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A Farrow-Structure-Based Multi-Mode Transmultiplexer

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Abstract—This paper introduces a multi-mode transmultiplexer (TMUX) consisting of Farrow-based variable integer sampling rate conversion (SRC) blocks. The polyphase components of general interpolation/decimation filters are realized by the Farrow structure making it possible to achieve different linear-phase finite-length impulse response (FIR) lowpass filters at the cost of a fixed set of subfilters and adjustable fractional delay values. Simultaneous design of the subfilters, to achieve overall approximately Nyquist (M th-band) filters, are treated in this paper. By means of an example, it is shown that the subfilters can be designed so that for any desired range of integer SRC ratios, the TMUX can approximate perfect recovery as close as desired.

Index Terms—Multi-mode communications, transmultiplexers, sampling rate conversion.

I. INTRODUCTION

One of the main aims in communications engineering is to constructing flexible radio systems (e.g., software defined radios) so that services among different telecommunications standards can be handled [1]. As the number of communications standards (or modes) increases, the requirements on flexibility and cost-efficiency of these systems increase as well. Consequently, it is vital to develop new low-cost multi-mode terminals. Transmultiplexers (TMUXs) allow different users to share a common channel and hence, constitute one of the main building blocks in communications systems [2]. The importance of TMUXs gets pronounced by considering the fact that well known multiple access schemes such as code division multiple access (CDMA), time division multiple access (TDMA), and frequency division multiple access (FDMA) are special cases of a general TMUX theory [3].

Multi-mode communications require multi-mode TMUXs that support different bandwidths for various telecommunications standards. As an example, the bit rate of the wireless standards IS-54/136, GSM, and IS-95 are 48.6, 271, and 1228.8 Kbps, respectively [4]. Furthermore, in each of these standards, respectively, 3, 8, and 798 users share one channel where the channel spacing is 30, 200, and 1250 KHz. In conclusion, to support multi-mode communications, there is a need for a system which can allow various users with different bit rates to share a common channel.

TMUXs are composed of a synthesis filter bank (SFB) followed by an analysis FB (AFB) with both the AFB and SFB being a parallel connection of a number of branches [2]. Each branch is realized by digital bandpass interpolators/decimators where in the case of a uniform TMUX, the bandwidths and center frequencies of the bandpass interpolators/decimators are fixed. However, multi-mode TMUXs require interpolators/decimators with variable bandwidths and center frequencies.

A. Contribution of the Paper

In this paper, we introduce a multi-mode TMUX which consists of Farrow-based variable integer sampling rate conversion (SRC) and variable frequency shifters. To be more specific, each integer SRC block is designed using the Farrow structure [5] resulting in a fixed set of subfilters. To perform any integer SRC, there is only

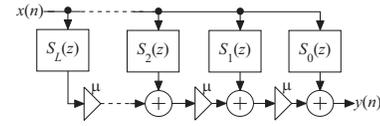


Fig. 1. Farrow structure with fixed subfilters and fractional delay μ .

a need to modify the fractional delay values required by the Farrow structure and, consequently, it is possible to use one set of subfilters to perform any integer SRC. The Farrow structure is generally designed to approximate an allpass transfer function in the frequency range of interest [6]. In this paper, the Farrow structure realizes the polyphase components of general interpolation/decimation filters. In addition, it is designed such that the cascade of interpolation and decimation filters in the SFB and AFB, approximates a Nyquist (M th-band) filter. This method of designing the Farrow structure has not been treated before. Using the design method in this paper, Nyquist filters with arbitrarily small approximation errors and different passband edges can be achieved. Therefore, the TMUX can approximate perfect recovery (PR) [7] as close as desired via proper design of the subfilters. Previous design techniques [8], [9] cannot achieve this as they have no constraints on a band near π which is considered as the don't-care band for the Farrow structure.

In comparison with the TMUX in this paper, [8], [9] propose a multi-mode TMUX where a cascade of the Farrow structure and a lowpass filter is used in the SFB and AFB. The advantage of the TMUX in this paper over that of [8] is the elimination of the lowpass filter, which also results in a different way to design the subfilters of the Farrow structure, with constraints in the whole frequency range $[-\pi, \pi]$. However, by utilizing the Farrow structure, both these approaches eliminate the need to design different sets of subfilters which would be required for general non-uniform TMUX structures, e.g., [10].

B. Paper Outline

Section II discusses the Farrow structure and how it can be used to obtain SRC blocks. In Section III, the structure of the multi-mode TMUX is introduced and the design and implementation of its building blocks are considered. Then, the simultaneous design of the subfilters is discussed and illustrated by an example. Section IV deals with the functionality and performance of the TMUX which is followed by concluding remarks in Section V.

II. FARROW STRUCTURE FOR SRC

As shown in Fig. 1, the Farrow structure is composed of fixed linear-phase finite-length impulse response (FIR) subfilters $S_k(z)$, $k = 0, 1, \dots, L$ with either a symmetric (for k even) or anti-symmetric (for k odd) impulse response. Furthermore, the subfilters can have even or odd orders and in the case of odd order, all the subfilters are general filters whereas for the even-order case, $S_0(z)$

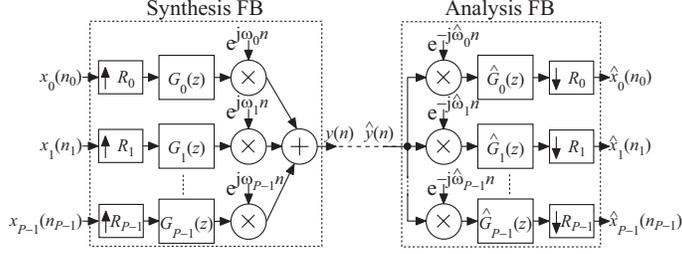


Fig. 2. Multi-mode TMUX composed of variable integer SRC and adjustable frequency shifters.

reduces to a pure delay. The transfer function of the Farrow structure is given by [6]

$$H(z) = \sum_{k=0}^L S_k(z) \mu^k, |\mu| \leq 0.5 \quad (1)$$

where μ is the fractional delay value. Assuming T_{in} (T_{out}) to be the input (output) sampling period and considering even/odd order subfilters, μ is defined as [11]

$$\begin{aligned} \text{Even order} &: [n_{in} + \mu(n_{in})]T_{in} = n_{out}T_{out}, \\ \text{Odd order} &: [n_{in} + 0.5 + \mu(n_{in})]T_{in} = n_{out}T_{out} \end{aligned} \quad (2)$$

where n_{in} (n_{out}) is the input (output) sample index. If μ is constant for all input samples, the Farrow structure generates a delayed (with a delay of μ) version of the input signal. However, if μ changes for every input sample, as in (2), the Farrow structure can perform SRC. The subfilters in Fig. 1 can be designed such that the Farrow structure 1) approximates an allpass transfer function having a fractional delay [6], or 2) can realize the polyphase components of general interpolation/decimation filters (with the Nyquist filter [12] being a special case). In the latter case, lowpass filters with different passband edges can be obtained through a fixed set of subfilters and variable multipliers [13].

III. PROPOSED MULTI-MODE TMUX

The discussion in Section II reveals that it is possible to use the Farrow structure to obtain general lowpass filters at the cost of a fixed set of subfilters and variable multipliers. In other words, the Farrow structure can realize the polyphase components of a general lowpass filter which means that a fixed set of subfilters can be used to implement interpolators/decimators with different integer SRC ratios. Hence, these general filters can be used to construct a multi-mode TMUX as shown in Fig. 2. The TMUX consists of upsampling/downsampling by R_p ; lowpass interpolation/decimation filters, i.e., $G_p(z)$ for interpolation and $\hat{G}_p(z)$ for decimation; and adjustable frequency shifters, i.e., frequency shifts by ω_p and $\hat{\omega}_p$. Assuming the sampling period at branch p of the TMUX to be T_p , we have

$$\frac{T_0}{R_0} = \frac{T_1}{R_1} = \dots = T_y \quad (3)$$

where T_y is the sampling period of $y(n)$.

In the SFB, the TMUX generates the required bandwidths through upsampling by R_p followed by a lowpass filter $G_p(z)$. To place the users in appropriate positions in the frequency spectrum, variable frequency shifters are utilized. Finally, all users are summed to form $y(n)$ for transmission¹. In the AFB, to recover a specific user signal, the received signal $\hat{y}(n)$ is first frequency shifted such that the

¹Like e.g., OFDM-based TMUXs, the output of the TMUX is complex.

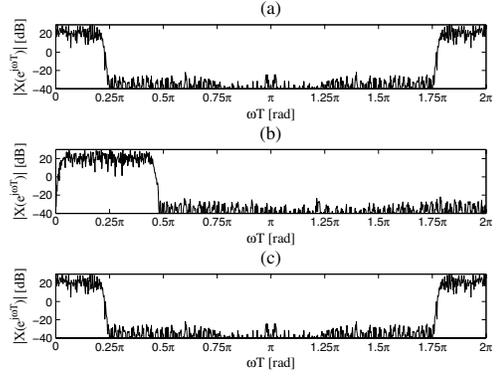


Fig. 3. TMUX outputs. (a) Integer interpolator. (b) and (c) Frequency shifters. $R_p = 5$, $\omega_p = 0.24\pi$, $\hat{\omega}_p = 0.24\pi$.

desired user signal can be processed in the baseband. Then, a lowpass filter $\hat{G}_p(z)$ followed by downsampling by R_p is used to obtain the desired signal. Figure 3 illustrates the principle of the structure by plotting the frequency spectrum at the output of the interpolator and the frequency shifters with a Gaussian input. It is noted that the procedure to compute ω_p ensures that the user signals do not overlap and hence, the TMUX is slightly redundant². However, redundancy is needed anyhow to achieve 1) a multi-mode TMUX with a fixed set of subfilters and without the need to redesign them for each new configuration of standards, and 2) high quality transmission in communications systems [2].

A. Implementation of $G_p(z)$ and $\hat{G}_p(z)$

General linear-phase FIR interpolation and decimation filters can be realized using the Farrow structure [13]. To do so, each polyphase branch is realized by a Farrow structure having a distinct fractional delay value and, thus, integer SRC blocks can be implemented using a fixed set of subfilters and variable multipliers. In other words, if the SRC ratio is to be changed, there is only a need to change the set of multipliers as they correspond to the set of fractional delays required for SRC. Assuming that $G_p(z)$ is used for SRC by R_p , its polyphase representation can be written as [2]

$$G_p(z) = \sum_{m=0}^{R_p-1} z^{-m} G_{p,m}(z^{R_p}), \quad (4)$$

²Specifically, the values of ω_p are chosen such that the cutoff frequencies of the filters, in different branches, do not overlap. Details can be found in [8] with a difference that the present paper does not assume any guard bands.

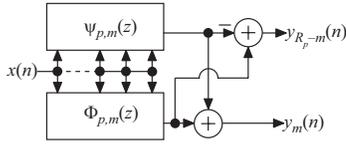


Fig. 4. Realization of polyphase components $G_{p,m}(z)$ and $G_{p,R_p-m}(z)$ utilizing the symmetry of $\mu_{p,m}$.

where $G_{p,m}(z)$ denote the polyphase components of $G_p(z)$. In order to make $G_p(z)$ a general interpolation/decimation filter of order N , it should approximate $z^{-N/2}$ in the passband and zero in the stopband. Consequently, in the passband, each term $z^{-m}G_{p,m}(z^{R_p})$ should have a delay of $z^{-N/2}$ which means that $G_{p,m}(z)$ should approximate an allpass transfer function with a fractional delay of $(\frac{N}{2} - m)/R_p$ [13].

To conclude, a general interpolation/decimation filter of order N can be designed by choosing its zeroth polyphase component, i.e., $G_{p,0}(z)$ to be a Type I linear-phase FIR filter of order $N_0 = \frac{N}{R_p}$ and utilizing the Farrow structure to realize the polyphase components $G_{p,m}(z), m = 1, 2, \dots, R_p - 1$ so that they have an odd order³ of $N_1 = \frac{N}{R_p} - 1$ as

$$G_{p,m}(z) = \sum_{k=0}^L S_k(z) \mu_{p,m}^k, \quad \mu_{p,m} = \frac{-m}{R_p} + \frac{1}{2}. \quad (5)$$

By choosing the values of $\mu_{p,m}$ as in (5), they possess antisymmetry according to $\mu_{p,m} = -\mu_{p,R_p-m}$. Considering the antisymmetry of $\mu_{p,m}$, and as shown in Fig. 4, the polyphase components $G_{p,m}(z)$ and $G_{p,R_p-m}(z)$ can be written as

$$\begin{aligned} G_{p,m}(z) &= \Phi_{p,m}(z) + \Psi_{p,m}(z), \\ G_{p,R_p-m}(z) &= \Phi_{p,m}(z) - \Psi_{p,m}(z), \end{aligned} \quad (6)$$

where $\Phi_{p,m}(z)$ and $\Psi_{p,m}(z)$ are shown in Fig. 5 and defined as

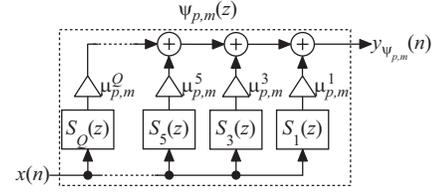
$$\begin{aligned} \Phi_{p,m}(z) &= \sum_{k=0}^{\lfloor \frac{L}{2} \rfloor} G_{p,2k}(z) \mu_{p,m}^{2k}, \\ \Psi_{p,m}(z) &= \sum_{k=1}^{\lfloor \frac{L+1}{2} \rfloor} G_{p,2k-1}(z) \mu_{p,m}^{2k-1}. \end{aligned} \quad (7)$$

Hence, interpolation by R_p can be performed as shown in Fig. 6 which consists of a fixed set of subfilters, viz. the zeroth polyphase component $G_{p,0}(z)$ and the Farrow subfilters $S_k(z)$; multipliers due to the fractional delays $\mu_{p,m}$; and the output commutator [2]. The structure for the decimator can be derived by transposing the interpolator structure of Fig. 6.

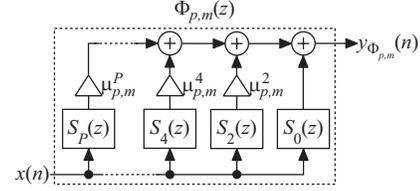
B. Design of $G_p(z)$ and $\hat{G}_p(z)$

In this section, we will discuss the design of the interpolation/decimation filters $G_p(z)$ and $\hat{G}_p(z)$ used in the TMUX of Fig. 2. Assuming one branch of the TMUX between users $x_p(n_p)$ and $\hat{x}_p(n_p)$, to approximate PR as close as desired, the filter $G_p(z)\hat{G}_p(z)$ should approximate an R_p th-band filter as close as desired. As the TMUX is redundant, the level of cross talk is determined by the stopband attenuation of the interpolation/decimation filters. In each branch of the TMUX, the filter $G_p(z)\hat{G}_p(z)$ is sandwiched between upsamplers and downsamplers by R_p . This means that the overall transfer function, for each branch, is equal to the 0th polyphase

³With proper modifications, even-order filters can also be designed [13].



(a) Realization of $\Psi_{p,m}(z)$. $Q = 2\lfloor \frac{L+1}{2} \rfloor - 1$.



(b) Realization of $\Phi_{p,m}(z)$. $P = 2\lfloor \frac{L}{2} \rfloor$.

Fig. 5. Realizations of $\Psi_{p,m}(z)$ and $\Phi_{p,m}(z)$ according to (7).

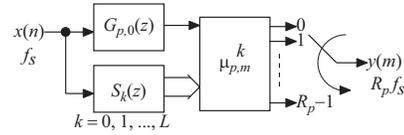


Fig. 6. Interpolator with fixed subfilters, multipliers, and commutator.

component of $G_p(z)\hat{G}_p(z)$. Consequently, the filters $G_p(z)$ and $\hat{G}_p(z)$ should be designed such that

- They have sufficiently small ripples in their stop bands to control the cross talk.
- The 0th polyphase component of $G_p(z)\hat{G}_p(z)$ approximates an allpass transfer function in the whole frequency band.

In other words, we should meet⁴

$$\begin{aligned} |[G_p(e^{j\omega T})\hat{G}_p(e^{j\omega T})e^{\frac{jN\omega T}{2}}]_{0th} - 1| &\leq \delta_1, \quad \omega T \in [0, \pi], \\ |G_p(e^{j\omega T})| &\leq \delta_2, \quad \omega T \in [\omega_s T, \pi], \\ |\hat{G}_p(e^{j\omega T})| &\leq \delta_3, \quad \omega T \in [\omega_s T, \pi] \end{aligned} \quad (8)$$

where the passband and stopband edges are given by

$$\omega_c T = \frac{\pi(1-\rho)}{R_p}, \quad \omega_s T = \frac{\pi(1+\rho)}{R_p}. \quad (9)$$

In addition, δ_2 and δ_3 are the stopband ripples (to control cross talk) with ρ being the roll-off factor of the R_p th-band filter. Furthermore, δ_1 is the deviation of the 0th polyphase component $[G_p(e^{j\omega T})\hat{G}_p(e^{j\omega T})]_{0th}$ from an allpass transfer function, and therefore it controls the distortion.

To use the fixed set of subfilters (as mentioned in the previous subsection) in the TMUX of Fig. 2, it is necessary that the subfilters are designed such that (8) is satisfied over the range of R_p values of interest. Assuming $G_p(z) = \hat{G}_p(z)$, due to the fact that there is only one fixed set of subfilters, the 0th polyphase component of the filter $G_p(z)\hat{G}_p(z)$ can be written as

$$F_p(e^{j\omega T}) = [G_p(e^{j\omega T})\hat{G}_p(e^{j\omega T})e^{\frac{jN\omega T}{2}}]_{0th} = \sum_{n=0}^{R_p-1} [G_p(e^{j(\omega T - \frac{2n\pi}{R_p})})e^{j(\omega T - \frac{2n\pi}{R_p})\frac{N}{2}}]^2. \quad (10)$$

⁴For convenience in design, the term $e^{\frac{jN\omega T}{2}}$ constructs a non-causal filter and through (10), the center tap belongs to the 0th polyphase component.

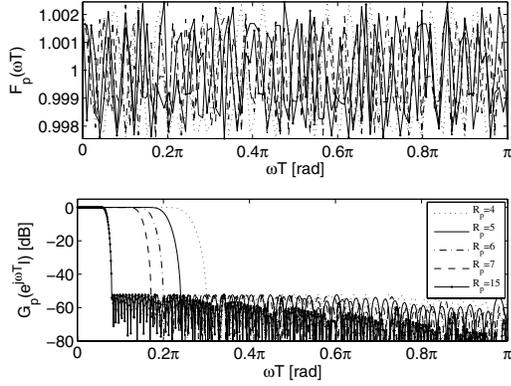


Fig. 7. Approximate R_p -th band filters and their 0th polyphase components.

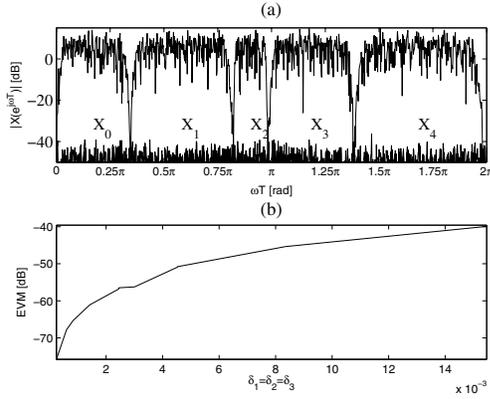


Fig. 8. Functionality and performance of the TMUX in Fig. 2.

In order to use a fixed set of subfilters to perform any desired integer SRC, a simultaneous optimization to minimize δ_i , $i = 1, 2, 3$ in (8) over R_p values of interest needs to be performed. Figure 7 shows the characteristics of the approximately R_p -th-band filters and their corresponding 0th polyphase components resulting from a simultaneous optimization for $R_p = \{7, 5, 15, 6, 4\}$ and $\delta_1 = \delta_2 = \delta_3 = 0.0024$. In this design example, the values for ρ , L , N_1 , and N_0 were 0.2, 5, 17, and 18 respectively, and furthermore, the design problem has been formulated in the minimax sense. However, other design methods such as least-squares to minimize the energy can alternatively be used, *mutatis mutandis*.

IV. TMUX FUNCTIONALITY AND PERFORMANCE

To verify the functionality of the proposed TMUX, a multi-mode setup consisting of five different user signals $\{X_0, X_1, X_2, X_3, X_4\}$ with $R_p = \{7, 5, 15, 6, 4\}$ resulting in $\omega_p = \{0.1714\pi, 0.5829\pi, 0.9029\pi, 1.1829\pi, 1.6829\pi\}$ is assumed⁵. Figure 8(a) shows the spectrum of the SFB outputs at different branches of the TMUX in Fig. 2.

To illustrate the performance of the proposed TMUX with respect to the values of δ_i , $i = 1, 2, 3$ in (8), the error vector magnitude (EVM), a metric of transmitter signal quality, is used [8]. EVM provides a statistical estimate of the error vector normalized by the

⁵For illustration purposes, the values of R_p are chosen such that 99% of the frequency range $[0, 2\pi]$ is occupied by the spectrum of the users.

magnitude of the ideal signal and is defined as

$$EVM_{rms} = \sqrt{\frac{\sum_{k=0}^{N_s-1} |s(k) - s_{ref}(k)|^2}{\sum_{k=0}^{N_s-1} |s_{ref}(k)|^2}}, \quad (11)$$

where $s(k)$ and $s_{ref}(k)$ represent the length- N_s measured and ideal complex sequences, respectively. Using the filters depicted in Fig. 7 and the values of R_p mentioned above, the mean value for EVM_{rms} and EVM_{dB} in a 16-QAM signal are 0.0015 and -56.4384 , respectively. The trend of EVM for different filter designs is shown in Fig. 8(b) and it can be seen that the error in approximating PR can be made as small as possible⁶ by reducing δ_i , $i = 1, 2, 3$.

V. CONCLUSION

In this paper, a multi-mode TMUX consisting of Farrow-based variable integer SRC and variable frequency shifters was introduced. Using the Farrow structure to realize the polyphase components of general interpolation/decimation filters, it is possible to perform any integer SRC by the use of a fixed set of subfilters. The Farrow structure is designed such that the cascade of interpolation and decimation filters approximates a Nyquist filter. By means of examples, the functionality and performance of the proposed TMUX is illustrated. It is possible to extend the idea such that both rational and integer SRC ratios can be handled through one set of subfilters resulting in a TMUX that supports arbitrary SRC ratios. This will be treated in another paper.

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⁶The least-squares formulation results in smaller EVM values than the minimax approach. However, it is the application that defines the approach to be used. Further discussion on this is out of the scope of this paper.