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Outage Rate Regions for the MISO IFC

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Abstract—We consider the two-user multiple-input single-output (MISO) interference channel (IFC) and assume that the receivers treat the interference as additive Gaussian noise. We study the rates that can be achieved in a slow-fading scenario, allowing an outage probability. We introduce three definitions for the outage region of the IFC. The definitions differ on whether the rates are declared in outage jointly or individually and whether there is perfect or statistical information about the channels. Even for the broadcast and the multiple-access channels, which are special cases of the IFC, there exist several definitions of the outage rate regions. We provide interpretations of the definitions and compare the corresponding regions via numerical simulations. Also, we discuss methods for finding the regions. This includes a characterization of the beamforming strategies, which are optimal in the sense that achieve rate pairs on the Pareto boundary of the outage rate region.

I. INTRODUCTION

In this paper, we consider the two-user multiple-input single-output (MISO) interference channel (IFC), consisting of two base station (BS) - mobile station (MS) pairs. The BSs employ n transmit antennas and the MSs a single receive antenna. The transmissions are concurrent and cochannel; hence, they interfere with each other. The BSs choose their beamforming vectors in either a coordinated or an uncoordinated manner. The fundamental question raised is which rates can be simultaneously achieved.

The MISO IFC was studied in [1], where the authors characterized the transmit strategies, which yield Pareto-optimal operating points, assuming that the BSs have channel state information (CSI). Herein, CSI refers to the scenario that the BSs perfectly know the channel realizations. In [2], we extended the characterization in [1] to the ergodic rate region, assuming that the BSs have channel distribution information (CDI). That is, the BSs know that the channels are zero-mean complex Gaussian random variables with given covariance matrices. The ergodic rate is a long-term achievable rate, averaged over the time-varying channel. Hence, the optimal coding is across an infinite number of channel realizations. However, for real-time applications, we cannot tolerate the delays introduced by the ergodic-rate achieving codes. If we accept that the transmission is in outage during severe fading, then we can achieve higher rates when the channel conditions are good. For some applications, e.g., voice communication

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systems, packet error rates up to 10% yield only a hardly noticeable degradation of the call quality [3].

Outage capacity regions have been previously studied for the broadcast channel (BC) and multiple-access channel (MAC), which are important special cases of the IFC. The outage capacity regions for the BC were studied in [4] for the case of single-antenna transmitters with CSI and an average transmit power constraint. First, the authors determined the zero-outage regions for code division (CD), with and without successive decoding, and time division (TD). TD means that transmissions are separated in time, while CD means simultaneous and cochannel transmissions separated with orthogonal codes. Second, outage capacity regions were determined for both individually and simultaneously declared outage probability specifications (individual and common outage, respectively). It was shown that the outage capacity regions are implicitly obtained from the outage probability regions for a given rate vector. For each outage scenario, an optimal power allocation was determined.

Outage capacity regions for the MAC were studied in [5], again for single-antenna transmitters with CSI. Given a required rate and an average power constraint, a successive decoding strategy and an optimal power allocation policy for achieving points on the boundary were determined. Both common and individual outage were discussed. Also, the case when the transmitters have no CSI was treated in [6].

For the single-input single-output (SISO) IFC, various methods have been proposed to find optimal power allocations given transmission rates and outage probability specifications, or to minimize outage probabilities given power constraints and rates. But not much effort has been spent on characterizations of the outage rate regions. Optimal power control strategies were derived in [7], given outage specifications for the interference-limited SISO IFC. The results of [7] were derived under the assumption of CDI at the transmitters. For SISO channels this assumption implies that the channel gains have general mean.

In [8], the asymptotic behavior of the outage probability was studied for the two-user block-fading SISO IFC in the interference-limited regime. Upper and lower bounds of the diversity for asymmetric networks were derived. It was shown that, for a symmetric channel with strong interference, sending all common information is optimal, while when the interference is weak, achievable schemes without rate splitting do not meet the diversity upper bound in general.

A. Contributions

We study the achievable rates of the MISO IFC, when the receivers treat the interference as additive Gaussian noise, the transmit power is peak constrained, the channels experience slow fading and a probability of outage is allowed. In order to simplify the analysis, we make the practical assumption that the BSs use beamforming to send a single data stream. Beamforming is the optimal transmit strategy for CSI, but for CDI it can be suboptimal.

Up to the authors' knowledge, there is not much work on outage rate regions for the IFC; especially, not for the MISO IFC. As for the MAC and BC, there exist several definitions for the outage rate region of the IFC. The main contribution of this paper is three different definitions, which depend on the specification of outage probability (common or individual) and on the amount of channel knowledge (CSI or CDI).

After defining the regions, we characterize the beamforming vectors which achieve operating points on the northeast boundary of the rate region. Especially, we are interested in the Pareto-optimal points of the boundary, for which it is impossible to improve the rate of one link without simultaneously decreasing the rate of the other. We also provide closed-form expressions for the outage probabilities for CDI. Finally, we compare the single-user points of the proposed outage rate regions and show an illustration of the regions.

B. Notation

$\text{rank}\{\cdot\}$, $\mathcal{S}\{\cdot\}$, and $\mathcal{K}\{\cdot\}$ denote the rank, span, and kernel, respectively, of a matrix. $\mathbf{\Pi}_{\mathbf{Z}} \triangleq \mathbf{Z}(\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H$ is the orthogonal projection onto the column space of \mathbf{Z} . $\mathbf{\Pi}_{\mathbf{Z}}^\perp \triangleq \mathbf{I} - \mathbf{\Pi}_{\mathbf{Z}}$ is the orthogonal projection onto the orthogonal complement of the column space of \mathbf{Z} , where \mathbf{I} is the identity matrix. $\mathbb{E}\{\cdot\}$ is the expectation operator. We define the set

$$\mathcal{I} \triangleq \{(i, j) : i, j \in \{1, 2\}, i \neq j\}. \quad (1)$$

II. PRELIMINARIES

A. System Model

We assume that transmission consists of scalar coding (single-stream transmission) followed by beamforming and that all propagation channels are frequency-flat. The matched-filtered symbol-sampled complex baseband data received by MS_{*i*} is modeled as

$$y_i = \mathbf{h}_{ii}^H \mathbf{w}_i s_i + \mathbf{h}_{ji}^H \mathbf{w}_j s_j + e_i \quad (i, j) \in \mathcal{I}, \quad (2)$$

where s_i is the i.i.d. unit-energy symbol transmitted by BS_{*i*}, \mathbf{w}_i is the associated employed beamforming vector, and e_i is i.i.d zero-mean Gaussian noise with variance σ_i^2 . The conjugated¹ channel vector \mathbf{h}_{ij} between BS_{*i*} and MS_{*j*} is modeled as $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ij})$. The transmission power is bounded due to regulatory and hardware constraints. Without loss of generality we set this bound to 1. Hence, the set of feasible beamforming vectors is

$$\mathcal{W} \triangleq \{\mathbf{w} \in \mathbb{C}^n : \|\mathbf{w}\|^2 \leq 1\}. \quad (3)$$

¹We incorporate conjugation in definition to simplify subsequent notation.

We assume that the receivers treat the interference as noise. Then, the achievable rate on link i is given by

$$R_i(\mathbf{h}_{ii}, \mathbf{h}_{ji}, \mathbf{w}_i, \mathbf{w}_j) = \log_2 \left(1 + \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2} \right). \quad (4)$$

We note that the power terms $|\mathbf{h}_{ji}^H \mathbf{w}_j|^2$ are exponentially distributed with mean

$$p_{ji} \triangleq \mathbb{E}\{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2\} = \left\| \mathbf{Q}_{ji}^{1/2} \mathbf{w}_j \right\|^2, \quad (5)$$

which is the average power received by MS_{*i*} from BS_{*j*}.

III. OUTAGE RATE REGIONS FOR CDI

In this section, we assume that the BSs have CDI, so that they can only adapt their beamforming vectors to the statistical distributions of the channels. Under this assumption, we would like to find the outage rate region, which consists of all the rate pairs (r_1, r_2) that can be simultaneously achieved given an outage specification. We say that a rate pair (r_1, r_2) has *individual* outage probabilities ϵ_1 and ϵ_2 when there exists a pair of beamforming vectors, such that r_1 is achieved in at least a fraction $1 - \epsilon_1$ of the possible fading states or r_2 is achieved in at least a fraction $1 - \epsilon_2$ of the possible fading states. We say that a rate pair (r_1, r_2) has a *common* outage probability ϵ if there exists a pair of beamforming vectors such that r_1 and r_2 are achieved simultaneously with a probability at least $1 - \epsilon$.

We determine the outage rate region in two steps. Given a pair of fixed beamforming vectors, the rate in (4) is a function of the random channels. Hence, the rate is a random variable. Given the outage specification, we can find the rate points corresponding to the fixed beamforming vectors. It is apparent that each choice of beamforming vectors yields a different rate region $\mathcal{R}_{\mathbf{w}}$. Second, we define the outage rate region as the union of all these fixed-beamforming regions $\mathcal{R}_{\mathbf{w}}$. In Defs. 1 and 2, we consider the cases of common and individual outage probabilities, respectively.

Definition 1. Let $\epsilon_1 > 0$ and $\epsilon_2 > 0$ denote the *individual* outage probability specifications. Assuming that a fixed pair of beamforming vectors $(\mathbf{w}_1, \mathbf{w}_2)$ is used for transmission, we define the region $\mathcal{R}_{\mathbf{w}}^{\text{ind}}(\epsilon_1, \epsilon_2)$ as the set of all rate pairs (r_1, r_2) for which

$$\Pr\{R_i(\mathbf{h}_{ii}, \mathbf{h}_{ji}) > r_i\} \geq 1 - \epsilon_i, \quad (i, j) \in \mathcal{I}. \quad (6)$$

Considering all possible choices for the beamforming vectors, we define the outage rate region as

$$\mathcal{R}_1(\epsilon_1, \epsilon_2) = \bigcup_{(\mathbf{w}_1, \mathbf{w}_2) \in \mathcal{W}^2} \mathcal{R}_{\mathbf{w}}^{\text{ind}}(\epsilon_1, \epsilon_2). \quad (7)$$

□

In Def. 1, we characterize the scenario where one user might be in outage while the other one is able to decode the received message.

Next, we express in closed form the probability in (6).

$$f_i(p_{ii}, p_{ji}, \gamma_i) \triangleq \Pr \{R_i(\mathbf{h}_{ii}, \mathbf{h}_{ji}) > r_i\} \quad (8)$$

$$= \Pr \left\{ \log_2 \left(1 + \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2} \right) > r_i \right\} \quad (9)$$

$$= \Pr \left\{ |\mathbf{h}_{ii}^H \mathbf{w}_i|^2 - (2^{r_i} - 1)|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 > (2^{r_i} - 1)\sigma_i^2 \right\} \quad (10)$$

$$= \Pr \left\{ |\mathbf{h}_{ii}^H \mathbf{w}_i|^2 - \gamma_i |\mathbf{h}_{ji}^H \mathbf{w}_j|^2 > \gamma_i \sigma_i^2 \right\} \quad (11)$$

$$= \frac{p_{ii}}{p_{ii} + \gamma_i p_{ji}} e^{-\frac{\gamma_i \sigma_i^2}{p_{ii}}}. \quad (12)$$

In (11), we defined $\gamma_i \triangleq 2^{r_i} - 1$ to be the signal-to-interference-plus-noise ratio (SINR) that corresponds, due to (4), to rate r_i . From Section II-A and (5) we know that $|\mathbf{h}_{ii}^H \mathbf{w}_i|^2$ is exponentially distributed with mean p_{ii} . Also, we note that $\gamma_i |\mathbf{h}_{ji}^H \mathbf{w}_j|^2$ is exponentially distributed with mean $\gamma_i p_{ji}$. Using (4.5) in [9] we can write (11) as (12).

Definition 2. Let $\epsilon > 0$ denote the *common* outage probability specification. Assuming that a fixed set of beamforming vectors $(\mathbf{w}_1, \mathbf{w}_2)$ is used for transmission, we define the region $\mathcal{R}_w^{\text{com}}(\epsilon)$ as the set of all rate pairs (r_1, r_2) for which

$$\Pr \left\{ \bigcap_{(i,j) \in \mathcal{I}} R_i(\mathbf{h}_{ii}, \mathbf{h}_{ji}) > r_i \right\} \geq 1 - \epsilon. \quad (13)$$

Considering all possible choices for the beamforming vectors, we define the outage rate region as

$$\mathcal{R}_2(\epsilon) = \bigcup_{(\mathbf{w}_1, \mathbf{w}_2) \in \mathcal{W}^2} \mathcal{R}_w^{\text{com}}(\epsilon). \quad (14)$$

□

According to Def. 2, an outage is declared when either (or both) of the systems cannot decode the received message.

Since the channels are independent, the events intersected in the probability term in (13) are independent too. Hence, (13) can be rewritten as

$$\prod_{(i,j) \in \mathcal{I}} \Pr \{R_i(\mathbf{h}_{ii}, \mathbf{h}_{ji}) > r_i\} \geq 1 - \epsilon. \quad (15)$$

Using the result of (12), we can express (15) in closed form.

IV. OUTAGE RATE REGION FOR CSI

In this section, we assume that the BSs have CSI and, therefore, are able to adapt their beamforming vectors to the current fading state. Based on this, we provide an alternative definition for the outage rate region of the MISO IFC. We again follow a two-step approach. First, we consider a given realization of the channels; thus, the rate in (4) is a function of the beamforming vectors. Then, we define the region \mathcal{R}_h consisting of the rate points that can be achieved using all possible pairs of beamforming vectors. It is apparent that for each channel realization we yield a different rate region \mathcal{R}_h . Second, we define the outage rate region as the set of rate pairs that can be achieved with the common outage probability $1 - \epsilon$.

These are the rate pairs that lie into any of the regions $\{\mathcal{R}_h\}$ with probability $1 - \epsilon$.

Definition 3. Given a realization of the channels $\{\mathbf{h}_{ij}\}_{i,j=1}^2$, we define the region of achievable rate pairs as

$$\mathcal{R}_h = \bigcup_{(\mathbf{w}_1, \mathbf{w}_2) \in \mathcal{W}^2} (R_1(\mathbf{w}_1, \mathbf{w}_2), R_2(\mathbf{w}_2, \mathbf{w}_1)). \quad (16)$$

Let $\epsilon > 0$ denote the common outage probability specification. We define the outage rate region as

$$\mathcal{R}_3(\epsilon) = \{(r_1, r_2) : \Pr \{(r_1, r_2) \in \mathcal{R}_h\} \geq 1 - \epsilon\}. \quad (17)$$

□

We say that a rate point is in $\mathcal{R}_3(\epsilon)$ if the point lies in a randomly drawn (i.e. the channel vectors are randomly drawn) rate region with probability $1 - \epsilon$.

V. THE SINGLE-USER POINTS

In this section, we study the single-user (SU) points of the regions corresponding to common outage. The SU points are the points where one of the BSs is quiet and the other transmits with the maximum-ratio transmission (MRT) strategy. We show that there are scenarios where the SU rates for CSI are larger than those for CDI.

We assume that $\mathbf{w}_j = \mathbf{0}$ and BS_{*i*} uses its MRT strategy. For CDI we have that $\mathbf{w}_i^{\text{MRT}} = \mathbf{v}_1$, where \mathbf{v}_1 is the dominant eigenvector of \mathbf{Q}_{ii} [10]. For CSI we have $\mathbf{w}_i^{\text{MRT}} = \mathbf{h}_{ii}/\|\mathbf{h}_{ii}\|$ [1]. We define $r_{ii} \triangleq \text{rank}\{\mathbf{Q}_{ii}\}$ and let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{r_{ii}} > 0$ be the nonzero eigenvalues of \mathbf{Q}_{ii} and x be a unit-mean exponentially distributed random variable. For the case of CDI the following rates are achievable

$$\begin{aligned} \mathcal{R}_{CDI}^{SU} &= \left\{ r_i : \Pr \left\{ r_i < \log_2 \left(1 + \frac{|\mathbf{h}_{ii}^H \mathbf{v}_1|^2}{\sigma_i^2} \right) \right\} \geq 1 - \epsilon \right\} \\ &= \left\{ r_i : \Pr \left\{ r_i < \log_2 \left(1 + \frac{\lambda_1 x}{\sigma_i^2} \right) \right\} \geq 1 - \epsilon \right\} \\ &= \left\{ r_i : \Pr \{ \lambda_1 x > \gamma_i \sigma_i^2 \} \geq 1 - \epsilon \right\}. \end{aligned}$$

For CSI, we first define $\tilde{\mathbf{h}}_{ij}, \bar{\mathbf{h}}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and let $\mathbf{Q}_{ii} = \mathbf{T}_{ii} \Delta_{ii} \mathbf{T}_{ii}^H$ be the eigenvalue decomposition of \mathbf{Q}_{ii} . Then, we write $\|\mathbf{h}_{ii}\|^2 = \mathbf{h}_{ii}^H \mathbf{h}_{ii} = \tilde{\mathbf{h}}_{ii}^H \mathbf{Q}_{ii} \tilde{\mathbf{h}}_{ii} = \tilde{\mathbf{h}}_{ii}^H \mathbf{T}_{ii} \Delta_{ii} \mathbf{T}_{ii}^H \tilde{\mathbf{h}}_{ii} = \bar{\mathbf{h}}_{ii}^H \Delta_{ii} \bar{\mathbf{h}}_{ii} = \sum_{k=1}^{r_{ii}} \lambda_k x_k$. Then, the variables $\{x_k\}_{k=1}^{r_{ii}}$ are exponentially distributed unit-mean random variables and we have the possible outcome

$$\begin{aligned} \mathcal{R}_{CSI}^{SU} &= \left\{ r_i : \Pr \left\{ r_i < \log_2 \left(1 + \frac{\|\mathbf{h}_{ii}\|^2}{\sigma^2} \right) \right\} \geq 1 - \epsilon \right\} \\ &= \left\{ r_i : \Pr \left\{ r_i < \log_2 \left(1 + \frac{\sum_{k=1}^{r_{ii}} \lambda_k x_k}{\sigma_i^2} \right) \right\} \geq 1 - \epsilon \right\} \\ &= \left\{ r_i : \Pr \left\{ \sum_{k=1}^{r_{ii}} \lambda_k x_k > \gamma_i \sigma_i^2 \right\} \geq 1 - \epsilon \right\} \\ &= \left\{ r_i : \Pr \left\{ \lambda_1 x_1 > \gamma_i \sigma_i^2 - \sum_{k=2}^{r_{ii}} \lambda_k x_k \right\} \geq 1 - \epsilon \right\}. \end{aligned}$$

Since $\gamma_i \sigma_i^2 \geq \gamma_i \sigma_i^2 - \sum_{k=2}^{r_{ii}} \lambda_k x_k$ with equality if and only if $r_{ii} = 1$, it is clear that $\mathcal{R}_{CDI}^{SU} \subseteq \mathcal{R}_{CSI}^{SU}$. This shows that if $\text{rank}\{Q_{ii}\} > 1$, then the SU rates for the scenario of CSI are larger than those for the scenario of CDI.

VI. FINDING THE PARETO BOUNDARIES

In this section, we describe how the boundaries of the defined regions can be obtained.

A. CDI

We characterize the Pareto-optimal beamforming strategies for the case of CDI. Interestingly, the characterization given by Prop. 1 in [2], for the case of ergodic rates, applies here too.

Proposition 1. *Assume that $\mathcal{S}\{\Pi_{\mathcal{K}\{Q_{ij}\}} Q_{ii}\} \neq \emptyset$. Then, the beamforming vectors that achieve operating points on the Pareto boundary of the outage rate regions satisfy*

- w_i^{PO} lies into the subspace spanned by Q_{ii} and Q_{ij}
- $\|w_i^{PO}\|^2 = 1$.

The assumption that $\mathcal{S}\{\Pi_{\mathcal{K}\{Q_{ij}\}} Q_{ii}\} \neq \emptyset$ implies that Q_{ij} does not have full rank.

The proof of Prop. 1 is by contradiction and follows the proof of Prop. 1 in [2]. In the Appendix, we give the proof for system i . The proof for system j goes in a similar manner for $(i, j) \in \mathcal{I}$. In the proof we will use the following Lemma, which states three properties for the function $f_i(p_{ii}, p_{ji}, \gamma_i)$:

Lemma 1. *The function $f_i(p_{ii}, p_{ji}, \gamma_i)$ is*

- monotonously increasing with p_{ii} , for fixed p_{ji} and γ_i
- monotonously decreasing with p_{ji} , for fixed p_{ii} and γ_i
- monotonously decreasing with γ_i , for fixed p_{ii} and p_{ji} .

Proof: The proof is omitted, but the lemma is easily shown by studying the derivatives of $f_i(p_{ii}, p_{ji}, \gamma_i)$ with respect to p_{ii} , p_{ji} , and γ_i , respectively. ■

Based on the characterization in Prop. 1 and the methods described in [10] we can efficiently find the boundaries of the regions. In [10] we showed that the problem of finding the boundary can be split into either of two optimization problems. These problems can be stated as semidefinite programming problems.

B. CSI

Up to the authors knowledge, there does not exist any sophisticated method for finding the Pareto-boundary for the scenario of CSI. Therefore, we use a brute-force method. First, we draw a number of channel realization from their distributions. Second, we make a grid over the rate pairs. Third, for each pair in the grid we calculate how many times it has been contained in the regions. Finally, we keep the rate pairs that were contained in at least $1 - \epsilon$ of the regions. To find the region corresponding to each channel realization, we use the characterization in [1].

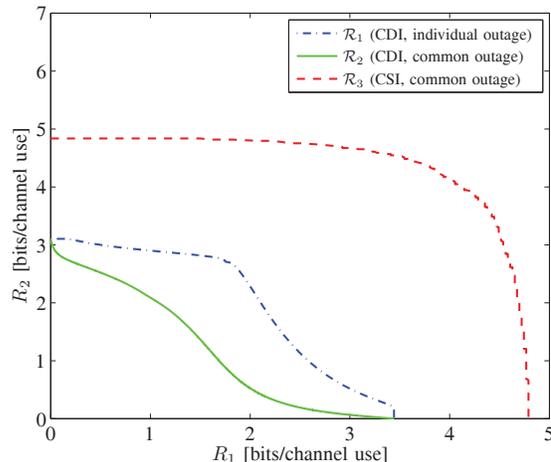


Fig. 1. Pareto boundary of the outage rate region for the MISO IFC.

VII. NUMERICAL RESULTS

In Fig. 1, we illustrate the three outage rate regions, defined in Sections III–IV. The methods to obtain the regions were described in Section VI. The covariance matrices are all rank-two, the number of transmit antennas is $n = 5$, and the noise variance is $\sigma_1^2 = \sigma_2^2 = 0.1$. In order to be able to compare the regions we have to choose the outage probabilities such that each user experiences the same outage probability for both individual and common outage. Therefore, we choose $\epsilon = \epsilon_1 = \epsilon_2 = 0.1$.

Def. 2 is more restrictive than Def. 1, since for common outage specification both channels must be able to support the desired rate simultaneously. This implies that the common outage rate region \mathcal{R}_2 is contained in the individual outage rate region \mathcal{R}_1 , as evidenced in Fig. 1.

Furthermore, the outage rate region \mathcal{R}_3 is larger than \mathcal{R}_2 . We can explain this as follows: If a point belongs to \mathcal{R}_2 , then there is a beamforming vector such that the rate point is achieved with probability $1 - \epsilon$. So, with probability at least $1 - \epsilon$, there is a channel such that this rate point is achieved for some beamforming vector. In Section V we showed that the rate at the SU points of \mathcal{R}_3 is larger than the corresponding points in \mathcal{R}_2 .

VIII. CONCLUSIONS

In this paper, we discussed the outage rate region of the MISO IFC. We proposed three different definitions which correspond to different scenarios of channel knowledge and outage specification. We justified these definitions by the fact that similar definitions exist for the MAC and the BC. Also, we described the methods we used to find the regions and characterized the Pareto-optimal beamforming vectors for CDI. Finally, we illustrated the differences between the regions via a numerical example.

Proof of Prop. 1 a): In order to arrive at a contradiction, suppose the statement in the proposition is false. Then there exists a \mathbf{w}_i^{PO} , $\|\mathbf{w}_i^{\text{PO}}\| \leq 1$, that corresponds to a rate point on the boundary but for which $\mathbf{w}_i^{\text{PO}} \notin \mathcal{S}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}$. Then we can write $\mathbf{w}_i^{\text{PO}} = \mathbf{w}'_i + \mathbf{u}_i$, where $\mathbf{w}'_i \in \mathcal{S}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}$, $\mathbf{u}_i \in \mathcal{K}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}$, and $\mathbf{u}_i \neq \mathbf{0}$. Note that $\mathbf{u}_i \perp \mathbf{w}'_i$. We define $p_{ii} = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}_i^{\text{PO}}\|^2$, $p_{ij} = \|\mathbf{Q}_{ij}^{1/2} \mathbf{w}_i^{\text{PO}}\|^2$, $p'_{ii} = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}'_i\|^2$, and $p'_{ij} = \|\mathbf{Q}_{ij}^{1/2} \mathbf{w}'_i\|^2$. For fixed \mathbf{w}_j (implies fixed p_{ji}) we show that

- i) $f_i(p'_{ii}, p_{ji}, \gamma_i) = f_i(p_{ii}, p_{ji}, \gamma_i)$ (outage probability for system i is unchanged),
- ii) $f_j(p_{jj}, p'_{ij}, \gamma_j) = f_j(p_{jj}, p_{ij}, \gamma_j)$ (outage probability for system j is unchanged)
- iii) $\|\mathbf{w}'_i\| < \|\mathbf{w}_i^{\text{PO}}\| \leq 1$.

Item i) follows because $\mathbf{Q}_{ii}^{1/2} \mathbf{u}_i = \mathbf{0}$, so that

$$p'_{ii} = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}'_i\|^2 = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}_i^{\text{PO}}\|^2 = p_{ii}.$$

Item ii) follows because $\mathbf{Q}_{ij}^{1/2} \mathbf{u}_i = \mathbf{0}$, so that

$$p'_{ij} = \|\mathbf{Q}_{ij}^{1/2} \mathbf{w}'_i\|^2 = \|\mathbf{Q}_{ij}^{1/2} \mathbf{w}_i^{\text{PO}}\|^2 = p_{ij}.$$

Item iii) follows because

$$\begin{aligned} \|\mathbf{w}_i^{\text{PO}}\|^2 &= \|\mathbf{\Pi}_{\mathcal{S}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}} \mathbf{w}_i^{\text{PO}}\|^2 + \|\mathbf{\Pi}_{\mathcal{S}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\}}^\perp \mathbf{w}_i^{\text{PO}}\|^2 \\ &= \|\mathbf{w}'_i\|^2 + \|\mathbf{u}_i\|^2 > \|\mathbf{w}'_i\|^2. \end{aligned}$$

The saved power $\|\mathbf{u}_i\|^2$ can be used to increase p_{ii} without affecting p_{ij} . An increase in p_{ii} increases γ_i , while the outage probability is fixed (see Lemma 1). For given δ_i , we define

$$\mathbf{w}''_i \triangleq \mathbf{w}'_i + \delta_i.$$

We define $p''_{ii} = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}''_i\|^2$ and $p''_{ij} = \|\mathbf{Q}_{ij}^{1/2} \mathbf{w}''_i\|^2$ and show that there exists a δ_i such that

- iv) $p''_{ii} > p_{ii}$ (gives $f_i(p''_{ii}, p_{ji}, \gamma_i) > f_i(p_{ii}, p_{ji}, \gamma_i)$),
- v) $p''_{ij} = p_{ij}$ (gives $f_j(p_{jj}, p''_{ij}, \gamma_j) = f_j(p_{jj}, p_{ij}, \gamma_j)$),
- vi) \mathbf{w}''_i satisfies the power constraint.

Item iv) is satisfied if

$$p''_{ii} = \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}''_i\|^2 \stackrel{(*)}{=} \|\mathbf{Q}_{ii}^{1/2} (\mathbf{w}_i^{\text{PO}} + \delta_i)\|^2 > \|\mathbf{Q}_{ii}^{1/2} \mathbf{w}_i^{\text{PO}}\|^2 \quad (18)$$

where the equality (*) holds since $\mathbf{Q}_{ii}^{1/2} \mathbf{u}_i = \mathbf{0}$ with probability 1 (cf. i) above). The inequality in (18) is satisfied if δ_i is chosen such that

$$2 \operatorname{Re}\{\delta_i^H \mathbf{Q}_{ii} \mathbf{w}_i\} > -\delta_i^H \mathbf{Q}_{ii} \delta_i. \quad (19)$$

Next, note that item v) is satisfied if

$$\mathbf{Q}_{ij}^{1/2} \delta_i = \mathbf{0} \Leftrightarrow \delta_i \in \mathcal{K}\{\mathbf{Q}_{ij}\}. \quad (20)$$

To construct δ_i , we first choose $\bar{\delta}_i$ such that (20) is satisfied and such that $\|\bar{\delta}_i\|^2 = 1$. We do this by solving

$$\begin{cases} \mathbf{Q}_{ii}^{1/2} \bar{\delta}_i \neq \mathbf{0} \\ \mathbf{Q}_{ij}^{1/2} \bar{\delta}_i = \mathbf{0}. \end{cases} \quad (21)$$

One solution is $\bar{\delta}_i \in \mathcal{S}\{\mathbf{\Pi}_{\mathcal{K}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii}\}$. Note that we cannot find any solution of (21) if $\mathcal{S}\{\mathbf{Q}_{ii}\} \subseteq \mathcal{S}\{\mathbf{Q}_{ij}\}$. Then we normalize $\bar{\delta}_i$ by setting $\tilde{\delta}_i = \bar{\delta}_i / \|\bar{\delta}_i\|$, and choose $\delta_i = \beta e^{i\phi} \tilde{\delta}_i$ where $\beta > 0$ (to be chosen later) and $\phi = -\arg \tilde{\delta}_i^H \mathbf{Q}_{ii} \mathbf{w}_i$. This choice will make $2 \operatorname{Re}\{\delta_i^H \mathbf{Q}_{ii} \mathbf{w}_i\} > 0$.

It remains to choose $\beta > 0$ such that $\|\mathbf{w}''_i\|^2 \leq 1$. But

$$\|\mathbf{w}''_i\| = \|\mathbf{w}'_i + \delta_i\| \leq \|\mathbf{w}'_i\| + \|\delta_i\| = \|\mathbf{w}'_i\| + \beta \leq 1,$$

which gives that $\beta \leq 1 - \|\mathbf{w}'_i\|$. So we take $\beta = 1 - \|\mathbf{w}'_i\|$.

Since we increased $f_i(p''_{ii}, p_{ji}, \gamma_i) > f_i(p_{ii}, p_{ji}, \gamma_i)$, it is possible to increase γ_i to γ'_i while keeping $f_j(p''_{ij}, p_{jj}, \gamma'_i) = f_j(p_{ii}, p_{ji}, \gamma_i)$. Hence we showed that $(\mathbf{w}''_i, \mathbf{w}_j)$ achieves (γ'_i, γ_j) , where $\gamma'_i > \gamma_i$. Hence, (γ_i, γ_j) cannot correspond to a point on the Pareto boundary, so we have a contradiction. ■

Proof of Prop 1 b): To show that we must have $\|\mathbf{w}_i^{\text{PO}}\|^2 = 1$ at the boundary, assume that $\|\mathbf{w}_i^{\text{PO}}\|^2 < 1$. Let $\mathbf{w}'_i = \mathbf{w}_i^{\text{PO}} + \delta_i$ where δ_i is chosen according to the recipe above. This shows that if $\|\mathbf{w}_i^{\text{PO}}\|^2 < 1$ then it is possible to choose a new beamforming vector \mathbf{w}'_i such that $\|\mathbf{w}'_i\|^2 = 1$, γ_i is increased, γ_j is unchanged, and the outage probabilities are kept constant. ■

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