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# Solving a multi-period supply chain problem for a pulp company using heuristics – an application to Södra Cell AB

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**Abstract**

In this paper, the integrated planning of production and distribution for a pulp company is considered. The tactical decisions included regard transportation of raw materials from harvest areas to pulp mills; production mix and contents at pulp mills; inventory; distribution of pulp products from mills to customers and the selection of potential orders and their levels at customers. The planning period is one year and several time periods are included. As a solution approach we make use of two different heuristic approaches. The main reason to use heuristics is the need for quick solution times. The first heuristic is based on a rolling planning horizon where iteratively a fixed number of time periods is taken into consideration. The second heuristic is based on Lagrangian decomposition and subgradient optimization. This provides optimistic bounds of the optimal objective function value, that are better than the LP relaxation value, which can be used as a measure of the heuristic (pessimistic) solution quality. In addition we apply the proposed rolling horizon heuristic in each iteration of the subgradient optimization. A number of cases based on real data is analyzed which shows that the proposed solution approach is simple and provides high quality solutions.

**Keywords**: Supply chain modelling, production planning, heuristics, Lagrangian decomposition

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# 1 Introduction

The problem addressed in this paper is the supply chain for one of the world's largest suppliers of market pulp, Södra Cell AB. The supply chain considered starts at the supply sources, that is forest districts and saw mills, located in southern Sweden, passes through production units, that is pulp mills and distribution centers, that is terminals, and ends at the customers' paper mills, located mainly in central Europe. The import of logs is also a potential source of raw material. Transportation and distribution are carried out by vessels, trains, lorries and barges. Decisions about production mix, terminal use, and contracts are considered. A general description of supply chain management can be found in Stadtler [30] and a survey of supply chain management with regard to Swedish manufacturing firms, can be found in Olhager and Selldin [26]. A mixed-integer linear programming model for bulk grain blending and shipping is presented in Bilgen and Ozkarahan [4], where the goal is to minimize the total costs for blending, loading, transportation and inventory costs. The model considers different type of vessels, several time periods and products [4]. An overview of supply chain management in the pulp and paper industry is found in Carlsson et al. [8] and the problem of planning for the wood fibre flow is presented in Carlsson et al. [6]. The supply chain for Södra Cell has been studied in a number of articles. A general overview of supply chain applications can be found in Carlsson and Rönnqvist [7]. Gunnarsson et al. [18] considered the second part of the supply chain, from pulp mills to customers. The problem involved different kinds of ship routes, terminal location and only one time period. In Gunnarsson et al. [19], the whole supply chain was considered, but only one time period and only one type of a more simple ship route were taken into account. Bredström et al. [5] investigated the first part of the supply chain. This starts with supplying the wood and ends at the shipment ports for the pulp mills. The main focus was detailed production planning together with transportation and storage. The planning period was three months and daily time periods were used. As a solution method, a special column generation was used to generate mill specific production plans and constraint branching was used to control the branch and bound algorithm.

In the work done earlier, there is a lack of models that take the overall supply chain into consideration, and where several time periods are used in order to consider variations in demand and supply and controlling which products that are produced at mills in different time periods. In this paper we have developed a model for this purpose. The model is very detailed, since it is to be used in real planning situations. The model becomes quite large and it is necessary to be able to decompose it in order to get an acceptable solution for the entire planning period. Thus, we have developed a rolling time heuristic and also a more complex Lagrangian heuristic method based on Lagrangian decomposition and combined them in order to take full advantage of them both. General presentations and early papers describing Lagrangian heuristics can be found in Geoffrion [15], Fisher [14, 13], Lemaréchal [24], Everett [11], Shapiro [28, 29] and Beasley [2]. We have split the variables concerning storage and then relaxed the constraints linking together the old and new variables. In Guignard [17] this method is called Lagrangian decomposition while it is known as variable layering in Glover and Klingmann [16] or variable splitting in Näsberg [25]. This relaxation of constraints will lead to a subproblem for each time period. The heuristic is based on solving the time periods in a rolling horizon, time period by time period. In Gunnarsson and Rönnqvist [20] a Lagrangian heuristic method that decomposes the physical supply chain into two subproblems was presented. A disadvantage of that decomposition scheme was that the subproblems have to be of similar sizes to work well. The outline of the paper is as follows. In Section 2 the problem is decribed and in Section 3 the mathematical model is presented. In Section 4 the solution methods are described. Thereafter, in Section 5 the computational results are presented and finally, in Section 6 some concluding remarks are made.

# 2 Problem description

The supply chain for Södra Cell AB is illustrated in Figure 1. It starts by the acquisition and transportation of raw materials, pulp wood from forest districts and wood chips from saw mills, to the pulp mills to be used in the pulp production. Different pulp products are produced according to different recipes at five pulp mills. The final pulp products are then further distributed by shipping vessels, trains and lorries, either direct or via terminals, to the final customers, consisting of paper mills, most of them located in central Europe. The planning horizon is normally one year but is divided into several time periods, for example, 12 months.

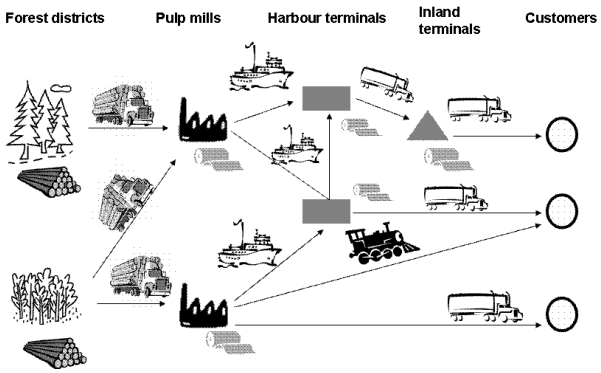


Figure 1: Illustration of the supply chain for Södra Cell AB.

## Procurement

The wood used in the pulp production originates either from domestic sources or is imported. In Sweden the domestic procurement is organized in forest districts, located in the southern part of the country, purchasing pulp wood from members of Södra. In addition, wood chips are purchased from different sawmills. In Norway an external supplier carries out the domestic procurement. Imports of wood originate mainly from the Baltic States. Some import is also carried out from Scotland and Ireland. The domestic wood is transported by lorries from the forests and saw mills to the pulp mills. Imported wood is carried by shipping vessels. The procured wood is classified into different assortments. Each assortment has its unique properties which is used either individually or mixed with other assortments when producing different pulp products.

## Production

Production is carried out at five pulp mills. Södra Cell owns three pulp mills in Sweden (Mönsterås, Värö, and Mörrum) and two in Norway (Tofte and Folla). The amount of the different assortments needed to produce one metric ton of pulp varies. For softwood the conversion factor is around 5 to 1 (i.e. five cubic meters of softwood is used to produce one metric ton of softwood pulp). When producing hardwood pulp less wood is needed; the conversion factor for birch is approximately 4 to 1 and for eucalyptus 3 to 1 (depends very much on the specific eucalyptus specie used). The conversion factor wood to pulp depends also on the specific process and machinery used at the individual pulp mill. Different shares of raw materials are used to form one unit of pulp according to different recipes. An example of a recipe is illustrated in Table 1. All recipes cannot be used at all pulp mills because of that different processes and chemicals are used at the pulp mills. An *alternative* describes which recipes will be used at which pulp mill. A specific cost is related to each alternative. The costs are for

example consisting of change-over (or set-up) costs from using one recipe at a pulp mill to using another recipe. Only one recipe can be used at the same time at the pulp mills.

Table 1: An example of a recipe used at the pulp mill in Värö.

| Recipe | Raw materials needed for production (tonnes) | Outcome of products (in tonnes) |
|---|---|---|
| R SCV blue Z | Softwood 3,800<br>Wood chips 1,200 | blue Z 950<br>blue FZ 150 |

## Distribution

There are two alternatives to transport pulp from the mills to customers. The most important is to use shipping vessels for part of the transportation. Here, the first step is to transport pulp to the nearest harbour. Then, vessels takes the pulp to terminals for intermediate storage. This is then further transported by trains, lorries or barges to the final customers. The alternative is to use trains or lorries directly from mills to final customers.

Södra Cell uses three shipping vessels chartered long term. They are also called TC-vessels (i.e., time chartered vessels). The vessels deliver about 0.7 million tonnes of pulp products to international terminals. The vessel routes vary in journey time from a few days for short routes to about 25 days for longer routes (e.g., to Italy). Loading and unloading times at harbours are included in these journey times. The ship capacity of a TC-vessel is 5,600 tonnes. The routes used by the TC vessels are divided into two groups, A-routes and B-routes. The A-routes, or simple routes, are routes from the relevant pulp mill to one terminal. The B-routes, or composite routes, are routes that start from a pulp mill, go to one terminal for unloading some of the pulp products and continue to one or more terminals for unloading the rest of the pulp products. The B-routes can also go from one pulp mill to another pulp mill for additional loading before they go to the terminals. In addition to the fleet chartered long term, short–term vessels (i.e., spot vessels) are used. The spot vessels are chartered from one origin to one destination and their ship capacity is ranging from 1,500 tonnes until 1,600.

The terminals which are the destinations for the shipping vessels are close to a harbour. Apart from these harbour terminals, there are terminals located in the interior of the continent, denoted inland terminals. They are reached by barges, trains, or lorries from harbour terminals. Each terminal has restricted capacity to receive products, depending on its size. From each terminal, the pulp products are transported to a number of customers. Terminals with storage capacities are rented on annual agreements.

Trains or lorries mainly make the deliveries in Sweden and Norway. To supply the foreign customers, shipping vessels are often used, but in some cases trains and lorries can be used. There can exist agreements making these transportation modes profitable. The internal transportation from harbour terminals to inland terminals can be handled by trains, lorries or barges. The possible transportation modes used for distribution are presented in Table 2.

## Customer demand

The customers are paper mills, which use the pulp products in order to make final paper products. An order is existing for the overall need of products at a customer. The demand of a product in the order is given within specific limits. Each order that is fulfilled will result in a value depending on the order and the customer. A contract can include one or several orders. If a contract is accepted all the included orders have to be fulfilled. In addition there exists fixed orders that always have to be fulfilled independent of which contract they belong to. Revenues depend on the price of the products. The pulp is priced according to a global market price.

Table 2: Distribution possibilities.

| From | To | With | Transportation mode |
|---|---|---|---|
| Forest district | Pulp Mill | Raw material | Lorry |
| The Baltic States | Pulp Mill | Raw material | Shipping vessel |
| Pulp Mill | Harbour terminal | Pulp product | Shipping vessel |
| Pulp Mill | Customer | Pulp product | Lorry, Train |
| Harbour terminal | Inland terminal | Pulp product | Barge, Lorry, Train |
| Harbour terminal | Customer | Pulp product | Lorry, Train |
| Inland terminal | Customer | Pulp product | Lorry, Train |

There is one price for short fiber (hardwood) and one for long fiber (softwood). However, the net price different customers pay may vary due to different commercial agreements. In Table 3 we give an example of a contract.

Table 3: An example of a contract consisting of four orders.

| Order | Product | Demand (min) | Demand (max) | Value |
|---|---|---|---|---|
| 1 | gold BZ | 6,100 tonnes | 6,100 tonnes | 3,800 (SEK per ton) |
| 2 | green 85Z | 35,000 | 35,000 | 4,100 |
| 3 | blue FZ | 500 | 1,000 | 3,700 |
| 4 | green 85FZ | 2,000 | 2,500 | 3,900 |

Each order is placed at a delivery point connected from pulp mills by trains and lorries or from terminals by barges, trains and lorries. The delivery points are spread across Europe. About 80% of the volume is delivered outside Sweden and Norway. The ten largest customers purchase half of the volume.

# 3 Mathematical model

The original problem presented in the mathematical model below will be referred to as problem [P1]. We first describe the sets of variables, then follows the constraints and the objective function. The model is developed in close collaboration with Södra Cell to be used in practice, which explains the size of the model and the high level of detail. The description of the supply chain is based on conditions for Södra Cell AB, but can be generalized to other pulp companies.

Let $M$ be the set of raw materials, $D$ the set of districts, $I$ the set of pulp mills, $A$ the set of alternatives, $P$ the set of products, $R$ the set of recipes, $J$ the set of terminals, $R_A$ the set of simple routes, $R_B$ the set of composite routes, $S$ the set of delivery points, $C$ the set of contracts, $Q$ the set of orders, and $T$ the set of time periods. The set of pulp mills includes a subset for pulp mills in route $k$, $I_k$. The set of terminals includes subsets for harbour terminals, $J_H$, inland terminals, $J_L$, and terminals in route $k$, $J_k$. The set of orders includes subsets for fixed orders, $Q_F$, and for orders belonging to a contract, $Q_C$ and finally $Q_S$ for orders at delivery point $s$. In general, we will use index $m$ for raw materials, $d$ for districts, $i$ for pulp mills, $a$ for alternatives, $p$ for products, $r$ for recipes, $j$ for terminals, $k$ for routes, $s$ for delivery point, $c$ for contracts, $q$ for orders, and $t$ for time periods. Unless otherwise stated, we assume that definitions for example using index $i$, are valid for all $i \in I$. Restrictions on individual flow variables are that they are non-negative.

## 3.1 Variables

The binary variables presented below represent the strategic decisions about production, terminals, routes and contracts.

$$z_a = \begin{cases} 1, & \text{if production alternative } a \text{ is used at the pulp mills,} \\ 0, & \text{otherwise.} \end{cases}$$

$$z_c = \begin{cases} 1, & \text{if contract } c \text{ is accepted,} \\ 0, & \text{otherwise.} \end{cases}$$

$$z_j^T = \begin{cases} 1, & \text{if terminal } j \text{ is used,} \\ 0, & \text{otherwise.} \end{cases}$$

$$u_{kt}^A = \begin{cases} 1, & \text{if A-route } k \text{ is used in time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

The continuous variables will be presented below, in the same order as they are presented from left to right in the supply chain illustrated in Figure 1. There is a possibility to store raw materials from one time period to another. The storing of raw materials is expressed in the variables

$L_{dmt}^F$ = volume of raw material $m$ stored at forest district $d$ at the end of time period $t$.

The flow of raw materials from the forest districts to the pulp mills is represented by the variables

$w_{mdiprt}$ = volume of raw materials $m$, transported from forest district $d$, to pulp mill $i$ for making product $p$ according to recipe $r$ in time period $t$.

The production is expressed in the variables

$u_{iprt}$ = production of product $p$ at pulp mill $i$ according to recipe $r$ in time period $t$.

A limited amount of products can be stored at the pulp mills. This possibility is expressed in the variables

$L_{ipt}^M$ = storing of product $p$ at pulp mill $i$ at the end of time period $t$.

The above variables connected to flow to pulp mills are illustrated in Figure 3.

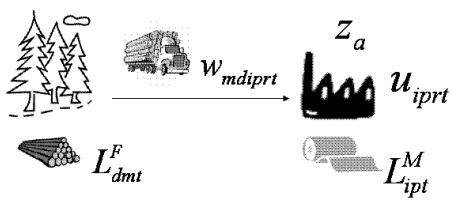


Figure 2: Flow to pulp mills.

Terminals located close to harbours are used for further transportation to the customers or transportation via inland terminals to customers. There are costs concerned with using terminals, both fixed costs and costs related to the flow through the terminals. The fixed cost arises if the terminal is used and the variable costs are defined by the amount of products passing the terminal. To keep a check on the total flow through the terminals, the variables

$y_{jt}^{tot}$ = total flow of products at terminal $j$ in time period $t$

are used. The variables expressing the storing of pulp products at harbour and inland terminals are

$L^{HT}_{jpt}$ = storing of product $p$ at harbour terminal $j$ at the end of time period $t$, and

$L^{LT}_{jpt}$ = storing of product $p$ at inland terminal $j$ at the end of time period $t$, respectively.

The flow routes are expressed by the variables

$x^{A}_{kijpt}$ = flow of product $p$ on A-route $k$ from pulp mill $i$ to terminal $j$ in time period $t$,

$x^{B}_{kijpt}$ = flow of product $p$ on B-route $k$ from pulp mill $i$ to terminal $j$ in time period $t$ and

$x^{S}_{ijpt}$ = flow of product $p$ from pulp mill $i$ to terminal $j$.

We use routes to represent the flow between pulp mills and terminals. In our case we make use of routes with a unique coupling between pulp mills and terminals. In order to model the time restriction for the routes, we introduce a variable describing the return flows on routes from harbour terminals back to pulp mills. As the shipping vessel is empty this flow does not include any products. The return flows on routes are expressed in the variables

$x^{R}_{jit}$ = return flows from terminal $j$ to pulp mill $i$ in time period $t$.

Sometimes the pulp products are transported via an inland terminal, before they arrive at the final customer. This transportation is often done by barge, but trains and lorries can also be used. This flow of products is described in the variables

$y^{T}_{hlpt}$ = flow of product $p$ from harbour terminal $h$ to inland terminal $l$ in time period $t$.

The $h$ and $l$ indexes are used instead of $j$ to separate harbour terminals from inland terminals. The above variables connected to flow to terminals are illustrated in Figure 4.

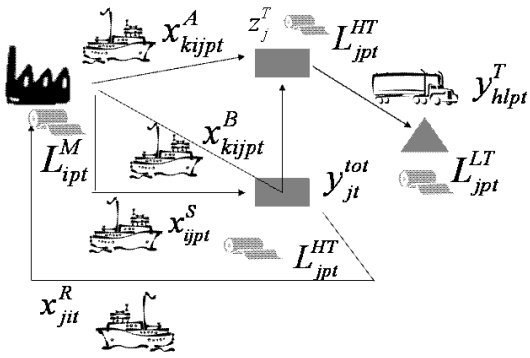


Figure 3: Flow to terminals.

The last flow in the supply chain is the flow to the customers, the paper mills. The customers can be reached by train or by lorry if they are located close to the pulp mills, or if for any reasons these transportation modes are more convenient, although trains or lorries are mostly used to reach customers in the Nordic countries. In the mathematical model, this flow is expressed by the variables

$y^{train}_{isqpt}$ = flow of product $p$ to delivery point $s$ according to order $q$ from pulp mill $i$ transported by trains in time period $t$ and

$y^{lorry}_{isqpt}$ = flow of product $p$ to delivery point $s$ according to order $q$ from pulp mill $i$ transported by lorries in time period $t$.

The above variables connected to flow to customers are illustrated in Figure 5. Trains and lorries transport the pulp products from the terminals to the customers. The flows from harbour and

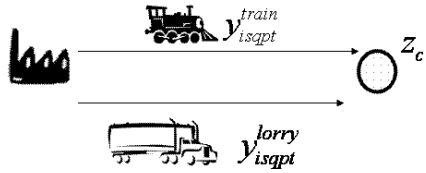


Figure 4: Flow from pulp mills to customers.

inland terminals to customers are expressed by the variables

$y^{Q}_{jsqpt}$ = flow of product $p$ from terminal $j$ to delivery point $s$ according to order $q$ in time period $t$.

This flow is illustrated below in Figure 6.

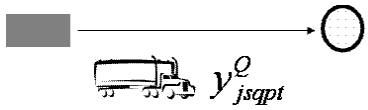


Figure 5: Flow from terminals to customers.

## 3.2 Constraints

The supply of raw materials is limited. To make sure that the supply of logs at districts is not exceeded, the constraints

$$\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} w_{mdiprt} \quad \leq \quad s_{md},\ \forall m \in M,\ d \in D \tag{1}$$

are used. The total supply of raw materials at the forest districts for the entire planning period is represented by the parameter $s_{md}$, supply of raw material $m$ at forest district $d$. There is a possibility to store raw materials in the forest. This is expressed in the constraints

$$L^{F}_{dmt-1} + s_t s_{md} \quad = \quad \sum_{i\in I}\sum_{p\in P}\sum_{r\in R} w_{mdiprt} + L^{F}_{dmt}, \forall d \in D, m \in M, t \in T. \tag{2}$$

The constants $s_t$ show how much of the total supply is available in the time period $t$. Most of the harvesting is done in the winter so the constants $s_t$ are typically larger during the first time periods (assuming that the periods are January, February, ..., December). The left hand side of the equation represents the supply of raw materials, the amount stored from the last time period and the new supply available for this time period. The right hand side of the equation shows the use of the raw materials, savings to the next time period or transportation of raw materials to be used in the pulp production to the pulp mills. The amount needed to make pulp products depends on the kind of raw materials used for the production. There are restrictions regarding the level of the different raw materials used in the different pulp products. The products are produced according to different recipes. To get the right level of raw materials in the recipes, we need the constraints

$$\sum_{d\in D}\sum_{p\in P} w_{mdiprt}/a_{mi} \quad \geq \quad n^{min}_{mr}\sum_{p\in P} u_{iprt}, \forall m \in M, i \in I, r \in R, t \in T, \tag{3}$$

and

$$\sum_{d\in D}\sum_{p\in P} w_{mdiprt}/a_{mi} \quad \leq \quad n^{max}_{mr}\sum_{p\in P} u_{iprt}, \forall m \in M, i \in I, r \in R, t \in T. \tag{4}$$

The amount of each raw material in each recipe can vary within certain limits and is described by constraints (3) and (4). The first constraints make sure that the level of raw materials

in the pulp products will exceed the minimum level and the last constraints make sure that the level will not exceed the maximum level. The parameters $a_{mi}$ show the amount of raw material $m$ consumed at pulp mill $i$ to get one unit. The parameters $n_{mr}^{min}$ and $n_{mr}^{max}$ express the minimum and maximum level of allowed raw materials $m$ used in recipe $r$ for making pulp products. Different capacities at the pulp mills need to be taken into account. The constraints with regard to the capacities to produce at pulp mills are

$$\sum_{r \in R}((\sum_{p \in P} u_{iprt})/\sum_{p \in P} r_{rp}) \quad = \quad g_i/ \mid T \mid, \ \forall i \in I, \ t \in T. \tag{5}$$

The constants $r_{rp}$ express the production in a 24 hour period of product $p$ according to recipe $r$. The constants $g_i$ define the total production capacity at pulp mill $i$ for a planning period expressed in 24 hours periods. The constraints (5) ensure that the total production of pulp products will not exceed the capacity at each of the pulp mills. There are costs involved if there is a change of recipe at a pulp mill. The constants $e_{ira}$ express which recipes are used at which pulp mills. If $e_{ira}$ equals 1, the use of recipe $r$ at the pulp mill $i$ is included in the alternative $a$, otherwise the constant is 0. In order to connect the production with the binary variables, $z_a$ for using alternatives, the constraints

$$(\sum_{p \in P} u_{iprt})/\sum_{p \in P} r_{rp} \quad \leq \quad g_i/ \mid T \mid \sum_{a \in A} e_{ira} z_a, \ \forall i \in I, \ r \in R, \ t \in T \tag{6}$$

are needed. The constraints (6) ensure that no products are produced unless their related recipes are involved in the chosen alternative. To ensure that each recipe consists of the right total sum of raw materials we use constraints

$$\sum_{m \in M} \sum_{d \in D} \sum_{p \in P} w_{mdiprt}/a_{mi} \quad = \quad \sum_{p \in P} u_{iprt}, \ \forall i \in I, \ r \in R, \ t \in T. \tag{7}$$

The recipes give one or several products. To ensure that the right level of products using a special recipe is reached the constraints

$$p_{pr} \sum_{p \in P} u_{iprt} \quad = \quad u_{iprt}, \ \forall i \in I, \ p \in P, \ r \in R, \ t \in T \tag{8}$$

are used. The constants $p_{pr}$ represent the outcome of product $p$ running recipe $r$. If the $p_{pr}$ is 1, only one kind of product can be made using the recipe. If the $p_{pr}$ is between 0 and 1, several products are produced using the recipe. The model will choose the most profitable alternative. An alternative can be to use some recipes to produce some kinds of products at some pulp mills and use some recipes which produce other kinds of products at other pulp mills. To make sure only one alternative is chosen the constraint

$$\sum_{a \in A} z_a \quad = \quad 1 \tag{9}$$

is used. The storage possibility at pulp mills is restricted. This is described by the constraints

$$\sum_{p \in P} L_{ipt}^{M} \quad \leq \quad L_{max}^{M}, \ \forall i \in I, \ t \in T, \tag{10}$$

where the constant $L_{max}^{M}$ expresses the maximum storage allowed at pulp mills.

The A-routes are the cheapest routes and they should be used if the flow of products corresponds to the use of at least one vessel monthly, that is 5600 tonnes per month. If the flow is less than the ship capacity the other kinds of routes can be used, B-routes or spot trips. In order to make sure that the flows of products on A-routes are at least one full ship capacity

tonnes monthly, the constraints

$$\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} \quad \geq \quad (Ns^H/\mid T\mid)u^A_{kt},\ \forall k\in R_A,\ t\in T, \tag{11}$$

and

$$\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} \quad \leq \quad \widetilde{M}u^A_{kt},\ \forall k\in R_A,\ t\in T \tag{12}$$

are used. The constant $s^H$ denotes the shipping vessel capacity, which is 5600 tonnes and the constant $N$ denotes the minimum number of vessels that have to be used for flow on A-routes in a planning horizon. The planning horizon in our problem is one year so we use $N = 12$ in the model. The constraints (11) make sure that if the A-route $k$ is used, the flow on the route is at least 5600 tonnes. The constraints (12) couple the continuous variables $x^A_{kijpt}$ and the binary variables $u^A_{kt}$, in order to make the binary variables to become one if there is any flow on A-routes. The constant $\widetilde{M}$ is a large constant and in our cases we have used $\widetilde{M}$ = maximum production capacity at pulp mills. There are time restrictions for TC-vessels and this is described in constraint

$$\sum_{k\in R_A}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} t^A_k x^A_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} t^B_k x^B_{kijpt} +$$

$$\sum_{j\in J_H}\sum_{i\in I} t^R_{ji} x^R_{jit} \quad \leq \quad t^{tot}s^H m/\mid T\mid,\ \forall t\in T. \tag{13}$$

The above constraint is an approximation, as parts of a full shipping vessel will be modelled in relation to the time used by a full shipping vessel. The constant $m$ expresses the number of shipping vessels chartered long term, TC-vessels. The constant $t^{tot}$ expresses the total time available for shipping vessels. The constants $t^A_k$ and $t^B_k$ express the time for A-routes and B-routes $k$, respectively. The constants $t^R_{ji}$ express the unit time for return routes between terminal $j$ and pulp mill $i$. In order to get the right return flow for the time constraints, we need the constraints

$$\sum_{k\in R_A}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} + \sum_{k\in R_B}\sum_{j\in J_k}\sum_{p\in P} x^B_{kijpt} = \sum_{j\in J_H} x^R_{jit}, \forall i\in I, t\in T, \tag{14}$$

and

$$\sum_{k\in R_A}\sum_{i\in I_k}\sum_{p\in P} x^A_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k}\sum_{p\in P} x^B_{kijpt} = \sum_{i\in I_k} x^R_{jit},\ \forall j\in J_H,\ t\in T. \tag{15}$$

Constraints (14) ensure that the outflow from a pulp mill equals the return flow to the same pulp mill and constraints (15) ensure that the inflow to a harbour terminal equals the return flow from the same harbour terminal. To assure that the B-routes are used as a B-route we need the constraints

$$\sum_{p\in P} x^B_{kijpt} \quad \geq n\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^B_{kijpt},\ \forall k\in R_B,\ i\in I_k,\ j\in J_k,\ t\in T. \tag{16}$$

The constant $n$ shows the share of the flow on a link in a route compared to the total flow on the given route. The constraints (16) will guarantee flow on each link of the B-route. If the flow of some link in the B-route was zero, the B-route would be used as an A-route. The capacity constraints at terminals are given by constraints

$$y^{tot}_{jt} \quad \leq \quad b_j z^T_j/\mid T\mid,\ \ \forall j\in J,\ t\in T. \tag{17}$$

The constants $b_j$ show the capacity at terminal $j$. The constraints (17) also make sure that there is no flow through terminals that are not used. The capacity for receiving products at

the terminals is restricted by agreements and contracts. The storage possibilities at harbour and inland terminals are restricted. These conditions are expressed in the constraints

$$\sum_{p\in P} L^{HT}_{jpt} \leq L^{HT}_{max},\ \forall j \in J_H, t \in T \qquad \text{and} \tag{18}$$

$$\sum_{p\in P} L^{LT}_{jpt} \leq L^{LT}_{max},\ \forall j \in J_L, t \in T. \tag{19}$$

The constants $L^{HT}_{max}$ and $L^{LT}_{max}$ express the maximum storage at harbour and inland terminals, respectively.

The delivery points can be reached by flow from the pulp mills or by flow from the terminals. The orders can belong to a contract. If the contract is accepted, the related orders have to be fulfilled. In order to deliver the right amounts of products according to each order and contract, the constraints

$$\sum_{j\in J} y^{Q}_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \leq z_c d^{max}_{qpt},\ \forall c \in C,\ s \in S,\ q \in Q_C \cap Q_S,$$
$$p \in P,\ t \in T, \quad \text{and} \tag{20}$$

$$\sum_{j\in J} y^{Q}_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \geq z_c d^{min}_{qpt},\ \forall c \in C,\ s \in S,$$
$$q \in Q_C \cap Q_S,\ p \in P,\ t \in T \tag{21}$$

are used. The constants $d^{min}_{qpt}$ and $d^{max}_{qpt}$ express the minimum and maximum demand according to order $q$ for product $p$ in time period $t$, respectively. Some of the orders are fixed, meaning that the demand for these orders has to be fulfilled within given limits. This is modelled by the constraints

$$\sum_{j\in J} y^{Q}_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \leq d^{max}_{qpt},\ \forall s \in S,\ q \in Q_F \cap Q_S,$$
$$p \in P,\ t \in T, \text{ and} \tag{22}$$

$$\sum_{j\in J} y^{Q}_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \geq d^{min}_{qpt},\ \forall s \in S,\ q \in Q_F \cap Q_S,$$
$$p \in P,\ t \in T. \tag{23}$$

To make sure that there is balance equilibrium at each pulp mill, the constraints

$$\begin{aligned} L^{M}_{ipt-1} + \sum_{r\in R} u_{iprt} = & \sum_{s\in S}\sum_{q\in Q_S} y^{train}_{isqpt} + \sum_{s\in S}\sum_{q\in Q_S} y^{lorry}_{isqpt} + \\ & \sum_{k\in R_A}\sum_{j\in J_k} x^{A}_{kijpt} + \sum_{k\in R_B}\sum_{j\in J_k} x^{B}_{kijpt} + \\ & \sum_{j\in J_H} x^{S}_{ijpt} + L^{M}_{ipt},\ \forall i \in I,\ p \in P,\ t \in T \end{aligned} \tag{24}$$

are needed. The left hand side of the above equation shows the supply of products, the products stored from the previous time periods and the products produced during the current time period, and the right hand side shows the use of the products, different kinds of transportation flows from the pulp mill by vessels, trains and lorries and the products stored for future time periods. Everything produced at the pulp mills has to be transported further to customers. The flow balance at pulp mills is formulated in constraints (24). The trains from some of the pulp mills have limited ability to transport pulp products. These conditions are expressed by the constraints

$$\sum_{s\in S}\sum_{q\in Q}\sum_{p\in P} y^{train}_{isqpt} \leq h_i / \mid T \mid,\ \forall i \in I,\ t \in T. \tag{25}$$

The constants $h_i$ describe the train capacity from pulp mill $i$. In order to get the right flow through harbour terminals, we need the balancing constraints

$$\begin{aligned}\sum_{p\in P} L^{HT}_{jpt-1} + \sum_{k\in R_A}\sum_{i\in I_k}\sum_{p\in P} x^A_{kijpt} + \\ \sum_{k\in R_B}\sum_{i\in I_k}\sum_{p\in P} x^B_{kijpt} + \sum_{i\in I}\sum_{p\in P} x^S_{ijpt} &= y^{tot}_{jt},\ \forall j\in J_H,\ t\in T, \end{aligned} \tag{26}$$

and

$$\begin{aligned} y^{tot}_{jt} &= \sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P} y^Q_{jsqpt} + \sum_{h\in J_L}\sum_{p\in P} y^T_{jhpt} + \sum_{p\in P} L^{HT}_{jpt},\ \forall j\in J_H, \\ & t\in T. \end{aligned} \tag{27}$$

Constraints (26) make sure that the total inflow to a harbour terminal equals the flow of products on the routes and the flow of products on the spot vessels passing the terminal. The flow out from a harbour terminal can be transported either to an inland terminal or directly to a order. The corresponding constraints for inland terminals are

$$\sum_{p\in P} L^{LT}_{jpt-1} + \sum_{h\in J_H}\sum_{p\in P} y^T_{hjpt} = y^{tot}_{jt},\ \forall j\in J_L,\ t\in T, \text{ and} \tag{28}$$

$$y^{tot}_{jt} = \sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P} y^Q_{jsqpt} + \sum_{p\in P} L^{LT}_{jpt},\ \forall j\in J_L,\ t\in T. \tag{29}$$

Constraints (28) ensure that all flow of products transported from harbour terminals to an inland terminal equals the total flow at the inland terminal and constraints (29) ensure the total flow at the inland terminals is equal to the total flow to customers from inland terminals. The potential difference consists of the storage at terminals.

$$\begin{aligned} L^{HT}_{jpt-1} + \sum_{k\in R_A}\sum_{i\in I_k} x^A_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k} x^B_{kijpt} + \sum_{i\in I} x^S_{ijpt} &= \\ \sum_{s\in S}\sum_{q\in Q_S} y^Q_{jsqpt} + \sum_{l\in J_L} y^T_{jlpt} + L^{HT}_{jpt},\ \forall j\in J_H,\ p\in P,\ t\in T. \end{aligned} \tag{30}$$

The constraints above assure that the inflow of each product to every harbour terminal equals the outflow of each product.

$$\begin{aligned} L^{LT}_{jpt-1} + \sum_{h\in J_H} y^T_{hjpt} &= \sum_{s\in S}\sum_{q\in Q_S} y^Q_{jsqpt} + L^{LT}_{jpt},\ \ \forall j\in J_L,\ p\in P, \\ & t\in T. \end{aligned} \tag{31}$$

Constraints (31) make the flow of each product to each inland terminal equal the flow of each product from the inland terminal to the orders. There are no pulp products stored in the beginning of the planning horizon and therefore there are no possibilities to have pulp products at store at the end of the last time period. We expect that all products produced in the planning horizon will be distributed to the customers in the same period. To state that there are no pulp products in store in the last time period, we set the variables for storage in time period T to zero,

$$L^M_{ipt} = 0,\ \forall i\in I,\ p\in P,\ t=\mid T\mid, \tag{32}$$

$$L^{HT}_{jpt} = 0,\ \forall j\in J_H,\ p\in P,\ t=\mid T\mid, \text{ and} \tag{33}$$

$$L^{LT}_{jpt} = 0,\ \forall j\in J_L,\ p\in P,\ t=\mid T\mid. \tag{34}$$

### 3.3 Objective function

The objective is to maximize the total profit. The total profit can be expressed as the total sales minus the total costs. The total profit is denoted $z$ and can be expressed as

$$\begin{aligned} Z^{P1} &= S - (C^{rawm} + C^{rec} + C^{alt} + C^{train/lorry} + C^{routes} + C^{return} + \\ &\quad C^{spot} + C^{flow} + C^{fix-term} + C^{term-term} + C^{distr} + C^{stor}). \qquad (I) \end{aligned}$$

The total sales, $S$, can be expressed as

$$\sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{train}_{isqpt} + \sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{lorry}_{isqpt} + \sum_{j\in J}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{Q}_{jsqpt},$$

The coefficients $p_{qp}$ denote the price of product $p$ in order $q$. The components of $C^{rawm}$ consist of

$$\sum_{m\in M}\sum_{d\in D}\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T}(c^{TR}_{mdi} + c^{P}_{md}) w_{mdiprt},$$

where the cost coefficients $c^{TR}_{mdi}$ denote the transportation cost of raw material $m$ from forest district $d$ to pulp mill $i$ and $c^{P}_{md}$ denote the purchasing cost of raw material $m$ from forest district $d$. The component $C^{rec}$ consists of

$$\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} c^{rec}_{ir} u_{iprt},$$

where the cost coefficients $c^{rec}_{ir}$ denote the cost of producing according to recipe $r$ at pulp mill $i$. The component $C^{alt}$ consists of

$$\sum_{a\in A} c^{alt}_{a} z_a,$$

where the cost coefficients $c^{alt}_{a}$ denote the change over costs using alternative $a$. The component $C^{train/lorry}$ consists of

$$\sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{train}_{is} y^{train}_{isqpt} + \sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{lorry}_{is} y^{lorry}_{isqpt},$$

where the cost coefficients $c^{train}_{is}$ denote the train cost between pulp mill $i$ and delivery point s and the cost coefficients $c^{lorry}_{is}$ denote the lorry cost between pulp mill $i$ and delivery point $s$. The component $C^{routes}$ consists of

$$\sum_{k\in R_A}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P}\sum_{t\in T}(c^{A}_{k} + c^{MP}_{i}) x^{A}_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P}\sum_{t\in T}(c^{B}_{k} + c^{MP}_{i}) x^{B}_{kijpt},$$

where the cost coefficients $c^{MP}_{i}$ denote the unit cost for transportation from pulp mill $i$ to the corresponding harbour. The cost coefficients $c^{A}_{k}$ and $c^{B}_{k}$ denote the unit cost for A-routes $k$ and B-routes $k$, respectively. The component $C^{return}$ consists of

$$\sum_{j\in J_H}\sum_{i\in I}\sum_{t\in T} c^{R}_{ji} x^{R}_{jit},$$

where the cost coefficients $c^{R}_{ji}$ denote the unit cost for return route between terminal $j$ and pulp mill $i$. The component $C^{spot}$ consists of

$$\sum_{i\in I}\sum_{j\in J_H}\sum_{p\in P}\sum_{t\in T}(c^{S}_{ij} + c^{MP}_{i}) x^{S}_{ijpt},$$

where the cost coefficients $c^S_{ij}$ denote the transportation spot cost between pulp mill $i$ and terminal $j$. The component $C^{flow}$ consists of

$$\sum_{j\in J}\sum_{t\in T} c^T_j y^{tot}_{jt},$$

where the cost coefficients $c^T_j$ denote the unit cost for flow at terminal $j$. The component $C^{fix-term}$ consists of

$$\sum_{j\in J} c^{FT}_j z^T_j,$$

where the cost coefficients $c^{FT}_j$ denote the fixed cost for using terminal $j$. The component $C^{term-term}$ consists of

$$\sum_{h\in J_H}\sum_{l\in J_L}\sum_{p\in P}\sum_{t\in T} c^T_{hl} y^T_{hlpt},$$

where the cost coefficients $c^T_{hl}$ denote the transportation cost between harbour terminal $h$ and inland terminal $l$. The component $C^{distr}$ consists of

$$\sum_{j\in J}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{TQ}_{js} y^Q_{jsqpt},$$

where the cost coefficients $c^{TQ}_{js}$ denote the transportation cost between terminal $j$ and delivery point $s$, and finally the component $C^{stor}$ consists of

$$\sum_{d\in D}\sum_{m\in M}\sum_{t\in T} c^{Fstor}_d L^F_{dmt} + \sum_{i\in I}\sum_{p\in P}\sum_{t\in T} c^{Mstor}_i L^M_{ipt} + \sum_{j\in J_H}\sum_{p\in P}\sum_{t\in T} c^{Tstor}_j L^{HT}_{jpt} + \sum_{j\in J_L}\sum_{p\in P}\sum_{t\in T} c^{Tstor}_j L^{LT}_{jpt},$$

where the cost coefficients $c^{Fstor}_d$ denote the storage cost per volume unit at district $d$, and the cost coefficients $c^{Mstor}_i$ the storage cost per volume unit at pulp mill $i$. The cost coefficients $c^{Tstor}_j$ denote the storage cost per volume unit at terminal $j$.

# 4 Solution methods

We propose two heuristic approaches. The first is based on a rolling planning horizon where only a part of the planning periods is active. The solution for one period is fixed and then the heuristic moves one period ahead and resolves the current planning period. We limit us to a look ahead of one and two periods. This approach will generate one solution and no bound to measure its quality. A similar approach can be found in Federgruen et al. [12], where it is classified as a progressive interval heuristic. Other kinds of rolling horizon and fix-and-relax heuristics can be found in Clark [9] and Beraldi et al. [3]. The second proposal is based on a Lagrangian decomposition. It will generate an optimistic bound. In addition it can be used as a multistart procedure for the first heuristic. By applying this approach we can generate several solutions as well as bounds.

## 4.1 Heuristic to generate solutions

The problem is divided into one problem for each time period and the problem is solved sequentially over the total time periods, denoted $T$. The constant $k$ expresses how much future information will be used when solving the problem. If the constant $k = 0$, only one time period at a time will be considered and if $k =\mid T \mid -1$ the problem will be solved including all time periods simultaneously. The problem including time period $t$ to $t + k$ will be denoted $\mathrm{P1}_{t,t+k}$. The main steps of the heuristic can be summarized as follows.

**Step 0.** Let time period $t = 1$. Decide the value of $k$.

**Step 1.** Solve problem $\text{P1}_{t,t+k}$.

**Step 2.** Fixation step

- **(i)** If $t = 1$ then fix the binary variables for the alternatives and the contracts.
- **(ii)** Fix the continuous variables for time period $t$.
- **(iii)** Fix the one-valued binary terminal variables for time period $t$.

**Step 3.** If $t =\mid T \mid -k$ then stop, otherwise let $t = t + 1$ and go to Step 1.

When $k = 0$ the objective function denoted $z^{H0}$ is produced in the last problem solved. When $k \geq 1$ we also have to find the right value of the objective function, $z^{H1}$, $z^{H2}$ and so on, by putting all the variables into the objective function. We have used the heuristic for $k = 0$, heuristic (a), and for $k = 1$, heuristic (b). The decision about which alternative to use is made in time period one. Whether or not this choice is relevant depends on the demand structure. The demand can be exactly the same for every time period and then the choice of alternative will be good, but if the demand changes during the year, growing or declining, the chosen alternative could lead to an infeasible solution. The same conditions apply to the contracts. The decisions with regard to whether or not to accept contracts are made in the first time period. If the optimal solution was to store some of the pulp until the next time period, the heuristic (a) would probably not give us a high quality solution. On the other hand, if the demand is the same in each time period the solution to this heuristic can be rather good. The number of terminals used can be high depending on the fact that the decisions in the earlier time periods have a bigger influence on the choices. Heuristic (b) uses more time than heuristic (a), as the problem is about twice as large. The strong advantage is that heuristic (b) can take care of storing, as information about the next coming time period is included. However, as there is no information about the two next time periods, the heuristic can not deal with growing demand, requiring much storage in the beginning of the planning horizon.

## 4.2 Lagrangian heuristic method

The other solution method in this paper is a Lagrangian heuristic method. The procedure for the method can be described as follows. First some reformulations of [P1] are made in order to be able to use the Lagrangian decomposition approach. After the reformulations have been made to [P1], we call the model [P2]. The model [P2] is presented in the Appendix. The optimal solution of [P2] will, of course, be the same as for [P1]. Next step is to relax constraints leading to a division into one subproblem per time period. Then, each subproblem is solved separately. Due to the fact that we have a maximization problem, the total objective function value of the subproblems will be a higher value compared to the optimal objective value for the original problem, and can therefore be used as an optimistic estimate. A heuristic uses the solutions for the subproblems in order to produce feasible solutions. This feasible solution can be used as a pessimistic estimate. The subgradient method uses the solution to decide the values of the Lagrange multipliers. Then the subproblems are solved using the new multiplier values and this result in new estimates. The process of updating the multipliers is iterative and will continue until the estimates are close enough to each-other or after a fixed number of iterations. The procedure for using the Lagrangian heuristic method in this paper is presented in Figure 6.

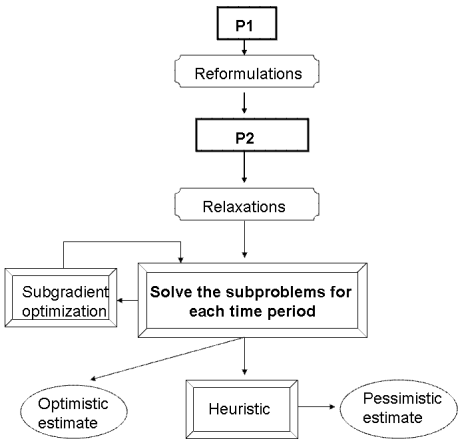


Figure 6: The procedure of the Lagrangian heuristic.

The Lagrangian decomposition method [17, 25], is used. The procedure in the method is to duplicate variables and relax the constraints holding the original and the duplicated ones at the same level. The duplication of variables will make the problem larger, but on the other hand, it can enable the division of the problem into appropriate parts. The main principle behind this methodology is described below in the following steps.

**(i)** Assume we have the problem

$$min \;\; \mathbf{c}^T\mathbf{x}, s.t. \;\; \mathbf{Ax} = \mathbf{b}, \mathbf{Bx} = \mathbf{d}, \mathbf{x} \in X$$

**(ii)** Duplicate the variables and add constraints with ($\mathbf{c} = \mathbf{c1} + \mathbf{c2}$)

$$min \;\; \mathbf{c1}^T\mathbf{x} + \mathbf{c2}^T\mathbf{y}, s.t. \;\; \mathbf{Ax} = \mathbf{b}, \mathbf{By} = \mathbf{d}, \mathbf{x} = \mathbf{y}, \mathbf{x} \in X, \mathbf{y} \in X$$

**(iii)** Relax $\mathbf{x} = \mathbf{y}$ with multipliers $\mathbf{u}$

$$min \;\; \mathbf{c1}^T\mathbf{x} + \mathbf{c2}^T\mathbf{y} + \mathbf{u}^T(\mathbf{x} - \mathbf{y}), s.t. \;\; \mathbf{Ax} = \mathbf{b}, \mathbf{By} = \mathbf{d}, \mathbf{x} = \mathbf{y}, \mathbf{x} \in X, \mathbf{y} \in X$$

**(iv)** The problem is separable in $\mathbf{x}$ and $\mathbf{y}$. We can use a subgradient method to find the optimal values of the multipliers $\mathbf{u}$.

$$min \;\; (\mathbf{c1} + \mathbf{u})^T\mathbf{x}, s.t. \;\; \mathbf{Ax} = \mathbf{b}, \mathbf{x} \in X$$

$$min \;\; (\mathbf{c2} - \mathbf{u})^T\mathbf{y}, s.t. \;\; \mathbf{By} = \mathbf{d}, \mathbf{y} \in X$$

#### 4.2.1 Reformulations of the model

In the first part of this section, new variables and constraints added to the model are presented. Thereafter, constraints added to the model in order to get better bounds are presented. The variables which concern choosing alternatives, terminals, and contracts have been increased by the index $t$ for time periods in order to get separate subproblems for each time period. The new variables will be:

$$z_{at} = \begin{cases} 1, & \text{if production alternative } a \text{ is used at the pulp mills,} \\ & \text{in time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{ct} = \begin{cases} 1, & \text{if contract } c \text{ is accepted in time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

$$z^T_{jt} = \begin{cases} 1, & \text{if terminal } j \text{ is used in time period } t, \\ 0, & \text{otherwise.} \end{cases}$$

In the presented problem, the variables for storing have been duplicated in order to decompose the problem into subproblems for each time period. The old storage variables concerned the storage level at the end of the time period, $L^F_{dmt}, L^M_{ipt}, L^{HT}_{jpt}$ and $L^{LT}_{jpt}$, and the new, duplicated ones will present the storage level at the beginning of the next time period. The new storage variables are expressed as

$$\begin{aligned}
L2^F_{dmt} &= \text{volume of raw material } m \text{ that is stored at district } d \\
&\quad \text{at the beginning of time period } t,\ t \geq 2, \\
L2^M_{ipt} &= \text{volume of product } p \text{ that is stored at pulp mill } i \\
&\quad \text{at the beginning of time period } t,\ t \geq 2, \\
L2^{HT}_{jpt} &= \text{volume of product } p \text{ that is stored at harbour terminal } j \\
&\quad \text{at the beginning of time period } t,\ t \geq 2, \text{ and} \\
L2^{LT}_{jpt} &= \text{volume of product } p \text{ that is stored at inland terminal } j \\
&\quad \text{at the beginning of time period } t,\ t \geq 2.
\end{aligned}$$

The following new constraints will appear in the model after the reformulations have been made.

$$L2^F_{dmt} = L^F_{dmt-1},\ \forall d \in D,\ m \in M,\ t \geq 2. \tag{35}$$
$$L2^M_{ipt} = L^M_{ipt-1},\ \forall i \in I,\ p \in P,\ t \geq 2. \tag{36}$$
$$L2^{HT}_{jpt} = L^{HT}_{jpt-1},\ \forall j \in J_H,\ p \in P,\ t \geq 2. \tag{37}$$
$$2L^{LT}_{lpt} = L^{LT}_{lpt-1},\ \forall l \in J_L,\ p \in P,\ t \geq 2. \tag{38}$$
$$z_{c1} = z_{ct},\ \forall c \in C,\ t \geq 2. \tag{39}$$
$$z_{a1} = z_{at},\ \forall a \in A,\ t \geq 2. \tag{40}$$
$$z^T_{j1} = z^T_{jt},\ \forall j \in J,\ t \geq 2. \tag{41}$$

In order to avoid exceeding the storage capacity at the pulp mills and terminals, we have to add the constraints

$$L2^M_{ipt} \leq L^M_{max},\ \forall i \in I,\ p \in P,\ t \geq 2, \tag{42}$$
$$L2^{HT}_{jpt} \leq L^{HT}_{max},\ \forall j \in J_H,\ p \in P,\ t \geq 2, \quad \text{and} \tag{43}$$
$$2L^{LT}_{jpt} \leq L^{LT}_{max},\ \forall j \in J_L,\ p \in P,\ t \geq 2. \tag{44}$$

for the new storage variables. All the balancing constraints have also been modified in such a way that the variables expressing the time period, $t-1$, are exchanged for the new storing variables. The objective function has also been modified in some ways. The fixed terminal costs, as well as the alternative costs, are divided by T in order to get the correct fixed costs for each time period. Half of the costs for storage is charged to the storage variables concerning storage at the end of the time period, $L^F_{dmt}$, $L^M_{ipt}$, $L^{TH}_{jpt}$, and $L^{LT}_{jpt}$ and the other half of the costs is charged to the storage at the beginning of the time period, $L2^F_{dmt}$, $L2^M_{ipt}$, $L2^{TH}_{jpt}$, and $L2^{LT}_{jpt}$. To strengthen the bounds, the following constraints were added to the problem.

$$L^F_{dmt} \leq s_{md},\ \forall d \in D, m \in M, t \in T \tag{45}$$
$$L2^F_{dmt} \leq s_{md},\ \forall d \in D, m \in M, t \in T \tag{46}$$
$$\sum_{i \in I} \sum_{p \in P} \sum_{r \in R} w_{mdiprt} \leq s_{md},\ \forall d \in D, m \in M, t \in T \tag{47}$$
$$L^F_{dmt} \leq s_t \sum_{m \in M} s_{md},\ \forall d \in D, t \in T, t \geq 2 \tag{48}$$
$$L2^F_{dmt} \leq s_{t-1} \sum_{m \in M} s_{md},\ \forall d \in D, t \in T, t \leq \mid T \mid -1 \tag{49}$$
$$L^M_{ipt} \leq k \sum_{r \in R} u_{iprt} \tag{50}$$

$$
\begin{aligned}
L2^M_{ipt} &\leq k \sum_{k \in R_A} \sum_{j \in J_k} x^A_{kijpt} + \sum_{k \in R_B} \sum_{j \in J_k} x^B_{kijpt} + \\
&\sum_{j \in J_H} x^S_{ijpt} + \sum_{s \in Q_S} \sum_{q \in Q} y^{train}_{isqpt} + \sum_{s \in Q_S} \sum_{q \in Q} y^{lorry}_{isqpt} \qquad (51)\\
L^{HT}_{jpt} &\leq k \sum_{k \in R_A} \sum_{i \in I_k} x^A_{kijpt} + \sum_{k \in R_B} \sum_{i \in I_k} x^B_{kijpt} + \sum_{i \in I} x^S_{ijpt} \qquad (52)\\
L2^{HT}_{jpt} &\leq k \sum_{h \in J_L} y^T_{hlpt} + \sum_{s \in S} \sum_{q \in Q_S} y^Q_{isqpt} \qquad (53)\\
L^{LT}_{jpt} &\leq k \sum_{h \in J_H} y^T_{hlpt} \qquad (54)\\
L2^{LT}_{jpt} &\leq k \sum_{s \in S} \sum_{q \in Q_S} y^Q_{isqpt} \qquad (55)
\end{aligned}
$$

The constraints (45,47) are merely to limit the sizes of the variables concerned and they are redundant in the original problem. The constraints (48) and (49) are redundant in the original problem as the demand and supply are well matched in the cases. That means that parts of the supply available in each time period have to be used and almost all of the total supply has used by the end of the planning horizon. The constraints (50) - (55) are used to prevent the storing of products that have not been produced in the last time period. These constraints express the fact that products have to be turned over and that products that are produced for example in time period one and not in periods two or three, can not be stored from time period one to time period 4. The value for constant $k$ is 0.05 in our cases. The new modified model, [P2], before relaxations can be found in the Appendix.

### 4.2.2 Relaxations

The new constraints above, (35–41), are relaxed together with the constraints concerning the supply presented earlier;

$$\sum_{i \in I} \sum_{p \in P} \sum_{r \in R} \sum_{t \in T} w_{mdiprt} \leq s_{md}, \ \forall d \in D, \ m \in M. \quad (1)$$

The Lagrange multipliers connected to each of the relaxed constraints are presented below:

$$
\begin{aligned}
\lambda^F_{dmt} &= \text{storage of raw material } m, \text{ at district } d, \text{ in time period } t, \ t \geq 2,\\
\lambda^M_{ipt} &= \text{storage of product } p, \text{ at pulp mill } i, \text{ in time period } t, \ t \geq 2,\\
\lambda^{HT}_{jpt} &= \text{storage of product } p, \text{ at harbour terminal } i, \text{ in time period } t, \ t \geq 2,\\
\lambda^{LT}_{jpt} &= \text{for storage of product } p, \text{ at inland terminal } i, \text{ in time period } t, \ t \geq 2,\\
\gamma_{md} &= \text{supply of raw material } m, \text{ at district } d,\\
\delta^T_{jt} &= \text{binary variables for using terminal } j, \text{ in time period } t, \ t \geq 2,\\
\delta^C_{ct} &= \text{binary variables for accepting contract } c, \text{ in time period } t, \ t \geq 2,\\
&\text{and}\\
\delta^A_{at} &= \text{binary variables for choosing alternative } a, \text{ in time period } t, \ t \geq 2.
\end{aligned}
$$

The restrictions on the multipliers concerning supply of raw materials, $\gamma_{md}$ are that they are non-negative. All the other multipliers are not restricted. The constraints that will be relaxed are marked with their relevant multipliers in the model [P2] in the Appendix.

### 4.2.3 Lagrangian relaxation

The relaxations of the constraints will decompose the model into one problem for each time period, $t$. The Lagrangian relaxed objective function can now be expressed as

$$\theta(\lambda,\gamma,\delta) = \sum_{m\in M}\sum_{d\in D}\gamma_{md}s_{md} + \sum_{t\in T}\theta_t(\lambda,\gamma,\delta), \text{ where}$$

$$\begin{aligned}\theta_t(\lambda,\gamma,\delta) &= S_t - C_t^{train/lorry} - C_t^{routes} - C_t^{return} - C_t^{spot} - C_t^{flow} - \\ &\quad C_t^{term-term} - C_t^{distr} - C_t^{rec}\end{aligned}$$

$$- \sum_{m\in M}\sum_{d\in D}\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}(c_{mdi}^{TR} + c_{md}^{P} + \gamma_{md})w_{mdiprt} - \sum_{a\in A}(c_a^{alt}/T + \sum_{t\geq 2}\delta_{at}^{A})z_{at} -$$
$$\sum_{j\in J}(c_j^{FT}/T + \sum_{t\geq 2}\delta_{jt}^{T})z_{jt}^{T} - \sum_{c\in C}\sum_{t\geq 2}\delta_{ct}^{C}z_{ct} +$$

$$\sum_{d\in D}\sum_{m\in M}(c_d^{Fstor}/2 + \lambda_{dmt+1}^{F})L_{dmt}^{F} + \sum_{i\in I}\sum_{p\in P}(c_i^{Mstor}/2 + \lambda_{ipt+1}^{M})L_{ipt}^{M}$$
$$+ \sum_{j\in J_H}\sum_{p\in P}(c_j^{Tstor}/2 + \lambda_{jpt+1}^{HT})L_{jpt}^{HT} + \sum_{j\in J_L}\sum_{p\in P}(c_j^{Tstor}/T + \lambda_{jpt+1}^{LT})L_{jpt}^{LT},$$

for $t = 1$ and

$$\begin{aligned}\theta_t(\lambda,\gamma,\delta) &= S_t - C_t^{train/lorry} - C_t^{routes} - C_t^{return} - C_t^{spot} - C_t^{flow} - \\ &\quad C_t^{term-term} - C_t^{distr} - C_t^{rec}\end{aligned}$$

$$- \sum_{m\in M}\sum_{d\in D}\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}(c_{mdi}^{TR} + c_{md}^{P} + \gamma_{md})w_{mdiprt} - \sum_{a\in A}(c_a^{alt}/T - \delta_{at}^{A})z_{at} -$$

$$\sum_{j\in J}(c_j^{FT}/T - \delta_{jt}^{T})z_{jt}^{T} + \sum_{c\in C}\delta_{ct}^{C}z_{ct} - \sum_{d\in D}\sum_{m\in M}(c_d^{Fstor}/2 - \lambda_{dmt+1}^{F})L_{dmt}^{F} -$$
$$\sum_{d\in D}\sum_{m\in M}(c_d^{Fstor}/2 + \lambda_{dmt}^{F})L2_{dmt}^{F} - \sum_{i\in I}\sum_{p\in P}(c_i^{Mstor}/2 - \lambda_{ipt+1}^{M})L_{ipt}^{M} -$$

$$\sum_{i\in I}\sum_{p\in P}(c_i^{Mstor}/2 + \lambda_{ipt}^{M})L2_{ipt}^{M} - \sum_{j\in J_H}\sum_{p\in P}(c_j^{Tstor}/2 - \lambda_{jpt+1}^{HT})L_{jpt}^{HT} -$$
$$\sum_{j\in J_H}\sum_{p\in P}(c_j^{Tstor}/2 + \lambda_{jpt}^{HT})L2_{jpt}^{HT} - \sum_{j\in J_L}\sum_{p\in P}(c_j^{Tstor}/2 - \lambda_{jpt+1}^{LT})L_{jpt}^{LT} -$$
$$\sum_{j\in J_L}\sum_{p\in P}(c_j^{Tstor}/2 + \lambda_{jpt}^{LT})L2_{jpt}^{LT} \quad \text{for } 2 \leq t \leq T-1,$$

and

$$\begin{aligned}\theta_t(\lambda,\gamma,\delta) &= S_t - C_t^{train/lorry} - C_t^{routes} - C_t^{return} - C_t^{spot} - C_t^{flow} - \\ &\quad C_t^{term-term} - C_t^{distr} - C_t^{rec}\end{aligned}$$

$$- \sum_{m\in M}\sum_{d\in D}\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}(c_{mdi}^{TR} + c_{md}^{P} + \gamma_{md})w_{mdiprt} - \sum_{a\in A}(c_a^{alt}/T - \delta_{at}^{A})z_{at} -$$
$$\sum_{j\in J}(c_j^{FT}/T - \delta_{jt}^{T})z_{jt}^{T} + \sum_{c\in C}\delta_{ct}^{C}z_{ct} - \sum_{d\in D}\sum_{m\in M}c_d^{Fstor}L_{dmt}^{F} -$$
$$\sum_{d\in D}\sum_{m\in M}(c_d^{Fstor}/2 + \lambda_{dmt}^{F})L2_{dmt}^{F} - \sum_{i\in I}\sum_{p\in P}(c_i^{Mstor}/2 + \lambda_{ipt}^{M})L2_{ipt}^{M} -$$
$$\sum_{j\in J_H}\sum_{p\in P}(c_j^{Tstor}/2 + \lambda_{jpt}^{HT})L2_{jpt}^{HT} - \sum_{j\in J_L}\sum_{p\in P}(c_j^{Tstor}/2 + \lambda_{jpt}^{LT})L2_{jpt}^{LT} \quad \text{for } t = T.$$

The components $S_t$, $C_t^{train/lorry}$, $C_t^{routes}$, $C_t^{return}$, $C_t^{spot}$, $C_t^{flow}$, $C_t^{term-term}$, $C_t^{distr}$, and $C_t^{rec}$ were presented in Section 2 including all time periods.

The dual problem can now be defined as

$\theta^* = \ min\ \theta(\lambda, \gamma, \delta)$.

For any fixed $\lambda, \gamma$, and $\delta$ we will get an optimistic estimate of the value of $z^{P2}$, which is $\theta(\lambda, \gamma, \delta) \geq z^*$. We want to find the best optimistic estimate, that is to say the lowest optimistic estimate, and this problem can be solved using subgradient optimization. The subgradient optimization method is often used for non-differentiable functions.

#### 4.2.4 Subgradient optimization

The procedure of the subgradient optimization is described in the following steps:

**1.** Start with all the multipliers being zero. Let the iteration number, $n = 0$. Let the lower bound, LBD $= -\infty$, and the upper bound, UBD $= \infty$.

**2.** Solve the Lagrangian subproblem for each $t$. Denote the value of the objective function, $\theta^n$. Update UBD = min (UBD, $\theta^n$).

**3.** Try to find a feasible solution from heuristic (a) using the current multipliers. Denote the value of the objective function, $\bar{z}$, if the corresponding solution is feasible. Update LBD = max (LBD, $\bar{z}$).

**5.** Evaluate a subgradient $\mu = (\mu_\lambda, \mu_\gamma, \mu_\delta)$ as

$$\mu_\lambda^{F(n)} = L2_{dmt}^F - L_{dmt-1}^F,\ \forall d \in D,\ m \in M,\ t \geq 2,$$

$$\mu_\lambda^{M(n)} = L2_{ipt}^M - L_{ipt-1}^M,\ \forall i \in I,\ p \in P,\ t \geq 2,$$

$$\mu_\lambda^{HT(n)} = L2_{jpt}^{HT} - L_{jpt-1}^{HT},\ \forall j \in J_H,\ p \in P,\ t \geq 2,$$

$$\mu_\lambda^{LT(n)} = L2_{jpt}^{LT} - L_{jpt-1}^{LT},\ \forall j \in J_L,\ p \in P,\ t \geq 2,$$

$$\mu_\gamma^{(n)} = \sum_{i \in I} \sum_{p \in P} \sum_{r \in R} \sum_{t \in T} w_{mdiprt} - s_{md},\quad \forall m \in M,\ d \in D,$$

$$\mu_\delta^{T(n)} = z_{j1}^T - z_{jt}^T,\ \forall j \in J,\ t \geq 2,$$

$$\mu_\delta^{A(n)} = z_{a1} - z_{at},\ \forall a \in A,\ t \geq 2,$$

$$\mu_\delta^{C(n)} = z_{c1} - z_{ct},\ \forall c \in C,\ t \geq 2,$$

where the variables have the values from the solution in step 2.

**6.** If the subgradient is the zero vector, or if the gap between the UBD and the LBD is sufficiently small, we terminate. We have then found the optimal or at least a near optimal solution. Additionally, if some maximum number of iterations is exceeded we terminate.

**7.** Update the multipliers as

$$\lambda_{dmt}^{F(n+1)} = \lambda_{dmt}^{F(n)} + step^{(n)} \mu_\lambda^{F(n)},\ \forall d \in D,\ m \in M,\ t \geq 2,$$

$$\lambda_{ipt}^{M(n+1)} = \lambda_{ipt}^{M(n)} + step^{(n)} \mu_\lambda^{M(n)},\ \forall i \in I,\ p \in P, t\ \geq 2,$$

$$\lambda_{jpt}^{HT(n+1)} = \lambda_{jpt}^{HT(n)} + step^{(n)} \mu_\lambda^{HT(n)},\ \forall j \in J_H,\ p \in P,\ t \geq 2,$$

$$\lambda_{jpt}^{LT(n+1)} = \lambda_{jpt}^{LT(n)} + step^{(n)}\mu_{\lambda}^{LT(n)},\ \forall j \in J_L,\ p \in P,\ t \geq 2,$$

$$\gamma_{md}^{(n+1)} = max(\gamma_{md}^{(n)} + step^{(n)}\mu_{\gamma}^{(n)}, 0),\ \forall d \in D,\ m \in M,$$

$$\delta_{jt}^{T(n+1)} = \delta_{jt}^{T(n)} + step^{(n)}\mu_{\delta}^{T(n)},\ \forall j \in J,\ t \geq 2,$$

$$\delta_{at}^{A(n+1)} = \delta_{at}^{A(n)} + step^{(n)}\mu_{\delta}^{A(n)},\ \forall a \in A,\ t \geq 2,$$

$$\delta_{ct}^{C(n+1)} = \delta_{ct}^{C(n)} + step^{(n)}\mu_{\delta}^{C(n)},\ \forall c \in C,\ t \geq 2,$$

using some step length, $step^{(n)}$.

**8.** Let n = n+1. Go to **2**.

The choice of step length when updating the Lagrangian multipliers follows the rule suggested by Poljak [27];

$$step^{(n)} = \frac{0.99^n \alpha(\theta^n - LBD)}{\|\mu^n\|^2},$$

where $\alpha$ is a scalar $0 \leq \alpha \leq 2$. We have used $\alpha = 0.5$. For every iteration, $n$, we decrease the step length according to $0.99^n$.

It can be shown that the optimistic estimate of the optimal objective function yielded by a Lagrangian relaxation is always at least as good as the estimate yielded by solving the linear program relaxation of the problem, see Geoffrion [15]. If the subproblems have the integrality property, the LP-relaxation will be at the same level as the Lagrangian relaxation of the problem. In this problem, we know that the subproblems do not have the integrality property, so we can expect a better optimistic estimate from the Lagrangian relaxation compared to the LP-relaxation estimate, although it also can produce the same value as the LP-relaxation.

#### 4.2.5 Lagrangian heuristic

The subproblems are solved one by one. The difference from the heuristic (a) is that the multipliers from the Lagrangian relaxation problem are used to get a feasible solution. The subproblems are also solved a lot of times and the heuristic (a) is only solved once. If the heuristic (a) is able to produce a feasible solution, this solution obtained will be the same as the first one from the Lagrangian heuristic. The last step in the Lagrangian heuristic is to fix all the variables and get the right costs based on the original coefficients.

## 5 Computational results

In all the methods, the modelling language AMPL [1], version 20051214, has been used and the commercial solver CPLEX [22], version 10.0, with default setting, has been used as solver. The MIP-gap expresses the accepted quality of the solution and the default setting in CPLEX is 0.01%. The instances have been solved on a 2.67 MHz Xeon processor with 2 GB RAM available.

The test cases are based on real data from Södra Cell AB. The levels and relations between the different coefficients are the same as in the real cases. The different test cases are presented in Table 1. For each case, different time periods have been tested, as well as different demand structures. The demand for each time period can be the same or vary. The supply, on the other hand, is larger in the first time periods compared to the rest of the time periods when the planning period is one year starting with January. The demand and supply structures in the instances are illustrated in Figure 8. The vertical axes show the amounts and the horizontal axes show the times. We have also tested different variants of the number of fixed orders.

Table 4: Description of the test cases and their sizes when 12 time periods are included.

| Number of | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 |
|---|---|---|---|---|---|---|
| Pulp mills | 4 | 4 | 4 | 4 | 4 | 4 |
| Harbour terminals | 10 | 10 | 10 | 10 | 10 | 10 |
| Inland terminals | 1 | 1 | 1 | 1 | 1 | 1 |
| Products | 5 | 7 | 10 | 12 | 19 | 19 |
| Assortments | 2 | 4 | 4 | 2 | 2 | 8 |
| Districts | 5 | 10 | 10 | 5 | 5 | 32 |
| Recipes | 9 | 19 | 34 | 23 | 33 | 33 |
| Alternatives | 2 | 7 | 15 | 5 | 5 | 5 |
| Delivery points | 10 | 15 | 30 | 10 | 10 | 193 |
| Orders | 72 | 72 | 145 | 84 | 91 | 360 |
| Contracts | 62 | 62 | 135 | 66 | 74 | 209 |
| Routes | 1464 | 1464 | 1464 | 1464 | 1464 | 1464 |
| Continuous variables | 370,811 | 559,115 | 998,052 | 955,284 | 1,545,552 | 3,489,036 |
| Binary variables | 675 | 680 | 761 | 682 | 690 | 825 |
| Constraints | 71,289 | 84,497 | 117,689 | 96,553 | 120,929 | 274,217 |

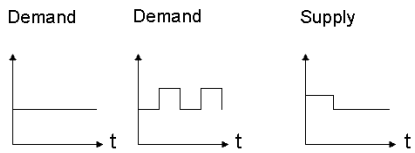


Figure 7: Demand and supply structures in the instances.

The results are presented in Tables 2–3, where the times are given in CPU. The term $\lambda_0$ expresses the Lagrangian heuristic started by the multipliers with the value set to zero. The term $\lambda_{LP}$ expresses the Lagrangian heuristic started by the values of the multipliers set to the dual of the relaxed constraints in the LP solution. The Lagrangian heuristic method is terminated after 200 iterations or if the time exceeds 20 hours. The results after 2, 10 and 200 iterations are presented. The dual objective value after one iteration is not presented in the tables but it gets the same value as the heuristic (a) if the start values of the multipliers are zero ($\lambda_0$). The denotation *inf* means that no feasible solution is found. The objective function values are given in profit units, in order not to reveal the actual values. The level of the maximum storage of pulp products is of importance for the level of the upper bounds. A lower level of maximum storage will lead to better (lower) upper bounds. The differences in prices between products, as well as the different outcomes from the recipes, are also important factors to consider. Smaller differences of these properties between products lead to better upper bounds, produced by the Lagrangian heuristics method.

Table 2 presents the results when 12 time periods, each with the same demand, are included. The terms AF and PF denote that all orders are free and that parts of the orders are free, respectively. When comparing the difference in percent between the produced feasible solutions and the optimal values, one has to remember that the size of the gap depends on the relation between revenues and costs, and that this value can be both positive and negative. The Lagrangian heuristic, ($\lambda_0$), often starts by producing optimistic estimates of the objective values which are of low qualities and the rate of convergence is slow. On the other hand, it often produces many good feasible solutions. The opposite conditions appeal to the Lagrangian heuristic, ($\lambda_{LP}$), which in many instances produces very good upper bounds, much below the LP-values, but the feasible solutions are of low quality. These conditions have been observed in Holmberg and Hellstrand [21]. Indeed, there exists exceptions, see for example Case 5a and Case 6a. The heuristic (a) and the heuristic (b) often produce quite good solutions. We can

Table 2: Results when the demand is the same in each of the time periods and 12 time periods are included.

| | Case 1a AF | Case 1b PF | Case 2a AF | Case 2b PF | Case 3a AF | Case 3b PF |
|---|---|---|---|---|---|---|
| Optimal obj. | 83,959 | 64,033 | 42,148 | 39,624 | 367,156 | 362,019 |
| Time | 15 min | 16 min | 45 min | 25 min | 3.5 h | 2.1 h |
| LP-value | 120,415 | 85,169 | 71,589 | 62,975 | 597,459 | 545,176 |
| Time | 2 min | 1 min | 2 min | 2,2 min | 5 min | 10 min |
| Heuristic (a) | 82,552 | 62,626 | 40,754 | 38,228 | *inf* | 349,152 |
| Time | 3 min | 3 min | 2 min | 3 min | - | 13 min |
| Heuristic (b) | 83,192 | 63,266 | 41,550 | 39,624 | *inf* | 360,806 |
| Time | 5 min | 5 min | 5 min | 6 min | - | 16 min |
| ***Lagrange,*** $\lambda_0$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 288,532 | 268,599 | 251,732 | 248,500 | 561,704 | 575,871 |
| Best primal obj. | 82,552 | 62,626 | 40,754 | 38,228 | *inf* | 360,806 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 243,331 | 223,396 | 208,483 | 204,764 | 536,499 | 531,586 |
| Best primal obj. | 83,959 | 64,033 | 42,148 | 39,624 | 344,590 | 362,019 |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 227,133 | 205,603 | 190,951 | 187,621 | 504,287 | 514,101 |
| Best primal obj. | 83,959 | 64,033 | 42,148 | 39,624 | 344,590 | 362,019 |
| ***Lagrange,*** $\lambda_{LP}$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 98,975 | 67,060 | 58,436 | 42,236 | 495,169 | 463,492 |
| Best primal obj. | 81,565 | *inf* | -970 | 39,124 | *inf* | 351,361 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 98,975 | 67,060 | 58,436 | 42,236 | 495,169 | 463,492 |
| Best primal obj. | 81,821 | *inf* | -953 | 39,124 | *inf* | 351,498 |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 98,975 | 67,060 | 58,436 | 42,236 | 495,169 | 463,492 |
| Best primal obj. | 81,821 | *inf* | -953 | 39,124 | *inf* | 351,498 |
| Average itr. time | 3.5 min | 2 min | 3 min | 3 min | 12 min | 13 min |

use the Lagrangian heuristic ($\lambda_0$), and the heuristics (a) and (b) to get feasible solutions and their qualities can be decided by comparing them to the values of the optimistic bounds we get from the Lagrangian heuristic ($\lambda_{LP}$). In Cases 3a, 5b, 6a, and 6b, the heuristics have problems finding a feasible solution. The reason behind this is that contracts have been accepted including too high minimum demand due to that the supply is larger in the beginning of the planning horizon. The raw materials needed for the orders may not be enough in the end of the planning horizon. Even the Lagrangian method has problems finding good feasible solutions. In Case 4a, all orders are free and the optimal function value is 101,824. In Case 4b, some of the unprofitable orders are fixed, which explains the negative optimal function value.

Table 3 presents the results when parts of the orders are free. The terms 4t and 12t denote the number of time periods included in the model, four and twelve, respectively. The demand is not the same in each time period when 12 time periods are included. However, when four time periods are included the demand is aggregated and therefore the demand could be the same in each time period. The solutions of the problems that include fewer time periods will be a relaxation of the solutions of the problems which include several time periods. To get a feasible solution, the products often have to be stored as early as in time period one to be able to fit the demand in time period three. Neither of the heuristics, (a) and (b) are able to produce a feasible solution in such cases. The choice of alternatives in time period one or in the two first periods can also lead to a bad or an infeasible solution in both variants of the heuristics. Under these circumstances the Lagrangian heuristic method can help us to get feasible solutions. In Larsson and Patriksson [23] the importance of the kind of heuristics used in the Lagrangian relaxation method is discussed and analyzed. They classified the heuristics in conservative and radical heuristics depending on how much the Lagrangian subproblem solution can be used in

Table 2 (contd.): Results when the demand is the same in each of the time periods and 12 time periods are included.

| | Case 4a AF | Case 4b PF | Case 5a AF | Case 5b PF | Case 6a AF | Case 6b PF |
|---|---|---|---|---|---|---|
| Optimal obj. | 101,824 | -581,340 | 81,124 | -46,617 | 169,119 | 4,496 |
| Time | 9.4 h | 2.5 h | 48 min | 3.2h | 4.1h | 83h |
| LP-value | 138,978 | -556,223 | 113,382 | -31,885 | 203,500 | 40,949 |
| Time | 12 min | 10 min | 5 min | 4 min | 10 min | 10 min |
| Heuristic (a) | 64,684 | -601,050 | 26,105 | *inf* | *inf* | *inf* |
| Time | 14 min | 8 min | 5 min | - | - | - |
| Heuristic (b) | 88,441 | -581,882 | 59,146 | *inf* | 154,512 | -8,074 |
| Time | 35 min | 17 min | 28 min | - | 1,5 h | 3 h |
| ***Lagrange,*** $\lambda_0$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 221,876 | -432,547 | 201,182 | 100,238 | 286,870 | 164,098 |
| Best primal obj. | 69,459 | -583,070 | 33,907 | *inf* | 115,540 | *inf* |
| *After 10 itr* | | | | | | |
| Best dual obj. | 221,876 | -440,485 | 201,182 | 100,238 | 286,870 | 164,098 |
| Best primal obj. | 69,459 | -581,340 | 33,908 | *inf* | 115,540 | *inf* |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 221,876 | -500,030 | 197,005 | 100,238 | 275,512 | 148,082 |
| Best primal obj. | 69,459 | -581,340 | 33,908 | *inf* | 115,540 | *inf* |
| ***Lagrange,*** $\lambda_{LP}$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 134,127 | -578,526 | 101,412 | -34,874 | 191,826 | 33,842 |
| Best primal obj. | 69,245 | -581,574 | 68,186 | *inf* | 155,533 | *inf* |
| *After 10 itr* | | | | | | |
| Best dual obj. | 134,127 | -578,526 | 101,412 | -34,874 | 191,826 | 33,842 |
| Best primal obj. | 69,245 | -581,573 | 68,186 | *inf* | 155,653 | *inf* |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 133,444 | -578,581 | 100,964 | -34,874 | 190,100 | 33,739 |
| Best primal obj. | 69,245 | -581,480 | 68,757 | *inf* | 159,600 | *inf* |
| Average itr. time | 40 min | 18 min | 6 min | 11 min | 15 min | 17 min |

order to find feasible solutions. They also relate the type of the heuristics to the size of the duality gap. In case of small gaps, it is sufficient to use conservative heuristics and if the gaps are large, it is necessary to use radical heuristics to get feasible solutions of high quality. In the Lagrangian heuristic presented in our paper only the solution of the first time period as well as the multipliers for all of the time periods is used to produce feasible solutions. Hence, our Lagrangian heuristic can be classified as radical, which may explain the fact that it often produces many feasible solutions of high quality. In order to get better optimistic estimates, we have tested different ways of deciding step-length when updating the Lagrangian multipliers. We have also tested other values of the scalar, $\alpha$. The testing has not led to any significant improvements.

Table 3: Results when parts of the orders are free and the demand is not the same in each time periods when 12 time periods are included.

| | Case 1c 4t | Case 1d 12t | Case 2c 4t | Case 2d 12t | Case 3c 4t | Case 3d 12t |
|---|---|---|---|---|---|---|
| Optimal obj. | 293,544 | 255,615 | 87,254 | 86,534 | 143,484 | 143,017 |
| Time | 10 min | 21 min | 12 min | 2 h | 30 min | 4.2 h |
| LP-value | 305,052 | 275,075 | 117,730 | 117,145 | 356,748 | 356,288 |
| Time | 3s | 1 min | 5 s | 6 min | 5 s | 5 min |
| Heuristic (a) | 292,821 | *inf* | 87,254 | 85,140 | 143,460 | *inf* |
| Time | 30 s | - | 25 s | 4 min | 1,5 min | - |
| Heuristic (b) | 293,544 | *inf* | 87,254 | 85,726 | 143,484 | 120,119 |
| Time | 1 min | - | 1,5 min | 6 min | 4 min | 20 min |
| ***Lagrange,*** $\lambda_0$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 373,461 | 453,907 | 169,408 | 293,409 | 235,166 | 343,173 |
| Best primal obj. | 292,821 | *inf* | 87,254 | 85,140 | 143,460 | 115,671 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 343,623 | 434,498 | 140,057 | 248,922 | 204,718 | 295,813 |
| Best primal obj. | 293,544 | *inf* | 87,254 | 86,534 | 143,484 | 116,829 |
| *After 200 itr or 10h* | | | | | | |
| Best dual obj. | 322,984 | 403,399 | 125,128 | 232,119 | 185,322 | 282,954 |
| Best primal obj. | 293,544 | *inf* | 87,254 | 86,534 | 143,484 | 116,829 |
| ***Lagrange,*** $\lambda_{LP}$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 298,499 | 263,599 | 98,145 | 103,536 | 170,978 | 152,563 |
| Best primal obj. | *inf* | 203,200 | 82,970 | *inf* | 128,840 | 101,522 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 298,499 | 263,599 | 98,145 | 103,536 | 170,978 | 152,563 |
| Best primal obj. | 292,786 | 203,629 | 82,970 | *inf* | 129,476 | 101,522 |
| *After 200 itr or 10h* | | | | | | |
| Best dual obj. | 298,499 | 263,599 | 95,654 | 103,009 | 170,978 | 152,563 |
| Best primal obj. | 292,786 | 203,629 | 85,089 | *inf* | 129,476 | 101,522 |
| Average itr. time | 30 s | 3 min | 30 s | 5 min | 3 min | 12 min |

Table 3 (contd.): Results when parts of the orders are free and the demand is not the same in each time periods when 12 time periods are included.

| | Case 4c 4t | Case 4d 12t | Case 5c 4t | Case 5d 12t | Case 6c 4t | Case 6d 12t |
|---|---|---|---|---|---|---|
| Optimal obj. | 93,590 | 79,390 | 84,784 | 75,155 | 173,648 | 163,298 |
| Time | 15 min | 288 h | 25 min | 6.5 h | 45 min | 10 h |
| LP-value | 119,268 | 117,870 | 114,602 | 107,077 | 204,677 | 197,100 |
| Time | 1 min | 10 min | 5 min | 15 min | 4 min | 15 min |
| Heuristic (a) | 90,520 | *inf* | 80,783 | 49,131 | 171,100 | *inf* |
| Time | 2 min | - | 7 min | 20 min | 7 min | - |
| Heuristic (b) | 93,589 | 71,206 | 84,742 | 56,545 | 173,573 | 144,112 |
| Time | 6 min | 20 min | 11 min | 25 min | 44 min | 50min |
| ***Lagrange,*** $\lambda_0$ | | | | | | |
| *After 2 itr* | | | | | | |
| Best dual obj. | 126,796 | 200,487 | 114,740 | 194,301 | 206,946 | 283,768 |
| Best primal obj. | 93,589 | 55,617 | 84,783 | 49,131 | 172,961 | 154,888 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 123,790 | 200,487 | 112,798 | 194,301 | 202,873 | 283,768 |
| Best primal obj. | 93,589 | 55,617 | 84,783 | 49,131 | 172,961 | 154,888 |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 117,678 | 200,487 | 108,701 | 194,301 | 193,382 | 267,947 |
| Best primal obj. | 93,590 | 66,502 | 84,784 | 49,131 | 172,962 | 154,889 |
| ***Lagrange,*** $\lambda_{LP}$ | | | | | | |
| Best dual obj. | 114,498 | 113,969 | 93,266 | 94,200 | 184,873 | 184,548 |
| Best primal obj. | 93,590 | 31,427 | 84,784 | 62,906 | 172,767 | 150,098 |
| *After 10 itr* | | | | | | |
| Best dual obj. | 114,498 | 113,969 | 93,266 | 94,200 | 184,873 | 184,548 |
| Best primal obj. | 93,590 | 31,427 | 84,784 | 64,010 | 172,767 | 151,669 |
| *After 200 itr or 20h* | | | | | | |
| Best dual obj. | 114,493 | 113,595 | 93,266 | 93,794 | 184,873 | 184,548 |
| Best primal obj. | 93,590 | 31,427 | 84,784 | 64,418 | 172,767 | 151,669 |
| Average itr. time | 2 min | 13 min | 5 min | 9 min | 6 min | 22 min |

# 6 Concluding remarks

We have developed a large-scale model for the supply chain planning problems arising at a pulp company. By combining the rolling planning horizon and the Lagrangian decomposition we mix the advantages from both to a powerful heuristic approach. We do generate several candidate solutions by starting from different dual solution by applying the rolling horizon heuristic. In addition we do generate bounds which are better than the LP relaxation value.

The proposed combined heuristic approach is general as it can be applied to any MIP model. There is no special adjustment for the general approach we propose. In general, the Lagrange heuristic ($\lambda_0$) provides good feasible solutions and the Lagrangian heuristic ($\lambda_{LP}$) provides stronger bounds than the LP-relaxation. The objective value from the Lagrange heuristic ($\lambda_{LP}$) can be used for evaluating the quality of the feasible solutions produced by the Lagrangian heuristic, as well as the feasible solutions produced by the two variants of the simple heuristic. Both the Lagrangian heuristics method and the simple heuristics can often find good feasible solutions independently of the number of fixed orders. The demand and supply structures are more inconvenient for the proposed heuristics. When the conditions are similar in each time period, the two variants of the proposed simple heuristics often produce solutions of high quality. However, the levels of the supply and demand often vary during the planning horizon. Under such circumstances the simple heuristics have problems to find feasible solutions. The proposed Lagrangian heuristic, on the other hand, is then able to produce solutions of higher quality, even if it has problems to find feasible solutions in some instances. One way of avoiding infeasible solutions can be to add more supply into the model. That could, on the other hand, lead to worse upper bounds produced by the Lagrangian heuristics, depending on the costs of the type of supply added. Expensive and unattractive raw materials can be added without making the upper bounds worse. Another suggestion for future research

is to find a way to include information about the demand and supply structure over the planning horizon, in the first time period. Both the simple heuristics and the Lagrangian heuristic may then be able produce solutions of higher quality. The proposed Lagrangian heuristic produces many solutions that can be used to initialize rolling horizon heuristics. These heuristics do, in turn, produce many high quality solutions. In this way, the Lagrangian based method can be viewed as an intelligent way to produce seeds for heuristics and there are many other tailor-made heuristics that can be developed using the same approach. We include no storage of raw materials at pulp mills. This is a shortcoming because there is probably always some amount of raw materials stored at pulp mills. On the other hand, we have no restrictions for transport from forest districts to pulp mills in the model. This means that the exact amount of raw materials can be transported to the pulp mills for inclusion into the production process. The lack of storage can also be viewed as the safety stock that is no storage corresponds to the safety stock. In our implementation, we make use of the AMPL environment together with CPLEX, which is relatively slow in this type of implementation. The solution time could be decreased by using direct implementation in, for example, C-routines connected to the callable CPLEX system. The rate of convergence is slow in the proposed Lagrangian heuristics method. In order to improve the rate of convergence, a modified subgradient can be used, for example the one suggested by Crowder [10], where the direction in an iteration includes information from directions in all earlier iterations. Scaling of relaxed constraints could also be done in order to get similar values of the different types of Lagrange multipliers. This may lead to better rate of convergence, at least in the beginning of the iteration process. Another idea in order to get better bounds is to add aggregated constraints for the storage per time period, and relax them. Other heuristics, which consider the strategical decisions for the entire planning period, can be combined with the ones suggested in this paper. The mathematical model is detailed since it is intended for direct usage. However, it is general enough to be applicable also for similar large-scale supply chain applications.

## Appendix: The complete optimization model, [P2]

$$\max \; Z^{P2} = (\sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{train}_{isqpt} + \sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{lorry}_{isqpt} +$$

$$\sum_{j\in J}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} p_{qp} y^{Q}_{jsqpt}) - (\sum_{m\in M}\sum_{d\in D}\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T}(c^{TR}_{mdi} + c^{P}_{md}) w_{mdiprt} +$$

$$\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} c^{rec}_{ir} u_{iprt} + (\sum_{a\in A}\sum_{t\in T} c^{alt}_{a} z_{at})/T + \sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{train}_{is} y^{train}_{isqpt} +$$

$$\sum_{i\in I}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{lorry}_{is} y^{lorry}_{isqpt} + \sum_{k\in R_A}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P}\sum_{t\in T}(c^{A}_{k} + c^{MP}_{i}) x^{A}_{kijpt} +$$

$$\sum_{k\in R_B}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P}\sum_{t\in T}(c^{B}_{k} + c^{MP}_{i}) x^{B}_{kijpt} + \sum_{j\in J_H}\sum_{i\in I}\sum_{t\in T} c^{R}_{ji} x^{R}_{jit} +$$

$$\sum_{i\in I}\sum_{j\in J_H}\sum_{p\in P}\sum_{t\in T}(c^{S}_{ij} + c^{MP}_{i}) x^{S}_{ijpt} + \sum_{j\in J}\sum_{t\in T} c^{T}_{j} y^{tot}_{jt} + (\sum_{j\in J}\sum_{t\in T} c^{FT}_{j} z^{T}_{jt})/T +$$

$$\sum_{h\in J_H}\sum_{l\in J_L}\sum_{p\in P}\sum_{t\in T} c^{T}_{hl} y^{T}_{hlpt} + \sum_{d\in D}\sum_{m\in M}\sum_{t\in T} c^{Fstor}_{d}/2L^{F}_{dmt} +$$

$$\sum_{i\in I}\sum_{p\in P}\sum_{t\in T} c^{Mstor}_{i} L^{M}_{ipt} + \sum_{j\in J_H}\sum_{p\in P}\sum_{t\in T} c^{Tstor}_{j} L^{HT}_{jpt} + \sum_{j\in J_L}\sum_{p\in P}\sum_{t\in T} c^{Tstor}_{j} L^{LT}_{jpt} +$$

$$\sum_{j\in J}\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P}\sum_{t\in T} c^{TQ}_{js} y^{Q}_{jsqpt} + \sum_{d\in D}\sum_{m\in M}\sum_{t\geq 2} c^{Fstor}_{d}/2L2^{F}_{dmt} +$$

$$\sum_{i\in I}\sum_{p\in P}\sum_{t\geq 2} c^{Mstor}_{i}/2L2^{M}_{ipt} + \sum_{j\in J_H}\sum_{p\in P}\sum_{t\geq 2} c^{Tstor}_{j}/2L2^{HT}_{jpt} +$$

$$\sum_{j\in J_L}\sum_{p\in P}\sum_{t\geq 2} c^{Tstor}_{j}/2L2^{LT}_{jpt} + \sum_{d\in D}\sum_{m\in M}\sum_{t=T} c^{Fstor}_{d}/2L^{F}_{dmt}) \qquad (II)$$

$$\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} w_{mdiprt} \leq s_{md},\ \forall d\in D,\ m\in M \quad | \quad \gamma_{md} \tag{1}$$

$$L2^F_{dmt} + s_t s_{md} = \sum_{i\in I}\sum_{p\in P}\sum_{r\in R} w_{mdiprt} + L^F_{dmt},$$
$$\forall d\in D,\ m\in M,\ t\in T \tag{2}$$

$$\sum_{d\in D}\sum_{p\in P} w_{mdiprt}/a_{mi} \geq n^{min}_{mr}\sum_{p\in P} u_{iprt},\ \forall m\in M,$$
$$i\in I,\ r\in R,\ t\in T \tag{3}$$

$$\sum_{d\in D}\sum_{p\in P} w_{mdiprt}/a_{mi} \leq n^{max}_{mr}\sum_{p\in P} u_{iprt}, \forall m\in M,$$
$$i\in I,\ r\in R\ t\in T \tag{4}$$

$$\sum_{r\in R}((\sum_{p\in P} u_{iprt})/\sum_{p\in P} r_{rp}) = g_i/\mid T\mid,\ \forall i\in I,\ \forall t\in T \tag{5}$$

$$(\sum_{p\in P} u_{iprt})/\sum_{p\in P} r_{rp} \leq g_i/\mid T\mid \sum_{a\in A} e_{ira} z_{at},\ \forall i\in I,$$
$$r\in R,\ t\in T \tag{6}$$

$$\sum_{m\in M}\sum_{d\in D}\sum_{p\in P} w_{mdiprt}/a_{mi} = \sum_{p\in P} u_{iprt},\ \forall i\in I,\ r\in R,$$
$$t\in T \tag{7}$$

$$p_{pr}\sum_{p\in P} u_{iprt} = u_{iprt},\ \forall i\in I,\ p\in P,\ r\in R,$$
$$t\in T \tag{8}$$

$$\sum_{a\in A} z_{at} = 1,\ \forall t\in T \tag{9}$$

$$\sum_{p\in P} L^M_{ipt} \leq L^M_{max},\ \forall i\in I,\ t\in T \tag{10}$$

$$\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} \geq (Ns^H/\mid T\mid)u^A_{kt},\ \forall k\in R_A,$$
$$\forall t\in T \tag{11}$$

$$\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} \leq \widetilde{M}u^A_{kt},\ \forall k\in R_A,\ \forall t\in T \tag{12}$$

$$\sum_{k\in R_A}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} t^A_k x^A_{kijpt} +$$
$$\sum_{k\in R_B}\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} t^B_k x^B_{kijpt} +$$
$$\sum_{j\in J_H}\sum_{i\in I} t^R_{ji} x^R_{jit} \leq t^{tot} s^H m/\mid T\mid,\ \forall t\in T \tag{13}$$

$$\sum_{k\in R_A}\sum_{j\in J_k}\sum_{p\in P} x^A_{kijpt} + \sum_{k\in R_B}\sum_{j\in J_k}\sum_{p\in P} x^B_{kijpt} = \sum_{j\in J_H} x^R_{jit},\ \forall i\in I,\ \forall t\in T \tag{14}$$

$$\sum_{k\in R_A}\sum_{i\in I_k}\sum_{p\in P} x^A_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k}\sum_{p\in P} x^B_{kijpt} = \sum_{i\in I_k} x^R_{jit},\ \forall j\in J_H,\ \forall t\in T \tag{15}$$

$$\sum_{p\in P} x^B_{kijpt} \geq n\sum_{i\in I_k}\sum_{j\in J_k}\sum_{p\in P} x^B_{kijpt},\quad \forall k\in R_B,\ i\in I_k,\ j\in J_k,\ t\in T \tag{16}$$

$$y^{tot}_{jt} \leq b_j z^T_{jt}/\mid T\mid,\ \forall j\in J,\ \forall t\in T \tag{17}$$

$$\sum_{p\in P} L^{HT}_{jpt} \leq L^{HT}_{max},\ \forall j\in J_H, t\in T \tag{18}$$

$$\sum_{p\in P} L^{LT}_{jpt} \leq L^{LT}_{max},\ \forall j\in J_L, t\in T \tag{19}$$

$$\sum_{j\in J} y^Q_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \leq z_{ct} d^{max}_{qpt},\ \forall c\in C,\ s\in S,\ q\in Q_C\cap Q_S,\ p\in P,\ t\in T \tag{20}$$

$$\sum_{j\in J} y^Q_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \geq z_{ct} d^{min}_{qpt},\ \forall c\in C,\ s\in S,\ q\in Q_C\cap Q_S,\ p\in P,\ t\in T \tag{21}$$

$$\sum_{j\in J} y^Q_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \leq d^{max}_{qpt},\ \forall s\in S,\ q\in Q_F\cap Q_S,\ p\in P,\ t\in T \tag{22}$$

$$\sum_{j\in J} y^Q_{jsqpt} + \sum_{i\in I} y^{train}_{isqpt} + \sum_{i\in I} y^{lorry}_{isqpt} \geq d^{min}_{qpt},\ \forall s\in S,\ q\in Q_F\cap Q_S,\ p\in P,\ t\in T \tag{23}$$

$$L2^M_{ipt} + \sum_{r\in R} u_{iprt} = \sum_{s\in S}\sum_{q\in Q_S} y^{train}_{isqpt} + \sum_{s\in S}\sum_{q\in Q_S} y^{lorry}_{isqpt} + \sum_{k\in R_A}\sum_{j\in J_k} x^A_{kijpt} + \sum_{k\in R_B}\sum_{j\in J_k} x^B_{kijpt} + \sum_{j\in J_H} x^S_{ijpt} + L^M_{ipt},\ \forall i\in I,\ p\in P,\ t\in T \tag{24}$$

$$\sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P} y^{train}_{isqpt} \leq h_i/\mid T\mid,\ \forall i\in I,\ t\in T \tag{25}$$

$$\sum_{p\in P} L2^{HT}_{jpt} + \sum_{k\in R_A}\sum_{i\in I_k}\sum_{p\in P} x^A_{kijpt} + \sum_{k\in R_B}\sum_{i\in I_k}\sum_{p\in P} x^B_{kijpt} + \sum_{i\in I}\sum_{p\in P} x^S_{ijpt} = y^{tot}_{jt},\ \forall j\in J_H,\ t\in T \tag{26}$$

$$
\begin{aligned}
y_{jt}^{tot} &= \sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P} y_{jsqpt}^{Q} + \\
&\quad \sum_{h\in J_L}\sum_{p\in P} y_{jhpt}^{T} + \sum_{p\in P} L_{jpt}^{HT}, \\
&\quad \forall j\in J_H,\ t\in T && (27)\\
\sum_{p\in P} L2_{jpt}^{LT} + \sum_{h\in J_H}\sum_{p\in P} y_{hjpt}^{T} &= y_{jt}^{tot},\ \forall j\in J_L,\ \forall t\in T && (28)\\
y_{jt}^{tot} &= \sum_{s\in S}\sum_{q\in Q_S}\sum_{p\in P} y_{jsqpt}^{Q} + \sum_{p\in P} L_{jpt}^{LT}, \\
&\quad \forall j\in J_L,\ t\in T && (29)
\end{aligned}
$$

$$
\begin{aligned}
L2_{jpt}^{HT} + \sum_{k\in R_A}\sum_{i\in I_k} x_{kijpt}^{A} + \\
\sum_{k\in R_B}\sum_{i\in I_k} x_{kijpt}^{B} + \sum_{i\in I} x_{ijpt}^{S} &= \sum_{s\in S}\sum_{q\in Q_S} y_{jsqpt}^{Q} + \sum_{l\in J_L} y_{jlpt}^{T} + L_{jpt}^{HT}, \\
&\quad \forall j\in J_H,\ p\in P,\ t\in T && (30)
\end{aligned}
$$

$$
\begin{aligned}
L2_{jpt}^{LT} + \sum_{h\in J_H} y_{hjpt}^{T} &= \sum_{s\in S}\sum_{q\in Q_S} y_{jsqpt}^{Q} + L_{jpt}^{LT}, \\
&\quad \forall j\in J_L,\ p\in P,\ t\in T && (31)\\
L_{ipt}^{M} &= 0,\ \ \forall i\in I,\ p\in P,\ t=\mid T\mid && (32)\\
L_{jpt}^{HT} &= 0,\ \ \forall j\in J_H,\ p\in P,\ t=\mid T\mid && (33)\\
L_{jpt}^{LT} &= 0,\ \ \forall j\in J_L,\ p\in P,\ t=\mid T\mid && (34)\\
L2_{dmt}^{F} &= L_{dmt-1}^{F},\ \forall d\in D,\ m\in M,\ t\geq 2 \quad \mid \lambda_{dmt}^{F} && (35)\\
L2_{ipt}^{M} &= L_{ipt-1}^{M},\ \forall i\in I,\ p\in P,\ t\geq 2 \quad \mid \lambda_{ipt}^{M} && (36)\\
L2_{jpt}^{HT} &= L_{jpt-1}^{HT},\ \forall j\in J_H,\ p\in P,\ t\geq 2 \quad \mid \lambda_{jpt}^{HT} && (37)\\
L2_{jpt}^{LT} &= L_{jpt-1}^{LT},\ \forall j\in J_L,\ p\in P,\ t\geq 2 \quad \mid \lambda_{jpt}^{LT} && (38)\\
z_{c1} &= z_{ct},\ \forall c\in C,\ t\geq 2 \quad \mid \delta_{ct}^{C} && (39)\\
z_{a1} &= z_{at},\ \forall a\in A,\ t\geq 2 \quad \mid \delta_{at}^{A} && (40)\\
z_{j1}^{T} &= z_{jt}^{T},\ \forall j\in J,\ t\geq 2 \quad \mid \delta_{jt}^{T} && (41)\\
L2_{ipt}^{M} &= L_{max}^{M},\ \forall i\in I,\ p\in P,\ t\geq 2 && (42)\\
L2_{jpt}^{HT} &= L_{max}^{HT},\ \forall j\in J_H,\ p\in P,\ t\geq 2 && (43)
\end{aligned}
$$

$$
\begin{aligned}
2L_{jpt}^{LT} &= L_{max}^{LT},\ \forall j\in J_L,\ p\in P,\ t\geq 2 && (44)\\
L_{dmt}^{F} &\leq s_{md},\ \forall d\in D, m\in M, t\in T && (45)\\
L2_{dmt}^{F} &\leq s_{md},\ \forall d\in D, m\in M, t\in T && (46)\\
\sum_{i\in I}\sum_{p\in P}\sum_{r\in R} w_{mdiprt} &\leq s_{md},\ \forall d\in D, m\in M, t\in T && (47)
\end{aligned}
$$

$$
\begin{aligned}
L^F_{dmt} &\leq s_t \sum_{m \in M} s_{md}, \ \forall d \in D, t \in T, t \geq 2 && (48)\\
L2^F_{dmt} &\leq s_{t-1} \sum_{m \in M} s_{md}, \ \forall d \in D, \\
& \quad t \in T, t \leq T-1 && (49)\\
L^M_{ipt} &\leq k \sum_{r \in R} u_{iprt}, \ \forall i \in I, \ p \in P, \ t \in T && (50)\\
L2^M_{ipt} &\leq k( \sum_{k \in R_A} \sum_{j \in J_k} x^A_{kijpt} + \sum_{k \in R_B} \sum_{j \in J_k} x^B_{kijpt} + \\
& \quad \sum_{j \in J_H} x^S_{ijpt} + \sum_{s \in S} \sum_{q \in Q_S} y^{train}_{isqpt} + \\
& \quad \sum_{s \in S} \sum_{q \in Q_S} y^{lorry}_{isqpt}), \ \forall i \in I, \ p \in P, \ t \in T && (51)\\
L^{HT}_{jpt} &\leq k( \sum_{k \in R_A} \sum_{i \in I_k} x^A_{kijpt} + \sum_{k \in R_B} \sum_{i \in I_k} x^B_{kijpt} + \\
& \quad \sum_{i \in I} x^S_{ijpt}), \ \forall j \in J_H, \ p \in P, \ t \in T && (52)\\
L2^{HT}_{jpt} &\leq k( \sum_{h \in J_L} y^T_{jhpt} + \sum_{s \in S} \sum_{q \in Q_S} y^Q_{jsqpt}), \\
& \quad \forall j \in J_H, \ p \in P, \ t \in T && (53)\\
L^{LT}_{jpt} &\leq k \sum_{h \in J_H} y^T_{hjpt}, \ \forall j \in J_L, \ p \in P, \ t \in T && (54)\\
L2^{LT}_{jpt} &\leq k \sum_{s \in S} \sum_{q \in Q_S} y^Q_{jsqpt}, \ \forall j \in J, \ p \in P, \ t \in T && (55)
\end{aligned}
$$

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