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Optimal Scheduling and QoS Power Control for Cognitive Underlay Networks

(Invited Paper)

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Abstract—We study the NP-hard problem of scheduling and power control with quality-of-service (QoS) constraints. We consider a generic wireless network comprising K mutually interfering links and $N < K$ orthogonal time or frequency slots. We formulate the joint resource allocation problem as a constrained optimization problem, specifically, as a mixed-integer programming (MIP) problem. This enables us to solve the problem *exactly*, and relatively efficiently for the vast majority of instances, using off-the-shelf algorithms. We also apply our formulation to the paradigm of cognitive underlay networks.

I. INTRODUCTION

The motivation for this work is the increased sophistication of wireless communication systems envisioned for the future. For example, we witness a number of emerging applications, such as streaming video and interactive gaming, which bring high requirements on the quality-of-service (QoS). This puts additional demands on the resource allocation algorithms used in the network. Another trend is that links that belong to different infrastructure (operators) and that exist under different regulatory constraints (e.g., both licensed and unlicensed operation) may coexist in the same spectrum. Collectively, the envisioned technology that will be able to offer such coexistence of different links in the same spectrum is often called *cognitive radio*. Cognitive radio will require a wide range of new enabling technology and algorithms, which includes, among others, algorithms for optimally solving scheduling and power control problems.

The fundamental aspects of resource allocation in wireless networks can be understood by considering a generic model comprising K transmitter-receiver pairs. The transmissions on the K links under study take place concurrently on the same channel. Due to the broadcast nature of the wireless medium there is coupling between the transmitter-receiver pairs. The effect is that each receiver listens to a superposition of the desired signal and of all the other $K - 1$ transmitted signals, which constitute interference. The communication quality of the k th link can be quantified via the received signal-to-

interference-and-noise ratio (SINR)

$$\text{SINR}_k \triangleq \frac{G_{kk}p_k}{\sum_{\ell \neq k} G_{\ell k}p_\ell + V_k}. \quad (1)$$

In (1), p_k is the transmit power used on the k th link, $G_{\ell k}$ is the gain of the channel between the ℓ th transmitter and the k th receiver, and V_k is the variance of the noise (assumed to be AWGN throughout). The channel gains include the effects of propagation loss, shadowing and fading.

We assume that the receivers treat the interference as noise. Hence, the maximum achievable rate, that a link can support, is dictated by the SINR. In practice, a rate requirement, that is imposed by an application, can be directly translated to an SINR requirement, for a given modulation/coding scheme and a given target bit-error rate. Thus, QoS can be guaranteed to the k th link when the SINR_k exceeds a predetermined threshold T_k . In order to ensure this condition, the physical-layer protocol may adjust the transmit power p_k up to a bound P , which is typically determined by regulatory and/or hardware constraints. However, while boosting p_k increases SINR_k in (1), it reduces at the same time $\text{SINR}_\ell \forall \ell \neq k$. Hence, the transmission powers need to be determined jointly. This can be accomplished by the following QoS power control problem

$$\begin{aligned} \min_{\{p_k \in [0, P]\}_{k=1}^K} & \sum_{k=1}^K p_k & (2) \\ \text{s.t.} & \text{SINR}_k \geq T_k \quad \forall k \in \mathcal{K}, & (3) \end{aligned}$$

where SINR_k is defined in (1) and $\mathcal{K} \triangleq \{1, \dots, K\}$ is the set of all direct links. The objective function in (2) strives to minimize the total transmission power subject to a QoS constraint (3) for each link. This minimizes the overall interference emitted by the network, and at the same time prolongs the operating lifetime of energy-starved transmitters.

Problem (2)–(3) has been extensively studied in the past. From an optimization viewpoint, it is a linear programming (LP) problem. Hence, the optimum solution can be efficiently found by the simplex method or the interior-point algorithms, provided that there exists a central controller which knows all the channel gains $\{G_{\ell k}\}$ and QoS requirements $\{T_k\}$. Owing

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to the linearity of the problem, even in the absence of a central controller efficient algorithms have been proposed to find the optimum solution in a distributed manner [1], [2].

Note that (2)–(3) may be infeasible. This typically happens when the requested SINR thresholds $\{T_k\}$ are large or when the coupling channel gains $\{G_{\ell k}\}_{\ell \neq k}$ are large relative to the corresponding direct link gains $\{G_{kk}\}$. If the QoS constraints for all K links cannot be simultaneously satisfied by power control, the different links need to be scheduled to more than one orthogonal degree of freedom (DoF) (for example, time slots or frequency channels), in order to decrease the overall interference level. Algorithms available to date take such access-control decisions based mostly on the channel gains and disregard valuable insights gained by the attempt to solve the QoS power control problem on the physical layer. Significant improvements are expected by a cross-layer approach to the resource (joint scheduling and power) allocation problem. Previous, related results in this direction include the joint power and admission control¹ problems considered in [4], [5]. Therein, modifications to the distributed power control algorithm of [1] are proposed to determine whether another link can be served in the same DoF without yielding the problem infeasible.

A different, more disciplined, approach is proposed in [6]–[8]. This line of work follows a common methodology. Initially, the two-layer resource allocation problem of interest is formulated as a joint optimization problem, which is inherently NP-hard. The trick that makes this possible is the introduction of auxiliary binary variables to model the scheduling decisions. Then, the original (intractable) optimization problem is relaxed to a convex (tractable) one. Finally, the convex problem is used in the core of iterative heuristic algorithms. These algorithms are solved centrally and provide high-quality suboptimal solutions with polynomially-bounded worst-case complexity. The case of admission control (one DoF) is treated in [6] and [7], jointly with beamforming and power control, respectively. The joint formulations are relaxed to semidefinite programming problems. The general case of scheduling (many DoF) and power control is treated in [8]. Therein, the joint formulation is relaxed to a geometric programming problem, which is effectively convex.

Here, we revisit the problem that was considered in [8], for a slightly different scenario, and propose an alternative formulation. The key difference is that the advocated formulation maintains the linearity of the original QoS power control problem (2)–(3). Specifically, we model the joint scheduling and QoS power control problem as a so-called mixed-integer programming (MIP) problem. MIP problems are NP-hard, but they can be efficiently solved, in most instances, by means of branch-and-bound techniques. Hence, contrary to the approach in [6]–[8], the formulation we propose here enables us to find the optimal solution, yet at the cost of occasional high complexity.

¹Some authors refer to this problem as scheduling. Herein, we reserve the term scheduling for the explicit allocation of links to DoF.

II. JOINT SCHEDULING AND POWER CONTROL

The fundamental question is how to *jointly* allocate DoF and power *optimally*, in order to minimize the overall interference and maximize the number of links that can be served. To state this problem formally, we assume that there are $N < K$ available orthogonal DoF and denote the set of their indexes as $\mathcal{N} \triangleq \{1, \dots, N\}$. The main assumption is that the K links requesting service can be potentially scheduled to any DoF. We denote the transmission powers and the channel gains in the n th DoF as $\{p_k^n\}_{k=1}^K$ and $\{G_{\ell k}^n\}_{\ell, k=1}^K$, respectively. The SINR that the k th receiver experiences when tuned to the n th DoF is then equal to

$$\text{SINR}_k^n \triangleq \frac{G_{kk}^n p_k^n}{\sum_{\ell \neq k} G_{\ell k}^n p_\ell^n + V_k}. \quad (4)$$

We say that the k th link is *assigned* to the n th DoF when there exist feasible powers $\{p_k^n\}_k$ such that $\text{SINR}_k^n \geq T_k$. We call the k th link *served* or *admitted* when it is assigned to some DoF. The problem is then to find the optimum (i) scheduling, i.e., assignment of links to DoF; and (ii) transmission powers, that maximize the number of admitted receivers and minimize the total transmission power required to serve them.

In order to solve the joint QoS problem, we formulate it as a constrained optimization problem. To proceed, we introduce the auxiliary binary variables $\{s_k^n \in \{0, 1\}\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$, one per DoF and link. Each binary variable s_k^n models the following scheduling question: Can the k th link be assigned to the n th DoF? The answer is “yes” when $s_k^n = 1$ and “no” otherwise. It is evident that the primary goal of the optimization should be to maximize the number of positive answers. Hence, using the auxiliary variables $\{s_k^n\}$, we formulate the problem as

$$\max_{\substack{p_k^n \in [0, P] \\ s_k^n \in \{0, 1\}}}^{n \in \mathcal{N}} \sum_{k=1}^K \sum_{n=1}^N s_k^n - W \sum_{k=1}^K \sum_{n=1}^N p_k^n \quad (5)$$

subject to

$$\frac{G_{kk}^n p_k^n + M(1 - s_k^n)}{\sum_{\ell \neq k} G_{\ell k}^n p_\ell^n + V_k} \geq T_k \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (6a)$$

$$p_k^n - P s_k^n \leq 0 \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (6b)$$

$$\sum_{n=1}^N s_k^n \leq 1 \quad \forall k \in \mathcal{K}. \quad (6c)$$

In what follows, we will explain the role of each equation in (5)–(6), starting the discussion with the conditions (6) and concluding it with the motivation of the objective function (5). In the process, we will also elaborate on the operational meaning of the (yet undefined) nonnegative, real, scalar parameters W and M that appear in (5) and (6a).

Equation (6a) defines N SINR constraints, one per available DoF, for each of the K links. Effectively, the binary variable s_k^n acts as a switch that determines whether the (k, n) th inequality of (6a) is active in the power control problem. When $s_k^n = 1$, the (k, n) th inequality falls back to defining a standard SINR constraint. When $s_k^n = 0$, the (k, n) th inequality does not

impose any constraint on $\{p_k^n\}_k$, provided that M is large enough to satisfy the inequality for all feasible values of $\{p_k^n\}_k$.

Hence, we need to choose the parameter M so that all KN inequalities (6a) are fulfilled when $\{s_k^n = 0\}$, irrespective of the values for $\{p_k^n\}$. It suffices to consider the worst-case scenario, where all the interfering transmitters use full power, whereas the transmitter in the direct link is silent. Setting $\{p_\ell^n = P\}_{\ell \neq k}$ and $p_k^n = 0$ in each inequality in (6a), and selecting the maximum resulting lower bound we have

$$M \geq \max_{k,n} T_k \sum_{\ell \neq k} G_{\ell k}^n P + T_k V_k. \quad (7)$$

Note that with $s_k^n = 0$, the optimization (5)–(6) will yield $p_k^n = 0$. This is so because then p_k^n appears only as interference in the denominator of the “active” SINR constraints in (6a), while the second term of the objective function (5) seeks to minimize the total transmission power. Hence, there is no need to explicitly account for the links that do not transmit in the denominator of SINR_k^n .

Equation (6b) is a technical condition that is, strictly speaking, redundant in the sense that the optimization problem will yield the same solution without this constraint. When $s_k^n = 1$, (6b) basically restates the power constraint, and when $s_k^n = 0$, it effectively sets the optimal p_k^n to zero. This will however be the result even without requiring it explicitly in a separate constraint, as discussed above. While the constraint (6b) does not affect the optimal solution, including it may speed up the algorithms used to find a numerical solution.

Equation (6c) makes sure that each link is assigned to at most one DoF. Since QoS is already guaranteed to the link when just one out of N respective constraints (6a) is active, multiple assignments would solely increase the interference in the wireless network. If there are extra DoF (i.e., other feasible assignments), the system would rather utilize them to serve more links or to decrease the total transmission power.

Note that by letting the sum in (6c) take on values smaller than 1 (actually 0), admission control functionality is included in the joint optimization problem (5)–(6). This means that when there are not enough resources, the system may deny service to some links in order to ensure service to the remaining ones. For a denied link k , we get $\{s_k^n = 0\}^n$. This insight leads to the following result.

Claim 1. *Optimization problem (5)–(6) is always feasible.*

Proof: A trivial feasible solution is always $\{s_k^n = 0\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ for any $\{p_k^n \in [0, P]\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$. However, this is the worst possible solution from QoS perspective, since it corresponds to the case that none of the K links is served. ■

If the inequalities (6c) were replaced with equalities, the resulting problem would be a restriction of (5)–(6), since the set of feasible solutions would be a subset of the original one. This restricted problem would become infeasible when it is impossible to admit all links.

The objective function in (5) is a sum of two terms and the second term is scaled with a weight $W \geq 0$. The first

term effectively counts the number of links served; hence, it rewards solutions that provide service to many users. The second term represents the total amount of power spent. Due to the minus sign, it penalizes solutions that are power-inefficient. By choosing W , we can trade off between the two conflicting objectives of saving power, and serving many links. In the special case that $W = 0$, we obtain the solution that maximizes the number of served users, subject to the power constraints.

Remarkably, exploiting the special properties of the two terms, we can simultaneously achieve the best of both objectives, by fine-tuning the parameter W . This is possible because the first term is discrete (with step size 1) and the second term is bounded by KP (when all K transmitters use maximum power P). Hence, adapting the ruler analogy argument of [6, p. 2684], we can ensure that the objectives do not overlap when W is chosen such that $1 > WKP$. The interpretation of this choice is that the scheduling objective is prioritized over the power minimization, since the reward for serving one link is higher than the maximum potential power saving. Thus, we have the following result

Claim 2. *If W is chosen according to*

$$0 < W < 1/KP \quad (8)$$

then the solution has the following properties: (i) The maximum possible number of links will be served, as if $W = 0$; and (ii) No other solution that serves this set of links can operate with less power.

Proof: A formal proof will be published elsewhere. ■

III. MIXED-INTEGER PROGRAMMING REFORMULATION

The proposed formulation of the joint power control and scheduling problem in (5)–(6) is nonconvex, owing to the binary variables $\{s_k^n\}$. In fact, it can be shown that (5)–(6) is NP-hard. The computationally intensive components of the problem are the scheduling and admission control. Due to its combinatorial nature, the optimization with respect to the variables $\{s_k^n\}_{k \in \mathcal{K}}^{n \in \mathcal{N}}$ has a worst-case complexity that is exponential in the number KN of optimization variables.

In what follows, we show that (5)–(6) admits a MIP problem representation. In fact, we have a priori designed it so that it does. MIP is a linear problem that comprises both integers and continuous variables. Even though the problem is NP-hard, there are many algorithms and software packages that find the global solution *exactly* very efficiently in the vast majority of the instances. The resulting method can be directly used in practice for small-scale problems, where “small-scale” is of course relative to the computational capacity of the resource allocation unit. In addition, as the MIP reformulation allows us to solve the problem exactly, much more efficiently than a brute-force search, it also provides a benchmark for performance evaluation of competing, suboptimal algorithms in offline simulations.

It can be easily seen that constraints (6a) are actually linear inequalities. Since the denominator of the fraction is positive,

(6a) can be equivalently rewritten as

$$\begin{aligned} G_{kk}^n p_k^n + M(1 - s_k^n) &\geq T_k \sum_{\ell \neq k} G_{\ell k}^n p_\ell^n + T_k V_k \Leftrightarrow \\ T_k \sum_{\ell \neq k} G_{\ell k}^n p_\ell^n - G_{kk}^n p_k^n + M s_k^n &\leq M - T_k V_k \Leftrightarrow \\ \sum_{\ell=1}^K A_{\ell k}^n p_\ell^n + M s_k^n &\leq B_k, \end{aligned} \quad (9)$$

where we have defined $B_k \triangleq M - T_k V_k$ and

$$A_{\ell k}^n \triangleq \begin{cases} -G_{kk}^n & \text{if } \ell = k, \\ T_k G_{\ell k}^n & \text{if } \ell \neq k. \end{cases} \quad (10)$$

Replacing (6a) with (9), the optimization (5)–(6) is equivalently recast in the following standard MIP form

$$\begin{aligned} \max_{\substack{\{p_k^n \in [0, P]\} \\ \{s_k^n \in \{0, 1\}\}}} & \sum_{k=1}^K \sum_{n=1}^N s_k^n - W \sum_{k=1}^K \sum_{n=1}^N p_k^n \end{aligned} \quad (11)$$

subject to

$$\sum_{\ell=1}^K A_{\ell k}^n p_\ell^n + M s_k^n \leq B_k \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (12a)$$

$$p_k^n - P s_k^n \leq 0 \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}, \quad (12b)$$

$$\sum_{n=1}^N s_k^n \leq 1 \quad \forall k \in \mathcal{K}. \quad (12c)$$

IV. APPLICATION IN COGNITIVE RADIO UNDERLAY

Here, we generalize, to the case of many DoF, the treatment of the joint resource allocation problem for spectrum underlay networks that was presented in [7]. The core idea is that the links are divided into two user sets. The set of primary users consists of links that are guaranteed to receive service, for example, because they have a license to operate in a specific band. The secondary users compete for access rights. They are allowed to transmit in the same DoF as the primary users, provided that they do not cause interference that cannot be compensated for by power control, i.e., that will yield the problem infeasible. The assumption is that primary users are willing to boost their transmission level to cope with the extra interference from the secondary ones, but will not compromise their access rights. We can incorporate this functionality in the proposed formulation by setting $s_k^n = 1$ for the primary users.

V. NUMERICAL RESULTS

We present preliminary numerical results for $N = 1$ time slot that validate the optimality of the proposed formulation. In the simulated scenario $K = 10$ links request a common SINR threshold of $\{T_k = 0 \text{ dB}\}_k$. The channel gains account only for the propagation loss with an attenuation exponent of 4. The link distances that determine the channel gains are chosen uniformly in $[10, 100]$. The power bound is $P = 1$ and the noise variance $\{V_k = 10^{-9}\}_k$. The number of links served is shown in Fig. 1 for 50 different network instances. The solution obtained by the MIP formulation is consistent with the result of a brute-force enumeration baseline. We used the GNU linear programming kit (GLPK) package for the simulation.

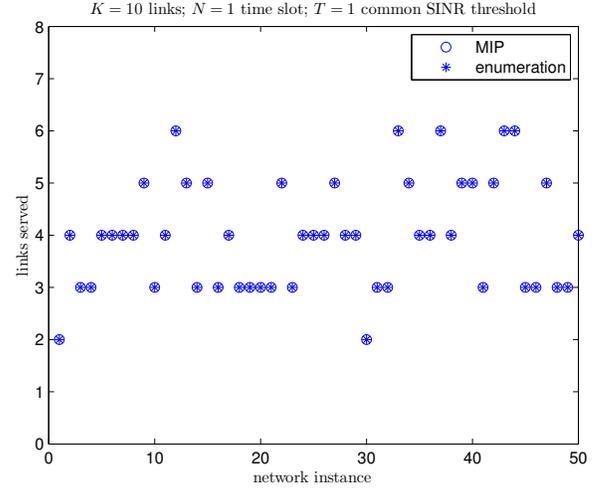


Fig. 1. MIP versus brute-force enumeration

VI. CONCLUSIONS

The proposed formulation of the joint scheduling and power control problem enables us to solve it *exactly*, using off-the-shelf algorithms for mixed-integer programming. A basic prerequisite is that there is a central controller that is aware of all the channel gains and requested SINR thresholds. This renders the solution directly applicable in systems that have such a controller. Our proposed method can also be used to provide fundamental limits for decentralized solutions, assuming that the signaling overhead is negligible and that the optimal feedback can be fed back under tight latency constraints (which is NP-hard in practice).

REFERENCES

- [1] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641–646, Nov. 1993.
- [2] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.
- [3] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.
- [4] N. Bambos, C. Chen, and G. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE/ACM Trans. Networking*, vol. 8, no. 5, pp. 583–597, Oct. 2000.
- [5] T. ElBatt and A. Ephremides, "Joint scheduling and power control for wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 74–85, Jan. 2004.
- [6] E. Matakani, N. D. Sidiropoulos, and L. Tassioulas, "Convex approximation techniques for joint multiuser downlink beamforming and admission control," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2682–2693, Jul. 2008.
- [7] I. Mitliagkas, N. D. Sidiropoulos, and A. Swami, "Convex approximation-based joint power and admission control for cognitive underlay networks," in *Proc. IEEE Int. Wireless Commun. and Mobile Computing Conf. (IWCMC)*, Crete island, Greece, Aug. 6–8, 2008, pp. 28–32.
- [8] E. Karipidis, N. D. Sidiropoulos, and L. Tassioulas, "Joint QoS multicast power / admission control and base station assignment: A geometric programming approach," in *Proc. 5th IEEE Workshop on Sensor Array and Multi-Channel Signal Processing (SAM)*, Darmstadt, Germany, Jul. 21–23, 2008, pp. 155–159.