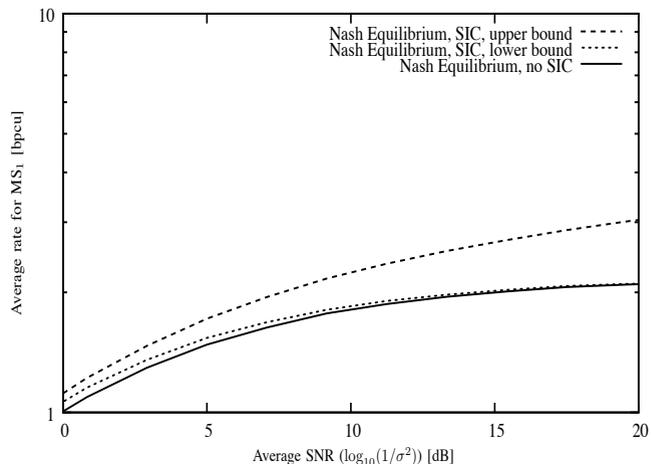


Fig. 5. Upper and lower bounds on the average rates with and without SIC at the Nash equilibrium, for the 2-user MISO IFC with $n = 2$ transmit antennas with at the base stations, in the symmetric scenario (all channels i.i.d. Rayleigh fading). (The scale is the same as in Figure 4.)

with multiple antennas. Second, in numerical experiments, we found that the outcome of a Nash bargaining between the two systems can on the average be close to the sum-rate bound. This indicates that in reality, selfish but cooperating systems may achieve close to the max-sum-rate performance. It remains to develop protocols that the systems can use to communicate and actually reach the bargaining agreements whose existence we have predicted theoretically.

The work here can be extended in several directions. One may study the case where only partial (for example, long-term) channel state information is available. The theory may also be extended to multiple antennas at the receivers (i.e., MIMO). Another direction concerns the specific choice of cooperative game-theoretic axiomatic framework. We have chosen the Nash bargaining theory because it is well-established, and since it enabled us to compute the bargaining point numerically, with relative ease. Other approaches to cooperative games (e.g., λ -transfer theory) may also be possible.

A number of open problems remain. For example, numerical results indicate that the rates of the bargaining solution grow logarithmically with SNR, although we do not have an analytical proof at this point. Also, a more precise characterization of the ZF solution and its distance to the Pareto boundary may be of interest. We leave these issues for future work.

APPENDIX

Proof of Proposition 1: We need to show the existence of a nonzero perturbation vector δ such that

$$|\mathbf{w}_1^T \mathbf{h}_{11} + \delta^T \mathbf{h}_{11}|^2 > |\mathbf{w}_1^T \mathbf{h}_{11}|^2 \quad (19)$$

$$\|\mathbf{w}_1 + \delta\|^2 > \|\mathbf{w}_1\|^2 \quad (20)$$

$$|\mathbf{w}_1^T \mathbf{h}_{12}|^2 = |\mathbf{w}_1^T \mathbf{h}_{12} + \delta^T \mathbf{h}_{12}|^2 \quad (21)$$

$$\|\mathbf{w}_1 + \delta\|^2 \leq 1. \quad (22)$$

A sufficient (but not necessary) condition to satisfy (21) is that δ has the form

$$\delta = \alpha e^{j\phi} \mathbf{h}_{12}^\perp \quad (23)$$

where α is real-valued and strictly positive, $\phi \in \mathbb{R}$ and \mathbf{h}_{12}^\perp is an arbitrary vector that satisfies $\mathbf{h}_{12}^{\perp H} \mathbf{h}_{12} = 0$ and $\|\mathbf{h}_{12}^\perp\| = 1$. (Such a vector \mathbf{h}_{12}^\perp always exists although it is not unique.)

If δ has the form of (23) then condition (20) is equivalent to

$$\begin{aligned} & \|\mathbf{w}_1 + \alpha e^{j\phi} \mathbf{h}_{12}^\perp\|^2 > \|\mathbf{w}_1\|^2 \\ \Leftrightarrow & \alpha^2 + 2\alpha \operatorname{Re} \left\{ \mathbf{w}_1^H \mathbf{h}_{12}^\perp e^{j\phi} \right\} > 0 \\ \Leftrightarrow & \operatorname{Re} \left\{ e^{j\phi} \mathbf{w}_1^H \mathbf{h}_{12}^\perp \right\} > -\frac{\alpha}{2} \\ \Leftrightarrow & |\mathbf{w}_1^H \mathbf{h}_{12}^\perp| \cos(\phi + \rho_1) > -\frac{\alpha}{2} \\ \Leftrightarrow & \cos(\phi + \rho_1) > -\frac{1}{2|\mathbf{w}_1^H \mathbf{h}_{12}^\perp|} \alpha \end{aligned} \quad (24)$$

where $\rho_1 \triangleq \arg(\mathbf{w}_1^H \mathbf{h}_{12}^\perp)$.

At the same time, if δ has the form of (23) then condition (19) can be written

$$\begin{aligned} & |\delta^T \mathbf{h}_{11}|^2 + 2\operatorname{Re} \left\{ \delta^T \mathbf{h}_{11} \mathbf{h}_{11}^H \mathbf{w}_1^* \right\} > 0 \\ \Leftrightarrow & \alpha^2 |\mathbf{h}_{12}^{\perp T} \mathbf{h}_{11}|^2 + 2\alpha \operatorname{Re} \left\{ e^{j\phi} \mathbf{h}_{12}^{\perp T} \mathbf{h}_{11} \mathbf{h}_{11}^H \mathbf{w}_1^* \right\} > 0 \\ \Leftrightarrow & \frac{\alpha}{2} |\mathbf{h}_{12}^{\perp T} \mathbf{h}_{11}| + |\mathbf{h}_{11}^H \mathbf{w}_1^*| \cos(\phi + \rho_2) > 0 \\ \Leftrightarrow & |\mathbf{h}_{11}^H \mathbf{w}_1^*| \cos(\phi + \rho_2) > -\frac{\alpha}{2} |\mathbf{h}_{12}^{\perp T} \mathbf{h}_{11}| \\ \Leftrightarrow & \cos(\phi + \rho_2) > -\frac{|\mathbf{h}_{12}^{\perp T} \mathbf{h}_{11}|}{2|\mathbf{h}_{11}^H \mathbf{w}_1^*|} \alpha \end{aligned} \quad (25)$$

where $\rho_2 \triangleq \arg(\mathbf{h}_{12}^{\perp T} \mathbf{h}_{11}) + \arg(\mathbf{h}_{11}^H \mathbf{w}_1^*)$.

We need to show that one can choose α and ϕ such that (22), (24) and (25) are satisfied. Take

$$\alpha = \frac{1 - \|\mathbf{w}\|}{2} \quad (26)$$

Then (22) is satisfied.⁵ With this choice of α , the right hand side of (24) is negative. Thus, there exists an angular range $[\theta_1, \theta_2]$ for which (24) is satisfied if $\phi \in [\theta_1, \theta_2]$. Also, this range is strictly wider than π , i.e., $\theta_2 - \theta_1 > \pi$. Similarly, there exists an angular range $[\psi_1, \psi_2]$ such that (25) is satisfied if $\phi \in [\psi_1, \psi_2]$ and this range is strictly wider than π as well. Therefore, the intersection of these two regions must form a new angular region $[\theta_1, \theta_2] \cap [\psi_1, \psi_2]$ which has a nonzero size. Hence, by taking ϕ to lie in the angular region $[\theta_1, \theta_2] \cap [\psi_1, \psi_2]$, and α according to (26), conditions (19)–(22) are satisfied.

To see that the statement holds also for the convex hull, suppose there was a point (R_1, R_2) on the boundary of $\bar{\mathcal{R}}$ which could be reached with less than maximal power. This point must then be a convex combination of two rate points, which lie on the boundary of \mathcal{R} . But this is a contradiction since no point on the boundary of \mathcal{R} can be reached with less than full power.

⁵To see this, note the following inequality:

$$\|\mathbf{w}_1 + \delta\| \leq \|\mathbf{w}_1\| + \|\delta\| = \|\mathbf{w}_1\| + \frac{1 - \|\mathbf{w}_1\|}{2} = \frac{1}{2} + \frac{\|\mathbf{w}_1\|}{2} \leq 1$$

Proof of Proposition 2: We are interested in the statistics of

$$|\mathbf{w}_1^T \mathbf{h}_{11}|^2 = \left| \frac{(\Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11})^H}{\|\Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11}\|} \mathbf{h}_{11} \right|^2 = \mathbf{h}_{11}^H \Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11}.$$

Let \mathbf{U} be an $n \times n - 1$ semi-unitary matrix that satisfies $\mathbf{U}\mathbf{U}^H = \Pi_{\mathbf{h}_{12}}^\perp$. Note that \mathbf{U} is a function of \mathbf{h}_{12} but independent of \mathbf{h}_{11} . Since \mathbf{h}_{11} has a rotationally invariant distribution, $\mathbf{U}^H \mathbf{h}_{11}$ is a vector of length $n - 1$ with i.i.d. zero mean, unit-variance Gaussian elements. Hence the random variable

$$x \triangleq \mathbf{h}_{11}^H \Pi_{\mathbf{h}_{12}}^\perp \mathbf{h}_{11} = \|\mathbf{U}^H \mathbf{h}_{11}\|^2$$

has a χ_{n-1}^2 distribution. Therefore, by applying the results in [20] we find that the average rate per user is

$$E_x \left[\log \left(1 + \frac{x}{\sigma^2} \right) \right] = \mathcal{G}(\sigma^2, n - 1). \quad (27)$$

Proof of Proposition 4: First, consider the probability distribution of the expression in the denominator of (14). Define the random variable

$$t \triangleq \frac{|\mathbf{h}_{22}^H \mathbf{h}_{21}|^2}{\|\mathbf{h}_{22}\|^2}.$$

Note that

$$t = \left| \frac{\mathbf{h}_{22}^H}{\|\mathbf{h}_{22}\|} \mathbf{U} \mathbf{U}^H \mathbf{h}_{21} \right|^2$$

for any unitary matrix \mathbf{U} . Now choose \mathbf{U} as a function of \mathbf{h}_{22} , such that

$$\frac{\mathbf{h}_{22}^H}{\|\mathbf{h}_{22}\|} \mathbf{U} = [1, 0, \dots, 0].$$

Since \mathbf{h}_{21} is isotropically distributed by assumption, $\mathbf{U}^H \mathbf{h}_{21}$ has the same statistics as \mathbf{h}_{21} . Thus t is standard exponentially distributed

$$t = |h_{21,1}|^2 \sim \exp(-t)$$

(Here $h_{21,1}$ denotes the first entry of the channel vector \mathbf{h}_{21} .) The individual user rates are then given by

$$E_{x,y} \left[\log_2 \left(1 + \frac{x}{y + \sigma^2} \right) \right]$$

where x is independent of y , x is χ^2 distributed with n complex degrees of freedom, i.e. $p(x) = x^{n-1} \exp(-x) \Gamma(n)^{-1}$ and y is standard exponentially distributed, i.e. $p(y) = \exp(-y)$. First, we evaluate the expectation with respect to y for $\sigma^2 \rightarrow 0$ to obtain

$$\begin{aligned} & E_{x,y} \left[\log_2 \left(1 + \frac{x}{y} \right) \right] \\ &= E_x \left[\frac{1}{\log(2)} (\log(x) + \gamma + e^x \operatorname{Ei}(1, x)) \right] \end{aligned} \quad (28)$$

where γ is Euler's constant and $\operatorname{Ei}(1, x)$ denotes the exponential integral. Finally, computing the expectation with respect to x gives

$$\begin{aligned} E[R_i^{ZF}] &\leq E_x \left[\frac{1}{\log(2)} (\Psi(n) + \gamma + e^x \operatorname{Ei}(1, x)) \right] \\ &= \frac{\Psi(n) + \gamma + 1/n}{\log(2)}. \end{aligned} \quad (29)$$

REFERENCES

- [1] R. Ahlswede, "The capacity region of a channel with two senders and two receivers," *Ann. Prob.*, vol. 2, pp. 805–814, Oct. 1974.
- [2] A. B. Carleial, "Interference channels," *IEEE Trans. Inform. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.
- [3] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inform. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.
- [4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge University Press, 2005.
- [5] M. H. M. Costa, "On the Gaussian interference channel," *IEEE Trans. Inform. Theory*, vol. 31, pp. 607–615, Sept. 1985.
- [6] X. Shang, B. Chen, and M. J. Gans, "On the achievable sum rate for MIMO interference channels" *IEEE Trans. Inform. Theory*, vol. 52., no. 9, pp. 4313–4320, Sept. 2006.
- [7] S. A. Jafar and M. Fakhreddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inform. Theory*, vol. 53, no. 7, pp. 2637–2642, July 2007.
- [8] A. MacKenzie and L. DaSilva, *Game Theory for Wireless Engineers*, Morgan & Claypool Publishers, 2006.
- [9] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, second edition, SIAM, 1998.
- [10] W. Yu, W. Rhee, S. Boyd and J. Cioffi, "Iterative water-filling for Gaussian vector multiple access channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 145–151, Jan. 2004.
- [11] J. Huang, R. A. Berry and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 1074–1084, May 2006.
- [12] R. Etkin, A. Parekh and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Select. Areas Commun.*, vol. 25, pp. 517–528, Apr. 2007.
- [13] G. Arslan, M. F. Demirkol, and Y. Song, "Equilibrium efficiency improvement in MIMO interference systems: a decentralized stream control approach," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2984–2993, Aug. 2007.
- [14] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband non-cooperative systems based on game-theory - Part I: Nash equilibria," *IEEE Trans. Signal Processing*, vol. 56, pp. 1230–1249, Mar. 2008.
- [15] A. Leshem and E. Zehavi, "Bargaining over the interference channel," *Proc. IEEE ISIT*, pp. 2225–2229, July 2006.
- [16] J. E. Suris, L. A. DaSilva, Z. Han, and A. B. MacKenzie, "Cooperative game theory for distributed spectrum sharing," in *Proc. IEEE ICC*, 2007.
- [17] R. J. La and V. Anantharam, "A game-theoretic look at the Gaussian multiaccess channel," *DIMACS series in discrete mathematics and theoretical computer science*, 2003.
- [18] J. Kovacevic, "How to encourage and publish reproducible research," in *Proc. IEEE ICASSP*, May 2007.
- [19] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*, Cambridge University Press, 2003.
- [20] M.-S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques" *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1165–1181, July 1999.
- [21] G. Owen, *Game Theory*, Academic Press 1995 (3rd edition).
- [22] P. Dubey, "Inefficiency of Nash equilibria," *Math. Oper. Res.*, vol. 11, no. 1, pp. 1–8, 1986.
- [23] J. Nash, "The bargaining problem," *Econometrica*, vol. 18, pp. 152–162, 1950.
- [24] M. Schubert and H. Boche, "Properties and operational characterization of proportionally fair resource allocation," in *Proc. IEEE SPAWC*, 2007.
- [25] G. Rabow, "The social implications of non-zero-sum games," *IEEE Technology and Society Magazine*, pp. 12–18, March 1988.
- [26] A. Lozano, A. M. Tulino, and S. Verdú, "High-SNR power offset in multiantenna communication," *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4134–4151, Dec. 2005.



Erik G. Larsson is Professor and Head of the Division for Communication Systems in the Department of Electrical Engineering (ISY) at Linköping University (LiU) in Linköping, Sweden (www.commsys.isy.liu.se). He joined LiU in September 2007. He has previously been Associate Professor (Docent) at the Royal Institute of Technology (KTH) in Stockholm, Sweden, and Assistant Professor at the University of Florida and the George Washington University, USA.

His main professional interests are within the areas of wireless communications and signal processing. He has published some 50 papers on these topics, he is co-author of the textbook *Space-Time Block Coding for Wireless Communications* (Cambridge Univ. Press, 2003) and he holds 10 patents on wireless technology.

He is Associate Editor for the *IEEE Transactions on Signal Processing* and the *IEEE Signal Processing Letters* and a member of the IEEE Signal Processing Society SAM and SPCOM technical committees.



Eduard A. Jorswieck was born in 1975 in Berlin, Germany. He received his Diplom-Ingenieur (M.S.) degree and Doktor-Ingenieur (Ph.D.) degree, both in electrical engineering and computer science from the Technische Universität Berlin, Germany, in 2000 and 2004, respectively. In 2008, he accepted a position as the Head of the Chair of Communication Theory (www.ifn.et.tu-dresden.de/tnt) and Full Professor at Technische Universität Dresden (TUD), Germany. He has previously been with the Fraunhofer Institute for Telecommunications, Heinrich-

Hertz-Institut (HHI) Berlin, in the Broadband Mobile Communication Networks Department from 2001 to 2006. In 2006, he joined the Signal Processing Department at the Royal Institute of Technology (KTH) as post-doc and in 2007 as research associate.

Eduard's main research interests are in signal processing for communications and networks, communication theory, and applied information theory. He has published over 100 papers in these fields, he is co-author of the monograph *Majorization and Matrix Monotone Functions in Wireless Communications* (Now publishers, 2007), and holds three patents. He is member of the IEEE SPCOM Technical Committee. In 2006, he was co-recipient of the IEEE Signal Processing Society Best Paper Award.