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Original Publication:

Bo Sernelius, Reply to Comment on "Effects of spatial dispersion on electromagnetic surface modes and on modes associated with a gap between two half spaces", 2007, Physical Review B. Condensed Matter and Materials Physics, (75), 036102.

<http://dx.doi.org/10.1103/PhysRevB.75.036102>

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Postprint available at: Linköping University Electronic Press

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-37456>

Reply to “Comment on ‘Effects of spatial dispersion on electromagnetic surface modes and on modes associated with a gap between two half spaces’ ”

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(Received 14 September 2006; published 25 January 2007)

The two different versions of the theory of electromagnetism in metallic systems, used in the professional literature and in textbooks, are reviewed. The difference in the resulting boundary conditions for the auxiliary functions in the two versions is pointed out. The topic of energy conservation is addressed, and it is demonstrated that energy is conserved in the formalism used in the original work.

DOI: [10.1103/PhysRevB.75.036102](https://doi.org/10.1103/PhysRevB.75.036102)

PACS number(s): 71.10.-w, 68.90.+g, 71.36.+c, 41.20.-q

One of the main objections to the original work¹ in the Comment² is the boundary conditions I use, so I start with this issue. There are basically two versions of the theory of electromagnetism for metallic systems in the professional literature as well as in textbooks. They differ in the bookkeeping of the induced charge and current densities of the carriers, the unbound charges. Maybe the most common treatment (here I call it the first treatment) is to lump these charges together with the external charges and call this group of charges free charges as opposed to the bound polarization charges. The second treatment, the one I used in the original work,¹ is to treat the carrier charges on the same footing as the bound charges. In these treatments the fundamental fields $\mathbf{E}$ and $\mathbf{B}$ are of course the same but the auxiliary fields $\mathbf{D}$ and $\mathbf{H}$ are different; so are the boundary conditions. To avoid confusing the reader I feel I have to go through the transformation between these different treatments. Let me start with Maxwell's equations in the first treatment. They are

$$\nabla \cdot \mathbf{D} = 4\pi\rho_f = 4\pi\rho_{ext} + 4\pi\rho_c, \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (3)$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{J}_f + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c} \mathbf{J}_{ext} + \frac{4\pi}{c} \mathbf{J}_c + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \quad (4)$$

where the sources are free charge and current densities, which can be divided into external and carrier contributions. The external charges are charges that do not belong to the material. They are absent in the present problem but I keep them here for the sake of completeness.

Next I express the equations in terms of the fundamental fields,

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi\rho_t \\ &= 4\pi\rho_{ext} + 4\pi\rho_b + 4\pi\rho_c \\ &= 4\pi\rho_{ext} - \nabla \cdot \mathbf{P}_b - \nabla \cdot \mathbf{P}_c, \end{aligned} \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (6)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (7)$$

$$\begin{aligned} \nabla \times \mathbf{B} &= 4\pi \nabla \times \mathbf{M}_b + 4\pi \nabla \times \mathbf{M}_c \\ &\quad + \frac{4\pi}{c} [\mathbf{J}_{ext} + \mathbf{J}_c + \mathbf{J}_b] + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \\ &= 4\pi \nabla \times \mathbf{M}_b + 4\pi \nabla \times \mathbf{M}_c + \frac{4\pi}{c} \mathbf{J}_{ext} + \frac{4\pi}{c} \frac{\partial \mathbf{P}_c}{\partial t} \\ &\quad + \frac{4\pi}{c} \frac{\partial \mathbf{P}_b}{\partial t} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}. \end{aligned} \quad (8)$$

Here also bound densities appear as sources and the different contributions to the magnetization $\mathbf{M}$ and polarization $\mathbf{P}$ in the system, are recognized.

I introduce the new auxiliary fields $\tilde{\mathbf{D}}$ and $\tilde{\mathbf{H}}$,

$$\begin{aligned} \mathbf{D} &= \mathbf{E} + 4\pi\mathbf{P}_b = \varepsilon\mathbf{E}, \quad \tilde{\mathbf{D}} = \mathbf{E} + 4\pi\mathbf{P}_b + 4\pi\mathbf{P}_c = \varepsilon\mathbf{E} + 4\pi\mathbf{P}_c \\ &= \tilde{\varepsilon}\mathbf{E}, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathbf{H} &= \mathbf{B} - 4\pi\mathbf{M}_b = \mathbf{B}/\mu, \quad \tilde{\mathbf{H}} = \mathbf{B} - 4\pi\mathbf{M}_b - 4\pi\mathbf{M}_c = \mathbf{B}/\mu \\ &\quad - 4\pi\mathbf{M}_c = \mathbf{B}/\tilde{\mu}. \end{aligned} \quad (10)$$

These relations are strictly valid only in the Fourier space since the constitutive relations in case of temporal and/or spatial dispersion are in the form of convolution integrals in real space. I have here not distinguished between transverse and longitudinal material parameters. It should be noted that the screenings of transverse and longitudinal fields are in general different. In cubic systems these fields do not mix. All fields can be divided into a transverse and a longitudinal part. In all relations between transverse (longitudinal) fields the transverse (longitudinal) version of the dielectric function and magnetic permeability should be used.

I may now write down Maxwell's equations from the second treatment,

$$\nabla \cdot \tilde{\mathbf{D}} = 4\pi\rho_{ext}, \quad (11)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (12)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (13)$$

$$\nabla \times \tilde{\mathbf{H}} = \frac{4\pi}{c} \mathbf{J}_{ext} + \frac{1}{c} \frac{\partial \tilde{\mathbf{D}}}{\partial t}. \quad (14)$$

They are identical in form to the ones in the first treatment; the difference lies in that the sources are only the external charge and current densities; the auxiliary fields are also different.

Let me now derive the expression for the dielectric function $\tilde{\varepsilon}(\mathbf{q}, \omega)$. I start with the Fourier-transformed Maxwell equations,

$$i\tilde{\varepsilon}(\mathbf{q}, \omega) \mathbf{q} \cdot \mathbf{E}(\mathbf{q}, \omega) = 4\pi \rho_{ext}(\mathbf{q}, \omega), \quad (15)$$

$$i\tilde{\mu}(\mathbf{q}, \omega) \mathbf{q} \cdot \tilde{\mathbf{H}}(\mathbf{q}, \omega) = 0, \quad (16)$$

$$i\mathbf{q} \times \mathbf{E} = i\omega \frac{1}{c} \tilde{\mu}(\mathbf{q}, \omega) \tilde{\mathbf{H}}(\mathbf{q}, \omega), \quad (17)$$

$$i\mathbf{q} \times \tilde{\mathbf{H}} = \frac{4\pi}{c} \mathbf{J}_{ext}(\mathbf{q}, \omega) - i\omega \tilde{\varepsilon}(\mathbf{q}, \omega) \frac{1}{c} \mathbf{E}(\mathbf{q}, \omega), \quad (18)$$

and make use of the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \quad (19)$$

This equation is valid for the total densities, or for the separate parts of the densities. Then in particular

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot \mathbf{J}_c = 0. \quad (20)$$

The Fourier-transformed version is

$$-i\omega \rho_c(\mathbf{q}, \omega) = -i\mathbf{q} \cdot \mathbf{J}_c(\mathbf{q}, \omega). \quad (21)$$

I furthermore need Ohm's law,

$$\mathbf{J}_c(\mathbf{q}, \omega) = \sigma(\mathbf{q}, \omega) \mathbf{E}(\mathbf{q}, \omega). \quad (22)$$

Combining the equation of continuity and Ohm's law gives

$$\rho_c(\mathbf{q}, \omega) = \frac{1}{\omega} \mathbf{q} \cdot \mathbf{J}_c(\mathbf{q}, \omega) = \frac{\sigma(\mathbf{q}, \omega)}{\omega} \mathbf{q} \cdot \mathbf{E}(\mathbf{q}, \omega). \quad (23)$$

The charge and current densities due to the carriers may now be eliminated from Maxwell's equations,

$$i\varepsilon(\mathbf{q}, \omega) \mathbf{q} \cdot \mathbf{E}(\mathbf{q}, \omega) = 4\pi \rho_{ext}(\mathbf{q}, \omega) + \frac{4\pi \sigma(\mathbf{q}, \omega)}{\omega} \mathbf{q} \cdot \mathbf{E}(\mathbf{q}, \omega), \quad (24)$$

$$i\tilde{\mu}(\mathbf{q}, \omega) \mathbf{q} \cdot \tilde{\mathbf{H}}(\mathbf{q}, \omega) = 0, \quad (25)$$

$$i\mathbf{q} \times \mathbf{E} = i\omega \frac{1}{c} \tilde{\mu}(\mathbf{q}, \omega) \tilde{\mathbf{H}}(\mathbf{q}, \omega), \quad (26)$$

$$i\mathbf{q} \times \tilde{\mathbf{H}} = \frac{4\pi \sigma(\mathbf{q}, \omega)}{c} \mathbf{E}(\mathbf{q}, \omega) + \frac{4\pi}{c} \mathbf{J}_{ext}(\mathbf{q}, \omega) - i\omega \varepsilon(\mathbf{q}, \omega) \frac{1}{c} \mathbf{E}(\mathbf{q}, \omega), \quad (27)$$

and the dielectric function is identified as

$$\tilde{\varepsilon}(\mathbf{q}, \omega) = \varepsilon(\mathbf{q}, \omega) + i4\pi \sigma(\mathbf{q}, \omega)/\omega. \quad (28)$$

For the boundary conditions we use Maxwell's equations in the form of Eqs. (11)–(14). In the present problem there are no external charge and current densities so there are no sources in these equations. To find the boundary conditions for the normal components of the $\tilde{\mathbf{D}}$ and $\mathbf{B}$ fields one places a Gaussian pillbox at the interface and applies Gauss' theorem to this with Eqs. (11) and (12), respectively, as integrands. The result is that the normal components are continuous across the boundary. To find the boundary conditions for the in-plane components of the $\mathbf{E}$ and $\tilde{\mathbf{H}}$ fields one places a Stokesian rectangle at the interface and applies Stokes' theorem to this with Eqs. (13) and (14), respectively, as integrands. The result is that the in-plane components are continuous across the boundary; the flux of the $\mathbf{B}$ and $\tilde{\mathbf{D}}$ fields through the Stokesian rectangles vanish in the limit when the widths of the rectangles go toward zero. In the first version of the theory one finds from Eq. (1) that the normal component of the $\mathbf{D}$ field is discontinuous at the boundaries.

Another objection to the original work put forward in the Comment is that energy might not be conserved. In the first version of the theory one can show the following relation for a volume V :

$$\frac{\partial}{\partial t} \int_V \left(\frac{1}{8\pi} (\mathbf{H} \cdot \mathbf{B} + \mathbf{E} \cdot \mathbf{D}) \right) dv + \int_V \mathbf{E} \cdot \mathbf{J}_c dv + \oint_S \mathbf{S} \cdot \mathbf{n} da = 0, \quad (29)$$

where the first term is the rate of change of the energy stored in the fields (the Helmholtz free energy); the second, the rate of change of the energy of the accelerated carriers or Joule heat; the third, the energy leaving the volume per time unit. In the second version, which is the one I use, the relation is

$$\frac{\partial}{\partial t} \int_V \left(\frac{1}{8\pi} (\tilde{\mathbf{H}} \cdot \mathbf{B} + \mathbf{E} \cdot \tilde{\mathbf{D}}) \right) dv + \oint_S \tilde{\mathbf{S}} \cdot \mathbf{n} da = 0. \quad (30)$$

Here the Joule heat is part of the energy in the fields, i.e., the energy of the normal modes. Both relations show that energy is conserved. The increase (decrease) of energy in V is equal to the energy entering (leaving) the volume. Now, what the authors of the Comment suggest is probably that energy might not be conserved across an interface. Since in my treatment the in-plane components of the $\mathbf{E}$ and $\tilde{\mathbf{H}}$ fields are continuous across the interfaces in our problem the normal component of the Poynting vector,

$$\tilde{\mathbf{S}} = \frac{c}{4\pi} \mathbf{E} \times \tilde{\mathbf{H}}, \quad (31)$$

is as well. Thus the energy leaving a region through an interface is entering the region on the other side. Energy is fully conserved.

The authors of the Comment claim that I use “the usual Lifshitz formula for the free energy derived in the presence of only temporal dispersion, to compute the effects of spatial dispersion...” I should point out that I am not using the Lifshitz formalism³ and derivation. The derivation and formalism are fully relying on the electromagnetic normal modes of the system.^{4,5} There is no problem with the present formalism.

Furthermore the authors of the Comment state that I “apply the phenomenological dielectric permittivity which depends on frequency and wave vector to the calculation of the Casimir force, a phenomenon primarily determined by virtual photons.” In the calculation¹ of the Casimir force I used the random phase approximation dielectric function which is not phenomenological and is known to represent quite a good approximation of the true dielectric function of a metal. It takes into account the interaction between the charges in the form of both real and virtual Coulomb interactions as well as real and virtual photons.

To fully solve the electromagnetic problem near the interface between two materials is practically impossible. The concepts of dielectric functions and macroscopic fields rely on spatial averaging procedures that break down near a surface or an interface. This is the case also in the neglect of spatial dispersion. One has to make an approximate approach. In the original work¹ the approximation is stated in a sentence on the second page: “We use, consistently, the idealization that the interface is perfectly sharp; the dielectric function on each side is represented by the bulk function all the way up to the interface...” The effects of the abrupt change in dielectric properties at an interface and the deviation of the properties on the other side of the interface are in my work represented by mutually consistent, induced charge and current densities at the interface. This is of course an approximate approach but there are no problems with the boundary conditions; no problems with the convolution inte-

grals that relates the auxiliary fields to the true fields; no problems with energy conservation. It has several benefits: it returns a result in the absence of spatial dispersion when the spatial dispersion is turned off; the calculations are cumbersome but feasible even for complicated dielectric functions; it has the great advantage that it does not leave room for arbitrariness; one is not tempted to introduce prescriptions that lead to preferred results.

In conclusion I find that the criticism of the original work as put forward in the Comment is unwarranted. All claimed problems with boundary conditions, energy conservation, convolution integrals in the constitutive relations, and so on just are not there. However, the results in the original work¹ and in our earlier work⁶ showed that the TE modes give negligible and zero contribution, respectively, to one of the contributions to the temporal Casimir force—the contribution from zero imaginary frequencies. The dilemma is that this does not seem to be verified experimentally, as pointed out and stressed in the Comment. Nobody would have been happier than I, had the original work¹ resulted in important contributions from the TE modes. The work that so far has been presented, giving contributions from the TE modes, I find not to be satisfactory. The authors either neglect the inevitable fact that all real metals have dissipation, and use the so-called plasma model; or they use model dielectric functions including dissipation, or experimental dielectric data, and then introduce some so-called prescription, i.e., they manipulate the theoretical expressions in order to obtain the same result as if neglecting dissipation; or they use the surface impedance approach instead of the standard boundary conditions and then take the liberty of assuming different analytical expressions for the dielectric function in different frequency ranges. The last approach boils down to neglecting dissipation at low frequencies where the effect of dissipation is important for the Casimir force. This fragmentation of the dielectric function leads to a nonphysical function that does not obey the Kramers-Kronig dispersion relations. So I am not content with the present discrepancy between theory and experiment. We all have to dig deeper to find out what goes wrong.

Financial support was obtained from EU within the EC Contract No. 012142-NANOCASE.

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